IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received 16 June 2022, accepted 28 June 2022, date of publication 12 July 2022, date of current version 20 July 2022.
Digital Object Identifier 10.1109/ACCESS.2022.3189989

RESEARCH ARTICLE

Design of Multihop Communication Systems Under Hardware Manufacturing Defects and Self Interference

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This work was supported by the Deanship of Scientific Research, Taif University Researchers Supporting Project, Taif University, Taif, Saudi Arabia, under Grant TURSP-2020/214.

ABSTRACT Most studies on multi-hop communication systems are actually based on the assumption that hardware parts are built with high quality transmit/receive radio frequency chains. In practice, low cost hardware components are employed, which are prone to hardware manufacturing defects. This paper focuses on the design of the transmitter, relay stations (RSs) and receiver for multi-hop communication systems under hardware defects. This paper analyzes the hardware defects and their impacts on the multi-hop communication system performance. In addition, the impact of self-interference at the RSs is considered. The effect of hardware defects is modeled using distortion noises. A closed-form expression for the signal to noise and distortion ratios (SNDRs) is mathematically derived, and then the performance metrics (i.e., SNDR, outage probability (OP) and ergodic capacity) are calculated, accounting for hardware defects. In addition, exact form expression of the average bit error probability is derived. To facilitate comparison, performance metrics of the proposed multi-hop relaying system and ideal system are illustrated. We also propose a linear minimum mean square error (LMMSE) and self-interference cancellation technique to mitigate the hardware defects. The results reveal that hardware defects can degrade the system performance. In addition, SNDR ceiling is inversely proportional to the summation of the square of hardware defect level. Also, the proposed mitigation technique can enhance the system performance.

INDEX TERMS Multi-hop system, hardware defects, self-interference, outage, ergodic capacity, bit error probability.

I. INTRODUCTION

A. BACKGROUND

Transmission-based multi-hop becomes an effective scheme to obtain wider coverage and overcome wireless channels impairment. The primary goal is to verify the communication via relaying the data from the transmitter to receiver through several relays. Multi-hop communication systems have several features over classical communication systems like capacity, deployment, and connectivity, enhanced coverage, throughput, and power/battery life [1]–[2].

Relaying schemes enable network connectivity where classical architectures are difficult to implement because of constraints of the location, and may also be implemented to wireless local area networks and cellular networks. With multi-hop schemes, the transmitter can communicate with receiver via relay stations (RSs). So, multi-hop systems can extend the coverage without applying higher transmission power.

The utilization of RSs for enhancing reliability, coverage, and quality of service in wireless communication systems

The associate editor coordinating the review of this manuscript and approving it for publication was Kathiravan Srinivasan.

interesting topic in recent years, both in industry and in research. Unlike macro base stations, RSs can be easily implemented with low cost and, hence, enhance the agility of the network. The most studies in relaying schemes consider ideal transceiver hardware [1].

Decode-and-forward (DF) and amplify-and-forward (AF) RSs are the utmost common relaying techniques. The DF strategy decodes, re-modulates and resends the received signal, while AF strategy amplifies and resends the received signal without decoding process. In addition, AF RSs can receive and transmit without delay in the transmissions.

The complexity of DF RS is higher than AF RS because of its full processing capability. It is nearly as complex as a base station. In addition, a sophisticated media access control layer is required for the protocol of DF, while in the case of AF protocol, this layer is unnecessary [3]–[10].

The study of multi-hop MIMO relay system in the case of AF and DF protocols is illustrated. For N-hop channel of linear topology with many antenna at the transmitter and receiver and RSs with single antenna, the capacity gain of the DF over the AF does not exceed $\log_2 N$ bit/s/Hz. Also, for any channel realization, signal-to-noise ratio gain does not exceed number of hops.

A fixed DF protocol cannot give diversity [2]. But, the adaptive DF protocol can provide a second order diversity in the high SNR region. In this protocol, the transmitter utilizes either the transmitter-RS channel state information (CSI) or the feedback from RS to choose among retransmitting the data or allowing the RS to send the data. On the other hand, fixed AF RS, in which the RS simply amplifies and sends the data at all times, does achieve diversity.

Cooperative diversity concept is studied, where mobile users can cooperate to extract the spatial diversity advantages without applying physical antenna arrays. In general, networks-based cooperative strategies are multi-hop, where the receiver collects the received signals from the transmitter and RSs [6]–[9].

Researches on multi-hop communication systems consider that the hardware parts are designed with high quality transmit/receive radio frequency chains of high cost and power-hungry.

Practically, hardware parts with bad manufacturing quality can cause nonlinearities of high power amplifier (HPA), imbalance of in and quadrature (I/Q) phase component and phase noise.

B. MOTIVATION AND LITERATURE SURVEY

Multiple-input-multiple-output (MIMO) communication systems were studied in the presence of transceiver with hardware defects [1], [2].

The study of different hardware defects on different communication systems were illustrated in [3]–[13]. I/Q imbalance was illustrated. The I/Q imbalance effect decreases the amplitude, rotates the desired constellation phase and causes floor in the symbol error rate. In [3], a dual hop AF cooperative system under I/Q imbalance was studied. In order to detect the transmitted signal, a compensation algorithm employs the signals and their conjugations at the transmitter, at the RSs and at the receiver.

The half-duplex AF outage performance in orthogonal frequency division multiplexing (OFDM) with I/Q imbalance has been explained. It is showed that the direct mode can give better performance than AF mode with I/Q imbalance [4]. Also, the levels of I/Q imbalance is proportional to $(1/\text{signal constellation size})^3$.

The direct conversion beamforming OFDM transceivers and joint transmit-receive I/Q imbalance impacts have been investigated in [5]. In this study, the average subcarrier signal to interference and noise ratio (SINR) depends on I/Q imbalance level, the beamforming array size and the input level of SNR.

In [6], the impact of hardware impairments on dual hop relaying system is studied. The performance loss is small at low rates. In addition, at high SNR, signal to noise and distortion ratio (SNDR) approaches to a constant, which is called SNDR ceiling and proportional to $(1/\text{level of impairments})$.

Another study of AF full-duplex (FD) relay transceivers with I/Q imbalance were explained in [7]. The authors determine the relay power amplifier with optimal input back-off in order to maximize the bit error rate. The distortion of power amplifier is the dominant hardware impairments at high SNR. Also, the authors in [8] studied the massive MIMO FD relaying, where transmitter and receiver multiple pairs can simultaneously communicate with a common relay station. The effect of hardware impairments has been modeled as transmit-receive distortion noises. Also, transceiver scheme in the case of hardware impairments and with low complexity has been introduced to reduce effect of noises of distortion by obtaining the statistical information of channels and antenna arrays at transmitters and receivers [9].

In addition, the problem of I/Q imbalance and residual loopback self-interference has been studied for dual-hop full-duplex OFDM system [10], which shows outage performance depends on I/Q level and self-interference average power level. The hardware imperfections can greatly affect the system performance of two-way satellite terrestrial relay network when the impairment level is larger [12]. Energy detectors with single and multi channel under I/Q imbalance for cooperative and non-cooperative spectrum sensing have been quantified [13]. I/Q imbalance can also affect the spectrum sensing accuracy in cognitive radio network. Also, error of spectrum sensing can increase, leading to a negative effect on the performance of primary network.

Recently, researchers have tended to find ways to overcome such problems. Hardware defects are mitigated by compensation techniques, but there are always residual defects [5], [6]. There are not enough studies to study this residual effect. Some methods have been used to overcome the hardware defects. Results showed that hardware defects decrease the system performance [3]–[13].

On contrary, multi-hop communication systems have attracted attention recently because it can improve transmission reliability, increase quality of service, mitigate wireless channels impairment and increase coverage area. The data transmission is provided by sending the data from the transmitter to the receiver with the help of RSs.

There are limitations in the performance of multi-hop systems due to practical issues, including hardware manufacturing defects, signal processing complexity, propagation environment and accuracy of the channel estimation.

Various types of multi-hop communication systems have been investigated [14]–[19]. The wireless transmission for internet of things under Nakagami channels has been studied. The caching wireless transmission with direct link and multi-hop relaying from the transmitter to the receiver has been considered [14]. Another study that shows the impact of high power amplifiers and phase noise issues on the performance of multi-pair massive antenna relaying communication is investigated in [15]. The impact of hardware issues are illustrated at transmitters, relay station, and receivers. Hardware problems can reduce system performance.

In addition, a relay selection scheme based MIMO system for reliable multi-hop transmission has been studied [16]. This technique applies a MIMO channel matrix between the transmitters and relays to choose a suitable relay station. It can obtain the singular values of channel matrix through the singular value decomposition. After that, the sum of the singular values is determined and the relay station having the largest values has been chosen.

Also, one way and multiple relay stations with optimal training design and maximum likelihood channel estimation has been studied [17]. The destination can estimate the residual self-interference (RSI) channel plus the channel of the relay system, to overcome the RSI using equalization.

In [19], multi-hop relay network was illustrated, where two users can communicate with each other through a set of clusters of RSs. A set of protocols have been proposed to improve the symbol error probability performance. A multi-hop MIMO relay network under self interference has been demonstrated in [20].

In order to enhance the coverage range and signal transmission, hybrid beamforming technique for multi-hop reconfigurable intelligent surface-aided communication system was studied [11]. In this study, many passive and controllable reconfigurable intelligent surfaces are used to aid the transmissions between the BS and multiple single-antenna receivers. The matrices design at the BS and the reconfigurable intelligent surfaces are calculated. This technique can improve 50% more coverage range of communications compared with the classical techniques.

Another work that illustrates a communication system with intelligent reflecting surface is explained, where many intelligent reflecting surfaces aid in the communication between BS and a remote receiver via multi-hop transmission [18]. Also, a multi-hop cascaded direct path between the BS and receiver is set up where a group of intelligent reflecting surfaces are chosen to successively reflect the signal from BS. By doing this, the power of the received signal is maximized at the receiver. The optimal active and passive beamforming at the BS and intelligent reflecting surface are investigated.

However, none of these works studied the impact of hardware defects in multi-hop communication systems. Motivated by the above works, we study the performance of multi-hop communication systems under hardware defects and self-interference.

C. CONTRIBUTIONS

In this paper, a multi-hop communication system under hardware manufacturing defects is proposed. Data transmission is done through multiple full-duplex AF (FD-AF) RSs. In addition, RSs under hardware defects and self interference in the proposed system is considered. To the best of our information, there is no previous studies that illustrate the impact of hardware defects and self interference in the multi-hop communication system. The contributions may be arranged as follows:

- A multi-hop communication system in the presence of hardware manufacturing defects and self interference at different transmission hops number, N is proposed.
- The outage performances of the proposed system and ideal system are compared, which illustrate that the SNDR ceiling is proportional to $1/\sum_{i=1}^{N-1}\kappa_i^2$, where κ is the hardware defect level.
- Mathematical formulas of the performance metrics (i.e., SNDR, outage probability, ergodic capacity and average bit error probability) are demonstrated in approximated form for the proposed system and compared with the ideal system. We also characterize the impact of hardware defects to any value of SNR. The proposed system outage probability and capacity bounds are obtained.
- The fundamental impact of hardware defects is illustrated when SNR approaches to infinity. The ceiling of the proposed system capacity is also calculated due to hardware defects and show that capacity is inversely proportional to the defect level of hardware manufacturing.
- In addition, a mitigation technique based on LMMSE and self-interference cancellation to overcome the hardware defects is also proposed. This technique can improve system performance.
- Guidelines for designing the proposed system for optimal performance are explained mathematically.

D. ORGANIZATION

The paper is organized as follows. The multi-hop communication system under hardware manufacturing defects and self interference is defined in Section II. The performance matrices for the proposed system are mathematically derived in Section III. In Section IV, the fundamental impact of hardware defects is presented. The proposed mitigation technique to overcome these defects is described in Section V. Results are demonstrated in Section VI. Finally, the conclusion is provided in Section VII.

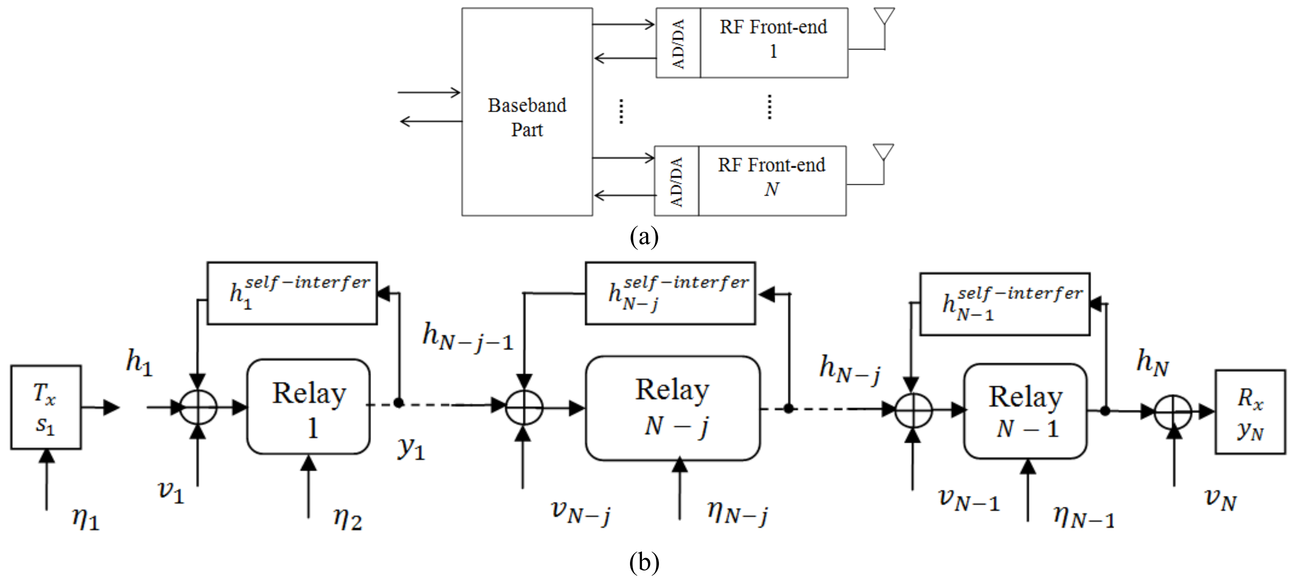

FIGURE 1. (a) The physical structure of RF and baseband for a general transceiver [7] and (b) The proposed multi-hop communication system under hardware defects and self-interference.

E. NOTATION

The symbol $x \sim CN(a, b)$ represents a circularly symmetric complex Gaussian distributed variables, where (a, b) denotes the values of mean and the variance, respectively. Gamma distributed parameters are referred as $\rho \sim \text{Gamma}(\alpha, \beta)$, where α and β are the parameters of shape and the scale, respectively. The expectation operator is referred to as $\mathbb{E}\{\cdot\}$. The operator $\triangleq$ denotes a definition. The symbols $\mathcal{L}^{-1}(\cdot)$, $\mathrm{M}(\cdot)$, $K_m(\cdot)$, $G[\cdot]$, $\operatorname{erfc}(\cdot)$ and $f_\gamma(\gamma)$ denote the inverse Laplace transform, the moment generating function (MGF), the *mth*-order modified Bessel function, the Meijer G-function, the complementary error function and the probability density functions (PDF) of γ, respectively.

II. THE PROPOSED MULT-HOP COMMUNICATION SYSTEM UNDER HARDWARE MANUFACTURING DEFECTS AND SELF INTERFERENCE

The application of a wireless communication transceiver using MIMO-concept is illustrated in Fig. 1(a). The figure depicts the physical layer of the system [7].

The transmitter baseband part modulates the signals for transmitting. The receiver front- and down-end amplifies converts the signals to the digital baseband.

A general multi-hop communication system with multiple FD-AF RSs is depicted in Fig. 1(b). The system contains many RSs that forward the data. We consider N-transmission hops over i.n.i.d. Nakagami-m fading channels. The transmitter sends its data signal $\mathbf{s} \in \mathbb{C}$ to the receiver with an average signal power $P = \mathbb{E}_{\mathbf{s}}\left\{|\mathbf{s}|^2\right\}$ through $N-1$ nodes terminals $R_1, R_2, \ldots, R_{N-1}$. Assume that the self-interference at each RS is Gaussian, $\mathbf{h}_i^{self-interfer} \sim CN\left(0, N_i^{self-interfer}\right)$ [23]. Also, assume that the receiver noise is i.i.d complex Gaussian noise, $\boldsymbol{v}_i \sim CN(0, N_i), i = 1, 2, \ldots, N$.

Let $\mathbf{h} \in \mathbb{C}$ represents the channel fading and $\boldsymbol{v} \in \mathbb{C}$ represents the additive noise. The transceiver chains of the proposed multi-hop communication system are transmitter hardware at the source, receiver and the relay station, and receiver hardware at the destination. Radio-frequency (RF) transceivers system model describes the receiving signal with the problem of hardware as [6]–[8]:

$$\mathbf{y} = \mathbf{h}(\mathbf{s} + \eta_t) + \eta_r + \boldsymbol{v}. \tag{1}$$

where the parameters η_t and η_r are the distortion noises due to manufacturing defects at the transmitter and receiver [29], η_t satisfy $CN\left(0, \kappa_t^2 P\right)$ and η_r satisfy $CN\left(0, \kappa_r^2 P|h|^2\right)$.

This distortion noise is an unknown noise, like interference signal. The overall distortion noise at the receiving part has power of, $\mathbb{E}_{\eta_t,\eta_r}\left\{|\mathbf{h}\eta_t + \eta_r|^2\right\} = P|\mathbf{h}|^2\left(\kappa_t^2 + \kappa_r^2\right)$.

This distortion is based on the mean signal power $P = \mathbb{E}_s\left\{|s|^2\right\}$ and the instantaneous channel gain $|\mathbf{h}|^2$ [30].

The parameters, κ_t and κ_r evaluate the defect level of the impairments. Let $\kappa = \sqrt{\kappa_t^2 + \kappa_r^2}$ refers to the overall level of manufacturing defects, without taking the transmitter and the receiver defect levels. The received signal with hardware manufacturing defects, is equivalent to

$$\mathbf{y} = \mathbf{h}(s + \eta) + \boldsymbol{v}. \tag{2}$$

where $\eta \sim CN\left(0, \kappa^2 P\right)$ and $P = \mathbb{E}\left\{|s|^2\right\}$ are the independent distortion noise and the average signal power, respectively. The parameter κ represents the hardware manufacturing effect on the proposed system performance. When its value equals to zero (i.e., $\kappa = 0$), the proposed system is in its ideal case.

The noise of distortion from hardware impairments represents an unknown noise η that enters the channel $\mathbf{h}_i$.

The channel amplitudes $|\mathbf{h}_i|, i = 1, 2, \ldots, N$ can be described as non-identically and independent distributed Nakagami-m fading, with shape and scale parameters $\alpha_i \geq 1$ and $\beta_i > 0$. The channel gains are $\rho_i = \mathbb{E}\left\{|\mathbf{h}_i|^2\right\}$. The channel gains probability density functions (PDFs) and cumulative distribution functions (CDFs), ρ_i are [24], [25]:

$$f_{\rho_i}(x) = \frac{x^{\alpha_i - 1} e^{\frac{-x}{\beta_i}}}{\Gamma(\alpha_i) \beta_i^{\alpha_i}}, \quad x \geq 0 \tag{3}$$

$$F_{\rho_i}(x) = 1 - \sum_{j=0}^{\alpha_i - 1} \frac{e^{-\frac{x}{\beta_i}}}{j!} \left(\frac{x}{\beta_i}\right)^j, \quad x \geq 0 \tag{4}$$

For any distribution of fading, the quantity $SNR_i = P_i \mathbb{E}_{\rho_i}\{\rho_i\}/N_i$ denotes the mean SNR. Let $\mathbb{E}_{\rho_i}\{\rho_i\} = \alpha_i \beta_i$ refers to the average fading power for Nakagami-m fading case. In this paper, the impact of the hardware manufacturing defects at the transmitter, RSs and receiver is the main focus, since it employs lower quality hardware. Also, a general case where the RS suffers from both hardware defects and self-interference is considered. To derive a general form of the SNDR for the proposed system under hardware defects as in Fig. 1(b), first the end-to-end SNR at the receiver is calculated. The model in (2) describes the hardware defects.

A. RECEIVED SIGNAL ANALYSIS AT RS $(N - j)$

At the RS $(N - j)$ as in Fig. 1(b), the received signal is the sum of the transmitted signal, the self interference and the noise, which is expressed by:

$$\begin{aligned} &\mathbf{y}_{N-j,input}[i] \\ &= \mathbf{h}_{N-j}\left(\mathbf{y}_{N-j-1}[i] + \eta_{N-j-1}[i]\right) + \mathbf{h}_{N-j}^{self-interfer} \mathbf{y}_{N-j}[i] \\ &\quad + \mathbf{v}_{N-j}[i]. \end{aligned} \tag{5}$$

where $\mathbf{y}_{N-j-1}$ is the received signal from the previous $(N - j - 1)$ RS, $\left|\mathbf{h}_{N-j}\right|, j = 0, 1, \ldots, N - 1$ represents the channel amplitudes of non-identically and independent distributed Nakagami-m fading, $\eta_{N-j-1} \sim CN\left(0, \kappa_{N-j-1}^2 P_{N-j-1}\right)$ is the distortion noise from the previous $(N - j - 1)$ RS as in Fig. 1(b), $\mathbf{h}_{N-j}^{self-interfer}$ denotes the residual self-interference and $\mathbf{v}_{N-j} \sim CN\left(0, N_{N-j}\right), j = 0, 1, \ldots, N - 1$ represents the i.i.d complex Gaussian noise. The RS in the full-duplex mode receives the signal $\mathbf{y}_{N-j,input}[i]$ and transmits the signal $\mathbf{y}_{N-j}[i]$ on the same frequency.

Then, RS at $(N - j)$ amplifies the input signal $\mathbf{y}_{N-j,input}$ in (5) by amplification factor G_{N-j}. After the amplification process, the transmitted signal from RS at $(N - j)$ with an average power P_{N-j} is:

$$\begin{aligned} \mathbf{y}_{N-j}[i] = G_{N-j} \sum_{j=1}^{\infty} &\left(G_{N-j} \mathbf{h}_{N-j}^{self-interfer}\right)^{j-1} \\ &\times\left(\mathbf{h}_{N-j} \mathbf{y}_{N-j-1}[i - j\tau] + \mathbf{h}_{N-j} \eta_{N-j-1}[i - j\tau]\right. \\ &\left.+ \mathbf{v}_{N-j}[i - j\tau]\right) + \eta_{N-j}[i]. \end{aligned} \tag{6}$$

where G_{N-j} is the amplification factor of RS at $(N - j)$. The amplification factor G_{N-j}, is chosen at $(N - j)$ RS to realize its power constraint. $\eta_{N-j} \sim CN\left(0, \kappa_{N-j}^2 P_{N-j}\right)$ is the distortion noise at $(N - j)$ RS. The average signal power of the $(N - j)$ RS after making the expectation process of (6) is:

$$\begin{aligned} \mathbb{E}\left\{\left|\mathbf{y}_{N-j}\right|^2\right\} = G_{N-j}^2 &\frac{\rho_{N-j}\left(P_{N-j-1} + \kappa_{N-j-1}^2\right) + N_{N-j}}{1 - N_{N-j}^{self-interfer} G_{N-j}^2} \\ &+ \kappa_{N-j}^2 P_{N-j} \end{aligned} \tag{7}$$

B. RECEIVED SIGNAL ANALYSIS AT RS $(N - j + 1)$

For the next $(N - j + 1)$ RS as in Fig. 1(b), the input signal at this RS is calculated as:

$$\begin{aligned} \mathbf{y}_{N-j+1,input}[i] = \mathbf{h}_{N-j+1}&\left(\mathbf{y}_{N-j}[i] + \eta_{N-j}[i]\right) \\ &+ \mathbf{h}_{N-j+1}^{self-interfer} \mathbf{y}_{N-j+1}[i] + \mathbf{v}_{N-j+1}[i]. \end{aligned} \tag{8}$$

where $\mathbf{y}_{N-j}$ is the received signal from the $(N - j)$ RS as in (6) and $\left|\mathbf{h}_{N-j+1}\right|, j = 0, 1, \ldots, N - 1$ is the non-identically and independent distributed Nakagami-m fading channel magnitudes. The channel gains are $\rho_{N-j+1} = \mathbb{E}\left\{\left|\mathbf{h}_{N-j+1}\right|^2\right\}$. After that, the $(N - j + 1)$ RS amplifies the input signal $\mathbf{y}_{N-j+1,input}$ and induces a processing delay τ, as follows:

$$\mathbf{y}_{N-j+1}[i] = G_{N-j+1} \mathbf{y}_{N-j+1,input}[i - \tau]. \tag{9}$$

where G_{N-j+1} is the amplification factor of $(N - j + 1)$ RS. The total received signal from $(N - j)$ RS to the next $(N - j + 1)$ RS and after plugging (6) and (9), is:

$$\begin{aligned} &\mathbf{y}_{N-j+1}[i] \\ &= G_{N-j} G_{N-j+1} \sum_{j=1}^{\infty} \begin{pmatrix} G_{N-j} G_{N-j+1} \mathbf{h}_{N-j}^{self-interfer} \\ \mathbf{h}_{N-j+1}^{self-interfer} \end{pmatrix}^{j-1} \\ &\quad \times \left(\left(\mathbf{h}_{N-j} \mathbf{h}_{N-j+1} \mathbf{y}_{N-j}[i - 2j\tau]\right.\right. \\ &\quad + \mathbf{h}_{N-j} \mathbf{h}_{N-j+1} \eta_{N-j}[i - 2j\tau] \\ &\quad \left.+ \mathbf{h}_{N-j+1} \mathbf{v}_{N-j}[i - 2j\tau]\right) + \mathbf{h}_{N-j+1} \eta_{N-j+1}[i - j\tau] \\ &\quad \left.+ \mathbf{v}_{N-j+1}[i - j\tau]\right) + \eta_{N-j+1}[i] \end{aligned} \tag{10}$$

where $\eta_{N-j+1} \sim CN\left(0, \kappa_{N-j+1}^2 P_{N-j+1}\right)$ is the distortion noise of $(N - j + 1)$ RS. The parameters $\kappa_{N-j+1}, j = 0, 1, \ldots, N$ characterize the hardware defect level for the proposed system. Then, the total received signal from the first RS to the $(N - j + 1)$ RS and after arranging terms is:

$$\begin{aligned} &\mathbf{y}_{N-j+1}[i] \\ &= G_1 \ldots G_{N-j+1} \sum_{j=1}^{\infty} \begin{pmatrix} G_1 \ldots G_{N-j+1} \mathbf{h}_1^{self-interfer} \\ \ldots \mathbf{h}_{N-j+1}^{self-interfer} \end{pmatrix}^{j-1} \\ &\quad \times \left(\left(\mathbf{h}_1 \ldots \mathbf{h}_{N-j+1} \mathbf{y}_{N-j}[i - 2j\tau]\right.\right. \\ &\quad + \mathbf{h}_1 \ldots \mathbf{h}_{N-j+1} \eta_{N-j}[i - 2j\tau] \\ &\quad \left.+ \mathbf{h}_2 \ldots \mathbf{h}_{N-j+1} \mathbf{v}_{N-j}[i - 2j\tau]\right) \end{aligned}$$

$$+\mathbf{h}_2 \ldots \mathbf{h}_{N-j+1}\eta_{N-j+1}[i-j\tau] + \boldsymbol{\nu}_{N-j+1}[i-j\tau]\big) + \eta_{N-j+1}[i] \quad (11)$$

C. RECEIVED SIGNAL ANALYSIS AT RS $(N-1)$

After that, the received signal at the last $(N-1)$ RS as in Fig. 1(b) is:

$$\mathbf{y}_{N-1,input}[i] = \mathbf{h}_{N-1}(\mathbf{y}_{N-2}[i] + \eta_{N-2}[i]) + \mathbf{h}_{N-1}^{self-interfer}\mathbf{y}_{N-1}[i] + \boldsymbol{\nu}_{N-1}[i]. \quad (12)$$

The delayed transmitted signal of the $(N-1)$ RS after the amplification process is given by:

$$\begin{aligned}\mathbf{y}_{N-1}[i] &= G_1 \ldots G_{N-j} \ldots G_{N-1} \times \sum_{j=1}^{\infty}\begin{pmatrix} G_1\mathbf{h}_1^{self-interfer} \ldots G_{N-j}\mathbf{h}_{N-j}^{self-interfer} \\ \ldots G_{N-1}\mathbf{h}_{N-1}^{self-interfer}\end{pmatrix}^{j-1} \\ &\times \big((\mathbf{h}_1 \ldots \mathbf{h}_{N-j} \ldots \mathbf{h}_{N-1}s[i-j\tau] \\ &+ \mathbf{h}_1 \ldots \mathbf{h}_{N-j} \ldots \mathbf{h}_{N-1}\eta_1[i-2j\tau] \\ &+ \mathbf{h}_2 \ldots \mathbf{h}_{N-j} \ldots \mathbf{h}_{N-1}\boldsymbol{\nu}_1[i-j\tau] \\ &+ \mathbf{h}_2 \ldots \mathbf{h}_{N-j} \ldots \mathbf{h}_{N-1}\eta_2[i-j\tau] \\ &+ \mathbf{h}_3 \ldots \mathbf{h}_{N-j} \ldots \mathbf{h}_{N-1}\boldsymbol{\nu}_2[i-j\tau] \\ &+ \ldots + \mathbf{h}_{N-1}\eta_{N-2}[i-2j\tau] \\ &+ \mathbf{h}_{N-1}\boldsymbol{\nu}_{N-2}[i-j\tau] + \boldsymbol{\nu}_{N-1}[i-j\tau])\big) + \eta_{N-1}[i]\end{aligned} \quad (13)$$

The index $N-j$ lies in the range 1 and $N-1$, then the above equation can be rewritten as:

$$\begin{aligned}\mathbf{y}_{N-1}[i] &= G_1 \ldots G_{N-1}\sum_{j=1}^{\infty}\Big(G_1\mathbf{h}_1^{self-interfer} \ldots G_{N-1}\mathbf{h}_{N-1}^{self-interfer}\Big)^{j-1} \\ &\times ((\mathbf{h}_1 \ldots \mathbf{h}_{N-1}s[i-j\tau] + \mathbf{h}_1 \ldots \mathbf{h}_{N-1}\eta_1[i-j\tau] \\ &+ \mathbf{h}_2 \ldots \mathbf{h}_{N-1}\nu_1[i-j\tau]) \\ &+ \mathbf{h}_2 \ldots \mathbf{h}_{N-1}\eta_2[i-j\tau] \\ &+ \mathbf{h}_3 \ldots \mathbf{h}_{N-1}\boldsymbol{\nu}_2[i-j\tau] + \ldots + \mathbf{h}_{N-1}\eta_{N-2}[i-2j\tau] \\ &+ \mathbf{h}_{N-1}\boldsymbol{\nu}_{N-2}[i-j\tau] + \boldsymbol{\nu}_{N-1}[i-j\tau]) + \eta_{N-1}[i]\end{aligned} \quad (14)$$

Finally, the received signal at the receiver part as in Fig. 1(b) after N-hops is given by:

$$\mathbf{y}_N[i] = \mathbf{h}_N\mathbf{y}_{N-1}[i] + \boldsymbol{\nu}_N[i]. \quad (15)$$

Plugging (14) into (15), the received signal after N-hops is:

$$\begin{aligned}\mathbf{y}_N[i] &= G_1G_2 \ldots G_{N-1}\sum_{j=1}^{\infty}\begin{pmatrix} G_1G_2h_1^{self-interfer}\mathbf{h}_2^{self-interfer} \\ \ldots G_{N-1}\mathbf{h}_{N-1}^{self-interfer}\end{pmatrix}^{j-1} \\ &\times (\mathbf{h}_1 \ldots \mathbf{h}_N s[i-j\tau] + \mathbf{h}_1\mathbf{h}_2 \ldots \mathbf{h}_N\eta_1[i-j\tau] \\ &+ \mathbf{h}_2\mathbf{h}_3 \ldots \mathbf{h}_N\boldsymbol{\nu}_1[i-j\tau] + \mathbf{h}_2\mathbf{h}_3 \ldots \mathbf{h}_N\eta_2[i-j\tau] \\ &+ \mathbf{h}_3 \ldots \mathbf{h}_N\boldsymbol{\nu}_2[i-j\tau] + \ldots + \mathbf{h}_{N-1}\mathbf{h}_N\eta_{N-2}[i-2j\tau] \\ &+ \mathbf{h}_{N-1}\mathbf{h}_N\boldsymbol{\nu}_{N-2}[i-j\tau] + \mathbf{h}_N\eta_{N-1}[i-j\tau] \\ &+ \mathbf{h}_N\boldsymbol{\nu}_{N-1}[i-j\tau]) + \boldsymbol{\nu}_N[i]\end{aligned} \quad (16)$$

After obtaining the final expression of the received signal after N-hops, we use it to derive a closed form of signal-to-noise and distortion ratios (SNDRs) for the proposed multi-hop communication system under hardware defects and self-interference as illustrated in the following ***Theorem* 1**.

Theorem 1: A closed form end-to-end SNDR expression for the proposed N-hop communication system under hardware manufacturing defects and self interference is given by:

$$\gamma_{non,Nhops}^{FD-AF} = \frac{\prod_{i=1}^{N}\rho_i}{\sum_{i=1}^{N-1}\Big(a_i\prod_{i=j-1}^{N}\rho_i\Big) + \sum_{i=1}^{N-1}\Big(b_i\prod_{i=j}^{N}\rho_i\Big) + c_i} \quad \text{for } j = 2, \ldots, N-1 \quad (17)$$

and the coefficients in (17) a_i, b_i and c_i are given by:

$$-a_i = \kappa_i^2 P_i/P_1 \quad (18)$$

$$-b_i = N_i/P_1 \quad (19)$$

$$-c_i = N_N^{noise}\left(1 - \prod_{i=0}^{N-2}G_{i+1}^2 N_{i+1}^{self-interfer}\right)\Big/ P\prod_{i=0}^{N-2} \times G_{i+1}^2 \quad (20)$$

where ρ_i denotes the channel gains of the ith-hop, N_i denotes the Gaussian noise of the ith-hop, N_N^{noise} is the i.i.d complex Gaussian noise of the last N-hop, G_i^2 is the amplification factor of ith hop, $N_i^{self-interfer}$ is the self-interference of the ith hop, $\kappa_i$is the level of hardware defects at the ith relay and P_i is the average signal power at the ith hop. The derivations details are given in Appendix A.

Proof: The proof is given in Appendix A.

In order to measure the impact of hardware defects on the proposed system, we compare with the ideal system. In the following Proposition 1, we obtained a closed form for SNDRs.

Proposition 1 (Ideal Hardware): A closed form SNDR expression in *theorem* 1 becomes SNDR for ideal multi-hop system when setting $\kappa_i = 0$ and cancelling the self interference impact, then the SNDR expression becomes:

$$\gamma_{non,Nhops}^{FD-AF} = \frac{\prod_{i=1}^{N}\rho_i}{\sum_{i=1}^{N-1}\Big(b_i\prod_{i=j}^{N}\rho_i\Big) + c_i} \quad \text{for } j = 2, \ldots, N-1 \quad (21)$$

where the coefficients b_i and c_i in (21) are:

$$-b_i = N_i/P_1 \quad (22)$$

$$-c_i = \left(1 - \prod_{i=0}^{N-2}G_{i+1}^2\right)/P\prod_{i=0}^{N-2}G_{i+1}^2 \quad (23)$$

III. PERFORMANCE METRICS

The performance metrics for the proposed system is studied as follows. In Section III-A, the outage probability (OP) is analyzed while ergodic capacity and average bit error probability are analyzed in Section III-B and Section III-C, respectively to show the impact of hardware defects on the system performance.

A. OUTAGE PROBABILITY

OP measures the quality of service. Based on the received SNDR expression derived in (17), OP for the proposed system with N-hops under hardware defects and self-interference can be approximated as [26], [27]:

$$\begin{aligned} P_{out} &= \Pr\left(\gamma_{non,i,N}^{FD-AF} < \gamma_{\text{th}}\right) \\ &= 1 - \Pr\left(\frac{1}{\gamma_{non,i,N}^{FD-AF}} < \frac{1}{\gamma_{\text{th}}}\right) \\ &= 1 - \mathcal{L}^{-1}\left(\frac{\mathcal{M}_{1/\gamma_{non,i,N}^{FD-AF}}(-S)}{S}\right)\Bigg|_{1/\gamma_{non,i,N}^{FD-AF}=1/\gamma_{\text{th}}}. \end{aligned} \tag{24}$$

where $\mathcal{L}^{-1}(\cdot)$ and $\mathcal{M}_{1/\gamma_{non,i,N}^{FD-AF}}(\cdot)$ denote the inverse Laplace transformation and moment generating function (MGF) of the end-to-end SNDR inverse. The Euler numerical technique in [28, appendix 9B] is applied to solve the inverse Laplace transform in (24). A general form OP expression for the proposed multi-hop communication system after some algebraic manipulations is given by:

$$\begin{aligned} &\Pr\left(\frac{1}{\gamma_{non,i,N}^{FD-AF}} < \frac{1}{\gamma_{\text{th}}}\right) \\ &= \frac{2^{-k}e^{A/2}}{1/\gamma_{\text{th}}}\sum_{k=0}^{K}\binom{K}{k}\sum_{n=0}^{N+k}\frac{(-1)^n}{\alpha_n} \\ &\quad\times \mathrm{Re}\left(\frac{\mathcal{M}_{1/\gamma_{non,i,N}^{FD-AF}}\left(-\frac{A+2\pi jn}{2/\gamma_{\text{th}}}\right)}{\frac{A+2\pi jn}{2/\gamma_{\text{th}}}}\right) + \mathrm{E}(A,K,N). \end{aligned} \tag{25}$$

and the approximated term, $\mathbb{E}(A,K,N)$is given by [26], [28]:

$$\begin{aligned} \mathrm{E}(A,K,N) &= \frac{e^{-A}}{1-e^{-A}} \\ &\quad + \frac{2^{-k}e^{A/2}}{1/\gamma_{\text{th}}}\sum_{k=0}^{K}(-1)^{N+k+1}\binom{K}{k} \\ &\quad \times \mathrm{Re}\left(\frac{\mathcal{M}_{1/\gamma_{non,i,N}^{FD-AF}}\left(-\frac{A+2\pi j(N+k+1)}{2/\gamma_{\text{th}}}\right)}{\frac{A+2\pi j(N+k+1)}{2/\gamma_{\text{th}}}}\right) \end{aligned} \tag{26}$$

where $\alpha_n = 1$ for $n = 1, 2, \ldots, N$ and $\alpha_n = 2$ for $n = 0$. We set $A = 10\ln(10)$ in order to have discretization error less than 10^{-10}. The MGF can be expressed with the help of [29, eq. (3.471.9)] as follows:

$$\begin{aligned} &\mathcal{M}_{1/\gamma_{n,non,i}^{FD-AF}}(-s) \\ &= \frac{2}{\Gamma(m)}\left(\frac{s}{\gamma_{n,non,i}^{FD-AF}}\right)^{m/2} K_m\left(2\sqrt{\frac{s}{\gamma_{n,non,i}^{FD-AF}}}\right) \end{aligned} \tag{27}$$

where $K_m(\cdot)$ denotes the second kind mth-order modified Bessel function [29, eq. (8.432.1)]. For comparison, OP is analyzed for the proposed system and ideal system in Subsection III-1 and Section III-2, respectively.

1) OP FOR THE IDEAL MULTI-HOP SYSTEM

The OP for the ideal system using approximation method in (25) is calculated as follows. By means of [29, eq. (3.471.9)], the MGF's of the end-to-end SNDR, γ_{Nhop}^{FD-AF} in *remark* 1 is calculated as follows:

$$\begin{aligned} &\mathcal{M}_{1/\gamma_{Nhop}^{FD-AF}}(-s) \\ &= \frac{2}{\Gamma(m)}\left(\frac{s}{\gamma_{Nhop}^{FD-AF}}\right)^{m/2} K_m\left(2\sqrt{\frac{s}{\gamma_{Nhop}^{FD-AF}}}\right) \end{aligned} \tag{28}$$

where γ_{Nhop}^{FD-AF} is the end-to-end SNDR for the ideal system, which is given in (21). Putting (28) into (25) and (26), then the OP in (25) becomes:

$$\begin{aligned} &\Pr\left(\frac{1}{\gamma_{non,i,N}^{FD-AF}} < \frac{1}{\gamma_{\text{th}}}\right) \\ &= \frac{2^{2-k}e^{A2}}{\Gamma(m)\left(\gamma_{Nhop}^{FD-AF}\right)^{m/2}}\left(\frac{A+2\pi jn}{2/\gamma_{\text{th}}}\right)^{m/2} \\ &\quad\times \sum_{k=0}^{K}\binom{K}{k}\sum_{n=0}^{N+k}\frac{(-1)^n}{\alpha_n}\mathrm{Re}\left(\frac{1}{A+2\pi jn}\right) \\ &\quad\times \mathrm{Re}\left(K_m\left(\sqrt{\frac{2(A+2\pi jn)}{\gamma_{Nhop}^{FD-AF}/\gamma_{\text{th}}}}\right)\right) + \mathrm{E}(A,K,N) \end{aligned} \tag{29}$$

After arranging the terms and making some algebraic processing, the OP becomes as follows:

$$\begin{aligned} P_{out} &\approx 1 - \frac{2^{2-k}e^{A/2}}{\Gamma(m)\left(\gamma_{Nhop}^{FD-AF}\right)^{m/2}}\left(\frac{A+2\pi jn}{2/\gamma_{\text{th}}}\right)^{m/2} \\ &\quad\times \sum_{k=0}^{K}\binom{K}{k}\sum_{n=0}^{N+k}\frac{(-1)^n}{\alpha_n}\mathrm{Re}\left(\frac{1}{A+2\pi jn}\right) \\ &\quad\times \mathrm{Re}\left(K_m\left(\sqrt{\frac{2(A+2\pi jn)}{\gamma_{Nhop}^{FD-AF}/\gamma_{\text{th}}}}\right)\right) + \mathrm{E}(A,K,N) \end{aligned} \tag{30}$$

2) OP FOR THE PROPOSED MULTI-HOP SYSTEM

The approximation method in (25) is used to obtain the OP. First, we find the MGF's of the SNDR, $\gamma_{non,Nhops}^{FD-AF}$

in *theorem* 1 using the formula in [29, eq. (3.471.9)], as follows:

$$\mathcal{M}_{1/\gamma^{FD-AF}_{non,Nhops}}(-s) = \frac{2}{\Gamma(m)}\left(\frac{s}{\gamma^{FD-AF}_{non,Nhops}}\right)^{m/2} K_m\left(2\sqrt{\frac{s}{\gamma^{FD-AF}_{non,Nhops}}}\right). \quad (31)$$

where $\gamma^{FD-AF}_{non,Nhops}$ is the end-to-end SNDR for the proposed communication system, which is given in (17). Putting (31) into (25) and (26), then

$$\Pr\left(\frac{1}{\gamma^{FD-AF}_{non,Nhops}} < \frac{1}{\gamma_{th}}\right) = \frac{2^{2-k}e^{A/2}}{\Gamma(m)\left(\gamma^{FD-AF}_{non,Nhops}\right)^{m/2}}\left(\frac{A+2\pi jn}{2/\gamma_{th}}\right)^{m/2} \times \sum_{k=0}^{K}\binom{K}{k}\sum_{n=0}^{N+k}\frac{(-1)^n}{\alpha_n}\mathrm{Re}\left(\frac{1}{A+2\pi jn}\right) \times Re\left(K_m\left(\sqrt{\frac{2(A+2\pi jn)}{\gamma^{FD-AF}_{non,Nhops}/\gamma_{th}}}\right)\right) + \mathrm{E}(A,K,N) \quad (32)$$

After arranging the terms and making algebraic processing, the closed form expression of OP becomes as follows:

$$P_{out} \approx 1 - \frac{2^{2-k}e^{A/2}}{\Gamma(m)\left(\gamma^{FD-AF}_{non,Nhops}\right)^{m/2}}\left(\frac{A+2\pi jn}{2/\gamma_{th}}\right)^{m/2} \times \sum_{k=0}^{K}\binom{K}{k}\sum_{n=0}^{N+k}\frac{(-1)^n}{\alpha_n}\mathrm{Re}\left(\frac{1}{A+2\pi jn}\right) \times \mathrm{Re}\left(K_m\left(\sqrt{\frac{2(A+2\pi jn)}{\gamma^{FD-AF}_{non,Nhops}/\gamma_{th}}}\right)\right) + \mathrm{E}(A,K,N) \quad (33)$$

The coefficients of hardware defects appear in $\gamma^{FD-AF}_{non,Nhops}$. We analyze the effect of these coefficients on the OP performance and compare with the ideal system.

B. ERGODIC CAPACITY

The ergodic channel capacity, expressed in bits/channel is used to measure the system performance. While the capacity of the relaying channel under ideal case are described in previous works [30]–[32], the case of AF relaying with hardware defects is scarcely addressed. The ergodic capacity for multi-hop communication system under hardware defects and self-interference can be expressed as [1], [30]–[32]:

$$C^{FD-AF}_{non-ideal,N} \triangleq \frac{1}{2}\mathbb{E}\left\{\log_2\left(1+\gamma^{FD-AF}_{non,Nhops}\right)\right\} \quad (34)$$

where $\gamma^{FD-AF}_{non,Nhops}$ is the end-to end SNDR for the proposed system and the term non-ideal refers to multi-hop communication system under hardware defects and self-interference. The value of $\gamma^{FD-AF}_{non,Nhops}$ is given in (17).

Theorem 2: For Nakagami-*m* fading channels, the suggested multi-hop communication system ergodic capacity under hardware manufacturing defects and self interference is upper bounded as:

$$C^{FD-AF}_{non-ideal,N} \leq \frac{1}{2}\log_2\left(1+\frac{\mathcal{M}_1}{a\mathcal{M}_2+b\mathcal{M}_3+c_i}\right) \quad (35)$$

and the coefficients $\mathcal{M}_1$, $\mathcal{M}_2$ and $\mathcal{M}_3$ are given by:

$$\mathcal{M}_1 = \frac{1}{\prod_{i=1}^{N}\Gamma(\alpha_i)}G^{1,N}_{N,1} \times \left[-s\prod_{i=1}^{N}\beta_i\left|\begin{matrix}\Delta(1,1-\alpha_1),\ldots,\Delta(1,1-\alpha_N)\\ \Delta(1,0)\end{matrix}\right.\right]. \quad (36)$$

$$\mathcal{M}_2 = \frac{\sqrt{N}\prod_{i=1}^{N}(N+1-i)^{\alpha_i-1/2}}{\left(\sqrt{2\pi}\right)^{r-1}\prod_{i=1}^{N}\Gamma(\alpha_i)} \times G^{N,r}_{r,N}\left[\frac{\left(\frac{-s}{N}\right)^N}{\prod_{i=1}^{N}[\beta_i(N+1-i)]^{(i-N-1)}} \times \left|\begin{matrix}\Delta(N,1-\alpha_1),\ldots,\Delta(1,1-\alpha_N)\\ \Delta(N,0)\end{matrix}\right.\right]. \quad (37)$$

$$\mathcal{M}_3 = \frac{\sqrt{N}\prod_{i=2}^{N}(N+1-i)^{\alpha_i-1/2}}{\left(\sqrt{2\pi}\right)^{r-1}\prod_{i=2}^{N}\Gamma(\alpha_i)} \times G^{N,r}_{r,N}\left[\frac{\left(\frac{-s}{N}\right)^N}{\prod_{i=2}^{N}[\beta_i(N+1-i)]^{(i-N-1)}} \times \left|\begin{matrix}\Delta(N,1-\alpha_1),\ldots,\Delta(1,1-\alpha_N)\\ \Delta(N,0)\end{matrix}\right.\right]. \quad (38)$$

where $G[\cdot]$ is the Meijer G-function [33].

Proof: The proof is given in Appendix B.

The ergodic capacity can be obtained by employing the approximation [9] to (34) as follows:

$$\mathbb{E}\left\{\log_2\left(1+\frac{x}{y}\right)\right\} \approx \log_2\left(1+\frac{\mathbb{E}\{x\}}{\mathbb{E}\{y\}}\right). \quad (39)$$

We can obtain the following formula (40), as shown at the bottom of the next page, where the parameters b_{i+1}, c_{i+1} and d are defined in *Theorem* 1. Despite the approximation of ergodic capacity, we illustrate numerically in the results that ergodic capacity approximation is an upper bound. We also study the impact of these parameters on the capacity performance.

C. AVERAGE BIT ERROR PROBABILITY

In this subsection, we examine the last performance matrix, which is average bit error probability. It is one of the critical performance measures of communication systems. The average bit error probability for coherent binary signal constellations is given by [34]:

$$\bar{P}_e = \frac{1}{2}\int_0^{\infty} \mathrm{erfc}\left(\sqrt{a\gamma}\right) f_{\gamma}\left(\gamma\right) d\gamma. \tag{41}$$

where $\mathrm{erfc}\left(\cdot\right)$ denotes the complementary error function, $f_{\gamma}\left(\gamma\right)$ denotes the PDF of γ and $a = \left(1-\epsilon\right)/2$, denotes the correlation factor. The value of ϵ for coherent BPSK and coherent orthogonal BFSK is $\epsilon = -1$ and $\epsilon = 0$, respectively. To compute the above integral, we need first to get the PDF of γ.

$$\gamma_{non,Nhops}^{FD-AF} \leq \gamma = \frac{1}{N}\prod_{i=1}^{N} \gamma_i^{1/N} \tag{42}$$

where $\gamma_i = P_i \left|H_i\right|^2 / \left(\kappa_i^2 P_i \left|H_i\right|^2 + N_i\right)$ denotes the SNR of i-th hop. The PDF of γ is defined as:

$$f_{Y_1}\left(y\right) = \mathcal{L}^{-1}\left\{M_{Y_1}\left(-s\right); y\right\} \tag{43}$$

where $\mathcal{L}^{-1}\left(\cdot;\cdot\right)$ is the inverse Laplace transform. We apply an expression for the inverse Laplace transformation of the Meijer G-function [35, eq. (3.38.1)].

$$f_{\gamma}\left(\gamma\right) = \frac{N\gamma^{-1}}{\prod_{i=1}^{N}\Gamma\left(\alpha_i\right)} G_{0,N}^{N,0}\left[\left(\gamma N\right)^N \prod_{i=1}^{N}\frac{\alpha_i}{\bar{\gamma}_i} \middle| \begin{matrix} - \\ \alpha_1, \alpha_2, \ldots, \alpha_N \end{matrix}\right] \tag{44}$$

Plugging (44) into (41), the average bit error probability becomes as follows:

$$\bar{P}_e = \frac{N}{2}\int_0^{\infty} \mathrm{erfc}\left(\sqrt{a\gamma}\right) \frac{\gamma^{-1}}{\prod_{i=1}^{N}\Gamma\left(\alpha_i\right)} \times G_{0,N}^{N,0}\left[\left(\gamma N\right)^N \prod_{i=1}^{N}\frac{\alpha_i}{\bar{\gamma}_i} \middle| \begin{matrix} - \\ \alpha_1, \alpha_2, \ldots, \alpha_N \end{matrix}\right] d\gamma. \tag{45}$$

We represent the $\mathrm{erfc}\left(\cdot\right)$ function with the Meijer G-function [36, eq. (12)], and then we use the integral for the product of Meijer G-function [36, eq. (21)]. After arranging the terms and some algebra, a lower bound exact form for the average bit error probability becomes:

$$\bar{P}_e = \frac{1}{\left(\sqrt{2}\right)^{N+1}\left(\sqrt{\pi}\right)^N \prod_{i=1}^{N}\Gamma\left(\alpha_i\right)} \times G_{2N,N}^{N,2N}\left[\left(\frac{N^2}{a}\right)^N \prod_{i=1}^{N}\frac{\alpha_i}{\bar{\gamma}_i} \middle| \begin{matrix} \Delta\left(N,1\right), \Delta(N,1/2) \\ \alpha_1, \alpha_2, \ldots, \alpha_N, \Delta\left(N,0\right) \end{matrix}\right] \tag{46}$$

IV. THE FUNDAMENTAL IMPACT OF HARDWARE MANUFACTURING DEFECTS

We illustrate the impact of hardware manufacturing defects in the high region of SNR. The average SNR_i at any fading distribution is:

$$\mathrm{SNR}_i = \frac{P_i \mathbb{E}_{\rho_i}\left\{\rho_i\right\}}{N_i} \quad \text{for } i = 1, 2, \ldots, N. \tag{47}$$

where $\mathbb{E}_{\rho_i}\left\{\rho_i\right\} = \alpha_i\beta_i$ is average power of fading in the case of Nakagami-m fading. The level of hardware defects, κ_i depends on the SNR [37]–[39]. We assume that SNR_i increase large with $\mathrm{SNR}_1 = \varepsilon_2\mathrm{SNR}_2 = \ldots = \varepsilon_N\mathrm{SNR}_N$ for ratio $0 < \varepsilon_j < \infty, j = 2, \ldots, N$, such that the gain of the RS is constant.

Corollary 1: Let $\mathrm{SNR}_1, \ldots, \mathrm{SNR}_N$ increase with a constant ratio and assume any independent fading distributions on $\rho_i$that are strictly positive. The OP for multi-hop communication system under hardware defects satisfies

$$\lim_{\mathrm{SNR}_1,\ldots,\mathrm{SNR}_N\to\infty} P_{out} = \begin{cases} 0, & \gamma_{non,N\,hops}^{FD-AF} \leq \frac{1}{\sum_{i=1}^{N-1}\kappa_i^2}, \\ 1, & \gamma_{non,N\,hops}^{FD-AF} > \frac{1}{\sum_{i=1}^{N-1}\kappa_i^2}, \end{cases} \tag{48}$$

Proof: From (17), we rewrite $\gamma_{non,Nhops}^{FD-AF}$ using $\mathrm{SNR}_1, \ldots, \mathrm{SNR}_N$ by simplifying the average fading power as $\mathbb{E}_{\rho_i}\left\{\rho_i\right\}\tilde{\rho}_i$ (i.e., $\tilde{\rho}_i$ refers to the normalized channel gain). After the limit process, $\mathrm{SNR}_1, \ldots, \mathrm{SNR}_N \to \infty$, $\gamma_{non,Nhops}^{FD-AF}$ leads to

$$\lim_{\mathrm{SNR}_1,\ldots,\mathrm{SNR}_N\to\infty} \gamma_{non,Nhops}^{FD-AF} = \frac{1}{a} = \frac{1}{\sum_{i=1}^{N-1}\kappa_i^2} \tag{49}$$

for any non-zero realization of $\tilde{\rho}_1, \ldots, \tilde{\rho}_N$. With high values of SNR, OP decreases to zero when $\gamma_{non,Nhops}^{FD-AF}$ is low compared to the ceiling. The SNDR ceiling of the proposed system is:

$$\gamma_{non,Nhops}^{FD-AF} \triangleq \frac{1}{\sum_{i=1}^{N-1}\kappa_i^2} \tag{50}$$

We notice that the SNDR ceiling is inversely proportional to the summation of the square of hardware defect level. This validates that hardware defects can limit the system performance and must be considered at the stage of manufacturing the devices of multi-hop system.

Corollary 2: Suppose $\mathrm{SNR}_1, \ldots, \mathrm{SNR}_N$ increase with a constant ratio and assume independent fading distributions on ρ_i that are strictly positive. The SNDR is maximized as $\gamma_{non,Nhops}^{FD-AF} \leq 1/\sum_{i=1}^{N-1}\kappa_i^2$. The ergodic capacity satisfies:

$$\lim_{\mathrm{SNR}_1,\ldots,\mathrm{SNR}_N\to\infty} C_{non-ideal,N}^{FD-AF} \leq \frac{1}{2}\log_2\left(1 + \frac{1}{\sum_{i=1}^{N-1}\kappa_i^2}\right) \tag{51}$$

$$C_{non-ideal,N}^{FD-AF} \approx \frac{1}{2}\log_2\left(1 + \frac{\prod_{i=1}^{N}\rho_i}{\sum_{i=1}^{N-1}\left(a_i\prod_{i=j-1}^{N}\alpha_i\beta_i\right) + \sum_{i=1}^{N-1}\left(b_i\prod_{i=j}^{N}\alpha_i\beta_i\right) + c_i}\right) \tag{40}$$

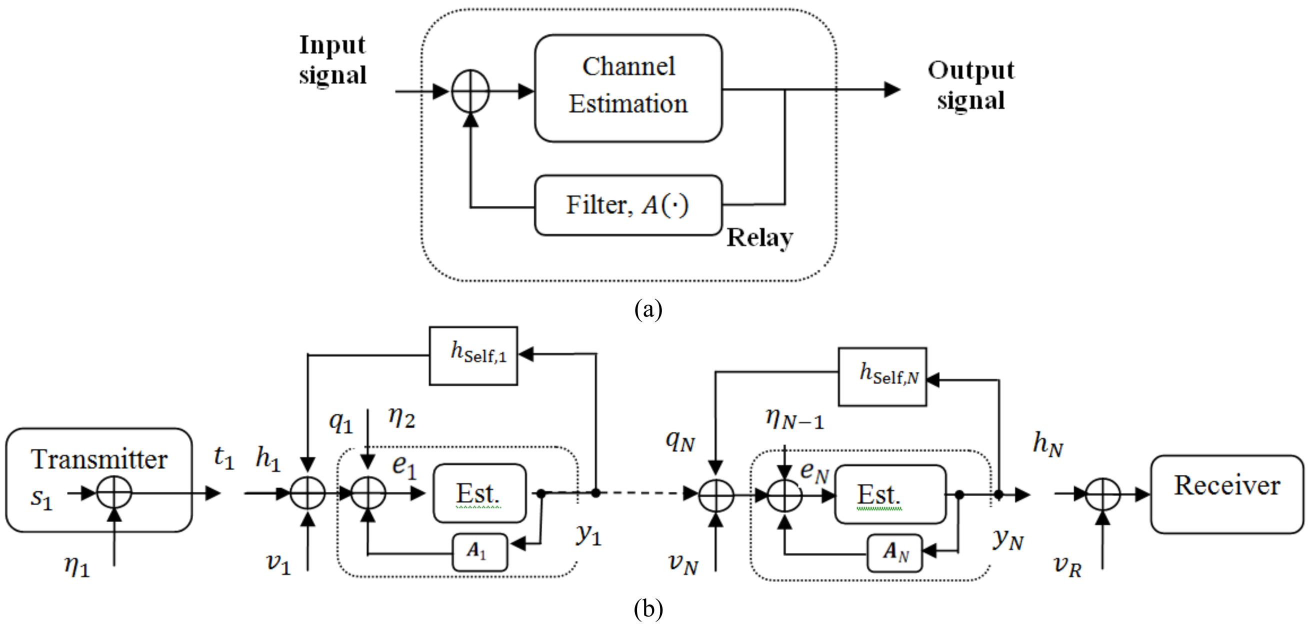

FIGURE 2. (a) A mitigation technique and (b) The proposed multi-hop communication system with a mitigation technique.

A. DESIGN FOR HARDWARE DEVICES

The total distortion noises after N-hops for a given channel realization h, has power:

$$\mathbb{E}_{\eta_1,\ldots,\eta_{N-1}}\left\{|h\eta_1 + h\eta_2 + \ldots + \eta_{N-1}|^2\right\}$$
$$= P\,|h|^2\left(\kappa_1^2 + \kappa_2^2 + .. + \kappa_{N-1}^2\right) = P\,|h|^2 \sum_{i=1}^{N-1} \kappa_i^2 \quad (52)$$

The design parameters $\kappa_1, \kappa_2, \ldots, \kappa_{N-1} \geq 0$ characterize the level of hardware defects at the ith hop. These parameters are expressed as the error vector magnitudes (EVMs). EVM is employed to evaluate the radio frequency transceivers quality. It can be calculated in practice [40]. From (52), it is sufficient to characterize the total level of hardware defects as

$$\kappa = \sqrt{\kappa_1^2 + \kappa_2^2 + .. + \kappa_{N-1}^2}. \quad (53)$$

The low cost devices (i.e., transmitter, RSs and receiver) have poor quality and thus higher values of EVMs. So, the cost of devices is a decreasing function of the EVMs. The question is how to calculate the EVMs that maximizes the system performance for a constant cost. We denote the devices hardware cost as $\sum_{i=1}^{N-1} \varphi(\kappa_i)$, where $\varphi(\cdot)$ is a continuously decreasing, twice differentiable, and convex function.

Corollary 3: Suppose $\sum_{i=1}^{N-1} \varphi(\kappa_i) = S_{max}$ for a given device cost $S_{max} \geq 0$. Assume the EVMs $\kappa_1^*, \kappa_2^*, .., \kappa_{N-1}^*$ are not equal and optimal solution. The devices hardware cost represents a Schur-convex function [41], thus the solution is:

$$\kappa_1 = \kappa_2 = \ldots = \kappa_{N-1} = \frac{\sum_{i=1}^{N-1} \kappa_1^* + \kappa_2^* + \ldots + \kappa_{N-1}^*}{N-1} \quad (54)$$

As a result, the SNDR ceiling in (50) is maximized by:

$$\kappa_1 = \kappa_2 = \ldots = \kappa_{N-1} = \varphi^{-1}\left(S_{max}/N - 1\right) \quad (55)$$

This solution reduces the cost and enhance the SNDR ceilings, thus the values of EVMs should be equal at the optimum. From *Corollary* 3, the level of hardware defects is equal at each device. Also, the quality of the RS hardware is same for transmitter and receiver hardware.

V. THE PROPOSED MITIGATION TECHNIQUE

In this section, the proposed mitigation technique to overcome hardware defects and self-interference is shown in Fig. 2(a). It consists of channel estimation and a filter. In this technique, linear minimum mean square error (LMMSE) method is employed to overcome the hardware defects and a filter is used to cancel the self-interference. First, the channel estimation is obtained, then these channel estimates is used to design a receive filters.

A. CHANNEL ESTIMATION

The conventional methods from [42], [43], cannot be employed due to the fact that the system model in (2) has non-standard characteristics, which are the data transmission is affected by hardware defects and self interference. Therefore, LMMSE technique at each RS for the proposed system is first derived. Fig. 2(b) shows the proposed mitigation technique for multi-hop communication system under hardware defects and self-interference. The mitigation technique is applied at each RS. After algebraic processing, the estimation of $\mathrm{h}_i[i]$ at the ith RS using LMMSE techniqueis given by:

$$\hat{\mathrm{h}}_i[i] = \frac{\rho_i G P_i}{1 - N_{i+1}^{self-interfer} G}$$

$$\times\left(\frac{G^2\rho_i}{1-G^2N_{i+1}^{self-interfer}}\left[P_i+\kappa_i^2P_i+N_i\right]+\kappa_{i+1}^2P_{i+1}\right)^{-1}y_i. \quad (56)$$

Proof: The proof is listed in Appendix C.

B. SELF-INTERFERENCE CANCELLATION

To mitigate the self-interference signal q_i, the proposed cancellation architecture is depicted in Fig. 2(b). It consists of Lth-order filter, $A_i[n]$, with adjustable parameters.

At time instant n, the transmitter sends its signal, $t_1[n]$. The ith RS receives the signal, $q_i[n]$, while transmitting the signal $y_i[n]$. The receiver receives the signal, $y_R[n]$. The received signal at ith RS, $q_i[n]$, may be expressed as:

$$q_i[n]=h_{i-1}[n]\,t_i[n]+h_i[n]\,y_i[n]+v_i[n] \quad (57)$$

Let $x_i[n]=h_{i-1}[n]\,t_i[n]$ and $\mathrm{f_i}[n]=h_i[n]\,y_i[n]$, and setting these values in (57), the received signal at the RS becomes:

$$q_i[n]=x_i[n]+\mathrm{f_i}[n]+v_i[n] \quad (58)$$

The self interference term $\mathrm{f_i}[n]$ may be seen as an extra noise source [44]. For this purpose, the cancellation architecture as shown in Fig. 2(b) is employed, which consists of the strictly causal Lth-order filter, $A_i[n]$. The signal $e_i[n]$ is given by:

$$e_i[n]=x_i[n]+\underbrace{(h_i[n]+A_i[n])}_{i(n)}y_i[n]+\eta_i[n]+v_i[n] \quad (59)$$

where $\mathrm{i}(n)$ denotes the residual self interference after mitigation. If $\tilde{h}_i(n)$ denotes the L_ith-order impulse response of the equivalent self interference channel, i.e.,

$$\tilde{h}_i(n)=-h_i[n] \quad (60)$$

Then, perfect cancellation is obtained when $A_i[n]=\tilde{h}_i(n)$. A rule for adapting the coefficients of $A_i[n]$ is determined as:

$$A[k](n+1)=A[k](n)+\mu\left(\mathrm{R}[k](n)-e[n]\,y^H(n-k)\right) \quad (61)$$

where μ is the adaptation rate and $\mathrm{R}[k](n)$ is the cross-correlation between two signals, $e[n]$ and $y[n]$.

After the process of LMMSE method and self-interference cancellation, the output signal from the ith RS, $y_i[n]$ is:

$$y_i[n]=Gt_i[n]+G(\eta_i[n]+v_i[n])\,\hat{h}_i^{-1} \quad (62)$$

Finally, the received signal at the receiver is given by:

$$y_R[n]=h_{N+1}Gt_i[n]+\mathrm{h}_{N+1}G(\eta_i[n]+v_i[n])\,\hat{\mathrm{h}}_i^{-1}+v_{i+1}[n] \quad (63)$$

and the end-to-end SNDR of the proposed system with a mitigation technique can be expressed as:

$$\gamma_{ihop}^{FD-AF}=\frac{\mathbb{E}\left\{|Gh_{N+1}t_i[n]|^2\right\}}{\mathbb{E}\left\{\left|h_{N+1}G(\eta_i[n]+v_i[n])\,\hat{h}_i^{-1}+v_{i+1}[n]\right|^2\right\}} \quad (64)$$

After arranging the terms and some algebraic processing, the SNDRs becomes as follows:

$$\gamma_{ihop}^{FD-AF}=\frac{\rho_{i+2}}{\boldsymbol{M}_1\times\boldsymbol{M}_2+\left(N_{i+1}/P_iG^2\right)}. \quad (65)$$

where

$$-\boldsymbol{M}_1=\frac{\rho_i^2\rho_{i+2}P_iG^2\left(\kappa_{i+1}^2P_{i+1}+N_i\right)}{\left(1-N_{i+1}^{self-interfer}G\right)^2}. \quad (66)$$

$$-\boldsymbol{M}_2=\left(\frac{G^2\rho_i}{1-N_{i+1}^{self-interfer}G^2}\times\left[P_i\left(1+\kappa_i^2\right)+N_i\right]+\kappa_{i+1}^2P_{i+1}\right)^{-1}. \quad (67)$$

The comparison of the performance metrics for the proposed system with and without a mitigation techniqueis analyzed.

VI. RESULTS

We evaluate the system performance with the increase of hardware defect level at different transmission hops. Fig. 3 and Fig. 4 show the OP of the ideal and proposed multi-hop communication systems, respectively at different hardware defect levels and transmission hops. The results of theoretical analysis and simulation are the same that show the accuracy of the analysis results.

A. ANALYSIS OF OUTAGE PROBABILITY

Fig. 3 and Fig. 4 illustrate the OP as a function of the average SNR. Fig. 3 shows the OP versus SNR for the ideal system with different transmission hops. With SNR = 25 dB, the OP are 3×10^{-4}, 3×10^{-3}, 2×10^{-2} and 3×10^{-2} at transmission hops $N=2, N=3, N=4$ and $N=5$, respectively as shown in Fig. 3(a). We can see that, as the effect of self interference increases from 3 dB to 5 dB as in Fig. 3(b), the performance of the ideal system decreases. We illustrate the impact of hardware manufacturing defects on the OP performance with different transmission hops. The low and high values of defect levels are $\kappa=0.1$ and $\kappa=0.2$, respectively. Hardware defect levels are characterized by $\kappa = 0.1$, 0.2 and 0.25, where $\kappa=0.1$ and $\kappa=0.25$ reflect the lowest and highest effect of the hardware defects, respectively [45], [46].

The increase in the number of transmission hops in multi-hop communication system requires more aggregated power to transmit a signal with sufficient low outage. A multi-hop communication system with $N = 5$ as shown in Fig. 3(a) needs a SNR penalty of about 7 dB due to the increase in the number of transmission hops from 3 to 5 to ensure sufficient low outage. We find that the OP of the proposed system is high compared to the ideal system. At SNR = 25 dB, the OP of the proposed system are 1×10^{-3}, 6×10^{-3}, 2×10^{-2} and 6×10^{-2} at the transmission hops $N=2, N=3, N=4$ and $N=5$, respectively as shown in Fig. 4(a). We see that the OP decreases as the effect of the hardware defects increases from $\kappa=0.1$ to $\kappa=0.2$ as plotted in Fig. 4(b) and Fig. 4(c).

As the number of transmission hops increases with high values of hardware defects, the system performance

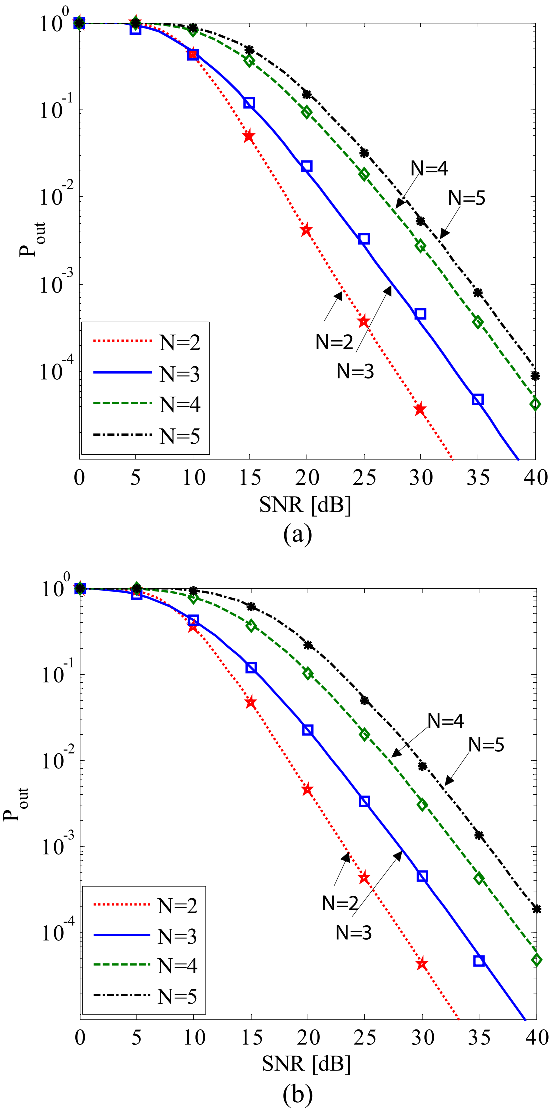

FIGURE 3. OP vs. SNR for the ideal multi-hop communication system at different transmission hops equal 2, 3, 4 and 5 and Nakagami-*m* fading (a) SNIR threshold x = 7 and self interference=3dB and (b) SNIR threshold x = 15 and self-interference=5dB.

decreases. At SNIR threshold $x = 7$, self interference=5dB, $N = 4$ and SNR = 30 dB, the OP for the ideal system is 3×10^{-3} as in Fig. 3(b), which is low compared to 1×10^{-2} for the proposed system at hardware defect level, $\kappa = 0.25$ as in Fig. 4(c).

At OP $= 10^{-3}$and $\kappa = 0.1$, we find that the values of loss for the proposed system are 1.5, 1.7, 0.7 and 1dB at transmission hops $N = 2$, $N = 3$, $N = 4$ and $N = 5$, respectively. We see that the performance loss increases as the hardware defect level increases. With self interference = 5dB, the OP increases as in Fig. 4(c). The proposed system becomes worse when the number of hops increases. This is because the effect of the hardware defects is repeated for each hop, so the cumulative effect of the hardware defects affects the system performance. So, we recommend the use of devices with good manufacturing quality.

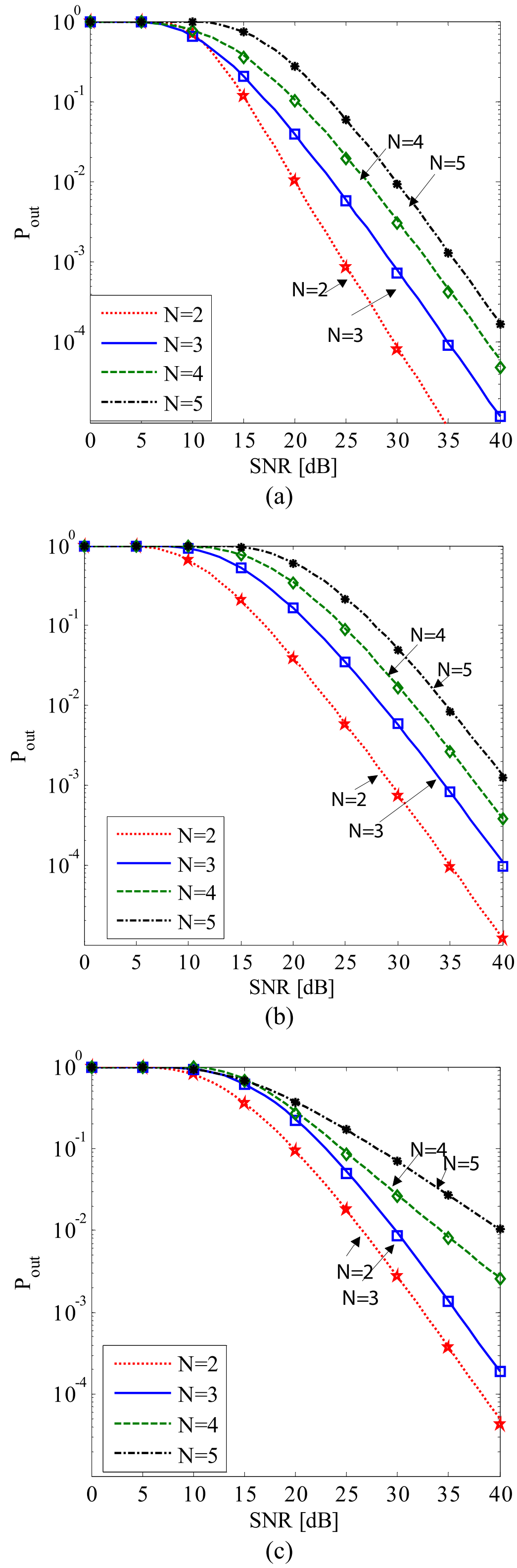

FIGURE 4. OP vs. SNR for the proposed system at different transmission hops equal 2, 3, 4 and 5, SNDR threshold x = 7, Nakagami-*m* fading, self interference=3dB and hardware defect level(a)$\kappa = 0.1$, (b) $\kappa = 0.2$ and (c) $\kappa = 0.25$.

Next, we show the effect of the shape parameters α_i of the Nakagami-*m* fading distributions. Fig. 5 depicts the OP

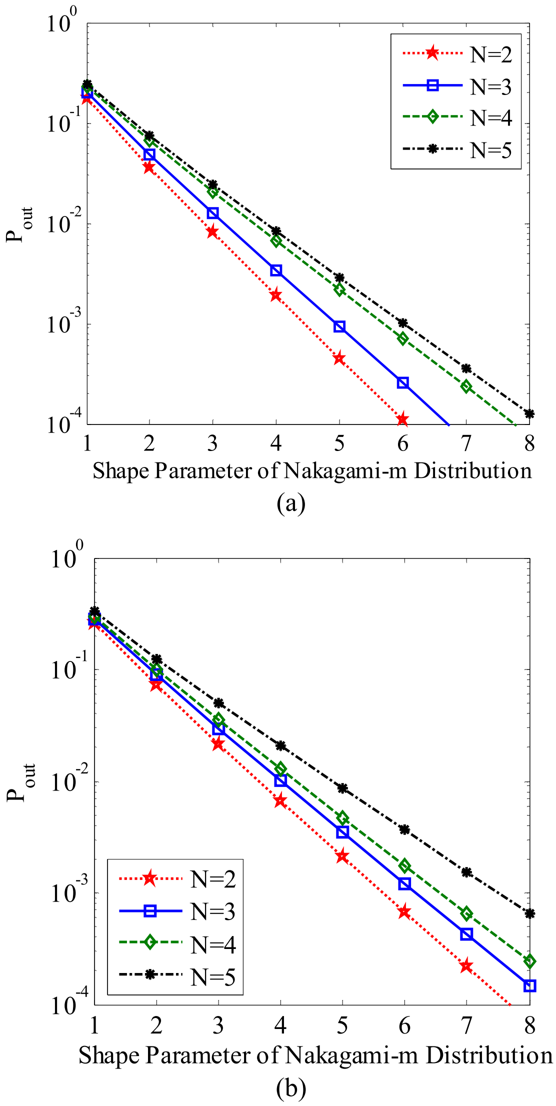

FIGURE 5. OP vs. shape parameters α of Nakagami-m fading for the (a) ideal system and (b) proposed system with $\kappa = 0.1$.

with ideal hardware and hardware defects characterized by $\kappa = 0.1$. Increasing the shape parameters will reduce the OP and enhance the system performance. This is due to that the channel gain variance ρ_i decreases when increasing the shape parameters α_i, while we keep the average SNR constant. Fig. 5(a) shows OP versus shape parameters of Nakagami-m fading for ideal system at different transmission hops.

At $\alpha = 5$and $N = 2$, the OP of the ideal system is 4.5×10^{-4}, which is low compared to 9×10^{-4}, $2 \times 10^{-3}$and 2.5×10^{-3} for three, four and five transmission hops, respectively. Fig. 5(b) shows the OP vs. shape parameters for the proposed system at$\kappa = 0.1$. At $\alpha = 5$and $N = 5$, the ideal system has OP $= 2.5 \times 10^{-3}$ as in Fig. 5(a), which is low compared to 9×10^{-3} for the proposed system at $\kappa = 0.1$.

B. ANALYSIS OF ERGODIC CAPACITY

Next, we analyze the ergodic capacity performance for the ideal and proposed system in the presence of hardware manufacturing defects and self interference. The capacity vs. SNR for the ideal and proposed multi-hop communication system at different transmission hops, SNDR threshold x = 7 and self interference=3dB is plotted in Fig. 6.

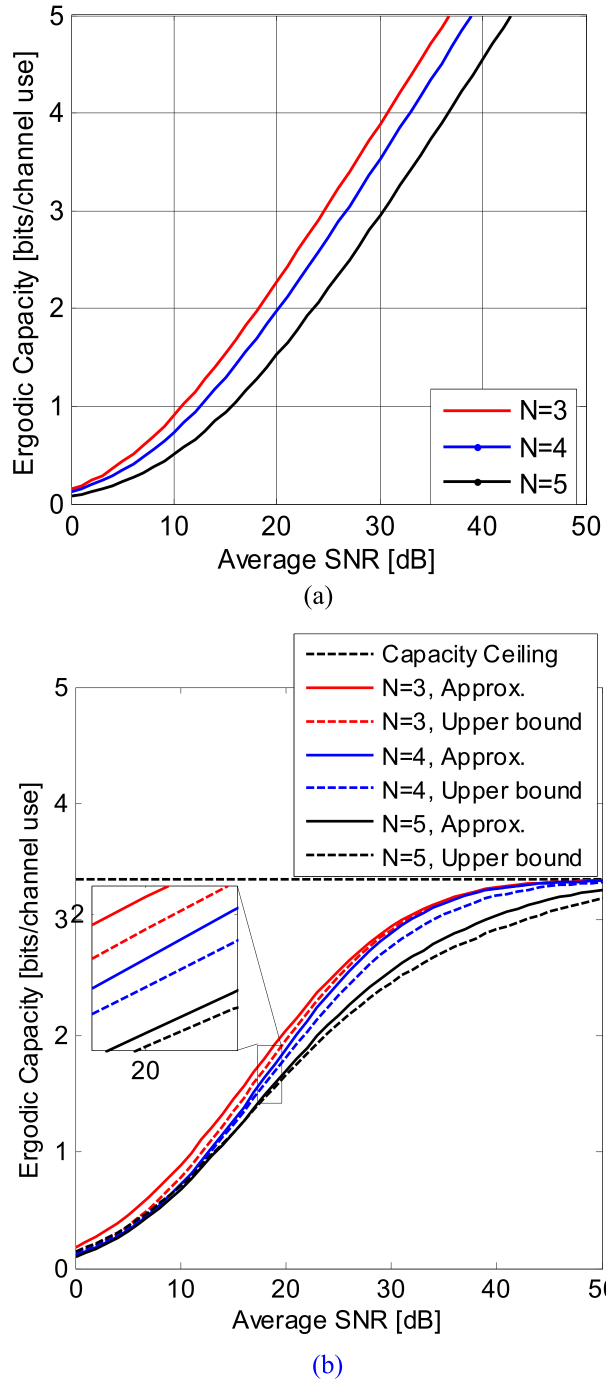

FIGURE 6. Ergodic capacity vs. SNR for the (a) ideal system and (b) proposed system with $\kappa = 0.1$ at different transmission hops, SNDR threshold x = 7 and self interference=3dB.

The capacity performance for the ideal and proposed multi-hop communication system at SNDR threshold x = 7 and self interference=3dB is illustrated in Fig. 6. We illustrate three scenarios as follows:

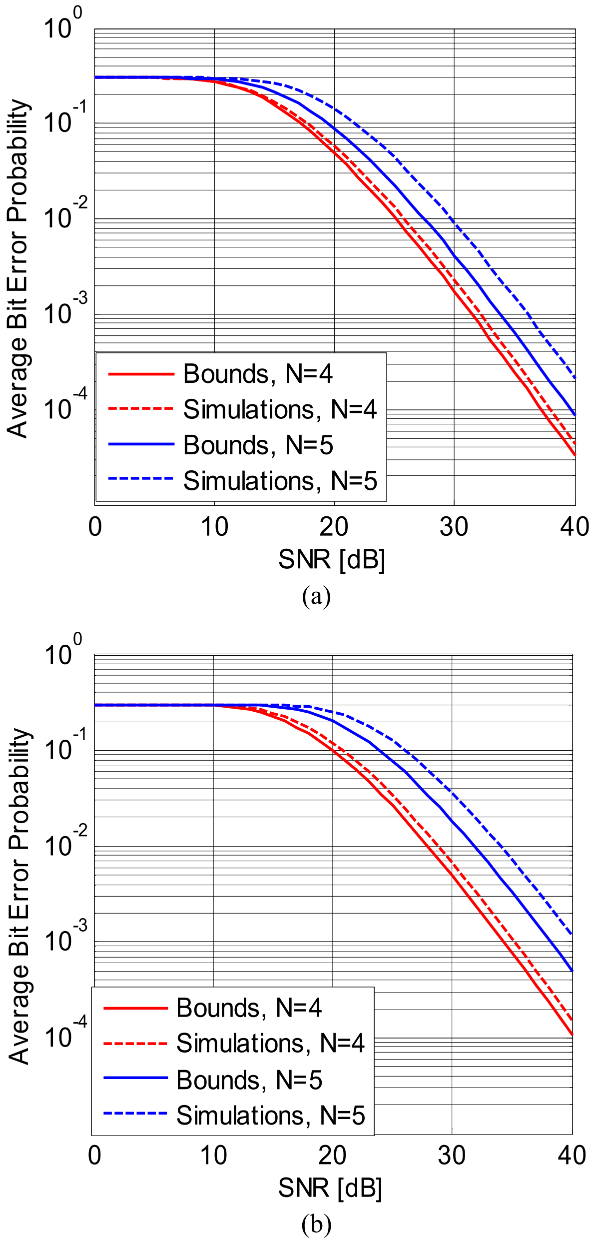

FIGURE 7. BPSK error bounds of the proposed system at four and five hops with Nakagami-*m* fading channels ($\alpha_i = \alpha = 2.6$) and self interference=3dB (a) $\kappa = 0.1$ and (b)$\kappa = 0.2$.

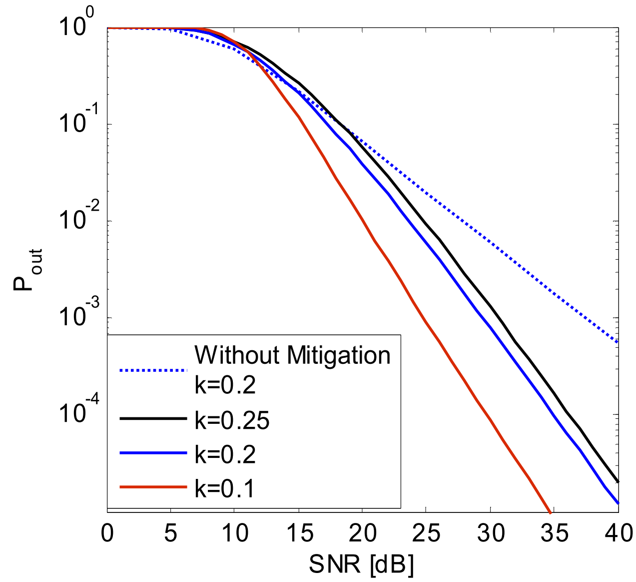

FIGURE 8. OP vs. SNR for the proposed system at different hardware defect levels after mitigation technique with adaptation rate $\mu = 0.01$.

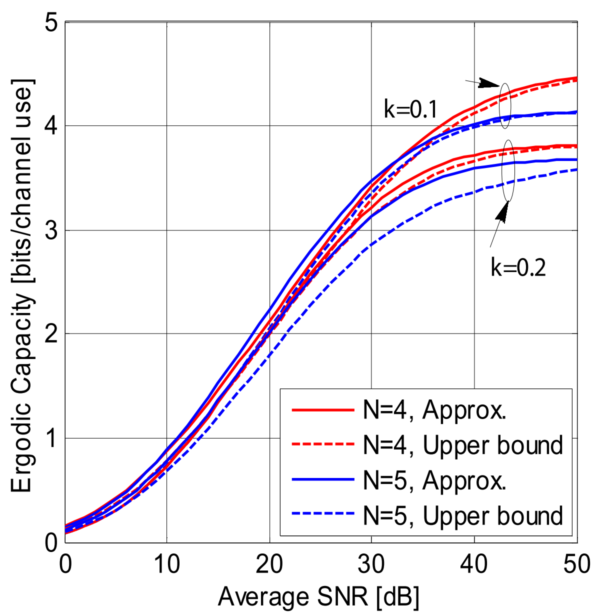

FIGURE 9. Ergodic capacity vs. SNR at different hops and hardware defect levels, $\kappa = 0.1$ and $\kappa = 0.2$ after mitigation technique with adaptation rate $\mu = 0.01$.

1- The number of hops is three (i.e., $N = 3$), as in red curve of Fig. 6.

2- The transmission hop is four (i.e., $N = 4$), as in blue curve.

3- When the transmission hop is five (i.e., $N = 5$), as in black curve of Fig. 6.

Fig. 6 indicates that the hardware defects are only influential at high SNRs. These defects give severe degradation in the capacity performance compared with the ideal system. The proposed system capacity is saturated and approximated to $\log_2\left(1 + 1/\sum_{i=1}^{N-1} \kappa_i^2\right)$, as confirmed in Corollary 2. Fig. 6(b) illustrates the upper bound and approximation of the capacity bound, which gives a tight result. In the high SNR, the defects have a large impact on the capacity performance. Capacity performance is more degraded and the gap between ideal and proposed system gets larger at high SNRs.

C. ANALYSIS OF BIT ERROR PROBABILITY

The average bit error probability (BEP) lower bound for the proposed multi-hop communication system is illustrated in

Fig. 7. We consider the case of four and five transmission hops. The analytic and simulation bounds for the bit error probability are tight. We see that the bit error probability increases as the transmission hop increases. Fig. 7(a) shows the small impact of hardware defects on the proposed system. When the impact of hardware defects is increased as in Fig. 7(b), the BEP is more degraded and the performance gap is small for all ranges of average SNRs.

D. ANALYSIS OF THE PROPOSED MITIGATION TECHNIQUE

Without mitigation technique, the impact of hardware defects in the previous stages will be propagated to the next stages of the RSs. To reduce the propagation of hardware defects, a mitigation technique is proposed at each RS. This technique is based on LMMSE technique for the channel estimation and a filter for self-interference cancellation. The OP performance can be enhanced by decreasing the impact of hardware defects and self interference at each RS.

Fig. 8 shows OP versus SNR for the proposed system at different hardware defect levels. We apply a mitigation technique with adaptation rate, $\mu = 0.01$. We notice from Fig. 8 and Fig. 9 that the mitigation technique can enhance the system performance. With $\kappa = 0.2$ and OP $= 10^{-2}$, 4dB gain is obtained after applying mitigation technique. Fig. 9 shows ergodic capacity versus SNR for the proposed system with mitigation technique at adaptation rate $\mu = 0.01$. The proposed technique improves system capacity and provides a better performance. Utilization of the proposed mitigation technique in multi-hop systems under hardware problems can enhance overall system performance. In the future work, we will propose mitigation algorithms with the ability of tracking, identifying and canceling the self interference distortion while the RS is operated in its normal mode.

VII. CONCLUSION

In this paper, a multi-hop communication system under hardware defects and self interference has been proposed. A closed form expression for the performance metrics has been provided. Hardware defects can give a ceiling in the capacity performance. Also, the SNDR ceiling is inversely proportional to the summation of the square of hardware defect level. The system performance is degraded under high number of transmission hops and high values of hardware defects. In addition, a mitigation technique has been proposed. The proposed mitigation technique is based on the LMMSE scheme to mitigate the hardware defects and a filter to cancel the self-interference. The channel estimates is calculated and then, it's used to design a receive filters. This technique can enhance the system performance and can reduce the impact of hardware defects.

APPENDIX A: PROOF OF THEOREM 1

From (16), the SNDR of the proposed multi-hop communication system after the expectation manipulations is:

$$\gamma_{Nhop}^{FD-AF} = \frac{\boldsymbol{\varphi}_1}{\boldsymbol{\varphi}_2 + N_N^{noise}}. \tag{68}$$

where the values of $\boldsymbol{\varphi}_1$ and $\boldsymbol{\varphi}_2$, respectively are given by (69) and (70), as shown at the bottom of the page. After some algebraic processing, the above equation becomes (71), as shown at the bottom of the page. We rewrite the product terms in a general form as (72), shown at the bottom of the next page. Rearrange the terms of the above equation, thus the SNDR reduces to (17) as illustrated in *theorem* 1.

APPENDIX B: PROOF OF THEOREM 2

We put the value of $\gamma_{non,Nhops}^{FD-AF}$ into (34), the ergodic capacity becomes (73), as shown at the bottom of the next page. We can

$$\boldsymbol{\varphi}_1 = \frac{(\rho_1\rho_2\ldots\rho_N)\,P_1\left(G_1^2G_2^2\ldots G_{N-1}^2\right)}{\left[1-\left(G_1^2G_2^2N_1^{self-interfer}N_2^{self-interfer}\ldots G_{N-1}^2N_{N-1}^{self-interfer}\right)\right]}. \tag{69}$$

$$\boldsymbol{\varphi}_2 = \frac{\left(G_1^2G_2^2\ldots G_{N-1}^2\right)\begin{bmatrix}\rho_1\rho_2\ldots\rho_N\kappa_1^2P_1+\rho_2\rho_3\ldots\rho_NN_1+\rho_2\rho_3\ldots\rho_N\kappa_2^2P_2\\ +\rho_3\ldots\rho_NN_2+\ldots+\rho_{N-1}\rho_N\kappa_{N-2}^2P_{N-2}\\ +\rho_{N-1}\rho_NN_{N-2}+\rho_N\kappa_{N-1}^2P_{N-1}+\rho_NN_{N-1}\end{bmatrix}}{\left[1-\left(G_1^2G_2^2N_1^{self-interfer}N_2^{self-interfer}\ldots G_{N-1}^2N_{N-1}^{self-interfer}\right)\right]}. \tag{70}$$

$$\frac{\rho_1\rho_2\ldots\rho_N}{\left\{\begin{array}{l}\frac{1}{P_1}\begin{bmatrix}\rho_1\rho_2\ldots\rho_N\kappa_1^2P_1+\rho_2\rho_3\ldots\rho_NN_1+\rho_2\rho_3\ldots\rho_N\kappa_2^2P_2\\ +\rho_3\ldots\rho_NN_2+\cdots+\rho_{N-1}\rho_N\kappa_{N-2}^2P_{N-2}\\ +\rho_{N-1}\rho_NN_{N-2}+\rho_N\kappa_{N-1}^2P_{N-1}+\rho_NN_{N-1}\end{bmatrix}\\ +\dfrac{N_N^{noise}\left[1-\left(G_1^2G_2^2N_1^{self-interfer}N_2^{self-interfer}\ldots G_{N-1}^2N_{N-1}^{self-interfer}\right)\right]}{P_1\left(G_1^2G_2^2\ldots G_{N-1}^2\right)}\end{array}\right\}} \tag{71}$$

apply the Jensen's inequality in [13] to obtain (74), as shown at the bottom of the page, where the coefficient $a = \sum_{i=1}^{N-1} a_i$ and $b \sum_{i=1}^{N-1} b_i$. An upper bound for the ergodic capacitycan be rewritten in a simple form as (75), shown at the bottom of the page. The expectation in the above equation is calculated using the MGF of the product of random variable. Let $\{\rho_i\}_{i=1}^{N}$ be N random variable with PDF given in (1). The MGF of the new random variable Y_1 is the product of powers of N random variables ρ_i, i.e.,

$$Y_1 = \prod_{i=1}^{N} \rho_i^{l_i/k} \tag{76}$$

with $l_1, l_2, \ldots, l_N$ and k being positive integers, is defined as:

$$\mathcal{M}_{Y_1}(-s) = \int_0^\infty \int_0^\infty \ldots \int_0^\infty e^{-s x_1^{l_1/k} \cdots x_N^{l_N/k}} \times f_{X_1}(x_1) \ldots f_{X_N}(x_N)\, dx_1 \ldots dx_N. \tag{77}$$

where $f_X(x)$ is the PDF given in (1). The first integral in (77) can be rewritten as:

$$I_1 = \int_0^\infty e^{-s x_1^{l_1/k} f_{X_1}(x_1)} dx_1 = \int_0^\infty x_1^{\alpha_1 - 1} e^{-x_1/\beta_1} e^{-s W_2 x_1^{l_1/k}} dx_1 \tag{78}$$

where $W_i = x_i^{l_i/k} x_{i+1}^{l_{i+1}/k} \ldots x_N^{l_{i+N}/k}$. Using [36], the first integral can be expressed as:

$$I_1 = \int_0^\infty x_1^{\alpha_1 - 1} G_{0,1}^{1,0}\left[\frac{x_1}{\beta_1}\middle| \begin{matrix} - \\ 0 \end{matrix}\right] G_{0,1}^{1,0}\left[s W_2 x_1^{l_1/k}\middle| \begin{matrix} - \\ 0 \end{matrix}\right] dx_1 \tag{79}$$

Using the solution of integral in [36, eq. (21)], thus the integral, I_1 can be solved as follows:

$$\frac{\sqrt{k}\,\ell_1^{\alpha_1 - 1/2}}{\beta_1^{-\alpha_1}\left(\sqrt{2\pi}\right)^{\ell_1 - 1 + k - 1}} G_{\ell_1,k}^{k,\ell_1}\left[\frac{(sW_2)^k k^{-k}}{(\beta_1 \ell_1)^{-\ell_1}}\middle| \begin{matrix} \Delta(\ell_1, 1-\alpha_1) \\ \Delta(k, 0) \end{matrix}\right] \tag{80}$$

In similar, the second integral can be written as:

$$I_2 = \frac{\sqrt{k}\,\ell_1^{\alpha_1 - 1/2}}{\beta_1^{-\alpha_1}\left(\sqrt{2\pi}\right)^{\ell_1 - 1 + k - 1}} \int_0^\infty x_2^{\alpha_2 - 1} G_{0,1}^{1,0}\left[\frac{x_2}{\beta_2}\middle| \begin{matrix} - \\ 0 \end{matrix}\right] \times G_{\ell_1,k}^{k,\ell_1}\left[\frac{(sW_3)^k k^{-k}}{(\beta_1 \ell_1)^{-\ell_1}} x_2^{\ell_2}\middle| \begin{matrix} \Delta(\ell_1, 1-\alpha_1) \\ \Delta(k, 0) \end{matrix}\right] dx_2. \tag{81}$$

By using the solution in [36], the integral, I_2 becomes:

$$I_2 = \frac{\sqrt{k}\,\ell_1^{\alpha_1 - 1/2} \ell_2^{\alpha_2 - 1/2}}{\beta_1^{-\alpha_1} \beta_2^{-\alpha_2}\left(\sqrt{2\pi}\right)^{\ell_1 + \ell_2 - 2 + k - 1}} \times G_{\ell_1+\ell_2,k}^{k,\ell_1+\ell_2}\left[\frac{(sW_3)^k k^{-k}}{(\beta_1 \ell_1)^{-\ell_1} (\beta_2 \ell_2)^{-\ell_2}} \times \middle| \begin{matrix} \Delta(\ell_1, 1-\alpha_1), \Delta(\ell_2, 1-\alpha_2) \\ \Delta(k, 0) \end{matrix}\right]. \tag{82}$$

$$\gamma_{Nhop}^{FD-AF} = \frac{\prod_{n=1}^{N} \rho_n}{\left\{ \begin{matrix} \frac{1}{P_1}\left[\begin{matrix} \kappa_1^2 P_1 \prod_{n=1}^{N} \rho_n + N_1 \prod_{n=2}^{N} \rho_n + \kappa_2^2 P_2 \prod_{n=2}^{N} \rho_n \\ + N_2 \prod_{n=3}^{N} \rho_n + \cdots + \kappa_{N-2}^2 P_{N-2} \prod_{n=N-1}^{N} \rho_n \\ + N_{N-2} \prod_{n=N-1}^{N} \rho_n + \kappa_{N-1}^2 P_{N-1} \rho_N + N_{N-1} \rho_N \end{matrix} \right] \\ + \dfrac{N_N^{noise}\left(1 - \prod_{j=0}^{N-2} G_{i+1}^2 N_{i+1}^{self-interfer}\right)}{P_1 \prod_{i=0}^{N-2} G_{i+1}^2} \end{matrix} \right\}}. \tag{72}$$

$$C_{non-ideal,N}^{FD-AF} \triangleq \frac{1}{2} \mathbb{E}\left\{ \log_2\left(1 + \frac{\prod_{i=1}^{N} \rho_i}{\sum_{i=1}^{N-1}\left(a_i \prod_{i=j-1}^{N} \rho_i\right) + \sum_{i=1}^{N-1}\left(b_i \prod_{i=j}^{N} \rho_i\right) + c_i}\right)\right\} \tag{73}$$

$$C_{non-ideal,N}^{FD-AF} \leq \frac{1}{2} \log_2\left(1 + \frac{\mathbb{E}\left\{\prod_{i=1}^{N} \rho_i\right\}}{a\mathbb{E}\left\{\sum_{i=1}^{N-1} \prod_{i=j-1}^{N} \rho_i\right\} + b\mathbb{E}\left\{\sum_{i=1}^{N-1} \prod_{i=j}^{N} \rho_i\right\} + c_i}\right). \tag{74}$$

$$\frac{1}{2} \log_2\left(1 + \frac{\mathbb{E}\left\{\prod_{i=1}^{N} \rho_i\right\}}{a\mathbb{E}\left\{N \prod_{i=1}^{N} \rho_i^{(N+1-i)/N}\right\} + b\mathbb{E}\left\{N \prod_{i=2}^{N} \rho_i^{(N+1-i)/N}\right\} + c_i}\right) \tag{75}$$

$$\mathbb{E}\left\{\mathrm{h}_i y_i^{\mathrm{H}}\right\} = \mathbb{E}\left\{\mathrm{h}_i\left[h_i^H G\sum_{j=1}^{\infty}\left(h_{\mathrm{self},i}G\right)^{j-1} \times \left(s_i^*\left[i-j\tau\right]+\eta_i\left[i-j\tau\right]+v_i\left[i-j\tau\right]\right)+\eta_{i+1}\left[i\right]\right]\right\} = \frac{\rho_i G P_i}{1-N_{i+1}^{self-interfer}G}. \quad (87)$$

Following the same steps, the final result of the integral in (77) can be formulated as:

$$\mathcal{M}_{Y_1}(-s) = \frac{\sqrt{k}\prod_{i=1}^{N}\ell_i^{\alpha_i-1/2}}{\left(\sqrt{2\pi}\right)^{r-N+k-1}\prod_{i=1}^{N}\Gamma(\alpha_i)} \times G_{r,k}^{k,r}\left[\frac{(-1)^k\left(\frac{s}{k}\right)^k}{\prod_{i=1}^{N}(\beta_i\ell_i)^{-\ell_i}}\,\middle|\,\begin{matrix}\Delta(\ell_1,1-\alpha_1),\ldots,\Delta(\ell_N,1-\alpha_N)\\ \Delta(k,0)\end{matrix}\right] \quad (83)$$

where the values of r and $\Delta(k,u)$ are given by:

$$-r = \sum_{i=1}^{N}\ell_i \quad (84)$$

$$-\Delta(k,u) \triangleq u/k, (u+1)/k, \ldots, (u+k-1)/k \quad (85)$$

APPENDIX C

The general expression of an LMMSE estimator at the ith RS is given by [42, Ch. 12]:

$$\hat{h}_i = \mathbb{E}\left\{\mathrm{h}_i y_i^{\mathrm{H}}\right\}\left(\mathbb{E}\left\{y_i y_i^{\mathrm{H}}\right\}\right)^{-1} y_i \quad (86)$$

where y_i is the transmitted signal at $i$thRS and h_i is the channel estimate at the $i$thRS (87), as shown at the top of the page. In addition, we have that

$$\mathbb{E}\left\{y_i y_i^{\mathrm{H}}\right\} = \mathbb{E}\left\{h_i G\sum_{j=1}^{\infty}\left(h_{\mathrm{self},i}G\right)^{j-1}\left(s_i\left[i-j\tau\right]+\eta_i\left[i-j\tau\right] + v_i\left[i-j\tau\right]\right)+\eta_{i+1}\left[i\right] \times h_i^H G\sum_{j=1}^{\infty}\left(h_{\mathrm{self},i}G\right)^{j-1} \times \left(s_i^*\left[i-j\tau\right]+\eta_i^H\left[i-j\tau\right] + v_i^H\left[i-j\tau\right]\right)+\eta_{i+1}^H\left[i\right]\right\} \quad (88)$$

After some algebraic processing, the expectation becomes:

$$\mathbb{E}\left\{y_i y_i^{\mathrm{H}}\right\} = \frac{G^2\rho_i}{1-G^2 N_{i+1}^{self-interfer}} \times\left[P_i+\kappa_i^2 P_i+N_i\right]+\kappa_{i+1}^2 P_{i+1}. \quad (89)$$

Plugging (87) and (89) into (86), the final expression in (56) is obtained.

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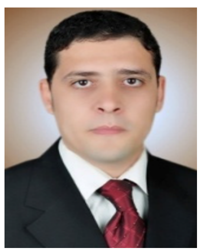

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