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RESEARCH ARTICLE

A Hybrid Smart Quantum Particle Swarm Optimization for Multimodal Electromagnetic Design Problems

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ABSTRACT Quantum behaved particle swarm optimization (QPSO) has been one of the most widely used algorithm in engineering world. Since its debut in 2004, QPSO has been used for resolving numerous complicated multimodal problems. Moreover, considering the adaptability and versatility, it has resolved a variety of real-world and test problems. To tackle numerical and engineering optimization problems, we introduce novel hybrid algorithm QPSODE. The novel hybrid algorithm integrates Quantum behaved particle swarm optimization (QPSO) with differential evolution (DE) strategy. A crossover and selection (influenced by DE) is used in the QPSODE's position updating mechanism. During the selection process, the Boltzmann operator is applied to the position vectors of two randomly chosen particles, not to their individual optimum placements. Therefore, unlike the QPSO, a particle is only relocated to a new position if it has a higher fitness value, implying the application of a selection strategy across the whole search space. Additionally, the hybrid algorithm is improved by introducing proper parameters tuning, control parameter, path disparity. The hybrid algorithm enhances the algorithm's performance by speeding up the convergence and avoiding the premature convergence, the main flaw in the earlier algorithms. The proposed algorithm is put to test, by using 19 well-known benchmark test functions and the engineering optimization problem for superconducting magnetic energy storage (SMES). In terms of the quality of the resulting outputs, QPSODE outperforms various state-of-the-art approaches.

INDEX TERMS Smart quantum particle swarm, DE, hybridization, electromagnetic device, SMES.

I. INTRODUCTION

In fields of electrical engineering and mathematics, the electromagnetic design problems are interesting but at the same time offer complications enough to deal with numerical techniques. The traditional deterministic techniques are facing the same problem that they do not have the potential to find the global optimal solutions for the complex electromagnetic design problems as well. In the early 1970s first time multi-agent evolutionary algorithms inspired from the biological process known as genetic algorithm (GA) was applied in electromagnetic inverse scattering [1]. Subsequently, other optimal technique namely differential evolution (DE) and PSO have been used for the solution of electromagnetic inverse problems [2],[3],[4]. The elimination of analytical evaluation of cost function in the process of optimization and swarm intelligence algorithm as well as its biological procedure was explained and described in details in [5], [6]. More importantly, the randomization step in evolutionary algorithms magnifies the search constraints in the solution space and assists the evolutionary algorithms to escape from the local optima. Thus, evolutionary algorithms are known as stochastic or heuristic optimization algorithms. As, the best quality of stochastic optimization algorithms is flexibility and tractability. Moreover, in the searching process, the control parameters of an optimal algorithm also play an imperative

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role. Consequently, one should fix the trail base values range of an algorithm in order to avoid false solutions while it provides the information about inverse problems and the requirement is not comprised on the bases of a given problem. Also, the stochastic optimization algorithm is more precise and convenient for the purpose to take advantage with the prior information about problem. As a consequence, a wealth of stochastic optimization algorithms was incorporated in various fields of engineering. Moreover, PSO algorithm perform effectively and is much more efficient in finding the global optimal solution [7]. PSO was inspired from the group of birds and schooling fishes, first introduced by two scientists in 1995 namely "Kennedy and Eberhart"[8]. Due to the presence of metaheuristic properties of PSO, it is more suitable for solving real world and benchmark problems. Also, different variants of PSOs were developed that proved useful in many research areas.

A. INCEPTION OF THE QUANTIZED PSO

Nevertheless, PSO encounters premature convergence problems because of lack of diversity of the individuals (particles) at the final stages of the optimization process and improper balance between the global and local searches [9]. Real world problems, have been recognized as an active research topic in the fields of academia and engineering sciences, and the optimal solution to such kinds of problems is difficult and hard due to the presence of multi-modal cost functions. Because traditional optimization methods are incapable of resolving complex or real-world problems, a wealth of studies has consequently contributed to the development of nature-inspired algorithmic models, to improve computational capabilities and diversity of the search space in engineering complex and complicated problems. At the same time, researchers have tried to design various nature-inspired algorithmic models in the state of the art to enhance the computational capabilities as well as increase the diversity of search space in engineering optimization problems. Electromagnetic problem has been investigated for more than a decade. In general, it sometimes refers to the optimal electromagnetic device design that occurs naturally in many engineering disciplines. This work focused on the optimization of a Superconducting Magnetic Energy Storage (SMES) system, employing a well-known meta-heuristic optimization approaches like PSO to address the problem in electromagnetic devices. According to the previous works, the searching process of PSO method is limited at every generation and it is hard to cover the whole region of the optimization problem [10], due to which the traditional PSO algorithm has a hard time convergence to a global optimal solution. To tackle and control such issues, Sun *et al.* [11] developed a quantum-behaved particle swarm optimization (QPSO) algorithm using the basic procedure of PSO method as describe bellow. Study of the convergence of the classical PSO, quantum system gave the foundation for the formulation of QPSO. In quantum physics, the wave function can be used to represent the state of a particle with momentum and energy. QPSO, suppose that each particles is in a quantum state instead of position and velocity like in PSO, and its formulation is determined by its wave function. The probability of a particle occurring at a specific position can be calculated from the probability according to the wave function.

In the QPSO process, a wave function $\Psi(\boldsymbol{x}, \boldsymbol{t})$ is associated with the particles in spite of position and velocity $(\boldsymbol{x}(\boldsymbol{t}), \boldsymbol{v}(\boldsymbol{t}))$. The author presented a delta potential well model by using time dependent Schrodinger equation depicted by

$$i\hbar(x,t) = \frac{-\hbar}{2m}\frac{d^2(x,t)}{dx^2} + V(x)(x,t) \tag{1}$$

where, $\hbar$ is Planck constant, $\boldsymbol{V}(\boldsymbol{x})$ is the potential energy and $\Psi(\boldsymbol{x}, \boldsymbol{t})$ is a quantum state known as normalized wave state vector. The $\Psi(\boldsymbol{x}, \boldsymbol{t})$ likeness of particle in mechanics is expressed as

$$\Psi(x) = \frac{1}{\sqrt{L}} e^{\left(\frac{-|z-x|}{L}\right)} \tag{2}$$

where $\boldsymbol{z}$ is a converge point in the search space. Max Born gives the interpretation of particle appearance in search space by using a probability density function of quantum state as given by

$$\text{Probability Density Function} = |\Psi(x_t)|^2 = \frac{1}{L} e^{\left(\frac{-2|z-x_t|}{L}\right)} \tag{3}$$

Subsequently, the position function obtained by the Monto Carlo stochastic model is given as follows

$$X_i(t+1) = \begin{cases} p(t) + \beta * |M_{best} - X_i(t)| * \ln\frac{1}{u}, & if\ u \geq 0.5 \\ p(t) - \beta * |M_{best} - X_i(t)| * \ln\frac{1}{u}, & otherwise \end{cases} \tag{4}$$

where $\boldsymbol{\beta}$ is a contraction expansion coefficient and $\boldsymbol{M_{best}}$ is the mean best written as follows

$$M_{best} = \frac{1}{M}\sum_{i=1}^{M} P_{best_i}(t) \tag{5}$$

Recently, QPSO has been successfully applied in various fields including electromagnetic design [12], [13] semiconductor design [14], bioinformatics [15], [16], and so on. Moreover, according to the statement "no free lunch theorem" none of the optimization methods or algorithms have the ability to solve all kinds of problems. In the past few years, QPSO and DE have emerged as powerful optimization tools for solving complex optimization problems. Despite the fact that DE and QPSO have been effectively applied to a wide range of problems, including test and real-world problems, both include shortcomings that might cause to effect the algorithms' performance. Consequently, a novel QPSODE model is presented in this research work. The main proposal of the novel approach is that we have integrated QPSO with the standard basic DE algorithm.

The rest of this paper is structured as follows. Sections 2 discuss the hybridized QPSODE and associated research. The proposed methodology of QPSODE is described in

Section 3. Section 4 presents numerical data and analysis proving the performance of QPSODE in contrast to the basic QPSO and DE and many state-of-the-art algorithms across a suite of 19 numerical optimization problems as well as an electromagnetic problem, followed by the conclusion in Section 5.

II. RELATED WORKS

Previously, a hybrid QPSO algorithm was developed by Nirmal Kumar *et al.* by integrating the advanced quantum behaved particle swarm optimization (QPSO) method and the global quantum algorithm which employed the binary tournamenting process for the purpose to enhance the basic QPSO performance [17]. The main idea of the proposal is to keep the balance between the global search process and local search process. Kusum Deep *et al.* hybridized the PSO method to the quadratic approximation operator (QA). According to their approach, the whole swarm is divided into sub swarms [18]. Also, the QA operator is indulged into sub swarms for the purpose to modify the leader of the candidates among the sub swarms. Shahin Pourbahrami developed an improved seeding particle swarm optimization method. The main proposal of the mechanism is to generate noticeable features while storing all information in the storage list [19]. According to the strategy, the best objective function value will participate for the coming generation and also to search and find the enlightened features in order to improve exploration searches. Also, the novel method improves the searching space and uses the chaos theory to enhance the swarm of PSO procedure, so in this way the new formula optimizes the feature size in the said algorithm. The different control parameters values in the stochastic search methods have a key and primary role in the process of evolution. As, in this paper the values should be static or varied during the search process, as explained and described in details [20]. Also, various ideas on the bases of complementary features were paid special devotions to fix the factors and parameters formulation or model (values) according to optimization design problems. Thresholding techniques are most popular in the field of image processing due to its reliability and low computational cost. In the mentioned approach the author explained in detail the multi-threshold segmentation of 2D Kapur's entropy which is incorporated a hybrid adaptive quantum behaved particle swarm optimization (HAQPSO) algorithm [21]. However, Gaussian distribution function and its chaotic model and levy flight are used in the mentioned algorithm for the purpose of controlling the diversity loss and keeping a proper balance between the exploration and exploitation searches. Considering the drawbacks and deficiencies of the traditional PSO method, the author incorporated the integration of the simulated annealing (SA), co-evolution theory, quantum behavior theory and diversity-guided mutation strategy (MSCQPSO) [22]. The central approach of the mechanism to divide the swarms into sub swarms on the bases of best fitness value improves the global search ability. The introducing of quantum behavior theory will vary the mode of the particles and as a result the method avoids premature convergence. In order to improve the preceding approach a more capable QPSO method by introducing a novel strategy combining the social learning and Lévy flights (LSL-QPSO) is introduced [23]. According to this approach the social learning strategy is employed in order to modernize the slow particle and increase the global search ability. According to the Lévy flights mechanism, the premature process is controlled and further improves the convergence accuracy and search efficiency of the algorithm. T. M. Habash *et al.* researched different optimal methods and its hybridization techniques for the purpose to enhance the algorithms performance using more capable methodologies for solving complicated benchmark problems [24]. A especial kind of portfolio models having fuzzy return rates regarding its risk were employed by fuzzy theory and incorporated in the traditional QPSO process [25]. Also, a novel hybrid probability distribution method as well as beta parameter having non-linear plot are introduced for the purpose to improve the diversity of the searching process and avoid a premature convergence. A new algorithm that combines the concepts of QPSO and BFA is developed for the balance between local and global searches. P-spline is interpolated in which MR images is tested and several additional medical images using the RMI similarity index improved by BF-QPSO [26]. To solve electromagnetic design problems, a QPSO method having randomized mean formulation was adapted [27], [28]. The central idea of the selection process is simple and logical which choses randomly on the bases of the objective function, as all these global functions were employed in the QPSO. The authors introduced new set in the QPSO process which incorporated personal /local or global worst terms to the attractor model in order to control the diversity of the swarm at the latter stages of the optimization process and keep a good balance between the exploration and exploitation. Min *et al.* integrated the EO method with the PSO algorithm for the purpose to keep a decent balance between the global and local searches [29]. According to their idea the EO algorithm showed best performance on the local search while the PSO technique perform well on global searching, as in this way the diversity of the swarm is preserved at the final stages of optimization process. Manja *et al.* present an exhaustive survey of various applications of Quantum-inspired computational intelligence (QCI) algorithms [30]. Moreover, in the said paper the authors clearly explain the applications of grouping as well as its importance which is useful for researchers on Quantum computing in exploring this upcoming and novel discipline. R. logesh *et al.* introduce a new cluster method in the basic QPSO for filtering based recommender system. Also, the addition of novel segment of ABC technique was explained in [31], where the innovative idea integrates the PSO technique for the purpose to address the issue of the unbalance and lacking of diversity among the candidates. Min- Liang Huang introduces a novel strategy in the traditional PSO process to empower every candidate which possess quantum behavior, for the purpose to widen its search space, and as a result it will improve the forecasting accuracy. According to the strategy the QPSO algorithm is integrated with the chaotic mapping function including load forecasting model namely the support vector regression with a chaotic QPSO model [32]. The main proposal of the novel mechanism is to achieve more precise forecasting performance. Bessem *et al.* develop a new technique with the purpose to design a model for the electromagnetic disturbances. Authors proposed approach in this paper by incorporating the PSO method [33]. From the above discussions we conclude that our novel approach will enhance the particles exploration capability and the particles will be more aware as compared to the basic process stated in detail in section 3.

III. METHODOLOGY OF HYBRIDIZED QPSODE ALGORITHM

Despite the fact that the QPSO and DE algorithms have been effectively applied to a variety of optimization problems in several disciplines of engineering. The basic QPSO offers a variety of advantages, including ease in implementation, less computational time, robustness, rapid convergence, low processing time; but from the previous work we know that this basic algorithms have limitations and premature convergence problem, and as results the algorithm converge to local optima. Consequently, the traditional QPSO and DE algorithms can easily converge to local minima in complicated and multimodal optimization problems, resulting in a slow and premature convergence. In this respect, a novel approach by comprising the integration of QPSO and DE algorithms is proposed. In the evolution process the imperative role of QPSO algorithm is to improve the exploration process of particles while the DE algorithm is to enhance the local searches of the particles. According to the proposed approach, the novel crossover strategy and the best selection strategy are introduced into the basic QPSO process. An exploration and exploitation balancing strategy is used to tackle this issue. QPSO parameters are initially chosen to maximize explorations while minimizing exploitation. To get the better of this issue, the crossover and selection phenomenon are plagiarized from DE and incorporated into QPSO design to avoid premature phase of electromagnetic optimization problems. The detailed steps of parameter setting and hybridized smart particle selection for solving premature convergence are described as follows.

A. PARAMETER SETTING

1) PARAMETERS TUNING

Recently, a wealth of global optimization methods and models was parameterized, which allows them to be fine-tuned for a particular situation. At the same time researchers attempted to figure out what parameters to use for various type algorithms. In the search process, some parameters in the suggested model can be fine-tuned in order to find the best optimal solution in the search space. In addition, the proposed algorithm employs revised sets of random numbers $\boldsymbol{\varphi}$ and phi $\boldsymbol{\rho}$ with a 0.5 offset instead of a pure random integer, as illustrated in Equations (6) and (7).

$$\varphi = u + n_{offset} \tag{6}$$

$$\rho = u + n_{offset} \tag{7}$$

2) ADAPTIVE PARAMETER CONTROL

The proposed mechanism added adaptive parameters in order to improve the algorithm's performance. In the adaptive parameters, the search feedback is utilized as input to the system which determines the best optimal solution during the search process. A novel formulation was adapted for $\boldsymbol{\alpha_i}$ parameter in this research paper, while the values of adaptive parameter are determined by the ratio of absolute difference of the $\boldsymbol{i^{th}}$ particle fitness with the neighbor fitness divided by the square of the worst fitness in the same swarm generation,

$$\alpha_i = \frac{\left|f(X_i)-f(X_j)\right|}{(f(X_{worst}))^2} + rand() \tag{8}$$

$\boldsymbol{X_j}$ is a neighbor particle of $\boldsymbol{X_i}$

B. MODIFIED MEAN BEST POSITION

In this paper we introduced a novel mathematical equation for the mean best particles position to enhance the proposed mechanism. The $\boldsymbol{M_{best}}$ denotes the mean best position which is defined as the average of the $\boldsymbol{P_{best}}$ positions of all the particles incorporated with path disparity:

$$M_{best} = \frac{1}{M}\sum_{i=1}^{M} P_{best_i}(t) * \alpha_i\{i = 1, \ldots .N\} \tag{9}$$

C. HYBRIDIZED SMART PARTICLE SELECTION USING BOLTZMANN PHENOMENON

In this section, we discuss how to increase the searching capabilities of an optimization algorithm by using the best particle nomination (smart particle) in the swarm. The new study of QPSODE uses a memory bank called archive to store the current and the past P_{best} for an improved selection of the global best particle G_{best}. Previous work mainly focused on selecting the global best particle G_{best} from the P_{best} of the current iteration of the population. The mechanism is represented mathematically as follows:

$$P_{best_i}(t) = \begin{cases} X_i \text{ if } f(X_i(t)) < f(P_{X_{i-1}}(t)) \\ P_{X_{i-1}}(t) \quad otherwise \end{cases} \tag{10}$$

In this process, the particle's current $\boldsymbol{P_{best}}$ location is compared to its previously stored $\boldsymbol{P_{best}}$ position, and if it is better, the prior $\boldsymbol{P_{best}}$ is replaced; otherwise, the previous value is retained for future use. To further improve the global search and decrease the local search of solution candidates in the search space, we employed the Boltzmann selection to pick smart particles, taking into account the hybridization algorithm's balance between exploration and exploitation searches. We present QPSODE, a hybrid algorithm framework that integrates DE's crossover and selection operators with QPSO.

In the search space, the initial population $X_{initial} = [x_i, \ldots.., x_M]$(M stands for swarm size) is generated randomly. In crossover, each particle x_i(t) ($i = 1, \ldots\ldots; N$) is copied from v_i with the probability $Cr \in [0, 1]$ (crossover rate) and is taken 0.9 because it shows good response to higher dimension search problems where i is equals an index $J_{rand} \in [1, N]$

$$x_i(t+1) = \begin{cases} v(t)\, P_{boltzman} \leq Crori = i_{rand} \\ x(t) \quad otherwise \end{cases} \tag{11}$$

An investigation of Cr and $P_{boltzman}$ demonstrates how their values influence QPSODE performance by using low selection probability $P_{boltzman}$ that can be used to enhance the performance of the proposed hybridized algorithm. The $P_{boltzman}$ value is generated according to the following formula,

$$P_{boltzman} = 1 - \frac{1}{D}\frac{\exp(\sqrt{f(X_i)}/T)}{\sum_{i=1}^{N}\exp(f(X_w)^2/T)} \tag{12}$$

$$T = T_0(\mu^t) \tag{13}$$

where $P_{boltzman}$ is the selection probability of particle Xi. If the particle fitness $f(Xi)$ is small then the selection probability of Xi will be maximum. T and T_0 are the temperature and the initial temperature as set between 100 and 200 respectively. μ is a constant smaller than 1 and acts as a control parameter to change the speed of the selection process. For better results, the previous experimental researches validate that its value should lies between 0.991 and 0.999. Therefore, the proposed algorithm also used same value of μ to be effective and having a good convergence for complex problems without premature occurrences.

IV. NUMERICAL RESULTS AND ANALYSIS

A. BENCHMARK FUNCTIONS

Numerous benchmark functions are used as test problems in the article to examine the newly proposed algorithms from CEC2005. In this study, a set of 19 of them were used for the numerical experiments to assess the performance of the hybridized QPSODE. The functions are composed of unimodal, multi-modal, shifted, and rotated functions, as given in Table 1. Even though a lot of the benchmark functions are tough and complex.

B. COMPARISON OF QPSODE WITH OTHERS METHODS

In this session the proposed hybridized QPSODE is compared to several standard and well-known algorithms which are MPSOEG [34], standard QPSO [11], original DE [35], and RMPSO presented in [36] to evaluate its effectiveness. Experientially, for all algorithms the evaluation conditions e.g. the population size is N=40, the number of generations is 500, dimensions 10D, 30D and benchmark functions reported in Table 1 are supposed to be the same for judicious comparison. For MPSOEG, and RMPSO the learning parameters, C_1 (cognitive) are actually pulling particles toward P_b and C_2 (social) trying to push particles to G_b, are set as 2.

TABLE 1. Mathematical test problems.

No	Test Problems	Modal Class	Dimension	Search range	F(x*)
1	Weierstrass Function	Shifted Rotated	10	[-0.5,0.5]	+90
2	Griewank's Function	Rotated	10	[0,600]	-180
3	Rosenbrock Function	Shifted	30	[-100,100]	+390
4	Rastrigin Function	Multi-modal	30	[-5,5]	--
5	Griewank's Function	Multi-modal	10	[0,600]	--
6	Griewank's Function	Multi-modal	30	[0,600]	--
7	Schwefel's 2.22 Function	Uni-modal	30	[-100,100]	--
8	Rastrigin Function	Multi-modal	10	[-5,5]	--
9	Schwefel's 2.22 Function	Uni-modal	10	[-100,100]	--
10	Michalewicz Function	Multi-modal	10	[0,π]	--
11	Michalewicz Function	Multi-modal	30	[0,π]	--
12	Sphere Function	Uni-modal	10	[-100,100]	--
13	Sphere Function	Uni-modal	30	[-100,100]	--
14	Hyper-Ellipsoid Function	Rotated	10	[-65, 65]	--
15	Hyper-Ellipsoid Function	Rotated	30	[-65, 65]	--
16	Schumer Steiglitz Function	Uni-modal	30	[-100,100]	--
17	Chung Reynolds Function	Uni-modal	10	[-100,100]	--
18	Chung Reynolds Function	Uni-modal	30	[-100,100]	--
19	Schumer Steiglitz Function	Uni-modal	10	[-100,100]	--

Crossover rate CR is set to 0.9 and mutation factor F is set at 0.6 for DE and QPSODE.

In order to compare the different approaches, an appropriate measuring parameters metrics should be used. An evolutionary algorithm's credibility is also determined by its ability to how greatly they handle optimization problems. All implementations are carried out under the identical conditions as presented in Table 1 in order to generate fair outcomes. Tables 2, 3 and 4 present a comparative analysis of the proposed QPSODE against those of other algorithms by using four indicators: minimum f_b, maximum f_w, mean f_{mean}, and standard deviation (SD).

On the testing suite, the QPSODE outperforms the MPSOEG, standard QPSO, DE, and RMPSO in terms of optimization results. Particles stopping exploration attempts as QPSO swiftly converges to local optima like other

TABLE 2. Numerical comparison of best f_b, max f_w, mean f_{mean}, and sd value of test problems.

Problem	Algorithm	f_b	f_w	f_{mean}	SD
f_1	QPSODE	-8.71	2.10	-1.45	2.06
	MPSOEG	-0.52	2.29	0.43	1.08
	Standard QPSO	-5.36	-0.91	-3.41	1.53
	Standard DE	1.65	4.51	1.68	0.23
	RMPSO	-4.31	4.15	-2.63	2.59
f_2	QPSODE	-6.30	5.19	0.17	2.38
	MPSOEG	3.95	5.19	4.27	0.54
	Standard QPSO	5.19	5.19	5.19	0.00
	Standard DE	4.68	5.19	5.10	0.20
	RMPSO	5.19	5.19	5.19	0.00
f_3	QPSODE	-11.14	8.00	-3.01	3.90
	MPSOEG	-9.69	0.24	-9.50	1.14
	Standard QPSO	-9.15	2.63	-7.45	3.16
	Standard DE	-1.33	-1.33	-1.33	0.00
	RMPSO	3.56	7.65	3.76	0.63
f_4	QPSODE	-30.56	5.27	-21.18	14.96
	MPSOEG	-7.89	2.65	-5.30	2.56
	Standard QPSO	-22.21	5.73	-15.49	9.83
	Standard DE	4.58	5.76	4.62	0.17
	RMPSO	-0.40	6.19	1.69	1.66
f_5	QPSODE	-39.96	-6.30	-17.77	12.44
	MPSOEG	-22.12	-11.7	-19.07	3.43
	Standard QPSO	-35.39	-1.47	-13.11	9.80
	Standard DE	-27.98	3.10	-11.07	10.12
	RMPSO	-15.90	-5.72	-15.00	1.53

algorithms. For the rest of evolution, the QPSO becomes trapped. QPSODE quickly escapes local optima and converges to the global optimum after incorporated DE. As a result, the proposed QPSODE is proven to be successful and efficient in global searches.

C. GRAPHICAL RESULTS AND DISCUSSION

The aim of this section is also to be more descriptive on the efficiency and effectiveness of the proposed approach. Numerical experimental results of the proposed approach are graphically compared with those of MPSOEG, standard QPSO, original DE, and RMPSO. We further investigate and witness the QPSO and DE executed self-sufficiently in this work in order to measure the effectiveness of QPSODE.

The performance of QPSODE are graphically reported in Figures 1-5. The results show that the hybridization of QPSO with DE (QPSODE) increase the performance of effectiveness to tackle different optimization problems. We may conclude that DE enhances the QPSO's exploration and exploitation ability to fulfill the needs of optimization of electromagnetic problems.

TABLE 3. Numerical comparison of best f_b, max f_w, mean f_{mean}, and sd value of test problems.

Problem	algorithm	f_b	f_w	f_{mean}	SD
f_6	QPSODE	-36.98	-6.30	-20.41	13.72
	MPSOEG	-20.83	-9.53	-18.93	2.47
	Standard QPSO	-30.23	-12.47	-19.95	5.62
	Standard DE	-15.31	-1.40	-12.62	3.19
	RMPSO	-10.66	-5.02	-10.54	0.58
f_7	QPSODE	-242.37	-0.47	-159.46	77.59
	MPSOEG	-5.97	0.33	-5.47	0.62
	Standard QPSO	-226.6	9.18	-119.7	71.41
	Standard DE	-1.10	4.76	-0.05	1.68
	RMPSO	-12.75	4.05	-7.24	5.15
f_8	QPSODE	-33.66	3.13	-31.66	7.54
	MPSOEG	-12.16	2.75	-10.68	3.59
	Standard QPSO	-24.32	2.10	-23.19	4.60
	Standard DE	2.68	4.62	2.74	0.27
	RMPSO	-8.20	2.34	-3.79	2.44
f_9	QPSODE	-401.47	9.19	-255.76	121.69
	MPSOEG	-6.06	0.10	-5.23	0.86
	Standard QPSO	-321.41	0.19	-222.66	99.16
	Standard DE	-10.61	3.60	-9.67	3.20
	RMPSO	-13.96	3.33	-12.62	4.19
f_{10}	QPSODE	-739.40	-30.40	-601.05	210.03
	MPSOEG	-63.90	-34.26	-58.40	7.15
	Standard QPSO	-509.63	-35.51	-386.22	133.02
	Standard DE	-12.55	-12.55	-12.55	0.00
	RMPSO	-93.59	-7.58	-84.95	14.50

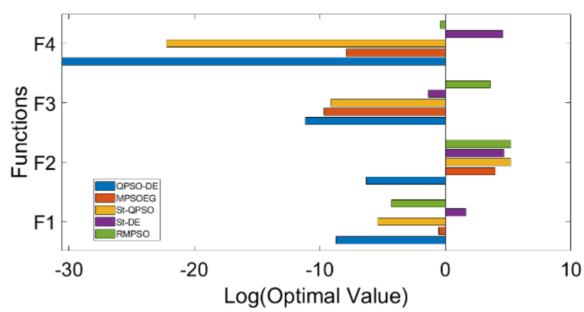

FIGURE 1. Performance comparison of functions f_1, f_2, f_3 and f_4.

D. NUMERICAL VALIDATION BY ENGINEERING DESIGN PROBLEMS

Numerical study on Benchmark functions in previous section validated the proposed QPSODE optimization algorithm's efficacy. To further verify it on practical problems, we used TEAM workshop problem 22 (SMES), a well-known electromagnetic optimization problem [37], as a test suite for QPSODE. A superconducting magnetic energy

TABLE 4. Numerical comparison of best f_b, max f_w, mean f_{mean}, and sd value of test problems.

Problem	algorithm	f_b	f_w	f_{mean}	SD
f_{11}	QPSODE	-492.20	-18.76	-322.63	109.44
	MPSOEG	-56.86	-21.94	-38.02	5.54
	Standard QPSO	-165.59	-33.69	-130.03	33.26
	Standard DE	-2.27	-2.27	-2.27	0.00
	RMPSO	-87.41	-21.01	-79.87	16.16
f_{12}	QPSODE	-489.11	2.10	-227.81	140.13
	MPSOEG	-19.99	-0.66	-11.49	6.92
	Standard QPSO	-412.40	1.97	-253.09	124.74
	Standard DE	-13.62	6.72	-12.03	4.98
	RMPSO	-5.14	3.05	-4.56	1.22
f_{13}	QPSODE	-390.38	-0.67	-239.80	131.52
	MPSOEG	-9.12	-1.24	-8.41	0.92
	Standard QPSO	-171.51	8.63	-115.20	61.12
	Standard DE	-4.63	9.49	-1.83	4.60
	RMPSO	-7.74	3.64	-5.92	2.51
f_{14}	QPSODE	-581.58	-2.69	-337.07	178.14
	MPSOEG	-15.36	-1.86	-13.62	2.98
	Standard QPSO	-272.92	-3.08	-174.54	83.74
	Standard DE	-3.68	3.92	-1.86	2.04
	RMPSO	-4.48	2.32	-3.68	1.33
f_{15}	QPSODE	-415.73	-4.43	-299.84	129.99
	MPSOEG	-19.04	-1.60	-17.34	3.91
	Standard QPSO	-160.73	-4.13	-118.12	36.94
	Standard DE	3.55	5.40	3.98	0.42
	RMPSO	-2.84	4.02	-2.03	1.52

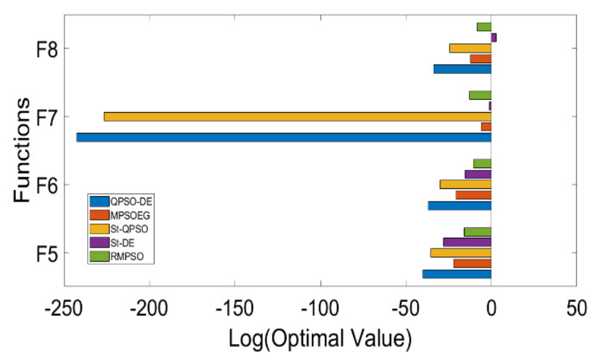

FIGURE 2. Performance comparison of functions f_5, f_6, f_7 and f_8.

storage (SMES) arrangement, as shown in Fig. 6, was considered as a benchmark problem for assessing various optimization algorithms, both deterministic and stochastic, in the last decade of the twentieth century.

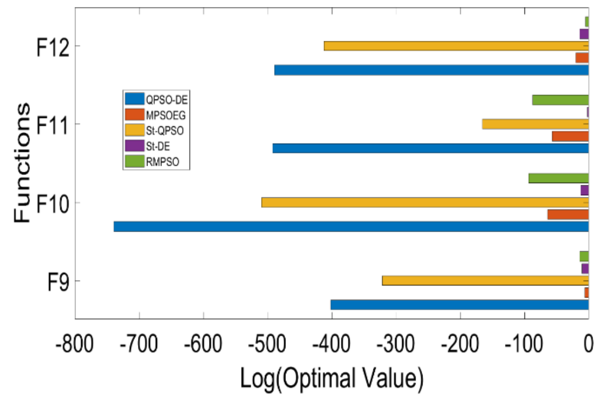

FIGURE 3. Performance comparison of functions f_9, f_{10}, f_{11} and f_{12}.

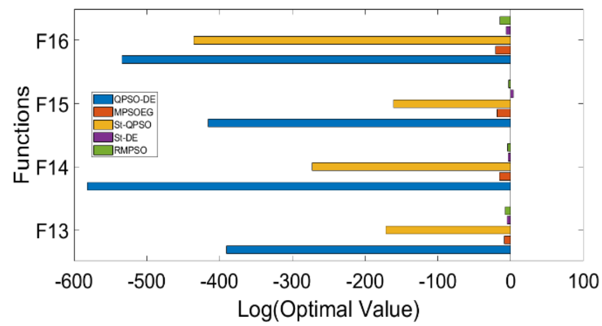

FIGURE 4. Performance comparisons of functions f_{13}, f_{14}, f_{15} and f_{16}.

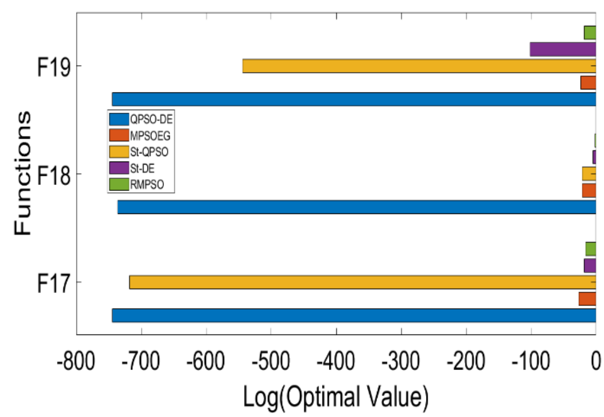

FIGURE 5. Performance comparisons of functions f_{17}, f_{18} and f_{19}.

In this electromagnetic design problem, the inner solenoid parameters are persistent, that is, is $R_1 = 2.0$ m, $d_1 = 0.27$ m, and $h_1/2 = 0.8$ m, whereas the outer-solenoid geometrical dimensions are limited to $0.204 < h_2/2 < 1.1$ m, $0.1 < d_2 < 0.4$ m and $2.6 < R_2 < 3.4$ m, and will be optimized.

The objective functions include both the energy requirements (the stored magnetic energy E should be as close to 180 MJ as possible) and the stray field requirements.

Consequently, the objective function is

$$OF = \frac{B^2_{stray}}{B^2_{norm}} + \frac{|E - E_{ref}|}{E_{ref}} \tag{14}$$

where $\boldsymbol{E_{ref}} = 180\boldsymbol{MJ}, \boldsymbol{B_{norm}} = 3 \times \boldsymbol{10^{-3}T}$ and $\boldsymbol{B^2_{stray}}$ is evaluated along 22 equidistant points along lines $\boldsymbol{a}$ and $\boldsymbol{b}$ in

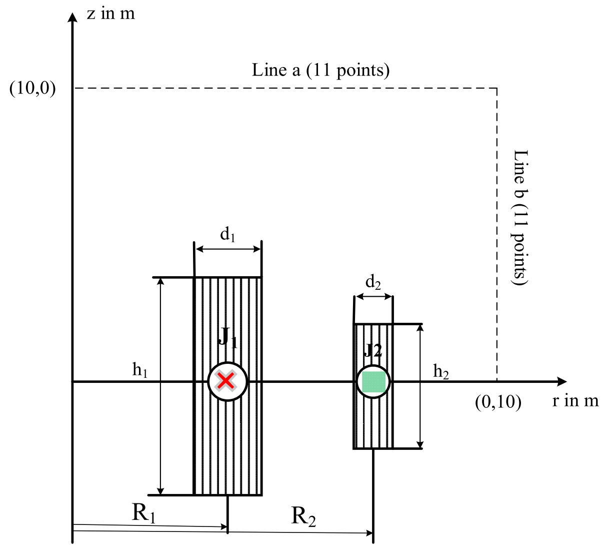

FIGURE 6. Schematic diagram of SMES optimization TEAM problem.

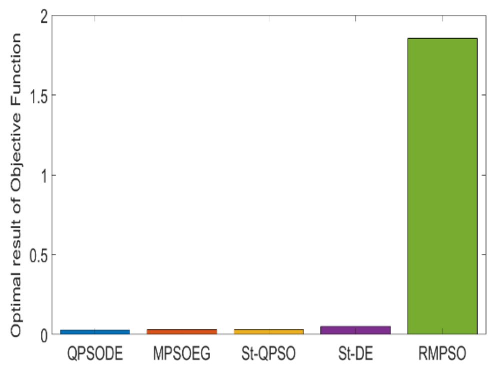

FIGURE 7. Optimized fitness of different algorithms for TEAM workshop problem 22.

Figure 6 by:

$$B_{stray}^{2}=\frac{\sum_{i=1}^{22}\left|B_{stray,i}\right|^{2}}{22} \quad (15)$$

According to TEAM workshop problem 22, the problem was solved by linking the value of the current density of coils and the maximum value of the generated flux density in continuous states with a dynamic current density following through the quench condition, which guarantees that the superconducting material should work safely.

$$|J_i| \leq (-6.4|B_i|+56)(A/\mathrm{mm}^2)(i=1,2)$$

Figure 7 illustrates the comparison on the optimized results of the proposed QPSODE and other approaches for this case study. Again, the numerical results demonstrate the advantages of the proposed QPSODE over others.

V. CONCLUSION

Problems of optimizations can be observed in almost every field of engineering like slow convergence to optimal solution, higher computational time and balance between exploration and exploitation. In this study we introduced a QPSODE, a new algorithm that incorporates differential evolution DE to increase the performance of quantum behaved particle swarm optimization used for better exploration and to achieve speedy convergence. The study also explores and compares different algorithms used for controlling the premature convergence and exploration of an optimization algorithm. To speed up convergence, avoid premature convergence and increase exploration capability, the hybridized QPSODE algorithm introduces proper parameters tuning, control parameter, path disparity and the Boltzmann selection being adopted alongside with other smart behavior of particle. To verify and validate the newly designed algorithm, various well-known benchmark functions and the engineering optimization problem like superconducting magnetic energy storage (SMES) are used for comparison of the performance and quality of different algorithms.

Future research should concentrate on more practical methods to solve electromagnetic design issues like Loney's solenoid problem. The results of the proposed hybrid models technique are quite promising and could greatly enhance QPSO performance for various electromagnetic issues.

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