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SURVEY

A Survey of Decomposition Based Evolutionary Algorithms for Many-Objective Optimization Problems

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ABSTRACT The framework of decomposition-based multi-objective evolutionary algorithms(MOEA/D) has evolved for more than ten years, and it has become irreplaceable tool for solving multi-objective optimization problems. In recent years, many scholars have investigated improved strategies from different directions. This paper gives a systematic comparison of six different components for decomposition-based algorithms, including framework analysis, weight vector generation scheme, aggregation evaluation function construction, reproduction operator, individual selection and update strategy, and the characteristics and application scope of various algorithms are also analyzed in detail in the survey. Different from previous survey on decomposition-based multi-objective evolutionary algorithms, a more detailed classification and experimental comparison are elaborated in the proposed paper.

INDEX TERMS Many-objective, decomposition, evolutionary algorithm.

I. INTRODUCTION

Without the loss of generality, multi-objective optimization problems (MOPs) commonly used in practical engineering are defined as formula (1), where x is decision variable vector, and Ω and F are search space and objective space consisting of ^{m} objective functions. Simultaneously, when the size of objectives is more than three, they are considered as many-objective problems (MaOPs).

$$\min F(x) = (f_1(x), \cdots f_m(x))^T$$
$$s.t. x \in \Omega \tag{1}$$

Due to the conflicting characteristics of objective functions in MOPs and MaOPs, there is no unique optimal solution. Instead, a number of compromise individuals that non-dominant with each other are obtained as the best solutions, which are called Pareto optimal solutions. So far, evolutionary algorithms (EAs) have become the mainstream algorithm [1], [2] for MOPs and MaOPs because of population based characteristics, and have formed an independent algorithm branch, named multi-objective evolutionary algorithm (MOEA).

Although MOEA can successfully obtain satisfactory optimal solution set for problems with only 2 or 3 objectives, its performance will be greatly degraded for problems with irregular Pareto front (PF) shapes [3] or problems with many objectives. The reasons are mainly reflected in two aspects. Firstly, when the number of objectives increases, the traditional individual comparison methods based on dominance relationship cannot quantitatively distinguish individuals in the population, so that the selection pressure of individuals will be eliminated, and search ability will be declined [4]. Secondly, the number of individuals covering the complete PF will increase dramatically as the number of objectives increase [5], so it is not easy to acquire Pareto optimal solution set performing well in both convergence and diversity at limited computing resources.

With the increasing complexity of engineering optimization problems, the research results of multi-objective evolutionary algorithm have been widely used in various fields,

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including ship type parameter optimization in ship design, filter optimization design, power system operation scheduling optimization, and logistics distribution center location problem. These complex optimization problems often contain a large number of optimization objectives, and it is difficult to obtain a set of satisfactory solutions.

In recent years, scholars have carried out a lot of research to enhance the performance of the obtained solution set for MaOPs, and the main research work can be classified into the four types, including modifying dominance relation to improve selective pressure of solutions [6]–[11], optimizing performance indicator [12]–[14] as objective, objective reduction strategy [15]–[22] and decomposition based algorithms. This paper will focus on the last category. The decomposition based algorithm divides the original MaOPs into a number of sub-problems via a set of weight vectors, and utilizes a population-based method to solve sub-problems collaboratively. Due to its effectiveness and simplicity, decomposition based algorithms have become a promising mechanism for handing MaOPs. Now decomposition based MOEAs, particularly after MOEA/D presented by Zhang and Li [23], have been demonstrated promising performance for dealing with MaOPs. Nevertheless, convergence or diversity will deteriorate for problems with complicated shape of Pareto front. The improved studies have been conducted on the following aspects including weight vector design, individual fitness evaluation mechanism, mating selection and environmental selection strategy.

Recently, there are several review literatures about decomposition-based MOEAs, e.g. Ma *et al.* [24] systematically compares the basic principles and experimental performance of several typical algorithms based on decomposition, including MSOPS, MOEA/D, NSGAIII, MOEA/DD, DBEA MOEA/D-DU and EFR-RR. Trivedi and Srinivasan [25] and Xu *et al.* [26] summarize the improvement strategies in different directions, such as the variant of novel aggregation functions, weight vector generation approaches, mating and environmental selection strategy, etal. In addition, there are also some reviews that summarize and compare the strategies in single direction for decomposition based algorithm. For instance, in literature [27], a detailed survey of reference vector adjustment approaches is presented. The authors focus on the adjustment strategy of weight vector on simplex, and analyze the adaptation strategy, adjustment frequency. Also, the advantages and disadvantages of each weight vector adjustment method are presented in detail. A survey of various types of decomposition-based aggregation functions [28] for MOEA/D is discussed, and the influence of various aggregation functions on convergence and diversity is concluded.

However, there is little literature to analyze the applicability of different strategies by comparing experiments. The motivation of the proposed paper can be described as follows. Starting from comparing the framework of different decomposition algorithm, the author summarizes the characteristics of several types of algorithm framework based on decomposition, containing individual evaluation methods, diversity maintenance and selection strategies, and also compares their performance through algorithm simulation. Next, this paper will investigate a comprehensive survey on decomposition based MOEAs in different strategies, including weight vector generation scheme, aggregation evaluation function construction, reproduction operator, individual selection and update strategy. Notably, the effect of experimental performance comparison with different strategies has also been conducted in the paper. To compare the performance of the algorithms, four type test cases for MaOPs, DTLZ, WFG, MOP and UF are chosen in the experiments.

The remainder of the paper is organized as follows. The framework of MOEA/D proposed by Zhang is presented in Section II, Section III generalizes the framework of decomposition based MaEAs, and five different directions are described respectively in Section IV to VIII. Finally, Section IX concludes the paper.

II. RELATED WORK

A. THE FRAMEWORK OF MOEA/D

Original decomposition based multi-objective evolutionary algorithm (MOEA/D) is proposed by Qingfu Zhang, and decomposition and collaboration are two important elements. It employs a number of reference vectors $\omega^1, \omega^2, \cdots, \omega^N$ to decompose the original problem into a set of single-objective sub-problems. In each sub-problem, MOEA/D adopts evolutionary algorithm to find unique optimal solution, which corresponds to one non-dominated solution of the MOPs. Globally, the diversity of the non-dominated solution set largely depends on the setting of the reference vectors. Meanwhile, the assessment of individuals is completely relied on the scalarization function, which will play a vital role in the update of the individuals.

Besides, the evolution of individuals is not only restricted in its subpopulation, but also involves the neighborhood of the sub-problem. T closet neighbors based on the Euclidean distances are exploited to solve the corresponding SOP in a collaborative way, and each sub-problem primarily uses the information from its neighboring sub-problems to execute evolution and update. In a sense, two solutions have a probability to execute mating selection and replacement when they are for two neighboring sub-problems. The framework of MOEA/D is shown in Figure 1.

Although MOEA/D is potential when dealing with MOPs with regular PF, the performance will deteriorate with the increase of problem complexity. In fact, the original MOEA/D itself has some limitations: 1) simplex-lattice approach is used to generate reference vectors in MOEA/D, which is only suitable for MOPs with regular shape of Pareto front. If the shape of the true PF is irregular, for example, discontinued, degenerated, incomplete, the adopted weight vectors generation method can't guarantee the diversity of solutions. 2) The choice of aggregation function does not take into account the need for different problems with different Pareto front. e.g. WS method performs well in convex

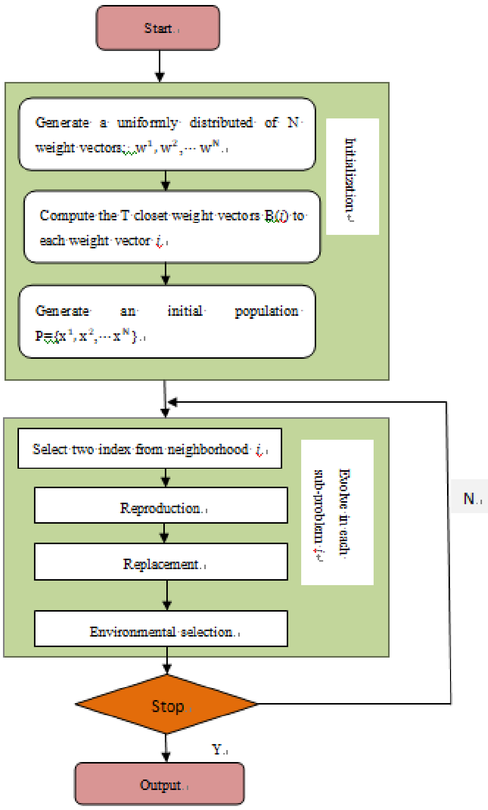

FIGURE 1. Framework of MOEA/D.

problems while TCH approach is better suitable for non-convex problems. 3) Individuals involved in mating selection in original MOEA/D have been always selected within the neighboring sub-problems without considering the population distribution of the individual's region. Here, the neighborhood and its size as two vital parameters determine the individuals participating the mating and entering in the next generation. However, the above parameters are fixed in the whole evolution, which will imbalance the convergence and diversity. Also, it may generate similar offspring thus resulting to the loss of diversity.

In order to retain the MOEA/D framework while improving the quality of obtained solutions, several variants have been proposed.

B. DIFFERENT IMPROVEMENT DIRECTIONS OF DECOMPOSITION BASED MOEA

After analyzing the limitation of MOEA/D, this section gives a framework of the variants in six aspects, which is illustrated in Figure 2. Next, we will investigate the different strategies in every direction in detail, and give a comprehensive analysis on experimental results of compared algorithms.

III. STUDIES ON THE FRAMEWORK OF DECOMPOSITION

A. THE ROUGH DESCRIPTION OF RELATED ALGORITHMS USING THE IDEA OF DCOMPOSITION

Before the MOEA/D proposed by Zhang, the idea of decomposition based approach for solving MOPs has emerged.

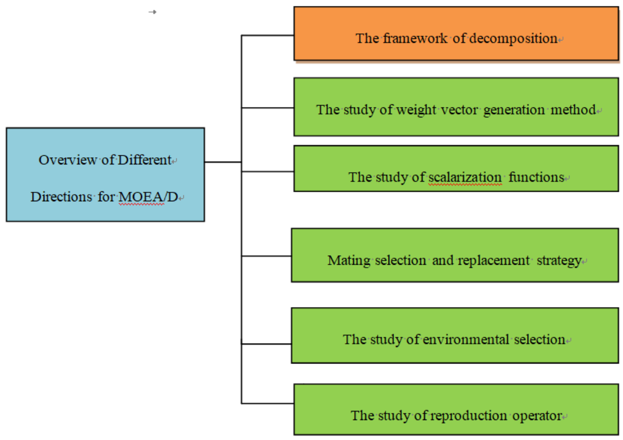

FIGURE 2. Overview the different directions for MOEA/D.

In fact, the origin of MOEA/D comes from cellular based multi-objective genetic algorithm (C-MOGA [29]), and the similarity is that both borrow the idea of reference vectors to transform the MOP into a set of SOPs. However, compared with MOEA/D, C-MOGA employs cellular structure to assign weight vectors. Besides, the replacement strategy in MOEA/D is different from C-MOGA, since local replacement adopted in MOEA/D is executed within its neighbors, while C-MOGA only with the current solution of the cell.

Multiple Single Objective Pareto Sampling (MSOPS [30]) and its variants (MSOPSII [31]) are another decomposition based method using predefined weight vectors to implement multiple single objective optimization in parallel. They use the score matrix to evaluate the ranking value of all individuals in the population for each weight vector. The size of rows and columns represents the number of population and weight vectors, respectively. In the process of ranking individuals in population, the elements of the score matrix are sorted by column and row, and finally the best individual for every reference vector is chosen as the output.

Instead of using reference vectors to preserve the diversity, NSGAIII [32] implicitly borrows the idea of decomposition with combination of NSGAII and MOEA/D, and a number of uniformly scattered reference points is employed to maintain the diversity of population. In NSGAIII, fast non-domination ranking is employed to partition the population into different fronts, with higher fronts indicating better convergence of individuals. The diversity of individuals is measured by the perpendicular distance of the reference points to which it is associated, using the principle of niche preservation.

Notably, Li *et al.* [33] combines the idea of decomposition with dominance relation and proposes a unified framework, named MOEA/DD. Starting with employing a set of weight vectors by two-layer generation approach to guide selection procedure, the evolution of population is executed in the whole population but not in sub-problems. In MOEA/DD,

TABLE 1. Summary of main characteristics for five decomposition based algorithms.

Characteristics	MOEA/D	NSGAIII	θ -DEA	MOEA/DD	SDEA
The evaluation mechanism of individuals	Scalarization function	Pareto dominance	θ - dominance	Scalarized based dominance	Non-domination level Scalarization function
Neighborhood selection	Yes	No	No	No	Yes
Diversity maintenance using reference vectors	No	Yes	Yes	Yes	Yes
Diversity preservation method	No	Distance -based	Distance -based	Angle-based	Niche-count

the non-dominated solutions with lower scalarization fitness values in the sub-region are preferentially selected. To maintain the balance of diversity and convergence, hierarchical manner is utilized for environmental selection considering Pareto dominance, scalarization fitness values and local density estimation respectively.

To alleviate the computational complexity, a scalarization-based dominance evolutionary algorithm (SDEA [34]), which combines the reference point method with the scalarization method, is proposed to develop a individual ranking strategy. Table 1 lists the characteristics of five decomposition based algorithms.

B. EXPERIMENTAL RESULTS OF RELATED ALGORITHMS USING THE FRAMEWORK OF DECOMPOSITION

In this part, four representative algorithms are adopted, including MOEA/D, MSOPSII, NSGAIII and MOEA/DD. Two types of test cases, MOPs with regular shape of PF, such as DTLZ1, DTLZ3, and MOPs with irregular shape of PF, such as DTLZ7 and WFG1, are adopted. In simulation, the number of objectives (m) is set to 5. For the sake of fairness, we use a MATLAB Platform for Evolutionary Multi-Objective Optimization (PlatEMO) [35] to complete the execution of all compared algorithms, and the setting of relevant parameters refers to literature [32].

Line graphs are used to show the average value of the IGD over 20 runs for the four compared algorithms under different generation in each test problem in Figure 3. In Figure 3(1), the Pareto front of DTLZ1 is linear triangular hyper-plane with $\sum_{i=1}^{m} f_i = 0.5$, and the purpose of choosing this test case is to verify the ability of the algorithm to search for global optimality in problems with multiple local optimum. It is observed that the quality of the solutions obtained by MSOPSII is inferior to other three algorithms under different generations. Specifically, MOEA/DD obtains the best results at the early stage of evolution, while with the increase of generation, the difference between the MOEA/D, NSGAIII and MOEA/DD become less apparent.

Figure 3(2) shows the comparison results of the average IGD values on test case DTLZ3, and the Pareto front of DTLZ3 is a sphere hyper-plane with $\sum_{i=1}^{m} f_i^2 = 1$, which used to testify whether the comparison algorithm will fall into a local optimum before reaching the global optimum. At the early stage of evolution, MSOPSII outperforms than other three algorithms, however, its performance varies subtle in the whole evolution. In particular, The performance of the other three types of algorithms is significantly improved after 400 generations, and MOEA/DD shows better results in generation 400, 600 and 800, while NSGAIII obtains best IGD values in generation 1000 and 1500.

The comparison of IGD results of DTLZ7(5) in different algorithms is shown in Figure 3(3). Due to the disconnected PF of DTLZ7(5), this test case is employed to evaluate whether the compared algorithms are capable of dealing with disconnected segments. As shown in figure 3(3), the difference between four algorithms is apparent. NSGAIII obtains the best results in every stage of evolution, and the performance of MOEA/DD is worst.

Figure 3(4) displays the comparison of the average IGD values on WFG1, which is used to assess the ability of compared algorithms in MOPs of complicated mixed geometries. In general, NSGAIII and MSOPSII outperform than the other two algorithms in the whole stage of evolution. As for NSGAIII and MSOPSII, MSOPSII shows better results than NSGAIII in generation 200, 400 and 600, while NSGAIII obtains the best IGD values in generation 800, 1000 and 1500.

C. SUMMARY

In general, two types of test cases are adopted here to compare the performance of four representative decomposition based algorithms. It can be shown that the hybrid algorithm combining the dominance relation and decomposition can achieve better performance than the original MOEA/D in many-objective optimization problems. Besides, none algorithm will have obvious advantages over all the different problems. For DTLZ1 and DTLZ3 with regular shape of PF, MOEA/DD obtains the best results, while for test cases with irregular shape of PF, DTLZ7 and WFG1, NSGAIII outperform than other compared algorithms.

IV. THE STUDY OF GENERATION STRATEGY OF WEIGHT VECTOR

The diversity of weight vectors largely determines the quality of obtained solutions, thus, the study of weight vector or reference vector generation strategy is a crucial problem for decomposition based algorithms [36]. In practice, the actual shape of Pareto front is rather complex, and the uniformity

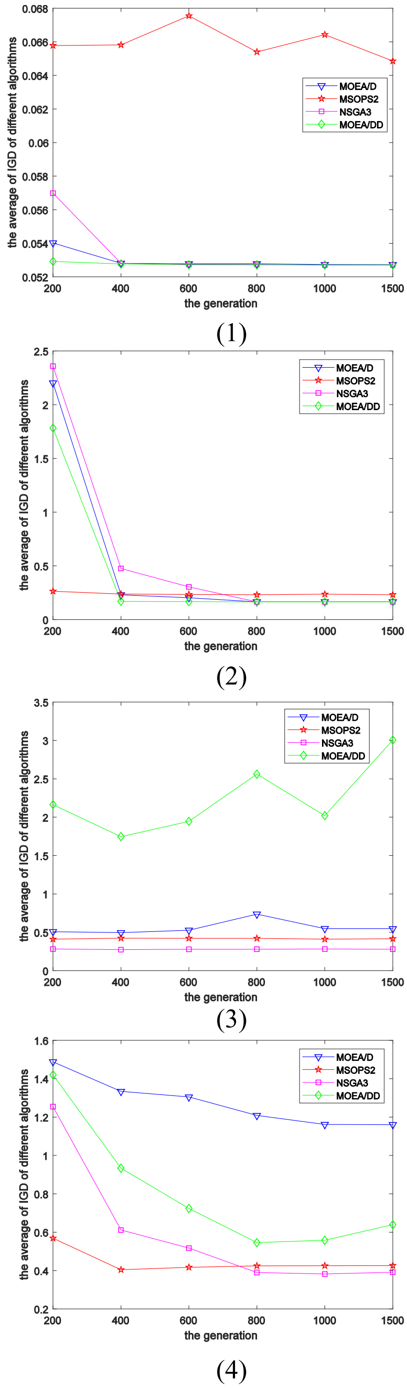

FIGURE 3. Average IGD of compared algorithms on some types of problems.

of weight vector distribution cannot guarantee the quality of obtained solutions. In summary, the generation approach of weight vector can be categorized into four types, the first type is aim to generate evenly distributed weight vector by using experimental design method for MaOPs with regular shape of Pareto front. The second type is an adaptive reference vector generation approach which updates the reference vectors periodically based on the information of current solutions. The second type adopts heuristic information and fitting-based generation method to obtain the most appropriate estimated PF shape. The last type employs machine learning technique to generate dynamically weight vector with the change of population. The classification and the corresponding algorithms are listed in Table 2.

TABLE 2. Classification of weight vector generation strategy.

Classification	Algorithms
Systematic design approaches	UDEM[37], MOEA/D-UMD[38], a two layer reference vector generation approach [33]
Adaptive weight vector generation methods	Generalized decomposition (gD)[41], PICEA-w[43], g-DBEA[45], MaOEA/D-2ADV[44], MOEA/D-AWA[46], AdaW[47]
Fitting based generation methods	W-MOEA/D[48], tw-MOEA/D[49]
Machine learning based methods	CLIA[51], SOM-MOEA/D[52]

A. WEIGHT VECTOR GENERATION BY SYSTEMATIC DESIGN APPROACH

1) THE IDEA OF WEIGHT VECTOR GENERATION BY SYSTEMATIC DESIGN

In MaOPs with regular shape of Pareto front, e.g., a sphere or a triangle plane, several systematic weight vectors are proposed. Initially, the original MOEA/D employs simplex-lattice design approach to create evenly distributed reference vectors. The size of objectives m and the obtained weight vectors N satisfies the relation $N = C_{H+m-1}^{m-1}$. However, the setting of N is not arbitrary, and it will restrict its use in MaOPs with more than five objectives.

In order to overcome the inability of arbitrarily set of weight vectors, Tan *et al.* [37] employed uniform design for experiments with mixtures (UDEM) to construct reference vectors. L_2-discrepancy is adopted to measure the uniformity of reference vector set, and good lattice point approach and transformation method with smallest discrepancy are taken to generate reference vectors. Also, UDEM is combined with MOEA/D as evolutionary framework, which is represented by UMOEA/D.

Since the weight vectors resulting from the above two methods lie on the boundary or inside the simplex, Ma *et al.* [38] proposed an improved MOEA/D with uniform decomposition measurement (MOEA/D-UMD) to make a complementary between the above methods. Initially, uniform design measurement with transformation method is adopted to choose reference vectors with smallest discrepancy to ensure uniformity. Then, the reference vectors in the boundary are selected by uniform design and simplex lattice method repeatedly.

Meanwhile, Li *et al.* developed a two layer reference vector generation approach [33] to ensure the diversity without increasing computational cost. Firstly, weight vector generation method proposed by Das and Dennis is employed to construct two different sets of weight vectors with different parameters setting in the interior and boundary layers of the simplex, respectively. Next, co-ordinate transformation is used to shrink the weight vectors of the inside layer.

2) SIMULATION SUMMARY OF COMPARED ALGORITHMS

In literature [38], the performance of three systematic uniformly designed weight vector methods, namely MOEA/D, UMOEA/D and MOEA/D-UMD respectively, is compared on two types of test cases with regular shape of Pareto front. DTLZ1 with a hyper-plane and DTLZ2-DTLZ4 with unit spheres are adopted as test problems, and the inverted generation distance (IGD [39]) and hyper-volume (HV [40]) are employed to evaluate the quality of the obtained solutions. In simulation, MOEA/D-UDM and UMOEA/D significantly outperform MOEA/D on DTLZ1–DTLZ4 with 3-6 objectives. This result can be further verified in the graphical representation of Pareto optimal set. MOEA/D-UDM obtains the best uniformly distributed solutions including the both boundary and the interior area of Pareto front, while UMOEA/D achieves good diversity but poor uniformity.

B. ADAPTIVE WEIGHT VECTOR GENERATION METHOD

1) THE IDEA OF ALGORITHMS BASED ON ADAPTIVE WEIGHT VECTOR GENERATION APPROACH

When facing with the problems with irregular PF, e.g. disconnected, degenerated, inverted simplex-like, weight generators struggle to adjust search directions dynamically to adapt the shape of the PF, and accordingly some adaptive weight vector generation methods have been developed. The adaptive weight vector design method is to estimate the shape of target Pareto front according to the current population, and then adjust the weight vector dynamically, so that the weight vector can change dynamically with different stages of evolution.

One trend of adaption is to analyze the relative relationship between candidate solutions and current predefined weight vectors, and adjust weight vectors according to the shape of PF. Generalized decomposition [41] (gD) is presented to generate an adaptive optimal weight vector on the assumption that the optimal set of reference vector can be obtained when a reference PF exists. The authors adopt Tchebycheff aggregation function to derive an optimal reference vector corresponding to one Pareto optimal solution of each sub-problem. It means that gD method needs some prior knowledge of the shape of the target PF, however, it seems unavailable to obtain in advance [42].

Wang *et al.* introduced a novel preference inspired co-evolutionary algorithm(PICEA-w) [43] to dynamically change the reference vectors with the co-evolution of candidate solutions, and only weight vectors that contribute to the candidate solutions are eligible to be reserved.

Cai *et al.* proposed two kinds of adjustments for the direction vectors [44] (MaOEA/D-2ADV), and the weight vector set can be categorized into effective weight vectors and ineffective ones via Pareto dominance mechanism. In execution, ineffective weight vectors are constantly repositioned and adjusted to cope with MOPs with irregular PF.

Asafuddoula *et al.* [45] developed a periodical adaptation approach for dynamically adjusting the reference vectors (g-DBEA). In their work, a number of uniformly distributed reference vectors are initially predefined as active reference vectors, and the insert or the deletion of the weight vectors is determined by the comprehensive information of solutions associated with these weight vectors over a period of learning time, including their feasibility, the non-dominance and presence. Different from other strategies to delete the useless reference vectors directly, the g-DBEA moves them to the inactive reference vector set.

The other trend of adaption is to employ an external population or archive to periodically adjust weight vectors and maintain consistency with the distribution of reference vectors and the shape of PF. Qi *et al.* developed a new adaptive adjustment strategy, named MOEA/D-AWA [46]. In order to obtain better uniformity in problems with complex Pareto front (discontinuous or PF with irregular shape), a external population is borrowed to evaluate the density of individuals, and the weight vectors are dynamically deleted and added by estimating the density information of population.

Li *et al.* presented another adaptive method(AdaW [47]) to update weight vectors periodically based on information of archive, which is used to locate potential undeveloped and crowded region by comparing evolving population with current weight vectors. In AdaW, five components of adaption, including weight vector generation, deleting, adding, archive maintenance and the update frequency of weight vectors, are elaborated in detail. Nevertheless, repeated addition and deletion of weight vectors will result in the increase of computation cost, and also deteriorate the uniformity of Pareto optimal set.

2) THE PERFORMANCE COMPARISON

In literature [44], two adaptive reference vectors adjustments algorithms, MaOEA/D-2ADV and MOEA/D-AWA respectively, are compared for MaOPs of DTLZ test case. The experimental results show that MaOEA/D-2ADV outperform MOEA/D-AWA in term of IGD in DTLZ1-4 with regular shape of Pareto front, however, for degenerate shape of Pareto front problems, DTLZ5, DTLZ6 and disconnected problem DTLZ7, MaOEA/D-2ADV is not obviously superior to MOEA/D-AWA. Also, in literature [47], two state-of-art weight adaptation method, AdaW and MOEA/D-AWA are considered as peer algorithm testing on MOPs with various Pareto fronts for 2-, 3- objectives. The simulation results demonstrate that MOEA/D-AWA is better than AdaW on problems with Simplex-like Pareto Fronts. In contrast, AdaW is superior to MOEA/D-AWA on problems with inverted simplex-like, highly nonlinear, disconnect and degenerated shape of PF. Besides, in literature [45], g-DBEA is compared with RVEA,$\theta-DEA$, NSGAIII, MOEA/DD and MOEA/D-PBI. For IDTLZ1 and IDTLZ2 with inverted fronts, both of NSGAIII and g-DBEA perform reasonably well, and g-DBEA obtains the best average HV value in DTLZ7 with irregular and disconnected shape of PF. For DTLZ 5 with degenerate POF, MOEA/D-PBI wins.

C. FITTING BASED GENERATION METHOD

To suit MOPs with the shape of PF approach $\sum_{i=1}^{m} f_i(x)^p = 1$, fitting based weight vector generation methods are proposed to approximate the PF shape by the fitting or interpolation of the individuals in population, and uniformly distributed reference weight vectors or reference points are re-positioned on the estimated PF.

In bi-objective MOPs, literature [48], [49] borrow a piecewise interpolation by linear function or cubic spline function respectively to approximate the shape of PF. Subsequently, a number of reference vectors are sampled on the estimated PF. For MOPs with more objectives, parameter p is estimated to fit the PF shape close to $\sum_{i=1}^{m} f_i(x)^p = 1$ [50], and even samples are generated to replace the original weight vectors. Such methods may lead to over-fitting because of the occurrence of outliers, so they are only suitable for problems with 2 or 3 objectives.

D. MACHINE LEARNING BASED WEIGHT VECTOR GENERATION METHOD

Recently, some scholars adopt machine learning technique to generate adaptive weight vector, e.g. Hongwei *et al.* [51] presented MOEA using cascade clustering and reference point incremental learning (CLIA). Firstly, cascade clustering based selection is employed to cluster and sort solutions, and an incremental learning based reference vector adaptation process repositions the weight vectors to adjust the effective areas of PF. In the dynamic adjustment process of the weight vector, the algorithm trains a class of sample classifiers to estimate the optimal timing for weight vector adjustment. Numerical studies on CEC2018 test suites show that CLIA outperform the other reference vector adaptation based MOEAs, e.g. A-NSGAIII, RVEA*, AR-MOEA.

Gu and Cheung employed the self-organizing mapping [52] (SOM) to find the topological structure of the population and update the reference vectors dynamically by training SOM network. In a sense, the above adaptation methods globally adjust the searching direction according to the distribution of individuals, which will ensure the satisfactory versatility. In the framework of evolutionary process, SOM is equipped with MOEA/D or M2M, respectively. The experimental results demonstrate the superiority of MOEA/D-SOM on both redundancy and non-redundancy problem with incomplete PF.

E. SUMMARY

Adaptive weight vector adjustment methods delete or increase the weight vector according to the current population distribution information, and machine learning based methods adjust weight vectors based on empirical or learning knowledge. These methods only improve the quality of solutions for limited kinds of irregular problem. However, their performance will be degraded for problems with regular PF. How to configure the adaptive frequency is a key factor to improve the quality of weight vector.

V. STUDY OF SCALARIZATION FUNCTIONS

The scalarization functions play a vital role in decomposition based MaOPs, which determine the selection of solutions and thereby affect the quality of the resulting solutions. The original MOEA/D employs different scalarization functions, e.g. weighted sum(WS), Tchebychedff method (TCH) or penalty boundary intersection (PBI) approach, to evaluate the fitness of individuals, and the above three functions are described briefly as follows.

$$Ming^{ws}(x|\lambda_i) = \sum_{j=1}^{m} \lambda_i^j f_j(x) \quad (2)$$

$$Ming^{\mathrm{tch}}(x|\lambda_i, z^*) = \max\{\lambda_i^j |f_j(x) - z_j^*|\} \quad (3)$$

$$Ming^{\mathrm{pbi}}(x|\lambda_i, z^*) = d_1 + \theta d_2 \quad (4)$$

$$d_1 = \frac{\left\|(F(x) - z^*)^T \lambda_i\right\|}{\|\lambda_i\|}$$

$$d_2 = \left\|F(x) - \left(z^* - d_1 \frac{\lambda_i}{\|\lambda_i\|}\right)\right\|$$

where z^*andλ_i are the ideal reference point and the weight vector, and θ is the penalty factor.

The simulation results indicate that different scalarization functions varies distinctive with different problems with different characteristics of PF. e.g. WS method performs well in convex problems while TCH approach is better suitable for non-convex problems. However, the shape of Pareto front is unknown beforehand, the selection of the proper aggregation function is a rather difficult problem. Several variants have been investigated for enhancing both convergence and diversity, and they can be grouped into the following three categories. Table 3 summarizes the variants.

A. THE COMBINATION OF WS AND TCH

One idea is to combine the two above aggregation function together to demonstrate their own characteristics. Ishibuchi *et al.* [53] developed an adaptive scalarization function (MOEA/D-AS) via automatically switching function from WS to TCH, once the optimal solution corresponding to WS function with different weight vectors is the same solution in non-convex region. Meanwhile, the same author also simultaneously used two types of scalarization

TABLE 3. Three variants of scalarization functions.

Classification	Algorithms
Combination of WS and TCH	MOEA/D-AS[53], MOEA/D-SS[54], NBI-style Tchebycheff[55], p-TCH[56] MOEA/D-PaS[57] Multiplicative SF, penalty SF[58],
Variants of PBI	APS, SPS[59], θ-DEA[60], PaP[61]
Others	RVEA[63] SPEA/R[64] ISPEA/R[65]

functions(MOEA/D-SS) [54], WS and TCH, to evaluate the fitness of individuals in one generation. Two implementation methods are adopted. One is multiple grids of weight vectors equipped with one scalarization function in one grid, and the other is one single grid with two kinds of scalarization functions. The experimental study shows that simultaneous using both WS and the TCH functions works better than using either WS or TCH function alone on multi-objective 0/1 knapsack problems with two-, four-, and six-objective. Zheng *et al.* [55] combined the features of TCH and NBI, and developed a NBI-style Tchebycheff. Ma *et al.* [56] presented a generalized objective function in subproblem, which combined Tchebycheff decomposition and Lp norm constraint to adjust the significance of the subproblem in competition by altering the p value.

Wang *et al.* [57] generalized the above WS and TCH approaches into Lp scalaring functions by letting p=1 or $p = \infty$ respectively, and proposed a generalized Pareto adaptive scalarizing approach(MOEA/D-PaS). In theory, a weighted Lp scalaring function can be formulated as (5).

$$Ming^{wd}(x|w,p) = (\sum_{i=1}^{m}(\frac{1}{w_i})^{p}(f_i(x) - z_i^*)^p)^{\frac{1}{p}} \quad (5)$$

The properties and their search abilities of Lp scalaring functions with different p value are analyzed, and it can be concluded that search ability of Lp scalaring functions decreases with the increase of value p. Besides, the author compares the robustness of the scalarizing function, and finds that as p increases, the robustness of the scalarizing function to problems with different PF shapes gradually increases. Meanwhile, the paper also discusses how to set the optimal value of p. In MOEA/D-PaS, in order to approximately obtain the best p value for each subproblem, the author usually borrows a series of reference curves to approximate the shape of PF, but the optimal solution corresponding to the optimal p-value of the scalarizing function is bound to be closer to the search direction than that obtained by other p-value functions. In the experiment, the four algorithms equipped with different scalarizing functions are compared in pairs, namely MOEA/D-SS with MOEA/D, and MOEA/D-PaS with MOEA/D-AS, and experimental results show that none of them can completely converge to the true PF, while PaS achieves the best convergence.

Jiang and Yang [58] presented two new scalarization functions (SF), multiplicative SF and penalty SF to control the contour lines to maintain the balance of convergence and diversity.

B. THE VARIANTS OF PBI

Compared with WS and TCH method, the PBI approach, which employs a simple penalty factor θ to coordinate the relationship between convergence and diversity, is aim to force the solutions to converge on the PF along the search direction and obtain a good distribution of solutions both in convex and concave problems. The performance of the algorithm will largely rely on the quality of the parameter θ settings. Mohammadi [59] launched a comprehensive analysis on the sensitivity of θ, including the effect of θ on convergence and diversity. The experimental results demonstrate that larger θ generally improves uniformity on the one hand but adversely affects convergence. Thus, small θ values are recommended in multi-modal problems while larger θ values in uni-modal problems. Also, the relationship between θ and the number of objectives is investigated and there is no evidence that the setting of θ will affect the performance of problems with different objectives number.

Two types of new penalty schemes (PS) for PBI scalarizing function were investigated by Yang *et al.* [60], named adaptive PS and sub-problem-based PS respectively. APS dynamically adjusts the value of θ at different stage of evolution, whereas SPS assigns independent penalty value θ to each subproblem by distinguishing the regions of the objective space where the weight vectors are located, such as the boundary and interior regions. Experimental studies verify that both APS and SPS schemes can effectively alleviate the loss of boundary individual reduction while improve the diversity. Ming *et al.* [61] proposed a configuration of an adaptively penalty value method by PBI method counters and approximated PF shape.

Motivated by PBI scalarizing function balancing both convergence and diversity, a hybrid version, θ dominance-based evolutionary algorithm (θ-DEA [62]) is proposed. Followed by allocating combined population into many clusters based on the distance from the nearest reference point, a new θ dominance relation, which borrows PBI scalarizing function within the same cluster, is employed to partition the combined population into many θ-nondomination levels. In θ dominance relation, there is no competitive relationship between solutions in different clusters, which ensures that the diversity of selected solutions between different clusters.

Here, MOEA/D-PBI and θ-DEA, both taking PBI scalarization function as tool of fitness evaluation, are compared with each other. Table 4 summarizes the significance test by HV metric on DTLZ and WFG with 80 test cases. Here, B, W and E represent the number of cases where θ-DEA is better than, worse than and equal to MOEA/D-PBI. It can be concluded that θ-DEA is significantly superior to MOEA/D-PBI, which indicates that PBI scalarizing function within the non-dominated cluster can effectively balance convergence and distribution.

TABLE 4. Significance test between θ-DEA and MOEA/D-PBI.

		MOEA/D-PBI
θ-DEA	B	66
	W	8
	E	6

C. OTHER SCALARIZATION FUNCTIONS

Some researchers develop a kind of parameter-free aggregate function methods in sub-problem to improve the quality of generated solutions.

Cheng *et al.* [63] proposed a reference vector guided EA (RVEA), which developed an angle penalized distance (APD) to maintain the convergence and diversity in different stage of evolution process via penalty function. Specifically, the APD is defined as follows. In expression (6), $\left\| f'_{t,i} \right\|$ represents the convergence measurement which is expressed as the distance from a translated objective vector $f'_{t,i}$ to the ideal point in subpopulation j, and $\theta_{t,i,j}$ represents the angle between $f'_{t,i}$ and reference vector $v_{t,j}$ for evaluating diversity. The penalty function (7) is precisely designed to apply a changing pressure on convergence and diversity at the different phase of search process. Another advantage of APD is that the normalized part $\frac{\theta_{t,i,j}}{\gamma_{v_{t,j}}}$ in penalty function offers a stable balancing between diversity and convergence without considering the distribution density of the reference vectors.

$$d_{t,i,j} = (1 + P(\theta_{t,i,j})) \cdot \left\| f'_{t,i} \right\| \tag{6}$$

$$P(\theta_{t,i,j}) = M \cdot (\frac{t}{t\max})^{\alpha} \cdot \frac{\theta_{t,i,j}}{\gamma_{v_{t,j}}}$$

$$\gamma_{v_{t,j}} = \min_{i \in \{1,2,\cdots,N\}, i \neq j} < v_{t,i}, v_{t,j} > \tag{7}$$

We compare the quality of the solutions generated by the decomposition algorithms in pairs, θ-DEA, RVEA and MOEA/D-PBI. The results are demonstrated in Table 5 and Table 6. In the following table, B, W and E represent the number of cases where the algorithm listed on the left side is superior to, worse than and equal to its counterparts. It can be concluded that θ-DEA wins, and RVEA is the second, followed by MOEA/D-PBI.

Jiang and Yang [64] and Junhua and Yuping [65] investigated a novel combined fitness assignment mechanism in the decomposed objective space, and proposed SPEA/R and ISPEA/R. For convergence measurement, local strength and global strength are employed to assign fitness values by calculating the number of solution which are dominated or dominated. The diversity is determined by the acute angle information and the nearest neighbor density estimation.

D. SUMMARY

Since different scalarization functions vary distinctive with different problems with different characteristics of PF, it is feasible to adopt multiple scalarization functions at the same time or composite individual evaluation method, instead of using one scalarization function.

TABLE 5. Significant test between θ-DEA and RVEA.

		RVEA
θ-DEA	B	38
	W	25
	E	1

TABLE 6. Significant test between RVEA and MOEA/D-PBI.

		MOEA/D-PBI
RVEA	B	56
	W	4
	E	0

VI. THE STUDY OF MATING SELECTION AND REPLACEMENT STRATEGY

In the original MOEA/D, individuals involved in mating and replacement have been always selected within the neighboring sub-problems without considering the population distribution of the individual's region. Here, the neighborhood and its size as two vital parameters determine the individuals mating and entering in the next generation. However, the above parameters are fixed in the whole evolution, which will imbalance the convergence and diversity [66], [67]. The configuration of neighborhoods in mating selection and replacement strategy will be discussed.

Jiang and Yang [68] developed a niche scheme to avoid duplicate individuals in his MOEA/D-TPN. In order to better maintain the diversity of population distribution, the mating selection area is set up by calculating the number of niches in each individual's neighborhood. Once the number of niches exceeds threshold, individuals from outside the neighborhood are selected as mating parents. Niche guided strategies can show better performance in maintaining population diversity when dealing with complex problems.

To equip an appropriate neighborhood size (NS) for various types of MaOPs, Zhao proposed an ensemble NS strategy, briefly named ENS-MOEA/D [69], to determine the optimal NS for each sub-problem. By calculating the historical performances of generating promising solutions, an ensemble of different NSs and their selection probability are dynamically adjusted during different stage of evolution.

Wang and Zhang [70] investigated the effect of the size of neighborhood in mating and replacement on the quality of the obtained solutions in MOEA/D, and concluded that the newly generated individual in the sub-problem may be inappropriate for replacing within its neighboring sub-problems. In his study, he proposes global replacement scheme, designated as MOEA/D-GR, to guarantee that each solution should be assigned to a suitable sub-problem within the scope of all sub-problems instead of neighboring of $B(i)$. The advantage of global replacement lies in achieving the optimal match

TABLE 7. Classification of environmental selection.

Classification	Algorithms
Separate distance measurements	DBEA-Eps[71], I-DBEA[72], ASEA[73]
Establishing a corresponding relationship between solution and sub-problem	MOEA/D-STM[74], Matching-based selection with incomplete lists[75], MOEA/D-SAS[76], ARA[77], DL[78]

between subproblem and solution. Notably, the effect of the size of replacement neighborhood T_r on the convergence and diversity is investigated, and a dynamic adjustment (AGR) of T_r is adopted to set small T_r at the early period of evolution and large value at the late period of evolution.

VII. THE STUDY OF ENVIRONMENT SELECTION

Environment selection selects the best individuals in the population to enter the next generation by using some evaluation strategies. In the decomposition based algorithm, researchers usually use the scalarizing function as the criteria for selecting individuals. To effectively balance the conflict between convergence and diversity, the variant algorithms also adopt the strategies of distance measurement and matching relationship for the assessment of individuals. Table 7 lists the two classifications of environmental selection and representative algorithms.

A. SEPARATE DISTANCE MEASUREMENTS TO EXECUTE ENVIRONMENTAL SELECTION

Replacing by aggregation function to perform environmental selection, many literatures adopt separate distance measurements to balance the convergence and diversity respectively. Asafuddoula proposed DBEA-Eps [71] and I-DBEA [72] via an adaptive epsilon level comparison to maintain the balance. Initially, each solution is associated with two distances. One is the distance along a reference direction $d1$, and the other is the distance perpendicular to the reference direction $d2$. Next, adaptive epsilon prioritized distance comparison is proposed, which advocates $d2$ precedence over $d1$.

Liu *et al.* designed a adaptive two stage sorting scheme (ASEA [73]) to use the reference direction more efficiently. The scheme follows the principle of convergence-then-diversity: solutions are firstly ranked by convergence, and only solutions whose convergence meets certain requirements are eligible for diversity sorting. Meanwhile, the number of individuals participating in diversity ordering is dynamically adjusted in different stages of evolution.

In order to investigate the effects of the two types of individual evaluation methods, adaptive two stage sorting scheme ASEA with three MaEAs equipped with different scalarization functions, named RVEA, MOEA/DD and θ-DEA are compared [73]. ASEA wins on DTLZ1, DTLZ3, WFG1 and WFG3. For DTLZ2, θ-DEA wins the leading position. For DTLZ5 and DTLZ6 with degenerate PFs and WFG2 with disconnected PF, the above algorithms fail to obtain a satisfactory IGD value.

B. ESTABLISHING A CORRESPONDING RELATIONSHIP BETWEEN SOLUTION AND SUBPROBLEM

Li *et al.* advocated a stable matching model (STM [74]) to select one single individual for each sub-problem in selection operator. For each sub-problem in MOEA/D-STM, scalarization function value is adopted to rank the individuals, while the solution with higher scalarization value is preferred to encourage convergence. While for each individual, the distance from individuals to each weight vector from all sub-problems is ranked to choose the optimal sub-problem with shortest distance for diversity. In MOEA/D-STM, the selection operator can be treated as one-to-one matching procedure between solutions and sub-problems. Although the STM model produces each individual for each sub-problem to make up the deficiencies of poor diversity in MOEA/D, it inevitably suffers from a situation where a solution will match an unfavorable sub-problem. The reason is that individuals with better scalarization function values are always at the top of the preference list and have a high probability of being selected, while those individuals close to the reference direction but far from PF are unlikely to be selected and matched in STM model, which will obviously deteriorate the diversity of population.

To alleviate the side effect of MOEA/D-STM, Wu *et al.* [75] imposed a restriction on the size of the subproblems associated with a solution, and two types of novel STM strategies by using incomplete preference lists are developed. The first approach is two level one-to-one matching which uses two stages to accomplish stable matching. In the first level, only the top ranked partial sub-problems are kept in the preference list of each solution. At this moment, to ensure diversity, each individual can only be matched with its adjacent preferred subproblems. Not all subproblems are assigned a stable solution after the first level of matching, and the remaining unmatched subproblems are continued to look for a stable solution by updated complete preference lists in the second level. The other approach is many-one stable matching (MOSTM) which means one sub-problem can be allowed to match with more than one solution. In the main loop of MOSTM, an unmatched solution initially matches with current preferred subproblem based on its preference list. Then, the matching pairs with the largest number of matching solutions and the worst rank of matching solutions in the subproblem preference list are adjusted by releasing matching relations to balance the selection. The restriction of the preference list length will cause the solution only to be matched with its favorite subproblems, which will repair the loss of diversity.

Meanwhile, each subproblem is associated with only one solution in STM based approach, which is not suitable for complex problems when the shape of Pareto front is irregular. Under this situation, a more elaborate environmental selection strategy is needed. Cai *et al.* [76] proposed one sorting before selecting approach MOEA/D-SAS, which employed two distinct components, decomposition-based sorting (DBS) and angle-based-selection (ABS) to evaluate the individuals. The proposed MOEA/D-SAS initially sorts L closest solutions to the weight vector in each subproblem and rank them based on its corresponding aggregation function respectively to accomplish decomposition based sorting. That is, if one individual with the k-th best aggregation value (k = 1,2,...L) is chosen in each subproblem, the chosen solutions in each time can be sorted into L layers, namely $Q_1, Q_2, \ldots Q_L$. Subsequently, the individuals in each front are selected to next generation until the number no more than the number of subproblems. Otherwise, angle-based-selection ensures individuals with the largest angle difference of the individuals in the selected fronts for more fine-grained diversity.

Wang *et al.* analyzed the relationship between solution and its neighborhood subproblems, and proposed an adaptive region adjustment strategy (ARA [77]) in MOEA/D. To enhance the balance of convergence and diversity, a new concept, contour point, is defined to divide the whole objective space into many sub-problems dynamically, and the selection of solutions only occurs in the most suitable sub-problem according to the relation between the solution and the sub-problem. In addition, Yu *et al.* [78] proposed the concept of loss of individual diversity to population diversity, and designed an online measure of diversity named maximum related diversity loss.

C. SUMMARY

In order to select good individuals into the next generation, more elaborate environmental selection strategies are needed for different shapes of PF. Selecting individuals by establishing the matching relation between individuals and corresponding sub-problems is more beneficial to improving the quality of population than aggregation function evaluation method alone.

VIII. REPRODUCTION OPERATOR

The offspring reproduction operator is another crucial factor, and classical operators, such as simulated binary crossover (SBX) operator and polynomial mutation operator, are used as genetic tools to generate offspring, whose advantage is strong at local search ability. However, single operator does not perform well in different problems with different Pareto shapes. Meanwhile, other evolution strategies are employed to replace SBX as reproduction operator. For example, differential evolution (DE) is embedded for handling complicated shape of Pareto set, which has superior global search ability. The simulation results reveal that the parameter setting has great influence on the performance, such as scaling factor. Afterwards, some studies have been investigated to discuss the effect of different DE variant settings in MOEA/D [79]–[82], such as DE/RAND/1, DE/RAND/2, DE/BEST/1, DE/BEST/2, and DE/RAND-BEST/1. Test cases F1-F9 with complicated Pareto sets, it can be concluded that DE/BEST/1 and DE/BEST/2 surpass the other DE settings, although none of DE settings can achieve the best performance in all test problems. Besides, some combination versions, such as MOEA/D with ant colony optimization [83] (ACO), MOEA/D with PSO, have been also used to generate new offspring.

The second direction is using probabilistic based estimation of distribution algorithms (EDA) to extract the information of population distribution and sample new solutions. Zhou *et al.* [84] proposed two new reproduction operators by employing multivariate Gaussian distribution model to balance subproblem exploitation and neighborhood exploration. The information of both neighborhood and history individuals is recorded. Specifically, the offspring generated by the information of neighborhood may be more likely to extend along the PS for exploration, while the offspring generated by historical individuals may push the offspring to optimal solutions with the subproblem. Venske *et al.* [85] combined probability matching (PM) with adaptive pursuit (AP) to execute adaptive strategy selection. It uses empirical quality estimation based on credit assignment to perform the selection of DE strategy based on the probability that relying on previous experience of relative fitness improvements in producing promising solutions.

Laumanns and Ocenasek [86] made use of the relationship of decision variables and developed a binary decision tree based Bayesian model. Karshenas *et al.* [87] established a joint probabilistic model of Bayesian network employing the information of objectives and variables. IM-MOEA [88] utilized the statistical information of population and established Gaussian process-based inverse models to generate offspring. In the procedure of reproduction, once the new test points in objective function space are given as input, the established Gaussian regression model will generate new points in decision space as output.

The third is adopting adaptive operator selection (AOS) method. Lin *et al.* [89] developed an adaptive composite operator selection strategy (MOEA/D-ACOS) to improve the robustness and effectiveness. As the performance of different DE mutation methods varies dramatically in different types of problems, multiple composite DE strategies are employed in MOEA/D-ACOS to improve exploratory capabilities for variety type of MOPs. For finding a preferred operator pool to generate new solutions, a operator pools selection approach based on bandit is proposed, which dynamically select the preferred operator pool by the fitness improvement rates (FIR) of historical search experience.

IX. CONCLUSION

We have conducted a research on decomposition based evolutionary algorithms, and six different directions have been summarized to improve the performance of obtained Pareto optimal solution set or extend the decomposition based algorithms to scalable complicated type of MaOPs. Different from current literatures, a more detailed classification under each direction is elaborated briefly, and pairwise comparison demonstrates the characteristics of the algorithms investigated. A list of conclusions can be conducted from the work, and it can be concluded as follows.

(1) With respect to different framework of decomposition based algorithms, different types have their own advantages, and none of the any type has an obvious superiority over others.

(2) According to the characteristics of population distribution, adaptive weight vector adjustment is an effective weight vector design method for MaOPs with complex or irregular Pareto front. However, how to configure the adaptive frequency is a key factor to improve the quality of weight vector.

(3) As for scalarization function, one of the current trends is integrating convergence and diversity effectively in decomposed objective space, which is verified to be able to balance convergence and diversity.

In the future, we will focus on more hybrid strategies, such as combining multiple genetic operators, adding machine learning algorithms into weight vector design methods, and adopting multiple mechanisms for evaluating individuals. At the same time, we will also study the design of many-objective evolutionary algorithm in more application problems, such as machine learning classification and resource scheduling problem.

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