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Golomb Patterns, Astrophysics, and Citizen Science Games

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ABSTRACT This paper presents an online citizen science game called "Game and Repository for Aperture Solutions and Patterns (GRASP)." GRASP is uniquely tailored to an important task of designing a small-aperture antenna array by leveraging a mathematical structure called the Golomb pattern. In this application of GRASP, an optimal aperture pattern leads to space images of desired spectral resolution observed with the least-cost antenna aperture array via interferometry imaging techniques. Finding near optimal Golomb patterns is computationally very hard. GRASP citizen scientists are tasked with finding (near-)optimal Golomb patterns by selecting cells on a grid that obeys the rules of the game, ensuring Golomb properties to be satisfied. They can also submit known Golomb solutions with their citations, display existing solutions in the GRASP repository, and employ GRASP solutions for non-redundant aperture design of the interferometry and radar applications. In this paper, we introduce the concept of Golomb patterns, and present current design and framework of our citizen science platform GRASP as a means to solve, store, and display such patterns, as well as the motivation behind our work. In particular, we discuss the application of Golomb patterns in astrophysics' inteferometry missions for small-aperture antenna array design.

INDEX TERMS Citizen science, online game, Golomb rectangles, Golomb rulers, Costas arrays, non-redundant apertures, aperture array, antenna, interferometry, radar, astrophysics, STEM education.

I. INTRODUCTION

Citizen science online games are securing their foothold in the citizen science project space, as manifested by the increasing number of those games [1], [2]; and, combined with emerging and accelerating attention on citizen science for mathematics [3], citizen science games are well-positioned to bring unique impacts to relevant domain sciences founded upon the mathematics.

This paper presents the framework of a web-based citizen science gaming platform called "Game and Repository for Aperture Solutions and Patterns (GRASP)", which provides a new and efficient way of finding solutions to a computationally hard mathematical problem of finding non-redundant patterns (i.e., Golomb patterns) that has applications in astrophysics — inteferometry in particular.

Different types of citizen scientist platforms have been developed as mentioned in a recent survey by Liu *et al.* [4], and GRASP is one of those developed for "specific scientific topics." GRASP is unique in the space of citizen science applications in that it aims to solve a problem — especially a computationally hard problem — through an online game for citizen scientists, unlike many others that primarily aim to collect data or observation reports from citizen scientists.

GRASP is founded upon a mathematical grid pattern called a "Golomb pattern" [5]. This is a non-redundant pattern of marks on grid cells, i.e., one with distinct displacements between any pair of marks. Figure 1 shows two types of such patterns: rectangular on a rectilinear grid and triangular on a hexagonal grid.

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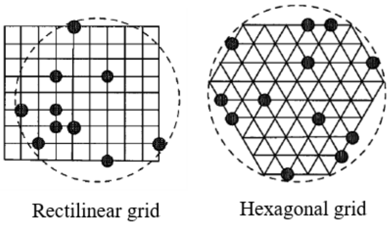


FIGURE 1. Golomb patterns (source: [6]).

The objectives of this paper are to discuss the GRASP framework from its foundation (i.e., Golomb patterns) to its applications in astrophysics antenna array design, and describe the software platform developed to make the connection. The GRASP platform, comprising the front-end web site and the back-end database, is operational, and a pilot citizen science project has been completed. The current implementation of GRASP is available at https://grasp-game.org.

The remainder of this paper is organized as follows. Section II presents the motivation behind the work and some background on GRASP's implementation. Section III introduces Golomb patterns and illustrates their relevant properties and variants. Section IV discusses the application to astrophysics, namely designing non-redundant antenna apertures. Section V presents GRASP as a platform for solving, storing, and retrieving non-redundant apertures, i.e., Golomb, patterns. Section VI discusses the current status and the future plans for GRASP. Section VII concludes the paper.

II. MOTIVATION AND BACKGROUND

NASA Science missions often involve sky observations that require high angular resolution imagery at wavelengths spanning the ultraviolet, visible, and infrared spectra. Achieving such desired resolutions with one mirror is often impractical or very costly. For example, resolution similar to that of Hubble Space Telescope's in the far infrared requires a single aperture telescope of order of one kilometer in diameter [7]. In contrast, multiple smaller apertures combined with interferometry techniques [8] enable high-resolution data in a cost-effective manner. To this end, finding the optimal number of apertures that provide non-redundant information as well as their geometric configuration for the required spatial and spectral resolution and best sky coverage, with minimal collecting area, is of paramount importance. The desired configuration of such apertures is directly related to the concept of a Golomb pattern [9], i.e., points with integer coordinates and unique pairwise-distances on a given grid. Golomb patterns have NASA applications in radio astronomy, interferometry, and radar missions. Other applications include radar, sonar synchronization, communication network labeling, encryption, X-ray crystallography, and coding theory [9], [10].

Finding an optimal Golomb pattern is conjectured to be NP-hard [11]. That is, the computation time to calculate an optimal Golomb pattern on a grid grows exponentially with the size of the grid. See the paper by Soliday *et al.* [12] for more discussion and some examples. Past work on solving restricted versions of the Golomb ruler problem includes using exhaustive parallel search [13], genetic algorithm [12], simulated annealing [14], and linear programming [15]. Distributed.net's OGR project crowdsources volunteer participants' computing resources to find optimal Golomb rulers via an exhaustive search [16]. It took them almost 5 years to find the longest known optimal Golomb ruler with 27 marks, that is 553 units long. Some optimal Golomb rulers are published in article by Shearer [17].

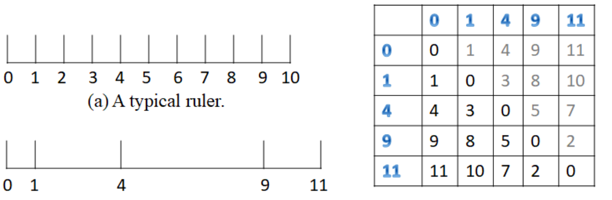
There are no polynomial time algorithm that solve for (near-)optimal Golomb, i.e., non-redundant, patterns [15], [18]. Verifying non-redundancy of a candidate solution is, however, easy. We conjecture that humans, with their vision and cognitive abilities, can find optimal Golomb patterns more efficiently than computers. This has been demonstrated to be the case for another difficult scientific optimization problem: protein folding, via Fold.It [19]. Fold.It is an online citizen science game site at which users can fold the structures of given proteins as perfectly as possible.

More relevant to GRASP, scientists at NASA Goddard Space Flight Center (GSFC) found Golomb solutions for hexagonal deformable mirrors of the Visible Nulling Coronagraph (VNC) testbed when preparing for exoplanet missions [8], [20]. They did so by improving the coverage of Golomb patterns in a previously published article [21] while satisfying the VNC requirements. The past experiences of finding Golomb patterns for the VNC testbed suggested that humans might be better at finding some Golomb solutions than computers.

These experiences and observations motivated us to employ a citizen science gaming approach for finding and collecting non-redundant aperture patterns for NASA astrophysics' applications and thereby develop GRASP. GRASP was initially proposed by Memarsadeghi to the NASA GSFC's Science Directorate, Science Innovation Fund (SIF) in 2013, which was awarded. From the SIF award a preliminary design and requirements for GRASP's game, web interface, and database were developed. After a long hiatus, through 2020 NASA Vermont EPSCoR award to Lee at the University of Vermont (UVM) and collaborations between NASA GSFC and UVM, a fully functional GRASP game and repository is now realized and supported by UVM's Enterprise Technology Services (ETS). ETS continues to provide the computing platform to host the project's web site and database and all related software tools including WebDB/MySQL DBMS and Silk web hosting, and big data accommodations as well as the hosting of a grasp-game.org domain name.

III. GOLOMB PATTERNS, PROPERTIES, AND VARIANTS

In mathematics, a *Golomb pattern* is defined as a set of marks on a grid such that pairwise non-zero distances between the marks are all distinct; there is no repeating pattern of marks in such a pattern. For better understanding, let us consider a



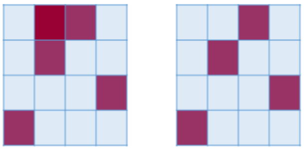
(a) A typical ruler.

(b) A Golomb ruler.

(c) Pairwise distance between marks.

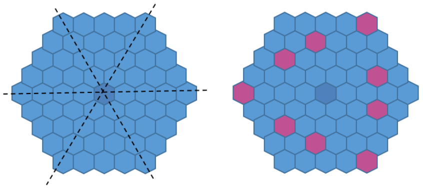
	0	1	4	9	11
0	0	1	4	9	11
1	1	0	3	8	10
4	4	3	0	5	7
9	9	8	5	0	2
11	11	10	7	2	0

FIGURE 2. An example golomb ruler.



(a) Golomb rectangle. (b) Costas array.

FIGURE 3. An example golomb rectangle.



(a) Hexagonal grid. (b) Honeycomb array.

FIGURE 4. An example golomb pattern in a honeycomb array.

Golomb pattern in a one-dimensional grid, called a "Golomb ruler" [9]. A typical ruler has consecutive marks one unit of length apart (e.g., 1 inch or centimeter) (see Fig. 2(a)), where many marks on the ruler have the same pairwise distances. In contrast, in a Golomb ruler, all pairwise non-zero distances among its marks are distinct. Figure 2(b) shows an example of a Golomb ruler with marks at locations 0, 1, 4, 9, and 11, and Figure 2(c) displays unique pairwise distances among these marks.

For a Golomb pattern in a two-dimensional grid, the vector distances (not scalar distances) between marks are all distinct. The grid is typically rectangular, and a Golomb pattern in such a grid is called a "Golomb rectangle" [22] (see Figure 3(a)). Given a Golomb rectangle with marks, its autocorrelation matrix has elements that are 0, 1, or K, where K is the number of marks in the grid. (See the page on "2-D cross correlation" [23] for a description of how an autocorrelation matrix can be calculated for any given matrix; this is equivalent to MATLAB's `xcorr2`.)

A special case of a Golomb rectangle, a square with exactly one mark in each row and each column, is called a "Costas array". For a square grid of size $N \times N$ to be a Costas array, it should have exactly N marks such that their pairwise vector distances are all distinct and there is exactly one mark in each row and each column [10] (see Figure 3(b)). Costas arrays have applications in phased arrays, computer-controlled antenna arrays as well as radar or sonar applications [24].

A Golomb pattern may be created in a *hexagonal* grid, which is defined on three primary axes (see Figure 4(a)). Particularly, a Costas array defined in a hexagonal grid, i.e., with exactly one mark in each of the three primary axes, is called a "honeycomb array" (see Figure 4(b)) [21].

All these patterns have practical use in aperture or antenna array configuration for interferometry and radar missions. The current GRASP implementation focuses on Golomb rectangles, which include Golomb rulers and Costas arrays as special cases.

A Golomb pattern of a fixed size is *optimal* if it contains the maximum possible number of marks. For a Golomb rectangle (G) of size $N \times M$, let $G(N, M)$ be the maximum number of marks that can be present in it. Robinson defined an optimal Golomb rectangle of size $N \times M$ as the one with $G(N, M)$ marks [22], whereas Shearer added the conditions that $G(N, M) > G(N-1, M)$ and $G(N, M) > G(N, M-1)$ [17].

A relevant metric for autocorrelation matrix of a Golomb rectangle, commonly used in astrophysics applications, is the *fill factor* (see Equation 1). Fill factor is defined as the percentage of "covered" (i.e., non-zero) cells, in the autocorrelation matrix [25]. For a Golomb rectangle G of size $M \times N$, the autocorrelation matrix C is of size $(2M-1)(2N-1)$. Let K be the number of non-zero elements in C. Then, the fill factor F is calculated as follows:

$$F = \frac{K}{(2M-1)(2N-1)} \times 100 \tag{1}$$

Note that for a Golomb rectangle G with m marks, the number K of non-zero elements in its corresponding autocorrelation matrix C is

$$K = m(m-1) + 1 \tag{2}$$

IV. APPLICATIONS IN ASTROPHYSICS

Golomb patterns find applications in many areas needing non-redundant patterns, such as coding theory [26], [27], sensor placement in X-Ray crystallography, communication network labeling, circuit layout [5], error correction codes [28], radio frequency allocation [29], radio antenna placement [30], and sonar sequences [24], [31].

The primary application of the current GRASP game is in designing non-redundant aperture arrays for interferometry or radar space missions by replacing a costly single large telescope or antenna by multiple smaller and less expensive ones. The world's largest ground-based telescope, Atacama Large Millimeter Array (ALMA) composed of 64 antennas located in Chile (see Figure 5, as an example of such antenna arrays). The Stellar Imager (SI) is an example of an interferometry mission concept, with an array of more than 20 mirrors of meter-class each to image dynamo activity of stars and to survey the universe [32].

In such missions, an interferometer is built up from an array of apertures such that each aperture is a smaller telescope. Then, the interferometer yields an image with the resolution

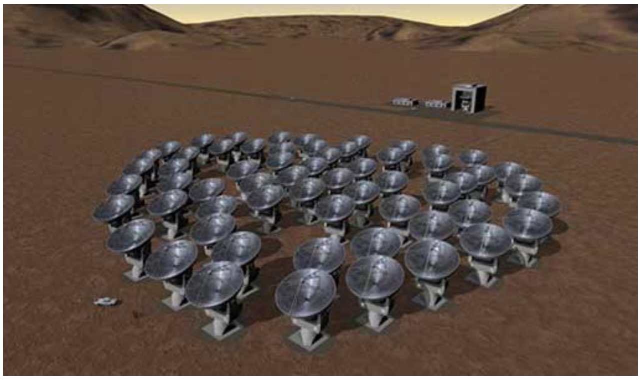


FIGURE 5. ALMA antenna array in chile (Credit: NRAO/AUI and ALMA/ESO/NRAO/NAOJ [33]).

of a larger telescope whose diameter is equal to the interferometer's diameter (not individual smaller apertures). The interferometer's diameter D is defined as the largest pairwise distance among the smaller apertures. In other words, an alternative to launching a large telescope to space is to design an array of smaller apertures such that when deployed on orbit, they make up a much larger synthetic aperture. A non-redundant pattern, such as a Golomb rectangle described in Section III, is desired for such interferometers' aperture array to avoid capturing redundant information and to reduce mission cost. Note that these smaller telescopes do not individually result in as high quality images as those that one could obtain with a much larger telescope that we aim to synthesize. This is where interferometry image reconstruction techniques play a key role [8], [20].

To understand the image synthesis and quality of an interferometer, we need to consider the following imaging concepts: the object or true image I_o that is being observed, the *Point Spread Function (PSF)* (or the impulse response), and the resulting as-seen image I_r generated by the interferometer. The basic imaging equation for I_r is

$$I_r = I_o * \mathrm{PSF} \tag{3}$$

where $*$ denotes 2D convolution. Applying the 2D Fourier Transform (FFT2) to this equation gives

$$\mathrm{FFT2}(I_r) = \mathrm{FFT2}(I_o) \circ \mathrm{FFT2}(\mathrm{PSF}) \tag{4}$$

where $\circ$ denotes the element-wise matrix product (or the Hadamard product). The FFT2 of the PSF is known as the *Optical Transfer Function (OTF)*, which filters the spatial frequencies of I_o to generate the resulting image I_r; that is,

$$\mathrm{OTF} = \mathrm{FFT2}(\mathrm{PSF}) \tag{5}$$

This OTF is the same as the autocorrelation matrix C of a Golomb grid G. An ideal OTF would be unity everywhere (no filtering), but this would require an aperture of infinite size. A real size circular aperture gives an OTF that only has values within a finite circle, while a non-redundant aperture pattern, like that in a Golomb rectangle, gives an OTF sampled within a circle or a grid. Having a non-redundant pattern implies that there is only one sample at each spatial frequency. The OTF is also called the uv-plane in astronomy, and is specifically denoted as OTF(u,v), where u and v are variables in the Fourier domain. Using the OTF definition in Equation 5, Equation 3 can be rewritten as

$$I_r = I_o * \mathrm{IFFT2}(\mathrm{OTF}) \tag{6}$$

where IFFT2 is the inverse 2D Fourier Transform, and Equation 4 can be rewritten as

$$\mathrm{FFT2}(I_r) = \mathrm{FFT2}(I_o) \circ \mathrm{OTF} \tag{7}$$

which in turn, by applying IFFT2, as

$$I_r = \mathrm{IFFT2}(\mathrm{FFT2}(I_o) \circ \mathrm{OTF}) \tag{8}$$

Equation 8 is equivalent to Equations 3 and 6 while being more computationally efficient for large matrices.

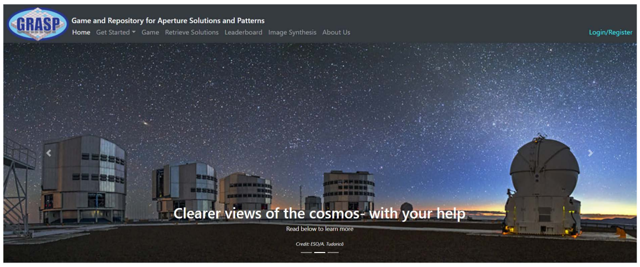
In GRASP, a good Golomb pattern produces a good antenna aperture array, which in turn results in an I_r image of high angular resolution at desired wavelengths, or (u, v) locations. The underlying theory behind it, briefly introduced above, lies in inteferometry image reconstruction techniques [8], [20], [34]. An interferometer measures the interference pattern produced by two apertures, which is directly related to the source brightness. A good image quality requires a good coverage of the uv-plane [34].

The cells selected in a grid $G_{i,j}$ represent the locations of apertures, and their pairwise distances and angles represent an interferometer's baselines $D_{i,j}$. These baselines are represented in the grid's autocorrelation matrix C, or the uv-plane. The angular resolution achieved with an interferometer is a function of the observation's wavelength λ and the baseline sampled, $D_{i,j}$, that is $\lambda/D_{i,j}$ approximately. The maximum baseline length is dictated by the grid dimension and cell size.

In general, the better the uv-plane coverage, the better the resulting image quality. Optimal or near-optimal Golomb patterns are of interest because they achieve their uv-plane coverage with a minimal number of sampled baselines, thereby significantly reducing the cost and complexity of space imagery missions. The desired optimality of a Golomb pattern in light of the uv-plane coverage involves more than the fill factor of the autocorrelation matrix C. Different Golomb patterns may result in autocorrelation matrices of the same fill factor but different uv-plane coverages, thereby resulting in images captured at different light wavelengths and consequently different kinds of scientific discoveries.

V. GRASP, A CITIZEN SCIENCE GAMING PLATFORM

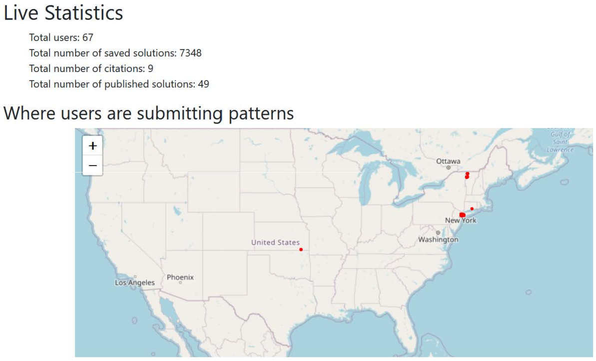
This section introduces the web-based citizen science game GRASP. The objectives of GRASP are (1) to showcase an online gaming approach for solving a complex mathematical problem with solutions contributed by the citizen scientist, (2) to bring a positive impact to the potential NASA astrophysics and radar mission in non-redundant aperture array designs for cost-effective high-resolution space images, (3) to build a repository of all known published and users' solutions



Clearer views of the cosmos- with your help

Gaming For A Clearer View of the Cosmos

(a) Rotating images. Image credit: European Southern Observatory (ESO)/Babak Tafreshi at the World at Night (TWAN) [35], [36].



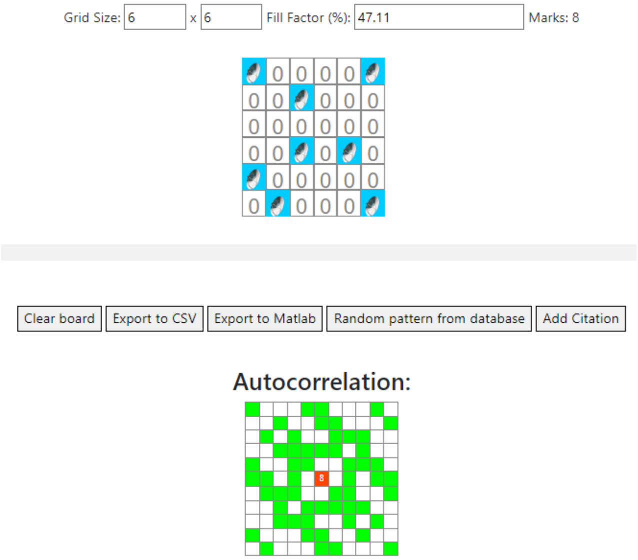
Live Statistics

Total users: 67
Total number of saved solutions: 7348
Total number of citations: 9
Total number of published solutions: 49

Where users are submitting patterns

(b) Live statistics.

FIGURE 6. **GRASP portal (domain name: grasp-game.org).**



Grid Size: 6 x 6 Fill Factor (%): 47.11 Marks: 8

Autocorrelation:

FIGURE 7. **GRASP game.**

available to the general public, and scientists and researchers in particular, and (4) to stimulate STEM education by providing a platform for STEM students in secondary and higher education. For the remainder of this section we give an overview of GRASP's user interface, platform, features and functionalities.

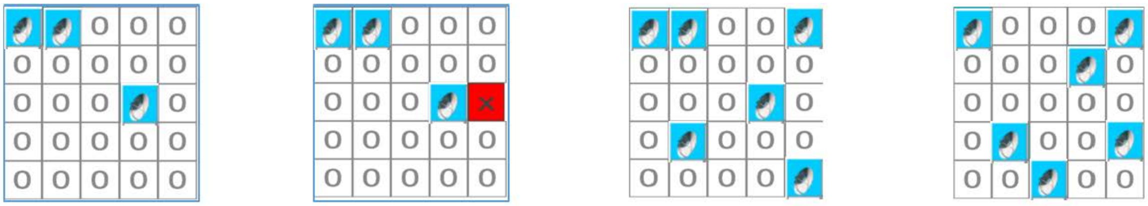
Figure 6 shows the GRASP home page displaying rotating images and live usage statistics. At the top of this home page and all other GRASP pages, there is a menu for users to access various functionalities and content of the GRASP's site that includes tutorials, the game, repository, user leaderboard, and image synthesis capabilities. Next, we describe each of these features.

A. GRASP GAME

Current GRASP game involves finding Golomb rectangles, i.e., non-redundant patterns on a rectilinear grid. Figure 7 shows an example for a 6×6 grid. When users start a game on the "Game" tab of the site, by default they are provided with a randomly generated empty grid, whose rows and columns are greater than 10 and less than 50. Alternatively, users can start playing the game from either an empty grid they initialize the size by specifying the dimensions or by loading an existing pattern from GRASP's database. Users' main goal is to improve the fill factor of the autocorrelation matrix to achieve a near-optimal pattern for a particular grid size. Users select cells as long as they do not repeat a pattern (blue cells with an antenna icon in Figure 7).

Every time the user selects a cell, the selection's validity is tested for non-redundancy by calculating and displaying the grid's autocorrelation matrix; an autocorrelation matrix cell that has a value equal to 1 is displayed in green (see Figure 7). An autocorrelation matrix is considered valid if all its elements are either 0 or 1, except in the center where it has a value equal to the total number of selected cells. A cell selection is rejected if it violates the game's criteria. A rejected cell, along with its corresponding autocorrelation cells that have values greater than 1, are displayed in red. The center cell in the autocorrelation matrix is an exception — since it always has a value equal to the total number of marks in the original grid, it is always displayed in red for a grid with more than one selected cell. The user can continue playing the game by deselecting the invalid cell. Figure 7 and Figures 8(a),(c),(d) are examples of valid solutions, while Figure 8(b) displays an invalid solution. Users can also enter existing solutions from publications with their citations. They earn credit for their contributions, have access to all solutions saved in the database, and can load and export them.

The fill factor is also calculated and displayed as a measure of the quality of each solution. For a given grid size, patterns that yield higher fill factor are desired. We currently give the option of saving a solution only if its fill factor is greater than a predetermined threshold of 10% to avoid saving many sparse and far-from-optimal patterns. Figure 8 shows four patterns in a grid and their corresponding autocorrelation matrices. In Figure 8(a), a user has selected three cells that form a sparse Golomb pattern with 8% of its autocorrelation matrix being filled. As the user selects a fourth cell in Figure 8(b), thereby repeating a pattern (two horizontally adjacent cells), the system marks it as red and rejects that pattern. Figure 8(c) and Figure 8(d) are more optimal valid patterns with six selected cells that can be saved in the repository. While these two patterns have the same fill factor of 38.27%, they provide different coverage of the autocorrelation matrix, or uv-plane for our interferometry applications, and enable different kinds of observations.

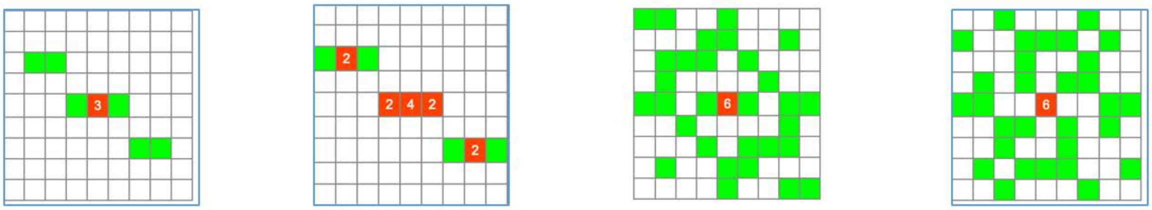


a) A valid, but sparse, Golomb pattern.

b) A redundant pattern; rejected cells turn red when selected.

c) A more optimal pattern with 6 aperture positions and 15 (= 6×(6-1)/2)) baselines.

d) A pattern with the same number of baselines as in (c) but different (u,v)-plane coverage.



Fill factor = 8%.

Fill factor = 16.05%.

Fill factor = 38.27%.

Fill factor = 38.27%.

FIGURE 8. Example golomb patterns (in a 5 × 5 grid) and the corresponding autocorrelation matrices (indicating the uv-plane coverage of the apertures).

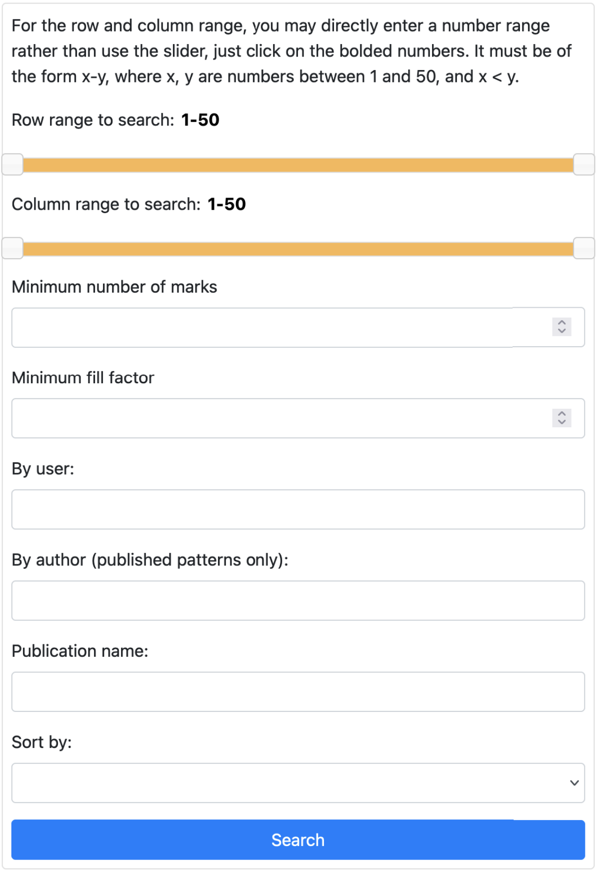
B. GRASP REPOSITORY

GRASP users may search through the existing repository of patterns generated by other GRASP users via ''Retrieve Solutions'' tab. Figure 9 displays the browsing page. The search can be filtered by one or more of the following conditions: the Golomb grid size (ranges of the number of rows and the number of columns), the minimum required number of marks in the Golomb grid, the minimum required fill factor of the resulting autocorrelation matrix, the username of the GRASP user that created the Golomb grid, and, if it is a published pattern with an associated citation, the publication author and the publication name. The search result may be sorted by the fill factor, the number of marks, or the date a solution was saved in the database. The user can then load one of the patterns in the search result to the Game page and start playing the game continued from it. It is our goal that, with the help of GRASP users, citizen scientists and researchers, GRASP will grow to become a complete repository of all published Golomb patterns that are maintained over time.

C. GAME TUTORIAL AND BACKGROUND INFORMATION

GRASP users are provided with a playable tutorial in the ''Game Tutorial'' menu option via the ''Get Started'' tab. The Game Tutorial is a version of the game that is decoupled from the database and added with pop-up prompts to guide the user interactively through all the available functionalities. This tutorial checks for pattern correctness, but not redundancy in the database, so that a user can experience finding a correct pattern without needing to produce an original one. Once the user has produced a correct pattern of sufficient fill factor, they are prompted to play the actual game.

In addition to the playable tutorial, GRASP users are provided with a written guide to the game, as well as educational pages introducing the Citizen Science and Golomb Patterns concepts, including links to outside resources, all under the ''Get Started'' tab's menu options.



For the row and column range, you may directly enter a number range rather than use the slider, just click on the bolded numbers. It must be of the form x-y, where x, y are numbers between 1 and 50, and x < y.

Row range to search: **1-50**

Column range to search: **1-50**

Minimum number of marks

Minimum fill factor

By user:

By author (published patterns only):

Publication name:

Sort by:

FIGURE 9. GRASP retrieve solutions.

User	Score (%)	Average Fill Factor (%)	Average Size of Solution	Total Patterns Contributed	Total Published Patterns Contributed
JaeP	22.32	25.83	30 x 42	2	0
shameeza	20.39	10.14	41 x 50	1	0
hhaider1	19.68	10.50	41 x 45	1	0
Basimah	19.68	16.01	34 x 36	12	0
yorgelis.100	18.27	10.72	38 x 41	1	0
KarenVG123	18.12	19.92	28 x 32	6	0
EmmanuelD	16.98	24.63	27 x 33	3	0
brenda8848	16.93	22.54	27 x 28	20	0
GenesisV	16.87	10.94	30 x 45	2	0
Ahmad	16.44	27.17	22 x 26	10	0

FIGURE 10. GRASP Leaderboard. This example shows top ten performers ranked by the score (first column).

D. USER LEADERBOARD

To foster a sense of competition and gamification for the discovery of high-quality Golomb patterns, GRASP users are presented with a user leaderboard. The page can be accessed via GRASP site's "Leaderboard" tab (see Figure 10). This page displays the top ten users based on different metrics, which can be selected by the user via clicking on the metric name in the first row. These metrics are also explained on the web page by clicking the 'i' button next to the metric name. By default, the leaderboard is ranked by the average of quality scores of each user's contributed solutions. For each solution, a quality score is calculated as

$$\text{Score} = F \times \frac{\sqrt{\text{Rows} \times \text{Cols}}}{50} \tag{9}$$

where F is the fill factor of the solution, Rows and Cols are the numbers of rows and columns, respectively, of the solution (see Equation 1), and 50 is the maximum allowed number of rows and number of columns set by GRASP. Users can rank top players also based on average grid size of their contributed solutions (solutions to larger grid sizes are preferred), their total number of contributed patterns to GRASP's repository, and the total number of previously published solutions.

E. IMAGE SYNTHESIS

GRASP demonstrates the application of Golomb patterns in astronomy via an Image Synthesis tool available from the site's menu. Users can apply different Golomb patterns to sky images (as if they were true-sky views) and see, through simulation, how those images would appear if synthesized by a telescope with an aperture array based on selected Golomb patterns. Users are allowed to upload their own sky images, or experiment with one of the provided sample images of different astronomical phenomena. Users need to select the Golomb pattern to be applied for image synthesis through an interface for retrieving solutions (similar to Figure 9). The image synthesis is then performed by an implementation of Equation 6 in Section IV, as summarized below.

```
import numpy as np
import scipy.signal as spsig
from scipy.signal.signaltools import correlate2d
from numpy.fft import fft2, ifft2, fftshift,
ifftshift

#assuming matrix g has been previously initialized
# auto_correlation C, or the OTF
otf = correlate2d(g, g, mode='full')
psf = ifftshift(abs(ifft2(otf)))
I_r = np.array(spsig.convolve2d(I_o,psf,mode='same'))
```

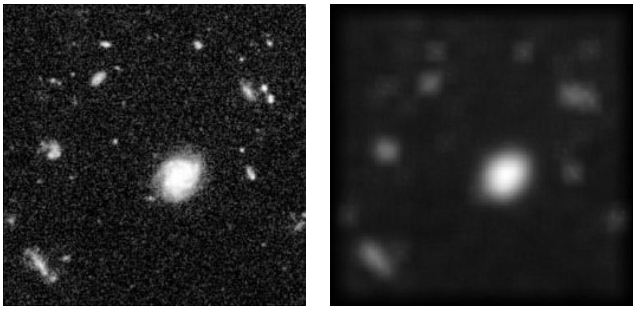
FIGURE 11. Python code for image synthesis of input image I_o with two dimensional Golomb grid g.

1) Calculate the modulus inverse Fourier transform of the auto correlation matrix (i.e., OTF). This results in the PSF.
2) Convolve the PSF with the input image. This is the synthesized image I_r. This step assumes that the cells in PSF's matrix have the same spatial resolution as the input image.
3) Normalize the I_r values to integers in the desired output range, in our case [0, 255] for jpeg images.

Figure 11 shows relevant parts of the Python code implementing the image synthesis steps; and Figure 12 demonstrates how the image would look when synthesized with the code, given a Golomb pattern of grid size 10 rows × 10 columns and fill factor of 58.45%. Note that this synthesis is a mathematical simulation, estimating the PSF of what would be a hypothetically one large telescope with some gaps in its uv-plane; a real astronomical interferometer would produce a higher angular resolution image via calculation of interference patterns of light wave signals from multiple telescopes. Some interferometry image reconstruction algorithms are outlined in an article by Lyon *et al.* [37].

F. GRASP ARCHITECTURE

The GRASP website is hosted on a Linux server at University of Vermont, using the Apache Web Server software. The actual program is written in the PHP scripting language,



(a) Raw image of the night sky (credit: NASA/Hubble).

(b) Synthesized image (Golomb grid size 10 × 10; with 15 marks and fill factor 58.45%).

FIGURE 12. **GRASP image synthesis.**

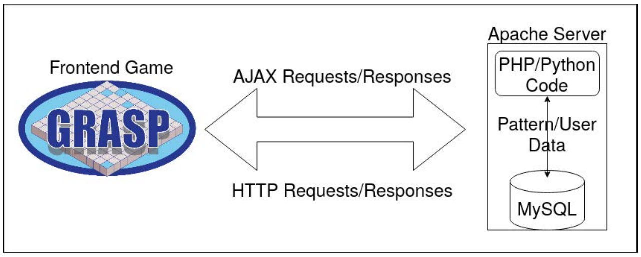


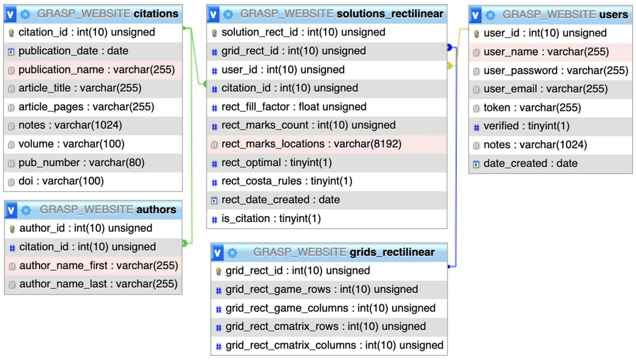
FIGURE 13. **GRASP overall configuration.**

with some Python, and JavaScript for user interactivity. The database persistence is provided by the MySQL relational database management system. This is commonly referred to as a LAMP (Linux, Apache, MySQL, PHP/Python) stack website [38].

Figure 13 shows the overall system configuration of GRASP. The front-end game runs on the client web-browser (primarily targeting desktop users), and the database runs on the back-end server. Communication between the front-end and back-end is accomplished via Asynchronous JavaScript and XML (AJAX) messages. The JavaScript game makes asynchronous HTTP requests to the back-end API to verify a pattern's uniqueness, validity, etc. as the user plays the game. This API is also used to export a current pattern to a file format for download (in MATLAB or CSV data formats), to load a pattern by its identification number (by clicking on a link on the Retrieve Solutions page), or to load a random pattern for a user to continue playing on.

The API also offers a mechanism for a user to "orphan" a pattern linked to their account. (Once saved, a pattern is not deleted but can be orphaned.) This allows their username to be removed from the pattern, no longer counting towards their user statistics, but the pattern remains in the database, credited to the "Anonymous" user.

The database consists of five relational tables managed by MySQL: users, citations, authors, grids_rectilinear, and solutions_rectilinear (i.e., Golomb grids). Figure 14 shows the current database schema. Future versions may include schema for non-rectilinear grids.



GRASP_WEBSITE **citations**
citation_id : int(10) unsigned
publication_date : date
publication_name : varchar(255)
article_title : varchar(255)
article_pages : varchar(255)
notes : varchar(1024)
volume : varchar(100)
pub_number : varchar(80)
doi : varchar(100)

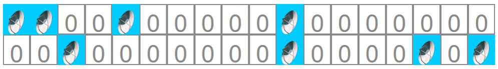
GRASP_WEBSITE **authors**
author_id : int(10) unsigned
citation_id : int(10) unsigned
author_name_first : varchar(255)
author_name_last : varchar(255)

GRASP_WEBSITE **solutions_rectilinear**
solution_rect_id : int(10) unsigned
grid_rect_id : int(10) unsigned
user_id : int(10) unsigned
citation_id : int(10) unsigned
rect_fill_factor : float unsigned
rect_marks_count : int(10) unsigned
rect_marks_locations : varchar(8192)
rect_optimal : tinyint(1)
rect_costa_rules : tinyint(1)
rect_date_created : date
is_citation : tinyint(1)

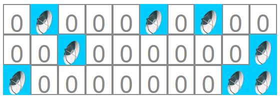
GRASP_WEBSITE **grids_rectilinear**
grid_rect_id : int(10) unsigned
grid_rect_game_rows : int(10) unsigned
grid_rect_game_columns : int(10) unsigned
grid_rect_cmatrix_rows : int(10) unsigned
grid_rect_cmatrix_columns : int(10) unsigned

GRASP_WEBSITE **users**
user_id : int(10) unsigned
user_name : varchar(255)
user_password : varchar(255)
user_email : varchar(255)
token : varchar(255)
verified : tinyint(1)
notes : varchar(1024)
date_created : date

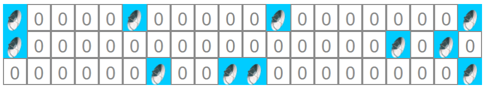
FIGURE 14. **Database schema (autogenerated using PHPMyAdmin).**



(a) A 2 × 18 solution with 8 marks and fill factor 54.29% by user dc on October 2, 2021.



(b) A 3 × 10 solution with 8 marks and fill factor 60% by user tstanley on January 22, 2021.



(c) A 3 × 20 solution with 11 marks and fill factor 56.92% by user nargess on September 7, 2021.

FIGURE 15. **Example solutions with fill factor greater than 50%. Grids replaced by screen snapshots.**

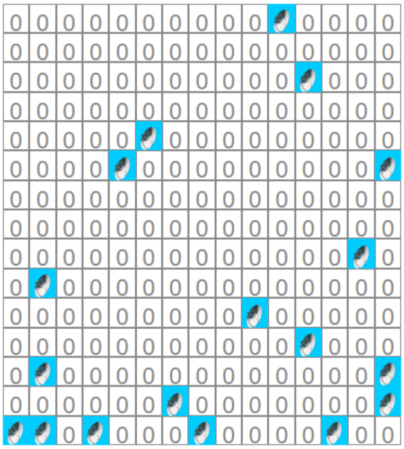
Each record in the "solutions_rectilinear" table has a required (i.e., not null) link (i.e., foreign key) to a record in the "grids_rectilinear" table, a required link to a user record, and an optional link to a citation record.

A solution record in the "solutions_rectilinear" table contains an encoded string of coordinates in the rectilinear Golomb grid, the fill factor in the autocorrelation matrix, the entry timestamp of the solution, a Boolean flag denoting whether it is an optimal pattern, and a Boolean flag denoting if it is a Costas array pattern.

A grid record in the "grids_rectilinear" table contains the height and width of the Golomb grid and the height and width of its autocorrelation matrix.

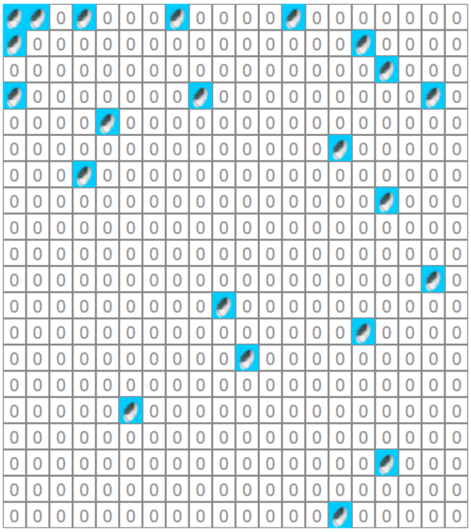
A user record in the "users" table contains a username, email, hashed password, account creation date, a Boolean flag denoting email verification, and a token used for account verification, password reset, and any other tasks calling for a one-time password (which is used to reset the password to a new random token).

A citation record in the "citations" table contains the publication name, publication date, title, page numbers of the article the pattern was published in, the volume of the article,



0	0	0	0	0	0	0	0	0	0	●	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	●	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	●	0	0	0	0	0	0	0	0	0
0	0	0	0	●	0	0	0	0	0	0	0	0	0	●
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	●	0
0	●	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	●	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	●	0	0	0
0	●	0	0	0	0	0	0	0	0	0	0	0	0	●
0	0	0	0	0	0	●	0	0	0	0	0	0	0	●
●	●	0	●	0	0	0	●	0	0	0	0	●	0	0

(a) A 15×15 solution with 18 marks and fill factor 36.50% by user brenda8848 on May 15, 2022.



●	●	0	●	0	0	0	●	0	0	0	0	●	0	0	0	0	0	0	0
●	0	0	0	0	0	0	0	0	0	0	0	0	0	0	●	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	●	0	0	0
●	0	0	0	0	0	0	0	●	0	0	0	0	0	0	0	0	0	●	0
0	0	0	0	●	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	●	0	0	0	0	0
0	0	0	●	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	●	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	●	0
0	0	0	0	0	0	0	0	0	●	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	●	0	0	0	0
0	0	0	0	0	0	0	0	0	0	●	0	0	0	0	0	0	0	0	0
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0	0	0	0	0	●	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	●	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	●	0	0	0	0	0

(b) A 20×20 solution with 22 marks and fill factor 30.44% by user dbijraj on May 12, 2022.

FIGURE 16. **Example solutions with grid size at least 15 × 15 and fill factor greater than 30%.**

the publication number, as well as the DOI. The authors of the publication are stored separately in the authors table, which has a required link (i.e., foreign key) to the citation record, thereby allowing for multiple authors per publication.

VI. CURRENT STATUS AND FUTURE PLANS

At the time of this writing, GRASP has 67 individual user accounts with 7,341 solutions in its repository. While there are several publications that report on known optimal Golomb rectangles [17], [22], [39], there are significantly more novel Golomb rectangles contributed by users that met the game's minimum fill factor requirement of 10%. Figure 15 shows a few of these Golomb rectangles that have a fill factor greater than 50%. During Spring 2022, a pilot citizen science project was conducted in collaboration with Queensborough Community College at the City University of New York (CUNY) to use GRASP in an astronomy course attended by approximately 40 students, who contributed solutions of various grid sizes. Figure 16 shows a couple of those solutions with grid size at least 15 × 15 and fill factor greater than 30%. We have observed that as the grid dimensions increases, it becomes harder to find Golomb solutions with more than 25% fill factor. GRASP was also featured during April 2022, the Citizen Science Month, by CitizenScience.gov's Twitter account.

Provided with funding, GRASP platform development will continue towards a public launch, including functionalities to support user engagement through email subscription, user forums, and ranking their solutions based on different scores and recognizing their contributions. Given the nature of the GRASP platform developed for "specific scientific topics" [4], approximately 100 citizen scientists will initially be recruited from target audience with interest in Science, Technology, Engineering, and Mathematics (STEM) fields, more specifically in mathematics, computer science, or astrophysics as well as a broader audience reachable via online social media.

We will form partnerships with academic collaborators and citizen science communities at NASA and across the nation to participate in various data entry campaigns. These campaigns would ask GRASP users to perform a task within a fixed time period (e.g., 1–2 months) — from a general task such as entering published solutions in the GRASP's repository and finding near-optimal solutions for a specific grid size (e.g., 30×30) to a more specific task such as finding solutions that result in coverage of specific regions in their corresponding autocorrelation matrices (or uv-planes).

A full-fledged GRASP would offer a variety of games with different grid sizes, grid types (e.g., rectangular, hexagonal), Golomb variant types (e.g., Costas), etc. and eventually for different applications beyond the antenna array design.

VII. CONCLUSION

This paper presented GRASP, an online citizen science gaming website, developed to find and store Golomb patterns. We gave an overview of Golomb patterns, their properties, and applications. In particular, we described their specific applications in non-redundant aperture array designs for radar and interferometry space missions with high angular resolution images at desired wavelengths. We also introduced the developed GRASP online platform along with its features, functionalities, current status and results. The development of GRASP can continue both as a science tool enabling new low-cost radar and interferometry space missions with maximum scientific return on investment and as an educational tool for STEM students in astronomy and other areas (e.g., information theory, cryptography).

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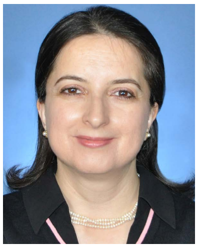

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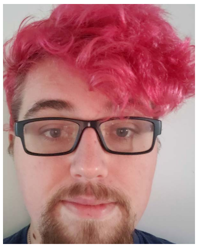

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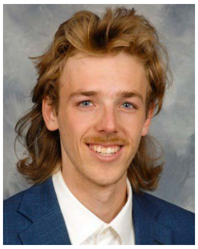

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