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RESEARCH ARTICLE

Dynamics of Discrete Memristor-Based Rulkov Neuron

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ABSTRACT Continuous-time memristor have been widely used in fields such as chaotic circuits and neuromorphic computing systems, however, research on the application of discrete memristors haven't been noticed yet. In this paper, a new chaotic neuron is firstly designed by applying the discrete memristor to two-dimensional Rulkov neuron. And then the dynamical behaviors of the discrete memristor-based neuron are analyzed by experiments including phase diagram, bifurcation, and spectral entropy complexity algorithm. The results show that the resistance of memristor has an important effect on the system dynamics, which delays the occurrence of bifurcation, in particular, the bifurcation disappears and the system reaches the fixed point of the neuron when the resistance is greater than a threshold. It is also found that with the increase of the current gain, the bursting activity becomes higher in frequency and wider range of high complexity is obtained. The results of our study show that the performance of Rulkov neuron is improved by applying the discrete memristor, and may provide new insights into the mechanism of memory and cognition in the nervous.

INDEX TERMS Discrete memristor, Rulkov neuron, spectral entropy complexity, bifurcation, phase diagram.

I. INTRODUCTION

Current, voltage, charge, and magnetic flux are the four basic variables in the circuit and there is a relationship between two of them. However, the relationship between electric charge and magnetic flux has never been discovered until Chua proposed memristor to describe the relationship between charge and magnetic flux, about 50 years ago [1]. Because the resistance of memristor is determined by the electric charge flowing through it, memristor has the function of memorizing electric charge. In 2008, a group at Hewlett-Packard Laboratories found and confirmed the existence of nanoelectronic memristor when studying TiO2 [2]. Owing to its nano size, powerful information storage capability, potential nonvolatility and high integration density, the memristor has been widely used in the field of nonlinear science, circuit device design and biological memory behavior simulation. For instance, [3] identified some unknown neuromorphic dynamics of the Chua corsage memristor (CCM), and showed that the CCM, when biased at the edge of chaos domain, can exhibit rich dynamics of biological neurons. Reference [4] investigated the mixed H∞ and passive projective synchronization problem for fractional-order (FO) memristor-based neural networks. Reference [5] designed a two-dimensional network of resonators and memristors coupled at the nearest neighbor connection. By carefully selecting the coupling strength, the network can be fully synchronized. Reference [6] proposed a multi-synaptic circuit (MSC) based on memristor, which realizes the multi-synapse connection between neurons and the multi-delay transmission of pulse signals. Reference [7] explored the effect of electromagnetic induction on a two-layer small-world neuronal network with electrical intra-layer connections and memristive inter-layer connections. Reference [8] built a memristor synapse-coupled neural network, which coupled the HR and FN neurons with the locally active memristor. Zhang *et al*. proposed a chaotic circuit based on a memristor-capacitor, which has abundant bifurcation paths to chaos,

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various periodic limit cycles and chaotic at-tractors [9]. Reference [10] proposed a fractional-order memristor component for designing memristor chaotic oscillators implemented in FPGA. Ma *et al*. proposed a new 4D HR model with a threshold flux control memristor to describe effect of the electromagnetic induction on the synchronization of memristive Rössler oscillators [11]–[13]. Reference [14], [15] studied the influence of electromagnetic radiation on the dynamics of spatiotemporal modes in neural networks based on memristors, which are based on nonlinear continuous-time mathematical models. The newly presented discrete memristor can also be widely used in the chaotic oscillations and neuromorphic computations. Existing research is more about applying discrete memristors to some famous chaotic maps, for example, Chen and He proposed a discrete integer-order and fractional-order memristor models, which were applied into the sine chaotic map and the Hénon map, respectively [16]–[19]. Reference [20] presented a new second-order discrete memristor-based chaotic map and found that it has hyper-chaotic behavior. However, these researches of the memristor-based discrete neuron are still rarely discussed. On the other hand, in recent years, discrete neurons have received extensive attention as an effective model for studying neural dynamics, because it shows high computational efficiency whose model is simple, reliable, numerically stable [21]–[24]. Currently, the well-known discrete-time models include Rulkov model [25]–[28], Kinouchi and Tragtenberg model [29], Courbage-Nekorkin-Vdovin model [30], etc. Especially, Rulkov neuron has shown very rich nonlinear dynamic behavior and remarkable biological neuron characteristics, and been widely used in the field of computational neuroscience. Cao *et al*. concerned the intermittent evolution routes to the asymptotic regimes in the Rulkov map and predicted successfully the evolution path using a three-layer feedforward neural network [31]. Wang *et al*. discussed the triggering mechanism of the chaotic discharge of the Rulkov neuron model and the mechanism of their chaotic-rest state transition [32]. Hu *et al*. studied the stability and synchronization of two coupled Rulkov neurons in the presence of electrical and chemical synapses [33]. Tanaka *et al*. used a map-based model to study the firing patterns in neural networks, clarifying the difference between in-phase and anti-phase synchronization patterns [34]. It can be seen that the Rulkov chaotic neuron is a neuron model with simple structure and rich dynamics. Therefore, it is of practical significance to introduce discrete memristor into traditional Rulkov neuron and study the influence of memristor on dynamics of neuron.

In this paper, a new chaotic neuron is firstly designed by applying the discrete memristor to Rulkov neuron. And then the dynamics of this memristor-based neuron are analyzed by experiments. The results show that the resistance of memristor can delays the occurrence of bifurcation, in particular, the bifurcation disappears and the system reaches the fixed point of the neuron when the resistance parameter is greater than a threshold. It is also found that with the increase of the current gain, the bursting activity becomes higher in frequency and wider range of high complexity is obtained. The use of discrete memristor has improved the performance of Rulkov neuron and may provide new insights for the memory and cognitive mechanisms of the nervous system.

II. THE DISCRETE HP MEMRISTOR MODEL

The classic HP memristor model is a semiconductor film sandwiched by two metals [2] with a certain thickness. This semiconductor film consists of two parts, one is a small resistance with a high concentration of dopants (R_{ON}), the remaining part is a large resistance with almost no dopant concentration (R_{OFF}). Its expression is as follows

$$M(r) = R_{OFF}\left(1 - \frac{\mu_v R_{ON}}{D^2} q(t)\right) \tag{1}$$

where μ_v is the mobility of doped ions, $R_A = R_{OFF}$ and $R_B = \frac{R_{OFF} u_v R_{ON}}{D^2}$. Then $M(r)$ can be written as

$$M(r) = R_A - R_B q(t) \tag{2}$$

According to Chua [1], the mathematical expression of continuous-time memristor is as follows

$$\begin{cases} v(t) = Mq(t)i(t) \\ \dfrac{dq}{dt} = ki(t) \end{cases} \tag{3}$$

where $v(t)$ is the voltage of the memristor, $i(t)$ is the input current and k is current gain. The ideal relationship between the charge and the input current is

$$q(t) = k\int_{-\infty}^{t} i(t)dt \tag{4}$$

Supposing the initial state of the memristor is $q(t_0)$, Eq. (4) can be written as,

$$q(t) = k\int_{-\infty}^{t} i(t)dt = q(t_0) + k\int_{t_0}^{t} i(t)dt \tag{5}$$

Let q_n, i_n, v_n represent the discrete value of $q(t)$, $i(t)$, $v(t)$, respectively, the discrete memristor equation is written as follows

$$\begin{cases} v_n = M(q_n)i_n \\ \nabla q_n = Ki_n \end{cases} \tag{6}$$

where $\nabla q_n = q_n - q_{n-1}$, $n = 0, 1, 2, \ldots N$ specify the discrete-time series. It can be derived from equation q_n-$q_{n-1} = ki_n$:

$$\begin{aligned} &ki_n + ki_{n-1} + k_{n-2} + \ldots + ki_3 + ki_2 + ki_1 + ki_0 \\ &\quad = q_n - q_{n-1} + q_{n-1} - q_{n-2} + q_{n-2} \\ &\qquad - \ldots + q_1 - q_0 + q_0 - q_{-1} \\ &\quad = q_n - q_{-1} \end{aligned} \tag{7}$$

The expression of q_n is obtained by shifting term

$$q_n = q_{-1} + k\sum_{j=0}^{n} i_j \tag{8}$$

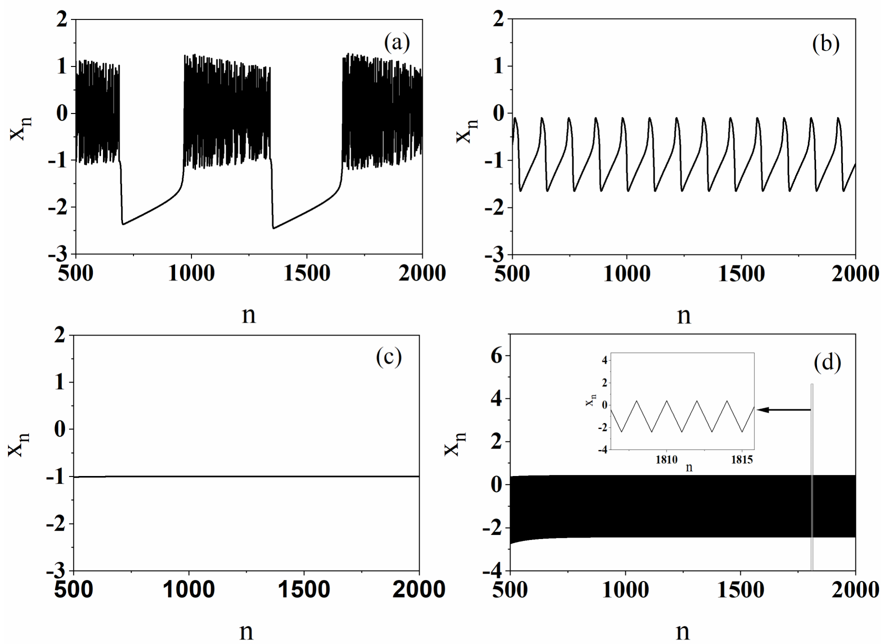

FIGURE 1. The phase portraits of Rulkov neuron with considering different values of α, where Figs.1(a)-(d) correspond to $\alpha = 4, 2, -1, -4$, respectively.

In the discrete domain, q_{-1} can be regarded as the initial electric charge of the memristor. According to the [12], the discrete HP memristor is obtained by

$$v_n = M(r)i_n \tag{9}$$

Then Eq. (9) is derived as

$$v_n = \left[R_A - R_B(q_0 + K \sum_{j=0}^{n} i_j) \right] i_n \tag{10}$$

III. THE ORIGINAL AND MEMRISTOR-BASED RULKOV MODEL

The original Rulkov neuron is a two-dimensional discrete model, and has been fully studied since it was proposed in 2003 [23]–[25]. We consider the Rulkov model with the following dynamic equation

$$x_{(n+1)} = \frac{\alpha}{1 + x_{(n)}^2} + y_{(n)} \tag{11}$$

$$y_{(n+1)} = y_{(n)} - \mu \left(x_{(n)} - \sigma \right) \tag{12}$$

where n is the discrete time ($n = 1, 2, \ldots$), x_n represents the fast variable of neuron transmembrane voltage and y_n represents the slow variable of the gating process. α, μ, σ are system parameters, where $0 < \mu < 1$. When the system parameters take different values, Rulkov neuron shows very rich nonlinear dynamic behavior and significant biological neuron characteristics, which is shown in Fig.1(a)-(d) corresponding to $\alpha = 4, 2, -1, -4$, respectively. Fig. 1(a) shows the square bursting firing. Fig. 1(b) shows the spike firing where the periodical action potential corresponding to a stable limit cycle attractor. Figs. 1(c) and (d) show the silent state and periodic pulse firing state, respectively.

In order to study the dynamic behaviors of the discrete memristor-based neuron, the discrete HP memristor is applied into the Rulkov neuron. Assuming $S_n = V_n, H_n = i_n$, the discrete memristor can be rewritten as:

$$S_n = \left[R_A - R_B(q_0 + K \sum_{j=0}^{n} H_j) \right] H_n \tag{13}$$

The memristor-based Rulkov neuron is derived as follows

$$\begin{cases} x_{(n+1)} = \dfrac{\alpha}{1 + x_{(n)}^2} + \left[R_A - R_B(q_0 + K \sum_{j=0}^{n} y_j) \right] y_{(n)} \\ y_{(n+1)} = y_{(n)} - \mu \left(x_{(n)} - \sigma \right) \end{cases} \tag{14}$$

Besides, the use of the memristor leads to an increase in dimensionality. Following [36], the equation of this three-dimensional memristive nervous system can be written as:

$$\begin{cases} z_{(n+1)} = z_{(n)} + y_{(n)} \\ x_{(n+1)} = \dfrac{\alpha}{1 + x_{(n)}^2} + \left[R_A - R_B(q_0 + kz_{(n)}) \right] y_{(n)} \\ y_{(n+1)} = y_{(n)} - \mu \left(x_{(n)} - \sigma \right) \end{cases} \tag{15}$$

IV. NONLINEAR DYNAMICS OF THE DISCRETE MEMRISTOR-BASED RULKOV MODEL

The dynamics of the memristor-based Rulkov neuron is firstly studied by bifurcation diagram. The initial state of

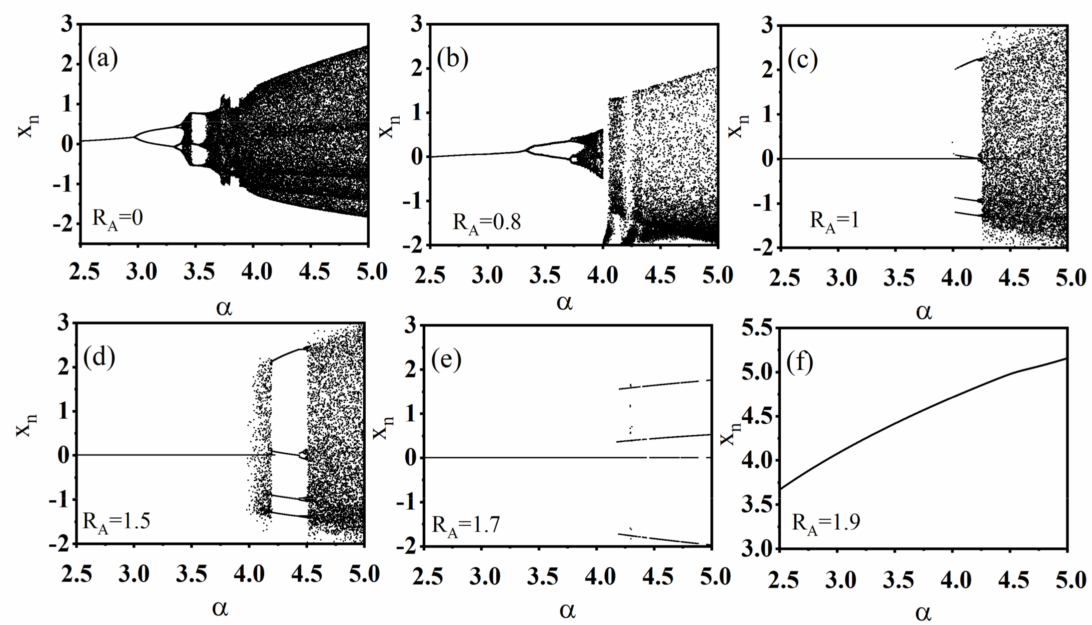

FIGURE 2. The bifurcation of the neuron with different values of R_A, where Figs. (a)-(f) correspond to $R_A = 0$, 0.8, 1, 1.5, 1.7, 1.9 respectively.

the system is set to $x_0 = 1$, $y_0 = -2.9$, $z_0 = 0$, the parameters of the neuron are set to $\mu = 0.001$, $\sigma = 0.001$, $R_B = 0.001$, $k = 10$. Fig. 2 shows bifurcation diagrams of the memristor-based neuron versus serveal different values of R_A, where Figs. 2(a)-(f) correspond to $R_A = 0$, 0.8, 1, 1.5, 1.7, 1.9, respectively. From Fig. 2, one can find two interesting information different from what happens in the original Rulkov neuron. First, the resistance parameter delays the occurrence of bifurcations, in particular, when the resistance parameter is greater than a threshold ($R_A = 1.8$), the bifurcation disappears and the system reaches the fixed point of the neuron. Secondly, it is found that the memristor have an important effect on the chaos in the memristor-based neuron. With increasing the resistance parameter of memristor, the chaotic regions shrink. On the further increase of the resistance parameter, chaotic regions eventually disappear.

The effect of current gain k on the firing pattern of memristor-based Rulkov neuron is further studied. The results are shown in Figs. 3(a)-(d), corresponding to $k = 0.2$, 0.3, 0.5, 1, respectively. From Fig. 3 one can observe that as the current gain increases, the bursting activity of memristor-based Rulkov neuron becomes more frequent, which indicates that an increase in current gain can induce and enhance the activity.

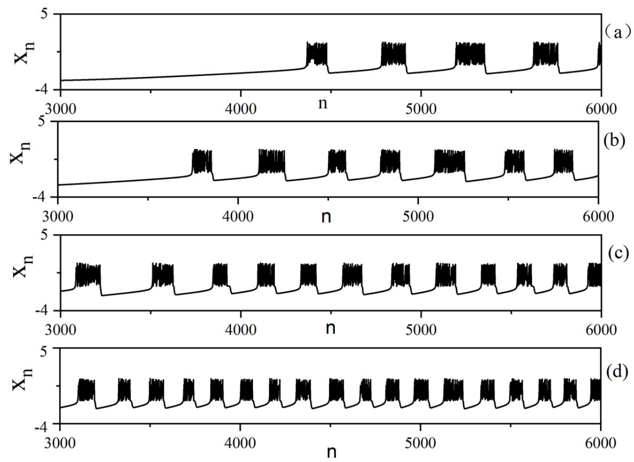

FIGURE 3. The firing pattern of the discrete memristive Rulkov neuron with different values of k, where Figs.(a)-(d) correspond to $k = 0.2$, 0.3, 0.5, 1, respectively.

Finally, the complexity of memristive Rulkov neuron is studied. Complexity algorithm of a nonlinear system is an indicator to measure how closely the time sequence approaches to random sequence. For a chaotic system, the higher the complexity is, the higher the randomness of the pseudo random sequence generated by the chaotic system is. On the other hand, proven by previous work, the chaos activity of EEG (electroencephalo-gram) is closely related to the functional state of the brain. In the normal state of the brain, the indicators of the chaotic activity of the brain, such as the dimensionality, Lyapunov exponent and complexity, are relatively larger, while the above-mentioned chaotic indicators will decrease if the brain is in the pathological state of impaired brain function. Among these chaotic indicators, complexity is of great significance to the normal activities of brain function[36]–[39]. In this paper, we use spectral entropy complexity (SE) algorithm to calculate the complexity of neuron [38]. Figs. 4(a) and (b) show the complexity of the original and memristive Rulkov neuron, respectively, where dark color area indicates that the area has high complexity.

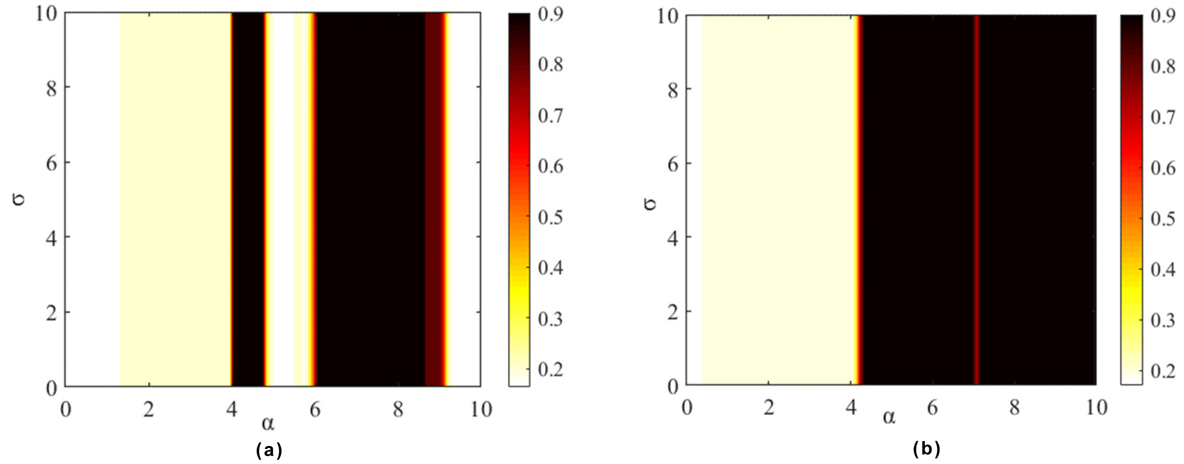

FIGURE 4. SE complexity with parameters α and σ for: (a) original Rulkov neuron; (b) discrete memristor-based Rulkov neuron.

Observe from Fig. 4 that the dark-colored area of the original Rulkov neuron is very narrow, which is mainly concentrated in parameter range $\alpha \in (4, 4.3) \cup (6, 8]$. On the contrary, the discrete memristive Rulkov neuron have a relatively wide parameter range with high complexity, where $\alpha \in (4, 10]$. Existing research results show that the complexity of EEG activity increases sequentially in three states: coma, deep sleep, and awake [40]–[42]. With the high complexity, the discrete memristive Rulkov neuron is more adaptable to internal and external environments, more reliable to accept and process external information, and more adaptable to extracellular factors such as neurotransmitters, temperature, etc. Therefore, discrete memristive Rulkov neurons have better prospects for medical applications, and have potential significance for the understanding of neurological diseases.

V. CONCLUSION

The discrete memristors are more suitable for discrete chaotic systems and digital circuits. We apply discrete HP memristors into two-dimensional Rulkov neurons, named discrete memristive Rulkov neurons. And then the dynamics of the discrete memristor-based neuron are analyzed by experiments including phase diagram, bifurcation structures, and spectral entropy complexity algorithm. It is shown that the resistance of the memristor delays the occurrence of bifurcation, especially, when the resistance parameter is greater than a threshold, the chaotic area gradually shrinks, and then the system reaches the fixed point of the neuron. The current gain also has an important influence on neuron activity. The increase in current gain can bring about a significant increase in the firing frequency of neurons. Compared with the original system, the spectral entropy complexity of this system has a wider range of complexity. The results of this study show that discrete memristor can improve the performance of Rulkov neuron and may provide new insights into the memory and cognitive mechanisms of the nervous system.

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