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RESEARCH ARTICLE

An Optimized Complex Fuzzy Hypersoft Set System Based Approach for the Evaluation of Strategic Procurement Techniques for Fuel Cell and Hydrogen Components

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ABSTRACT While fuel-cell conjunction with hydrogen (FCH) innovation is still in its infancy, the existing market progression towards corporatization is straddled by prevailing research, development, and advancement. The availability of spare components and auxiliary equipment for the FCH process is vital to the component's improvement. An optimal strategic procurement function can prosper protracted research and recognize the levels of technical understanding to produce high performance and affordably priced modules for a successful transformation. This study presents a multi-attribute decision-making approach based on Entropy (ENT), Similarity Measure (SM), and TOPSIS Approach to evaluate FCH supplier selection strategies. This research examined the novelty of the Complex Fuzzy HyperSoft Set (CFHSS), which may respond to instabilities, ambiguity, and vagueness of facts in knowledge by simultaneously putting into consideration the amplitude and phase terms (P-terms) of complex numbers (C-numbers). The presented structure is the most suitable option for exploring FCH concerns as it allows for a more comprehensive array of membership values, and the periodic nature of the content can be expressed in P-terms to widen the content to a unit circle in a dynamic reference frame through the specification of the Fuzzy Hypersoft Set (FHSS). Secondly, the features in CFHSS may be further sub-divided into attribute values for easier comprehension. The paper also illustrates the apparent connection between FCH and CFHSS SM, ENT, and TOPSIS. These strategies may be used to determine the best approach from a group of possibilities that have a variety of applications in the field of optimization. The recommended methodology's reliability and effectiveness are examined by evaluating the acquired findings from several prior studies. Some assessments have been done using various parameter values to validate the robustness of the suggested approaches. A scenario-based research project on the FCH modules decision problem is presumed to put the envisaged holistic strategy into action. Ultimately, comparisons are provided to demonstrate the usefulness of the proposed method.

INDEX TERMS Fuel cell, hydrogen, fuzzy hypersoft set (FHSS), complex fuzzy hypersoft set (CFHSS), entropy (ENT), similarity measures (SM).

I. INTRODUCTION

Fuel cell and Hydrogen alternatives have gained significant interest in the scientific community as it is proving to be a potential energy source for the future. Researchers have looked at hydrogen gas as a fuel source for fuel cells in the past. The main drawback of using hydrogen as a fuel source is that it is relatively difficult to store and transport large amounts of hydrogen. However, recent advancements in hydrogen storage have been made, and it is now possible

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to store large amounts of hydrogen for use in fuel cells. One of the most common methods of storing hydrogen is to use the solid form of the element, called hydrogels. Hydrogen is often used to improve the energy density of batteries, but it also has the potential to improve the conductivity of the electrodes. By mixing hydrogen with the electrodes, the conductivity of the electrodes can be further improved, which could help increase their energy density. While power generators stay a constraint to the impactful utilization of renewable energies (REs), evolving chemical-based storage technologies are becoming a realistic alternative with the possibility to store vast amounts of energy in a compact apartment [3]. The combination of a fuel cell (FC) and hydrogen is a practical approach that can determine the fate of sustainable energy requirements [1], [2]. An FC incorporates additional electricity, thermal, or even liquid. FCs, like battery systems, generate power through a reaction mechanism, but they never start losing their charges as long as fuel (i.e., hydrogen) is readily accessible [4]. It is estimated that electrolytes will drive growth in the electricity sector as a fuel source.

Multi-Criteria Decision-making (MCDM) has the ability to strengthen all aspects of making by putting, from design to design and production. Still, it's instrumental in high-tech markets, where slight improvements in properties and performance often achieve product strategy and comparative position. The versatility of MCDM methodologies to area incorporates material, process, and form for complex selection of construction issues illustrates its maximum capabilities. As a result, it's crucial to broaden the definition of MCDM procedures that include a variety of advanced applications and to use evaluation to increase the manufacturing process. The need to deal with unpredictability and make a compromise is a constant practical challenge task, and the ability to affect data ranges properly is essential in making better use of MCDM in resource selection and implementation. Some new MCDM techniques like COMET and SPOTIS [70], [71] have developed to face the recent challenges of uncertainty.

Nowadays, the system is in its infant stage and struggling with its grounding, but prognosticators are hopeful about the future life experience of the worldwide FCH market. The development moment of technological progress highlights the significance of research and innovation (RD) and prototype operations on the track to corporatization. This stage requires the availability of external components of the FCH system applications and accessories. The development moment of technological progress highlights the significance of research and innovation (RD) and prototype operations on the track to corporatization. This stage requires the availability of external components of the FCH system applications and accessories. For many education systems, procuring such facilities coming from external vendors is a requirement for effective and recurring innovation [5]. Given their inadequate technical insight to design and produce excellent performance and inexpensive FCs, developing local regions of the world with more constricted infrastructure facilities and mineral wealth, viable RD, and entrepreneurialism are undergirded by a qualified supply chain function. Emphasizing vendors can help coordinate and increase the speed of long-term investigation on FCH software by increasing RD intensity, establishing know-how devices, developing cost-effective components, and delivering state-of-the-art stack samples. A strategic relationship with an adequate FCH distributor may provide the benefit of feasible collaborative effort [6]. However, it becomes a tricky problem when the procurement includes financial and technical factors and strategic, ecologic, and even social effects.

Multiple criterion decision making (MCDM) is a well-established mechanism used for the significant primary decisions of industrial companies, particularly noticeably in logistics systems and acquisition [7]. In previous research, MCDM schemes have been developed to establish an effective solution for feasible judgment call issues.

The appropriate decision of the FCH technique involves thorough administration because the methodologies from which the increasing the weight is to be performed widely in terms of money, labour, features provided, trustworthiness, and the geographical spectrum in consequences. The preponderance of the posture and balance for this assignment has missing parameters, and poor treatment of this variability may result in unsatisfactory performance. Because when classification identifies the applicability of the specified requirements as an accurate reflection, the features are considered an example of a spectrum, or the data is indicated in a colloquial language such as decent, or above, and excellent, this leaves more room for volatility and, if not adequately addressed, may outcome in an erroneous decision stage. To overcome this problem, technologies from FS theory are used since they can avoid these situations precisely. The best resource for this type of event is goal setting, in which many decision-makers decide things on the existing solutions that are being characterized. In a sense, an ultimate choice is a collective approach of all judgment call groups. A review of the literature reveals the use of a diversity of MCDM strategies in FCH. Dynamic Electrochemical Impedance Spectroscopy (DEIS) measures the impedance changes of the fuel cell during its operation [68], [69]. Accordingly, there is a possibility to monitor the changes in individual components' characteristics in the operand.

There are some fundamental theories which have been developed in the last century that discussed the fuzzy nature of data [10]–[14], [16]–[23], [65] and can fulfill the deficiencies of parametrization techniques.

The ambiguous event's probabilistic indicator has significantly contributed to FS and its combination frames described in [25]. De Luca and Termini [26] proposed a particular set of possibilities for fuzzy ENT. On the other hand, ENT has received increasing attention than SM, a crucial method for evaluating the degree of SM between two surfaces. Pappis and his colleagues looked at the SM in detail in a succession of studies [27], [28]. The ENT and SM for internal and external determinants have been presented in [29], [31], [33] for strategic planning, intelligence, and future forecasting concerns.

The theories of CFS (Complex fuzzy set) and CFSS have been developed by Al-Qudah *et al.* [9], [32] that can facilitate the resolution of complicated two-dimensional representational characteristics. The variables should be sub-divided into data points in various pragmatic deployments for ease of interpretation. Smarandache [34] fulfilled this criterion by designing the Hypersoft set (HSS) as an enhancement of the SS. He extended this approach by re-envisioning SS as a multi-attribute procedure and expanding it to the HSS.

In a neutrosophic context, Saeed *et al.* [30], [35], [61], [64]–[67] presented the concepts of Hypersoft (HS) and its applications in real-life scenario. There are some other theories [60]–[62] which have been developed to discuss the hybrid structures of fuzzy-like environments. The strategies described in the articles above are inadequate for a meticulous evaluation of the data to better insight and make intelligent decisions. These theories face the challenge of managing 2D data content (the magnitude of the effects and the time needed to have an effect) when the criteria have sub-values. We develop it into a complex fuzzy hypersoft set to meet those goals by combining the complex set and fuzzy hypersoft set. This methodology is more customizable in three distinct ways. It must provide a broad range of membership function quantities to be allotted to the celestial sphere in an imaginary axis by extending the complex fuzzy hypersoft set to add an extra keyword, the Phase-term, to accommodate for the statement's seasonal character. Furthermore, the complex fuzzy hypersoft set attributes will be further segregated into sentence numbers to increase accuracy. This model helps handle problems with the properties of multidimensional characterization. Our proposed model can handle uncertainties, imprecision, and vagueness of two-dimensional fuzzy information by simultaneously capturing the amplitude and phase terms of the complex numbers. The recommended methodology's reliability and effectiveness are examined by evaluating the acquired findings from several prior studies. Some assessments have been done using various parameter values to validate the robustness of the suggested approaches. Also, convergence refers to the stable point found at the end of a sequence of solutions via an iterative optimization algorithm. This is not an iterative algorithm. This is different approach in which convergence means the data lies between (0,1), hence the convergence (optimum solution) lies between 0,1). A scenario-based research project of the fuel cell hydrogen modules decision problem is presumed to put the envisaged holistic strategy into action. Remarkably, the principle of procurement management has maintained sparse in energy technology, with just a few survey results dealing with the issue in the area. There has been no investigation on multi-criteria strategic planning for the FCH development issue, despite the reality that it is a significant problem and required in practice. As a result, the objective of this project is mainly on the comprehension of the numerical solution in an FCH multi-criterion decision utilizing an innovative imprecise Mathematical model approach based on wavelet estimates.

Our investigation's primary goals are as follows. First, we provided the terms of ENT and SM in CFHSS and the theoretical underpinnings. Mathematical models are also capable of testing the project's efficiency and dominance. Furthermore, detailed results are compared between traditional algorithms and accepted notions. Eventually, the mathematical components highlight the consistency and usefulness of adopting changes. The judgment call review panel will reassess the data in the form of CFHSS by assessing the amount of influence and time required of influence as a complex number; as well as the deep analysis of the knowledge by deciding to take sub modelling values of appointed attributes as a hypersoft structural system; where all data can be taken as a quantitative value between 0 (degree of zero per cent match) and 1 (degree of one per cent match) (degree of hundred per cent match).

A. MOTIVATION

Because it is hard to pinpoint the specific type of FCH approaches utilizing prior, existing understanding and methods [10], [17], [20], [32], and [9] because these equipment are constrained to attain combinations, the objectives of this research is to forecast plausible occasions for FCH schemes and their effectual recognizing outcome. The strategies described in [10], [17], [20], [32] and [9] are inadequate for a meticulous evaluation of the data to have a better insight and make intelligent decisions. These assumptions, according to [10], [17], [20], find it challenging to manage 2D data content (the magnitude of the effects and the time needed to have an effect) when the criteria have their sub-values. In [9], [32] shows that they can organize 2D relevant data but refuse to interact with specifications that have sub-parameter types of characteristics. To meet those goals, we blended these methodologies into a lengthy situation consisting of a blending of fuzzy sets and HSS. This methodology is more customizable in three distinct ways. It must provide a broad range of membership function quantities to be allotted to the celestial sphere in an imaginary axis by extending the CFHSS to add an extra keyword, the P-term, to accommodate the statement's seasonal character. Furthermore, the CFHS attributes will be further segregated into sentence numbers to increase accuracy. The proposed method analyses distinct FCH solutions based on financial, historical, intellectual, monetary, and ecological. When combined with modelling, these assumptions are as valuable and appropriate for attaining goals as any potential outcome.

B. ARTICLE COMPOSITION

Section II emphasizes the basic definitions and terminologies applied to address the problem. In Section III, we introduce, with an illustration, the metaphorical perception of ENT for CFHSS. Section IV will look into the SM with CFHSS and the interaction between ENT and the SM. Section V presented the evaluations of FCH using a TOPSIS-based

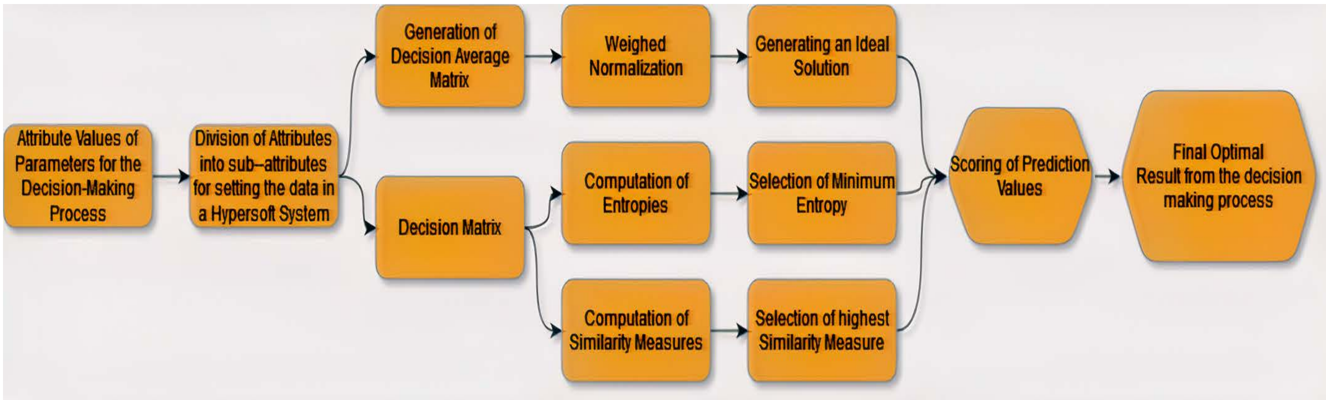

FIGURE 1. Frame diagram for proposed algorithms.

optimized FHS set classifier. Section VI refers to the article's findings; please see Fig 1 for the frame diagram to clarify the working of the algorithms.

II. PRELIMINARIES

The concepts primarily applied for designing the desired approach are discussed in this section.

Definition 1 ([10]): The FS, $R = \{(q, \varpi(q))|\ q \in Q\}$ such that

$$\varpi : Q \to [0, 1],$$

where Q indicates entities accumulating and $\varpi(q)$ symbolizes the rating of membership of $q \in Q$.

Definition 2 ([17]): SS is the pair (ϖ, N) over Q, where ϖ is a method that resembles in this manner:

$$\varpi : N \to P(Q),$$

for $\epsilon \in N$, $\varpi(\epsilon)$ can be construed as ϵ predictions of the SS (ϖ, N).

Definition 3 ([33]): An ENT is a map ψ from $FS(\varpi, N)$ to $[0, \infty)$ for FSS if ψ meets the required necessities:

1) $\psi(\varpi, N) = 0$ if (ϖ, N) is a SS,
2) $\psi(\varpi, N) = 1$ if $\varpi(e) = 0.5$.
3) Assume (ϖ, N) be crisp than that of (χ, M) that is, for $e \in N$ and $q \in Q$, $\varpi(e)(q) \leq \chi(e)(q)$ if $\chi(e)(q) \leq 0.5$ and $\varpi(e)(q) \geq \chi(e)(q)$ if $\chi(e)(q) \geq 0.5$. Then $\psi(\varpi, N) \leq \psi(\chi, M)$,
4) $\psi(\varpi, N) = \psi(\varpi^c, N)$, where (ϖ^c, N) is the complement of FSS (ϖ, N), which can be underlying as $\varpi^c(e) = (\varpi(e))^c$, for every $e \in N$.

Definition 4 ([33]): If a translation V between $FS(Q, E)$ towards $[0, 1]$ matches the requirements specified, this is certified as that of an SM for FSS.

1) $V(X_Q, \Phi_Q) = 0$, for any $Q \in E$, and $V((\varpi, N), (\varpi, N)) = 1$ for any $(\varpi, N) \in FS(Q, E)$,
2) $V((\varpi, N), (\chi, M)) = V((\chi, M), (\varpi, N))$, for any $(\varpi, N), (\chi, M) \in FS(Q, E)$,
3) For any $(\varpi, N), (\chi, M), (Q, O) \in FS(Q, E)$ if $(\varpi, N) \subseteq (\chi, M) \subseteq (Q, O)$, then $V((Q, O), (\varpi, N)) = min(V((Q, O), (\chi, M)), V((\chi, M), (\varpi, N)))$.

Definition 5 ([35]): Suppose G and $\varpi(G)$ be two sets, suppose $l_1, l_2, l_3, \ldots, l_n$ be different attributes which relates to $G_1, G_2, G_3, \ldots, G_n$, respectively. where $G_i \cap G_j = \Phi$ in which i and j is unequal, then the FHSS (Σ_L, L) over G represented in such a way that $\Sigma_L : L \to \varpi(G)$, where $L = G_1 \times G_2 \times G_3 \times \cdots \times G_n$.

Definition 6 ([61]): Suppose G and $\varpi(G)$ be two sets, suppose $l_1, l_2, l_3, \ldots, l_n$ be different attributes which relates to $G_1, G_2, G_3, \ldots, G_n$, respectively. where $G_i \cap G_j = \Phi$ in which i and j is unequal, then the CF-set represented in such a way that:

$$\xi_G = \{(\underline{o}, \delta(\underline{o})) : \underline{o} \in G, \delta(\underline{o}) \in M(G)\}.$$

Example 1: Consider an individual intended to withdraw funds from an accounts with a certain number of days. Let $G = \{c_1 = \text{Lloyds}, c_2 = \text{NatWest}, c_3 = \text{HSBC}\}$ contain three main London banks. A year is usually assumed to be partitioned into four components, each containing its own premiums. Let a_1 = amount of payback, a_2 = lending rate, and a_3 = rate of return a_4 = Contents, different components with properties some of which are set elements G_1, G_2, G_3. Let $G_1 = \{\eta_1 = \text{Flexible}, \eta_2 = \text{Difficult}\}$, $G_2 = \{\eta_3 = \text{High}, \eta_4 = Low\}$, $G_3 = \{\eta_5 = \text{Quick}\}$. Now, we are producing CFHSS facts provided earlier.

$$\chi(\eta_1, \eta_3, \eta_5) = \{c_1/(0.9e^{i2\pi(2/4)}), c_2/(0.8e^{i2\pi(1/4)}), c_3/(0.4e^{i2\pi(3/4)})\},$$
$$\chi(\eta_1, \eta_4, \eta_5) = \{c_1/(0.8e^{i2\pi(2/4)}), c_2/(0.5e^{i2\pi(1/4)}), c_3/(0.1e^{i2\pi(3/4)})\},$$
$$\chi(\eta_2, \eta_3, \eta_5) = \{c_1/(0.1e^{i2\pi(2/4)}), c_2/(0.8e^{i2\pi(2/4)}), c_3/(0.04e^{i2\pi(1/4)})\},$$
$$\chi(\eta_2, \eta_4, \eta_5) = \{c_1/(0.2e^{i2\pi(2/4)}), c_2/(0.7e^{i2\pi(1/4)}), c_3/(0.1e^{i2\pi(3/4)})\},$$

The A-terms inside this aspect characterize the quantities of adherence to the currency union, meanwhile the

P-terms represent the amounts of conformity to the summer months in relation to the study items. In the CFHSS value $c_1/(0.8e^{i2\pi(2=4)}, c_2/0.2e^{i2\pi(4=4)}, c_3/0.3e^{i2\pi(3=4)}$. The first amount $(0.8ei2\pi(2=4)$ reveals that perhaps the affects on bank interests are expanding during spring time, the A-term and P-term are 0.8 and $(2=4)$ accordingly, indicates the spring season with the phase w.r.t $(\eta 1, \eta 3, \eta_5)$. Although the aforementioned degree of participation $0.2e^{i2\pi(4=4)}$ shows that the incentivisation rates are so low in the cold weather since this P-term 0.2 is absolutely minimal and the P-term $(4=4)$ takes place at the end season of the last year (the winter season) in relation to the features value $(\eta 1, \eta 3, \eta 5)$. Otherwise we'll go through all the CFHSS's underlying concept and execution.

III. ENTROPY (ENT) ON CFHS-SETS

ENT is FS's most significant characteristic since it administers a vital FS regulatory environment. What is the intensity of uncertainty in an FS? The ENT procedure analyzes FS confusion. This report concentrates on the concept of ENT within the setting of the CFHSS perspective. Numerous additional principles and implementations emphasise the robustness and productivity of the prototype design ENT-based CFHSS.

Definition 7: A map $E : CFHSS(H) \to [0, 1]$ is said to be ENT on *CFHSS*, if E matches all of the standards

1) $E(\chi, \mathcal{T}) = 0 \Leftrightarrow \upsilon_{F(e)}(q) = 1$ and $\omega_{F(e)}(e)(q) = 2\pi$, $\forall e \in \mathcal{T}, q \in H,$.
2) $E(\chi, \mathcal{T}) = 1 \Leftrightarrow \upsilon_{\chi(e)}(q) = 0.5$ and $\omega^j_\chi(e)(q) = \pi$, $\forall e \in \mathcal{T}, q \in H$.
3) $E(\chi, \mathcal{T}) = E(\chi, \mathcal{T})^c$.
4) if $(\chi, \mathcal{T}) \subseteq (\xi, \mathcal{T})$, i.e, $\upsilon_{\chi(e)}(q) \leq \upsilon_{\xi(e)}(q)$ and $\omega_{\chi(e)}(q) \leq \omega_{\xi(e)}(q), e \in E, q \in H$, then $E(\chi, \mathcal{T}) \geq E(\xi, \mathcal{T})$.

Theorem 1: Let $H = \{c_1, c_2, \ldots, c_g\}$ be the set of elements and $\mathcal{T}$ be the set of parameters. Then $(\chi, \mathcal{T}) = \{\mathcal{T}(e) = r_{\chi(e)}(b).e^{i\omega_{\chi(e)}}(b)|l = 1, 2, 3, \ldots, m\}$, where $e \in \mathcal{T}$, is a class of *CFHSS*. Consider $E(\chi, \mathcal{T})$ given in such a way:

$$E(\chi, \mathcal{T}) = \frac{1}{2m}\Sigma_{l=1}^{m}[E_l^r(\chi, \mathcal{T}) + E_l^\omega\frac{(\chi, \mathcal{T})}{2\pi}], \quad (1)$$

here,

$$E_l^r(\chi, \mathcal{T}) = \frac{1}{n}\Sigma_{p=1}^{n}[1 - |\upsilon_{\chi(e_l)}(c_g) - \upsilon_{\chi^c(e_l)}(c_g)|], \quad (2)$$

and

$$E_l^\omega(\chi, \mathcal{T}) = \frac{1}{n}\Sigma_{p=1}^{n}[1 - |\omega_{\chi(e_l)}(c_g) - \omega_{\chi^c(e_l)}(c_g)|] \quad (3)$$

then $E(\chi, \mathcal{T})$ is an ENT of *CFHSS*.

Proof: This step illustrates that the $E(\chi, \mathcal{T})$ satisfies all of the conditions in requirement 7.

1) $$E(\chi, \mathcal{T}) = 0, \Leftrightarrow \frac{1}{2m}\Sigma_{l=1}^{m}[E_l^r(\chi, \mathcal{T}) + E_l^\omega\frac{(\chi, \mathcal{T})}{2\pi}] = 0, \Leftrightarrow E_l^r(\chi, \mathcal{T}) = 0$$

and

$$E_l^\omega(\chi, \mathcal{T}) = 0 \Leftrightarrow \forall\ e_l \in \mathcal{T}, c_g \in H, \quad \Sigma_{p=1}^{n}[1 - |\upsilon_{\chi(e_l)}(c_g) - \upsilon_{\chi^c(e_l)}(c_g)|] = 0,$$

and

$$\Sigma_{p=1}^{n}[1 - |\omega_{\chi(e_l)}(c_g) - \omega_{\chi^c(e_l)}(c_g)|] = 0, \Leftrightarrow \forall\ e_l \in \mathcal{T}, c_g \in H, |\upsilon_{\chi(e_l)}(c_g) - \upsilon_{\chi^c(e_l)}(c_g) = 1, |\omega_{\chi(e_l)}(c_g) - \omega_{\chi^c(e_l)}(c_g)| = 2\pi, \Leftrightarrow \forall\ e_l \in \mathcal{T}, c_g \in H, \upsilon_{\chi(e_l)}(c_g) = 1, \omega_{\chi(e_l)}(c_g) = 2\pi,$$

2) For $(\chi, \mathcal{T}) \in CFSS(H)$, we have $E(\chi, \mathcal{T}) = 1$,

$$\Sigma_{l=1}^{m}[E_l^r(\chi, \mathcal{T}) + E_l^\omega\frac{(\chi, \mathcal{T})}{2\pi}] = 2m, \Leftrightarrow E_l^r(\chi, \mathcal{T}) = 1,$$

and

$$E_l^\omega(\chi, \mathcal{T})] = 2\pi, \Leftrightarrow \forall\ e_l \in \mathcal{T}, c_g \in H, \frac{1}{n}\Sigma_{p=1}^{n}[1 - |\upsilon_{\chi(e_l)}(c_g) - \upsilon_{\chi^c(e_l)}(c_g)|] = 1,$$

and

$$\frac{1}{n}\Sigma_{p=1}^{n}[1 - |\omega_{\chi(e_l)}(c_g) - \omega_{\chi^c(e_l)}(c_g)|] = 2\pi, \Leftrightarrow \forall\ e_l \in \mathcal{T}, c_g \in H, \Sigma_{p=1}^{n}[1 - |\upsilon_{\chi(e_l)}(c_g) - \upsilon_{\chi^c(e_l)}(c_g)|] = n,$$

and

$$\Sigma_{p=1}^{n}[2\pi - |\omega_{\chi(e_l)}(c_g) - \omega_{\chi^c(e_l)}(c_g)|] = 2\pi(n), \Leftrightarrow \forall\ e_l \in \mathcal{T}, c_g \in H, [1 - |\upsilon_{\chi(e_l)}(c_g) - \upsilon_{\chi^c(e_l)}(c_g)|] = 1,$$

and

$$[2\pi - |\omega_{\chi(e_l)}(c_g) - \omega_{\chi^c(e_l)}(c_g)|] = 2\pi, \Leftrightarrow \forall\ e_l \in \mathcal{T}, c_g \in H, |\upsilon_{\chi^c(e_l)}(c_g) - \upsilon_{\chi(e_l)}(c_g)| = 0,$$

and

$$|\omega_{\chi^c(e_l)}(c_g) - \omega_{\chi(e_l)}(c_g)| = 0, \Leftrightarrow \forall\ e_l \in \mathcal{T}, c_g \in H,, \upsilon_{\chi(e_l)}(c_g) = \frac{1}{2}$$

and

$$\omega_{\chi(e_l)}(c_g) = \pi,$$

3) For $E(\chi, \mathcal{T}) \in CFSS(H)$,
we have,

$$E_l^r(\chi, \mathcal{T}) = \frac{1}{n}\Sigma_{p=1}^{n}[1 - |\upsilon_{\chi(e_l)}(c_g) - \upsilon_{\chi^c(e_l)}(c_g)|],$$

$$\frac{1}{n}\Sigma_{p=1}^{n}[1-|\upsilon_{\chi^{c}(e_l)}(c_g)-\upsilon_{\chi(e_l)}(c_g)|],$$
$$=E_l^r(\chi,\mathcal{T})^c,$$

Similarly, we show that $E_l^r(\chi,\mathcal{T}) = E_l^r(\chi,\mathcal{T})^c$ it is clear that $E(\chi,\mathcal{T}) = E(\chi,\mathcal{T})^c$.

4) Assume $(\chi,\mathcal{T})$ and $(\xi,\mathcal{T}) \in CFSS(H)$. If $(\chi,\mathcal{T}) \subseteq (\xi,\mathcal{T}), \Rightarrow \forall\ e_l \in \mathcal{T}, b \in H, , \upsilon_{\chi(e_l)}(c_g) \leq \upsilon_{\xi(e_l)}(c_g)$ and $\omega_{\chi(e_l)}(c_g) \leq \omega_{\xi(e_l)}(c_g) \Rightarrow \forall\ e_l \in \mathcal{T}, b \in H,$ $|\upsilon_{\chi(e_l)}(c_g)-\upsilon_{\chi^c(e_l)}(c_g)| \leq |\upsilon_{\xi(e_l)}(c_g)-\upsilon_{\xi^c(e_l)}(c_g)|,$ and

$$|\omega_{\chi(e_l)}(c_g)-\omega_{\chi^c(e_l)}(c_g)|$$
$$\leq |\omega_{\xi(e_l)}(c_g)-\omega_{\xi^c(e_l)}(c_g)|,$$
$$\Rightarrow \forall e_l \in \mathcal{T}, b \in H,$$
$$1-|\upsilon_{\chi(e_l)}(c_g)-\upsilon_{\chi^c(e_l)}(c_g)|$$
$$\geq 1-|\upsilon_{\xi(e_l)}(c_g)-\upsilon_{\xi^c(e_l)}(c_g)|,$$

and

$$2\pi-|\omega_{\chi(e_l)}(c_g)-\omega_{\chi^c(e_l)}(c_g)|$$
$$\geq 2\pi-|\omega_{\xi(e_l)}(c_g)-\omega_{\xi^c(e_l)}(c_g)|,$$
$$\Rightarrow \frac{1}{n}\Sigma_{p=1}^{n}([1-|r_{\chi(e_l)}(c_g)-\upsilon_{\chi^c(e_l)}(c_g)|]),$$
$$\geq \frac{1}{n}\Sigma_{p=1}^{n}([1-|\upsilon_{\xi(e_l)}(c_g)-\upsilon_{\xi^c(e_l)}(c_g)|]),$$

and

$$\frac{1}{n}\Sigma_{p=1}^{n}([2\pi-|\omega_{\chi(e_l)}(c_g)-\omega_{\chi^c(e_l)}(c_g)|])$$
$$\geq \frac{1}{n}\Sigma_{p=1}^{n}([2\pi-|\omega_{\xi(e_l)}(c_g)-\omega_{\xi^c(e_l)}(c_g)|]),$$
$$\Rightarrow E_l^r(\chi,\mathcal{T}) \geq E_l^r(\xi,\mathcal{T}),$$

and

$$\Rightarrow E_l^{\omega}(\chi,\mathcal{T}) \geq E_l^{\omega}(\xi,\mathcal{T}),$$
$$\Rightarrow E_l^r(\chi,\mathcal{T})+E_l^{\omega}(\chi,\mathcal{T}) \geq E_l^r(\xi,\mathcal{T})+E_l^{\omega}(\xi,\mathcal{T}),$$
$$\Rightarrow \frac{1}{2m}\Sigma_{l=1}^{m}[E_l^r(\chi,\mathcal{T})+E_l^{\omega}\frac{(\chi,\mathcal{T})}{2\pi}]$$
$$\geq \frac{1}{2m}\Sigma_{l=1}^{m}[E_l^r(\xi,\mathcal{T})+E_l^{\omega}\frac{(\xi,\mathcal{T})}{2\pi}],$$
$$\Rightarrow E(\chi,\mathcal{T}) \geq E(\xi,\mathcal{T}).$$

A. RANK THE FCH STRATEGY USING THE SPECIFIED APPROACH

This paper assesses different FCH methods using the preferred ENT-based CFHSS judging approach. We evaluated the proposed approach to various previous studies to establish the durability and usefulness of the functioning setup.

B. INVESTIGATION OF THE FCH APPROACH AND ITS COMPONENTS

Analytic FCH analysis and computational mathematics have a substantial environmental impact. There are three kinds of FCH techniques that are described.

- Polymer electrolyte membrane electrolysis
- Alkaline fuel cell
- Incineration

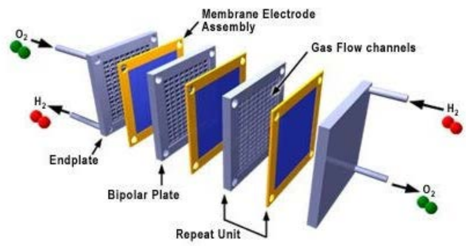

FIGURE 2. An expansive view of a polymer electrolyte membrane fuel cell stack. ***Source***: https://www.fuelcellstore.com/blog-section/mathematical-modeling-pem-membrane-dmfc.

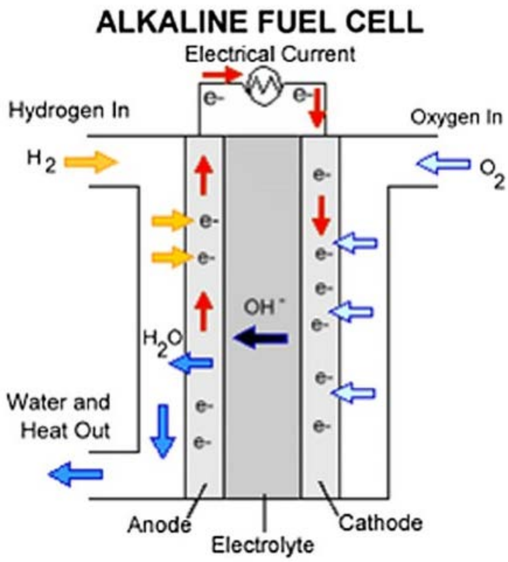

FIGURE 3. Schematic representation of an alkaline fuel cell (AFC). ***Source***: https://sustainablechemicalprocesses.springeropen.com/articles/10.1186/2043-7129-1-16.

1) POLYMER ELECTROLYTE MEMBRANE ELECTROLYSIS

Polymer electrolyte membrane (PEM) electrolysis is the electrolysis of water in a cell equipped with a solid polymer electrolyte (SPE) responsible for the conduction of protons, separation of product gases, and electrical insulation of the electrodes. The PEM electrolyzer was introduced to overcome partial load issues, low current density, and low-pressure operation currently plaguing the alkaline electrolyzer. For more detail see Fig 2.

2) ALKALINE FUEL CELL

The alkaline fuel cell (AFC), also known as the Bacon fuel cell after its British inventor, Francis Thomas Bacon, is one of the most developed fuel cell technologies. Alkaline fuel cells consume hydrogen and pure oxygen to produce potable water, heat, and electricity. They are among the most efficient fuel cells, potentially reaching 70 percent. For more detail see Fig 3.

3) DIRECT METHANOL FUEL CELLS

Direct-methanol fuel cells, or DMFCs, are proton-exchange fusion reactors that use methanol as fuel. Their primary aspect

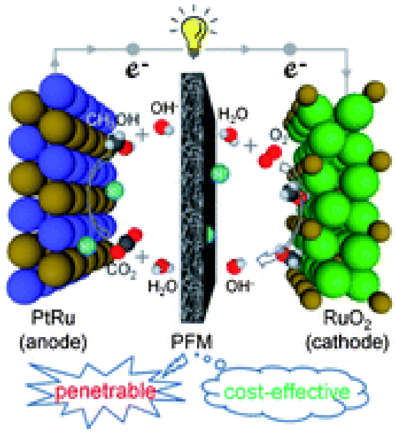

FIGURE 4. **A high-performance direct methanolfuel cell. *Source*: https://pubs.rsc.org/en/content/articlelanding/2013/ta/c2ta00095d.**

is the easiness with which methyl, a power generation but moderately robust solvent in all ecological parameters, can be transferred. For more detail see Fig 4.

4) ALGORITHM

Let $H = \{x_1, x_2, \ldots, x_m\}$ be the configuration of possibilities under review. Let $\mathcal{T} = G_1 \times G_2 \times \cdots \times G_n$ be sub values of attributes, the algorithm flow chart can be seen in 6.

1) The data sets in the form of CFHSS must be mentioned.
2) Find ENT by using $E(\chi, \mathcal{T}) = \frac{1}{2m}\Sigma_{l=1}^{m}[E_l^r(\chi, \mathcal{T}) + E_l^{\omega}\frac{(\chi, \mathcal{T})}{2\pi}]$, where $E_l^r(\chi, \mathcal{T}) = \frac{1}{n}\Sigma_{p=1}^{n}[1 - |\upsilon_{\chi(e_l)}(c_g) - \upsilon_{\chi^c(e_l)}(c_g)|]$, and $E_l^{\omega}(\chi, \mathcal{T}) = \frac{1}{n}\Sigma_{p=1}^{n}[1 - |\omega_{\chi(e_l)}(c_g) - \omega_{\chi^c(e_l)}(c_g)|]$.
3) Select minimal possible ENT.
4) Choose any of the maximums if it achieved more than one.

Example 2: Iran has a high hydrogen prospect due to its vast fossil fuels and diverse FCH infrastructure. Since the early 1990s, the administration has adopted crucial measures toward testing and building the FCH enterprise, emphasizing the transportation and industrial plant enterprises. The innovation is influenced by three main characters: government agencies, academic institutions, higher education institutions, and the corporate industry. The clean energy government organization manages and directs the actors' basic research (SUNA). SUNA, which was investigated in this work, is at the heart of most activities integrating Iran's sustainable energy facilities. SUNA has acknowledged basic strategies toward utilizing the FCH sector for emissions reduction, with the guideline plan concentrating solely on commuting and ancillary nuclear plant enterprises. SUNA has been the foremost FC supplier for these actors throughout the last decade, endorsed by corporates. SUNA preserves the requisite laboratory and trimming samples, support equipment, and FC related parts sorted by other cast members through east Asian and European conglomerates.

In the FCH domain, SUNA's properly functioning approach presents integration between actors, providers, and other national bodies [32]. The foundation's outlook on procurement management is short and long, but it does not rule out the potential of a long-term partnership. Even though several sorts of FCs are now being developed, participants are most often concerned with electrolytic (PEMFC) and supercapacitors (SOFC). Although Iran's FCH shows some common is still in its early phases (RD and implementation phase), the research focuses on improving the accuracy and survival of FC stacked. A trained provider capable of addressing the objectives of the participants enhances RD.

The established PF-Entropy-SWARA-COPRAS framework was used to appraise the FCH component suppliers in Iran, exhibiting the effectiveness and operability of the suggested scheme. To that end, the SUNA constituted a committee of panellists to interact with the life choice issue. The expert group started their work by expecting and explaining the evaluation methods.

Assume an industry that has three FCH's (Polymer electrolyte membrane, Alkaline, Direct methanol fuel cells), and three experts $X = \{x, y, z\}$, let a_1 = Economic, a_2 = Technology capability, a_3 = Organisation be different attributes with corresponding attribute values that are components of the groupings belongs to Q_1, Q_2, Q_3. The industry wants to choose the best optimal alternatives for FCH. Let $Q_1 = \{\eta_1 =$ Cost of commodity components, $\eta_2 =$ Cost of specialty components$\}$, $Q_2 = \{\eta_3 =$ Information disclosure$\}$, $Q_3 = \{\eta_4 =$ Green strategy direction, $\eta_5 =$ Degree of strategic cooperation$\}$. CEO can transmit this evidence in the form of CFHSS with the assistance of decision makers. $(\chi, \mathcal{T})$, $(\xi, \mathcal{T})$ and $(\psi, \mathcal{T})$ respectively, where $0 \leq \alpha \leq 2\pi$.

1) This is plausible with public approval.

$$\text{Polymer electrolyte membrane} = (\chi, \mathcal{T}) = \Bigg\{\chi(\eta_1, \eta_3, \eta_4) = \Bigg\{\frac{(0.3e^{i0.4\alpha})}{x}, \frac{(0.8e^{i0.2\alpha})}{y}, \frac{(0.8e^{i0.2\alpha})}{z}\Bigg\},$$

$$\Bigg\{\chi(\eta_1, \eta_3, \eta_5) = \Bigg\{\frac{(0.3e^{i0.3\alpha})}{x}, \frac{(0.8e^{i0.2\alpha})}{y}, \frac{(0.2e^{i0.9\alpha})}{z}\Bigg\},$$

$$\Bigg\{\chi(\eta_2, \eta_3, \eta_4) = \Bigg\{\frac{(0.3e^{i0.9\alpha})}{x}, \frac{(0.2e^{i0.4\alpha})}{y}, \frac{(0.9e^{i0.3\alpha})}{z}\Bigg\}$$

$$\Bigg\{\chi(\eta_2, \eta_3, \eta_5) = \Bigg\{\frac{(0.3e^{i0.5\alpha})}{x}, \frac{(0.3e^{i0.4\alpha})}{y}, \frac{(0.4e^{i0.7\alpha})}{z}\Bigg\}\Bigg\},$$

$$\text{Alkaline} = (\xi, \mathcal{T}) = \left\{\chi(\eta_1, \eta_3, \eta_4) = \left\{\frac{(0.2e^{i0.7\alpha})}{x}, \frac{(0.7e^{i0.5\alpha})}{y}, \frac{(0.4e^{i0.3\alpha})}{z}\right\},\right.$$

$$\left\{\chi(\eta_1, \eta_3, \eta_5) = \left\{\frac{(0.6e^{i0.9\alpha})}{x}, \frac{(0.2e^{i0.8\alpha})}{y}, \frac{(0.3e^{i0.6\alpha})}{z}\right\},\right.$$

$$\left\{\chi(\eta_2, \eta_3, \eta_4) = \left\{\frac{(0.3e^{i0.9\alpha})}{x}, \frac{(0.8e^{i0.5\alpha})}{y}, \frac{(0.6e^{i0.9\alpha})}{z}\right\}\right.$$

$$\left.\left\{\chi(\eta_2, \eta_3, \eta_5) = \left\{\frac{(0.6e^{i0.5\alpha})}{x}, \frac{(0.3e^{i0.4\alpha})}{y}, \frac{(0.4e^{i0.7\alpha})}{z}\right\}\right\}\right\},$$

$$\text{Direct methanol fuel cells} = (\psi, \mathcal{T}) = \left\{\chi(\eta_1, \eta_3, \eta_4) = \left\{\frac{(0.4e^{i0.8\alpha})}{x}, \frac{(0.2e^{i0.9\alpha})}{y}, \frac{(0.3e^{i0.2\alpha})}{z}\right\},\right.$$

$$\left\{\chi(\eta_1, \eta_3, \eta_5) = \left\{\frac{(0.7e^{i0.2\alpha})}{x}, \frac{(0.5e^{i0.2\alpha})}{y}, \frac{(0.2e^{i0.6\alpha})}{z}\right\},\right.$$

$$\left\{\chi(\eta_2, \eta_3, \eta_4) = \left\{\frac{(0.5e^{i0.9\alpha})}{x}, \frac{(0.8e^{i0.6\alpha})}{y}, \frac{(0.2e^{i0.9\alpha})}{z}\right\}\right.$$

$$\left.\left\{\chi(\eta_2, \eta_3, \eta_5) = \left\{\frac{(0.2e^{i0.5\alpha})}{x}, \frac{(0.5e^{i0.4\alpha})}{y}, \frac{(0.2e^{i0.5\alpha})}{z}\right\}\right\}\right\}.$$

2) Calculating the Entropies of $(\chi, \mathcal{T})$, $(\xi, \mathcal{T})$ and $(\psi, \mathcal{T})$ employing the algorithm's methodology, see Table 1. Hence the Entropies of the *CFSSs* $(\chi, \mathcal{T})$, $(\xi, \mathcal{T})$ and $(\psi, \mathcal{T})$ are underlying as $E(\chi, \mathcal{T}) = 0.31$, $E(\xi, \mathcal{T}) = 0.4148$, $E(\psi, \mathcal{T}) = 0.42$ respectively.
3) The ideal solution is to use $((\chi, \mathcal{T})$ as the minimum amount of ENT.
4) Polymer electrolyte membrane is best FCH strategy.

C. COMPARATIVE STUDIES

Some reviews of the methods and techniques with imperfections are explored to assess the suggested algorithm's validity and dominance. Besides, we compare our ENT-based CFHSS to nine distinct entropies currently being used. Szmidt *et al.* [49] proposed ENT for the non-probabilistic measure. Majumdar *et al.* [50] investigated ENT using a neutrosophic set, Zhang *et al.* [29] provided the explanation of ENT for the IVFSS set, and ENT for interval-valued neutrosophic sets developed by Ye *et al.* [51], and the concept of ENT and SM for interval-valued neutrosophic sets developed by Aydodu *et al.* [52]. Lvqing *et al.* [55] displayed an accusation focused on classes of ENT-based complex FS. The entropies are based on complex intuitionistic FSS made by Kumar *et al.* [56], as well as the suggestion made by Athira *et al.* [54] based on ENT for Pythagorean FSS. The concept recommended by Selvachandran *et al.* [57] for intelligence relies on a confusingly worded ENT measure; when the features are further subdivided into input variables and concerns that contain two-dimensional information, all constraints as mentioned above are eliminated. The predicted ENT-based CFHSS will encounter this requirement. See 5 for more information.

TABLE 1. Entropies.

$E_1^r(\chi, \mathcal{T})$	0.46
$E_2^r(\chi, \mathcal{T})$	0.46
$E_3^r(\chi, \mathcal{T})$	0.4
$E_4^r(\chi, \mathcal{T})$	0.66
$E_1^\omega(\chi, \mathcal{T})$	0.53
$E_2^\omega(\chi, \mathcal{T})$	0.933
$E_3^\omega(\chi, \mathcal{T})$	1.066
$E_4^\omega(\chi, \mathcal{T})$	1.066
$E_1^r(\xi, \mathcal{T})$	0.6
$E_2^r(\xi, \mathcal{T})$	0.6
$E_3^r(\xi, \mathcal{T})$	0.6
$E_4^r(\xi, \mathcal{T})$	0.73
$E_1^\omega(\xi, \mathcal{T})$	1
$E_2^\omega(\xi, \mathcal{T})$	1.533
$E_3^\omega(\xi, \mathcal{T})$	1.533
$E_4^\omega(\xi, \mathcal{T})$	1.066
$E_1^r(\psi, \mathcal{T})$	0.6
$E_2^r(\psi, \mathcal{T})$	0.6
$E_3^r(\psi, \mathcal{T})$	0.6
$E_4^r(\psi, \mathcal{T})$	0.6
$E_1^\omega(\psi, \mathcal{T})$	1.26
$E_2^\omega(\psi, \mathcal{T})$	0.66
$E_3^\omega(\psi, \mathcal{T})$	1.6
$E_4^\omega(\psi, \mathcal{T})$	2.51

IV. SIMILARITY MEASURE AMONG CFHS-SETS

SM denotes the relationship between various forms, images, or configurations. These types of indices are frequently used in FSS proposals. The SM derivation of *CFHSS* is as follows.

Definition 8: A function from $S : CFHSS(H) \times CFHSS(H) \to [0, 1]$ is called SM, if S passes the conceptual constraints specified here:

TABLE 2. The planned ENT-based CFHSS is compared with the conventional entropies.

SN	References	Entropies	Ranking
1	[49]	assertion is incorrect	$\times$
2	[29]	assertion is incorrect	$\times$
3	[50]	assertion is incorrect	$\times$
4	[51]	assertion is incorrect	$\times$
5	[52]	assertion is incorrect	$\times$
6	[54]	assertion is incorrect	$\times$
7	[55]	assertion is incorrect	$\times$
8	[56]	assertion is incorrect	$\times$
9	[57]	assertion is incorrect	$\times$
10	Proposed Method in this paper	$E(\chi,\mathcal{T}) = 0.31$, $E(\xi,\mathcal{T}) = 0.4148$, $E(\psi,\mathcal{T}) = 0.42$	$E(\chi,\mathcal{T}) \geq E(\xi,\mathcal{T}) \geq E(\psi,\mathcal{T})$

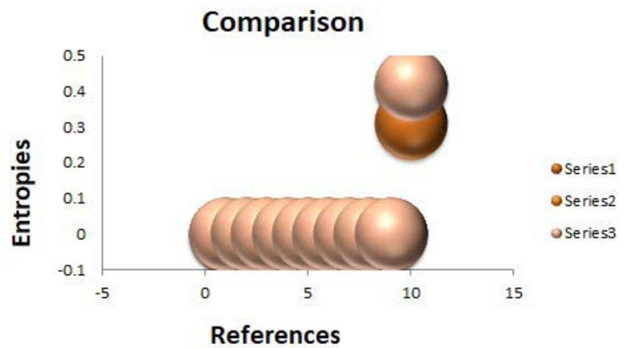

FIGURE 5. The recommended ENT-based CFHSS is evaluated by comparing to known entropies.

1) $S((\chi,\mathcal{T}),(\xi,\mathcal{T})) = S((\xi,\mathcal{T}),(\chi,\mathcal{T}))$,
2) $S((\chi,\mathcal{T}),(\xi,\mathcal{T})) = 1 \Leftrightarrow (\chi,\mathcal{T}) = (\xi,\mathcal{T})$,
3) $S((\chi,\mathcal{T}),(\xi,\mathcal{T})) = 0 \Leftrightarrow \forall\ e \in \mathcal{T},\ x \in H,$, the the following rules are met: $\upsilon_{\chi(e)} = 1, \upsilon_{\xi(e)} = 0$ or $\upsilon_{\chi(e)} = 0, \upsilon_{\xi(e)} = 1$ and $\omega_{\chi(e)} = 2\pi, \omega_{\xi(e)} = 0$ or $\omega_{\chi(e)} = 0, \omega_{\xi(e)} = 2\pi$,
4) $\forall\ (\chi,\mathcal{T}),(\xi,\mathcal{T})$ and $(\psi,\mathcal{T}) \in$ *CFHSS*, if $(\chi,\mathcal{T}) \subseteq (\xi,\mathcal{T}) \subseteq (\psi,\mathcal{T})$, then $S((\chi,\mathcal{T}),(\psi,\mathcal{T})) \leq S((\chi,\mathcal{T}),(\xi,\mathcal{T}))$ and $S((\chi,\mathcal{T}),(\psi,\mathcal{T})) \leq S((\xi,\mathcal{T}),(\psi,\mathcal{T}))$.

Theorem 2: Let $H = \{c_1, c_2, \ldots, c_g\}$ be the alternatives and $\mathcal{T}$ the set of parameters.

$(\chi,\mathcal{T}) = \{\mathcal{T}(e) = \upsilon_{\chi(e)}(b).e^{i\omega_{\chi(e)}}(b)|\}$, and $(\xi,\mathcal{T}) = \{\mathcal{T}(e) = \upsilon_{\xi(e)}(b).e^{i\omega_{\xi(e)}}(b)|\}$, are two families of *CFHSS*.

Define $S((\chi,\mathcal{T}),(\xi,\mathcal{T}))$ as follows,

$$S((\chi,\mathcal{T}),(\xi,\mathcal{T})) = \frac{1}{2m}\Sigma_{l=1}^{m}[S_l^r((\chi,\mathcal{T}),(\xi,\mathcal{T})) + \frac{S_l^{\omega}((\chi,\mathcal{T}),(\xi,\mathcal{T}))}{2\pi}], \tag{4}$$

where,

$$S_{l=1}^r((\chi,\mathcal{T}),(\xi,\mathcal{T})) = 1 - \frac{1}{n}\Sigma_{l=1}^{n} max\{(|\upsilon_{\chi(e)}(c_g) - r_{\xi(e)}(c_g)|)\}, \tag{5}$$

and

$$S_{l=1}^{\omega}((\chi,\mathcal{T}),(\xi,\mathcal{T})) = 2\pi - \frac{1}{n}\Sigma_{l=1}^{n} max\{(|\omega_{\chi(e)}(c_g) - \omega_{\xi(e)}(c_g)|)\}. \tag{6}$$

then $S((\chi,\mathcal{T}),(\xi,\mathcal{T}))$ is a SM between two *CFHSS* $(\chi,\mathcal{T})$ and $(\xi,\mathcal{T})$.

Proof: It is worth emphasizing that $S((\chi,\mathcal{T}),(\xi,\mathcal{T}))$ meet the definition's identified attributes 8.

1) For

$$\begin{aligned} S_{l=1}^r((\chi,\mathcal{T}),(\xi,\mathcal{T})) &= 1 - \frac{1}{n}\Sigma_{p=1}^{n} max\{(|\upsilon_{\chi(e_l)}(c_g) - \upsilon_{\xi(e_l)}(c_g)|)\},\\ &= 1 - \frac{1}{n}\Sigma_{p=1}^{n} max\{(|\upsilon_{\xi(e_l)}(c_g) - \upsilon_{\chi(e_l)}(c_g)|)\}\\ &= S_{l=1}^r((\xi,\mathcal{T}),(\chi,\mathcal{T})), \end{aligned}$$

and

$$\begin{aligned} S_{l=1}^{\omega}((\chi,\mathcal{T}),(\xi,\mathcal{T})) &= 2\pi - \frac{1}{n}\Sigma_{p=1}^{n} max\{(|\omega_{\chi(e_l)}(c_g) - \omega_{\xi(e_l)}(c_g)|)\},\\ &= 2\pi - \frac{1}{n}\Sigma_{p=1}^{n} max\{(|\omega_{\xi(e_l)}(c_g) - \omega_{\chi(e_l)}(c_g)|)\}\\ &= S_{l=1}^{\omega}((\xi,\mathcal{T}),(\chi,\mathcal{T})), \end{aligned}$$

So we have

$$\begin{aligned} S((\chi,\mathcal{T}),(\xi,\mathcal{T})) &= \frac{1}{2m}\Sigma_{l=1}^{m}[S_l^r((\chi,\mathcal{T}),(\xi,\mathcal{T})) + \frac{S_l^{\omega}((\chi,\mathcal{T}),(\xi,\mathcal{T}))}{2\pi}],\\ &= \frac{1}{2m}\Sigma_{l=1}^{m}[S_l^r((\xi,\mathcal{T}),(\chi,\mathcal{T})) + \frac{S_l^{\omega}((\xi,\mathcal{T}),(\chi,\mathcal{T}))}{2\pi}]\\ &= S((\xi,\mathcal{T}),(\chi,\mathcal{T})). \end{aligned}$$

2)

$$\begin{aligned} S((\chi,\mathcal{T}),(\xi,\mathcal{T})) &= 1\\ \Leftrightarrow \frac{1}{2m}\Sigma_{l=1}^{m}[S_l^r((\chi,\mathcal{T}),(\xi,\mathcal{T})) &+ \frac{S_l^{\omega}((\chi,\mathcal{T}),(\xi,\mathcal{T}))}{2\pi}] = 1, \end{aligned}$$

$$\Leftrightarrow S_l^r((\chi,\mathcal{T}),(\xi,\mathcal{T}))=1,$$
$$\Leftrightarrow S_l^\omega((\chi,\mathcal{T}),(\xi,\mathcal{T}))=2\pi,$$
$$\Leftrightarrow S_{l=1}^r((\chi,\mathcal{T}),(\xi,\mathcal{T}))=1 - \frac{1}{n}\Sigma_{p=1}^n max\{(|\upsilon_{\chi(e_l)}(c_g)-\upsilon_{\xi(e_l)}(c_g)|)\},$$
$$2\pi-\frac{1}{n}\Sigma_{p=1}^n max\{(|\omega_{\chi(e_l)}(c_g)-\omega_{\xi(e_l)}(c_g)|)\} = 2\pi,$$
$$\forall e_l\in\mathcal{T}, b\in H,$$
$$\Leftrightarrow \frac{1}{n}\Sigma_{p=1}^n max\{(|\upsilon_{\chi(e_l)}(c_g)-\upsilon_{\xi(e_l)}(c_g)|)=0,$$

and

$$\Leftrightarrow \frac{1}{n}\Sigma_{p=1}^n max\{(|\omega_{\chi(e_l)}(c_g)-\omega_{\xi(e_l)}(c_g)|)=0,$$
$$\forall e_l\in\mathcal{T}, b\in H,$$
$$\Leftrightarrow \Sigma_{p=1}^n max\{(|\upsilon_{\chi(e_l)}(c_g)-\upsilon_{\xi(e_l)}(c_g)|)=0,$$
$$\Leftrightarrow \Sigma_{p=1}^n max\{(|\omega_{\chi(e_l)}(c_g)-\omega_{\xi(e_l)}(c_g)|)=0,$$
$$\forall e_l\in\mathcal{T}, b\in H,$$
$$\Leftrightarrow \upsilon_{\chi(e_l)}(c_g)=\upsilon_{\xi(e_l)}(c_g),$$
$$\Leftrightarrow \omega_{\chi(e_l)}(c_g)=\omega_{\xi(e_l)}(c_g),$$
$$\forall e_l\in\mathcal{T}, b\in H,$$
$$\Leftrightarrow (\chi,\mathcal{T})=(\xi,\mathcal{T}).$$

3)

$$S((\chi,\mathcal{T}),(\xi,\mathcal{T}))=0,$$
$$\frac{1}{2m}\Sigma_{l=1}^m[S_l^r((\chi,\mathcal{T}),(\xi,\mathcal{T})) + \frac{S_l^\omega((\chi,\mathcal{T}),(\xi,\mathcal{T}))}{2\pi}]=0,$$
$$\Leftrightarrow S_l^r((\chi,\mathcal{T}),(\xi,\mathcal{T}))=0,$$

and

$$\Leftrightarrow S_l^\omega((\chi,\mathcal{T}),(\xi,\mathcal{T}))=0,$$
$$\Leftrightarrow 1-\frac{1}{n}\Sigma_{p=1}^n max\{(|\upsilon_{\chi(e_l)}(c_g)-\upsilon_{\xi(e_l)}(c_g)|)\}=0,$$

and

$$\Leftrightarrow 2\pi-\frac{1}{n}\Sigma_{p=1}^n max\{(|\omega_{\chi(e_l)}(c_g)-\omega_{\xi(e_l)}(c_g)|)\}=0,$$
$$\forall e_l\in\mathcal{T}, b\in H,$$
$$\Leftrightarrow \frac{1}{n}\Sigma_{p=1}^n max\{(|\upsilon_{\chi(e_l)}(c_g)-\upsilon_{\xi(e_l)}(c_g)|)\}=1,$$
$$\Leftrightarrow \frac{1}{n}\Sigma_{p=1}^n max\{(|\omega_{\chi(e_l)}(c_g)-\omega_{\xi(e_l)}(c_g)|)\}=2\pi,$$
$$\forall e_l\in\mathcal{T}, b\in H,$$
$$\Leftrightarrow max\{(|\upsilon_{\chi(e_l)}(c_g)-\upsilon_{\xi(e_l)}(c_g)|)\}=1,$$

and

$$\Leftrightarrow max\{(|\omega_{\chi(e_l)}(c_g)-\omega_{\xi(e_l)}(c_g)|)\}=2\pi,$$
$$\forall e_l\in\mathcal{T}, b\in H,$$
$$\Leftrightarrow \upsilon_{\chi(e_l)}=0, \upsilon_{\xi(e_l)}=1, \upsilon_{\chi(e_l)}=1, \upsilon_{\xi(e_l)}=0$$

and $\omega_{\chi(e_l)}=0, \omega_{\xi(e_l)}=2\pi$ or $\omega_{\chi(e_l)}=2\pi, \omega_{\xi(e_l)}=0$.

4)

$$(\chi,\mathcal{T})\subseteq(\xi,\mathcal{T})\subseteq(\psi,\mathcal{T}),$$
$$\Rightarrow \upsilon_{\chi(e_l)}(c_g)\le\upsilon_{\xi(e_l)}(c_g)\le\upsilon_{\psi(e_l)}(c_g)$$

and

$$\omega_{\chi(e_l)}(c_g)\le\omega_{\xi(e_l)}(c_g)\le\omega_{\psi(e_l)}(c_g),$$
$$\forall e_l\in\mathcal{T}, b\in H,$$
$$\Rightarrow |\upsilon_{\chi(e_l)}(c_g)-\upsilon_{\psi(e_l)}(c_g)|\le|\upsilon_{\chi(e_l)}(c_g)-r_{\xi(e_l)}(c_g)|,$$

and

$$\Rightarrow |\omega_{\chi(e_l)}(c_g)-\omega_{\psi(e_l)}(c_g)| \le |\omega_{\chi(e_l)}(c_g)-\omega_{\xi(e_l)}(c_g)|,$$
$$\forall e_l\in\mathcal{T}, b\in H,$$
$$\Leftrightarrow 1-\frac{1}{n}\Sigma_{p=1}^n max\{(|\upsilon_{\chi(e_l)}(c_g)-\upsilon_{\psi(e_l)}(c_g)|)\} \le 1-\frac{1}{n}\Sigma_{p=1}^n max\{(|\upsilon_{\chi(e_l)}(c_g)-\upsilon_{\xi(e_l)}(c_g)|)\},$$
$$\Leftrightarrow 2\pi-\frac{1}{n}\Sigma_{p=1}^n max\{(|\omega_{\chi(e_l)}(c_g)-\omega_{\psi(e_l)}(c_g)|)\} \le 2\pi-\frac{1}{n}\Sigma_{p=1}^n max\{(|\omega_{\chi(e_l)}(c_g)-\omega_{\xi(e_l)}(c_g)|)\},$$
$$\Rightarrow S_{l=1}^r((\chi,\mathcal{T}),(\psi,\mathcal{T}))\le S_{l=1}^r((\chi,\mathcal{T}),(\xi,\mathcal{T})),$$

and

$$\Rightarrow S_{l=1}^\omega((\chi,\mathcal{T}),(\psi,\mathcal{T}))\le S_{l=1}^\omega((\chi,\mathcal{T}),(\xi,\mathcal{T})),$$
$$\Rightarrow S_{l=1}^r((\chi,\mathcal{T}),(\psi,\mathcal{T}))+S_{l=1}^\omega((\chi,\mathcal{T}),(\psi,\mathcal{T})) \le S_{l=1}^r((\chi,\mathcal{T}),(\xi,\mathcal{T}))+S_{l=1}^\omega((\chi,\mathcal{T}),(\xi,\mathcal{T})),$$
$$\Rightarrow \frac{1}{2m}\Sigma_{l=1}^m[S_l^r((\chi,\mathcal{T}),(\psi,\mathcal{T})) + \frac{S_l^\omega((\chi,\mathcal{T}),(\psi,\mathcal{T}))}{2\pi}] \le \frac{1}{2m}\Sigma_{l=1}^m[S_l^r((\chi,\mathcal{T}),(\xi,\mathcal{T}))+\frac{S_l^\omega((\chi,\mathcal{T}),(\xi,\mathcal{T}))}{2\pi}],$$
$$S((\chi,\mathcal{T}),(\psi,\mathcal{T}))\le S((\chi,\mathcal{T}),(\xi,\mathcal{T})).$$

A. IMPLEMENTATION OF THE ENVISAGED SM-BASED CFHSS

We expanded SM by applying CFHSS in a blurry predicament. In this subsection, a CFHSS theoretical model is developed to generate a new conceptual model and specific methodology widely recognized as SM-based CFHSS. A practical choosing difficulty is also devised to reflect the importance and imperative of the recently evolved ENT-based CFHSS.

1) ALGORITHM

Let $H=\{c_1,c_2,\ldots,c_m\}$ be the universe. Let $\mathcal{T}=G_1\times G_2\times\cdots\times G_n$, where $n\ge 1$ and Ai is the acquisition of all the concept's characteristics a_i, $i=1,2,3,\ldots,n$. The methodologies for the envisaged CFHSS-based similarity are as outlined, or see fig 7:

1) Input each of the CFHSS.
2) Evaluate the similarity measure for each CFHSS using a strategy, $S((\chi,\mathcal{T}),(\xi,\mathcal{T})) = \frac{1}{2m}\Sigma_{l=1}^{m}[S_l^r((\chi,\mathcal{T}),(\xi,\mathcal{T})) + \frac{S_l^{\omega}((\chi,\mathcal{T}),(\xi,\mathcal{T}))}{2\pi}]$, where $S_{l=1}^r((\chi,\mathcal{T}),(\xi,\mathcal{T})) = 1-\frac{1}{n}\Sigma_{l=1}^{n}max\{(|\upsilon_{\chi(e)}(c_g)-r_{\xi(e)}(c_g)|)\}$, and $S_{l=1}^{\omega}((\chi,\mathcal{T}),(\xi,\mathcal{T})) = 2\pi-\frac{1}{n}\Sigma_{l=1}^{n}max\{(|\omega_{\chi(e)}(c_g)-\omega_{\xi(e)}(c_g)|)\}$.
3) Chose the CFHSS with some of the most similarities.
4) Use one of the optimums if it obtained more than one.

Example 3: From Example 2,

1) The task at hand is to identify the ideal FCH methods consisting of a set of criteria. The CFHSS mindset should be included in the tables below.

$$\text{Alkaline} = (\xi,\mathcal{T}) = \left\{\xi(\eta_1,\eta_3,\eta_4) = \left\{\frac{(0.2e^{i0.7\alpha})}{x}, \frac{(0.7e^{i0.5\alpha})}{y}, \frac{(0.4e^{i0.3\alpha})}{z}\right\},\right.$$
$$\xi(\eta_1,\eta_3,\eta_5) = \left\{\frac{(0.6e^{i0.9\alpha})}{x}, \frac{(0.2e^{i0.8\alpha})}{y}, \frac{(0.3e^{i0.6\alpha})}{z}\right\},$$
$$\xi(\eta_2,\eta_3,\eta_4) = \left\{\frac{(0.3e^{i0.7\alpha})}{x}, \frac{(0.9e^{i0.2\alpha})}{y}, \frac{(0.3e^{i0.6\alpha})}{z}\right\},$$
$$\left.\xi(\eta_2,\eta_3,\eta_5) = \left\{\frac{(0.6e^{i0.9\alpha})}{x}, \frac{(0.6e^{i0.8\alpha})}{y}, \frac{(0.4e^{i0.7\alpha})}{z}\right\}\right\},$$

Polymer electrolyte membrane

$$= (\psi,\mathcal{T}) = \left\{\psi(\eta_1,\eta_3,\eta_4) = \left\{\frac{(0.2e^{i0.7\alpha})}{x}, \frac{(0.3e^{i0.9\alpha})}{y}, \frac{(0.2e^{i0.5\alpha})}{z}\right\},\right.$$
$$\psi(\eta_1,\eta_3,\eta_5) = \left\{\frac{(0.7e^{i0.9\alpha})}{x}, \frac{(0.1e^{i0.7\alpha})}{y}, \frac{(0.1e^{i0.6\alpha})}{z}\right\},$$
$$\psi(\eta_2,\eta_3,\eta_4) = \left\{\frac{(0.8e^{i0.9\alpha})}{x}, \frac{(0.8e^{i0.8\alpha})}{y}, \frac{(0.2e^{i0.6\alpha})}{z}\right\},$$
$$\left.\psi(\eta_2,\eta_3,\eta_5) = \left\{\frac{(0.6e^{i0.9\alpha})}{x}, \frac{(0.2e^{i0.8\alpha})}{y}, \frac{(0.3e^{i0.6\alpha})}{z}\right\}\right\},$$

Direct methanol fuel cells

$$= (\mu,\mathcal{T}) = \left\{\mu(\eta_1,\eta_3,\eta_4) = \left\{\frac{(0.2e^{i0.7\alpha})}{x}, \frac{(0.2e^{i0.6\alpha})}{y}, \frac{(0.7e^{i0.6\alpha})}{z}\right\},\right.$$
$$\mu(\eta_1,\eta_3,\eta_5) = \left\{\frac{(0.6e^{i0.9\alpha})}{x}, \frac{(0.2e^{i0.8\alpha})}{y}, \frac{(0.2e^{i0.6\alpha})}{z}\right\},$$
$$\mu(\eta_2,\eta_3,\eta_4) = \left\{\frac{(0.6e^{i0.9\alpha})}{x}, \frac{(0.2e^{i0.8\alpha})}{y}, \frac{(0.3e^{i0.6\alpha})}{z}\right\},$$
$$\left.\mu(\eta_2,\eta_3,\eta_5) = \left\{\frac{(0.3e^{i0.4\alpha})}{x}, \frac{(0.2e^{i0.8\alpha})}{y}, \frac{(0.3e^{i0.6\alpha})}{z}\right\}\right\},$$

and ideal FCH strategy in the form of CFHSS is

$$(\chi,\mathcal{T}) = \left\{\chi(\eta_1,\eta_3,\eta_4) = \left\{\frac{(0.3e^{i0.4\alpha})}{x}, \frac{(0.8e^{i0.2\alpha})}{y}, \frac{(0.8e^{i0.2\alpha})}{z}\right\},\right.$$
$$\chi(\eta_1,\eta_3,\eta_5) = \left\{\frac{(0.3e^{i0.3\alpha})}{x}, \frac{(0.8e^{i0.2\alpha})}{y}, \frac{(0.2e^{i0.9\alpha})}{z}\right\},$$
$$\chi(\eta_2,\eta_3,\eta_4) = \left\{\frac{(0.4e^{i0.8\alpha})}{x}, \frac{(0.2e^{i0.4\alpha})}{y}, \frac{(0.6e^{i0.9\alpha})}{z}\right\},$$
$$\left.\chi(\eta_2,\eta_3,\eta_5) = \left\{\frac{(0.3e^{i0.3\alpha})}{x}, \frac{(0.8e^{i0.2\alpha})}{y}, \frac{(0.7e^{i0.3\alpha})}{z}\right\}\right\},$$

2) Quantify the SM of $(\chi,\mathcal{T})$, $(\xi,\mathcal{T})$ and $(\psi,\mathcal{T})$ by using formula in Step (2), see Table 3.
Hence the degree of similarity between $(\chi,\mathcal{T})$ and $(\xi,\mathcal{T})$, $(\psi,\mathcal{T})$, $(\mu,\mathcal{T})$ respectively is given by $S_1 = S((\chi,\mathcal{T}),(\xi,\mathcal{T})) = 0.82$,
$S_2 = S((\chi,\mathcal{T}),(\psi,\mathcal{T})) = 0.89$,
$S_3 = S((\chi,\mathcal{T}),(\mu,\mathcal{T})) = 0.82$.
3) Thus, $(\psi,\mathcal{T})$ have highest similarity measure so Polymer electrolyte membrane is most optimal FCH strategy.

The association between the proposed changes and the prevailing increase quality are outlined in table 4.

TABLE 3. Similarity measures.

$S^{\tau}_{l=1}((\chi,\mathcal{T}),(\xi,\mathcal{T}))$	0.8
$S^{\tau}_{l=2}((\chi,\mathcal{T}),(\xi,\mathcal{T}))$	0.66
$S^{\tau}_{l=3}((\chi,\mathcal{T}),(\xi,\mathcal{T}))$	0.63
$S^{\tau}_{l=4}((\chi,\mathcal{T}),(\xi,\mathcal{T}))$	0.73
$S^{\omega}_{l=1}((\chi,\mathcal{T}),(\xi,\mathcal{T}))$	6.01
$S^{\omega}_{l=2}((\chi,\mathcal{T}),(\xi,\mathcal{T}))$	5.78
$S^{\omega}_{l=3}((\chi,\mathcal{T}),(\xi,\mathcal{T}))$	6.08
$S^{\omega}_{l=4}((\chi,\mathcal{T}),(\xi,\mathcal{T}))$	5.74
$S^{\tau}_{l=1}((\chi,\mathcal{T}),(\psi,\mathcal{T}))$	0.8
$S^{\tau}_{l=2}((\chi,\mathcal{T}),(\psi,\mathcal{T}))$	0.86
$S^{\tau}_{l=3}((\chi,\mathcal{T}),(\psi,\mathcal{T}))$	0.76
$S^{\tau}_{l=4}((\chi,\mathcal{T}),(\psi,\mathcal{T}))$	0.83
$S^{\omega}_{l=1}((\chi,\mathcal{T}),(\psi,\mathcal{T}))$	6.08
$S^{\omega}_{l=2}((\chi,\mathcal{T}),(\psi,\mathcal{T}))$	6.24
$S^{\omega}_{l=3}((\chi,\mathcal{T}),(\psi,\mathcal{T}))$	6.01
$S^{\omega}_{l=4}((\chi,\mathcal{T}),(\psi,\mathcal{T}))$	6.24
$S^{\tau}_{l=1}((\chi,\mathcal{T}),(\mu,\mathcal{T}))$	0.73
$S^{\tau}_{l=2}((\chi,\mathcal{T}),(\mu,\mathcal{T}))$	0.7
$S^{\tau}_{l=3}((\chi,\mathcal{T}),(\mu,\mathcal{T}))$	0.83
$S^{\tau}_{l=4}((\chi,\mathcal{T}),(\mu,\mathcal{T}))$	0.56
$S^{\omega}_{l=1}((\chi,\mathcal{T}),(\mu,\mathcal{T}))$	5.91
$S^{\omega}_{l=2}((\chi,\mathcal{T}),(\mu,\mathcal{T}))$	5.78
$S^{\omega}_{l=3}((\chi,\mathcal{T}),(\mu,\mathcal{T}))$	6.013
$S^{\omega}_{l=4}((\chi,\mathcal{T}),(\mu,\mathcal{T}))$	5.94

TABLE 4. The recommended similarity measure-based CFHSS is compared with the conventional SM.

References	SM
[37]	unsupported
[38]	unsupported
[39]	unsupported
[58]	unsupported
[58]	unsupported
[58]	unsupported
[40]	unsupported
[41]	unsupported
[42]	unsupported
[43]	unsupported
[43]	unsupported
[43]	unsupported
[44]	unsupported
[51]	unsupported
[59]	unsupported
[45]	unsupported
[46]	unsupported
[46]	unsupported
[46]	unsupported
[47]	unsupported
[48]	unsupported
[70]	unsupported
[71]	unsupported
our proposed method	$S_1 = 0.82, S_2 = 0.89, S_3 = 0.82.$

B. THE ATTRIBUTES OF THE CFHSS AND A COMPARATIVE ANALYSIS

In the following sections, a few assessment methods of the introduced methodologies with imperfections are explained to check the feasibility and superiority of the preferred method. Furthermore, the envisioned SM is evaluated by comparing other vital indicators and realised to have apparent drawbacks, which are tried to explain with an example, implementing the points raised in the earlier section. However, all emerging flaws fail to adapt to hurdles with 2D observation, i.e., two distinct forms of observation defined as the ratio to the model aspects. The approximation of the strategic planning process is depicted using scenario 3; see the results in table 5.

Example 4: Assess 3 if all we have is one-dimensional records

$$(\xi,\mathcal{T}) = \Big\{\xi(\eta_1,\eta_3,\eta_4) = \left\{\frac{0.2e^{i2\alpha(0.0)}}{x}, \frac{0.7e^{i2\alpha(0.0)}}{y}, \frac{0.4e^{i2\alpha(0.0)}}{z}\right\},$$

$$\xi(\eta_1,\eta_3,\eta_5) = \left\{\frac{0.6e^{i2\alpha(0.0)}}{x}, \frac{0.2e^{i2\alpha(0.0)}}{y}, \frac{0.3e^{i2\alpha(0.0)}}{z}\right\},$$

$$\xi(\eta_2,\eta_3,\eta_4) = \left\{\frac{0.3e^{i2\alpha(0.0)}}{x}, \frac{0.9e^{i2\alpha(0.0)}}{y}, \frac{0.3e^{i2\alpha(0.0)}}{z}\right\},$$

$$\xi(\eta_2,\eta_3,\eta_5) = \left\{\frac{0.6e^{i2\alpha(0.0)}}{x}, \frac{0.6e^{i2\alpha(0.0)}}{y}, \frac{0.4e^{i2\alpha(0.0)}}{z}\right\}, \Big\},$$

$$(\psi,\mathcal{T}) = \Big\{\psi(\eta_1,\eta_3,\eta_4) = \left\{\frac{0.2e^{i2\alpha(0.0)}}{x}, \frac{0.3e^{i2\alpha(0.0)}}{y}, \frac{0.2e^{i2\alpha(0.0)}}{z}\right\},$$

$$\psi(\eta_1,\eta_3,\eta_5) = \left\{\frac{0.7e^{i2\alpha(0.0)}}{x}, \frac{0.1e^{i2\alpha(0.0)}}{y}, \frac{0.1e^{i2\alpha(0.0)}}{z}\right\},$$

$$\psi(\eta_2,\eta_3,\eta_4) = \left\{\frac{0.8e^{i2\alpha(0.0)}}{x}, \frac{0.8e^{i2\alpha(0.0)}}{y}, \frac{0.2e^{i2\alpha(0.0)}}{z}\right\},$$

$$\psi(\eta_2,\eta_3,\eta_5) = \left\{\frac{0.6e^{i2\alpha(0.0)}}{x}, \frac{0.2e^{i2\alpha(0.0)}}{y}, \frac{0.3e^{i2\alpha(0.0)}}{z}\right\}, \Big\},$$

$$(\mu,\mathcal{T}) = \Big\{\mu(\eta_1,\eta_3,\eta_4) = \left\{\frac{0.2e^{i2\alpha(0.0)}}{x}, \frac{0.2e^{i2\alpha(0.0)}}{y}, \frac{0.7e^{i2\alpha(0.0)}}{z}\right\},$$

$$\mu(\eta_1,\eta_3,\eta_5) = \left\{\frac{0.6e^{i2\alpha(0.0)}}{x}, \frac{0.2e^{i2\alpha(0.0)}}{y}, \frac{0.2e^{i2\alpha(0.0)}}{z}\right\},$$

$$\mu(\eta_2,\eta_3,\eta_4) = \left\{\frac{0.6e^{i2\alpha(0.0)}}{x}, \frac{0.2e^{i2\alpha(0.0)}}{y}, \frac{0.3e^{i2\alpha(0.0)}}{z}\right\},$$

$$\mu(\eta_2,\eta_3,\eta_5) = \left\{\frac{0.3e^{i2\alpha(0.0)}}{x}, \frac{0.2e^{i2\alpha(0.0)}}{y}, \frac{0.3e^{i2\alpha(0.0)}}{z}\right\}, \Big\},$$

and ideal CFHSS are

$$(\chi,\mathcal{T}) = \Big\{\chi(\eta_1,\eta_3,\eta_4) = \left\{\frac{0.3e^{i2\alpha(0.0)}}{x}, \frac{0.8e^{i2\alpha(0.0)}}{y}, \frac{0.8e^{i2\alpha(0.0)}}{z}\right\},$$

$$\chi(\eta_1,\eta_3,\eta_5) = \left\{\frac{0.3e^{i2\alpha(0.0)}}{x}, \frac{0.8e^{i2\alpha(0.0)}}{y}, \frac{0.2e^{i2\alpha(0.0)}}{z}\right\},$$

$$\chi(\eta_2,\eta_3,\eta_4) = \left\{\frac{0.4e^{i2\alpha(0.0)}}{x}, \frac{0.2e^{i2\alpha(0.0)}}{y}, \frac{0.6e^{i2\alpha(0.0)}}{z}\right\},$$

$$\chi(\eta_2,\eta_3,\eta_5) = \left\{\frac{0.3e^{i2\alpha(0.0)}}{x}, \frac{0.8e^{i2\alpha(0.0)}}{y}, \frac{0.7e^{i2\alpha(0.0)}}{z}\right\}, \Big\},$$

TABLE 5. Comparison of the proposed and the observed SM.

SN	References	information in two dimensions	Sub-values	SM	Ranking
1	[37]	×	×	assertion is incorrect	×
2	[38]	×	×	assertion is incorrect	×
3	[39]	×	×	assertion is incorrect	×
4	[58]	×	×	assertion is incorrect	×
5	[40]	×	×	assertion is incorrect	×
6	[41]	×	×	assertion is incorrect	×
7	[42]	×	×	assertion is incorrect	×
8	[43]	×	×	assertion is incorrect	×
9	[44]	×	×	assertion is incorrect	×
10	[51]	×	×	assertion is incorrect	×
11	[59]	×	×	assertion is incorrect	×
12	[45]	×	×	assertion is incorrect	×
13	[46]	×	×	assertion is incorrect	×
14	[47]	×	×	assertion is incorrect	×
15	[48]	×	×	assertion is incorrect	×
16	[70]	×	×	assertion is incorrect	×
17	[71]	×	×	assertion is incorrect	×
18	[56]	✓	×	assertion is incorrect	×
19	[55]	✓	×	assertion is incorrect	×
20	[9]	✓	×	assertion is incorrect	×
21	[32]	✓	×	assertion is incorrect	×
22	[34]	×	✓	assertion is incorrect	×
23	Proposed Method	✓	✓	$S_1 = 0.35, S_2 = 0.40, S_3 = 0.35$	$S_2 \geq S_1 \geq S_3$

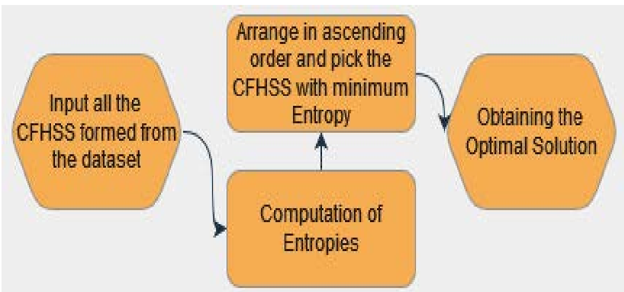

FIGURE 6. Construction steps for the proposed CFHSS-based ENT.

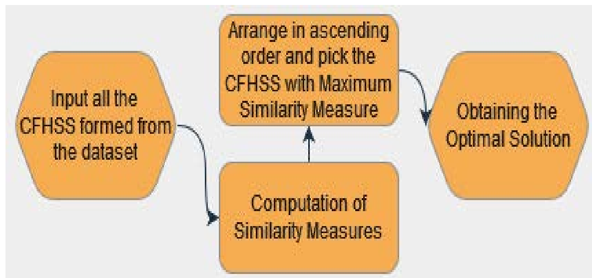

FIGURE 7. Steps in the design and construction CFHSS-based SM.

$$S_1 = S((\chi, \mathcal{T}), (\xi, \mathcal{T})) = 0.35,$$
$$S_2 = S((\chi, \mathcal{T}), (\psi, \mathcal{T})) = 0.40,$$
$$S_3 = S((\chi, \mathcal{T}), (\mu, \mathcal{T})) = 0.35.$$

C. SENSITIVITY ANALYSIS

1) By stripping away the imagined elements [34], the envisioned CFHSS was transmogrified into a fuzzy hypersoft set.
2) The proposed structure reduced to FSS [20] by discarding the complex elements and $G_1 = G_2 = G_3 \cdots = G_n$.
3) When $G_1 = G_2 = G_3 \cdots = G_n$, the envisaged CFHSS then diminished to CFSS [9].

As shown in [9], [22], the proposed CFHSS-based schemes are more dominant and detailed than traditional methodologies. We are actively investigating the founding of a more renowned structural manner for comparative indicators, with particular attention to build that with other forms of SM in the long term.

V. EVALUATIONS OF HYDROGEN FUEL CELL ELEMENTS USING A TOPSIS-BASED OPTIMIZED FHS SET CLASSIFIER

A. A REAL-LIFE EXAMPLE

One of the most crucial issues, such as environmental research, is the facets of FCH. Renewable force is generated as an unpreventable natural result of domestic, enterprise, and organizational activities. A merged fuzzy MCDM framework depending on the TOPSIS method is suggested in this section to appraise FCH options for Iran. A detailed examination is realized by means of specialized, contemporary social, institutional, innovation, and monetary and ecological aspects. Parameters are characteristics of populations-that describes the population. A clear understanding of the objectives of the study will influence the parameter(s) of importance. The parameter which we have been selected in such a way that can represent the systematic scenario that minimize the cost benefits and maximize the profit. We divide the parameters into sub-attributes to make make an optimal decision support system that is superior to the methods in literature in the following ways:

- Save time and make better use of resources
- Professional development is enhanced

TABLE 6. All experts opinions collectively.

Types of hydrogen fuel cells elements/Criteria	1	2	3	4	5	6	7	8
Solid acid	0.04	0.98	0.78	0.41	0.22	0.54	0.31	0.8
Alkaline	0.15	0.75	0.25	0.72	0.46	0.79	0.61	0.12
Solid oxide	0.27	0.57	0.92	0.39	0.37	0.87	0.33	0.42
Molten-carbonate	0.52	0.12	0.24	0.58	0.51	0.42	0.63	0.71
Direct-ethanol	0.83	0.99	0.42	0.21	0.39	0.62	0.61	0.64
Phosphoric acid	0.11	0.71	0.66	0.89	0.38	0.87	0.48	0.62
Metal hydride	0.63	0.67	0.78	0.06	0.6	0.07	0.89	0.38
Electro-galvanic	0.75	0.44	0.89	0.16	0.67	0.46	0.48	0.45

TABLE 7. Normalized matrix.

Types of hydrogen fuel cells elements/Criteria	1	2	3	4	5	6	7	8
Solid acid	0.028	0.49	0.41	0.28	0.16	0.30	0.19	0.50
Alkaline	0.10	0.37	0.13	0.50	0.34	0.44	0.37	0.076
Solid oxide	0.18	0.28	0.48	0.27	0.27	0.48	0.20	0.26
Molten-carbonate	0.36	0.060	0.12	0.40	0.38	0.23	0.39	0.45
Direct-ethanol	0.58	0.49	0.22	0.14	0.29	0.34	0.37	0.40
Phosphoric acid	0.07	0.35	0.34	0.62	0.28	0.48	0.29	0.39
Metal hydride	0.44	0.33	0.41	0.042	0.45	0.039	0.55	0.24
Electro-galvanic	0.52	0.22	0.46	0.11	0.50	0.25	0.29	0.28

TABLE 8. Weighted normalized matrix.

Types of hydrogen fuel cells elements/Criteria	1	2	3	4	5	6	7	8
Solid acid	0.0056	0.14	0.04	0.014	0.024	0.015	0.009	0.50
Alkaline	0.021	0.11	0.13	0.0250	0.05	0.022	0.018	0.007
Solid oxide	0.037	0.085	0.48	0.013	0.04	0.24	0.010	0.026
Molten-carbonate	0.07	0.018	0.12	0.020	0.05	0.011	0.19	0.45
Direct-ethanol	0.11	0.14	0.022	0.007	0.044	0.017	0.018	0.040
Phosphoric acid	0.015	0.010	0.0348	0.031	0.04	0.024	0.014	0.03
Metal hydride	0.088	0.100	0.04	0.002	0.067	0.0019	0.02	0.02
Electro-galvanic	0.10	0.066	0.04	0.005	0.07	0.012	0.014	0.28

- Delegation will be easier and more effective.
- Superiority in terms of functionality and optimal workflow

Actually, we used three parameters, a1 = Economic, a2 = Technology capability, a3 = Organization be different parameters with corresponding parametric values that are components of the groupings belongs to Q1 = Cost of commodity components, Cost of specialty components, Q2 = Information disclosure, Q3 = Green strategy direction, Degree of strategic cooperating, these are the major impactful factors during the fuel cell hydrogen elements selection process because it effects directly and indirectly in the optimization process. The parameters which we have been selected are comprehensive as they optimized the problem (cost minimized and profit maximized). One can change and add more parameters in the decision-making process depending upon the situation.Complexity has been determined in our example probelm by evaluating project attributes using the criteria listed below:

- Set of alternatives (Belonging to different technologies in terms of FCH)
- Set of attributes with sub attributes values
- Project organization complexity
- Technology readiness
- Economics and Long-Term Planning conditions

B. THE EXPLORATION OF HYDROGEN FUEL CELL ELEMENTS AND ITS ASPECTS

Analytic FCH energy exploration and machine learning mathematics have a huge impact on the environment. There are eight various kinds of FCH resources that are explained.

- Solid acid
- Alkaline
- Solid oxide
- Molten-carbonate
- Direct-ethanol
- Phosphoric acid

TABLE 9. Positive ideal solution.

C_1	0.116
C_2	0.148
C_3	0.048
C_4	0.031
C_5	0.075
C_6	0.024
C_7	0.027
C_8	0.05

- Metal hydride
- Electro-galvanic

1) SOLID ACID

Solid acid fuel cells (SAFCs) are carbon cells that use strong acid materials as electrodes. They function at temperatures up from 200 to 300 °C. Present SAFC programs depend on water vapour from various fuels, including advanced manufacturing propane and diesel.

2) ALKALINE

One of the most modern electrochemical devices is the alkaline fuel cell (AFC), usually known as the Bacon fuel cell after its British designer, Francis Thomas Bacon. Alkaline fuel cells produce clean water, steam, and electricity by burning helium and compressed air. It is one of the most improved fuel cells, with a prospective efficiency of 70.

3) SOLID OXIDE

A phosphoric acid fuel cell (SOFC) is an energy conversion system that transforms electricity by degrading a fuel. The electrolytic component of a fuel cell characterizes it; the SOFC has a solidified oxide or ceramic cathode.

4) MOLTEN-CARBONATE

Molten-carbonate fuel cells (MCFCs) are fuel costs cells that run at 600 degrees Celsius or higher temperatures. Magnesium carbonate fuel cells (MCFCs) were devised used for fossil fuels, biogas (generated by anaerobic digestion or biomass gasification), and coal-fired energy plants for an electrical company, industrial, and military purposes. MCFCs are high-temperature fuel cells that employ an electrolyte made up of a molten carbonate salt combination floating in a porous, chemically inert ceramic matrix of beta-alumina solid electrolyte (BASE).

5) DIRECT-ETHANOL

DEFCs, or specified battery storage, are continuous fuel cell that immediately utilises methanol. They consider it a precedent for designing a vast scope of anode material, notably PEM.

6) PHOSPHORIC ACID

PAFCs are fuel cells that use liquid phosphoric acid as an electrolyte. They were the first commercialised fuel cells.

TABLE 10. Negative ideal solution.

C_1	0.0056
C_2	0.018
C_3	0.012
C_4	0.0021
C_5	0.024
C_6	0.0019
C_7	0.0096
C_8	0.007

TABLE 11. Separation from positive ideal.

Solid acid	0.12
Alkaline	0.11
Hydroelectric Power	0.11
Molten-carbonate	0.14
Geothermal Sources	0.04
Phosphoric acid	0.116
Metal hydride	0.072
Electro-galvanic	0.09

TABLE 12. Separation from negative ideal.

Solid acid	0.14
Alkaline	0.10
Hydroelectric Power	0.09
Molten-carbonate	0.08
Geothermal Sources	0.17
Phosphoric acid	0.10
Metal hydride	0.13
Electro-galvanic	0.12

TABLE 13. Preference values.

Solid acid	0.52
Alkaline	0.46
Hydroelectric Power	0.44
Molten-carbonate	0.37
Geothermal Sources	0.77
Phosphoric acid	0.47
Metal hydride	0.64
Electro-galvanic	0.58

They were conceived in the mid-1960s and have been a profession since the 1970s, and they have been dramatically associated with the quality of stability, speed, and pricing. Because of all these qualities, the effective training programme was a fantastic opportunity for early power applications.

7) METAL HYDRIDE

The durability of hydrogen lithium batteries is a type of acidity energy production that has been massively improved in windows versions and has undergone major systematic

TABLE 14. Final ranking matrix.

Types of hydrogen fuel cells elements/Criteria	1	2	3	4	5	6	7	8	Rank
Solid acid	0.04	0.98	0.78	0.41	0.22	0.54	0.31	0.8	4
Alkaline	0.15	0.75	0.25	0.72	0.46	0.79	0.61	0.12	6
Solid oxide	0.27	0.57	0.92	0.39	0.37	0.87	0.33	0.42	7
Molten-carbonate	0.52	0.12	0.24	0.58	0.51	0.42	0.63	0.71	8
Direct-ethanol	0.83	0.99	0.42	0.21	0.39	0.62	0.61	0.64	1
Phosphoric acid	0.11	0.71	0.66	0.89	0.38	0.87	0.48	0.62	5
Metal hydride	0.63	0.67	0.78	0.06	0.6	0.07	0.89	0.38	2
Electro-galvanic	0.75	0.44	0.89	0.16	0.67	0.46	0.48	0.45	3

study. One unique aspect is their inclination to interact with and collect hydrogen within the gasoline dynamically.

8) ELECTRO-GALVANIC

An electro-galvanic fuel cell is an electrochemical device which consumes a fuel to produce an electrical output by a chemical reaction.

C. ALGORITHM

Step 1: Data sets are taken in the form of a CFHS set.First step involves the conversion of CFHS set to fuzzy hypersoft set to get weighted aggregation values by using the formula $\mathcal{T}_{z'(s')}(p) = w_1\mu_{z'(s')}(p) + w_2(\frac{1}{2\pi})\omega_{z'(s')}(p)$ [9], with weights $w_1 = 0.2$, $w_2 = 0.4$ to create a decision average matrix for each alternative premised on the general view of all professionals in the FHS set. The standardized precipitation fuzzy conceptual framework is a decision matrix that has been acknowledged TOPSIS demands that the efficiency of each option, in contrast, be ranked using the following equation:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_1^m x_{ij}^2}}; \tag{7}$$

with x = decision matrix; $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.

Step 2: Based on the weighted normalised rating (y_{ij}), the weighted normalised fuzzy control matrix can be computed as follows:

$$y_{ij} = w_i r_{ij} \tag{8}$$

with $i = 1, 2, \ldots, m$; and $j = 1, 2, \ldots, n$.

Step 3.

Identifying the best positive and negative solutions The positive ideal solution matrix is constructed using equation 9, while the negative ideal solution matrix is assessed using equation 10.

$$A^+ = (y_1^+, y_2^+, \ldots, y_n^+); \tag{9}$$
$$A^+ = (y_1^+, y_2^+, \ldots, y_n^+); \tag{10}$$

Step 4: The distance between each candidate and the ideal solution, both positive and negative. The distance between alternative G_i and the positive best solution can be demonstrated as equation 5:

$$D^+ = \sqrt{\sum_{j=1}^{n}(y_i^+ - y_{ij})^2}; \tag{11}$$

$i = 1, 2, 3 \ldots m$ The distance between alternative G_i with negative ideal solution can be formulated with equation 10:

$$D^- = \sqrt{\sum_{j=1}^{n}(y_i^- - y_{ij})^2}; \tag{12}$$

$i = 1, 2, 3 \ldots m$

Step 5: Determining the value of preference for each alternative The preference value for each alternative (V_i) is given as:

$$V_i = \frac{D_i^-}{D_i^- + D_i^+} \tag{13}$$

$i = 1, 2, 3 \ldots m$

Step 6: Sort the options and select the best one.

D. NUMERICAL EXAMPLE

Step 1: Let $X = \{a$ =Solid acid, b = Alkaline, c =Solid oxide, d =Molten-carbonate, e =Direct-ethanol, f =Phosphoric acid, g =Metal hydride, h =Electro-galvanic}, be a set of alternatives and $\{\delta_1, \delta_2, \delta_3, \delta_4\}$ is a set of experts who will evaluate the best alternative with weight vector $(0.2, 0.3, 0.1, 0.05, 0.15, 0.05, 0.05, 0.1)^T$, let a_1 = Environmental, a_2 = Technology capability, a_3 = Economic, be distinct features with respective feature values that are collections main components Q_1, Q_2, Q_3, let $Q_1 = \{\eta_1$ = Greenhouse gas emission, η_2 = Land requirement, η_3 = Need of waste disposal, η_4 = Environmental damage}, $Q_2 = \{\eta_5$ = Capacity expansion potential, η_6 = Sustainability}, $Q_3 = \{\eta_7$ = Affordability}, where $Q_1 \times Q_2 \times Q_3 = \{C_i, i = 1, 2, 3, \ldots 8\}$. With the help experts, develop decision average matrix for each alternative regarding all experts opinions collectively in FHS set by applying set of parameters with its sub-parametric values, please see Table 6. Normalized Table 7 by using Equation 7.

Step 2: Using Equation 8, it is able to develop a weighted decision matrix for each option, as shown in Table 8.

Step 3: Compute the positive ideal solution and negative ideal solution using Equations 9 and 10, respectively, see Tables 9,10.

Step 4: Compute the distance of each candidate from positive and negative ideal solution using Equations 11 and 12, see Tables 11,12.

Step 5: Compute the preference value for each alternative using Equation 13, see Table 13.

Step 6: Rank the alternatives and choose the best one, see Table 14.

VI. LIMITATION OF THE METHOD

1) The sets which we have been taken must be disjoint.
2) It is unable to manage if the frame lies in higher dimension.
3) The recommendations of decision maker must be known.

All these limitation can be tackled by involving more tocologies and experts.

VII. DISCUSSION AND CONCLUSION

The suggested conclusion has several managerial consequences for FCH strategic procurement analysts, managers, and policymakers. The provided methodology has specific implications for both the private sector and the government. Selecting the best source permits you to approach the market competitive by lowering exorbitant expenses and, as a result, galvanizing the manufacturers. The intended choice strategy permits selecting the environmental policy sector to pursue supplier infrastructure projects and identify each potential provider's strengths and unintended consequences. As a result, collaborating with a competent external provider fosters a thriving RD level while averting similar basic research currently undergoing elsewhere. This is a crucial goal for developing nations attempting to achieve trimming. The paradigm also enabled choice with a basic understanding of the performance of collaborative aspects. It is suitable for building a long-term collaboration with conceptual frameworks of sustainability reports, existing risks, and other cooperative elements. This study confirms a variety of meanings pertaining to the benefit derived from independent bodies for the FCH company's overall growth. Organizations at an advanced rank may prosper from a more favourable phase of development that goes outside depending exclusively on their endowments. It signifies that worldwide marketplaces are enduring a prominent globalized restructuring away from the domination of fossil fuel energy and toward alternative energy sources, signifying more economic growth and a decline in significant supplier bargaining power, signifying inclusiveness of overseas markets reformation. Consequently, most countries that extract liquid fuels, such as Iran, would face challenges because their macroeconomic strategy is based on fuel export revenues. Furthermore, environmental preservation is a significant motivator for the growth of novel FCH. To address such issues, fuel cell devices are being developed. Iran has committed to encouraging FCH as a feasible form of energy. FCH, on the other hand, is a viable technology and RD in this industry is challenging, especially given the present financial constraints. It is critical to understand the correct supplier to regulate the research and innovation process, especially when the methodology is ambiguous. This study examined 19 factors divided into four major groups to handle this issue: economic, managerial, supply concerns, and availability of technology. To solve MCDM concerns within the PFS setting, a combined MCDM technique known as the PF-entropy-SWARA-COPRAS sensor was proposed to cope with challenge space ambiguities. The objective ratings were estimated using an entropy measure, while the subjective weights were derived using SWARA in a state of flux. These two systems' outputs were then concatenated to generate objective weighted accumulation. The proposed decision-making paradigm was utilized to develop a use of the FCH modules strategic procurement issue, demonstrating the effectiveness of the proposed technique. A comparison with the present analysis and a risk assessment is performed to examine the legitimacy of the recommended procedure's outcome. Ignoring the fact that the overwhelming issues emerge to play an essential part in making a matrix, the data revealed that the financial components are more beneficial and impact decision inclinations. The finding was unsurprising, given that policymakers are apprehensive about financial measures experiencing significant concerns due to the recent financial sanctions crisis (particularly after the united states departed from the Global Action Initiatives in 2018). However, in terms of technological policy, the robustness of judgements is heavily dependent on balancing multiple elements, where overall organizational or intellectual abilities, for example, are just as crucial as economic demands. Due to the Iranian economy's increasing reliance on supplying fossil fuels and hydrocarbon goods, economic woes also impact the energy policy, particularly in the foreseeable future. Because FCH facilitates microgrids and intelligent grids instead of centralized natural grids, the FCH sector can improve Iran's supply security by diversifying energy sources, lowering main power costs to inaccessible places, and predictably exacerbating Iran's oil and coal output growth. According to study findings, FCH can increase but is hampered by budget pressures and macroeconomic appeals. The intended decision may be used to provide three possibilities for future advancement. To begin, the FCH sector is interlinked in various ways, and one industry, such as FCH, is linked to other industries at the same time. It will be essential to investigate FCH promotion concepts on a multidimensional height in collaboration with other FCH regions. This helps to evaluate the consequence of one source's priorities on some other and how one source's agile methodology affects something else, and the FCH segment's immunity to other fossil and FCH sectors. When researching various areas, the importance of the criteria in certain domains may be contrasted. They must be analyzed as a group since the government's emphasis on one issue has the overarching tendency to abandon another. Second, the model under review here was the dilemma model, signalling that the predominant feature and the macroeconomic levels would endure. It minimized exogenous potential options (such as political developments in the research study) that may significantly influence the state, especially the FCH segment. It would be interesting to explore how often impending political turmoil negatively influences the model's outcome by adjusting the relevance of the antecedent condition and the absence of the set requirements. In accordance with the above postulate, the framework would include the development of medium- to long-term possibilities to better address the unpredictability. Scientific investigations might include investigating the FCH supply chain theory in the context of plausible option opportunities rather than only the firm foundation and anticipating how and to what degrees included characteristics may affect perceptions. Real-life scenario building may also give the decision call framework long-term durability. Adequate and appropriate FCH methods must impact various pollutants, prevent infectious diseases, conserve natural resources, and reuse harmful chemicals. As a result, most scientists and academics have been focused on FCH. However, because uncertainty occurs in practically any appropriate system, researchers have turned to FS theory and its various versions to solve the FCH issues. This work has developed a fundamental innovative technique for revealing valid evidence of complications. The CFHSS set is created by combining an FS, and an HSS delineated in a complex manner. This viewpoint is adaptive in two ways. It broadens the member status, for starters, by moving it to a unit circle with transition stage and amplitude characteristics. Second, the CFHSS qualities will be further subdivided into data points to provide a more in-depth understanding. In this case, the presented structure can investigate the overall importance of each approach by weighing multiple factors that influence FCH methodologies based on attributes. This research will offer a scientific base for interfacing with variability and irregularity in development, healthcare, nanodevices, public transportation, and other companies. P-terms could represent a room temperature, tension, togetherness, or any other guideline that influences and collaborates with the various A-terms in the round of preferences. This improved understanding of P-terms broadens the range of options available. It is used in science content and specific other intercultural specializations. Furthermore, the metaphysical concept of ENT and SM of CFHSS and the related problems were discussed. Mathematical models are also provided to re-evaluate the layout methodologies' potential and priority. The implications of the proposed measures and their comparability to existing models are thoroughly discussed. The prospects of this method involve the expansion of the domains of the proposed methods in various other frameworks like Neutrosophic Hypersoft Set, Plithogenic Hypersoft Set, Hypersoft Set, Plithogenic Intuitionistic Fuzzy Hypersoft Set, and their hybrid structures. The applications of the proposed method are under words in medical imaging problems, image processing, and pattern recognition studies. Finally, analytical solutions illustrating the strategic planning's applicability are offered.

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