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## RESEARCH ARTICLE

# Two-Stage Multiclass Modeling Approach for Intermodal Rail-Road Transport Networks

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**ABSTRACT** In this paper, a two-stage modeling framework is proposed to represent the route choices and dynamics of two classes of traffic flows, namely passengers and freight, in a large-scale intermodal transportation network. More in details, the transport modes considered in this model are road transport, represented through a highway network, and rail transport. The methodology adopted in this work relies on a multi-class intermodal assignment model combined with a multi-class intermodal dynamic model. The proposed modeling scheme can be adopted for different purposes, such as to support decision makers who intend to utilize the full mobility capacity of a geographical area by allocating the mobility demand on different modes or by suggesting intermodal itineraries also when the network is affected by disruptive events. The proposed framework has been tested on a benchmark network with the aim of showing the mutual relations that occur between different modes of transport in case a connection of the intermodal network fails. Specifically, the analysis reported in the paper shows that the collapse of a railway arc can cause a high increase in travel times for some highway arcs, which result 30-40% higher than in the pre-disruption scenario, and an even greater growth in the average occupancy of many arcs.

**INDEX TERMS** Assignment model, dynamic discrete-time model, intermodal rail-road transport network, freight transport.

## I. INTRODUCTION

The need to move passengers and freight is becoming increasingly pervasive in conducting daily human activities and more generally in the social and economic advancement of countries. As a result, the availability and accessibility of transport infrastructures and mobility services allow this need to be met and, at the same time, increase the attractiveness of an area. In this regard, it is of utmost importance to quantify the capacity necessary to satisfy the transport demand of a geographical area and to try to predict how the transport systems included in it will react to unforeseen events. These latter may appear in different forms as changes in demand, e.g. the transfer of passengers from public to private transport that have occurred due to the ongoing pandemic, or modification

of the infrastructure network layout due to the verification of disruptive events.

Model-based approaches are the foremost methodologies for evaluating the performance of transportation networks because they allow:

- to quantify the efficiency of transport networks subject to different scenarios through the calculation of performance indexes (e.g., average travel times, fuel consumption, pollutant emissions, etc.);
- to evaluate the effects produced by the occurrence of critical events (e.g., infrastructure collapse, natural disasters limiting the functionality of transport systems, terrorist attacks, etc.);
- to evaluate the effects produced by the introduction of new systems (e.g., new infrastructure, modification of existing layout, etc.);
- to develop or test regulation policies.

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Our work is a model-based approach which relies on the combination of a traffic assignment model with a discrete-time dynamic model used for simulation. In particular, the assignment model is adopted to represent the route choices of users and to evaluate, in a static way, the distribution of flows on the network. The simulative model, instead, is used to represent the impact of the route choices on the transport network by describing the flow evolution over time. By analyzing the literature on these topics, it is possible to observe that most of the researches' efforts have been focused so far on the definition of assignment and simulative models on mono-modal transport networks, i.e. road networks or railway networks only (see for instance [1]–[3]). However, if the goal is to analyze the accessibility of a given geographic area or to give better information to users, it is necessary to adopt more extensive models which consider the intermodal transport network as a whole, by properly modeling the different modes of transport and the possibility of transfer among them.

Another aspect that cannot be overlooked in the analysis of transport networks is that, as in all critical infrastructure systems, the interdependencies increase the potential for cascading failures [4]. Referring specifically to intermodal transport networks [5], the interdependence among different transport modes is particularly relevant. It can be related to pursue the transport activity itself, in this case several transport services operate synergistically to allow the satisfaction of the mobility demand of users and the efficient distribution of goods and services. Interdependence can be related to physical reasons such as the overlap of different routes, even belonging to different transport modes, through bridges or tunnels, or to hidden relationships, such as those analyzed in [6]. Regardless of how these interdependencies occur, they imply an increased vulnerability of the transport network as a whole. In fact, it is possible that critical events affecting even one mode of transport could cause a ripple effect involving other transport modalities or, in the worst case, the entire transport network. For these reasons, the development of intermodal models represents an important step in the analysis of complex and highly interdependent systems such as transport networks.

This work, based on preliminary papers [7] and [8], falls in this field of research by representing large-scale transportation networks in which the considered transport modes are road and rail connected to each other through appropriate intermodal connections. Another peculiarity of this modeling framework, both in the assignment and in the dynamic model, is that the user demand is multi-class, hence it is distinguished into passengers and cargo units (e.g., containers). This distinction is of particular importance because passenger and freight flows may be characterized by different behaviors and may be subject to different restrictions, such as on route choices. Note that, compared with [7] and [8], in the present work the assignment model is multi-class (while in the previous versions there was no distinction of user classes) and the

dynamic model is improved. As a consequence, the results reported and discussed in this paper are completely new.

To summarize, the modeling framework proposed in this work is composed of two stages (see Fig. 1):

- a multi-class intermodal rail-road assignment model;
- a multi-class intermodal rail-road discrete-time dynamic model;

and has the following goals:

- to represent the route choices and the dynamic behavior of traffic flows in an intermodal rail-road network;
- to explicitly capture the behavior of passengers and freight in the network;
- to allow performance analysis of the intermodal network in different scenarios.

The present paper is organized as follows. A literature review on intermodal transport network models is presented in Section II. In Section III the general features of the proposed modeling framework and the basic notation are introduced. Then, the proposed two-stage modeling approach is presented: the assignment model is outlined in Section IV, while the discrete-time dynamic model is described in Section V. The application of the proposed methodology to a test network is shown in Section VI, while some conclusive remarks are gathered in Section VII.

## II. LITERATURE REVIEW

Intermodal transport networks have been studied extensively by operations researchers with the purpose of taking decisions referred to freight transport operations. The related planning problems are normally distinguished in strategic, tactical and operational ones, according to the length of the planning horizon (more details and classifications can be found in the review papers [9]–[11]). Location decisions are probably the most representative examples of strategic planning problems [12] and regard the definition of the optimal positioning of nodes of the intermodal network (see e.g. [13]–[15]). Tactical and operational decisions in intermodal networks regard medium-short term horizons and address different types of problems, such as, for example, the service network design [16], [17] or the selection of the best routes [18], [19].

Differently from these latter papers, in which the goal is to design and organize activities in an intermodal freight network, in this work we aim at defining a modeling framework for representing the spontaneous behavior of both freight and passengers in an intermodal network, not only in terms of alternative path choices but also including their dynamics. Hence, the scientific literature considered as a reference for this work is associated, firstly, with traffic assignment models and, secondly, with simulation-based approaches for intermodal networks.

The majority of the works addressing assignment models for intermodal transport networks are referred to urban areas and consider the choices of passengers rather than

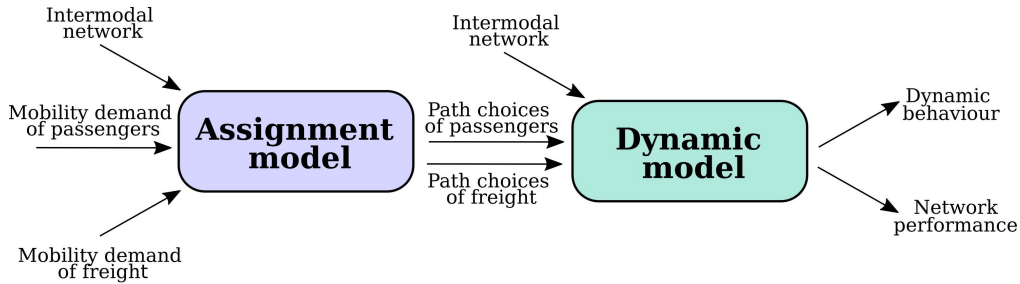


FIGURE 1. Sketch of the two-stage modeling framework.

freight flows. In particular, intermodal traffic assignment has been studied for some decades [20]–[23], by considering the so-called hypernetworks, which allow to include both links representing a portion of the real intermodal transport network (road, rail, private or public transport) and links associated with users' decisions. Intermodal traffic assignment approaches have also been studied more recently, e.g. in [24] the intermodal transport network of an urban area is represented as a graph composed of a number of sub-networks, each of which is associated with a transportation mode. In that work, the intermodal network is translated into an augmented-state network in which transfer rules and transfer probabilities between the different transport modes are defined. In [25], a stochastic model is proposed in order to account for the reliability of the chosen transport modes and paths, while in [26] different modes are considered, only-driving, carpooling, ride-hailing, public transit and park-and-ride, for a urban transportation system including private cars, freight trucks, buses, and so on.

Simulation-based approaches are either referred to passengers or freight separately. For instance, [27]–[29] provide simulative approaches for representing the choices of passengers in urban intermodal networks. In [27], three transport modes are considered, i.e. private vehicle transport, public vehicle transport and pedestrian transport, all represented through a three-dimensional macroscopic fundamental diagram (MFD) that allows to evaluate the accumulation of private and public vehicles according to different traffic scenarios. In [28], the considered transport modes are private vehicle transport and public road transport: private transportation is represented through the MFD, while constant transfer speeds are considered for public transportation. The derived intermodal model is used for the definition of adaptive pricing policies. The work presented in [29] proposes an agent-based simulation tool to test innovative planning methods involving pedestrians and autonomous shuttles in large urban areas. Regarding instead freight logistics, [30] reports a very detailed survey about simulation approaches for intermodal freight transportation systems, used to represent stakeholders, decision makers, operations, and planning activities. In [31], a dynamic model for an intermodal freight transport network is proposed to represent the mode changes at intermodal terminals, the physical capacity constraints of

the network, the time-dependent transport times on roads, and the time schedules for trains and barges. When dealing with freight intermodal networks, it is worth noting that the system performance must include different factors, such as costs, times, flexibility, reliability, quality and sustainability [32] and, then, multi-criteria analysis can be effectively applied in this context (see e.g. [33], [34]).

On the basis of the related literature review reported above, we can highlight that the main novelties and contributions of this work stand in:

- combining a traffic assignment model with a simulative model;
- capturing the travel choice and dynamic behavior of passengers and freight jointly in an intermodal network.
- representing a large-scale transport network covering a regional territory.

### III. GENERAL FEATURES AND BASIC NOTATION

As shown in Fig. 1, the proposed modeling scheme consists of two stages: an assignment model defined to allocate the demand of passengers and freight on a intermodal transport network, and a discrete-time dynamic model that allows to replicate the evolution of the system over time. It is worth noting that the discrete-time dynamic model receives as input the mobility demand and the path choices (route and modal choices) defined through the intermodal assignment model.

Both the assignment model and the dynamic model are based on a regional intermodal transport network in which the considered transport modes are road transport, represented by a highway network, and rail transport. The two modes of transport are connected with some intermodal arcs distinguished depending on the flow class, i.e., passengers or cargo units, which they can receive. The rail-road intermodal transport network is represented by means of an oriented graph, as depicted in Fig. 2, denoted with  $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ , in which  $\mathcal{N}$  indicates the set of nodes, whereas  $\mathcal{A} = \mathcal{A}^H \cup \mathcal{A}^R \cup \mathcal{A}^{fp} \cup \mathcal{A}^{ff}$  represents the set of arcs. Each subset of arcs is defined as follows:

- $\mathcal{A}^H$  is the set of highway connections;
- $\mathcal{A}^R$  is the set of railway connections;
- $\mathcal{A}^{fp}$  is the set of intermodal arcs for passengers;
- $\mathcal{A}^{ff}$  is the set of intermodal arcs for freight.

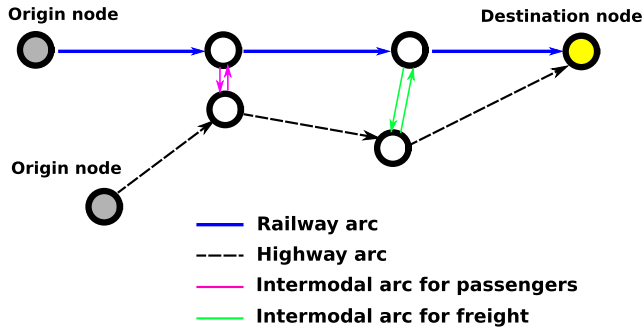


FIGURE 2. Sketch of the intermodal transport network.

Intermodal arcs are distinguished for passengers and for freight because modal shifts for passengers and freight typically occur at different locations and in different ways.

Let  $P(i)$  indicate the set of nodes preceding node  $i$  and  $S(i)$  the set of nodes succeeding node  $i$ ,  $i \in \mathcal{N}$ . The network is defined in an origin-destination-oriented mode in which  $J^O \subseteq \mathcal{N}$  represents the set of all possible origin nodes,  $J^D \subseteq \mathcal{N}$  represents the set of all possible destination nodes. The nodes which are not origins nor destinations simply allow the transit between successive arcs. The intermodal arcs, for both flow classes, are considered as fictitious arcs that allow modal transfers between road transport and rail transport and vice versa. For this reason, an origin node  $o \in J^O$  cannot be followed by intermodal arcs, and similarly, a destination node  $d \in J^D$  cannot be preceded by intermodal arcs.

In the proposed approach the flows of users are distinguished in passengers and freight, with superscript  $c$  denoting the class of users. In particular we define with  $c = 1$  the passenger flow and with  $c = 2$  the freight flow. Then,  $D^{od,c}$  indicates the demand of class  $c$  that originates at node  $o \in J^O$  and has destination at node  $d \in J^D$ : this is the total demand associated with the  $od$  pair for the whole simulation horizon. This demand defined for all  $od$  pairs represents the so-called “origin-destination matrix”. Let us also specify that the passenger demand, i.e., the demand for class  $c = 1$ , is expressed in number of passengers, while the demand of freight, corresponding to  $c = 2$ , is expressed in number of cargo units.

The main parameters and variables of the proposed approach are summarized in Tables 1-3. Specifically, Table 1 collects all the parameters that are used in both the assignment model and the dynamic simulation model. Table 2 collects the parameters and variables used in the assignment model only, while Table 3 reports parameters and variables used in the dynamic simulation model only.

As it will be described in Section V, the dynamic behavior of the whole network is represented with a discrete-time model in which the time horizon is divided in  $K$  time steps, where  $k = 1, \dots, K$  indicates the temporal stage, and  $T$  [h] represents the sample time interval. In order to ensure a correct time discretization, the length of the time interval  $T$  must allow a proper dynamic evolution of the system, therefore the length of the time step is chosen in order to fulfil, for any arc

$(i, j) \in \mathcal{A}^H \cup \mathcal{A}^R$ , the following condition:

$$T \leq \min \left\{ \frac{\Delta_{i,j}}{v_{i,j}^H}, \frac{\Delta_{i,j}}{v_{i,j}^R} \right\} \quad (1)$$

#### IV. THE INTERMODAL ASSIGNMENT MODEL FOR PASSENGER AND FREIGHT FLOWS

The purpose of the intermodal assignment model is to represent the spontaneous decisions of users about their routes. Before providing the details of the proposed approach, it is important to emphasize that the assignment model is a static approach in which all the mobility demand, estimated for a specific time window, is assumed to use the transport network simultaneously. As shown in Fig. 1, the result of this model is the redistribution of flows on the paths, i.e.,  $f_l^{od,1}$  for passengers and  $f_l^{od,2}$  for freight, which are obtained by the solution of some optimization problems. The flows thus obtained will be suitably transformed into splitting rates, see Section V, and used as input data for the dynamic model.

The passenger and freight assignment models are treated separately, firstly introducing the intermodal assignment model for passengers and, then, applying the intermodal assignment model for freight, since the latter uses the results of the passenger assignment to define the freight route choices. In particular, for the passenger assignment procedure we assume that the marginal impact on the network due to the presence of freight is sufficiently small so that it does not decisively influence the behavior of passengers. This assumption is quite reasonable since, referring to the overall flows on a transport network, the freight component typically constitutes a rather low percentage compared with the flow of passengers.

The assignment procedure presented below is conducted in an iterative manner in which at the first step the intermodal assignment model for passenger flows described in Section IV-A is run. Then the results of this assignment are used to estimate the average passenger travel times on the network using the dynamic model described in Section V. Finally, the intermodal freight assignment model, presented in Section IV-B, is run using the average travel times defined in the previous step.

##### A. THE INTERMODAL ASSIGNMENT MODEL FOR PASSENGER FLOWS

Modeling the behavior and therefore the mobility choices of users is a challenging problem for which several approaches have been developed by researchers. One of the most accepted, and widely adopted, methodologies concerns the use of traffic assignment models. Given an origin-destination matrix, representing the users’ demand, and knowing the functional characteristics of the network infrastructure, a traffic assignment model allows to estimate how users will be dispersed in the network, taking into account the relation between the infrastructure supply and the interaction of users mutual choices. The criteria underlying the mathematical representation of such behaviors can vary considerably

TABLE 1. Parameters common to the two models in the two-stage modeling framework.

Parameter	Description	Equations
$\Delta_{i,j}$	Arc length [km] for all $(i, j) \in \mathcal{A}^H \cup \mathcal{A}^R$	(1), (7), (8), (18), (21), (37), (38), (39)
$v_{i,j}^H$	Maximum freeway speed in [km/h] for all $(i, j) \in \mathcal{A}^H$	(1), (7), (38)
$\omega_{i,j}$	Congestion wave speed in freeway in [km/h] for all $(i, j) \in \mathcal{A}^H$	(7), (18), (38)
$n_{i,j}^{\max}$	Maximum number of vehicles in [PCE] for all $(i, j) \in \mathcal{A}^H$	(7), (18), (29), (38)
$v_{i,j}^R$	Maximum railway speed in [km/h] for all $(i, j) \in \mathcal{A}^R$	(1), (8), (39)
$h_{i,j}$	Average time headway in [h] for all $(i, j) \in \mathcal{A}^R$	(8), (19), (39)
$s_{i,j}^{\min}$	Minimum average space headway in [km] for all $(i, j) \in \mathcal{A}^R$	(8), (19), (39)
$C^p$	Capacity of a passenger train in number of passengers for all $(i, j) \in \mathcal{A}^R$	(8), (28), (31)
$C^f$	Capacity of a freight train in rail wagons for all $(i, j) \in \mathcal{A}^R$	(19), (28), (31), (35)
$L$	Length of a train in [km] for all $(i, j) \in \mathcal{A}^R$	(8), (19), (39)
$\alpha_{i,j}$	Number of time steps required to cross an intermodal passenger arc $(i, j) \in \mathcal{A}^{Ip}$	(9), (37)
$\gamma_{i,j}$	Number of time steps required to cross an intermodal freight arc $(i, j) \in \mathcal{A}^{If}$	(20), (37)
$\eta$	Average number of passengers per vehicle	(7), (25), (26)
$\varsigma$	Conversion factor used to convert trucks into passenger car equivalent	(18), (27), (31)

TABLE 2. Parameters and variables present only in the assignment model described in Section IV.

(a)

Parameter	Description	Equations
$\delta_{i,j,l}^{od,1}$	Binary parameter that defines the belonging of an arc $(i, j) \in \mathcal{A}$ to a passenger path associated to the $od$ pair	(5), (6), (12), (22)
$\delta_{i,j,l}^{od,2}$	Binary parameter that defines the belonging of an arc $(i, j) \in \mathcal{A}$ to a freight path associated to the $od$ pair	(16), (17), (22)
$C_{i,j}^{\text{time}}$	Cost per time unit used to estimate arc-traveling costs for freight flows for all arc $(i, j) \in \mathcal{A}$	(21)
$C_{i,j}^{\text{space}}$	Cost per space unit used to estimate arc-traveling costs for freight flows for all arc $(i, j) \in \mathcal{A}$	(21)
$C_{i,j}^{\text{fix}}$	Fixed cost used to estimate arc-traveling costs for freight flows for all arc $(i, j) \in \mathcal{A}$	(21)
$\bar{t}_{i,j}^1$	Average travel time over arc $(i, j) \in \mathcal{A}$ for passengers	(18), (19)
$\phi$	Parameter used to define a performance function for passengers	(7)
$M$	Large coefficient chosen arbitrarily	(9), (20)

(b)

Variable	Description	Equations
$f_l^{od,1}$	Decision variable related to passenger flow associated with the $od$ pair using a specific path $l$	(3), (4), (5), (10), (11), (12), (22)
$f_l^{od,2}$	Decision variable related to freight flow associated with the $od$ pair using a specific path $l$	(14), (15), (16), (22)
$x_{i,j}^1$	Decision variable related to the total passenger flow on arc $(i, j) \in \mathcal{A}$	(2), (5), (7), (8), (9)
$x_{i,j}^{1,UE}$	Decision variable related to the total passenger flow on arc $(i, j) \in \mathcal{A}$ satisfying User-Equilibrium conditions	(12)
$x_{i,j}^2$	Decision variable related to the total freight flow on arc $(i, j) \in \mathcal{A}$	(13), (16), (18), (19), (20)
$\tau_{i,j}^1(\cdot)$	Performance functions for passengers of arc $(i, j) \in \mathcal{A}$	(2), (7), (8), (9)
$\tau_{i,j}^2(\cdot)$	Performance functions for freight of arc $(i, j) \in \mathcal{A}$	(18), (19), (20), (21)
$c_{i,j}(\cdot)$	Total travel costs for freight on the arc $(i, j) \in \mathcal{A}$	(13), (21)

depending on the assignment model used. A comprehensive review of traffic assignment models is given in [35].

In the present work, the N-Path Restricted User-Equilibrium traffic assignment model has been applied. Generally speaking, the User Equilibrium traffic assignment model aims to estimate the network equilibrium state such that no user has unilaterally any interest in choosing an alternative path, since no other path would guarantee lower travel times. Such equilibrium is achieved if Wardrop’s first principle [36] is met, whereby “users choose the path that

at a given time minimizes their own travel time”. A way to compute the arc flows corresponding to such equilibrium involves finding the optimum solution of the Beckmann’s Transformation [37]. The N-Path Restricted variant of the aforementioned model [38] is obtained by constraining the assignment to a priori defined sets of admissible paths, which do not necessarily include all the possible ones for each origin-destination pair. This allows to avoid all the paths that are theoretically possible but quite implausible in practice. In this paper, all the paths involving more than one modal



TABLE 3. Parameters and variables present only in the dynamic model described in Section V.

(a)		
Parameter	Description	Equations
$N_{i,j}^{\max}$	Maximum number of trains for all $(i, j) \in \mathcal{A}^R$	(29)
$\epsilon_{i,j}^c$	Conversion factor defined for each class $c$ and for each arc $(i, j) \in \mathcal{A}$ to model the transition from a railway arc to a road arc and vice versa	(24), (25), (30)
$\xi_{i,j}^c$	Conversion factor defined for each class $c$ and for each arc $(i, j) \in \mathcal{A}$ to correctly computing the flow entering from an origin node	(24), (26), (30)

(b)		
Variable	Description	Equations
$n_{i,j}^{od,c}(k)$	Number of units of class $c$ in arc $(i, j) \in \mathcal{A}$ , associated with the $od$ pair at time step $k$	(23), (27), (28), (35), (36), (41), (42), (43)
$n_{i,j}^{\text{tot}}(k)$	Total number of vehicles in [PCE] in arc $(i, j) \in \mathcal{A}^H$	(27), (29), (38)
$N_{i,j}^{\text{tot}}(k)$	Total number of trains in arc $(i, j) \in \mathcal{A}^R$	(28), (29), (39)
$I_{i,j}^{od,c}(k)$	Number of units of class $c$ entering arc $(i, j) \in \mathcal{A}$ , associated with the $od$ pair at time step $k$	(23), (24)
$O_{i,j}^{od,c}(k)$	Number of units of class $c$ exiting arc $(i, j) \in \mathcal{A}$ associated with the $od$ pair at time step $k$	(23), (24), (40)
$w_{i,j}^{od,c}(k)$	Number of units of class $c$ that would like to enter arc $(i, j) \in \mathcal{A}$ associated with the $od$ pair at time step $k$	(30), (31)
$W_{i,j}^{\text{tot}}(k)$	Total number of units that would like to enter arc $(i, j) \in \mathcal{A}$ at time step $k$	(31), (32)
$\pi_{i,j}(k)$	Surplus rate of units that cannot enter at time step $k$ in the arc $(i, j) \in \mathcal{A}$	(32), (40)
$S_{i,j}^{od,c}(k)$	Number of units of class $c$ that would like to exit arc $(i, j) \in \mathcal{A}$ , associated with the $od$ pair at time step $k$	(30), (35), (36), (40)
$\beta_{i,j}^{od,c}(k)$	Splitting rates of class $c$ in arc $(i, j) \in \mathcal{A}$ , associated with the $od$ pair at time step $k$	(22), (24), (30), (33), (40)
$q^{od,c}(k)$	Number of units of class $c$ associated with the $od$ pair, that can actually enter the network from node $o \in J^O$ at time step $k$	(24), (33), (34)
$l^{od,c}(k)$	Queue length of class $c$ , associated with the $od$ pair, which waits at the origin node $o \in J^O$ at time step $k$	(30), (33), (34)
$q_{i,j}^{\text{res}}(k)$	Residual capacity of arc $(i, j) \in \mathcal{A}$ at time step $k$	(29), (31)
$t_{i,j}(k)$	Transfer time required to cover an arc $(i, j) \in \mathcal{A}$ at time step $k$	(36), (37)
$V_{i,j}(\cdot)$	Steady-state speed relationships defined for railway and roadway arcs $(i, j) \in \mathcal{A}^H \cup \mathcal{A}^R$ for each time step $k$	(37), (38), (39)

shift have been excluded. Then, denoted with  $l, l = 1, \dots, \mathcal{L}$ , a generic path existing in the network and with  $\mathcal{L}^{od}$  the set of all possible paths connecting the  $od$  pair, let  $\mathcal{P}^{od} \subseteq \mathcal{L}^{od}$  be the set of admissible paths from origin node  $o$  to destination node  $d$ . The resulting optimization problem for the traffic assignment of passengers demand in the intermodal transport network is the following.

Problem 1:

$$\min z(x) = \sum_{(i,j) \in \mathcal{A}} \int_0^{x_{i,j}^1} \tau_{i,j}^1(\omega) d\omega \quad (2)$$

subject to

$$\sum_{l \in \mathcal{P}^{od}} f_l^{od,1} = D^{od,1} \quad o \in J^O, d \in J^D \quad (3)$$

$$f_l^{od,1} \geq 0 \quad o \in J^O, d \in J^D, l \in \mathcal{P}^{od} \quad (4)$$

$$x_{i,j}^1 = \sum_{o \in J^O} \sum_{d \in J^D} \sum_{l \in \mathcal{P}^{od}} f_l^{od,1} \cdot \delta_{i,j,l}^{od,1} \quad (i, j) \in \mathcal{A} \quad (5)$$

where  $f_l^{od,1}$  and  $x_{i,j}^1$  are, respectively, the flow of class 1 of  $od$  pair using path  $l$  and the total flow of class 1 on arc  $(i, j)$ . Constraints (3) imply passengers' demand satisfaction for each  $od$  pair, while (4) are non-negativity constraints regarding traffic

flows. Moreover, constraints (5) define the relation among  $f_l^{od,1}$  and  $x_{i,j}^1$ , by means of the path-arc incidence matrix whose elements are defined as follows:

$$\delta_{i,j,l}^{od,1} = \begin{cases} 1 & \text{if } (i, j) \text{ belongs to path } l \text{ from } o \text{ to } d \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

In (2), the terms  $\tau_{i,j}^1(\cdot)$  are the performance functions of arcs related to class  $c = 1$ , i.e., passengers. These functions, representative of the functional characteristics of the network arcs, express the relation between the travel time spent by passengers traveling through an arc and the amount of congestion on the same arc, which is expected to perform worse (and thus resulting in increased travel times) if the number of users traveling on it increases. The performance functions adopted for this model are derived from (37), with  $c = 1$ , introduced in Section V, considering the definitions provided in (38) and (39). For highway and railway arcs, this equation computes the transit time as a function of the estimated speed. The average speed on these arcs, in turn, depends on the number of users who are using those arcs. Regarding intermodal passenger arcs, on the other hand, the transfer time is considered constant and independent of the number of users present on it. For further information the reader is referred to

the description of equations (37), (38) and (39) in Section V. It is worth noting that, with the exception of the intermodal case, the link performance functions are strictly increasing hyperbolic functions and therefore diverging in proximity to the arc theoretical maximum capacity.

In order to reduce the computational effort, appropriate linear versions of the same functions have been used within the assignment process. In the case of highway arcs, the resulting linear functions are obtained by interpolating two points: the first is obtained using the free-flow travel time corresponding to an arc completely empty, while the other uses the value assumed by the hyperbolic function when the number of users is equal to  $\phi \cdot n_{i,j}^{max}$ , where  $\phi \in [0, 1)$ . The linear performance functions in the highway case are therefore defined as follows

$$\tau_{i,j}^1(x_{i,j}^1) = \frac{\Delta_{i,j}}{\omega_{i,j} n_{i,j}^{max} (1 - \phi)} \cdot x_{i,j}^1 \frac{1}{\eta} + \frac{\Delta_{i,j}}{v_{i,j}^H} \quad (7)$$

for  $(i, j) \in \mathcal{A}^H$ .

Similarly, for the railway case, the second interpolating point is associated with the value assumed by the hyperbolic function when the number of users reaches the technical limit  $\frac{C^p \Delta_{i,j}}{s_{i,j}^{min}}$  (see (39)). The following linear performance function for the railway case is therefore obtained:

$$\tau_{i,j}^1(x_{i,j}^1) = \frac{h_{i,j} s_{i,j}^{min}}{(s_{i,j}^{min} - L) C^p} \cdot x_{i,j}^1 + \frac{\Delta_{i,j}}{v_{i,j}^R} \quad (8)$$

for  $(i, j) \in \mathcal{A}^R$ .

Finally, as mentioned above, the performance functions of intermodal arcs are constant functions, but it has been necessary to make them strictly increasing by introducing a (although very small) relation of direct proportionality between travel time and number of users on the arc in order to guarantee the uniqueness of the solution of the optimization problem, as detailed further on.

The performance functions for intermodal arcs are then defined as follows

$$\tau_{i,j}^1(x_{i,j}^1) = \alpha_{i,j} \cdot T + \frac{1}{M} x_{i,j}^1 \quad (9)$$

for  $(i, j) \in \mathcal{A}^{lp}$ , where  $\alpha_{i,j} \geq 1$  and  $M$  is a positive coefficient sufficiently big such that the impact of the number of users on the performance of an intermodal arc is negligible compared to the other types of arcs.

By defining the performance functions of the arcs as above, it can be proven that Problem 1 is strictly convex on a convex domain and therefore admits a unique optimal solution with respect to variables  $x_{i,j}^1$  but not for variables  $f_l^{od,1}$ . Since  $f_l^{od,1}$ , properly converted into splitting rates, represent the input of the dynamic model, their uniqueness is an essential requirement in this work. Several methodologies have been developed to overcome this issue associated with the User-Equilibrium model [39]. A possible solution is looking for a pattern of flows  $f$ , coherent with the distribution of users

at the equilibrium, maximizing an entropy function and for this reason more likely to occur [40].

The resulting optimization problem is as follows.

*Problem 2:*

$$\min h(f) = \sum_{o \in J^O} \sum_{d \in J^D} \sum_{l \in \mathcal{P}^{od}} f_l^{od,1} \cdot \ln(f_l^{od,1}) \quad (10)$$

subject to

$$\sum_{l \in \mathcal{P}^{od}} f_l^{od,1} = D^{od,1} \quad o \in J^O, d \in J^D \quad (11)$$

$$x_{i,j}^{1,UE} = \sum_{o \in J^O} \sum_{d \in J^D} \sum_{l \in \mathcal{P}^{od}} f_l^{od,1} \cdot \delta_{i,j,l}^{od,1} \quad (i, j) \in \mathcal{A} \quad (12)$$

Equations (11) – (12) convey the same constraints of Problem 1 with the only but fundamental difference that the flows on the arcs are now fixed and equal to  $x_{i,j}^{1,UE}$ , obtained as solution of Problem 1. Also, the non-negativity constraint (4) is now implicit in the fact that  $f_l^{od,1}$  appear in (10) as the argument of a logarithm.

## B. THE INTERMODAL ASSIGNMENT MODEL FOR FREIGHT FLOWS

In this section, the intermodal assignment model for freight flows is presented. The purpose of this assignment model is to replicate the average freight route choices by considering as objective the minimization of the total travel costs needed to satisfy a given demand. Given these premises, the intermodal assignment model for freight is given as follows

*Problem 3:*

$$\min y(x) = \sum_{(i,j) \in \mathcal{A}} x_{i,j}^2 \cdot c_{i,j}(x_{i,j}^2) \quad (13)$$

subject to

$$\sum_{l \in \mathcal{P}^{od}} f_l^{od,2} = D_{od,2} \quad o \in J^O, d \in J^D \quad (14)$$

$$f_l^{od,2} \geq 0 \quad o \in J^O, d \in J^D, l \in \mathcal{P}^{od} \quad (15)$$

$$x_{i,j}^2 = \sum_{o \in J^O} \sum_{d \in J^D} \sum_{l \in \mathcal{P}^{od,2}} f_l^{od,2} \cdot \delta_{i,j,l}^{od,2} \quad (i, j) \in \mathcal{A} \quad (16)$$

where  $f_l^{od,2}$  and  $x_{i,j}^2$  are, respectively, the flow of class  $c = 2$  of  $od$  pair using path  $l$  and the total flow of freight on arc  $(i, j)$ . Constraints (14)-(16) are analogous to constraints (3)-(5) included in Problem 1, and where  $\delta_{i,j,l}^{od,2}$  is defined as follows

$$\delta_{i,j,l}^{od,2} = \begin{cases} 1 & \text{if } (i, j) \text{ belongs to path } l \text{ from } o \text{ to } d \\ 0 & \text{otherwise} \end{cases} \quad (17)$$

As done for the passenger assignment, the performance functions that estimate the level of congestion and the travel time on arcs are defined by linearizing (37), with (38) and (39), where  $c = 2$ . Therefore, for highway arcs, the performance function for freight is defined as follows

$$\tau_{i,j}^2(x_{i,j}^2) = \frac{\Delta_{i,j}}{\omega_{i,j} n_{i,j}^{max} (1 - \phi)} \cdot \varsigma x_{i,j}^2 + \bar{t}_{i,j}^1 \quad (18)$$

where  $\zeta$  is the conversion factor adopted to express the number of trucks  $x_{i,j}^2$  in the arcs  $(i, j) \in \mathcal{A}^H$  in terms of passenger car equivalents. Equation (18) estimates the marginal contribution on travel times due to the presence of freight vehicles, which is summed to  $\bar{t}_{i,j}^1$ , that is the average travel time over arc  $(i, j)$  due exclusively to the presence of passengers, as calculated by the dynamic model. The term  $\frac{\Delta_{i,j}}{v_{i,j}^H}$  does not appear explicitly, since the free-flow travel time is contained in  $\bar{t}_{i,j}^1$ . In fact, it is assumed that the maximum speed  $v_{i,j}^H$  is the same for both freight and passenger vehicles.

In the case of railway arcs, a similar approach is adopted and the relative performance function is defined as described below

$$\tau_{i,j}^2(x_{i,j}^2) = \frac{h_{i,j}s_{i,j}^{min}}{(s_{i,j}^{min} - L)} \cdot \frac{x_{i,j}^2}{C^f} + \bar{t}_{i,j}^1 \quad (19)$$

which estimates the marginal contribution to the travel time of the arc due to the presence of freight, summed to the travel time due exclusively to the presence of passengers  $\bar{t}_{i,j}^1$ . The variable  $x_{i,j}^2$  expresses the amount of cargo units present on the arc. Therefore, it follows that  $\frac{x_{i,j}^2}{C^f}$  represents the equivalent number of freight trains. The free-flow travel time  $\frac{\Delta_{i,j}}{v_{i,j}^R}$  is implicitly contained in  $\bar{t}_{i,j}^1$ .

The performance functions for freight intermodal arcs  $(i, j) \in \mathcal{A}^{Hf}$  are defined as follows:

$$\tau_{i,j}^2(x_{i,j}^2) = \gamma_{i,j} \cdot T + \frac{1}{M} x_{i,j}^2 \quad (20)$$

where  $\gamma_{i,j} \geq 1$  and  $M$  is a positive coefficient large enough so that the impact due to the presence of cargo units on the arc is insignificant. Given the performance functions for the freight transport of each arc, the cost function in (13) associated with each arc of the network is defined as

$$c_{i,j}(x_{i,j}^2) = \tau_{i,j}^2(x_{i,j}^2)C_{i,j}^{time} + \Delta_{i,j}C_{i,j}^{space} + C_{i,j}^{fix} \quad (21)$$

where  $C_{i,j}^{time}$ ,  $C_{i,j}^{space}$  and  $C_{i,j}^{fix}$  are the cost per time unit, cost per space unit, and the fixed cost of arc  $(i, j)$ , respectively. Depending on the type of arc, the contribution made by each of the three parameters can change significantly. For example, in the case of an intermodal freight arc, it is reasonable to assume  $C_{i,j}^{space} = 0$ .

Being the performance functions (18)-(20) monotonically increasing with respect to the freight flows and being the cost function (21) linear with respect to the travel times, it follows that also the cost function of the arcs is monotonically increasing with respect to the freight flows. This makes the function  $y(\cdot)$  strictly convex and defined on a convex set (14)-(16) admitting a single optimal solution with respect to the variables  $x_{i,j}^2$ . However, also for the route choices for freight flows, the uniqueness of the solution is not guaranteed with respect to the flows on paths  $f_l^{od,2}$ .

To overcome this issue, a problem analogous to Problem 2 can be formulated to obtain the pattern of freight flows on

the routes most consistent with the routing problem described in Problem 3.

## V. THE INTERMODAL DYNAMIC MODEL FOR PASSENGER AND FREIGHT FLOWS

The dynamic model adopted in this work is used to capture the dynamic features of the overall system through a set of discrete-time equations. Aggregate discrete-time models have already been used for performance evaluation and optimization of specific intermodal transportation processes. In [41], [42], for instance, discrete-time models for freight movements by rail in maritime terminals are described. The model presented in this section is much more extensive considering the movement not only of freight but also of passengers and considering a more general applicative context, that is an intermodal rail-road multi-class transport network. Specifically, this dynamic model allows to evaluate the impact of the users' route choices and to analyze the behavior of the intermodal system at a more detailed level than the one provided by the assignment model.

The system evolution in time and in space is described by means of aggregate variables defined for each class  $c = 1, 2$ , for each arc  $(i, j) \in \mathcal{A}$ , for each  $od$  pair, with  $o \in \mathcal{J}^O, d \in \mathcal{J}^D$ , and for each time step  $k, k = 0, \dots, K$ . The complete list of parameters and variables adopted in the dynamic model is given in Tables 1 and 3; for further clarity, the main aggregate variables used to describe the dynamics of the intermodal road-rail system are recalled below:

- $n_{i,j}^{od,c}(k)$  is the number of units of class  $c$  in arc  $(i, j)$  associated with the  $od$  pair at time step  $k$ ;
- $I_{i,j}^{od,c}(k)$  is the number of units of class  $c$  entering arc  $(i, j)$  associated with the  $od$  pair at time step  $k$ ;
- $O_{i,j}^{od,c}(k)$  is the number of units of class  $c$  exiting arc  $(i, j)$  associated with the  $od$  pair at time step  $k$ ;
- $\beta_{i,j}^{od,c}(k)$  are the splitting rates of class  $c$  in arc  $(i, j)$  associated with the  $od$  pair at time step  $k$ ; note that the condition  $\sum_{j \in \mathcal{S}(i)} \beta_{i,j}^{od,c}(k) = 1$  must be verified  $\forall i, \forall o, \forall d, \forall c, \forall k$ .

It is worth clarifying that different units are adopted in the model, depending on the arc type and the flow class. Specifically:

- for class  $c = 1$ , i.e., passengers, the unit considered in railway arcs is the number of passengers, while in highway arcs is the number of vehicles; in intermodal arcs the units can be either passengers or vehicles depending on the type of arc preceding the intermodal one;
- for class  $c = 2$ , i.e., freight, the unit considered in railway arcs is the number of railway wagons, while in highway arcs is the number of trucks; in intermodal arcs, again, the units depend on the type of preceding arc. In this paper a single cargo unit corresponds to one rail wagon and to one truck. Extending this model for considering different load capacities in road and rail modes is straightforward and omitted here only for the sake of simplicity.



As mentioned in Section III, the dynamic model receives as inputs the route choices of passengers and freight, i.e., the splitting rates  $\beta_{i,j}^{od,c}(k)$ . These splitting rates  $\beta_{i,j}^{od,c}(k)$ , for each user class  $c$ , are obtained from the flows  $f_l^{od,c}$  resulting from the application of the intermodal and multi-class assignment procedure i.e.,

$$\beta_{i,j}^{od,c}(k) = \frac{\sum_{l \in \mathcal{P}^{od,c}} f_l^{od,c} \cdot \delta_{i,j,l}^{od,c}}{\sum_{p \in \mathcal{P}(i)} \sum_{l \in \mathcal{P}^{od,c}} f_l^{od,c} \cdot \delta_{p,i,l}^{od,c}} \quad (22)$$

for all  $k$ , with  $k = 0 \dots, K$ . Note that  $\beta_{i,j}^{od,c}(k)$  are constant along the simulation horizon since, in this work,  $D^{od,c}$  is the total demand of the whole horizon. In case the simulation horizon is divided in different time intervals, each one characterized by a different demand, the intermodal assignment model described in Section IV is applied for each time interval, resulting in different splitting rates  $\beta_{i,j}^{od,c}(k)$ .

Virtual queues at the origin nodes are considered in order to model the presence of flows that have to wait before entering the network. At this purpose, for each time step  $k$ ,  $k = 0, \dots, K$ , the following variables are introduced:

- $q^{od,c}(k)$  is the number of units of class  $c$  associated with the  $od$  pair, that can actually enter the network from node  $o \in J^O$ ;
- $l^{od,c}(k)$  is the queue length of class  $c$ , associated with the  $od$  pair, which waits at the origin node  $o \in J^O$ .

The dynamic evolution of the system is described, for each class  $c$  and for each  $k$ , with  $k = 0, \dots, K$ , by the following dynamic equation

$$n_{i,j}^{od,c}(k+1) = n_{i,j}^{od,c}(k) + I_{i,j}^{od,c}(k) - O_{i,j}^{od,c}(k) \quad (23)$$

for all  $(i,j) \in \mathcal{A}$ ,  $o \in J^O$ ,  $d \in J^D$ . Let us now describe separately the entering flows  $I_{i,j}^{od,c}(k)$  and the exiting flows  $O_{i,j}^{od,c}(k)$ .

### 1) ENTERING FLOWS

The entering flows  $I_{i,j}^{od,c}$  used in (23) are given by

$$I_{i,j}^{od,c} = \beta_{i,j}^{od,c}(k) \left[ \sum_{n \in \mathcal{P}(i)} \epsilon_{n,i}^c \cdot O_{n,i}^{od,c}(k) + \xi_{i,j}^c \cdot q^{od,c}(k) \right] \quad (24)$$

meaning that the flows of each user class  $c$  that enter a generic arc  $(i,j)$  of the network are the flows that actually succeed in exiting from the previous arcs plus the flows that actually manage to enter from node  $i$  if this is also an origin node  $o$ . These flows are then multiplied for  $\beta_{i,j}^{od,c}(k)$ , which indicates the portion that decides to use arc  $(i,j)$  to reach destination  $d$ .

Since we are describing the behavior of two class of users in an intermodal transport network, some parameters necessary to quantify the effective traffic load in the network have to be introduced, i.e., the two conversion factors  $\epsilon_{n,i}^c$  and  $\xi_{i,j}^c$  used in (24). Starting by class  $c = 1$ , i.e., passengers, and considering that the transition between two different modes of transport can only occur in an intermodal arc, the conversion

factor  $\epsilon_{n,i}^1$  is given by

$$\epsilon_{n,i}^1 = \begin{cases} 1 & \text{if } (n,i) \in \mathcal{A}^H \cup \mathcal{A}^R \\ \eta & \text{if } (n,i) \in \mathcal{A}^{lp} \text{ and } (i,j) \in \mathcal{A}^R \\ \frac{1}{\eta} & \text{if } (n,i) \in \mathcal{A}^{lp} \text{ and } (i,j) \in \mathcal{A}^H \end{cases} \quad (25)$$

where  $\eta$  is the average number of passengers per car, used to translate the number of vehicles in passengers and viceversa.

The conversion factor  $\xi_{i,j}^1$  is instead defined considering that the passenger demand is given in terms of number of passengers and that an origin node cannot be followed by an intermodal arc, therefore

$$\xi_{i,j}^1 = \begin{cases} 1 & \text{if } (i,j) \in \mathcal{A}^R \\ \frac{1}{\eta} & \text{if } (i,j) \in \mathcal{A}^H \end{cases} \quad (26)$$

As for class  $c = 2$ , i.e., the class referring to freight, both conversion factors  $\epsilon_{n,i}^2$  and  $\xi_{i,j}^2$  are set equal to 1 because, as mentioned above, a cargo unit is assumed to correspond to one truck and to one rail wagon.

Note that the flows that actually enter an arc  $(i,j)$  depend on the capability of the arc to receive flows, i.e., the residual capacity  $q_{i,j}^{res}(k)$ , calculated based on the total number of units in the arc  $(i,j)$  at the time step  $k$ . To this end let us define with  $n_{i,j}^{tot}(k)$  the total number of vehicles (expressed in terms of passenger car equivalents) present in arc  $(i,j) \in \mathcal{A}^H$  at time step  $k$  and with  $N_{i,j}^{tot}(k)$  the total number of trains in arc  $(i,j) \in \mathcal{A}^R$  at time step  $k$ , which are computed as follows

$$n_{i,j}^{tot}(k) = \sum_{o \in J^O} \sum_{d \in J^D} n_{i,j}^{od,1}(k) + \sum_{o \in J^O} \sum_{d \in J^D} \zeta n_{i,j}^{od,2}(k) \quad (27)$$

for all  $(i,j) \in \mathcal{A}^H$ , where  $\zeta$  is a coefficient introduced in order to translate the trucks in an equivalent number of cars, and

$$N_{i,j}^{tot}(k) = \frac{\sum_{o \in J^O} \sum_{d \in J^D} n_{i,j}^{od,1}(k)}{C^p} + \frac{\sum_{o \in J^O} \sum_{d \in J^D} n_{i,j}^{od,2}(k)}{C^f} \quad (28)$$

for all  $(i,j) \in \mathcal{A}^R$ . The total number of vehicles and the total number of trains present in each arc at each time step must ensure the following conditions:  $0 \leq n_{i,j}^{tot}(k) \leq n_{i,j}^{max}$  and  $0 \leq N_{i,j}^{tot}(k) \leq N_{i,j}^{max}$ .

The residual capacity of highway and railway arcs  $q_{i,j}^{res}(k)$  is given by

$$q_{i,j}^{res}(k) = \begin{cases} n_{i,j}^{max} - n_{i,j}^{tot}(k) & \text{if } (i,j) \in \mathcal{A}^H \\ N_{i,j}^{max} - N_{i,j}^{tot}(k) & \text{if } (i,j) \in \mathcal{A}^R \end{cases} \quad (29)$$

As for the intermodal arcs  $(i,j) \in \mathcal{A}^{lp} \cup \mathcal{A}^{lf}$ , being fictitious arcs, we chose to not impose bounds on the capacity.

Now let us analyze the receptive capacity of an arc according to the load conditions it is experiencing in a given time step. Then, let us define with  $w_{i,j}^{od,c}(k)$  the amount of users of

class  $c$  related to the  $od$  pair who want to enter arc  $(i, j)$  in time step  $k$

$$w_{i,j}^{od,c}(k) = \beta_{i,j}^{od,c}(k) \left[ \sum_{n \in P(i)} \epsilon_{n,i}^c \cdot S_{n,i}^{od,c}(k) + \xi_{i,j}^c \cdot \left( \frac{D^{od,c}}{K} + l^{od,c}(k) \right) \right] \quad (30)$$

In (30),  $w_{i,j}^{od,c}(k)$  includes the potential outflow from the previous arcs  $S_{n,i}^{od,c}(k)$ , better detailed in Section V-2, and the demand associated with the pair  $od$  and the eventual queue length at the node  $i$  if it coincides with the origin  $o$ .

Therefore, the total amount of units that potentially enter arc  $(i, j)$  at time step  $k$  is given by:

$$W_{i,j}^{tot}(k) = \begin{cases} \sum_{o \in J^O} \sum_{d \in J^D} w_{i,j}^{od,1}(k) + \zeta w_{i,j}^{od,2}(k) & \text{if } (i, j) \in \mathcal{A}^H \\ \sum_{o \in J^O} \sum_{d \in J^D} \frac{w_{i,j}^{od,1}(k)}{C^p} + \frac{w_{i,j}^{od,2}(k)}{C^f} & \text{if } (i, j) \in \mathcal{A}^R \end{cases} \quad (31)$$

Hence, thanks to the residual capacity defined in (29) we can determine the percentage of excess units  $\pi_{i,j}(k)$  that cannot enter at a given time step  $k$  in the arc  $(i, j)$

$$\pi_{i,j}(k) = \begin{cases} \frac{\max\{0, W_{i,j}^{tot}(k) - q^{res}(k)\}}{W_{i,j}^{tot}(k)} & \text{if } W_{i,j}^{tot}(k) > 0 \\ 0 & \text{otherwise} \end{cases} \quad (32)$$

where  $\pi_{i,j}(k) \in [0, 1]$ . Specifically,  $\pi_{i,j}(k)$  is equal to 0 when  $W_{i,j}^{tot} < q^{res}(k)$ , i.e., when the residual capacity is sufficient to host all units that want to enter arc  $(i, j)$ , while  $\pi_{i,j}(k)$  is equal to 1 when  $q^{res}(k) = 0$ , i.e., when the residual capacity is zero and all units that want to enter arc  $(i, j)$  cannot do so.

Having said that, the units that can effectively enter from an origin node is computed as follow

$$q^{od,c}(k) = \left[ \frac{D^{od,c}}{K} + l^{od,c}(k) \right] \sum_{n \in S(o)} \beta_{o,n}^{od,c}(k) \cdot (1 - \pi_{o,n}(k)) \quad (33)$$

whereas the relative queue of units at the origin node  $o$  is given by

$$l^{od,c}(k+1) = l^{od,c}(k) + \frac{D^{od,c}}{K} - q^{od,c}(k) \quad (34)$$

## 2) EXITING FLOWS

Even with respect to outflows from an arc, these are calculated based on the residual capacity of the arcs they wish to enter. For this reason, we distinguish  $O_{i,j}^{od,c}(k)$ , i.e., the units actually exiting arc  $(i, j)$ , from  $S_{i,j}^{od,c}(k)$ , i.e., the units that would like to exit arc  $(i, j)$  at time step  $k$ .

Let us start by describing the relation that defines the potential outflow from an intermodal freight arc  $(i, j) \in \mathcal{A}^{lf}$  preceded by a highway arc. We assume that cargo units can

only enter the rail network if they are enough to fully load at least a freight train with capacity  $C^f$ . Since the transfer of cargo units from road to rail is realized through a freight intermodal arc, we consider that this arc behaves as a buffer where cargo units wait until their number is sufficient to fill at least one train and then leave the intermodal arc. The number of cargo units possibly leaving a road-to-rail intermodal arc  $(i, j) \in \mathcal{A}^{lf}$  is then given by

$$S_{i,j}^{od,2}(k) = \left\lfloor \frac{n_{i,j}^{od,2}(k)}{C^f} \right\rfloor C^f \quad (35)$$

Now let us discuss the potential outflow for all classes  $c$  for the arcs  $(i, j) \in \mathcal{A}^H \cup \mathcal{A}^R$ , for passengers in arcs  $(i, j) \in \mathcal{A}^{lp}$  and for freight in rail-to-road intermodal arcs  $(i, j) \in \mathcal{A}^{lf}$ . The potential outflow  $S_{i,j}^{od,c}(k)$  is computed as

$$S_{i,j}^{od,c}(k) = \frac{T}{t_{i,j}(k)} n_{i,j}^{od,c}(k) \quad (36)$$

where  $t_{i,j}(k)$  is the transfer time required to cover arc  $(i, j)$  defined according to the following relation

$$t_{i,j}(k) = \begin{cases} \frac{\Delta_{i,j}}{V_{i,j}(n_{i,j}^{tot}(k))} & \text{if } (i, j) \in \mathcal{A}^H \\ \frac{\Delta_{i,j}}{V_{i,j}(N_{i,j}^{tot}(k))} & \text{if } (i, j) \in \mathcal{A}^R \\ \alpha_{i,j} \cdot T & \text{if } (i, j) \in \mathcal{A}^{lp} \\ \gamma_{i,j} \cdot T & \text{if } (i, j) \in \mathcal{A}^{lf} \end{cases} \quad (37)$$

For intermodal connections allowing the modal change from rail to road, the transfer time  $t_{i,j}(k)$  is considered constant and equal to  $\alpha_{i,j} \cdot T$ , with  $\alpha_{i,j} \geq 1$  for each  $(i, j) \in \mathcal{A}^{lp}$  and equal to  $\gamma_{i,j} \cdot T$ , with  $\gamma_{i,j} \geq 1$  for each arc  $(i, j) \in \mathcal{A}^{lf}$ . Instead, in intermodal connections allowing the road-to-rail modal change, the transfer time is not fixed and depends on the possibility to fill trains, i.e., on (35).

With regard to highway and railway arcs, it should be noted that, for both types of arc, the transfer time is estimated as a function of the total number of vehicles or trains present in the connection. More in details, for each highway arc  $(i, j) \in \mathcal{A}^H$ , the transfer time  $t_{i,j}(k)$  is computed according to the current traffic conditions through the steady-state relation between speed and number of vehicles given by

$$V_{i,j}(n_{i,j}^{tot}(k)) = \min \left\{ v_{i,j}^H, \frac{w_{i,j}}{n_{i,j}^{tot}(k)} \Delta_{i,j} \left[ \frac{n_{i,j}^{max}}{\Delta_{i,j}} - \frac{n_{i,j}^{tot}(k)}{\Delta_{i,j}} \right] \right\} \quad (38)$$

Relation (38) has been derived from a triangular fundamental diagram, as the one proposed in [43], and expressed in terms of number of vehicles. With regard to railway arcs, the steady-state speed relation  $V_{i,j}(N_{i,j}^{tot}(k))$  for all  $(i, j) \in \mathcal{A}^R$

may be formulated as

$$V_{i,j}(N_{i,j}^{tot}(k)) = \begin{cases} v_{i,j}^R & \text{if } \frac{N_{i,j}^{tot}(k)}{\Delta_{i,j}} \leq \frac{1}{h_{i,j}v_{i,j}^R + L} \\ \frac{1}{h_{i,j}} \left( \frac{\Delta_{i,j}}{N_{i,j}^{tot}(k)} - L \right) & \\ \frac{1}{h_{i,j}v_{i,j}^R + L} & \text{if } \frac{1}{h_{i,j}v_{i,j}^R + L} < \frac{N_{i,j}^{tot}(k)}{\Delta_{i,j}} \leq \frac{1}{s_{i,j}^{min}} \end{cases} \quad (39)$$

It is worth noting that, for each highway or railway arc, condition (1) with (38) and (39) implies that the transfer time  $t_{i,j}(k)$  is never lower than  $T$ , ensuring the validity of the conservation equations. For further details concerning the development of this relationship, please refer to Appendix.

Finally, given  $S_{i,j}^{od,c}(k)$  and  $\pi_{i,j}(k)$ , we can compute the outflow  $O_{i,j}^{od,c}(k)$  which represent the units of class  $c$  referred to the pair  $od$  that actually exit the arc  $(i, j)$  as

$$O_{i,j}^{od,c}(k) = S_{i,j}^{od,c}(k) \sum_{n \in S(j)} \beta_{j,n}^{od,c}(k) (1 - \pi_{j,n}(k)) \quad (40)$$

## VI. RESULTS AND DISCUSSION

The focus of this section is to show the potential benefits that can be obtained by adopting the proposed multi-class intermodal modeling scheme, simulating a perturbation on the network and then analyzing how this event can affect the rest of the network. Specifically, the considered perturbation is the failure of a connection in the intermodal network. The experimental tests have been performed in two distinct scenarios:

- *Scenario pre-disruption*: network at the equilibrium before the advent of the disruption;
- *Scenario post-disruption*: network once a new equilibrium is reached some time after the initial perturbation.

The results have been obtained by adopting a test network derived from the well-known Nguyen-Dupuis network properly modified to consider the intermodal case (for further information see [44]). This network is composed of 14 nodes and 21 arcs, as depicted in Fig. 3. The critical event is simulated considering the total loss of functionality of the railway arc 12-8.

The main parameters of the highway and railway arcs are reported in Table 4 and Table 5, respectively. The other parameters have been set as follows: the conversion factor  $\eta$ , representing the average number of passengers per car, is equal to 1.45, the congestion wave speed  $\omega_{i,j}$  is equal to 30 [km/h],  $\forall(i, j) \in \mathcal{A}^H$ , the average time headway  $h_{i,j}$  is 15 minutes and the minimum average space headway  $s_{i,j}^{min}$  is equal to 2 [km],  $\forall(i, j) \in \mathcal{A}^R$ , while the constant parameter  $\alpha_{i,j}$  has been set equal to 15,  $\forall(i, j) \in \mathcal{A}^{lp}$  and  $\gamma_{i,j}$  equal to 30,  $\forall(i, j) \in \mathcal{A}^{lf}$ . The capacity of a freight train  $C^f$  is 25 rail wagons, while the capacity of a passenger train  $C^p$  is chosen equal to 700 passengers.

As for the dynamic model, a sample time  $T$  equal to one minute has been adopted, while the time horizon of the

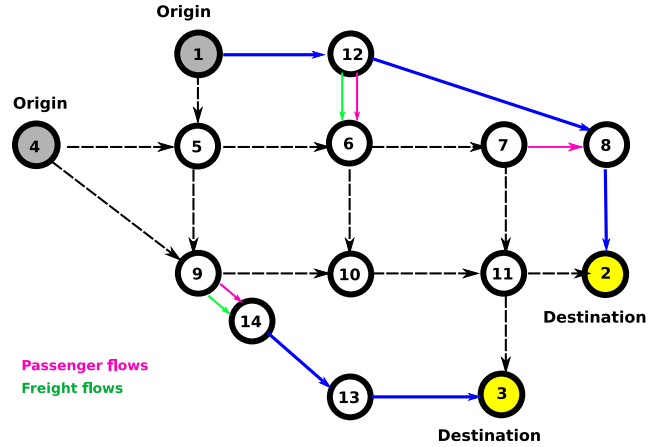


FIGURE 3. Sketch of the test intermodal transport network.

TABLE 4. Main parameters referred to highway arcs.

	$\Delta_{i,j}$ [km]	$v_{i,j}^H$ [km/h]
arc 1-5	6	70
arc 4-5	5	70
arc 4-9	12	70
arc 5-6	40	120
arc 5-9	6	70
arc 6-7	40	120
arc 6-10	8	70
arc 7-11	7	70
arc 9-10	40	120
arc 10-11	40	120
arc 11-2	6	70
arc 11-3	4	70

TABLE 5. Main parameters referred to railway arcs.

	$\Delta_{i,j}$ [km]	$v_{i,j}^R$ [km/h]
arc 1-12	40	120
arc 8-2	6	120
arc 12-8	50	120
arc 13-3	40	120
arc 14-13	45	120

simulation has been set equal to one hour, that corresponds to  $K = 60$  time steps. The transportation demand is expressed with two origin-destination matrices, one referring to passengers, as shown in Table 6, and one referring to freight, as indicated in Table 7. The mobility demand of both passengers and freight is assumed to be perfectly inelastic and therefore it is the same in the two tested scenarios.

Note finally that, in both scenarios described below, the splitting rates  $\beta_{i,j}^{od,c}(k)$  computed by the intermodal traffic assignment are constant throughout the simulation period, and, then,  $k$  is omitted for the sake of simplicity, i.e.,  $\beta_{i,j}^{od,c}(k) = \beta_{i,j}^{od,c}$ .

### A. PERFORMANCE INDICATORS

In order to evaluate the effects of the mobility demand on the intermodal transport network, some performance indicators

TABLE 6. Passengers origin-destination matrix.

$D^{od,1}$	Destination node 2	Destination node 4
Origin node 1	1800	1500
Origin node 4	2500	2000

TABLE 7. Freight origin-destination matrix.

$D^{od,2}$	Destination node 2	Destination node 4
Origin node 1	150	80
Origin node 4	40	25

have been developed using some of the dynamic variables introduced in Section V to describe the behavior of the system over time and space. Specifically, the proposed indicators are the *Total Travel Time*, the *Mean Arc Occupancy*, and the *Mean Arc Saturation*. Let us begin with the *Total Travel Time*, that is an indicator that computes the total time spent by each unit, both of passengers and freight type, in a connection, considering that an arc can belong to multiple paths at the same time. This indicator refers to each arc  $(i, j) \in \mathcal{A}$  and is calculated as follows:

$$TTT_{i,j} = T \sum_{k=1}^K \sum_{c=1}^2 \sum_{o \in J^O} \sum_{d \in J^D} n_{i,j}^{od,c}(k) \quad (41)$$

The *Mean Arc Occupancy* is an indicator for assessing the average occupancy of an arc, either road or rail, given the total number of vehicles or trains occupying it on average, therefore the *Mean Arc Occupancy* is given by

$$MAO_{i,j} = \begin{cases} \frac{1}{K} \sum_{k=1}^K n_{i,j}^{tot}(k) & \text{if } (i, j) \in \mathcal{A}^H \\ \frac{1}{K} \sum_{k=1}^K N_{i,j}^{tot}(k) & \text{if } (i, j) \in \mathcal{A}^R \end{cases} \quad (42)$$

Finally, the *Mean Arc Saturation* relates the *Mean Arc Occupancy* to the capacity of an arc, i.e.

$$MAS_{i,j} = \begin{cases} \frac{MAO_{i,j}}{n_{i,j}^{max}} \cdot 100 & \text{if } (i, j) \in \mathcal{A}^H \\ \frac{MAO_{i,j}}{N_{i,j}^{max}} \cdot 100 & \text{if } (i, j) \in \mathcal{A}^R \end{cases} \quad (43)$$

Note that the *Mean Arc Occupancy* and *Mean Arc Saturation* indicators are calculated for highway and rail arcs only, as for intermodal arcs we assume no capacity restrictions.

### B. SCENARIO PRE-DISRUPTION

The methodology presented in Section IV has been used to allocate the passengers demand on possible routes and the resulting assignment is shown in Fig. 4. It is worth reminding that the feasible paths are those that have at most one modal shift. The paths used at the equilibrium, before the disruption, are reported in Table 8: one path adopts only the rail mode, five are highway routes, while the remaining eight require the use of both modes of transport.

The non-zero splitting rates are in this case:

TABLE 8. Paths used by passengers in the pre-disruption scenario.

od pair	paths
1-2	[1 12 8 2]
1-3	[1 5 9 10 11 3], [1 5 6 7 11 3], [1 12 6 7 11 3], [1 5 9 14 13 3]
4-2	[4 9 10 11 2], [4 5 9 10 11 2], [4 5 6 10 11 2], [4 5 6 7 11 2],
4-3	[4 9 10 11 3], [4 5 6 7 11 3], [4 9 14 13 3]

TABLE 9. Paths used by freight in the pre-disruption scenario.

od pair	paths
1-2	[1 12 8 2]
1-3	[1 5 9 14 13 3]
4-2	[4 5 6 7 11 2], [4 5 9 10 11 2], [4 9 10 11 2]
4-3	[4 9 14 13 3], [4 5 9 14 13 3]

- od pair 1-2:  $\beta_{1,12}^{12,1} = 1, \beta_{5,6}^{12,1} = 1, \beta_{6,7}^{12,1} = 1, \beta_{7,11}^{12,1} = 1, \beta_{8,2}^{12,1} = 1, \beta_{11,2}^{12,1} = 1, \beta_{12,8}^{12,1} = 1;$
- od pair 1-3:  $\beta_{1,5}^{13,1} = 0.92, \beta_{1,12}^{13,1} = 0.92, \beta_{5,6}^{13,1} = 0.36, \beta_{5,9}^{13,1} = 0.64, \beta_{6,7}^{13,1} = 1, \beta_{7,11}^{13,1} = 1, \beta_{9,10}^{13,1} = 0.12, \beta_{9,14}^{13,1} = 1, \beta_{10,11}^{13,1} = 1, \beta_{11,3}^{13,1} = 1, \beta_{12,6}^{13,1} = 1, \beta_{13,3}^{13,1} = 1, \beta_{14,3}^{13,1} = 1;$
- od pair 4-2:  $\beta_{4,5}^{42,1} = 0.62, \beta_{4,9}^{42,1} = 0.38, \beta_{5,6}^{42,1} = 0.75, \beta_{5,9}^{42,1} = 0.25, \beta_{6,7}^{42,1} = 0.67, \beta_{6,10}^{42,1} = 0.33, \beta_{7,11}^{42,1} = 1, \beta_{9,10}^{42,1} = 1, \beta_{10,11}^{42,1} = 1, \beta_{11,2}^{42,1} = 1;$
- od pair 4-3:  $\beta_{4,5}^{43,1} = 0.18, \beta_{4,9}^{43,1} = 0.82, \beta_{5,6}^{43,1} = 1, \beta_{6,7}^{43,1} = 1, \beta_{7,11}^{43,1} = 1, \beta_{9,10}^{43,1} = 0.28, \beta_{9,14}^{43,1} = 0.72, \beta_{10,11}^{43,1} = 1, \beta_{11,3}^{43,1} = 1, \beta_{13,3}^{43,1} = 1, \beta_{14,13}^{43,1} = 1.$

The freight flows are fixed and their distribution is shown again in Fig. 4, while the paths are reported in Table 9. The corresponding non-zero splitting rates, again considered constant throughout the simulation, are the following:

- od pair 1-2:  $\beta_{1,12}^{12,2} = 1, \beta_{8,2}^{12,2} = 1, \beta_{12,8}^{12,2} = 1;$
- od pair 1-3:  $\beta_{1,5}^{13,2} = 1, \beta_{5,9}^{13,2} = 1, \beta_{9,14}^{13,2} = 1, \beta_{13,3}^{13,2} = 1, \beta_{14,13}^{13,2} = 1;$
- od pair 4-2:  $\beta_{4,5}^{42,2} = 0.75, \beta_{4,9}^{42,2} = 0.24, \beta_{5,6}^{42,2} = 0.17, \beta_{5,9}^{42,2} = 0.82, \beta_{6,7}^{42,2} = 1, \beta_{7,11}^{42,2} = 1, \beta_{11,2}^{42,2} = 1, \beta_{9,10}^{42,2} = 1, \beta_{10,11}^{42,2} = 1;$
- od pair 4-3:  $\beta_{4,9}^{43,2} = 0.21, \beta_{4,5}^{43,2} = 0.78, \beta_{5,9}^{43,2} = 1, \beta_{9,14}^{43,2} = 1, \beta_{13,3}^{43,2} = 1, \beta_{14,13}^{43,2} = 1.$

It should be noted that among these routes, only one is of intermodal type, with a change from road to rail, one route is entirely by railway, while the remaining ones involve only the use of highway arcs.

### C. SCENARIO POST-DISRUPTION

As mentioned earlier, the disruption is represented in this example by removing the railway arc 12-8. Regarding passengers, the new path configuration for each origin-destination





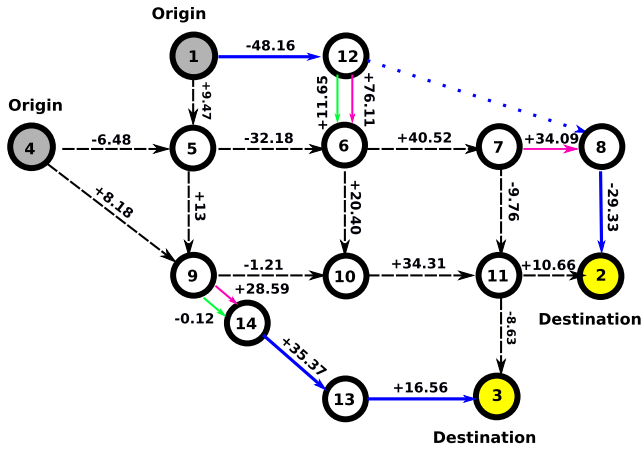


FIGURE 6. Total Travel Time absolute variations.

TABLE 12. The Total Travel Time for each arc.

	$TTT_{i,j}^E$	$TTT_{i,j}^D$	$\Delta TTT_{i,j}$
Arc 1-5	101.33	110.80	+9.47
Arc 1-12	629.54	581.38	-48.16
Arc 4-5	129.16	122.68	-6.48
Arc 4-9	219.33	227.51	+8.18
Arc 5-6	392.94	360.76	-32.18
Arc 5-9	88.99	101.99	+13.00
Arc 6-7	227.88	268.40	+40.52
Arc 6-10	20.57	40.97	+20.40
Arc 7-8	0.00	34.09	+34.09
Arc 7-11	40.92	31.17	-9.76
Arc 8-2	38.46	9.13	-29.33
Arc 9-10	360.60	359.39	-1.21
Arc 10-11	251.67	285.98	+34.31
Arc 11-2	52.57	63.23	+10.66
Arc 11-3	22.41	13.78	-8.63
Arc 12-8	184.64	-	-
Arc 13-3	107.92	124.49	+16.56
Arc 14-13	230.44	265.81	+35.37
Arc 9-14 If	37.96	37.84	-0.12
Arc 9-14 Ip	186.27	214.86	+28.59
Arc 12-6 If	0.00	+11.65	+11.65
Arc 12-6 Ip	5.41	81.52	+76.11

the values of the Mean Arc Saturation for each roadway and railway arc, in the pre-disruption scenario, i.e.,  $MAS_{i,j}^E$ , and in the post-disruption scenario, i.e.,  $MAS_{i,j}^D$ . The deviation from the unperturbed case is not reported for this indicator because, being derived from the Mean Arc Occupancy, this indicator has the same variations.

As can be seen, the disruption of 12-8 arc implies that freight and passenger flows, which previously used the peripheral rail route, are re-assigned in more internal routes, particularly intermodal routes. Consistently with an overall load increase on the central arcs of the network, it is possible to observe a shift, though small, of flows of the 4-3 pair in favor of the outermost intermodal route [4-9-14-13-3]. Not surprisingly, the arcs relatively most affected by the perturbation are those in proximity to the disrupted arc, such as arcs 12-6, 6-7 and 7-8. However, it can be seen that even the central arc 6-10 experiences a 30% increase in Total Travel Time. The computation of a metric such as the Total Travel

TABLE 13. The Mean Arc Occupancy for highway and railway arcs.

	$MAO_{i,j}^E$	$MAO_{i,j}^D$	$\Delta MAO_{i,j}$
Arc 1-5	77.15	83.69	+8.47
Arc 1-12	2.66	2.57	-3.33
Arc 4-5	93.41	88.80	-4.94
Arc 4-9	152.92	158.74	+3.81
Arc 5-6	272.34	248.79	-8.65
Arc 5-9	71.80	81.07	+12.90
Arc 6-7	157.36	191.49	+21.69
Arc 6-10	14.47	28.80	+99.01
Arc 7-11	28.26	22.59	-20.06
Arc 8-2	0.17	0.01	-92.29
Arc 9-10	256.66	257.11	+0.18
Arc 10-11	178.73	203.49	+13.85
Arc 11-2	36.96	45.63	+23.44
Arc 11-3	15.74	+9.49	-39.70
Arc 12-8	0.81	-	-
Arc 13-3	0.15	0.18	+15.19
Arc 14-13	0.33	0.38	+15.23

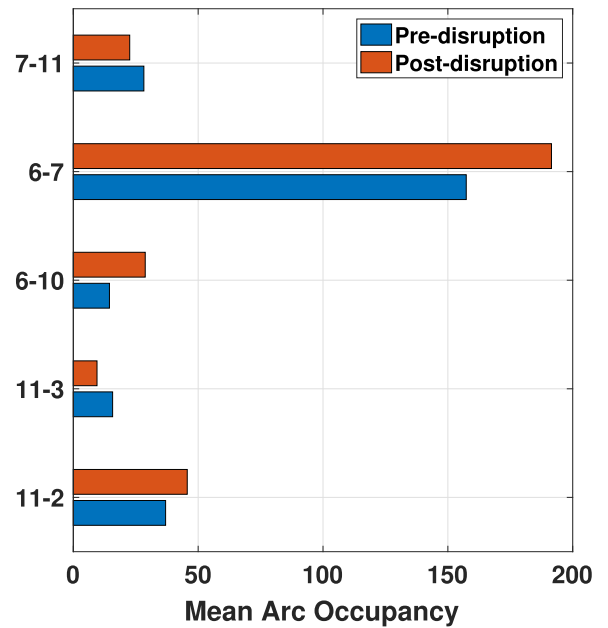


FIGURE 7. Mean Arc Occupancy for some of the most affected highway arcs.

Time, by means of the dynamic model presented in this paper, allows therefore to identify the elements of the network that will be more stressed after the perturbation.

Analyzing Tables 13 and 14, it is possible to observe that the disruptive event has effects on Mean Arc Occupancy and Mean Arc Saturation even for arcs that are not connected with the collapsed one. In addition, it can be seen that the disturbance affects not only the railway arcs but also the highway arcs, as highlighted in Fig. 7. Referring to the highway arcs, the most significant change in the average occupancy evaluated in a time interval of one minute is experienced for arc 6-10 which approximately doubles this indicator.

From the analysis of these results, it can then be concluded that the adoption of a modeling framework of this type permits to grasp the interdependencies that exist between the road and rail modes of transport, enabling a more accurate

**TABLE 14.** The Mean Arc Saturation for highway and railway arcs.

	$MAS_{i,j}^E$	$MAS_{i,j}^D$
Arc 1-5	1.93	2.09
Arc 1-12	13.30	12.85
Arc 4-5	2.34	2.22
Arc 4-9	2.55	2.65
Arc 5-6	4.54	4.15
Arc 5-9	1.80	2.03
Arc 6-7	2.62	3.19
Arc 6-10	0.36	0.72
Arc 7-11	0.71	0.56
Arc 8-2	5.64	0.43
Arc 9-10	4.28	4.29
Arc 10-11	2.98	3.39
Arc 11-2	0.92	1.14
Arc 11-3	0.39	0.24
Arc 12-8	3.25	-
Arc 13-3	0.77	0.89
Arc 14-13	1.32	1.52

analysis than the one which can be conducted using modeling frameworks that contemplate a single mode of transport only.

## VII. CONCLUSION

In this paper, a two-stage modeling approach is proposed to represent passenger and freight flows on a intermodal transportation network, i.e., a network in which there are roadways, railways, and connections that allow the modal shifts. The modeling scheme consists of an assignment model and a discrete-time dynamic model. The assignment model allows to represent the choices of the users, both passengers and freight, in terms of routes and transport modes. These route choices provide the input to the dynamic model that allows to represent the evolution in time and space of user flows, allowing to evaluate some dynamic characteristics such as speed and travel time on the network arcs. In order to provide some insights about the proposed approach, this concluding section has been structured into four parts, in order to further discuss the results produced in this paper, to illustrate some possible applications of the proposed modeling framework for scenario evaluation, on the one hand, and for testing regulatory policies and control actions, on the other hand, and, finally, to argue about the possible weaknesses of this methodology.

### 1) DISCUSSION OF THE PROPOSED TEST CASE

The objective of the proposed case study has been to illustrate the benefits of the proposed intermodal modeling scheme by analyzing the behavior of a network subject to a disruptive event, specifically the loss of functionality of a railway arc. This application revealed the ability of the intermodal model to capture the ripple effects of such events that cannot be gained if analyzed with models representing a single transportation mode only.

The analysis showed that the collapse of a railway arc can bring a substantial increase in travel time for some highway arcs, which can reach a 30-40% increase, and an even greater growth in the average occupancy of highway and railway arcs that are not directly connected with the

collapsed arc. Specifically, some arcs, both of road and rail type, have almost doubled in volume, making them obviously more critical to any further disruption. In addition, the post-disruption scenario showed a significant increase in the use of intermodal arcs, both for passengers and freight, suggesting to a possible decision maker that the adoption of more connections of this type can make the transport network more robust to the occurrence of unexpected events.

### 2) POSSIBLE APPLICATIONS FOR SCENARIO EVALUATION

The proposed model can be adopted to evaluate the effects of the decisions of the users on a transport network. These effects can be quantified through the adoption of specific performance indexes, such as those introduced above, or through the development of other indicators. For example, through the evaluation of the *Mean Arc Saturation* index, a preliminary analysis can be conducted to identify, under specific mobility demand conditions, which arcs are closest to saturation and assess the effects of their failure. Furthermore, this model can also be used to assess the sustainability of user choices by integrating this modeling scheme with models that estimate emissions or energy consumptions.

Scenarios of particular relevance are those concerning the occurrence of critical events. Indeed, large transport networks are anyway susceptible to critical events that may have severe implications on the whole activity system of a territory. These disruptive events may be caused by natural phenomena (such as floods, earthquakes, pandemics, etc.) or anthropogenic causes (such as terrorist attacks, infrastructure failures, but also planned maintenance works): whatever the cause, they may affect the transport network as a variation of mobility demand or as events that partially or totally deteriorate the capacity of a transport network. In this regard, a topic that is attracting particular attention in scientific research is the evaluation of the resilience of a transport network, i.e., its ability to resist, adapt or change in order to maintain acceptable performance in case of critical events. Although the concept of resilience has been initially introduced to describe a property of natural systems [45], recently it has been applied to transport networks and in particular to road networks [46]–[48] and railway networks [49]–[51]. However, as shown in the case study presented in this paper, transport networks are complex and highly interdependent systems and a critical event affecting one mode of transport can have an impact on other modes giving rise to a chain effect. What is lacking in the literature, and what this paper has aimed to address, is the possibility of using a tool that allows to quantify the effects of these interdependencies and to evaluate the ability of a transport network to maintain acceptable performance even when it is affected by disruptions.

### 3) POSSIBLE APPLICATIONS FOR DEVELOPMENT AND TESTING OF REGULATORY POLICIES AND CONTROL ACTIONS

The modeling framework proposed in this paper may constitute the basis for regulation and control approaches finalized

at defining routing and modal indications to be provided to the users. First of all, this model can be used to provide more detailed information to users about travel times or route choices for improving sustainability. Moreover, specific routing instructions can be defined for the users and, since the modeling scheme is multi-class, such instructions can be suitably defined for each class of users.

The proposed two-stage modeling framework may be adopted to test control policies that aim at fully exploiting the mobility capacity of a large-scale intermodal transport network by suggesting routes that may include one or more transport modes. This can be done both in nominal conditions of the network or when the transport system is affected by an event which changes its structure or the mobility demand.

Finally, this approach can be used to develop online journey re-planning strategies such as the one developed in [52] for the re-routing of freight during disruptive events.

#### 4) POSSIBLE WEAKNESSES OF THE PROPOSED METHODOLOGY

The major weakness concerning the proposed methodology is related to the collection and elaboration of data that must describe both the transport supply, i.e., the infrastructural and technical characteristics of the intermodal network, and the mobility demand that must be expressed both in terms of origin/destination matrix, to represent the willingness of freight and passengers to move, and punctual measurements on the network for the validation of the modeling scheme. This phase is particularly challenging because, since a large-scale intermodal network is considered, the involvement of different actors over a large territory is required and, then, it is necessary to integrate data from different, and possibly heterogeneous, data sources, as thoroughly discussed in [53].

#### APPENDIX

In the following, the main steps that have led to the definition of the function (39) expressing the steady-state speed with respect to the number of trains are briefly sketched. Let us start by considering a generic graphical train timetable and let us define with  $\Delta X$  the spatial interval,  $\Delta T$  the time interval and with  $N$  the number of trains present in that frame, hence the average space headway  $\bar{s}$  can be defined as  $\bar{s} = \frac{\Delta X}{N}$  while the average time headway  $\bar{h}$  is given by  $\bar{h} = \frac{\Delta T}{N}$ . Let us also define the relation between the average space headway and the average time headway that is given by  $\bar{s} = \bar{h} \cdot v + L$ , where  $v$  is the train speed and  $L$  is the average length of trains.

Inspired by the traffic fundamental diagram, we can assume that, for a given average time headway, the maximum flow of trains in an arc corresponds to an average space headway equal to  $\bar{s} = \bar{h} \cdot v^{max} + L$ , where  $v^{max}$  is the maximum speed allowed. At the same time, it has to be remembered that trains have to maintain a minimum distance  $s^{min}$  that allows them to stop safely; consequently, as the number of trains on an arc increases, they have to reduce their speed in order to guarantee this safety distance. Then, a triangular traffic

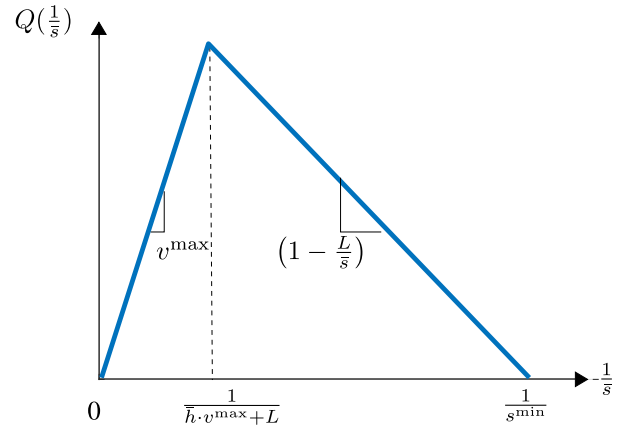


FIGURE 8. Triangular fundamental diagram for railway traffic.

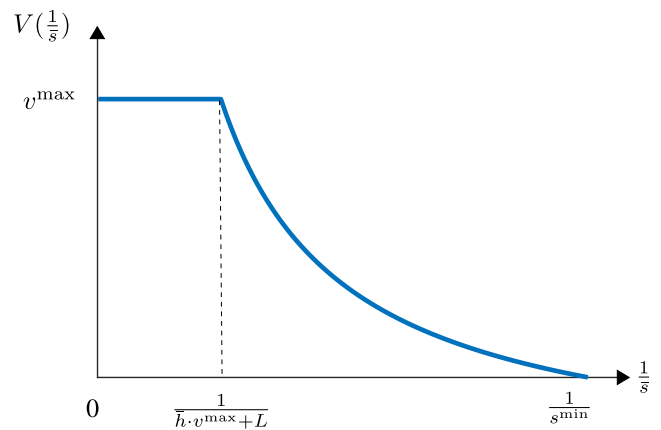


FIGURE 9. Steady-state speed-headway functions for railway traffic.

fundamental diagram in the railway context can be defined as follows

$$Q\left(\frac{1}{\bar{s}}\right) = \begin{cases} \frac{1}{\bar{s}} \cdot v^{max} & \text{if } \frac{1}{\bar{s}} \leq \frac{1}{\bar{h} \cdot v^{max} + L} \\ \frac{1}{\bar{h}} \left(1 - \frac{L}{\bar{s}}\right) & \text{if } \frac{1}{\bar{h} \cdot v^{max} + L} < \frac{1}{\bar{s}} \leq \frac{1}{s^{min}} \end{cases} \quad (44)$$

where  $Q\left(\frac{1}{\bar{s}}\right)$  is the ‘‘flow of trains’’ in function of the average space headway. The corresponding steady-state speed-headway relation  $V\left(\frac{1}{\bar{s}}\right)$  is given by

$$V\left(\frac{1}{\bar{s}}\right) = \begin{cases} v^{max} & \text{if } \frac{1}{\bar{s}} \leq \frac{1}{\bar{h} \cdot v^{max} + L} \\ \frac{1}{\bar{h}} (\bar{s} - L) & \text{if } \frac{1}{\bar{h} \cdot v^{max} + L} < \frac{1}{\bar{s}} \leq \frac{1}{s^{min}} \end{cases} \quad (45)$$

Thus, equation (39), has been derived on the basis of (45) and by considering that the inverse of the average space headway has an equivalent meaning of ‘‘density of trains’’ in an arc that is  $\frac{N_{i,j}^{tot}(k)}{\Delta_{i,j}}$ .



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