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RESEARCH ARTICLE

Identification and Prioritization of DevOps Success Factors Using Bipolar Complex Fuzzy Setting With Frank Aggregation Operators and Analytical Hierarchy Process

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ABSTRACT DevOps ("development and operations") is a dominant and collaborative organizational effort to computerize simultaneously the supply of a software strategy with a theme to enhance software features. The usage of the DevOps technique is not an easy task as there is various vagueness involved with it. The main goal of this analysis is to employ the Frank t-norm and t-conorm (which is an accommodating class of norms and conorms and due to the parameters the Frank t-norm and t-conorm become more flexible in the combining the data and appropriate to address the real-life issues) in the environment of bipolar complex fuzzy theory to handle the DevOps procedures. Moreover, we define Frank's operational laws and their appropriate result employing BCF information. Then, we establish, the bipolar complex fuzzy (BCF) Frank weight averaging (BCFFWA), BCF Frank ordered weight averaging (BCFFOWA), BCF Frank hybrid weight averaging (BCFFHWA), BCF Frank weight geometric (BCFFWG), BCF Frank ordered weight geometric (BCFFOWG), BCF Frank hybrid weight geometric (BCFFHWG) operators and assess certain properties and results. Furthermore, we find the major factors that positively influence the DevOps technique in software organizations. In the presence of the prevailing information, nineteen factors were diagnosed. The diagnosed features were moreover authorized by intellectuals via hypothetical information. Finally, BCF analytical hierarchy process (BCFAHP) technique is invented to prioritize the classification success features. The final ranking mentioned that "DevOps security pipeline", "use system orchestration", and "attempt matrix organization and transparency" features are the beneficial ranked success features for the valuable utilization of DevOps techniques. Finally, we compare the diagnosed operators with various existing theories to improve the worth of the invented approaches.

INDEX TERMS Bipolar complex fuzzy sets, frank aggregation operators, artificial intelligence, analytical hierarchy process, DevOps systems.

I. INTRODUCTION

DevOps is almost a new type of software used in many companies for computing the relationship between development

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and information technology operations. The major goal of DevOps is to replace and enhance the interrelationship by encouraging beneficial and dominant communication and collaboration among these two industries. The main role of DevOps in the companies there is a require separate decrease silos, where industries units deal as combined terms within

the companies where administration, procedure, and data are modified. On the software investigation side and for those working in information technology operations there requires to be beneficial communication and collaboration to beneficial thing the information technology industry requires of the company. Similarly, DevOps also plays an important and feasible role in the development of culture in our society. One main thing can break down because of the DevOps-based culture, in experts' investigators with operations staff to guarantee the company achieves beneficial running of software with a minimal dilemma. This culture encourages a commitment to work together and share. DevOps is not dependent on the stringent techniques and procedures: it is dependent on the professional rules and regulations that help company units work collectively inside the company and break down the old silos. DevOps is the massive dominant and well-developed idea which is extensively employed development techniques with a theme to reduce the expense of software progress by taking simultaneous delivery, as diagnosed by Sack [1] and Ce'spedas et al. [2]. In DevOps, the management of investigation and operation teams work collectively to enhance their supply procedures. The DevOps main theme is to share the main target and accountabilities within a team from investigation to implementation and assistance. Farid et al. [3] diagnosed the modified learning form software with the help of DevOps practices. Jabbari et al. [4] invented the beneficial dependency network for DevOps using systematic existing theories.

Decision-making skills can be distinct for illustrating the beneficial decision that enhances the expert industry. The strategy is select the decision is a very complicated task, which needed a leadership quality in very decision-maker who think objectively and critically. The capacity of the decision-maker for selecting the best decision can help to diagnose a strong and dominant concept. The Decision-making technique represents your feasibility in selecting among two or more decisions in the form of the opinion of the decision-maker, called alternatives. Decisionmaking tools are widely utilized in economics, computer science, network systems, and software engineering. Based on the capacity of the decision-making procedure for choosing the beneficial optimal is very fast and many scholars have used it to resolve has problems under the presence of classical information. But one of the most important questions asked by different scholars is what happen if we changed the range of the classical set into a unit interval. For this, the theory of fuzzy set (FS) [5] was invented with a new structure and improved from classical sets, which contained a mapping defined from universal set to unit interval. FS very effective idea for evaluating awkward and unreliable information which cannot be handled from classical information. With the help of FS, experts very easily determined our main target, which was impossible under the consideration of classical information. Certain applications of the FS have been done by distinct intellectuals in the form of a generalized form of FS [6], [7], software engineering [8], and decision-making (DM) such as Abdullah *et al.* [9] presented DM in the setting of FS, Lin *et al.* [10] presented risk evaluation of excavation based on fuzzy DM, Verma and Maheshwari [11] diagnosed a new measure divergence and DM in FS theory, Peng and Huang [12] presented CoCoSo technique in the FS theory, Verma and Sharma [13] defined measure of inaccuracy among two FS and Wang and Li [14] explored fuzzy DM technique based on cross-entropy.

Certain features with the perspective of bipolarity in the genuine Chinese food system control with sorts of food continue to have a stable body. The main theme of this paragraph is to recall the well-known and dominant idea of bipolar FS (BFS) which was diagnosed by Zhang [15]. The main mathematical shape of BFS is dependent on two basic terms called positive and negative supporting grades whose values lie among [0, 1] and [-1, 0]. In a BFS, the supporting grade of an element considered that the term is unrelated to the correspondence property, the supporting grade of a term mentioned that the term somewhat holds the property, and the supporting grade of a term mentioned that the somewhat holds the implicit counter-property. BFS very effective idea for evaluating awkward and unreliable information which cannot be handled from fuzzy information. With the help of BFS, experts very easily determined their main target, which was impossible under the consideration of classical information. Certain applications of the BFS have been done by distinct intellectuals, for instance, hybrid aggregation operators [16], [17], utilization of soft sets [18], [19], simple aggregation operators [20]-[22], uniforms [23], 2-tuple linguistic sets [24], ordered weighted averaging [25], decisionmaking [26]-[29], Bipolar fuzzy (BF) graph [30], [31], multi-criteria DM (MCDM) in the setting of BF theory [32], BF TOPSIS and ELECTRE-I methods [33], and BF hypergraphs [34].

Noticed from decision-making scenario and described information available in the above paragraph, mentioned that the existing theories are limited features to handle awkward and unworthy information, but continuously unsuccessful to handle its fluctuations at a provided phase of time. Yet, in some situations, if we obtained data, called from medical research, the information related to biometrics will be changed with time. Thus, handling such sort of information is very complicated for a decision-maker, for this, every expert has needed to change the range of FS, therefore, the main theory of complex FS was diagnosed by Ramot et al. [35]. Certain applications of CFS have been done in shape, called distance measures [36], complex fuzzy hypersoft sets [37], neighborhood operators for CFS [38], algebraic structure [39], and entropy measures [40]-[42]. By utilizing the two-dimension data, the comprehensive data can be expected in one set, and hence, loss of data can be prevented. Certain features with the perspective of bipolarity in the genuine chinses food system control with sorts of food continues to have a stable body. The main theme of this paragraph is to recall the well-known and dominant idea of bipolar CFS (BCFS) which was diagnosed by Mahmood and

Ur Rehman [43]. The main mathematical shape of BCFS is dependent on two basic terms called positive and negative supporting grades whose values lie among $[0, 1] \times [0, 1]$ and $[-1, 0] \times [-1, 0]$. In a BCFS, the supporting grade of an element considered that the term is unrelated to the correspondence property, the supporting grade of a term mentioned that the term somewhat holds the property, and the supporting grade of a term mentioned that the somewhat holds the implicit counter-property. BCFS very effective idea for evaluating awkward and unreliable information which cannot be handled from fuzzy information. With the help of BCFS, experts very easily determined our main target, which was impossible under the consideration of BFS information. Certain applications of the BCFS have been done by distinct intellectuals [44], [45].

Based on the above-cited and available information, noticed that the decision-making procedure contained three major problems:

- 1. How to aggregate a bundle of information into a singleton set.
- 2. How to utilize a new method for evaluating genuine life dilemmas.
- 3. How do we diagnose our required result using our invented operators or methods.

To evaluate genuine life information, we considered a numerical example, if an organization wants to install a new internet system, then the CEO of the organization should keep these four things in mind (i) benefits of the organization by new internet system (ii) easiness for the employees in work (iii) extra burden on the organization (in term of billing) (iv) distraction of employees. In these circumstances, the existing theories can't model this kind of information because some prevailing theories are not able to discuss the negative aspect and some are unable to discuss the unreal part of the information. To evaluate this sort of data we need the conception of BCFS which is very valuable and a handful to handle such sort of data. Numerous authors have defined aggregation operators (AOs) in the setting of BCF theory such as [44], [45], [46] but these AOs based on t-norm and t-conorm are not very accommodating to contain all data to structure real-life issues. Due to this, Frank [47] introduced Frank t-norms and Frank t-conorms by modifying the Lukasiewicz and Probabilistic t-norm and t-conorm which is an accommodating class of norms and conorms. By including the parameters Frank introduced t-norm and t-conorm become more flexible in combining the data and appropriate to address the real-life issues and DM issues. After that a lot of intellectuals employed Frank t-norm and t-conorm in various theories such as Zhang [48] utilized them in the interval-valued intuitionistic FS, Qin et al. [49] employed them in triangular interval type 2 FS, Tang et al. [50] used them in the theory of dual hesitant fuzzy set. The abovementioned discussion shows the great importance of the theory of BCF set and the concept of Frank t-norm and t-conorm and until now, no one employed Frank t-norm and Frank t-conorm in the theory of BCF set, which is the basic need and necessity for the decision-makers or experts, thus in this analysis we utilize the concept of Frank t-norm and t-conorm in the setting of BCF set. Now come back to the above-raised questions, for managing the above question, the major investigation of this manuscript is explained below:

- 1. To aggregate a bundle of information into a singleton set, we define the frank operational laws and their influential results using bipolar complex fuzzy (BCF) information. We diagnose the BCFFWA, BCFFOWA, BCFFHWA, BCFFWG, BCFFOWG, and BCFFHWG operators and evaluated certain properties and results.
- 2. To utilize a new method for evaluating genuine life dilemmas, we diagnose the BCFAHP technique is invented to prioritize the classification success features.
- 3. To diagnose our required result using our invented operators or methods, we discover and prioritize the major factors that positively influence the DevOps technique in software organizations. In the presence of the prevailing information, nineteen factors were diagnosed. The diagnosed features were moreover authorized by intellectuals via hypothetical information. The final ranking mentioned that "DevOps security pipeline", "use system orchestration", and "attempt matrix organization and transparency" features are the beneficial ranked success features for the valuable utilization of DevOps techniques. Finally, we compared the diagnosed operators with various existing theories is to improve the worth of the invented approaches.

The construction of this theory is available in the shape: In section 2, firstly, we revise the concept of Frank t-norms and t-conorms and then BCFS and their important laws. In section 3, we defined the frank operational laws and their influential results using BCF information. In section 4, we diagnosed the BCFFWA, BCFFOWA, BCFFHWA, BCF-FWG, BCFFOWG, BCFFHWG operators and evaluated certain properties and results. In section 5, we diagnosed the BCFAHP technique invented to prioritize the classification success features. In section 6, we discovered and prioritized the major factors that positively influence the DevOps technique in software organizations. In the presence of the prevailing information, nineteen factors were diagnosed. The diagnosed features were moreover authorized by intellectuals via hypothetical information. The final ranking mentioned that "DevOps security pipeline", "use system orchestration", and "attempt matrix organization and transparency" features are the beneficial ranked success features for the valuable utilization of DevOps techniques. Finally, we compared the diagnosed operators with various existing theories to improve the worth of the invented approaches. The conclusion of this manuscript is available in section 7.

II. PRELIMINARIES

In this scenario, firstly, we revise the concept of Frank t-norms and t-conorms and then BCFS and their important laws.

A. FRANK T-NORM AND T-CONORM

Schweizer and Sklar [51]-[53] diagnosed the conception of triangular norms (t-norms) depending on the theory firstly described by Menger [54] and then employed in numerous areas such as in FS [5], Fuzzy logic (FL) [55], [56], and their applications, however as well in the notion of modified measures [57] and difference equations. The logical disjunction and logical conjunction to FL were modified by the t-norms and t-conorms. After that, various intellectuals described numerous t-norms and t-conorms. But they were not very accommodating to contain all data to structure real-life issues. Due to this, Frank [47] introduced Frank t-norms and Frank t-conorms by modifying the Lukasiewicz and Probabilistic t-norm and t-conorm which is an accommodating class of norms and conorms. By including the parameters Frank introduced t-norm and t-conorm to become more flexible in combining the data and appropriate to address the real-life issues. The Frank t-norm and t-conorm are presented as

Definition 1 [47]: The Frank t-norm and Frank t-conorm are diagnosed as

$$\begin{aligned} \mathfrak{T}_{\mathcal{F}}\left(\mathfrak{b}_{1},\ \mathfrak{b}_{2}\right) &= \log_{\wp}\left(1 + \frac{\left(\wp^{\mathfrak{b}_{1}} - 1\right)\left(\wp^{\mathfrak{b}_{2}} - 1\right)}{\wp - 1}\right)\\ & \wp \in (0,\ +\infty)\\ \mathfrak{S}_{\mathcal{F}}\left(\mathfrak{b}_{1},\ \mathfrak{b}_{2}\right) &= 1 - \log_{\wp}\left(1 + \frac{\left(\wp^{1 - \mathfrak{b}_{1}} - 1\right)\left(\wp^{1 - \mathfrak{b}_{2}} - 1\right)}{\wp - 1}\right)\\ & \wp \in (0,\ +\infty) \end{aligned}$$

B. BCF SET

Definition 2 [43]: The conception of BCFS over a fixed set \mathfrak{B} is described as follows

$$\acute{\mathbf{K}} = \left\{ \left(\mathfrak{b}, \ \mathbf{H}_{P-\acute{\mathbf{K}}} \left(\mathfrak{b} \right), \ \mathbf{H}_{N-\acute{\mathbf{K}}} \left(\mathfrak{b} \right) \right) \mid \mathfrak{b} \in \mathfrak{B} \right\}$$
(1)

where, $\mathbf{H}_{P-\acute{\mathbf{K}}}(\mathfrak{b}) = \mathbf{H}_{RP-\acute{\mathbf{K}}}(\mathfrak{b}) + \iota \mathbf{H}_{IP-\acute{\mathbf{K}}}(\mathfrak{b})$ and $\mathbf{H}_{N-\acute{\mathbf{K}}}(\mathfrak{b}) = \mathbf{H}_{RN-\acute{\mathbf{K}}}(\mathfrak{b}) + \iota \mathbf{H}_{IN-\acute{\mathbf{K}}}(\mathfrak{b})$, labeled the positive and negative supporting grades with $H_{RP-K}(b)$, $\mathbf{H}_{IP-\acute{K}}(\mathfrak{b}) \in [0, 1] \text{ and } \mathbf{H}_{RN-\acute{K}}(\mathfrak{b}), \mathbf{H}_{IN-\acute{K}}(\mathfrak{b}) \in [-1, 0].$ In this article, the set $\acute{K} = (H_{P-\acute{K}}(\mathfrak{b}), H_{N-\acute{K}}(\mathfrak{b})) =$ $\left(\mathrm{H}_{RP-\acute{\mathrm{K}}}\left(\mathfrak{b}\right)+\iota\mathrm{H}_{IP-\acute{\mathrm{K}}}\left(\mathfrak{b}\right), \mathrm{H}_{RN-\acute{\mathrm{K}}}\left(\mathfrak{b}\right)+\iota\mathrm{H}_{IN-\acute{\mathrm{K}}}\left(\mathfrak{b}\right)\right)$ will appear for BCF numbers (BCFNs).

Definition 3 [44]: The score value (SV) of any BCFN is declared as

$$\begin{split} S_{SF}\left(\acute{\mathbf{K}}\right) &= \frac{1}{4} \begin{pmatrix} 2 + \mathbf{H}_{RP-\acute{\mathbf{K}}}\left(\mathfrak{b}\right) + \mathbf{H}_{IP-\acute{\mathbf{K}}}\left(\mathfrak{b}\right) \\ + \mathbf{H}_{RN-\acute{\mathbf{K}}}\left(\mathfrak{b}\right) + \mathbf{H}_{IN-\acute{\mathbf{K}}}\left(\mathfrak{b}\right) \\ S_{SF} &\in [0, 1] \end{split}$$
(2)

Definition 4 [44]: The accuracy value (AV) of any BCFN is declared as

$$\mathcal{H}_{AF}\left(\acute{K}\right) = \frac{\begin{pmatrix} \mathcal{H}_{RP-\acute{K}}\left(\mathfrak{b}\right) + \mathcal{H}_{IP-\acute{K}}\left(\mathfrak{b}\right) \\ -\mathcal{H}_{RN-\acute{K}}\left(\mathfrak{b}\right) - \mathcal{H}_{IN-\acute{K}}\left(\mathfrak{b}\right) \end{pmatrix}}{4}, \qquad (3)$$

From Def (2) and (3) we have (4)

1. if
$$\S_{SF}(\check{K}_{1}) < \S_{SF}(\check{K}_{2})$$
, then $\check{K}_{1} < \check{K}_{2}$;
2. if $\S_{SF}(\check{K}_{1}) > \S_{SF}(\check{K}_{2})$, then $\check{K}_{1} > \check{K}_{2}$;
3. if $\S_{SF}(\check{K}_{1}) = \S_{SF}(\check{K}_{2})$, then
i) if $H_{AF}(\check{K}_{1}) < H_{AF}(\check{K}_{2})$, then $\check{K}_{1} < \check{K}_{2}$;
ii) if $H_{AF}(\check{K}_{1}) > H_{AF}(\check{K}_{2})$, then $\check{K}_{1} > \check{K}_{2}$;
iii) if $H_{AF}(\check{K}_{1}) = H_{AF}(\check{K}_{2})$, then $\check{K}_{1} = \check{K}_{2}$.
Definition 5 [44]: For any two BCFNs
 $\check{K}_{1} = (H_{P-\check{K}_{1}}, H_{N-\check{K}_{1}})$
 $= (H_{RP-\check{K}_{1}} + \iota H_{IP-\check{K}_{1}}, H_{RN-\check{K}_{1}} + \iota H_{IN-\check{K}_{1}})$ and $\check{K}_{2} = (H_{P-\check{K}_{2}}, H_{N-\check{K}_{2}})$
 $= (H_{RP-\check{K}_{2}} + \iota H_{IP-\check{K}_{2}}, H_{RN-\check{K}_{2}} + \iota H_{IN-\check{K}_{2}})$ and $\varrho > 0$, we have

=

=

$$= \begin{pmatrix} H_{RP-\dot{K}_{1}} + H_{RP-\dot{K}_{2}} - H_{RP-\dot{K}_{1}}H_{RP-\dot{K}_{2}} \\ +\iota \left(H_{IP-\dot{K}_{1}} + H_{RP-\dot{K}_{2}} - H_{IP-\dot{K}_{1}}H_{IP-\dot{K}_{2}} \right), \\ - \left(H_{RN-\dot{K}_{1}}H_{RN-\dot{K}_{2}} \right) + \iota \left(- \left(H_{IN-\dot{K}_{1}}H_{IN-\dot{K}_{2}} \right) \right) \end{pmatrix}$$
(4)

$$= \begin{pmatrix} H_{RP-\dot{K}_{1}}H_{RP-\dot{K}_{2}} + \iota H_{IP-\dot{K}_{1}}H_{IP-\dot{K}_{2}}, \\ H_{RN-\dot{K}_{1}} + H_{RN-\dot{K}_{2}}H_{RN-\dot{K}_{1}} + H_{RN-\dot{K}_{2}} \\ + \iota \left(H_{IN-\dot{K}_{1}} + H_{IN-\dot{K}_{2}}H_{IN-\dot{K}_{1}} + H_{IN-\dot{K}_{2}} \right) \end{pmatrix}$$
(5)

$$= \begin{pmatrix} 1 - \left(1 - \mathbf{H}_{RP - \mathbf{K}_{1}}\right)^{\varrho} \\ + \iota \left(1 - \left(1 - \mathbf{H}_{IP - \mathbf{K}_{1}}\right)^{\varrho}\right), \\ - \left|\mathbf{H}_{RN - \mathbf{K}_{1}}\right|^{\varrho} + \iota \left(-\left|\mathbf{H}_{IN - \mathbf{K}_{1}}\right|^{\varrho}\right) \end{pmatrix}$$

$$\mathbf{\hat{K}}_{1}^{\varrho} \qquad (6)$$

$$= \left(\left(\left(\mathbf{H}_{RP-\mathbf{\acute{K}}_{1}} \right)^{\varrho} + \iota \left(\mathbf{H}_{IP-\mathbf{\acute{K}}_{1}} \right)^{\varrho}, -1 + \left(1 + \mathbf{H}_{RN-\mathbf{\acute{K}}_{1}} \right)^{\varrho} + \iota \left(-1 + \left(1 + \mathbf{H}_{IN-\mathbf{\acute{K}}_{1}} \right)^{\varrho} \right) \right) \right).$$

$$(7)$$

III. FRANK OPERATIONAL LAWS FOR BCFS

This analysis aims to define the frank operational laws and their influential results using BCF information.

Definition 6: By taking two BCFNs

$$\begin{split} \dot{\mathbf{K}}_{1} &= \left(\mathbf{H}_{P-\dot{\mathbf{K}}_{1}}, \ \mathbf{H}_{N-\dot{\mathbf{K}}_{1}}\right) = \left(\begin{array}{c} \mathbf{H}_{RP-\dot{\mathbf{K}}_{1}} + \iota \ \mathbf{H}_{IP-\dot{\mathbf{K}}_{1}}, \\ \mathbf{H}_{RN-\dot{\mathbf{K}}_{1}} + \iota \ \mathbf{H}_{IN-\dot{\mathbf{K}}_{1}} \end{array}\right) \text{ and} \\ \dot{\mathbf{K}}_{2} &= \left(\mathbf{H}_{P-\dot{\mathbf{K}}_{2}}, \ \mathbf{H}_{N-\dot{\mathbf{K}}_{2}}\right) = \left(\begin{array}{c} \mathbf{H}_{RP-\dot{\mathbf{K}}_{2}} + \iota \ \mathbf{H}_{IP-\dot{\mathbf{K}}_{2}}, \\ \mathbf{H}_{RN-\dot{\mathbf{K}}_{2}} + \iota \ \mathbf{H}_{IN-\dot{\mathbf{K}}_{2}} \end{array}\right), \& > 1, \\ \text{and} \ a > 0 \text{ as any real number, we have the following} \end{split}$$

and $\rho > 0$ as any real number, we have the following operations for BCFNs rely on the Frank t-norm and Frank t-conorm. Equations (8)-11, as shown at the bottom of the next page.

$$\begin{split} \tilde{\mathbf{K}}_{1} \oplus \tilde{\mathbf{K}}_{2} &= \begin{pmatrix} 1 - \log_{\wp} \left(1 + \frac{\left(\wp^{1-H_{\wp-\tilde{\mathbf{k}}_{1}}} - 1\right) \left(\wp^{1-H_{\wp-\tilde{\mathbf{k}}_{2}}} - 1\right)}{\wp - 1} \right) \\ + \iota \left(1 - \log_{\wp} \left(1 + \frac{\left(\wp^{1-H_{\wp-\tilde{\mathbf{k}}_{1}}} - 1\right) \left(\wp^{1-H_{\wp-\tilde{\mathbf{k}}_{2}}} - 1\right)}{\wp - 1} \right) \right) \\ - \left(\log_{\wp} \left(1 + \frac{\left(\wp^{-H_{\wp-\tilde{\mathbf{k}}_{1}}} - 1\right) \left(\wp^{-H_{\wp-\tilde{\mathbf{k}}_{2}}} - 1\right)}{\wp - 1} \right) \right) \\ + \iota \left(- \left(\log_{\wp} \left(1 + \frac{\left(\wp^{H_{\wp-\tilde{\mathbf{k}}_{1}}} - 1\right) \left(\wp^{H_{\wp-\tilde{\mathbf{k}}_{2}}} - 1\right)}{\wp - 1} \right) \right) \right) \\ + \iota \left(\log_{\wp} \left(1 + \frac{\left(\wp^{H_{\wp-\tilde{\mathbf{k}}_{1}}} - 1\right) \left(\wp^{H_{\wp-\tilde{\mathbf{k}}_{2}} - 1\right)}{\wp - 1} \right) \\ + \iota \left(\log_{\wp} \left(1 + \frac{\left(\wp^{H_{\wp-\tilde{\mathbf{k}}_{1}}} - 1\right) \left(\wp^{H_{\wp-\tilde{\mathbf{k}}_{2}}} - 1\right)}{\wp - 1} \right) \right) \\ - 1 + \log_{\wp} \left(1 + \frac{\left(\wp^{1-H_{\wp-\tilde{\mathbf{k}}_{1}}} - 1\right) \left(\wp^{1+H_{\varkappa-\tilde{\mathbf{k}}_{2}} - 1\right)}{\wp - 1} \right) \\ + \iota \left(- 1 + \log_{\wp} \left(1 + \frac{\left(\wp^{1-H_{\wp-\tilde{\mathbf{k}}_{1}}} - 1\right)^{\varrho} \right)}{\wp - 1} \right) \\ + \iota \left(- 1 + \log_{\wp} \left(1 + \frac{\left(\wp^{1-H_{\wp-\tilde{\mathbf{k}}_{1}}} - 1\right)^{\varrho}}{(\wp - 1)^{\varrho - 1}} \right) \\ + \iota \left(1 - \log_{\wp} \left(1 + \frac{\left(\wp^{1-H_{\wp-\tilde{\mathbf{k}}_{1}}} - 1\right)^{\varrho}}{(\wp - 1)^{\varrho - 1}} \right) \right) \\ + \iota \left(- \left(\log_{\wp} \left(1 + \frac{\left(\wp^{-H_{\wp-\tilde{\mathbf{k}}_{1}} - 1\right)^{\varrho}}{(\wp - 1)^{\varrho - 1}} \right) \\ + \iota \left(\log_{\wp} \left(1 + \frac{\left(\wp^{-H_{\wp-\tilde{\mathbf{k}_{1}}} - 1\right)^{\varrho}}{(\wp - 1)^{\varrho - 1}} \right) \\ + \iota \left(\log_{\wp} \left(1 + \frac{\left(\wp^{H_{\wp-\tilde{\mathbf{k}_{1}}} - 1\right)^{\varrho}}{(\wp - 1)^{\varrho - 1} \right) \\ + \iota \left(- 1 \log_{\wp} \left(1 + \frac{\left(\wp^{H_{\wp-\tilde{\mathbf{k}_{1}}} - 1\right)^{\varrho}}{(\wp - 1)^{\varrho - 1}} \right) \\ + \iota \left(- 1 + \log_{\wp} \left(1 + \frac{\left(\wp^{H_{\wp-\tilde{\mathbf{k}_{1}}} - 1\right)^{\varrho}}{(\wp - 1)^{\varrho - 1} \right) \\ + \iota \left(- 1 + \log_{\wp} \left(1 + \frac{\left(\wp^{H_{\wp-\tilde{\mathbf{k}_{1}}} - 1\right)^{\varrho}}{(\wp - 1)^{\varrho - 1}} \right) \\ \end{pmatrix} \end{pmatrix} \end{pmatrix} \end{pmatrix}$$

Theorem 1: By taking two BCFNs $\dot{\mathbf{K}}_1 = \left(\mathbf{H}_{P-\dot{\mathbf{K}}_1}, \ \mathbf{H}_{N-\dot{\mathbf{K}}_1}\right)$ $= \left(\mathbf{H}_{RP-\mathbf{K}_1} + \iota \mathbf{H}_{IP-\mathbf{K}_1}, \mathbf{H}_{RN-\mathbf{K}_1} + \iota \mathbf{H}_{IN-\mathbf{K}_1} \right) \text{ and } \mathbf{K}_2 =$ $\left(\mathrm{H}_{P-\mathrm{K}_{2}}, \mathrm{H}_{N-\mathrm{K}_{2}}\right)$ $= \left(\mathbf{H}_{RP-\dot{\mathbf{K}}_{2}} + \iota \mathbf{H}_{IP-\dot{\mathbf{K}}_{2}}, \mathbf{H}_{RN-\dot{\mathbf{K}}_{2}} + \iota \mathbf{H}_{IN-\dot{\mathbf{K}}_{2}} \right), \wp > 1, \text{ and}$ $\varrho, \ \varrho_{1}, \ \varrho_{2} > 0, \text{ we have}$ 1. $\acute{K}_1 \oplus \acute{K}_2 = \acute{K}_2 \oplus \acute{K}_1$ 1. $\mathbf{K}_{1} \otimes \mathbf{K}_{2} = \mathbf{K}_{2} \otimes \mathbf{K}_{1}$ 2. $\mathbf{K}_{1} \otimes \mathbf{K}_{2} = \mathbf{K}_{2} \otimes \mathbf{K}_{1}$ 3. $\varrho \left(\mathbf{K}_{1} \oplus \mathbf{K}_{2} \right) = \varrho \mathbf{K}_{1} \oplus \varrho \mathbf{K}_{2}$ 4. $\left(\mathbf{K}_{1} \otimes \mathbf{K}_{2} \right)^{\varrho} = \mathbf{K}_{1}^{\varrho} \otimes \mathbf{K}_{2}^{\varrho}$ 5. $\rho_{1}\dot{K}_{1} \oplus \rho_{2}\dot{K}_{1} = (\rho_{1} + \rho_{2})\dot{K}_{1}$ 6. $\dot{K}_{1}^{\rho_{1}} \otimes \dot{K}_{1}^{\rho_{2}} = \dot{K}_{1}^{\rho_{1}+\rho_{2}}$ 7. $(\dot{K}_{2}^{\rho_{1}})^{\rho_{2}} = \dot{K}_{2}^{\rho_{1}\rho_{2}}$.

- 1. We firstly, employ property 1 of Def (6) and then inter-change $\wp^{1-H_{RP}-\hat{K}_1} 1$ and $\wp^{1-H_{RP}-\hat{K}_2} 1$ in positive supportive grade and $\wp^{-H_{RN}-\hat{K}_1} 1$ and $\wp^{-H_{RN}-\hat{K}_2} 1$ in the negative supportive grade to get the required result.
- 2. Likewise 1, so omitted the proof.
- 3. We firstly, employ property 1 of Def (6) to find $K_1 \oplus K_2$ and then apply property 3 of Def (6) on the left hand side to find $\rho(\hat{K}_1 \oplus \hat{K}_2)$. After that on right hand side we firstly employ the property 3 of Def (6) to find ρK_1 and ρK_2 and then apply property 1 of Def (6) to find $\rho \acute{K}_1 \oplus \rho \acute{K}_2$. The result of $\rho \acute{K}_1 \oplus \rho \acute{K}_2$ and $\rho (\acute{K}_1 \oplus \acute{K}_2)$ are equal. Thus $\rho\left(\dot{K}_1 \oplus \dot{K}_2\right) = \rho \dot{K}_1 \oplus \rho \dot{K}_2$.
- 4. On the left hand side, we firstly, employ property 2 of Def (6) to find $K_1 \oplus K_2$ and then apply property 4 of Def (6) to find $\rho(\hat{K}_1 \oplus \hat{K}_2)$. After that on right hand side we firstly employ the property 4 of Def (6) to find \dot{K}_1^{ϱ} and \dot{K}_2^{ϱ} and then apply property 2 of Def (6) to find $\dot{K}_1^{\varrho} \otimes \dot{K}_2^{\varrho}$. The result of and $(\dot{K}_1 \oplus \dot{K}_2)^{\varrho}$ are equal. Thus $\left(\acute{\mathbf{K}}_1 \oplus \acute{\mathbf{K}}_2\right)^{\varrho} = \acute{\mathbf{K}}_1^{\varrho} \otimes \acute{\mathbf{K}}_2^{\varrho}.$
- 5. On the left hand side we firstly, employ property 3 of Def (6) to find $\rho_1 K_1$ and $\rho_2 K_1$ and then apply property 1 of Def (6) to find $\varrho_1 K_1 \oplus \varrho_2 K_1$. After that on right hand side we firstly employ add ρ_1 and ρ_2 to find $(\rho_1 + \rho_2)$ and then employ property 3 of Def (6) to find $(\varrho_1 + \varrho_2) \dot{K}_1$. The result of $\varrho_1 \dot{K}_1 \oplus \varrho_2 \dot{K}_1$ and $(\varrho_1 + \varrho_2) \dot{K}_1$ are equal. Thus, $\varrho_1 \dot{K}_1 \oplus \varrho_2 K_1 =$ $(\varrho_1 + \varrho_2) K_1.$
- 6. On the left hand side, we firstly, employ property 4 of Def (6) to find $\hat{K}_1^{\varrho_1}$ and $\hat{K}_1^{\varrho_2}$ then apply property 2 of Def (6) to find $\hat{K}_1^{\varrho_1} \otimes \hat{K}_1^{\varrho_2}$. After that on right hand side we firstly add ϱ_1 and ϱ_2 to find $(\varrho_1 + \varrho_2)$ and then apply property 4 of Def (6) to find $\hat{K}_{1}^{\varrho_{1}} + \hat{\varrho_{2}}^{\varrho_{1}}$. The result of $\hat{K}_{1}^{\varrho_{1}} \otimes \hat{K}_{1}^{\varrho_{2}}$ and $\hat{K}_{1}^{\varrho_{1}+\varrho_{2}}$ are equal. Thus $\hat{K}_{1}^{\varrho_{1}} \otimes \hat{K}_{1}^{\varrho_{2}} = \hat{K}_{1}^{\varrho_{1}+\varrho_{2}}$

IV. FRANK AGGREGATION OPERATORS FOR BCF INFORMATION

This analysis aims to use the frank operational laws for diagnosing the BCFFWA, BCFFOWA, BCFFHWA, BCFFWG, BCFFOWG, and BCFFHWG operators and evaluated certain properties and results. Here, we establish BCF Frank AOs. Throughout this manuscript, $\hat{K}_j = (H_{P-\hat{K}_i}, H_{N-\hat{K}_i}) =$

 $\begin{pmatrix} H_{RP-K_j} + \iota H_{IP-K_j}, \\ H_{RN-K_j} + \iota H_{IN-K_j} \end{pmatrix} (j = 1, 2, ..., n) \text{be a family of BCFNs, and p} = (p_1, p_2 ..., p_n) \text{be the weight vector (WV)}$ such that $0 \le p_j \le 1$, and $\sum_{j=1}^{n} p_j = 1$.

Definition 7: The BCFFWA operator is analyzed as

$$BCFFWA\left(\acute{\mathbf{K}}_{1}, \acute{\mathbf{K}}_{2}, \ldots, \acute{\mathbf{K}}_{n}\right) = \bigoplus_{j=1}^{n} p_{j}\acute{\mathbf{K}}_{j} \qquad (12)$$

Therefore, we acquire a significant theorem that observes the Frank operations on BCFNs

Theorem 2: The aggregating outcome by employing BCF-FWA operators is a BCFN and

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$$BCFFWA\left(\acute{K}_{1}, \acute{K}_{2}, ..., \acute{K}_{n}\right) = \begin{pmatrix} 1 - \log_{\wp} \left(1 + \prod_{j=1}^{n} \left(\wp^{1-H_{RP-\acute{K}_{j}}} - 1\right)^{P_{j}}\right) \\ +\iota \left(1 - \log_{\wp} \left(1 + \prod_{j=1}^{n} \left(\wp^{1-H_{IP-\acute{K}_{j}}} - 1\right)^{P_{j}}\right)\right) \\ - \left(\log_{\wp} \left(1 + \prod_{j=1}^{n} \left(\wp^{-H_{RN-\acute{K}_{j}}} - 1\right)^{P_{j}}\right)\right) \\ +\iota \left(-\left(\log_{\wp} \left(1 + \prod_{j=1}^{n} \left(\wp^{-H_{IN-\acute{K}_{j}}} - 1\right)^{P_{j}}\right)\right)\right) \end{pmatrix} \right)$$
(13)

Proof: In the accompanying, firstly, we prove that for any vector $\mathbf{p} = (\mathbf{p}_1, \mathbf{p}_2 \dots, \mathbf{p}_n)$ that is, with practically no limitation on p, the accompanying equations remains true.

$$BCFFWA\left(\hat{K}_{1}, \hat{K}_{2}, \dots, \hat{K}_{n}\right)$$

$$= \begin{pmatrix} 1 - \log_{\wp} \left(1 + \frac{\prod_{j=1}^{n} \left(\wp^{1-H_{RP}-\hat{K}_{j}} - 1\right)^{P_{j}}}{(\wp^{-1})^{\sum_{j=1}^{n} P_{j}-1}}\right) \\ + \iota \left(1 - \log_{\wp} \left(1 + \frac{\prod_{j=1}^{n} \left(\wp^{1-H_{IP}-\hat{K}_{j}} - 1\right)^{P_{j}}}{(\wp^{-1})^{\sum_{j=1}^{n} P_{j}-1}}\right) \right), \\ - \left(\log_{\wp} \left(1 + \frac{\prod_{j=1}^{n} \left(\wp^{-H_{RN}-\hat{K}_{j}} - 1\right)^{P_{j}}}{(\wp^{-1})^{\sum_{j=1}^{n} P_{j}-1}}\right) \right) \\ + \iota \left(- \left(\log_{\wp} \left(1 + \frac{\prod_{j=1}^{n} \left(\wp^{-H_{IN}-\hat{K}_{j}} - 1\right)^{P_{j}}}{(\wp^{-1})^{\sum_{j=1}^{n} P_{j}-1}}\right) \right) \right) \end{pmatrix}$$

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^{7.} It is obvious.

By employing mathematical induction, for n = 2, we have

$$\begin{split} & \textit{BCFFWA}\left(\hat{k}_{1}, \, \hat{k}_{2}\right) \\ &= p_{1}\hat{k}_{1} \oplus p_{2}\hat{k}_{2} \\ &= \begin{pmatrix} 1 - \log_{\wp}\left(1 + \frac{\left(\wp^{1-H_{RP}-\hat{k}_{1}} - 1\right)^{p_{1}}\right)}{(\wp - 1)^{p_{1}-1}}\right) \\ &+ \iota\left(1 - \log_{\wp}\left(1 + \frac{\left(\wp^{-H_{RP}-\hat{k}_{1}} - 1\right)^{p_{1}}\right)}{(\wp - 1)^{p_{1}-1}}\right) \end{pmatrix} \\ &- \left(\log_{\wp}\left(1 + \frac{\left(\wp^{-H_{RP}-\hat{k}_{1}} - 1\right)^{p_{1}}\right)}{(\wp - 1)^{p_{1}-1}}\right) \end{pmatrix} \\ &+ \iota\left(-\left(\log_{\wp}\left(1 + \frac{\left(\wp^{-H_{RP}-\hat{k}_{2}} - 1\right)^{p_{2}}\right)}{(\wp - 1)^{p_{2}-1}}\right) \right) \\ &+ \iota\left(1 - \log_{\wp}\left(1 + \frac{\left(\wp^{-H_{RV}-\hat{k}_{2}} - 1\right)^{p_{2}}\right)}{(\wp - 1)^{p_{2}-1}}\right) \\ &+ \iota\left(-\left(\log_{\wp}\left(1 + \frac{\left(\wp^{-H_{RV}-\hat{k}_{2}} - 1\right)^{p_{2}}\right)}{(\wp - 1)^{p_{2}-1}}\right) \right) \end{pmatrix} \\ &= \begin{pmatrix} 1 - \log_{\wp}\left(1 + \frac{\left(\wp^{-H_{RV}-\hat{k}_{2}} - 1\right)^{p_{2}}}{(\wp - 1)^{p_{2}-1}}\right) \\ &+ \iota\left(-\left(\log_{\wp}\left(1 + \frac{\left(\wp^{-H_{RV}-\hat{k}_{2}} - 1\right)^{p_{2}}\right)}{(\wp - 1)^{p_{2}-1}}\right) \end{pmatrix} \end{pmatrix} \end{pmatrix} \end{pmatrix} \\ &= \begin{pmatrix} 1 - \log_{\wp}\left(1 + \frac{\left(\wp^{-H_{RV}-\hat{k}_{2}} - 1\right)^{p_{2}}}{(\wp - 1)^{p_{2}-1}}\right) \\ &+ \iota\left(1 - \log_{\wp}\left(1 + \frac{\left(\wp^{-H_{RV}-\hat{k}_{2}} - 1\right)^{p_{2}}}{(\wp - 1)^{p_{2}-1}}\right) \end{pmatrix} \end{pmatrix} \end{pmatrix} \end{pmatrix} \end{pmatrix} \end{pmatrix}$$

 \Rightarrow Eq (13) is true for n = 2. Next assume that Eq (13) is true for some 3 i.e.

$$BCFFWA\left(\dot{K}_{1}, \dot{K}_{2}, ..., \dot{K}_{3}\right)$$

$$= \begin{pmatrix} 1 - \log_{\wp} \left(1 + \frac{\prod_{j=1}^{3} \left(\wp^{1-H_{RP-\dot{K}_{j}}} - 1\right)^{p_{j}}}{(\wp - 1)^{\sum_{j=1}^{3} p_{j}-1}}\right) \\ + \iota \left(1 - \log_{\wp} \left(1 + \frac{\prod_{j=1}^{3} \left(\wp^{1-H_{IP-\dot{K}_{j}}} - 1\right)^{p_{j}}}{(\wp - 1)^{\sum_{j=1}^{3} p_{j}-1}}\right) \right), \\ - \left(\log_{\wp} \left(1 + \frac{\prod_{j=1}^{3} \left(\wp^{-H_{RN-\dot{K}_{j}}} - 1\right)^{p_{j}}}{(\wp - 1)^{\sum_{j=1}^{3} p_{j}-1}}\right) \right) \\ + \iota \left(- \left(\log_{\wp} \left(1 + \frac{\prod_{j=1}^{3} \left(\wp^{-H_{IN-\dot{K}_{j}}} - 1\right)^{p_{j}}}{(\wp - 1)^{\sum_{j=1}^{3} p_{j}-1}}\right) \right) \right)$$

Next, we are going to show that Eq (13) is true for n = 3 + 1.

$$\begin{split} & \textit{BCFFWA}\left(\dot{k}_{1}, \, \dot{k}_{2}, \, \dots, \, \dot{k}_{3}, \, \dot{k}_{3+1}\right) \\ &= \textit{BCFFWA}\left(\dot{k}_{1}, \, \dot{k}_{2}, \, \dots, \, \dot{k}_{3}\right) \oplus p_{3+1}\dot{k}_{3+1} \\ &= \begin{pmatrix} 1 - \log_{p}\left(1 + \frac{j_{1}^{2}\left(\wp^{1-H_{pr},\dot{k}_{j}} - 1\right)^{\mathbf{P}_{j}}}{(\wp^{-1})^{p_{1}}(\wp^{-1})}\right) \\ + \iota\left(1 - \log_{p}\left(1 + \frac{j_{1}^{2}\left(\wp^{1-H_{pr},\dot{k}_{j}} - 1\right)^{\mathbf{P}_{j}}}{(\wp^{-1})^{p_{1}}(\wp^{-1})}\right) \end{pmatrix} \\ &= \begin{pmatrix} 1 - \log_{p}\left(1 + \frac{j_{1}^{2}\left(\wp^{1-H_{pr},\dot{k}_{j}} - 1\right)^{\mathbf{P}_{j}}}{(\wp^{-1})^{p_{1}}(\wp^{-1})}\right) \end{pmatrix} \\ + \iota\left(-\left(\log_{p}\left(1 + \frac{j_{1}^{2}\left(\wp^{1-H_{pr},\dot{k}_{j}} - 1\right)^{\mathbf{P}_{j}}{(\wp^{-1})^{p_{2}}(\wp^{-1})^{p_{3+1}}}\right) \end{pmatrix} \right) \end{pmatrix} \\ &+ \iota\left(1 - \log_{p}\left(1 + \frac{\left(\wp^{1-H_{pr},\dot{k}_{3+1}} - 1\right)^{\mathbf{P}_{3+1}}}{(\wp^{-1})^{\mathbf{P}_{3+1}-1}}\right) \right) \\ &+ \iota\left(1 - \log_{p}\left(1 + \frac{\left(\wp^{-H_{pr},\dot{k}_{3+1}} - 1\right)^{\mathbf{P}_{3+1}}}{(\wp^{-1})^{\mathbf{P}_{3+1}-1}}\right) \right) \end{pmatrix} \\ &+ \iota\left(-\left(\log_{p}\left(1 + \frac{\left(\wp^{-H_{pr},\dot{k}_{3+1}} - 1\right)^{\mathbf{P}_{3+1}}}{(\wp^{-1})^{\mathbf{P}_{3+1}-1}}\right) \right) \right) \end{pmatrix} \end{split}$$



$$= \begin{pmatrix} 1 - \log_{\wp} \left(1 + \frac{\prod_{j=1}^{3+1} \left(\wp^{1-H_{B^{r}},\hat{\kappa}_{j}} - 1 \right)^{P_{j}}}{(\wp - 1)^{\sum_{j=1}^{3+1} P_{j} - 1}} \right) \\ + \iota \left(1 - \log_{\wp} \left(1 + \frac{\prod_{j=1}^{3+1} \left(\wp^{1-H_{B^{r}},\hat{\kappa}_{j}} - 1 \right)^{P_{j}}}{(\wp - 1)^{\sum_{j=1}^{3+1} P_{j} - 1}} \right) \right) \\ - \left(\log_{\wp} \left(1 + \frac{\prod_{j=1}^{3+1} \left(\wp^{-H_{B^{r}},\hat{\kappa}_{j}} - 1 \right)^{P_{j}}}{(\wp - 1)^{\sum_{j=1}^{3+1} P_{j} - 1}} \right) \right) \\ + \iota \left(- \left(\log_{\wp} \left(1 + \frac{\prod_{j=1}^{3+1} \left(\wp^{-H_{B^{r}},\hat{\kappa}_{j}} - 1 \right)^{P_{j}}}{(\wp - 1)^{\sum_{j=1}^{3+1} P_{j} - 1}} \right) \right) \right) \end{pmatrix}$$

⇒ Eq (13) is true for n = 3+1. Therefore, Eq (13) is true $\forall n$. When we take that $0 \le p_j \le 1$, and $\sum_{j=1}^{n} p_j = 1$, then Eq (13) converts to Eq (13). This completes the proof.

Theorem 3: Let

$$\dot{\mathbf{K}}_{j} = \left(\mathbf{H}_{P-\dot{\mathbf{K}}_{j}}, \mathbf{H}_{N-\dot{\mathbf{K}}_{j}}\right)$$

$$= \left(\mathbf{H}_{RP-\dot{\mathbf{K}}_{j}} + \iota \mathbf{H}_{IP-\dot{\mathbf{K}}_{j}}, \mathbf{H}_{RN-\dot{\mathbf{K}}_{j}} + \iota \mathbf{H}_{IN-\dot{\mathbf{K}}_{j}}\right) \text{ and } \dot{\mathbf{K}}_{j}^{\#}$$

$$= \left(\mathbf{H}_{P-\dot{\mathbf{K}}_{j}}^{\#}, \mathbf{H}_{N-\dot{\mathbf{K}}_{j}}^{\#}\right)$$

$$= \left(\mathbf{H}_{RP-\dot{\mathbf{K}}_{j}}^{\#} + \iota \mathbf{H}_{IP-\dot{\mathbf{K}}_{j}}^{\#}, \mathbf{H}_{RN-\dot{\mathbf{K}}_{j}}^{\#} + \iota \mathbf{H}_{IN-\dot{\mathbf{K}}_{j}}^{\#}\right)$$

$$(j = 1, 2, ..., n) \text{ be two families of BCFNs, then}$$
1. **Idempotency:** If all $\dot{\mathbf{K}}_{j} = \dot{\mathbf{K}} \forall j$ then

$$BCFFWA\left(\acute{\mathbf{K}}_{1}, \ \acute{\mathbf{K}}_{2}, \ \ldots, \ \acute{\mathbf{K}}_{n}\right) = \acute{\mathbf{K}}$$
(14)

2. Boundedness: Let $\hat{\mathbf{K}}^{-} = \left(\min_{j} \left\{ \mathbf{H}_{RP-\hat{\mathbf{K}}_{j}} \right\} + \iota \min_{j} \left\{ \mathbf{H}_{IP-\hat{\mathbf{K}}_{j}} \right\}, \max_{j} \left\{ \mathbf{H}_{RN-\hat{\mathbf{K}}_{j}} \right\} + \iota \max_{j} \left\{ \mathbf{H}_{IP-\hat{\mathbf{K}}_{j}} \right\} \right), \text{ and } \hat{\mathbf{K}}^{+} = \left(\max_{j} \left\{ \mathbf{H}_{RP-\hat{\mathbf{K}}_{j}} \right\} + \iota \max_{j} \left\{ \mathbf{H}_{IP-\hat{\mathbf{K}}_{j}} \right\}, \min_{j} \left\{ \mathbf{H}_{RN-\hat{\mathbf{K}}_{j}} \right\} + \iota \min_{j} \left\{ \mathbf{H}_{IP-\hat{\mathbf{K}}_{j}} \right\} \right). \text{ Then}$

$$\dot{\mathbf{K}}^{-} \leq BCFFWA\left(\dot{\mathbf{K}}_{1}, \ \dot{\mathbf{K}}_{2}, \ \dots, \ \dot{\mathbf{K}}_{n}\right) \leq \dot{\mathbf{K}}^{+} \quad (15)$$

3. Monotonicity: If $H_{RP-\hat{K}_{j}} \leq H_{RP-\hat{K}'_{j}}$, $H_{IP-\hat{K}_{j}} \leq H_{IP-\hat{K}'_{j}}$, $H_{RN-\hat{K}_{j}} \leq H_{RN-\hat{K}'_{j}}$, $H_{IN-\hat{K}_{j}} \leq H_{IN-\hat{K}'_{j}}$ $|\forall j$, then $BCFFWA(\hat{K}_{1}, \hat{K}_{2}, \dots, \hat{K}_{n})$

$$\leq BCFFWA\left(\check{K}_{1}^{\#},\ \check{K}_{2}^{\#},\ \ldots,\ \check{K}_{n}^{\#}\right)$$
(16)

Next, we introduce the BCFFOWA operator as below *Definition 8:* The BCFFOWA operator is analyzed as

$$BCFFOWA\left(\acute{\mathbf{K}}_{1}, \acute{\mathbf{K}}_{2}, \ldots, \acute{\mathbf{K}}_{n}\right) = \bigoplus_{j=1}^{n} \mathbf{p}_{j}\acute{\mathbf{K}}_{\nu(j)} \quad (17)$$

where, $(\nu(1), \nu(2), \ldots, \nu(n))$ is the permutation of $j = 1, 2, \ldots, n$ with $\dot{K}_{\nu(j-1)} \ge \dot{K}_{\nu(j)} \forall j$.

Theorem 4: The aggregating outcome by employing BCF-FOWA operators is a BCFN and

$$BCFFWA\left(\acute{\mathbf{K}}_{1}, \acute{\mathbf{K}}_{2}, ..., \acute{\mathbf{K}}_{n}\right) = \begin{pmatrix} 1 - \log_{\wp} \left(1 + \prod_{j=1}^{n} \left(\wp^{1-\mathbf{H}_{R^{p}-\acute{\mathbf{K}}_{\nu(j)}} - 1\right)^{\mathbf{p}_{j}}\right) \\ + \iota \left(1 - \log_{\wp} \left(1 + \prod_{j=1}^{n} \left(\wp^{1-\mathbf{H}_{I^{p}-\acute{\mathbf{K}}_{\nu(j)}} - 1\right)^{\mathbf{p}_{j}}\right)\right) \\ - \left(\log_{\wp} \left(1 + \prod_{j=1}^{n} \left(\wp^{-\mathbf{H}_{R^{N}-\acute{\mathbf{K}}_{\nu(j)}} - 1\right)^{\mathbf{p}_{j}}\right) \right) \\ + \iota \left(-\left(\log_{\wp} \left(1 + \prod_{j=1}^{n} \left(\wp^{-\mathbf{H}_{I^{N}-\acute{\mathbf{K}}_{\nu(j)}} - 1\right)^{\mathbf{p}_{j}}\right)\right)\right) \right) \end{pmatrix}$$
(18)

Proof: Omitted.
Theorem 5: Let
$$\hat{K}_j = (H_{P-\hat{K}_j}, H_{N-\hat{K}_j})$$

 $= (H_{RP-\hat{K}_j} + \iota H_{IP-\hat{K}_j}, H_{RN-\hat{K}_j} + \iota H_{IN-\hat{K}_j})$ and $\hat{K}_j^{\#} = (H_{P-\hat{K}_j}^{\#}, H_{N-\hat{K}_j}^{\#})$
 $= (H_{RP-\hat{K}_j}^{\#} + \iota H_{IP-\hat{K}_j}^{\#}, H_{RN-\hat{K}_j}^{\#} + \iota H_{IN-\hat{K}_j}^{\#})$
 $(j = 1, 2, ..., n)$ be two families of BCFNs, then
1. **Idempotency:** If all $\hat{K}_j = \hat{K} \forall j$ then

$$BCFFOWA\left(\acute{K}_{1}, \acute{K}_{2}, \ldots, \acute{K}_{n}\right) = \acute{K}$$
(19)

2. Boundedness: Let
$$\hat{\mathbf{K}}^{-} = \left(\min_{j} \left\{ \mathbf{H}_{RP-\hat{\mathbf{K}}_{j}} \right\} + \iota \min_{j} \left\{ \mathbf{H}_{IP-\hat{\mathbf{K}}_{j}} \right\}, \max_{j} \left\{ \mathbf{H}_{RN-\hat{\mathbf{K}}_{j}} \right\} + \iota \max_{j} \left\{ \mathbf{H}_{IP-\hat{\mathbf{K}}_{j}} \right\} \right), \text{ and } \hat{\mathbf{K}}^{+} = \left(\max_{j} \left\{ \mathbf{H}_{RP-\hat{\mathbf{K}}_{j}} \right\} + \iota \max_{j} \left\{ \mathbf{H}_{IP-\hat{\mathbf{K}}_{j}} \right\}, \min_{j} \left\{ \mathbf{H}_{RN-\hat{\mathbf{K}}_{j}} \right\} + \iota \min_{j} \left\{ \mathbf{H}_{IP-\hat{\mathbf{K}}_{j}} \right\} \right). \text{ Then}$$

$$\hat{\mathbf{K}}^{-} \leq BCFFOWA \left(\hat{\mathbf{K}}_{1}, \hat{\mathbf{K}}_{2}, \dots, \hat{\mathbf{K}}_{n} \right) \leq \hat{\mathbf{K}}^{+} \quad (20)$$

3. Monotonicity: If $H_{RP-\acute{K}_{j}} \leq H_{RP-\acute{K}'_{j}}$, $H_{IP-\acute{K}_{j}} \leq H_{IP-\acute{K}'_{j}}$, $H_{RN-\acute{K}_{j}} \leq H_{RN-\acute{K}'_{j}}$, $H_{IN-\acute{K}_{j}} \leq H_{IN-\acute{K}'_{j}}$ $|\forall j$, then $BCFFOWA\left(\acute{K}_{1}, \acute{K}_{2}, \ldots, \acute{K}_{n}\right)$ $\leq BCFFOWA\left(\acute{K}_{1}^{\#}, \acute{K}_{2}^{\#}, \ldots, \acute{K}_{n}^{\#}\right)$ (21)

From Def (7), we observe that the weights related to the BCF-FWA operator are the most straightforward type of BCF values and from Def (8), the weights related to the BCFFOWA operator are the real arrangement of the ordered places of the BCF values. Thusly, the weights related to the BCF-FWA and BCFFOWA operators, describe different points of view which are clashing with each other. In any case, these points of view are thought to be something similar in an overall methodology. Just to be protected such downside, we presently interpret BCFFHA operator.

Definition 9: The BCFFHWA operator is analyzed as

$$BCFFHA\left(\dot{K}_{1}, \dot{K}_{2}, ..., \dot{K}_{n}\right) = \bigoplus_{j=1}^{n} p_{j}'\dot{K}_{\nu(j)}'$$

$$= \begin{pmatrix} 1 - \log_{\wp} \left(1 + \prod_{j=1}^{n} \left(\wp^{1-H_{RP-\dot{K}_{\nu(j)}}} - 1\right)^{p_{j}}\right) \\ + \iota \left(1 - \log_{\wp} \left(1 + \prod_{j=1}^{n} \left(\wp^{1-H_{IP-\dot{K}_{\nu(j)}}} - 1\right)^{p_{j}}\right)\right) \\ - \left(\log_{\wp} \left(1 + \prod_{j=1}^{n} \left(\wp^{-H_{PN-\dot{K}_{\nu(j)}}} - 1\right)^{p_{j}}\right)\right) \\ + \iota \left(-\left(\log_{\wp} \left(1 + \prod_{j=1}^{n} \left(\wp^{-H_{IN-\dot{K}_{\nu(j)}}} - 1\right)^{p_{j}}\right)\right)\right) \end{pmatrix}$$

$$(23)$$

where, $\mathbf{p}' = (\mathbf{p}'_1, \mathbf{p}'_2, \dots, \mathbf{p}'_n)$ is the aggregation linked WV such that $0 \le \mathbf{p}'_j \le 1$, and $\sum_{j=1}^n \mathbf{p}'_j = 1$, $\mathbf{p} = (\mathbf{p}_1, \mathbf{p}_2, \dots, \mathbf{p}_n)^t$ is the WV of \mathbf{K}_j (j = 1, 2, ..., n) such that $0 \le \mathbf{p}_j \le 1$, and $\sum_{j=1}^n \mathbf{p}_j = 1$. $\mathbf{K}'_{\nu(j)}$ is the *j*th major weighted BCF values of $\mathbf{K}'_j (\mathbf{K}'_j = n\mathbf{p}_j\mathbf{K}_j, j = 1, 2, ..., n)$, n is the balancing coefficient.

Theorem 6: Let
$$\hat{\mathbf{K}}_{j} = \left(\mathbf{H}_{P-\hat{\mathbf{K}}_{j}}, \mathbf{H}_{N-\hat{\mathbf{K}}_{j}}\right)$$

$$= \left(\mathbf{H}_{RP-\hat{\mathbf{K}}_{j}} + \iota \mathbf{H}_{IP-\hat{\mathbf{K}}_{j}}, \mathbf{H}_{RN-\hat{\mathbf{K}}_{j}} + \iota \mathbf{H}_{IN-\hat{\mathbf{K}}_{j}}\right) \text{ and } \hat{\mathbf{K}}_{j}^{\#} = \left(\mathbf{H}_{P-\hat{\mathbf{K}}_{j}}^{\#}, \mathbf{H}_{N-\hat{\mathbf{K}}_{j}}^{\#}\right)$$

$$= \left(\mathbf{H}_{RP-\hat{\mathbf{K}}_{j}}^{\#} + \iota \mathbf{H}_{IP-\hat{\mathbf{K}}_{j}}^{\#}, \mathbf{H}_{RN-\hat{\mathbf{K}}_{j}}^{\#} + \iota \mathbf{H}_{IN-\hat{\mathbf{K}}_{j}}^{\#}\right)$$

$$(j = 1, 2, ..., n) \text{ be two families of BCFNs, then}$$
1. **Idempotency:** If all $\hat{\mathbf{K}}_{j} = \hat{\mathbf{K}} \forall j$ then
$$BCFFHA\left(\hat{\mathbf{K}}_{1}, \hat{\mathbf{K}}_{2}, ..., \hat{\mathbf{K}}_{n}\right) = \hat{\mathbf{K}} \qquad (24)$$

2. Boundedness: Let
$$\hat{\mathbf{K}}^{-} = \left(\min_{j} \left\{ \mathbf{H}_{RP-\hat{\mathbf{K}}_{j}} \right\} + \iota \min_{j} \left\{ \mathbf{H}_{IP-\hat{\mathbf{K}}_{j}} \right\}, \quad \max_{j} \left\{ \mathbf{H}_{RN-\hat{\mathbf{K}}_{j}} \right\} + \iota \max_{j} \left\{ \mathbf{H}_{IP-\hat{\mathbf{K}}_{j}} \right\} \right), \text{ and } \hat{\mathbf{K}}^{+} = \left(\max_{j} \left\{ \mathbf{H}_{RP-\hat{\mathbf{K}}_{j}} \right\} + \iota \max_{j} \left\{ \mathbf{H}_{IP-\hat{\mathbf{K}}_{j}} \right\}, \quad \min_{j} \left\{ \mathbf{H}_{RN-\hat{\mathbf{K}}_{j}} \right\} + \iota \min_{j} \left\{ \mathbf{H}_{IP-\hat{\mathbf{K}}_{j}} \right\} \right). \text{ Then}$$
$$\hat{\mathbf{K}}^{-} \leq BCFFHA\left(\hat{\mathbf{K}}_{1}, \quad \hat{\mathbf{K}}_{2}, \quad \dots, \quad \hat{\mathbf{K}}_{n} \right) \leq \hat{\mathbf{K}}^{+} \quad (25)$$

3. **Monotonicity:** If $H_{RP-\check{K}_j} \leq H_{RP-\check{K}'_j}$, $H_{IP-\check{K}_j} \leq H_{IP-\check{K}'_j}$, $H_{RN-\check{K}_j} \leq H_{RN-\check{K}'_j}$, $H_{IN-\check{K}_j} \leq H_{IN-\check{K}'_j}$ $|\forall j$, then

$$BCFFHA\left(\acute{K}_{1}, \acute{K}_{2}, \dots, \acute{K}_{n}\right)$$

$$\leq BCFFHA\left(\acute{K}_{1}^{\#}, \acute{K}_{2}^{\#}, \dots, \acute{K}_{n}^{\#}\right) \qquad (26)$$

By taking $p = \left(\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n}\right)^{t}$ we have $\check{K}_{j} = n \times \frac{1}{n} \times \check{K}_{j} = \check{K}_{j}$ for $j = 1, 2, \dots, n$. Thus, the BCFFHA operator reduces to the BCFOWA operator. By taking $p' = \left(\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n}\right)^{t}$, the BCFFHA operator decreases to the BCFFWA operator. Therefore, BCFFWA and BCFFOWA operators are the particular cases of BCFFHA operators.

Definition 10: The BCFFWG operator is analyzed as

$$BCFFWG\left(\acute{\mathbf{K}}_{1}, \ \acute{\mathbf{K}}_{2}, \ \ldots, \ \acute{\mathbf{K}}_{n}\right) = \bigotimes_{j=1}^{n} \left(\acute{\mathbf{K}}_{j}\right)^{\mathbf{p}_{j}}$$
(27)

Therefore, we acquire a significant theorem that observes the Frank operations on BCFNs

Theorem 7: The aggregating outcome by employing BCF-FWG operators is a BCFN and

$$BCFFWG\left(\dot{K}_{1}, \dot{K}_{2}, ..., \dot{K}_{n}\right)$$

$$= \begin{pmatrix} \log_{\wp} \left(1 + \prod_{j=1}^{n} \left(\wp^{H_{RP-\dot{K}_{j}}} - 1\right)^{P_{j}}\right) \\ +\iota \left(\log_{\wp} \left(1 + \prod_{j=1}^{n} \left(\wp^{H_{IP-\dot{K}_{j}}} - 1\right)^{P_{j}}\right)\right) \\ -1 + \log_{\wp} \left(1 + \prod_{j=1}^{n} \left(\wp^{1+H_{RN-\dot{K}_{j}}} - 1\right)^{P_{j}}\right) \\ +\iota \left(-1 + \log_{\wp} \left(1 + \prod_{j=1}^{n} \left(\wp^{1+H_{IN-\dot{K}_{j}}} - 1\right)^{P_{j}}\right)\right) \end{pmatrix} \end{pmatrix}$$
(28)

Proof: Omitted.
Theorem 8: Let
$$\hat{K}_j = (H_{P-\hat{K}_j}, H_{N-\hat{K}_j})$$

 $= (H_{RP-\hat{K}_j} + \iota H_{IP-\hat{K}_j}, H_{RN-\hat{K}_j} + \iota H_{IN-\hat{K}_j})$ and $\hat{K}_j^{\#} = (H_{P-\hat{K}_j}^{\#}, H_{N-\hat{K}_j}^{\#})$
 $= (H_{RP-\hat{K}_j}^{\#} + \iota H_{IP-\hat{K}_j}^{\#}, H_{RN-\hat{K}_j}^{\#} + \iota H_{IN-\hat{K}_j}^{\#})$
 $(j = 1, 2, ..., n)$ be two families of BCFNs, then
1. **Idempotency:** If all $\hat{K}_j = \hat{K} \forall j$ then

$$BCFFWG\left(\acute{K}_{1}, \acute{K}_{2}, \ldots, \acute{K}_{n}\right) = \acute{K} \qquad (29)$$

2. Boundedness: Let
$$\hat{\mathbf{K}}^{-} = \left(\min_{j} \left\{ \mathbf{H}_{RP-\hat{\mathbf{K}}_{j}} \right\}$$

 $+\iota \min_{j} \left\{ \mathbf{H}_{IP-\hat{\mathbf{K}}_{j}} \right\}, \quad \max_{j} \left\{ \mathbf{H}_{RN-\hat{\mathbf{K}}_{j}} \right\}$
 $+\iota \max_{j} \left\{ \mathbf{H}_{IP-\hat{\mathbf{K}}_{j}} \right\}, \text{ and } \hat{\mathbf{K}}^{+} = \left(\max_{j} \left\{ \mathbf{H}_{RP-\hat{\mathbf{K}}_{j}} \right\}$
 $+\iota \max_{j} \left\{ \mathbf{H}_{IP-\hat{\mathbf{K}}_{j}} \right\}, \quad \min_{j} \left\{ \mathbf{H}_{RN-\hat{\mathbf{K}}_{j}} \right\}$
 $+\iota \min_{j} \left\{ \mathbf{H}_{IP-\hat{\mathbf{K}}_{j}} \right\}$. Then
 $\hat{\mathbf{K}}^{-} \leq BCFFWG\left(\hat{\mathbf{K}}_{1}, \ \hat{\mathbf{K}}_{2}, \ \dots, \ \hat{\mathbf{K}}_{n} \right) \leq \hat{\mathbf{K}}^{+}$ (30)

3. Monotonicity: If
$$H_{RP-\hat{K}_{j}} \leq H_{RP-\hat{K}_{j}'}, H_{IP-\hat{K}_{j}} \leq H_{IP-\hat{K}_{j}'}, H_{IP-\hat{K}_{j}} \leq H_{IP-\hat{K}_{j}'}, H_{IN-\hat{K}_{j}} \leq H_{IN-\hat{K}_{j}'} |\forall j, \text{ then}$$

$$BCFFWG\left(\hat{K}_{1}, \hat{K}_{2}, \dots, \hat{K}_{n}\right)$$

$$\leq BCFFWG\left(\hat{K}_{1}^{\#}, \hat{K}_{2}^{\#}, \dots, \hat{K}_{n}^{\#}\right) \qquad (31)$$

Definition 11: The BCFFOWG operator is analyzed as

$$BCFFWG\left(\acute{\mathbf{K}}_{1}, \ \acute{\mathbf{K}}_{2}, \ \ldots, \ \acute{\mathbf{K}}_{n}\right) = \bigotimes_{j=1}^{n} \left(\acute{\mathbf{K}}_{\nu(j)}\right)^{\mathbf{p}_{j}} \quad (32)$$

where, $(\nu(1), \nu(2), \ldots, \nu(n))$ is the permutation of $j = 1, 2, \ldots, n$ with $\check{K}_{\nu(j-1)} \ge \check{K}_{\nu(j)} \forall j$.

Theorem 9: The aggregating outcome by employing BCF-FOWG operators is a BCFN and

$$BCFFOWG\left(\acute{K}_{1}, \acute{K}_{2}, ..., \acute{K}_{n}\right)$$

$$= \begin{pmatrix} \log_{\wp} \left(1 + \prod_{j=1}^{n} \left(\wp^{H_{RP-\acute{K}_{\nu(j)}}} - 1\right)^{P_{j}}\right) \\ +\iota \left(\log_{\wp} \left(1 + \prod_{j=1}^{n} \left(\wp^{H_{IP-\acute{K}_{\nu(j)}}} - 1\right)^{P_{j}}\right)\right) \\ -1 + \log_{\wp} \left(1 + \prod_{j=1}^{n} \left(\wp^{1+H_{RN-\acute{K}_{\nu(j)}}} - 1\right)^{P_{j}}\right) \\ +\iota \left(-1 + \log_{\wp} \left(1 + \prod_{j=1}^{n} \left(\wp^{1+H_{IN-\acute{K}_{\nu(j)}}} - 1\right)^{P_{j}}\right)\right) \end{pmatrix}$$
(33)

Theorem 10: Let $\acute{K}_{j} = (H_{P-\acute{K}_{j}}, H_{N-\acute{K}_{j}}) = (H_{RP-\acute{K}_{j}} + \iota H_{IP-\acute{K}_{j}}, H_{RN-\acute{K}_{j}} + \iota H_{IN-\acute{K}_{j}})$ and $\acute{K}_{j}^{\#} = (H_{P-\acute{K}_{j}}^{\#}, H_{N-\acute{K}_{j}}^{\#})$ $= (H_{RP-\acute{K}_{j}}^{\#} + \iota H_{IP-\acute{K}_{j}}^{\#}, H_{RN-\acute{K}_{j}}^{\#} + \iota H_{IN-\acute{K}_{j}}^{\#})$ (j = 1, 2, ..., n) be two families of BCFNs, then 1. **Idempotency:** If all $\acute{K}_{j} = \acute{K} \forall j$ then

$$BCFFOWG(\mathbf{K}_1, \mathbf{K}_2, \dots, \mathbf{K}_n) = \mathbf{K}$$
(34)

2. Boundedness: Let
$$\mathbf{K} = \left(\min_{j} \left\{ \mathbf{H}_{RP-\hat{\mathbf{K}}_{j}} \right\} + \iota \min_{j} \left\{ \mathbf{H}_{IP-\hat{\mathbf{K}}_{j}} \right\}, \max_{j} \left\{ \mathbf{H}_{RN-\hat{\mathbf{K}}_{j}} \right\} + \iota \max_{j} \left\{ \mathbf{H}_{IP-\hat{\mathbf{K}}_{j}} \right\}, \text{ and } \hat{\mathbf{K}}^{+} = \left(\max_{j} \left\{ \mathbf{H}_{RP-\hat{\mathbf{K}}_{j}} \right\} + \iota \max_{j} \left\{ \mathbf{H}_{IP-\hat{\mathbf{K}}_{j}} \right\}, \min_{j} \left\{ \mathbf{H}_{RN-\hat{\mathbf{K}}_{j}} \right\} + \iota \min_{j} \left\{ \mathbf{H}_{IP-\hat{\mathbf{K}}_{j}} \right\} \right). \text{ Then}$$

$$\hat{\mathbf{K}}^{-} \leq BCFFOWG \left(\hat{\mathbf{K}}_{1}, \hat{\mathbf{K}}_{2}, \dots, \hat{\mathbf{K}}_{n} \right) \leq \hat{\mathbf{K}}^{+} \quad (35)$$

3. Monotonicity: If
$$H_{RP-\hat{K}_{j}} \leq H_{RP-\hat{K}_{j}}', H_{IP-\hat{K}_{j}} \leq H_{IP-\hat{K}_{j}'}, H_{IP-\hat{K}_{j}} \leq H_{IP-\hat{K}_{j}'}, H_{IN-\hat{K}_{j}} \leq H_{IN-\hat{K}_{j}'} | \forall j, \text{ then}$$

$$BCFFOWG\left(\hat{K}_{1}, \hat{K}_{2}, \dots, \hat{K}_{n}\right)$$

$$\leq BCFFOWG\left(\hat{K}_{1}^{\#}, \hat{K}_{2}^{\#}, \dots, \hat{K}_{n}^{\#}\right) \qquad (36)$$

From Def (10), we observe that the weights related to the BCFFWG operator are the most straightforward type of BCF values and from Def (11), the weights related to the BCF-FOWG operator are the real arrangement of the ordered places of the BCF values. Thusly, the weights related to the BCFFWA and BCFFOWG operators, describe different points of view which are clashing with each other. In any case, these points of view are thought to be something similar in an

overall methodology. Just to be protected such a downside, we presently interpret the BCFFHG operator.

Definition 12: The BCFFHWG operator is analyzed as

$$BCFFHA\left(\acute{\mathbf{K}}_{1}, \acute{\mathbf{K}}_{2}, ..., \acute{\mathbf{K}}_{n}\right) = \bigoplus_{j=1}^{n} \left(\acute{\mathbf{K}}_{\nu(j)}^{'}\right)^{\mathbf{p}_{j}^{'}} \\ = \left(\begin{array}{c} \log_{\wp} \left(1 + \prod_{j=1}^{n} \left(\wp^{\mathbf{H}_{RP-\acute{\mathbf{K}}_{\nu(j)}} - 1\right)^{\mathbf{p}_{j}}\right) \\ +\iota \left(\log_{\wp} \left(1 + \prod_{j=1}^{n} \left(\wp^{\mathbf{H}_{IP-\acute{\mathbf{K}}_{\nu(j)}} - 1\right)^{\mathbf{p}_{j}}\right)\right) \\ -1 + \log_{\wp} \left(1 + \prod_{j=1}^{n} \left(\wp^{1+\mathbf{H}_{RN-\acute{\mathbf{K}}_{\nu(j)}} - 1\right)^{\mathbf{p}_{j}}\right) \\ +\iota \left(-1 + \log_{\wp} \left(1 + \prod_{j=1}^{n} \left(\wp^{1+\mathbf{H}_{IN-\acute{\mathbf{K}}_{\nu(j)}} - 1\right)^{\mathbf{p}_{j}}\right)\right) \right) \\ (37)$$

where, $\mathbf{p}' = (\mathbf{p}'_1, \mathbf{p}'_2, \dots, \mathbf{p}'_n)$ is the aggregation linked WV such that $0 \le \mathbf{p}'_j \le 1$, and $\sum_{j=1}^n \mathbf{p}'_j = 1$, $\mathbf{p} = (\mathbf{p}_1, \mathbf{p}_2, \dots, \mathbf{p}_n)^t$ is the WV of \mathbf{K}_j (j = 1, 2, ..., n) such that $0 \le \mathbf{p}_j \le 1$, and $\sum_{j=1}^n \mathbf{p}_j = 1$. $\mathbf{K}_{\nu(j)}'$ is the *j*th major weighted BCF values of $\mathbf{K}'_j (\mathbf{K}'_j = n\mathbf{p}_j\mathbf{K}_j, j = 1, 2, ..., n)$, *n* is the balancing coefficient.

coefficient. Theorem 11: Let $\acute{K}_j = (H_{P-\acute{K}_j}, H_{N-\acute{K}_j}) = (H_{RP-\acute{K}_j} + \iota H_{IP-\acute{K}_j}, H_{RN-\acute{K}_j} + \iota H_{IN-\acute{K}_j})$ and $\acute{K}_j^{\#} = (H_{P-\acute{K}_j}^{\#}, H_{N-\acute{K}_j}^{\#})$ $= (H_{RP-\acute{K}_j}^{\#} + \iota H_{IP-\acute{K}_j}^{\#}, H_{RN-\acute{K}_j}^{\#} + \iota H_{IN-\acute{K}_j}^{\#})$ (j = 1, 2, ..., n) be two families of BCFNs, then

1. **Idempotency:** If all $K_j = K \forall j$ then

$$BCFFHG\left(\acute{K}_{1}, \acute{K}_{2}, \ldots, \acute{K}_{n}\right) = \acute{K} \qquad (38)$$

2. Boundedness: Let
$$\hat{\mathbf{K}}^{-} = \left(\min_{j} \left\{ \mathbf{H}_{RP-\hat{\mathbf{K}}_{j}} \right\} + \iota \min_{j} \left\{ \mathbf{H}_{IP-\hat{\mathbf{K}}_{j}} \right\}, \quad \max_{j} \left\{ \mathbf{H}_{RN-\hat{\mathbf{K}}_{j}} \right\} + \iota \max_{j} \left\{ \mathbf{H}_{IP-\hat{\mathbf{K}}_{j}} \right\} \right), \text{ and } \hat{\mathbf{K}}^{+} = \left(\max_{j} \left\{ \mathbf{H}_{RP-\hat{\mathbf{K}}_{j}} \right\} + \iota \max_{j} \left\{ \mathbf{H}_{IP-\hat{\mathbf{K}}_{j}} \right\}, \quad \min_{j} \left\{ \mathbf{H}_{RN-\hat{\mathbf{K}}_{j}} \right\} + \iota \min_{j} \left\{ \mathbf{H}_{IP-\hat{\mathbf{K}}_{j}} \right\} \right). \text{ Then}$$
$$\hat{\mathbf{K}}^{-} \leq BCFFHG\left(\hat{\mathbf{K}}_{1}, \quad \hat{\mathbf{K}}_{2}, \quad \dots, \quad \hat{\mathbf{K}}_{n} \right) \leq \hat{\mathbf{K}}^{+} \quad (39)$$

3. Monotonicity: If
$$H_{RP-\check{K}_j} \leq H_{RP-\check{K}'_j}$$
, $H_{IP-\check{K}_j} \leq H_{IP-\check{K}'_j}$,
 $H_{RN-\check{K}_j} \leq H_{RN-\check{K}'_j}$, $H_{IN-\check{K}_j} \leq H_{IN-\check{K}'_j}$ $\forall j$, then

$$BCFFHG\left(\acute{\mathbf{K}}_{1}, \acute{\mathbf{K}}_{2}, \ldots, \acute{\mathbf{K}}_{n}\right)$$
$$\leq BCFFHG\left(\acute{\mathbf{K}}_{1}^{\#}, \acute{\mathbf{K}}_{2}^{\#}, \ldots, \acute{\mathbf{K}}_{n}^{\#}\right) \qquad (40)$$

By taking $p = \left(\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n}\right)^{t}$ we have $\acute{K_{j}} = n \times \frac{1}{n} \times \acute{K_{j}} = \acute{K_{j}}$ for j = 1, 2, ..., n. Thus, the BCFFHA operator reduces to the BCFOWG operator. By taking $p' = \left(\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n}\right)^{t}$, the BCFFHG operator decreases to the BCFFWG operator. Therefore, BCFFWG and BCFFOWG operators are the particular cases of BCFFHA operators.

V. ANALYTICAL HIERARCHY PROCESS

DevOps ("development and operations") is a dominant and collaborative organizational effort to computerize simultaneously the supply of a software strategy with a theme to enhance software features. The utilization of DevOps procedures is not straightforward as there is certain ambiguity concerned with it. This analysis aims to define the analytical hierarchy process using BCF information. Therefore, provide a technique for the AHP tool using BCF information.

Step 1: Compute the objective, attribute, sub-attribute, and alternatives of the decision-making dilemma, and then develop a hierarchy of the selected dilemma.

Step 2: Compute the relation among the attribute and sub-attribute based on presence relations. Continually, based on the attribute and sub-attribute compared the alternative and then developed the BCF preference relations.

Step 3: Compute the consistency of every BCF preference relation based on the $d(R_7R) < \tau$. If we obtained all the BCF preference relation is acceptable then we proceed with the procedure.

$$d(R;R) = \frac{1}{2(n-1)(n-2)} \sum_{i=1}^{n} \sum_{k=1}^{n} \left(\frac{1}{2} \left(\left| \overline{\mathbf{H}}_{RP-ik} - \mathbf{H}_{RP-ik} \right| + \left| \overline{\mathbf{H}}_{IP-ik} - \mathbf{H}_{IP-ik} \right| \right) \right) + \left(\frac{1}{2} \left(\left| \overline{\mathbf{H}}_{RN-ik} - \mathbf{H}_{RN-ik} \right| + \left| \overline{\mathbf{H}}_{IN-ik} - \mathbf{H}_{IN-ik} \right| \right) \right) \right)$$

where τ , represented the consistency threshold.

Step 4: Compute the inconsistency BCF preference relation based on the Method-2.

Method-2

Sub-Step 1: Assume P, represented the iterations, and suppose p = 1 and develop the perfect multiplicative consistent BCF preference relation R and R^P from Method-1.

Method-1

Sub-Sub-Step 1: Assume k > i + 1 and suppose $\overline{R_{ik}} = (\overline{H}_{RP-ik} + \iota \overline{H}_{IP-ik}, \overline{H}_{RN-ik} + \iota \overline{H}_{IN-ik})$, where

$$\begin{split} \overline{\mathbf{H}}_{RP-ik} &= \frac{\left(\prod_{t=i+1}^{k-1} \mathbf{H}_{RP-it} \mathbf{H}_{RP-tk}\right)^{\frac{1}{k-i-1}}}{\left(\prod_{t=i+1}^{k-1} \mathbf{H}_{RP-it} \mathbf{H}_{RP-tk}\right)^{\frac{1}{k-i-1}}} \\ &+ \left(\prod_{t=i+1}^{k-1} (1 - \mathbf{H}_{RP-it}) (1 - \mathbf{H}_{RP-tk})\right)^{\frac{1}{k-i-1}} \\ \overline{\mathbf{H}}_{IP-ik} &= \frac{\left(\prod_{t=i+1}^{k-1} \mathbf{H}_{IP-it} \mathbf{H}_{IP-tk}\right)^{\frac{1}{k-i-1}}}{\left(\prod_{t=i+1}^{k-1} \mathbf{H}_{IP-it} \mathbf{H}_{IP-tk}\right)^{\frac{1}{k-i-1}}} \\ &+ \left(\prod_{t=i+1}^{k-1} (1 - \mathbf{H}_{IP-it}) (1 - \mathbf{H}_{IP-tk})\right)^{\frac{1}{k-i-1}} \\ \overline{\mathbf{H}}_{RN-ik} &= \frac{-\left(\prod_{t=i+1}^{k-1} \mathbf{H}_{RN-it} \mathbf{H}_{RN-tk}\right)^{\frac{1}{k-i-1}}}{\left(\prod_{t=i+1}^{k-1} \mathbf{H}_{RN-it} \mathbf{H}_{RN-tk}\right)^{\frac{1}{k-i-1}}} \\ &+ \left(\prod_{t=i+1}^{k-1} (1 + \mathbf{H}_{RN-it}) (1 + \mathbf{H}_{RN-tk})\right)^{\frac{1}{k-i-1}} \\ \overline{\mathbf{H}}_{IN-ik} &= \frac{-\left(\prod_{t=i+1}^{k-1} \mathbf{H}_{IN-it} \mathbf{H}_{IN-tk}\right)^{\frac{1}{k-i-1}}}{\left(\prod_{t=i+1}^{k-1} \mathbf{H}_{IN-it} \mathbf{H}_{IN-tk}\right)^{\frac{1}{k-i-1}}} \\ &+ \left(\prod_{t=i+1}^{k-1} (1 + \mathbf{H}_{IN-it}) (1 + \mathbf{H}_{IN-tk})\right)^{\frac{1}{k-i-1}} \\ &+ \left(\prod_{t=i+1}^{k-1} (1 + \mathbf{H}_{IN-it}) (1 + \mathbf{H}_{IN-tk})\right)^{\frac{1}{k-i-1}} \\ \end{split}$$

Sub-Sub-Step 2: Assume k = i + 1, let $\overline{\check{K}}_{ik} = \acute{K}_{i\underline{k}}$. Sub-Sub-Step 3: Assume k < i, let \check{K}_{ik}

 $(\overline{\mathrm{H}}_{RP-ik} + \iota \overline{\mathrm{H}}_{IP-ik}, \overline{\mathrm{H}}_{RN-ik} + \iota \overline{\mathrm{H}}_{IN-ik}).$ Sub-Step 2: Compute the distance $d(R, R^P)$ among R and R^P .

$$d\left(R,R^{P}\right)$$

$$=\frac{1}{2(n-1)(n-2)}\sum_{i=1}^{n}\sum_{k=1}^{n}$$

$$\times \left(\frac{\left(\frac{1}{2}\left(\left|\overline{\mathbf{H}}_{RP-ik}-\mathbf{H}_{RP-ik}^{P}\right|+\left|\overline{\mathbf{H}}_{IP-ik}-\mathbf{H}_{IP-ik}^{P}\right|\right)\right)}{\left(+\left(\frac{1}{2}\left(\left|\overline{\mathbf{H}}_{RN-ik}-\mathbf{H}_{RN-ik}^{P}\right|+\left|\overline{\mathbf{H}}_{IN-ik}-\mathbf{H}_{IN-ik}^{P}\right|\right)\right)\right)}$$

where $d(R, R^P) < \tau$.

Sub-Step 3: Compute the fused BCF preference relation $\overline{R^{P}} = \left(\overline{\mathrm{H}}_{RP-ik}^{P} + \iota \overline{\mathrm{H}}_{IP-ik}^{P}, \overline{\mathrm{H}}_{RN-ik}^{P} + \iota \overline{\mathrm{H}}_{IN-ik}^{P}\right)$, where,

$$\begin{split} \overline{\mathbf{H}}_{RP-ik}^{P} &= \frac{\left(\mathbf{H}_{RP-ik}^{P}\right)^{1-\sigma} \left(\mathbf{H}_{RP-ik}^{P}\right)^{\sigma}}{\left(\mathbf{H}_{RP-ik}^{P}\right)^{1-\sigma} \left(\mathbf{H}_{RP-ik}^{P}\right)^{\sigma}} \\ &+ \left(1-\mathbf{H}_{RP-ik}^{P}\right)^{1-\sigma} \left(1-\mathbf{H}_{RP-ik}^{P}\right)^{\sigma} \end{split}$$

$$\begin{split} \overline{\mathbf{H}}_{IP-ik}^{P} &= \frac{\left(\mathbf{H}_{IP-ik}^{P}\right)^{1-\sigma} \left(\mathbf{H}_{IP-ik}^{P}\right)^{\sigma}}{\left(\mathbf{H}_{IP-ik}^{P}\right)^{1-\sigma} \left(\mathbf{H}_{IP-ik}^{P}\right)^{\sigma}} \\ &+ \left(1 - \mathbf{H}_{IP-ik}^{P}\right)^{1-\sigma} \left(1 - \mathbf{H}_{IP-ik}^{P}\right)^{\sigma} \\ \overline{\mathbf{H}}_{RN-ik}^{P} &= \frac{-\left(-\mathbf{H}_{RN-ik}^{P}\right)^{1-\sigma} \left(-\mathbf{H}_{RN-ik}^{P}\right)^{\sigma}}{\left(-\mathbf{H}_{RN-ik}^{P}\right)^{1-\sigma} \left(-\mathbf{H}_{RN-ik}^{P}\right)^{\sigma}} \\ &+ \left(1 + \mathbf{H}_{RN-ik}^{P}\right)^{1-\sigma} \left(1 + \mathbf{H}_{RN-ik}^{P}\right)^{\sigma} \\ \overline{\mathbf{H}}_{IN-ik}^{P} &= \frac{-\left(-\mathbf{H}_{IN-ik}^{P}\right)^{1-\sigma} \left(-\mathbf{H}_{IN-ik}^{P}\right)^{\sigma}}{\left(-\mathbf{H}_{IN-ik}^{P}\right)^{1-\sigma} \left(-\mathbf{H}_{IN-ik}^{P}\right)^{\sigma}} \\ &+ \left(1 + \mathbf{H}_{IN-ik}^{P}\right)^{1-\sigma} \left(-\mathbf{H}_{IN-ik}^{P}\right)^{\sigma} \end{split}$$

where σ , represented the controlling parameter investigation by the decision-maker. The minimum value of σ .

Step 5: Compute the priority vector $\mathbf{p} = (\mathbf{p}_1, \mathbf{p}_2..., \mathbf{p}_n)^t$ for every BCF preference relation using the \mathbf{p}_i .

$$\mathbf{p}_{i} = \begin{pmatrix} \frac{\sum_{k=1}^{n} \mathbf{H}_{RP-ik}}{\sum_{i=1}^{n} \sum_{k=1}^{k} (1 + \mathbf{H}_{RN-ik})}, \\ \frac{\sum_{k=1}^{n} \mathbf{H}_{IP-ik}}{\sum_{i=1}^{n} \sum_{k=1}^{n} (1 + \mathbf{H}_{IN-ik})}, \\ \frac{\sum_{i=1}^{n} \sum_{k=1}^{n} (-1 - \mathbf{H}_{RN-ik})}{\sum_{i=1}^{n} \sum_{k=1}^{n} (\mathbf{H}_{RP-ik})}, \\ \frac{\sum_{k=1}^{n} (-1 - \mathbf{H}_{IN-ik})}{\sum_{i=1}^{n} \sum_{k=1}^{n} (\mathbf{H}_{IP-ik})}, \end{pmatrix}$$

Step 6: Rank the weight vector using Def. (3) and choose the beneficial alternatives. The main geometrical shape of the BCF-AHP technique is available in the shape in Figure 1.

VI. APPLICATION

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DevOps is also a new type of software used in many companies for computing the relationship between development and information technology operations. The major goal of DevOps is to replace and enhance the interrelationship by encouraging beneficial and dominant communication and collaboration among these two industries. The main role of DevOps in the companies there is a require separate decrease silos, where industries units deal as combined terms within the companies where administration, procedure, and data are modified. On the software investigation side and for those working in information technology operations there requires to be beneficial communication and collaboration to beneficial thing the information technology industry requires of the company. Similarly, DevOps also plays an important and feasible role in the development of culture in our society. One main thing can break down because of the DevOpsbased culture, in experts' investigators with operations staff to guarantee the company achieves beneficial running of software with a minimal dilemma. This culture encourages a commitment to work together and share. DevOps is not dependent on the stringent techniques and procedures: it is dependent on the professional rules and regulations that



FIGURE 1. The Main geometrical shape of the BCF-AHP technique.

help company units work collectively inside the company and break down the old silos. Decision-making skills can be distinct for illustrating the beneficial decision that enhances the expert industry. The strategy is select the decision is a very complicated task, which needed a leadership quality in very decision-maker who think objectively and critically. The capacity of the decision-maker for selecting the best decision can help to diagnose a strong and dominant concept. The decision-making technique represents your feasibility in selecting among two or more decisions in the form of the opinion of the decision-maker, called alternatives. Decisionmaking tools are widely utilized in economics, computer science, network systems, and software engineering. Based on the capacity of the decision-making procedure for choosing the beneficial optimal is very fast and many scholars have used it to resolve has problems under the presence of classical information.

DevOps is a dominant and collaborative organizational effort to computerize simultaneously the supply of a software strategy with a theme to enhance software features. The utilization of DevOps procedures is not straightforward as there is certain ambiguity concerned with it. The aim of this analysis is to discover and prioritize the major factors that positively influence the DevOps technique in software organizations. In the presence of the prevailing information, nineteen factors were diagnosed. The diagnosed features were moreover authorized by intellectuals via hypothetical information. Finally, BCF analytical hierarchy process (BCFAHP) technique is invented to prioritize the classification success features. The final ranking mentioned that "DevOps security pipeline", "use system orchestration", and "attempt matrix organization and transparency" features are the beneficial ranked success features for the valuable utilization of DevOps techniques. Finally, we compared the diagnosed operators with various existing theories to improve the worth of the invented approaches. We presently consider a multi-measures dynamic issue that concerns the worldwide provider advancement to show our method of the BCFAHP. The present globalized market pattern distinguishes the need for the foundation of long-haul business relationships with serious worldwide providers spread all over the planet. Instructions to choose different new global providers as indicated by the wide examination is extremely basic and have an immediate effect on the presentation of an association. Worldwide provider advancement is a multi-model's dynamic issue that incorporates both subjective and quantitative elements. It is more complex than a homegrown one and it needs more basic examination. Since most leaders can't deal with in excess of nine elements while going with choice, it is important to separate this complicated issue into more reasonable subproblems and hence the pecking order can be built. The fundamental goal is the determination of the best worldwide provider for assembling firm and the rules considered in accomplishing the goal is as follows.

- 1. C_1 : Total cost of the invention, which includes three sub-attributes: S_1 : Outcome value, S_2 : Transport price, and S_3 : Tax.
- 2. C_2 : Excellence of the invention, which includes four sub-attributes: S_4 : Rejection, S_5 : Lead time, S_6 : Quality and S_7 : Problem.
- 3. C_3 : Performance of evaluator, which includes four subattributes: S_8 : Delivery, S_9 : Technology, S_{10} : Response and S_{11} : Communication.
- 4. C_4 : Profile of evaluator, which includes four subattributes: S_{12} : Financial, S_{13} : Customer, S_{14} : History and S_{15} : Capacity.
- 5. C_5 : Risk of the evaluator, which includes four subattributes: S_{16} : Location, S_{17} : Stability, S_{18} : Economy and S_{19} : Terrorism.

Assume three providers are getting looked at. The progressive system comprises four levels. The general goal is set at Level 1, standards at Level 2, sub-attribute at Level 3, and alternatives at Level 4. In the wake of building the order, the pairwise examination of the significance of one standard, subattribute, or elective over one more should be possible with the assistance of the survey. It tends not entirely settled by the accessible examination, the current business situation, or the experience of the specialists. To improve on the show, in this paper, we would rather not focus on the correlation of the sub-attribute however take them overall. Practically speaking, we can do it exhaustively, however, the technique continues as before. Assume that the examination decisions are addressed in BCF numbers and displayed in Tables 1-4.

TABLE 1. BCF preference relation of the attribute.	
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R	<i>C</i> ₁	<i>C</i> ₂	<i>C</i> ₃	<i>C</i> ₄	<i>C</i> ₅
C_1	$(0.5 + \iota 0.5,)$	$(0.6 + \iota 0.2,)$	$(0.6 + \iota 0.2,)$	$(0.65 + \iota 0.25,)$	$(0.65 + \iota 0.25,)$
	$(-0.5 - \iota 0.5)$	$(-0.6 - \iota 0.2)$	$(-0.6 - \iota 0.2)$	$(-0.65 - \iota 0.25)$	$(-0.65 - \iota 0.25)$
C_2	$(0.2 + \iota 0.6,)$	$(0.5 + \iota 0.5,)$	$(0.6 + \iota 0.2,)$	$(0.65 + \iota 0.25,)$	(0.65 + <i>ι</i> 0.25,)
	$(-0.2 - \iota 0.6)$	$(-0.5 - \iota 0.5)$	$(-0.6 - \iota 0.2)$	$(-0.65 - \iota 0.25)$	$(-0.65 - \iota 0.25)$
C_3	$(0.2 + \iota 0.6,)$	$(0.2 + \iota 0.6,)$	$(0.5 + \iota 0.5,)$	$(0.6 + \iota 0.2,)$	$(0.6 + \iota 0.2,)$
	$(-0.2 - \iota 0.6)$	$(-0.2 - \iota 0.6)$	$(-0.5 - \iota 0.5)$	$(-0.6 - \iota 0.2)$	$(-0.6 - \iota 0.2)$
<i>C</i> ₄	(0.25 + ι0.65,)	$(0.25 + \iota 0.65,)$	$(0.2 + \iota 0.6,)$	$(0.5 + \iota 0.5,)$	$(0.6 + \iota 0.2,)$
	$(-0.25 - \iota 0.65)$	$(-0.25 - \iota 0.65)$	$(-0.2 - \iota 0.6)$	$(-0.5 - \iota 0.5)$	$(-0.6 - \iota 0.2)$
C_5	$(0.25 + \iota 0.65,)$	$(0.25 + \iota 0.65,)$	$(0.2 + \iota 0.6,)$	$(0.2 + \iota 0.6,)$	$(0.5 + \iota 0.5,)$
	$(-0.25 - \iota 0.65)$	$(-0.25 - \iota 0.65)$	$(-0.2 - \iota 0.6)$	$(-0.2 - \iota 0.6)$	$(-0.5 - \iota 0.5)$

TABLE 2. BCF preference relation of attribute for C₁.

R_1	A_1	A_2	A_3
A_1	$(0.5 + \iota 0.5,)$	$(0.6 + \iota 0.2,)$	$(0.4 + \iota 0.5,)$
	$(-0.5 - \iota 0.5)$	$(-0.6 - \iota 0.2)$	$(-0.4 - \iota 0.5)$
A_2	$(0.2 + \iota 0.6,)$	$(0.5 + \iota 0.5,)$	$(0.3 + \iota 0.4,)$
-	$(-0.2 - \iota 0.6)$	$(-0.5 - \iota 0.5)$	$(-0.3 - \iota 0.4)$
A_3	$(0.5 + \iota 0.4,)$	$(0.4 + \iota 0.3,)$	$(0.5 + \iota 0.5,)$
	$(-0.5 - \iota 0.4)$	$(-0.4 - \iota 0.3)$	$(-0.5 - \iota 0.5)$
113	$\binom{0.0}{-0.5 - \iota \ 0.4}$	$(-0.4 - \iota 0.3)$	$(-0.5 - \iota 0.5)$

TABLE 3. BCF preference relation of attribute for C₂.

R_2	A_1	A_2	A_3
A_1	$(0.5 + \iota 0.5,)$	$(0.6 + \iota 0.2,)$	$(0.45 + \iota 0.4,)$
	$(-0.5 - \iota 0.5)$	$(-0.6 - \iota 0.2)$	$(-0.45 - \iota 0.4)$
A_2	$(0.2 + \iota 0.6,)$	$(0.5 + \iota 0.5,)$	$(0.35 + \iota 0.2,)$
	$(-0.2 - \iota 0.6)$	$(-0.5 - \iota 0.5)$	$(-0.35 - \iota 0.2)$
A_3	$(0.4 + \iota 0.45,)$	$(0.2 + \iota 0.35,)$	(0.5 + <i>ι</i> 0.5,)
	<u>$(-0.4 - \iota 0.45)$</u>	$(-0.2 - \iota 0.35)$	$(-0.5 - \iota 0.5)$

TABLE 4. BCF preference relation of attribute for C₃.

R_3	A_1	A_2	A_3
A_1	(0.5 + ι0.5,)	$(0.55 + \iota 0.25,)$	$(0.65 + \iota 0.1,)$
	$(-0.5 - \iota 0.5)$	$(-0.55 - \iota 0.25)$	$(-0.65 - \iota 0.1)$
A_2	$(0.25 + \iota 0.55,)$	(0.5 + ι0.5,)	$(0.6 + \iota 0.3,)$
	$(-0.25 - \iota 0.55)$	$(-0.5 - \iota 0.5)$	$(-0.6 - \iota 0.3)$
A_3	$(0.1 + \iota 0.65,)$	$(0.3 + \iota 0.6,)$	$(0.5 + \iota 0.5,)$
	$(-0.1 - \iota 0.65)$	$(-0.3 - \iota 0.6)$	$(-0.5 - \iota 0.5)$

TABLE 5. BCF preference relation of attribute for C_4 .

R_4	A_1	A_2	A_3
A_1	$(0.5 + \iota 0.5,)$	$(0.8 + \iota 0.1,)$	$(0.7 + \iota 0.2,)$
	$(-0.5 - \iota 0.5)$	$(-0.8 - \iota 0.1)$	$(-0.7 - \iota 0.2)$
A_2	$(0.1 + \iota 0.8,)$	$(0.5 + \iota 0.5,)$	$(0.55 + \iota 0.4,)$
-	$(-0.1 - \iota 0.8)$	$(-0.5 - \iota 0.5)$	$(-0.55 - \iota 0.4)$
A_3	$(0.2 + \iota 0.7,)$	$(0.4 + \iota 0.55,)$	$(0.5 + \iota 0.5,)$
5	$(-0.2 - \iota 0.7)$	$(-0.4 - \iota 0.55)$	$(-0.5 - \iota 0.5)$

First, we represent the dominancy of the pairwise decisions for each BCF preference relation and restoration of the unpredictable decisions via Method-2, yet it is satisfactory. Assume demonstrate the procedure of dominancy representing the BCF preference relation *R*of the attribute as an example, **TABLE 6.** BCF preference relation of attribute for C_5 .

R_5	A_1	A_2	A_3
A_1	$(0.5 + \iota 0.5,)$	$(0.7 + \iota 0.2,)$	$(0.65 + \iota 0.2,)$
	$(-0.5 - \iota 0.5)$	$(-0.7 - \iota 0.2)$	$1-0.65 - \iota 0.2$
A_2	$(0.2 + \iota 0.7,)$	$(0.5 + \iota 0.5,)$	$(0.55 + \iota 0.25,)$
-	$(-0.2 - \iota 0.7)$	$(-0.5 - \iota 0.5)$	$(-0.55 - \iota 0.25)$
A_3	$(0.2 + \iota 0.65,)$	$(0.25 + \iota 0.55,)$	$(0.5 + \iota 0.5,)$
	<u>$1-0.2 - \iota 0.65$</u>	<u> </u>	$(-0.5 - \iota 0.5)$

based on the Method-1, we develop the perfect multiplicative dominant BCF preference relation $R = (\overline{r_{ik}})_{5\times 5}$, of the BCF preference relation *R*of the attribute, see Table 7.

Computing the values of $d(R_7R)$, we get the beneficial result $d_5(R_7R) = 0.09272 < 0.1$, which means that the final result is acceptable consistency. If not, then we need to repair it. Then we use Method-2. The main result is discussed below.

$$d_1(R_7R) = 0.08071, \quad d_2(R_7R) = 0.07127, d_3(R_7R) = 0.06679, \quad d_4(R_7R) = 0.07027, d_5(R_7R) = 0.09272$$

Using Method-2, for $\sigma = 0.3$, then see Table 8.

Computing the values of $d(R_{\tau}R^{P})$, we get the beneficial result $d_5(R, \overline{R^{P}}) = 0.09272 < 0.1$, which means that the final result is acceptable consistency. From Method-1, we noticed that the proposed work gives the same result. The main result is discussed below.

$$d_1\left(R, \overline{R^P}\right) = 0.08071, \quad d_2\left(R, \overline{R^P}\right) = 0.07127,$$

$$d_3\left(R, \overline{R^P}\right) = 0.06679, d_4\left(R, \overline{R^P}\right) = 0.07027,$$

$$d_5\left(R, \overline{R^P}\right) = 0.09272.$$

Additionally, we illustrate the priority vector of the BCF preference relation, such that

R	<i>C</i> ₁	<i>C</i> ₂	<i>C</i> ₃	<i>C</i> ₄	<i>C</i> ₅
C_1	$(0.1884 + \iota 0.1884,)$	$(0.3170 + \iota 0.1593,)$	$(0.4603 + \iota 0.0588,)$	$(0.6992 + \iota 0.0286,)$	$(0.8506 + \iota 0.0118,)$
	$(-0.1884 - \iota \ 0.1884)$	$(-0.3170 - \iota 0.1593)$	$(-0.4603 - \iota 0.0588)$	$(-0.6992 - \iota 0.0286)$	$(-0.8506 - \iota 0.0118)$
C_2	$(0.0865 + \iota 0.3625,)$	$(0.2748 + \iota 0.2748,)$	$(0.4105 + \iota 0.1111,)$	$(0.6549 + \iota 0.0556,)$	$(0.8229 + \iota 0.0234,)$
	$(-0.0865 - \iota 0.3625)$	$(-0.2748 - \iota 0.2748)$	$(-0.4105 - \iota 0.1111)$	$\sqrt{-0.6549 - \iota 0.0556}$	$(-0.8229 - \iota 0.0234)$
C_3	$(0.0303 + \iota 0.5109,)$	$(0.2 + \iota 0.3625,)$	$(0.1867 + \iota 0.1867,)$	$(0.3849 + \iota 0.0976,)$	$(0.6052 + \iota 0.0423,)$
	$(-0.0303 - \iota 0.5109)$	$(-0.2 - \iota \ 0.3625)$	$(-0.1867 - \iota 0.1867)$	$(-0.3849 - \iota 0.0976)$	$(-0.6052 - \iota 0.0423)$
<i>C</i> ₄	$(0.0167 + \iota 0.76,)$	$(0.2899 + \iota 0.4814,)$	$(0.0976 + \iota 0.3849,)$	$(0.2278 + \iota 0.2278,)$	$(0.4195 + \iota 0.1075,)$
-	$(-0.0167 - \iota 0.76)$	$(-0.2899 - \iota 0.4814)$	$(-0.0976 - \iota 0.3849)$	$(-0.2278 - \iota 0.2278)$	$(-0.4195 - \iota 0.1075)$
C_5	$(0.0069 + \iota 0.8858,)$	$(0.3660 + \iota 0.5767,)$	$(0.0423 + \iota 0.6052,)$	$(0.1075 + \iota 0.4195,)$	$(0.2278 + \iota 0.2278,)$
	$(-0.0069 - \iota 0.8858)$	$(-0.3660 - \iota 0.5767)$	$(-0.0423 - \iota 0.6052)$	$(-0.1075 - \iota 0.4195)$	$(-0.2278 - \iota \ 0.2278)$

TABLE 7. Using Method-1, we get the dominancy BCF preference relation.

TABLE 8. Using method-2, we get the dominancy BCF preference relation.

R	<i>C</i> ₁	<i>C</i> ₂	<i>C</i> ₃	<i>C</i> ₄	<i>C</i> ₅
<i>C</i> ₁	$(0.5 + \iota 0.5,)$	$(0.6 + \iota 0.2,)$	$(0.6 + \iota 0.2,)$	(0.65 + <i>ι</i> 0.25,)	$(0.65 + \iota 0.25,)$
	$(-0.5 - \iota 0.5)$	$(-0.6 - \iota 0.2)$	$(-0.6 - \iota 0.2)$	$(-0.65 - \iota 0.25)$	$(-0.65 - \iota 0.25)$
C_2	$(0.2 + \iota 0.6,)$	$(0.5 + \iota 0.5,)$	$(0.6 + \iota 0.2,)$	(0.65 + ι0.25,)	(0.65 + ι0.25,)
	$(-0.2 - \iota 0.6)$	$(-0.5 - \iota 0.5)$	$(-0.6 - \iota 0.2)$	$(-0.65 - \iota 0.25)$	$(-0.65 - \iota 0.25)$
C_3	$(0.2 + \iota 0.6,)$	$(0.2 + \iota 0.6,)$	(0.5 + ι0.5,)	$(0.6 + \iota 0.2,)$	$(0.6 + \iota 0.2,)$
	$(-0.2 - \iota 0.6)$	$(-0.2 - \iota 0.6)$	$(-0.5 - \iota 0.5)$	$(-0.6 - \iota 0.2)$	$(-0.6 - \iota 0.2)$
C ₄	$(0.25 + \iota 0.65,)$	(0.25 + ι0.65,)	$(0.2 + \iota 0.6,)$	$(0.5 + \iota 0.5,)$	$(0.6 + \iota 0.2,)$
	$(-0.25 - \iota 0.65)$	$(-0.25 - \iota 0.65)$	$(-0.2 - \iota 0.6)$	$(-0.5 - \iota 0.5)$	$(-0.6 - \iota 0.2)$
C_5	(0.25 + ι0.65,)	(0.25 + ι0.65,)	$(0.2 + \iota 0.6,)$	$(0.2 + \iota 0.6,)$	$(0.5 + \iota 0.5,)$
5	(-0.25 - i 0.65)	$(-0.25 - \iota 0.65)$	$(-0.2 - \iota 0.6)$	$(-0.2 - \iota 0.6)$	$(-0.5 - \iota 0.5)$

TABLE 9. Represented the aggregated values.

R	BCFFWA Operator
C_1	$(0.8302 + \iota 0.7182,)$
	$(-0.7130 - \iota 0.1331)$
C_2	$(0.8124 + \iota 0.7367,)$
	$(-0.6358 - \iota 0.2406)$
<i>C</i> ₃	$(0.7672 + \iota 0.7560,)$
	$(-0.4121 - \iota \ 0.3541)$
<i>C</i> ₄	$(0.7522 + \iota 0.7981,)$
	$(-0.2983 - \iota 0.5647)$
C_5	$(0.7402 + \iota 0.8446,)$
	<u>$-0.1999 - \iota 0.4184$</u>

TABLE 10. Represented the score values.

R	BCFFWA Operator
<i>C</i> ₁	0.6755
<i>C</i> ₂	0.6681
<i>C</i> ₃	0.6892
<i>C</i> ₄	0.6718
C_5	0.7416

Further, by using the information of the BCFFWA operator under the availability of the above weight vector, the required aggregated values are illustrated in Table 9.

To finalize the beneficial decision, we use the theory of score values, the main required results are illustrated in Table 10.

Using the information of BCFFWA operators, we get the best decision in the shape of C_5 ..

A. DECISION-MAKING STRATEGY

Decision-making skills can be distinct for illustrating the beneficial decision that enhances the expert industry. The strategy is select the decision is a very complicated task, which needed a leadership quality in very decision-maker who think objectively and critically. The capacity of the decision-maker for selecting the best decision can help to diagnose a strong and dominant concept. The decision-making technique represents your feasibility in selecting among two or more decisions in the form of the opinion of the decision-maker, called alternatives. Decision-making tools are widely utilized in economics, computer science, network systems, and software engineering. Based on the capacity of the decision-making procedure for choosing the beneficial optimal is very fast and many scholars have used it to resolve has problems under the presence of classical information. But one of the most important questions asked by different scholars is what happen if we changed the range of the classical set into a unit interval. To reduce the above dilemmas, in this section we used the finite family of alternatives $\hat{K} = \{\hat{K}_1, \hat{K}_2, \dots, \hat{K}_m\}$ and their attributes $\mathfrak{C} = \{\mathfrak{C}_1, \mathfrak{C}_2, \dots, \mathfrak{C}_n\}$ with $p = (p_1, p_2, \dots, p_n)^t$, representing the weight vectors such that $\sum_{i=1}^n p_i = 1$. For the information, we used the BCF information in the shape: $\acute{K} = (H_{P-\acute{K}}(\mathfrak{b}), H_{N-\acute{K}}(\mathfrak{b})) = (H_{RP-\acute{K}}(\mathfrak{b}) + \iota H_{IP-\acute{K}}(\mathfrak{b}), H_{RN-\acute{K}}(\mathfrak{b}) + \iota H_{IN-\acute{K}}(\mathfrak{b}), h_{RN-\acute{K}}(\mathfrak{b}) + \iota H_{IN-\acute{K}}(\mathfrak{b}), h_{N-\acute{K}}(\mathfrak{b}) = H_{RP-\acute{K}}(\mathfrak{b}) + \iota H_{IP-\acute{K}}(\mathfrak{b}) \text{ and } H_{N-\acute{K}}(\mathfrak{b}) = H_{RN-\acute{K}}(\mathfrak{b}) + \iota H_{IN-\acute{K}}(\mathfrak{b}), habeled the positive$ and negative supporting grades with $H_{RP-K}(\mathfrak{b})$, $H_{IP-K}(\mathfrak{b}) \in [0, 1]$ and $H_{RN-K}(\mathfrak{b})$, $H_{IN-K}(\mathfrak{b}) \in [-1, 0]$. The aim of this analysis is to define a new process using BCF information. Therefore, provide a technique for decision-making tool using BCF information.

Step 1: Compute the DM matrix by assessing the alternative by an expert in the structure of BCFNs. If the given attributes are the benefit type then no need to normalize the DM matrix but if the attributes are cost type then one needs to normalize the DM matrix by the following formula K, as shown at the bottom of the page.

Step 2: After step 1, compute the aggregated values for every BCF preference relation using one of the operators (BCFFWA, BCFFOWA, BCFFWG, and BCFFOWG).

Step 3: Compute the score value so the obtained aggregated values by utilizing the score value described in Def (3).

Step 4: Rank the score values achieved in the last step to illustrate the beneficial alternatives.

B. ILLUSTRATED EXAMPLE

Nowadays, there is one more country pleasant model in China countryside, i.e., the family farm common accommodating. Unlike customary nation pleasing, this family farm model accumulates incredible many segments of place where there are territories from government or town chambers. The chief in this accommodating truly manages this tremendous area of land. What the head needs to do isn't the family farm, yet the pleasing. Consequently, the chief gathers restricted scope farmers to join the enlisted accommodating and uses part of the agrarian workers to work. This kind of pleasing model is immovably related to public power, but it has inconceivable risks in genuine action. Ordinarily, the major plant task adventure of the family farm provincial support is picked by a few community people, whose limit is confined to specific conditions. As such, there exists a couple of risks and weakness in action. At the same time, the undertaking adventure as well as a couple of perils encountering exactly the same thing as the typical regular bet furthermore, the social bet. Before the execution of any major agrarian project, surveying its bet and sort out the most diminished risk is imperative plot. In this section, we use the proposed method based on BCF information is to survey the land adventure risk for the family farm country accommodating. For this, we used four alternatives $\dot{K} = \left\{ \dot{K}_1, \dot{K}_2, \dot{K}_3, \dot{K}_4 \right\}$, represented the external risk, internal risk, social risk and environmental risk and $\mathfrak{C} = {\mathfrak{C}_1, \mathfrak{C}_2, \mathfrak{C}_3, \mathfrak{C}_4}$ be set of four attributes, represented the economic exchange, bad weather, corporation negative impact and poor quality land. Therefore, provide a technique for decision-making tool using BCF information.

Step 1: Compute the objective, attribute, sub-attribute, and alternatives of the decision-making dilemma, and then develop a hierarchy of the selected dilemma, see Table 11.

Step 2: Compute the aggregated values for every BCF preference relation using the BCFFWA, BCFFOWA, BCFFWG, and BCFFOWG operators, see Table 12.

Step 3: Using the theory of score values of all obtained aggregated values, see Table 13.

Step 4: Rank the score values to illustrate the beneficial alternatives, see Table 14.

Noted that the operators explained in Table 15 are given the same result in the shape of K_2 .

C. COMPARATIVE ANALYSIS

Finally, we compared the diagnosed operators with various existing theories to improve the worth of the invented approaches. Enhancing the worth of the diagnosed operators with the help of comparative analysis, for this, we selected some prevailing theories explained in Ref [25], [43]–[45], [16], [17], [58]. The comparative result of the diagnosed operators with selected prevailing theories is explained in Table 15.

Evidently, in the above Table 15, Mesiar *et al.* [25], Mahmood and Ur Rehman [43], Jana *et al.* [16], Wei *et al.* [58], and Jana *et al.* [17] failed to provide any sort of ranking or outcome. Every above-mentioned prevailing theory has its drawbacks which is the reason for its failure, which is mentioned below

- 1. Mesiar *et al.* [25] defined the ordered weighted aggregation operators for BF set theory, but the theory of BF set can deal only with one-dimension information with both opinions. The theory discussed in the presented work can deal with two-dimension information with two opinions, and the theory available in Ref. [25] is not able to evaluate it. Therefore, the operators explained in Ref. [25] are very weak compared with the presented operators.
- 2. Mahmood and Ur Rehman [43] defined the similarity measures for BCF set theory, but the theory of similarity measures cannot deal to evaluate the selected information. The theory discussed in the presented work can deal with two-dimension information, and the theory available in Ref. [43] is not able to evaluate it.

$$\begin{split} \dot{\mathbf{K}} &= \left(\mathbf{H}_{P-\check{\mathbf{K}}}\left(\mathfrak{b} \right), \mathbf{H}_{N-\check{\mathbf{K}}}\left(\mathfrak{b} \right) \right) \\ &= \begin{cases} \left(\mathbf{H}_{RP-\check{\mathbf{K}}}\left(\mathfrak{b} \right) + \iota \ \mathbf{H}_{IP-\check{\mathbf{K}}}\left(\mathfrak{b} \right), \mathbf{H}_{RN-\check{\mathbf{K}}}\left(\mathfrak{b} \right) + \iota \ \mathbf{H}_{IN-\check{\mathbf{K}}}\left(\mathfrak{b} \right) \right) & \text{for benefit criterion} \\ \left(1 - \mathbf{H}_{RP-\check{\mathbf{K}}}\left(\mathfrak{b} \right) + \iota \ \left(1 - \mathbf{H}_{IP-\check{\mathbf{K}}}\left(\mathfrak{b} \right) \right), -1 - \mathbf{H}_{RN-\check{\mathbf{K}}}\left(\mathfrak{b} \right) + \iota \ \left(-1 - \mathbf{H}_{IN-\check{\mathbf{K}}}\left(\mathfrak{b} \right) \right)) & \text{for cost criterion} \end{cases} \end{split}$$

	\mathfrak{C}_1	\mathfrak{C}_2	\mathfrak{C}_3	\mathfrak{C}_4
Ќ1	$(0.21 + \iota 0.11,)$	$(0.57 + \iota 0.42,)$	$(0.14 + \iota 0.19)$	$(0.31 + \iota 0.51,)$
	$(-0.51 - \iota 0.61)$	$(-0.76 - \iota 0.12)$	$(-0.4 - \iota 0.5)$	$(-0.24 - \iota 0.34)$
Ќ ₂	$(0.22 + \iota 0.12,)$	$(0.76 + \iota 0.8)$	$(0.9 + \iota 0.8,)$	$(0.32 + \iota 0.52,)$
	$(-0.52 - \iota 0.62)$	$(-0.2 - \iota 0.4)$	$(-0.2 - \iota 0.3)$	$(-0.25 - \iota 0.35)$
Ќ ₃	$(0.23 + \iota 0.13,)$	$(0.67 + \iota 0.3,)$	(0.57 + ι 0.48,)	$(0.35 + \iota 0.53,)$
	$(-0.53 - \iota 0.63)$	$(-0.39 - \iota 0.8)$	$(-0.49 - \iota 0.93)$	$(-0.37 - \iota 0.36)$
Ќ₄	$(0.24 + \iota 0.14,)$	(0.55 + ι 0.67,)	$(0.29 + \iota 0.5,)$	$(0.27 + \iota 0.54,)$
	<u>$(-0.54 - \iota 0.64)$</u>	$(-0.19 - \iota 0.39)$	$(-0.15 - \iota 0.05)$	$(-0.46 - \iota 0.37)$

TABLE 11. Represented the decision-making information.

 TABLE 12. Represented the aggregated values.

Operators	Ќ ₁	Ќ ₂	Ќ ₃	Ќ4
BCFFWA	$(0.374 + \iota 0.327,)$	(0.705 + <i>ι</i> 0.686,)	$(0.533 + \iota 0.358,)$	$(0.395 + \iota 0.529,)$
	$(-0.283 - \iota 0.509)$	$(-0.252 - \iota 0.4)$	$(-0.436 - \iota 0.711)$	$(-0.255 - \iota 0.264)$
BCFFOWA	(0.379 + ι 0.347,)	(0.678 + ι 0.683,)	(0.537 + ι 0.377,)	$(0.338 + \iota 0.508,)$
	$(-0.491 - \iota 0.275)$	$(-0.245 - \iota 0.391)$	$(-0.428 - \iota 0.693)$	$(-0.256 - \iota 0.189)$
BCFFWG	$(0.305 + \iota 0.275)$	$(0.557 + \iota 0.531,)$	$(0.477 + \iota 0.314)$	$(0.36 + \iota 0.45,)$
	$(-0.58 - \iota 0.369)$	$(-0.282 - \iota 0.422)$	$(-0.443 - \iota 0.79)$	$(-0.307 - \iota 0.377)$
BCFFOWG	$(0.311 + \iota 0.297)$	(0.538 + ι 0.337,)	(0.487 + ι 0.337,)	$(0.316 + \iota 0.458)$
	$(-0.57 - \iota 0.353)$	$(-0.268 - \iota 0.409)$	$(-0.435 - \iota 0.783)$	<u>$(-0.31 - \iota 0.314)$</u>

TABLE 13. Represented the score values.

Operators	Ş _{SF} (K₁)	$S_{SF}(K_2)$	$S_{SF}(K_3)$	$S_{SF}(K_4)$
BCFFWA	0.477	0.685	0.436	0.601
BCFFOWA	0.49	0.681	0.448	0.6
BCFFWG	0.408	0.596	0.39	0.531
BCFFOWG	0.421	0.604	0.401	0.536

TABLE 14. Represented the ranking values.

Operators	Ranking
BCFFWA	$\acute{\mathrm{K}}_2>\acute{\mathrm{K}}_4>\acute{\mathrm{K}}_1>\acute{\mathrm{K}}_2$
BCFFOWA	$\acute{\mathrm{K}}_2>\acute{\mathrm{K}}_4>\acute{\mathrm{K}}_1>\acute{\mathrm{K}}_2$
BCFFWG	$\acute{\mathrm{K}}_2>\acute{\mathrm{K}}_4>\acute{\mathrm{K}}_1>\acute{\mathrm{K}}_2$
BCFFOWG	$\acute{\mathrm{K}}_2>\acute{\mathrm{K}}_4>\acute{\mathrm{K}}_1>\acute{\mathrm{K}}_2$

Therefore, the operators explained in Ref. [43] are very weak compared with the presented operators.

- 3. Jana *et al.* [16] defined Dombi AOs for the theory of BF set, but BF can cope with one-dimension data with both opinions. The theory discussed in the presented work is two-dimension with positive and negative opinions. Thus, the theory available in Ref [16] is not able to evaluate it. Consequently, the operators explained in Ref. [16] are very weak compared with the presented operators.
- 4. Wei *et al.* [58] defined Hamacher AOs for the theory of BF set, but BF can cope with one-dimension data with both opinions. The theory discussed in the presented work is two-dimension with positive and negative opinions. Thus, the theory available in Ref [58] is not able

to evaluate it. Consequently, the operators explained in Ref. [58] are very week is compared with the presented operators

5. Jana *et al.* [17] defined prioritized Dombi AOs for the theory of BF set, but as we know that BF can't deal with the data in two-dimension and with positive and negative opinions. Thus, the theory available in Ref [17] is not able to evaluate it. Therefore, the operators explained in Ref. [17] are very weak compared with the presented operators.

Next, notice from Table 15, that the prevailing operators Hamacher operators for BCF set theory was diagnosed by Mahmood et al. [44], and Dombi operators for BCF set theory were evaluated by Mahmood and Ur Rehman [35]. are providing the results here. Because the theory available in Ref [44] can handle the data in two dimensions with both opinions. Table 15 represented the different ranking results for different operators, the obtained results are in the form of K_2 , K_3 and K_1 . But most operators give their results in the shape of K_2 . Observed from the above analysis, we noticed that the operators based on BCF set theory evaluated in this manuscript are more powerful as compared to Ref [25], [16], [17], [43]–[45], [58]. The biggest advantage of the diagnosed work is to overcome the intricate and tricky data such as two-dimensions data with both positive and negative human opinions. The diagnosed work is also overcome with the data in the setting of the theory of FS, BFS, and CFS. This implies that the diagnosed work is more generalized than the selected prevailing theories. The disadvantage of the proposed work is that it can't handle the data presented in bipolar complex intuitionistic fuzzy, bipolar complex picture fuzzy, etc.

$S_{SF}(K_1)$	Ş _{SF} (K₂)	Ş _{SF} (Ḱ₃)	Ş _{sf} (Ќ₄)	Ranking
Failed	Failed	Failed	Failed	Failed
Failed	Failed	Failed	Failed	Failed
Failed	Failed	Failed	Failed	Failed
Failed	Failed	Failed	Failed	Failed
Failed	Failed	Failed	Failed	Failed
0.483	0.69	0.439	0.606	$\underline{\qquad}\dot{\mathrm{K}}_{2}>\dot{\mathrm{K}}_{4}>\dot{\mathrm{K}}_{1}>\dot{\mathrm{K}}_{3}$
0.517	0.31	0.561	0.394	$\acute{\mathrm{K}}_3>\acute{\mathrm{K}}_1>\acute{\mathrm{K}}_4>\acute{\mathrm{K}}_2$
0.519	0.724	0.46	0.651	$\acute{\mathrm{K}}_2>\acute{\mathrm{K}}_4>\acute{\mathrm{K}}_1>\acute{\mathrm{K}}_3$
0.409	0.214	0.395	0.324	$\acute{\mathrm{K}}_1>\acute{\mathrm{K}}_3>\acute{\mathrm{K}}_4>\acute{\mathrm{K}}_2$
0.477	0.685	0.436	0.601	$\acute{\mathrm{K}}_2>\acute{\mathrm{K}}_4>\acute{\mathrm{K}}_1>\acute{\mathrm{K}}_3$
0.49	0.681	0.448	0.6	$\acute{\mathrm{K}}_2>\acute{\mathrm{K}}_4>\acute{\mathrm{K}}_1>\acute{\mathrm{K}}_3$
0.408	0.596	0.39	0.531	$\acute{\mathrm{K}}_2>\acute{\mathrm{K}}_4>\acute{\mathrm{K}}_1>\acute{\mathrm{K}}_3$
0.421	0.604	0.401	0.536	$\acute{\mathrm{K}}_2>\acute{\mathrm{K}}_4>\acute{\mathrm{K}}_1>\acute{\mathrm{K}}_3$
	$\frac{S_{SF}(K_1)}{Failed}$ Failed Failed Failed Failed 0.483 0.517 0.519 0.409 0.477 0.49 0.49 0.408 0.421	$S_{SF}(K_1)$ $S_{SF}(K_2)$ Failed Failed 0.483 0.69 0.517 0.31 0.519 0.724 0.409 0.214 0.477 0.685 0.49 0.681 0.408 0.596 0.421 0.604	$S_{SF}(K_1)$ $S_{SF}(K_2)$ $S_{SF}(K_3)$ Failed0.4830.690.4390.5170.310.5610.5190.7240.460.4090.2140.3950.4770.6850.4360.490.6810.4480.4080.5960.390.4210.6040.401	$S_{SF}(K_1)$ $S_{SF}(K_2)$ $S_{SF}(K_3)$ $S_{SF}(K_4)$ Failed0.4830.690.4390.6060.5170.310.5610.3940.5190.7240.460.6510.4090.2140.3950.3240.4770.6850.4360.6010.490.6810.4480.60.4080.5960.390.5310.4210.6040.4010.536

TABLE 15. Represented the comparative analysis.

VII. CONCLUSION

Because of the advantages and significance of Frank t-norm and t-conorm and the theory of the BCF set, in the given analysis, we combined these two conceptions to interpret some new AOs. For this, firstly, we defined Frank operational laws and their influential results employing BCF information. After that, We diagnosed the AOs using Frank t-norm and Frank t-conorm such as BCFFWA, BCFFOWA, BCFFHWA, BCFFWG, BCFFOWG, and BCFFHWG operators and evaluated certain properties and results. Furthermore, We discovered and prioritized the major factors that positively influence the DevOps technique in software organizations. In the presence of the prevailing information, nineteen factors were diagnosed. The diagnosed features were moreover authorized by intellectuals via hypothetical information. After that, we developed an AHP method for the structure of the BCF set, and then we also developed a DM technique for the BCF set. We solved a numerical example by AHP and DM techniques. BCFAHP technique is invented to prioritize the classification success features. The final ranking mentioned that "DevOps security pipeline", "use system orchestration", and "attempt matrix organization and transparency" features are the beneficial ranked success features for the valuable utilization of DevOps techniques. At least, we compared the diagnosed operators with various existing theories to improve the worth of the invented approaches. By comparative study, we noticed that the majority of the selected prevailing theories failed to give any sort of result. The diagnosed operators and methods are generalized and can handle the information in the structure of FS, BFS, and CFS. In the future, we aim to expand our work in different domains such as complex hesitant fuzzy sets [61, 62], picture fuzzy settings [61], spherical fuzzy sets using Einstein aggregation operators [62], and complex spherical fuzzy [63].

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