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RESEARCH ARTICLE

Fuzzy Cognitive Maps Based on D-Number Theory

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ABSTRACT In real life, there will be a lot of uncertainty problems, one of which is due to the vagueness of the concept of things, that is, it is difficult to determine whether an object conforms to this concept. This situation widely exists in some states, phenomena, parameters and interrelationships between things. For such uncertain events with heavy subjective influencing factors and incomplete data, it is suitable to use fuzzy methods to deal with them. In present our work, we firstly introduce the notion of D-number cognitive maps (DCMs), which are intelligent framework models based on D-number theory and cognitive maps. Compared with Evidential Cognitive Maps (ECMs) and Fuzzy Cognitive Maps (FCMs), DCMs can fuse multiple sources of information with uncertainty and construct a cognitive map model with incomplete and conflicting information. To better solve the problem of knowledge combination, D-number fuzzy cognitive maps (DFCMs) are also constructed based on D-number theory and fuzzy cognitive maps. In many practical applications, the establishment of fuzzy cognitive maps is usually completed by using a simple arithmetic average method for multiple experts to obtain comprehensive fuzzy cognitive maps. This simple processing method may lead to the loss of some important information so that the synthesized results do not reflect reality. To overcome this challenge, we synthesize the knowledge of multiple experts by using D-number theory and the characteristics of the representation of expert knowledge in FCMs. It is based on expert knowledge, and the synthesized reliability distribution function is used as the basis of final weight synthesis.

INDEX TERMS Uncertainty, fuzzy cognitive maps, D-number cognitive maps, D-number fuzzy cognitive maps.

I. INTRODUCTION

Evidence cognitive maps (ECMs) [1] and fuzzy cognitive maps (FCMs) [2] are graph models proposed to deal with uncertain information. ECMs are uncertain graph structures that describe causal reasoning through cognitive maps (CMs) [3] and Dempster-Shafer theory and use basic probability assignment (BPA) and intervals to represent the relationship between concepts and the state of concepts. ECMs have been proven to be effective and convenient in modelling systems with subjective and objective uncertainties. However, when facing the problem of knowledge combination, ECMs may lead to unreasonable results in system modelling. To overcome the limitations of ECMs, this paper proposes D-number cognitive maps (DCMs) based on D-number theory. Compared with ECMs, in DCMs, some theoretical frameworks are redefined. A scheme combining information is established through D-number theory, and a method based on distance measurement is proposed to determine the weight of different DCMs. DCMs fully consider all the effects of various information. At the same time, this scheme can overcome the limitation of conflicting information generate aggregated ECMs to ensure that DCMs can effectively model complex systems.

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The concept of FCMs has attracted special attention in recent years as a powerful tool to manipulate knowledge by imitating human reasoning and thinking. For example, a study [4] proposes a short-term power load forecasting model based on K-means and FCM-BP. There are two ways to construct FCMs: artificial methods and computational methods. In many practical applications, the establishment of FCMs [5] is usually performed by experts. To overcome the limitations of a single assessment, data from multiple experts are usually used to construct the FCMs of the system separately, and then the arithmetic average of the weights is used to calculate the comprehensive FCMs. As the study of Dempster-Shafer theory develops [6]–[10], a synthetic method of weight evaluation is derived: with expert knowledge used as evidence, the possible values of the degree of influence of causality among concepts used as an identification framework, the membership degree of the fuzzy value evaluated by each weight used as the reliability distribution function m, and the combined reliability distribution function used as the basis for the final synthesis of weights, this method combines the knowledge of multiple experts.

Synthesizing the uncertain opinions of individual experts is a problem of uncertainty information fusion. Its purpose is to overcome the limitations in performance of using knowledge from a single expert, improve the internal quality of knowledge, and provide true, clear and complete available knowledge for decision-making. At present, the most commonly used method is individual probability aggregation [11]. Individual probability aggregation typically uses weighted aggregation, product aggregation, Bayesian aggregation, maximum direct model aggregation and other methods to aggregate the individual opinions of experts. To date, there is no clear method to determine the weight in the weighted aggregation model in synthesis research.

As an intelligent technology, D-number theory [12]–[15] provides a powerful tool for the synthesis of multiexpert knowledge. D-number theory is a numerical method of information reasoning that uses multivalued mapping to obtain the upper and lower bounds of probability. Based on the continuous accumulation of evidence, the hypothesis set is repeatedly reduced, thus gradually approaching the true value to inform scientific decision-making. This method incorporates the cognitive and comprehensive ability of the knowledge of multiple experts, resulting in effective integration. This paper uses D-number theory to carry out the effective synthesis of multiple expert knowledge, according to the characteristics expressed by FCMs. This research has great importance for knowledge base construction based on FCMs and the integration of new and old knowledge. As the applications of D-number theory, Nebojsa [16] proposed a novel multi-criteria methodology based on D-numbers which enables efficient analysis of the information used for decision making. Mo [17] proposed a new method to identify and evaluate the risk factors based on strengths weaknesses opportunities threats analysis and D-number theory. In addition, a new method [18] to solve the problem of emergency decision-making named D-PLTS is proposed, based on D-number theory and the probability linguistic term set.

The advantage of Dempster-Shafer, a reasoning theory in uncertain environments, is that it obtains the upper and lower bounds of probability through multivalued mapping and gradually approaches the true value to inform scientific decision-making. For example, Jia [19] proposed an extended intuitionistic fuzzy cognitive map via Dempster-Shafer theory. Yager [20] proposed the interval-valued entropies for Dempster-Shafer structures, which is based on Dempster-Shafer structures and classic Shannon entropy and is an interval entropy model. And a new classifier based on Dempster-Shafer [21] theory and a convolutional neural network architecture for set-valued classification is proposed.

However, some of its strong assumptions may not be relevant for certain applications in the real world. For example, the problem domain is represented by the concept of an identification framework, the elements of which are required to be mutually exclusive. The uncertain information is represented in the form of a BPA. Because the basic probability sum of a BPA is equal to 1, it is difficult to apply to incomplete information. If these assumptions are appropriately relaxed, a more realistic and comprehensive weight is obtained. Based on this point of view, this paper proposes a new method to construct cognitive maps based on D-number theory. Then, D-number theory is applied to FCMs, and an FCM construction method based on D-number theory and multiexpert knowledge synthesis is proposed. Compared with Dempster-Shafer, the D-number theory is more appropriate and capable of expressing uncertain information.

In order to overcome the limitations of FCMs and DCMs in dealing with uncertain information and solve the disadvantages of using Dempster-Shafer theory [22] to synthesize multiple experts knowledge, the main contributions of the cognitive maps based on D-number theory and the extended FCMs based on D-number theory are as follows.

- We propose that "$D\{-1, 1, 0\}$," the weight representation of edges in DCMs model, can solve the uncertainty in the estimation of the influence relationship between nodes due to the complexity of the relationship between concepts and the lack of expert knowledge and experience. In addition, the proposed DCMs model can integrate the knowledge of experts by constructing similarity matrix and defined D-number combination rules while maintaining consistency.
- We extend the D-number theory to FCMs. The proposed DFCMs model can effectively solve the limitations of some strong assumptions of Dempster-Shafer theory in dealing with practical problems when using Dempster-Shafer theory to synthesize multiple experts knowledge. DFCMs provide a reasonable method to determine the weight in the weighted aggregation model. This research is of great significance for the construction of knowledge

base based on FCM, the integration of old and new knowledge and the research field of synthesis.

The rest of this paper is arranged as follows. Section 2 introduces some basic concepts of fuzzy set theory, FCMs, Dempster-Shafer theory and D-number theory and then introduces the existing construction methods of FCMs based on Dempster-Shafer theory. In Section 3, DCMs are proposed as a new graph structure to deal with uncertain information, and an example is given to illustrate the advantages of this model for handling practical problems. In Section 4, a method of multiexpert knowledge synthesis based on D-number theory is used to construct FCMs. In Section 5, we compare and analyze the differences and advantages of DFCMs and existing research methods with examples. Finally, the conclusion of this paper is given in Section 6.

II. BACKGROUND KNOWLEDGE

A. FUZZY SET THEORY

Since Zadeh [23] first introduced the notion of fuzzy sets in 1965, the study of fuzzy sets has piqued the interest of numerous researchers. The definition of a fuzzy set given by Zadeh is as follows. A Fuzzy set is a class with a continuum of membership grades. So a fuzzy set μ in a referential X is characterized by a membership function μ in X which associates with each element $x \in U$ a real number $\mu(x) \in [0, 1]$, having the interpretation $\mu(x)$ is the "membership grade" of x in the fuzzy set μ. A fuzzy set μ is thus defined as a mapping:

$$\mu : X \to [0, 1]$$

where $\mu(x)$ is the membership degree.

Definition 1: If $\tilde{a} = \left[a^-, a^+\right] = \left\{x \mid a^- \leqslant x \leqslant a^+\right\}, a^-, a^+ \in R$, then $\tilde{a}$ is called an interval number, as shown in Figure 1 [24]. In particular, if $a^- = a^+$, then $\tilde{a}$ degenerates into a real number, and the interval number is a uniformly distributed fuzzy number.

FIGURE 1. Schematic diagram of interval numbers.

Let $\tilde{a}$ and $\tilde{b}$ be two interval numbers, where $\tilde{a} = \left[a^-, a^+\right], \tilde{b} = \left[b^-, b^+\right]$. The basic operators on interval numbers are described below.

Addition of interval numbers:

$$\tilde{a} \oplus \tilde{b} = \left[a^- + b^-, a^+ + b^+\right] \tag{1}$$

B. FUZZY COGNITIVE MAPS

Definition 2.: The model composed of concept nodes and directed arcs between concept nodes is called FCMs. Concept nodes represent the dynamic feature system to be modeled, and the directed arc between concept nodes represents the connection relationship between nodes [25].

Example 1: FCM is represented by a quaternion $G = (C, E, X, f)$. A FCM with 5 nodes is shown in Figure 2.

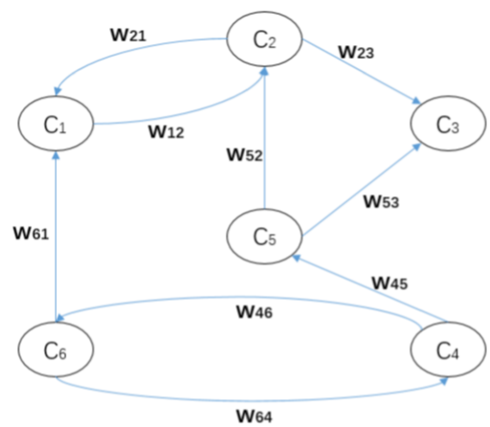

FIGURE 2. An examples of FCM.

$C = \{C_1, C_2, \ldots, C_n\}$ is the set of concepts that constitute the vertices of the directed graph.

$E : (C_i, C_j) \to \omega_{ij}$ is a mapping, $\omega_{ij} \in E, C_i, C_j \in C$. Use ω_{ij} to represent the degree of causal influence between C_i and C_j.

$\omega_{ij} > 0$ means that the increase in C_i will lead to the increase in C_j, and there is a positive causal relationship between C_i and C_j.

$\omega_{ij} < 0$ indicates that an increase in C_i will lead to a decrease in C_j, and there is a negative causal relationship between C_i and C_j.

$\omega_{ij} = 0$, it means that there is no causal relationship between C_i and C_j.

Then, $E(C \times C) = \left(\omega_{ij}\right)_{n \times n}$ is a connection matrix of the directed graph. For example, the adjacency matrix of Figure 2.

$$W = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{array} \begin{array}{c} \begin{array}{cccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \end{array} \\ \begin{bmatrix} 0 & \omega_{12} & 0 & 0 & 0 & 0 \\ \omega_{21} & 0 & \omega_{23} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega_{45} & \omega_{46} \\ 0 & \omega_{52} & \omega_{53} & 0 & 0 & 0 \\ \omega_{61} & 0 & 0 & \omega_{64} & 0 & 0 \end{bmatrix} \end{array}$$

$X : C_i \to x_i$ is a mapping, $x_i(t)$ represents the state of node C_i at time t, and $X(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T$ represents the state of G at time t; then,

$$x_i(t+1) = f[\sum_{\substack{i=1 \\ j \neq i}}^{n} \omega_{ij} x_j(t)] \tag{2}$$

or

$$X(t+1) = WX^T(t) \tag{3}$$

Remark 1: where $x_i(t+1)$ is the normalized ($x_i(t+1) \in [0, 1]$) value of concept C_i at time step t, and $f(x)$ is a threshold function. Generally, a sigmoidal function $f(x) = \frac{1}{1+e^{-Cx}}$ is used to constrain the value of $f(x)$ in the interval [0, 1], where $C > 0$ determines the steepness of $f(x)$.

Remark 2: Eq. (2) represents the model of FCM, which can also be called evolution equation. From the model, we can see that the dynamic behavior of the system [26] is formed by the interaction of concept nodes and their states in the system through causal arc. Each concept node in FCM transmits its output to other nodes through weight and receives the causal influence transmitted by other nodes at the same time. In short, when the adjacency matrix of FCM is known, the state $x(t+1)$ of the system at time $t+1$ can be obtained by the product of the adjacency matrix and the state $x(t)$ at time t.

The measure of the degree of causal influence is represented by a real number belonging to [0, 1], the magnitude of the number indicates the strength of the influence, and the sign (positive or negative) indicates the direction of the influence, resulting in a quantitative expression of the causal relationship. For example, the strength of the causal relationship can be categorized (none, very weak, weak, medium, strong, very strong), as well as quantified: (0, 0.2, 0.4, 0.6, 0.8, 1). The standard and accuracy of fuzzy quantification depend on the specific problem [27].

C. DEMPSTER-SHAFER THEORY

Definition 3: For a finite nonempty set $\Omega = H_1, H_2, \ldots, H_N$, Ω is called a frame of discernment (FOD) when satisfying

$$H_i \cap H_j = \emptyset, \quad \forall i, j = \{1, \ldots, N\}$$

The mapping

$$m : 2^{\Omega} \to [0, 1]$$

which satisfies the following condition:

$$m(\emptyset) = 0 \quad \text{and} \quad \sum_{A \in \Omega} m(A) = 1 \tag{4}$$

Then, the function m is called the basic credibility distribution on frame Ω. $\forall A \in \Omega$, m(A) is called the basic credibility number of A and reflects the degree of reliability of A [28]. The former condition indicates that an empty proposition does not produce reliability, and the latter condition indicates that a proposition can be assigned a reliability value of any magnitude, but the sum of the reliability values of all propositions is equal to 1 [29].

Definition 4: The mapping

$$Bel : 2^{\Omega} \to [0, 1]$$

defined by the formula

$$Bel(A) = \sum_{B \in A} m(B) \quad \forall A \in \Omega \tag{5}$$

is called the reliability function on Ω. That is, the reliability function $Bel(A)$ of A is the sum of the reliability values of each subset in A, which can be obtained:

$$\begin{cases} Bel(\emptyset) = 0 \\ Bel(\Omega) = 1 \end{cases} \tag{6}$$

The reliability function Bel and function m are mutually uniquely determined, so they can be understood as different representations of the same evidence. From the meaning of function m, reliability function $Bel(A)$ can be understood as the degree of trust in the evidence for proposition A and its subsets.

The suspicion function and the plausibility function of $Bel(A)$ are defined as:

$$\begin{aligned} Dou(A) &= Bel(\bar{A}) \\ Pl(A) &= 1 - Bel(\bar{A}), \quad \forall A \in \Omega \end{aligned} \tag{7}$$

Dou(A) is the suspicion degree of A and *Pl(A)* is the plausibility degree of A, meaning the highest degree of trust in proposition A.

In summary, the uncertainty of information is shown in Figure 3.

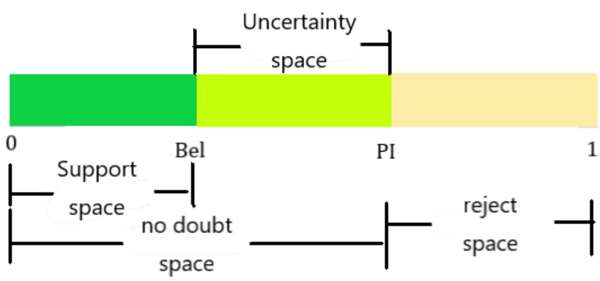

FIGURE 3. **Uncertainty representation of information.**

We can use $A[Bel(A), Pl(A)]$ to analyse the uncertainty of the proposition.

$A[0, 0]$ indicates that proposition A is false.

$A[0, 1]$ means proposition A cannot be affirmed or denied, indicating no knowledge about proposition A.

$A[1, 1]$ indicates that the proposition is true.

It can be seen from the above analysis that propositions can be analysed through $Pl(A)$ and $Bel(A)$ distinguishing between unknown and uncertain, which is very meaningful distinction in a decision problem.

D. D-NUMBER THEORY

D-number theory is based on people's understanding and knowledge of the objective world and provides uncertainty measures for uncertain events. For the first time, the probability of not satisfying additivity was provided, and the principle of combining two independent information sources was applied for statistical problems. Later, it was extended to the general case, which is a generalization of classical probability theory.

In the mathematical framework of Dempster-Shafer theory, it is required that an FOD must be a mutually exclusive and collectively exhaustive set, and the sum of basic probabilities of a BPA must be equal to 1, as shown in Eq. (4). When constructing an FCM, these two assumptions and constraints are inappropriate in many cases.

First, experts' estimates of the causal relationship between concepts are often qualitative, such as "very weak," "weak," and "strong," and then the fuzzy value is used to quantify the assessment in [0, 1]. Due to the arbitrary setting, there is inevitably an intersection between these qualitative categories. Therefore, the exclusion assumption cannot be accurately guaranteed.

Second, in some cases, due to a lack of knowledge and information, some experts have an unknown influence on certain causal relationships in FCMs and may obtain incomplete BPA with the sum of basic probabilities less than 1. Moreover, the incompleteness of FOD may also lead to the incompleteness of the BPA. Therefore, in some cases, it is difficult to guarantee that BPA is always equal to 1.

To solve these problems, the Dempster-Shafer theory is applied to enhance the ability to express the degree of influence of the causal relationship between concepts, give a more reasonable and comprehensive weight for the comprehensive knowledge of multiple experts, and systematically implement D-number theory [12].

Definition 5: Let U be a finite nonempty set; the D-number is a mapping

$$D : 2^U \to [0, 1]$$

such that

$$D(\emptyset) = 0 \quad \text{and} \sum_{B \in U} D(B) \leq 1 \tag{8}$$

where $\emptyset$ is an empty set and B is a subset of U.

Note that the elements in U of the D-number theory do not require mutual exclusivity, in contrast to the Ω of Dempster-Shafer theory. In D-number theory, BPA is not required to be equal to 1.

An example is given to show the difference between D-number theory and Dempster-Shafer theory as follows.

Suppose that in an FCM, an expert judges the causal influence intensity between two conceptual nodes C_1 and C_2 as follows: the possibility that the influence intensity is "weak (0.4)" is 0.7, and the influence intensity is the possibility of "strong (0.6)" is 0.1. The remaining 0.2 is unknown, and it may belong to one of the following categories: "very weak," "weak," and "strong." In the framework of Dempster-Shafer theory, the expert could give a BPA to express his estimate result by

$$m(0.4) = 0.7$$
$$m(0.6) = 0.1$$
$$m(0.2, 0.4, 0.6) = 0.2$$

However, if the expert gives his estimate result by using D-numbers, possible values are as follows.

$$m(0.4) = 0.7$$
$$m(0.6) = 0.1$$

Note that the set of $\{0.2, 0.4, 0.6\}$ is not actually an identification framework because the elements in the set of $\{0.2, 0.4, 0.6\}$ are not mutually exclusive, and BPA $=$ 0.8 based on the D-number theory. The example illustrates the difference between Dempster-Shafer and D-number theories.

E. CONSTRUCTING FCM BASED ON DEMPSTER-SHAFER THEORY

FCM has unique advantages for knowledge representation and reasoning. Its construction usually relies on the experience and knowledge of experts. To overcome the limitations of a single estimation, information from multiple experts is usually used to build systematic FCMs separately and then synthesize them. Dempster-Shafer theory, as an intelligent technology, provides a powerful tool for the synthesis of multiexpert knowledge. This method can incorporate the cognitive ability and comprehensive knowledge of multiple experts and, thus, effectively integrate the knowledge of multiple experts [30], [31].

Next, we introduce the specific synthetic formula of FCMs constructed by Dempster-Shafer theory and its importance.

Let Bel_1 and Bel_2 be two reliability functions on the same recognition framework, the corresponding basic reliability distributions are m_1 and m_2, respectively, and the focal elements are $X_1, X_2, \ldots, X_k$ and $Y_1, Y_2, \ldots, Y_k$, as shown in Figure 4.

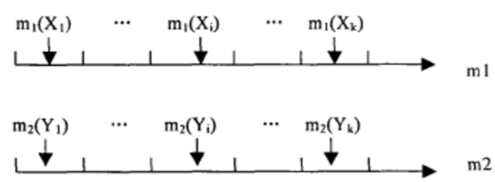

FIGURE 4. **Basic credibility distribution.**

The combination of the two types of evidence is as follows:

$$m(A) = m_1 \oplus m_2 = \begin{cases} 0, & A = \emptyset \\ \dfrac{1}{1-k} \displaystyle\sum_{X \cap Y = A} m_1(X)\, m_2(Y), & A \neq \emptyset \end{cases} \tag{9}$$

with

$$k = \sum_{X \cap Y = \emptyset} m_1(X)\, m_2(Y)$$

When multiple pieces of evidence are synthesized, a step-by-step method of synthesizing two at a time can be implemented:

$$Bel = \{[[Bel_1 \oplus Bel_2] \oplus Bel_3] \oplus \cdots\} \oplus Bel_n$$

It can also be solved by the formula given below.

Suppose $Bel_1, Bel_2, \ldots, Bel_n$ is the reliability function on the same recognition framework, and $m_1, m_2, \ldots, m_n$ is the corresponding basic reliability distribution. If

$$Bel_1 \oplus Bel_2 \oplus \cdots \oplus Bel_{\mathrm{n}}$$

exists and the basic reliability distribution is m, then the combination of n reliability functions is:

$$\begin{aligned} m(A) &= m_1 \oplus m_2 \oplus \cdots \oplus m_n \\ &= \begin{cases} 0, & A = \emptyset \\ \dfrac{\sum\limits_{A_1 \cap A_2 \cap \cdots \cap A_n} \prod\limits_{i=1}^{n} m_i(A_i)}{1-k}, & A \neq \emptyset \end{cases} \end{aligned} \tag{10}$$

where

$$k = \sum_{A_1 \cap A_2 \cap \cdots \cap A_n = \emptyset} \prod_{i=1}^{n} m_i(A_i).$$

III. COGNITIVE MAPS BASED ON D-NUMBER AND ITS APPLICATIONS

In this section, we systematically propose the graph structure of DCMs by introducing four aspects of DCMs, and illustrate the calculation method of edge weights in DCMs with an example.

A. D-NUMBER COGNITIVE MAPS MODEL

1) EDGE WEIGHT REPRESENTATION IN DCM

In the CMs, the knowledge and opinions of experts are incorporated via weight estimation, that is, an effective estimation of the degree of causal relationship between nodes in the reference concept set. Generally, due to the complexity of the relationships between concepts and the lack of expert knowledge and experience, the estimation of the influence relationship between nodes will be uncertain, and based on D-number theory, this can be captured by assigning a probability. If $\tilde{C}_i$ has a positive impact on $\tilde{C}_j$, it is expressed as $D\{1\}$; if it is a negative impact, it is expressed as $D\{-1\}$; if $\tilde{C}_i$ has no effect on $\tilde{C}_j$, it is expressed as $D\{0\}$.

As mentioned above, the degree of influence of $\tilde{C}_i$ on $\tilde{C}_j$ can be expressed by the basic probability assignment,

$$BPA_{ij} = \begin{pmatrix} D\{-1\} = a \\ D\{1\} = b \\ D\{0\} = c \end{pmatrix}$$

such that $a \geqslant 0$, $b \geqslant 0$, and $c \geqslant 0$.

According to this rule, the uncertainty of the causal relationship between two conceptual nodes can be expressed in a straightforward manner. The FCM of Figure 2 can be represented as a DCM, as shown in Figure 5, and the correlation matrix is as follows:

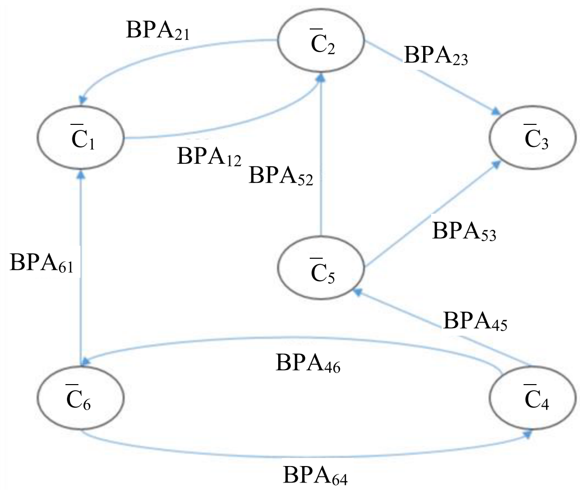

FIGURE 5. Examples of DCM.

$$W = \begin{array}{c} \\ \bar{C}_1 \\ \bar{C}_2 \\ \bar{C}_3 \\ \bar{C}_4 \\ \bar{C}_5 \\ \bar{C}_6 \end{array} \begin{array}{c} \begin{array}{cccccc} \tilde{C}_1 & \tilde{C}_2 & \tilde{C}_3 & \tilde{C}_4 & \tilde{C}_5 & \tilde{C}_6 \end{array} \\ \begin{bmatrix} 0 & BPA_{12} & 0 & 0 & 0 & 0 \\ BPA_{21} & 0 & BPA_{23} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & BPA_{45} & BPA_{46} \\ 0 & BPA_{52} & BPA_{53} & 0 & 0 & 0 \\ BPA_{61} & 0 & 0 & BPA_{64} & 0 & 0 \end{bmatrix} \end{array}$$

2) TRANSFORMATION USING THE BELIEF FUNCTION AND PLAUSIBILITY FUNCTION IN DCM

In order to evaluate the edge weights of the DCM in Figure 5, taking the information $\mathrm{BPA_{ij}}$ as an example. It can be denoted explicitly by

$$BPA_{ij} = \begin{pmatrix} D\{-1\} = a \\ D\{1\} = b \\ D\{0\} = c \end{pmatrix} \tag{11}$$

such that $a \geqslant 0$, $b \geqslant 0$, and $c \geqslant 0$. According to Eq. (5)

$$\begin{aligned} Bel\{-1\} &= D\{-1\} = a \\ Pl\{-1\} &= D\{-1\} + D\{u\} = 1 - b - c \end{aligned}$$

where u represents the case with unknown information, that is, $a + b + c \neq 1$. Therefore, the probability of a negative relationship between concept nodes is:

$$P\{-1\} = [Bel\{-1\}, \quad Pl\{-1\}] = [a, 1 - b - c]$$

The same can be obtained:

$$\begin{aligned} P\{1\} &= [b, 1 - a - c] \\ P\{0\} &= [c, 1 - a - b] \end{aligned}$$

Therefore, the connection weight between node i and node j is:

$$\begin{aligned} \tilde{\omega}_{ij} &= P\{1\} \times 1 \oplus P\{0\} \times 0 \oplus P\{-1\} \times (-1) \\ &= 1 \times [b, 1 - a - c] \oplus 0 \times [c, 1 - a - b] \end{aligned}$$

$$\begin{aligned} &\oplus (-1) \times [a, 1-b-c] \\ &= [b, 1-a-c] \oplus [-1+b+c, -a] \\ &= [2b+c-1, 1-2a-c] \end{aligned}$$

Therefore, the weight matrix between concepts is:

$$W = \begin{array}{c} \overline{C}_1 \\ \overline{C}_2 \\ \overline{C}_3 \\ \overline{C}_4 \\ \overline{C}_5 \\ \overline{C}_6 \end{array} \overset{\begin{array}{cccccc} \tilde{C}_1 & \tilde{C}_2 & \tilde{C}_3 & \tilde{C}_4 & \tilde{C}_5 & \tilde{C}_6 \end{array}}{\begin{bmatrix} 0 & \tilde{\omega}_{12} & 0 & 0 & 0 & 0 \\ \tilde{\omega}_{21} & 0 & \tilde{\omega}_{23} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{\omega}_{45} & \tilde{\omega}_{46} \\ 0 & \tilde{\omega}_{52} & \tilde{\omega}_{53} & 0 & 0 & 0 \\ \tilde{\omega}_{61} & 0 & 0 & \tilde{\omega}_{64} & 0 & 0 \end{bmatrix}}$$

3) FUSION OF UNCERTAIN CONFLICT INFORMATION IN DCM

In the process of knowledge gathering and integration, the opinions of experts may conflict with one another. The question of how to integrate the knowledge of various experts while maintaining consistency is an important one. In this paper, we adopt the fusion method of the reliability function based on evidence distance proposed by Deng Yong [32]. The main process is as follows:

Assuming that there are two pieces of information (R_i, D_i) and information (R_j, D_j), the distance between the two pieces of information can be calculated by the following algorithm, namely, $d(D_i, D_j)$.

$$d(D_1, D_2) = \sqrt{\frac{1}{2}(\vec{D}_1 - \vec{D}_2)^T C(\vec{D}_1 - \vec{D}_2)} \quad (12)$$

where D_i is a $(2^N \times 2^N)$-dimensional matrice.

The elements of C are:

$$C(A, B) = \frac{|A \cap B|}{|A \cup B|}, \quad A, B \in 2^N \quad (13)$$

$|A \cup B|$ represent the first element in the set $|A \cup B|$, as does the set $|A \cap B|$. For example, the distance between $\{-1\}$ and $\{1\}$ can be calculated as follows:

$$D(\{-1\}, \{1\}) = \frac{\emptyset}{\{-1, 1\}} = 0$$

The similarity between information (R_i, D_i) and information (R_j, D_j) is defined as S_{ij}:

$$S_{ij}(D_i, D_j) = 1 - d(D_i, D_j) \quad (14)$$

Suppose there are k pieces of information. When the similarity between all the information is obtained, we can construct a similarity measure matrix, which allows us to take into account the consistency between the information.

$$SMM = \begin{pmatrix} 1 & \cdots & S_{1k} \\ \vdots & \ddots & \vdots \\ S_{k1} & \cdots & 1 \end{pmatrix}$$

The support degree between information (R_i, D_i) $(i = 1, 2, \ldots, k)$ is defined as:

$$Sup(D_i) = \sum_{j=1, j\neq i}^{k} S_{ij}(D_i, D_j) \quad (15)$$

The credibility degree Crd_i of the information (R_i, D_i) $(i = 1, 2, \ldots, k)$ is defined as:

$$Crd_i = \frac{Sup(D_i)}{\sum_{i=1}^{k} Sup(D_i)} \quad (16)$$

It can be easily seen that $\sum_{i=1}^{k} Crd_i = 1$; thus, the credibility degree is a weight that represents the relative importance of the collected information.

If the maximum of the credibility degree ($MaxCrd_i$) of the information is 1, the discounting coefficient for the ith piece of information can be defined as:

$$\alpha = \frac{Crd_i}{MaxCrd_i}, \quad (i = 1, 2, \ldots, k) \quad (17)$$

If a source information m has credibility weight α, then the discounted evidence D' is defined in the identification frame Θ as:

$$\begin{aligned} D' &= \alpha D(A), \quad \forall A \in \Theta, \ A \neq \Theta \\ D'(\Theta) &= 1 - \alpha + \alpha D(\Theta) \end{aligned} \quad (18)$$

After that, the discounted information is combined into the final fusion result through the D-number combination rule. This method is applied in specific examples in the following sections.

4) INFORMATION COGNITIVE MAP DYNAMIC FUNCTION IN DCM

DCMs can simulate dynamic system operating conditions. If the initial state of each concept node of the system is given, the state value of any concept node at any time can be calculated by the DCM conversion function.

$$\tilde{A}_j^t = f\left(k_{D1} \bigoplus_{i=1, i\neq j}^{n} (\tilde{A}_i^{t-1} \otimes \tilde{\omega}_{ij}) \oplus k_{D2}\tilde{A}_j^{t-1}\right)$$
$$0 \leqslant k_{D1} \leqslant 1, \quad 0 \leqslant k_{D2} \leq 1 \quad (19)$$

Let $\tilde{C} = \{\tilde{C}_1, \tilde{C}_2, \ldots, \tilde{C}_N\}$ be the set of conceptual nodes of the system, N is the number of conceptual nodes contained in the system, $\tilde{A}_i^t (i = 1, 2, \ldots, N)$ represents the state value of the ith node $\tilde{C}_i$ at time t, and $\tilde{\omega}_{ij}$ represents the causal influence of the jth node $\tilde{C}_j$ on the ith node $\tilde{C}_i$. The weight is converted from the fused information BPA_{ij} through reliability and likelihood. If

$$BPA_{ij} = \begin{pmatrix} D\{-1\} = a \\ D\{1\} = b \\ D\{0\} = c \end{pmatrix}$$

Then the converted weight is:

$$\tilde{\omega}_{ij} = [2b + c - 1, 1 - 2a - c]$$

where $a \geqslant 0, b \geqslant 0$, and $c \geqslant 0$. f is the threshold function, as shown in Figure 6, and its purpose is to convert the result of

$$k_{D1} \bigoplus_{i=1, i \neq j}^{n} (\tilde{A}_i^{t-1} \otimes \tilde{\omega}_{ij}) \oplus k_{D2} \tilde{A}_j^{t-1}$$

The initial system value of each conceptual node of the system is the interval number $\left[a^-, a^+\right]$, $a^- \leqslant a^+$.

① $a^- \in [0, 1]$ and $a^+ \in [0, 1]$

② $a^- \in [-1, 1]$ and $a^+ \in [-1, 1]$

We denote $\tilde{x}_i^t$ is converted by the threshold function f as $\tilde{A}_i^t$.

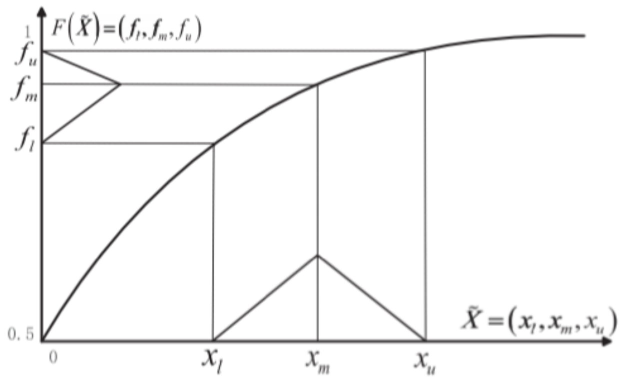

FIGURE 6. **Threshold function.**

In above formula, the k_{D1} and k_{D2} have the same physical meaning and represent the sum of all known fundamental probabilities (not necessarily 1). In this paper, we use the sigmoid function as the threshold function:

$$f\left(\tilde{X}\right) = \frac{1}{1 + e^{-\lambda \tilde{X}}} \tag{20}$$

where $\tilde{X}$ is an interval number, λ determines the speed of convergence, and the output of the function f is also an interval number. It can represent concept nodes and uncertain information between nodes.

From the above four points, it can be seen that DCMs, similar to ECMs and FCMs, are also a directed graph with feedback, consisting of concept nodes that describe the behaviour of the system and weighted arcs that represent the causal relationships that exist between the concepts. Each concept node $\tilde{C}_i$ has an interval $\tilde{a}_i$ representing its value obtained by the transformation of the fuzzy value of the system variable. Compared with FCMs, the representation of this concept is more flexible and practical than the representation using concrete, specific numbers. In contrast to ECMs, DCMs do not require the interval mutual exclusion of concept nodes and a cognitive map can be constructed even with incomplete information due to a limited amount of knowledge, thus providing a solution in the face of uncertainties in real life situations.

In summary, the above process can be divided into the following 5 steps:

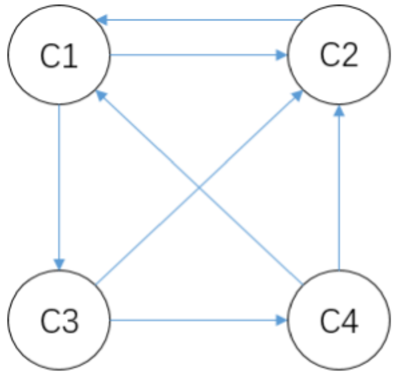

FIGURE 7. **National economic impact factors.**

Step 1: Calculate the distance matrix d of the evidences;

Step 2: Calculate the similarity matrix SMM of the evidences;

Step 3: Calculate the credibility degree Crd of the evidences;

Step 4: Calculate the discounting coeffificient a;

Step 5: Combine the evidences using D-number theory.

B. AN EXAMPLE OF ECONOMIC EVALUATION

Example 2: The economy is an important criterion for measuring the strength of a country. The national economy is manifested not only in its numerous and complex elements, but also in the mutual influence and interaction among the various elements of the system. Each element involves not only precise statistical information but also a large amount of uncertain information. The system is required not only to be able to model existing elements but also to be able to effectively expand to include new elements. The analysis of such a complex system requires new models that are compatible and account for these features.

Suppose this system has a total of 4 components, which are represented in the system as follows:

C_1: education C_2: science and technology

C_3: population C_4: labour

Assuming that three experts provide information, as shown in Figure 7, the following is a brief introduction to the construction process of the following DCM. Taking nodes C_1 and C_2 as an example, the relationship between nodes C_1 and C_2 is given by three experts, denoted E_1, E_2, and E_3, respectively, as follows:

$$E_1 : D_1\{-1\} = 0.6, \quad D_1\{1\} = 0.3,$$
$$D_1\{0\} = 0, \quad D_1\{-1, 1, 0\} = 0.1$$
$$E_2 : D_2\{-1\} = 0.7, \quad D_2\{1\} = 0,$$
$$D_2\{0\} = 0.1, \quad D_2\{-1, 1, 0\} = 0.2$$
$$E_3 : D_3\{-1\} = 0.2, \quad D_3\{1\} = 0.5,$$
$$D_3\{0\} = 0.1, \quad D_3\{-1, 1, 0\} = 0.2$$

The next five steps illustrate how to integrate multple experts knowledge in DCMs to calculate the degree of influence between nodes C_1 and C_2.

Step 1. Calculate the distance matrix between 3 pieces of information.

$$d = \begin{pmatrix} 0 & 0.11 & 0.21 \\ 0.11 & 0 & 0.50 \\ 0.21 & 0.50 & 0 \end{pmatrix}$$

For example, the calculation method of $d_{12} = 0.11$ is as follows:

$$\vec{D}_1 = \begin{pmatrix} 0.6 \\ 0.3 \\ 0 \\ 0.1 \end{pmatrix}, \quad \vec{D}_2 = \begin{pmatrix} 0.7 \\ 0 \\ 0.1 \\ 0.2 \end{pmatrix}, \quad \vec{D}_1 - \vec{D}_2 = \begin{pmatrix} -0.1 \\ 0.3 \\ -0.1 \\ -0.1 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & \frac{1}{3} \\ 0 & 0 & 1 & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 1 \end{pmatrix}$$

Step 2. Calculate the similarity matrix of matrix d

$$SMM = \begin{pmatrix} 1 & 0.89 & 0.79 \\ 0.89 & 1 & 0.50 \\ 0.79 & 0.50 & 1 \end{pmatrix}$$

Step 3. The credibility degree of the evidence is

$$Sup = (2.68, 2.39, 2.29)$$
$$Crd = (0.36, 0.32, 0.31)$$

Step 4. The discounting coefficient is

$$\alpha = (1.00, 0.89, 0.86)$$

Then the discounted expert information is

$$\begin{aligned} E_1 : D_1\{-1\} &= 0.6, \quad D_1\{1\} = 0.3, \\ D_1\{0\} &= 0, \quad D_1\{-1, 1, 0\} = 0.1 \\ E_2 : D_2\{-1\} &= 0.62, \quad D_2\{1\} = 0, \\ D_2\{0\} &= 0.09, \quad D_2\{-1, 1, 0\} = 0.18 \\ E_3 : D_3\{-1\} &= 0.17, \quad D_3\{1\} = 0.43, \\ D_3\{0\} &= 0.09, \quad D_3\{-1, 1, 0\} = 0.17 \end{aligned}$$

Step 5. Now we combine the information from these three experts using D-number theory.

First, we combine the information of Expert 1 and Expert 2.

$$\begin{aligned} Q_1 &= D_1\{-1\} + D_1\{1\} + D_1\{0\} = 0.9 \\ Q_2 &= D_2\{-1\} + D_2\{1\} + D_2\{0\} = 0.71 \\ K_{D_{12}} &= \frac{1}{Q_1 Q_2} \times (D_1\{-1\} D_2\{1\} + D_1\{-1\} D_2\{0\} \\ &\quad + D_1\{1\} D_2\{-1\} + D_1\{1\} D_2\{0\} \\ &\quad + D_1\{0\} D_2\{-1\} + D_1\{0\} D_2\{1\}) \\ &= 0.42 \end{aligned}$$

$$\begin{aligned} D_{12}\{-1\} &= \frac{1}{1 - K_{D_{12}}} D_1\{-1\} D_2\{-1\} = 0.64 \\ D_{12}\{1\} &= \frac{1}{1 - K_{D_{12}}} D_1\{1\} D_2\{1\} = 0 \\ D_{12}\{0\} &= \frac{1}{1 - K_{D_{12}}} D_1\{0\} D_2\{0\} = 0 \end{aligned}$$

with $Q_{12} = 0.64$

Then the result is combined with the information of Expert 3.

$$\begin{aligned} Q_3 &= D_3\{-1\} + D_3\{1\} + D_3\{0\} = 0.69 \\ K_{D_{123}} &= \frac{1}{Q_{12} Q_2} \times (D_{12}\{-1\} D_3\{1\} + D_{12}\{-1\} D_3\{0\} \\ &\quad + D_{12}\{1\} D_3\{-1\} + D_{12}\{1\} D_3\{0\} \\ &\quad + D_{12}\{0\} D_3\{-1\} + D_{12}\{0\} D_3\{1\}) \\ &= 0.04 \end{aligned}$$

$$\begin{aligned} D_{123}\{-1\} &= \frac{1}{1 - K_{D_{123}}} D_{12}\{-1\} D_3\{-1\} = 0.11 \\ D_{123}\{1\} &= \frac{1}{1 - K_{D_{123}}} D_{12}\{1\} D_3\{1\} = 0 \\ D_{123}\{0\} &= \frac{1}{1 - K_{D_{123}}} D_{12}\{0\} D_3\{0\} = 0 \end{aligned}$$

with $Q_{12} = 0.11$

Therefore, the final fusion information weight of C_1 and C_2 is

$$BPA_{12} = [0.11, 0, 0]$$

The result of $BPA_{12} = [0.11, 0, 0]$ shows that after fusing the uncertain information of three experts, the influence degree of C_1 on C_2 can be expressed in DCMs as follows: $D\{-1\} = 0.11$, $D\{1\} = 0$, $D\{0\} = 0$. It can be seen that DCMs solves the intersection of node intervals that cannot be solved in FCMs and DCMs and the incomplete mastery of information due to the limited level of knowledge. The application of step 1, step 4 and step 5 enables DCMs can deal with language fuzzy variables more reasonably and accurately, and can aggregate the knowledge of multiple experts.

IV. DFCM: AN EXTENDED FUZZY COGNITIVE MAPS BASED ON D-NUMBER

Considering that D-number not only have the ability to represent uncertainty, but also help to aggregate knowledge from different experts. Therefore, this section proposes a FCM construction method based on D-number theory that integrates multiple experts knowledge, and uses an example to illustrate how to synthesize the causal influence between two concept nodes based on multiple experts knowledge when constructing DFCMs. It can effectively solve the comprehensive problem of multiple experts knowledge.

A. MULTIEXPERT KNOWLEDGE SYNTHESIS BASED ON FCM KNOWLEDGE REPRESENTATION

1) PROBLEM DESCRIPTION

According to the formal definition of FCMs, the opinions of experts are reflected in the estimation of the strength of the causal relationship between the nodes of a given concept set, that is, the estimation of the weights. Therefore, the problem of expert knowledge synthesis is expressed in FCMs as the combination of corresponding elements in the adjacency matrix provided by each expert. This problem can be regarded as a multiexpert opinion aggregation problem, where each expert's assessment of a causal relationship based on his own knowledge and experience can be regarded as a piece of evidence, and the possible values of the degree of influence of the causal relationship between concepts constitute an identification framework, the membership degree of the fuzzy value evaluated by experts on a certain weight i used to determine the reliability distribution function m, and the synthesized reliability distribution function is used as the basis for the final weight synthesis.

2) MULTIEXPERT KNOWLEDGE SYNTHESIS

Under normal circumstances, there is a reasonable degree of consensus among experts based on professional knowledge, with deviations due to personal preferences, knowledge structure, and other reasons, which is reflected in the FCM model. The number of rows and columns included in the adjacency matrix is different; that is, the selected concept sets are different, and the description standards of the degree of causal influence are also different, which brings certain difficulties to the synthesis of expert knowledge, so standardization prior to the FCM synthesis is required.

There are n experts and, for a decision problem, they each establish their own FCM according to their experience and knowledge, and their adjacency matrices are denoted $F_1, F_2, \ldots, F_m$. The union of all the different concepts involved by each expert is taken as a concept set, and the adjacency matrix of each expert is extended to the dimension $m \times m$. The row and column of the original non-existing concept node are filled with zeros, and this process is called the normalization of the adjacency matrix.

Using D-number theory, the general process of fusing the FCMs of multiple experts is:

1. First, define the identification framework so that our research on propositions can be transformed into the research on sets;
2. Establish basic reliability assignments based on evidence;
3. Combine the basic reliability distribution functions according to the combination rules of the D-number theory, and then use the judgement principle based on the basic reliability assignment to determine the target type;
4. According to the comprehensive reliability, calculate the weighted average of each element in the framework.

3) SYNTHETIC COMPUTATIONAL COMPLEXITY ANALYSIS

Suppose the identification frame is $\Omega = \{\theta_1, \theta_2, \ldots, \theta_n\}$, and k experts provide k pieces of evidence. In the extreme case, each set of evidence has $2^n - 1$ mass function assignments. Namely: $m(\{\theta_1\})$, $m(\{\theta_2\})$, $\cdots$, $m(\{\theta_1, \theta_2\})$, $\cdots$, $m(\{\theta\})$. In this case, the complexity of the information is $O(k \times 2^n)$. The computational complexity of combining the knowledge of k experts with the two-evidence synthesis formula and the multievidence synthesis formula will be discussed separately below. For the synthesis formula, the main operation is the multiplication of two mass function values, so the computational complexity is $(2^n - 1) \times (2^n - 1) = O(2^{2n})$. For the knowledge of $k \times O\left(2^{2n}\right) = O\left(k \times 2^{2n}\right)$ experts, the computational complexity is k. For the multievidence synthesis formula, the main operation is k names. The mass function values are multiplied, so the computational complexity is $(2^n - 1)^k$, so the computational complexity is $O\left(2^{kn}\right)$.

B. CONSTRUCTING FCM BASED ON D-NUMBER THEORY

Definition 6: Let D be a D-number on a finite nonempty set U; the degree of information completeness in D is quantified by

$$Q = \sum_{B \in U} D(B) \tag{21}$$

In Dempster-Shafer theory, Dempster's rule of combination plays a central role in synthesizing all the knowledge of the initial BPAs. Correspondingly, in D-numbers theory, which is treated as a generalization of Dempster-Shafer theory, a D-numbers combination rule is proposed to combine the information indicated by D-numbers.

Definition 7: Let D_1 and D_2 be two D-numbers; the combination of D_1 and D_2, indicated by $D = D_1 \odot D_2$, is defined by

$$\begin{cases} D(\emptyset) = 0 \\ D(B) = \dfrac{1}{1-K_D} \displaystyle\sum_{B_1 \cap B_2 = \mathrm{B}} D_1(B_1) D_2(B_2), \quad B \neq \emptyset \end{cases} \tag{22}$$

where

$$\begin{aligned} K_D &= \frac{1}{Q_1 Q_2} \sum_{B_1 \cap B_2 = \emptyset} D_1(B_1) D_2(B_2) \\ Q_1 &= \sum_{B_1 \in U} D_1(B_1) \\ Q_2 &= \sum_{B_2 \in U} D_2(B_2) \end{aligned} \tag{23}$$

Similar to the Dempster-Shafer theory, when multiple expert evidence is synthesized, a step-by-step method of synthesizing two at a time can be implemented:

$$D = \{[[D_1 \odot D_2] \odot D_3] \odot \cdots\} \odot D_{\mathrm{n}} \tag{24}$$

It can also be solved by the formula given below:

$$\begin{aligned} D(B) &= D_1 \odot D_2 \odot \cdots \odot D_n \\ &= \begin{cases} 0, & B = \emptyset \\ \dfrac{\displaystyle\sum_{B_1 \cap B_2 \cap \cdots \cap B_n} \prod_{i=1}^{n} D_i(B_i)}{1 - K_D}, & B \neq \emptyset \end{cases} \end{aligned} \tag{25}$$

where

$$K_D = \frac{1}{\prod_{i=1}^{n} Q_i} \sum_{B_1 \cap B_2 \cap \cdots \cap B_n = \emptyset} \prod_{i=1}^{n} D_i(B_i)$$
$$Q_i = \sum_{B_i \in U} D_i(B_i) \quad i = 1, 2, \ldots, n \tag{26}$$

With the proposed combination rule of D-number is actually the generalization of multi-expert knowledge based on Dempster-Shafer theory. If we define D_1 and D_2 on FOD and $Q_1 = 1$, $Q_2 = 1$, the combination rule of the D-number will become the combination rule of Dempster-Shafer theory.

The construction process of DFCM can be summarized as the following 4 steps:

Step 1: Calculate the membership degree distribution of the evaluation value of the weight given by each expert;

Step 2: Calculate the K_D and $D(B)$ from the membership degree distribution of any two experts;

Step 3: Continue to iterate, and carry out knowledge synthesis based on D-number for other experts in turn;

Step 4: Calculate the final weight.

V. COMPARATIVE ANALYSIS

In this section, we compare and analyze DFCM with existing research through 2 examples. Example 4 and Example 5 are compared to illustrate the differences between the FCM construction method based on Dempster-Shafer theory and the FCM construction method based on D-number theory, so as to highlight that DFCMs is more suitable for dealing with incomplete and uncertain information.

A. AN EXAMPLE OF FCM CONSTRUCTION METHOD BASED ON DEMPSTER-SHAFER THEORY

Remark 3: The membership degrees given in Examples 4 and 5 are obtained according to the evaluation results of a certain weight on the framework by expert i. The following Example 3 is the expert 1 of Example 4 to illustrate the determination of the reliability distribution function.

Example 3: Expert 1's evaluation value of the weight between C_1 and C_2 is 0.46, and the membership function is:

$$\mu_0(x) = \begin{cases} \frac{0.2-x}{0.2}, & 0 < x \leqslant 0.2 \\ 0, & others \end{cases}$$

$$\mu_{0.2}(x) = \begin{cases} \frac{0.2-x}{0.2}, & 0 < x \leqslant 0.2 \\ \frac{0.4-x}{0.2}, & 0.2 < x \leqslant 0.4 \\ 0, & others \end{cases}$$

$$\mu_{0.4}(x) = \begin{cases} \frac{x-0.2}{0.2}, & 0.2 < x \leqslant 0.4 \\ \frac{0.6-x}{0.2}, & 0.4 < x \leqslant 0.6 \\ 0, & others \end{cases}$$

$$\mu_{0.6}(x) = \begin{cases} \frac{x-0.4}{0.2}, & 0.4 < x \leqslant 0.6 \\ \frac{0.8-x}{0.2}, & 0.6 < x \leqslant 0.8 \\ 0, & others \end{cases}$$

$$\mu_{0.8}(x) = \begin{cases} \frac{x-0.6}{0.2}, & 0.6 < x \leqslant 0.8 \\ \frac{1.0-x}{0.2}, & 0.8 < x \leqslant 1.0 \\ 0, & others \end{cases}$$

$$\mu_{1.0}(x) = \begin{cases} \frac{1.0-x}{0.2}, & 0.8 < x \leqslant 1.0 \\ 0, & others \end{cases}$$

So the weight of 0.46 has a membership of 0.7 in "weak (0.4)" and a membership of 0.3 in "strong (0.6)," so the reliability distribution is: $m = [0, 0, 0.7, 0.3, 0, 0]$.

Example 4: Three experts have the following judgements (see Remark 3) on the degree of causal influence between the conceptual nodes C_1 and C_2 in the FCM, as shown in Figure 1.

condition / expert	0	0.2	0.4	0.6	0.8	1
expert 1	0	0	0.7	0.3	0	0
expert 2	0	0.1	0.8	0.1	0	0
expert 3	0	0	0.9	0.1	0	0

Step 1. We calculate the composite result for expert 1 and expert 2:

$$\begin{aligned} k_{12} &= \sum_{A_1 \cap A_2 = \emptyset} m_1(A_1) m_2(A_2) \\ &= m_1(0.4) m_2(0.2) + m_1(0.4) m_2(0.6) \\ &\quad + m_1(0.6) m_2(0.2) + m_1(0.6) m_2(0.4) \\ &= 0.7 \times 0.1 + 0.7 \times 0.1 + 0.3 \times 0.1 + 0.3 \times 0.8 \\ &= 0.41 \end{aligned}$$

$$\begin{aligned} m_{12}(0.4) &= \frac{1}{1-k_{12}} \sum_{A_1 \cap A_2 = 0.4} m_1(0.4) m_2(0.4) \\ &= \frac{1}{1-0.41} \times (0.7 \times 0.8) \\ &= 0.95 \end{aligned}$$

$$\begin{aligned} m_{12}(0.6) &= \frac{1}{1-k_{12}} \sum_{A_1 \cap A_2 = 0.6} m_1(0.6) m_2(0.6) \\ &= \frac{1}{1-0.41} \times (0.3 \times 0.1) \\ &= 0.05 \end{aligned}$$

The synthesized result 1 of expert 1 and expert 2 is:

condition / expert	0	0.2	0.4	0.6	0.8	1
expert 1	0	0	0.7	0.3	0	0
expert 2	0	0.1	0.8	0.1	0	0
result 1	0	0	0.95	0.05	0	0

Step 2. We synthesize the results from expert 1 and expert 2 with expert 3:

$$\begin{aligned} k_{123} &= \sum_{A_{12}\cap A_3=\emptyset} m_{12}(A_{12})\, m_3(A_3) \\ &= m_{12}(0.4)\, m_3(0.6) \\ &\quad + m_{12}(0.6)\, m_3(0.4) \\ &= 0.95\times 0.1 + 0.05\times 0.9 \\ &= 0.14 \\ m_{123}(0.4) &= \frac{1}{1-k_{123}} \sum_{A_{12}\cap A_3=0.4} m_{12}(0.4)\, m_3(0.4) \\ &= \frac{1}{1-0.41}\times(0.95\times 0.9) \\ &= 0.9942 \\ m_{123}(0.6) &= \frac{1}{1-k_{123}} \sum_{A_{12}\cap A_3=0.6} m_{12}(0.6)\, m_3(0.6) \\ &= \frac{1}{1-0.41}\times(0.05\times 0.1) \\ &= 0.0058 \end{aligned}$$

The synthesized result of expert 1 and expert 2 with expert 3 is:

condition / expert	0	0.2	0.4	0.6	0.8	1
result 1	0	0	0.95	0.05	0	0
expert 3	0	0	0.9	0.1	0	0
result 2	0	0	0.9942	0.0058	0	0

Step 3. From the results of the above table, we can obtain the following: the result of the synthesis of the knowledge of the three experts is that the degree of causal influence is 0.4 with a reliability of 0.9942, and the degree of causal influence is 0.6 with a reliability of 0.0058. According to the final composite reliability, find the composite weight:

$$\omega_{12} = 0.4\times 0.9942 + 0.6\times 0.0058 = 0.4012$$

Therefore, the degree of causal influence between the two conceptual nodes C_1 and C_2 of the constructed FCM is $\omega_{12} = 0.4012$.

B. AN EXAMPLE OF FCM CONSTRUCTION METHOD BASED ON D-NUMBER THEORY AND ITS ADVANTAGE ANALYSIS

Example 5: Three experts have the following judgements on the degree of causal influence between the conceptual nodes C_4 and C_5 in the FCM, as shown in Figure 1.

condition / expert	0	0.2	0.4	0.6	0.8	1
expert 1	0	0	0.7	0.1	0	0
expert 2	0	0.1	0.8	0	0	0
expert 3	0	0	0.6	0.2	0	0

Expert 1 said that the probability that the degree of influence between these two nodes is weak (0.4) is 0.7, the probability that the degree of influence is medium (0.6) is 0.1, and the probability that the degree of influence is 0.2 is unknown. Expert 2 said that the probability that the degree of influence between these two nodes is very weak (0.2) is 0.1, the probability that the degree of influence is weak (0.4) is 0.8, and the probability of 0.1 is unknown. Expert 3 said that the probability that the degree of influence between these two nodes is weak (0.4) is 0.6, the probability that the degree of influence is medium (0.6) is 0.2, and the probability that the degree of influence is 0.2 is unknown.

We first consider constructing the influence degree relationship between the conceptual nodes C_4 and C_5. FCM based on the Dempster-Shafer theory.

Step 1. We first calculate the composite result for expert 1 and expert 2 based on Dempster-Shafer theory. According to the analysis of these three experts, two BPAs can be obtained

$$\begin{aligned} &m_1(0.4) = 0.7, \quad m_1(0.6) = 0.1, \quad m_1(S) = 0.2 \\ &m_2(0.2) = 0.1, \quad m_2(0.4) = 0.8, \quad m_2(S) = 0.1 \\ &m_3(0.4) = 0.6, \quad m_3(0.6) = 0.2, \quad m_3(S) = 0.2 \end{aligned}$$

where $S = (0, 0.2, 0.4, 0.6, 0.81)$.

$$\begin{aligned} k_{12} &= \sum_{A_1\cap A_2=\emptyset} m_1(A_1)\, m_2(A_2) \\ &= m_1(0.4)\, m_2(0.2) + m_1(0.6)\, m_2(0.2) \\ &\quad + m_1(0.6)\, m_2(0.4) \\ &= 0.7\times 0.1 + 0.1\times 0.1 + 0.1\times 0.8 \\ &= 0.16 \\ m(0.2) &= \frac{1}{1-k_{12}} \sum_{A_1\cap A_2=0.2} m_1(0.2, S)\, m_2(0.2, S) \\ &= \frac{1}{1-0.16}\times(0.2\times 0.1) \\ &= 0.0238 \\ m(0.4) &= \frac{1}{1-k_{12}} \sum_{A_1\cap A_2=0.4} m_1(0.4, S)\, m_2(0.4, S) \\ &= \frac{1}{1-0.16}\times(0.7\times 0.8 + 0.7\times 0.1 + 0.8\times 0.2) \\ &= 0.9405 \\ m(0.6) &= \frac{1}{1-k_{12}} \sum_{A_1\cap A_2=0.6} m_1(0.6, S)\, m_2(0.6, S) \\ &= \frac{1}{1-0.16}\times(0.1\times 0.1) \\ &= 0.0119 \end{aligned}$$

The synthesized result 1 of expert 1 and expert 2 is:

condition / expert	0	0.2	0.4	0.6	0.8	1
expert 1	0	0	0.7	0.1	0	0
expert 2	0	0.1	0.8	0	0	0
result 1	0	0.0238	0.9405	0.0119	0	0

Step 2. We synthesize the results from expert 1 and expert 2 with expert 3 based on Dempster-Shafer theory:

$$\begin{aligned}
k_{123} &= \sum_{A_{12}\cap A_3=\emptyset} m_{12}(A_{12})\, m_3(A_3) \\
&= m_{12}(0.2)\, m_3(0.4) + m_{12}(0.2)\, m_3(0.6) \\
&\quad + m_{12}(0.4)\, m_3(0.6) + m_{12}(0.6)\, m_3(0.4) \\
&= 0.0238 \times 0.6 + 0.0238 \times 0.2 + 0.9405 \times 0.2 \\
&\quad + 0.0119 \times 0.6 \\
&= 0.2143 \\
m(0.2) &= \frac{1}{1-k_{123}} \sum_{A_{12}\cap A_3=0.2} m_{12}(0.2, S)\, m_3(0.2, S) \\
&= \frac{1}{1-0.2143} \times (0.2 \times 0.0238) \\
&= 0.0061 \\
m(0.4) &= \frac{1}{1-k_{123}} \sum_{A_{12}\cap A_3=0.4} m_{12}(0.4, S)\, m_3(0.4, S) \\
&= \frac{1}{1-0.2143} \times (0.9405 \times 0.6 + 0.9405 \times 0.2 \\
&\quad + 0.0238 \times 0.6) \\
&= 0.9758 \\
m(0.6) &= \frac{1}{1-k_{123}} \sum_{A_{12}\cap A_3=0.6} m_{12}(0.6, S)\, m_3(0.6, S) \\
&= \frac{1}{1-0.2143} \times (0.0119 \times 0.2 + 0.0119 \times 0.2 \\
&\quad + 0.0238 \times 0.2) \\
&= 0.0121
\end{aligned}$$

The synthesized result of expert 1 and expert 2 with expert 3 is:

condition / expert	0	0.2	0.4	0.6	0.8	1
result 1	0	0.0238	0.9405	0.0119	0	0
expert 3	0	0	0.6	0.2	0	0
result 2	0	0.0061	0.9758	0.0121	0	0

Step 3. Therefore, the degree of causal influence between the two conceptual nodes C_4 and C_5 of FCM based on Dempster-Shafer theory is:

$$\begin{aligned}
\omega_{45} &= 0.2 \times 0.0061 + 0.4 \times 0.9758 + 0.6 \times 0.0121 \\
&= 0.3998
\end{aligned}$$

In the above calculation process, there is an unreasonable assumption; that is, the unknown possibility is attributed to the possibility equal to {0, 0.2, 0.4, 0.6, 0.8, 1}. In other words, the degree of causal influence between the two conceptual nodes of C_4 and C_5 is equivalent to the set {0, 0.2, 0.4, 0.6, 0.8, 1}, containing only these six possible weights. Obviously, this is not convincing. In fact, implicit assumptions are almost ignored in applications based on Dempster-Shafer theory to reduce complexity.

Next, let us consider this problem with D-number theory, which can handle the situation of incomplete information.

Step 1. We first calculate the composite result for expert 1 and expert 2 based on D-number theory. According to the analysis of these three experts, three D-numbers can be obtained

$$\begin{aligned}
D_1(0.4) &= 0.7, \quad D_1(0.6) = 0.1 \\
D_2(0.2) &= 0.1, \quad D_2(0.4) = 0.8 \\
D_3(0.4) &= 0.6, \quad D_3(0.6) = 0.2
\end{aligned}$$

It is worth noting that the unknown information of the three experts here is not assigned to any set, the sum of their respective basic probabilities is not equal to 1, and the three D-numbers constructed are incomplete forms of information.

$$\begin{aligned}
Q_1 &= D_1(0.4) + D_1(0.6) = 0.8 \\
Q_2 &= D_2(0.2) + D_2(0.4) = 0.9 \\
Q_3 &= D_3(0.4) + D_3(0.6) = 0.8 \\
K_{D_{12}} &= \frac{1}{Q_1 Q_2} \sum_{B_1\cap B_2=\emptyset} D_1(B_1)\, D_2(B_2) \\
&= \frac{1}{0.8 \times 0.9} \times [D_1(0.4)\, D_2(0.2) \\
&\quad + D_1(0.6)\, D_2(0.2) + D_1(0.6)\, D_2(0.4)] \\
&= \frac{1}{0.8 \times 0.9} \times (0.7 \times 0.1 + 0.1 \times 0.1 + 0.1 \times 0.8) \\
&= 0.2222 \\
D(0.4) &= \frac{1}{1-K_{D_{12}}} \sum_{B_1\cap B_2=B} D_1(B_1)\, D_2(B_2) \\
&= \frac{1}{1-K_{D_{12}}} D_1(0.4)\, D_2(0.4) \\
&= \frac{1}{1-0.2222} \times (0.7 \times 0.8) \\
&= 0.72
\end{aligned}$$

The synthesized result 1 of expert 1 and expert 2 is:

condition / expert	0	0.2	0.4	0.6	0.8	1
expert 1	0	0	0.7	0.1	0	0
expert 2	0	0.1	0.8	0	0	0
result 1	0	0	0.72	0	0	0

Step 2. We synthesize the results from expert 1 and expert 2 with expert 3 based on D-number theory.

$$\begin{aligned}
K_{D_{123}} &= \frac{1}{Q_1 Q_2 Q_3} \sum_{B_{12}\cap B_2=\emptyset} D_{12}(B_{12})\, D_3(B_3) \\
&= \frac{1}{0.8 \times 0.9 \times 0.8} \times [D_{12}(0.4)\, D_2(0.6)] \\
&= \frac{1}{0.8 \times 0.9} \times (0.72 \times 0.2) \\
&= 0.2 \\
D(0.4) &= \frac{1}{1-K_{D_{123}}} \sum_{B_{21}\cap B_3=B} D_{12}(B_{12})\, D_3(B_3)
\end{aligned}$$

$$= \frac{1}{1 - K_{D_{123}}} D_{12}(0.4)\, D_3(0.4)$$
$$= \frac{1}{1 - 0.2} \times (0.72 \times 0.6)$$
$$= 0.54$$

The synthesized result of expert 1 and expert 2 with expert 3 is:

condition / expert	0	0.2	0.4	0.6	0.8	1
result 1	0	0.72	0	0	0	0
expert 3	0	0	0.6	0.2	0	0
result 2	0	0	0.54	0	0	0

Step 3. Therefore, the degree of causal influence between the two conceptual nodes C_4 and C_5 of FCM based on the D-number is:

$$\omega_{45} = 0.4 \times 0.54 = 0.216$$

Comparing the two processing procedures and results, it is evident that, in contrast with the Dempster-Shafer theory, in the D-number theory proposed, the size of the weight representing the unknown situation is inherited in the inference process. D-number theory has inherent advantages in dealing with incomplete information. It relaxes the exclusive assumption of FOD and BPA integrity constraints in Dempster-Shafer theory, making the resulting FCM more natural and reasonable. And it can be seen from Example 5 that the calculation method of DFCM is more concise and clear.

VI. CONCLUSION

This paper analyses the deficiency of uncertain information representation and processing ability of ECMs and FCMs. Combined with the uncertain information representation framework of D-number theory and the latest research results of D-number theory in conflict information processing, a new cognitive graph model DCMs is proposed. DCMs expand the ability of ECMs to express uncertain information and have the flexibility to express and process uncertain information. As an intelligent technology, Dempster-Shafer theory in ECMs provides a powerful tool for integrating multi expert knowledge to obtain scientific calculation results, but its FOD exclusivity assumption and BPA integrity constraints cannot deal with uncertain information. Therefore, this paper systematically puts forward a new theory of constructing DCMs. In D-number theory, D-number is the extension of BPA and D-number combination rule that is an extension of the Dempster-Shafer combination rule.

When constructing FCMs, neither intuitionistic fuzzy cognitive maps (IFCMs) [33], [34] nor interval-valued fuzzy cognitive maps (IVFCMs) [35] fully consider the problem of knowledge composition. Therefore, this paper proposes a new theory for constructing DFCMs. DFCMs make full use of the characteristics of FCMs, use imprecise reasoning methods, effectively integrate the knowledge and experience of experts, and more intuitively reflect the comprehensive situation of many experts' opinions.

Future research will study other approaches to FCMs and ECMs, such as fuzzy logic, expert systems, and uncertainty theory. Additionally, we will further study the properties of D-number theory and extend it to more practical applications.

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