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Parametric Robust Generalized Minimum Variance Control

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ABSTRACT This work presents a hybridization of the Generalized Minimum Variance Control (GMVC) with Parametric Robust Control (RPC) for Linear Time-Invariant (LTI) Uncertain Systems. A linear parametric uncertainties model describes the system's dynamic behavior. The controller synthesis is based on a PID controller with a low-pass filter and formulated as a convex optimization problem that considers the desired closed-loop performance and the uncertainties of the model parameters. The robust controller gains represent the best solution for all possible models and assign the closed-loop poles within the desired region in the s-plane and are transferred to GMVC by Tustin's method, resulting in a parametric robust generalized minimum variance (PRGMV) controller. It was compared to two other approaches, carrying out several simulation essays in a Matlab environment. The performance index and sensitivity analysis highlight the controller's performance and efficiency. The results confirmed that the proposed controller ensures better robustness and performance for reference tracking and disturbance rejection.

INDEX TERMS GMVC, parametric uncertainties, convex optimization, PID control, sensitivity analysis.

I. INTRODUCTION

IN 1997, it was established that high order controllers were acutely sensitive to plant parameter perturbations, leading to attention to the synthesis and design of low order controllers, such as Proportional-Integral-Derivative (PID) [1], [2]. Indeed, in recent years, most industries still have employed such controllers due to their straightforward functionality and ease of use and tuning. However, many control practitioners pointed out that a PID tuned via a conventional approach was not robust. From there arose the need for robust control techniques for industrial applications employing PID with modifications in its original structure to guarantee the robustness of the control loop [3].

At first sight, such techniques proved to be better than the PID in its original form, but subsequently, these controllers became more complex in an uncertain environment [1], [4]. From there, the researchers felt there was a need to combine the simplicity of this controller with advanced control techniques [1]. In this context, GMVC (Generalized Minimum Variance Control) became attractive due to its properties to put different closed-loop characteristics under consideration in its optimization problem [5], [6].

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Proposed by [7], the GMVC uses a generalized output that is applied to a wider class of plants, being one of the simplest member of the Stochastic Control, differing to the most common Model-based Predictive Control (MPC) for being based on ARMAX (Auto-Regressive Moving Average with eXternal inputs) models while MPC on ARX (Auto-Regressive with eXternal inputs), i.e.., the GMVC considers a full description of system's deterministic and stochastic parts [8]. These important features enable the controller to distinguish between noise and uncertainty from deterministic cause and effect. In addition, the controlled plant presents a more stable behavior with less sensitivity to measurement and modeling uncertainty noises [9].

Several works about GMVC have been published in this context. For example, we have the work of [10], which presented a review of the controller design to eliminate the steady-state offset error and stabilize a nonlinear system. In [11] it was proposed a study on self-tuning control strategies with GMVC in a fixed two degree of freedom structure. This way, a considerable number of successful design studies have been carried out to hybridize PID and GMVC over the last years, like the one developed in [12] and [13].

Other applications can be seen in [14], in which presented a hybrid GMVC design with PID tuning, whose excessive actuator movements could be avoided and non-minimum phase plants were controlled, and in [15], that presented a new self-tuning PID controller design, in which the parameters were calculated online based on the relationship between the PID and GMVC laws by considering the pole assignment in the control system. Latest applications employing GMVC control were presented in [16], [17] and [18].

In control system design, the primary requirement is the asymptotic stability of controlled systems, which is guaranteed by designing an appropriate controller based on the process nominal model. However, in reality, several sources of uncertainties may make the nominal model inaccurate, leading to controller performance degradation [19]. This limitation is a drawback of designing GMVC based on the process nominal model.

Robust control deals explicitly with system uncertainties and guarantees the controlled system performance, primarily asymptotic stability when there are uncertainties [19], [20]. This design method has been studied extensively in [21], [22] and [23]. In addition, recent studies developed in [24]–[27] and [28] addressed an outstanding contribution for the current state-of-the-art on the study of parametric uncertainties. These works focused on robust controller synthesis and analysis for converters by using Parametric Robust Control (PRC), whose controller's parameters were obtained by solving a linear programming problem.

The work proposed in [29] presents a design of a fixed order controller for an uncertain system that guarantees closed-loop stability and a suitably adequate performance under a stated restricted frequency range. Then, an LMI condition ensures the robust stability of the closed-loop system in the presence of bounded parametric uncertainties.

Given the state-of-the-art in GMVC and PRC, it is possible to observe that there is a lack of a study on the hybridization of these two cited areas for parametric uncertainty plants. Furthermore, most studies in GMVC essentially still focus on inflicting a scalar λ-tuning weight over the control signal, such as in the positional case, with $Q(z^{-1}) = \lambda$, as was made in [30], and the incremental case for inclusion of a polynomial weight, in [11]. In these cases, the calibration of λ was mostly related to energy constraints regarding control effort in order to have good disturbance rejection and reference tracking, but without any consideration made on how this parameter would affect the relative stability of the control loop [11], [12].

In this context, this paper presents a hybridization of the GMVC with PRC, resulting in a parametric robust predictive control law which utilizes the predictive nature of the GMVC for parametric uncertainty plant. The proposed design is based on a PID controller with a low-pass filter, whose gains are obtained employing the robust pole placement technique and solved by formulating a convex optimization problem integrated to the Chebyshev theorem, which incorporates additional constraints on the system and desired performance and allows the designer to find the controller parameters that place closed-loop poles within desired intervals. PID gains are transferred to GMVC by Tustin's method, resulting in a parametric robust generalized minimum variance (PRGMV) controller.

The PRGMV is compared to two other controllers, one proposed in [22] and another in [31]. It is important to highlight that the PID gains and the robust stability analysis are obtained in the continuous frequency domain. In contrast, the sensitivity and robustness analysis of the PRGMV is performed in the discrete frequency domain, whose results confirm that the PRGMV outperforms the other controllers. Hence, the proposed design method uses continuous and discrete domain approaches, being one of this paper's contributions.

The significant contribution of this work lies in the nonexistence of a methodology directed to the hybridization of the stochastic control theory and parametric robust control for parametric uncertainty plants. Furthermore, the PRGMV provides a non-scalar $Q(z^{-1})$, which filters the control signal and reduces its variance, avoiding the need to tune this parameter based on trial and error, and still ensures better robustness and performance for reference tracking and disturbance rejection, allowing to figure out the closed-loop performance in terms of robustness analysis using sensitivity functions by means of magnitude plots in the frequency domain. From those functions, important indices are obtained to quantify the trade-off between robustness and performance to guarantee a suitable well-tuned controller.

In addition, a Parametric Robust Control approach of the type proposed in this paper represents a competitive option when dealing with some safety-critical plants, such as biomedical and electric power systems. In such systems, for safety reasons, it is not recommendable to use adaptive control approaches demanding the injection of disturbance test signals into the plant, a fact that may not be allowed in such safety-critical applications.

Therefore, this paper is organized as follows: Section 2 presents the Parametric Robust control (PRC) background, addressing the stability analysis, the Kharitonov theorem, and the interval pole placement design. Section 3 shows the Incremental GMVC Control. The synthesis of the PRGMV is developed in Section 4, followed by its robustness analysis in terms of sensitivity functions. In order to illustrate the effectiveness of the proposed controller, Section 5 presents simulation results. Finally, Section 6 gives conclusions and prospects.

II. PARAMETRIC ROBUST CONTROL (PRC)

Much of modern control theory addresses problems involving mathematical model uncertainty. When it involves various physical parameters specified within given bounds, it is called parametric or structured uncertainty, but when uncertainty concerns model structure or incorrect modeling, it is called non-parametric or unstructured uncertainty [20]. These uncertainties occur because a mathematical model cannot accurately represent natural systems, which compromises the system stability [19], [32]. In this case, control systems must consider model uncertainty so that the asymptotic stability of the controlled systems can be guaranteed.

Parametric Robust Control deals explicitly with uncertainties clarified above, providing satisfactory performance in a closed-loop system for all process uncertainties and bounded disturbances. Regarding stability, the robust control system should ensure closed-loop stability under nominal conditions and under uncertainties in the model [33], [34]. Therefore, this paper focuses on robust controller design for systems with parametric uncertainties.

A. PROBLEM STATEMENT

A Single Input Single Output (SISO) system, in the continuous frequency domain, can be represented by a transfer function $G(s)$. The basic structure for uncertain systems is generally described by uncertain polynomials $N(s, n)$ and $D(s, d)$, restricted within pre-specified closed real intervals [35], [36]. Thus, the transfer function of the uncertain system is given by:

$$G(s) = \frac{N(s,\mathrm{n})}{D(s,\mathrm{d})} = \frac{n_m s^m + n_{m-1}s^{m-1} + \ldots + n_0}{d_l s^l + d_{l-1}s^{l-1} + \ldots + d_0} \quad (1)$$

where n_i and d_i are bounded by intervals $\left[n_i^-, n_i^+\right]$ and $\left[d_i^-, d_i^+\right]$, respectively. Such a set of polynomials is called an interval polynomial family [37]. Then, once the minimum and maximum values of the various coefficients are assigned, the parametric uncertainty is completely characterized by letting n_i and d_i lie in a box defined by

$$\begin{aligned} &n_i \in \mathrm{R};\ n_i^- \le n_i \le n_i^+ \quad \text{for } i = 0, 1, \ldots, m \\ &d_i \in \mathrm{R};\ d_i^- \le d_i \le d_i^+ \quad \text{for } i = 0, 1, \ldots, l \end{aligned} \quad (2)$$

So, $G(s)$ can be represented by

$$G(s) = \frac{N(s,\mathrm{n})}{D(s,\mathrm{d})} = \frac{\sum_{i=0}^{m}\left[n_i^-, n_i^+\right]s^i}{\sum_{i=0}^{l}\left[d_i^-, d_i^+\right]s^i} \quad (3)$$

B. ROBUST POLE ASSIGNMENT

Consider the closed-loop system as depicted in Fig. 1. The controller C(s) is given by:

$$C(s) = \frac{B(s)}{A(s)} = \frac{b_r s^r + b_{r-1}s^{r-1} + \ldots + b_0}{a_t s^t + a_{t-1}s^{t-1} + \ldots + a_0} \quad (4)$$

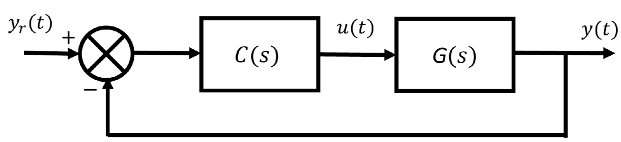


FIGURE 1. Schematic of the closed loop system.

In the case of a low-order controller, we have $r \le t$. By using (3) and (4) the closed-loop transfer function will be given by

$$[H(s)] = \frac{B(s)N(s,\mathrm{n})}{A(s)D(s,\mathrm{d}) + B(s)N(s,\mathrm{n})} \quad (5)$$

and the closed-loop characteristic polynomial family is as follows:

$$[\Delta(s)] = A(s)D(s,\mathrm{d}) + B(s)N(s,\mathrm{n}) \quad (6)$$

Let us now introduce a desired characteristic polynomial $\delta(s)$ of degree $l+t$ which is Hurwitz and which has the desired set of closed-loop characteristic roots:

$$\delta(s) = \delta_{l+t}s^{l+t} + \delta_{l+t-1}s^{l+t-1} + \ldots + \delta_0 \quad (7)$$

In order to attain the desired characteristic polynomial in (7), it is necessary and sufficient to solve a set of liner inequations [21], [38]. However, when $t = l - 1$ the linear equations admit a solution for the controller coefficients, for arbitrary $\delta(s)$ whenever the plant has no common pole zero pairs, and for $t < l - 1$ the exact attainment of the desired polynomial is impossible [22]. So, for $t \le l - 1$, (7) is enlarged into a box in the coefficient space, containing the point representing the original desired characteristic polynomial. This corresponds to choosing an interval desired polynomial family given by (8), which facilitates a solution from a root space point of view, where $\delta_n^- \le \delta_n \le \delta_n^+$ for $n = 0, 1, \ldots, l + t$.

$$[\delta(s)] = \delta_n s^n + \delta_{n-1}s^{n-1} + \ldots + \delta_0 \quad (8)$$

To guarantee robust stability and robust performance, the roots clusters of $[\Delta(s)]$ must be inside the roots clusters of $[\delta(s)]$, i.e.,

$$\Re\,[\Delta(s)] \subseteq \Re\,[\delta(s)] \quad (9)$$

where $\Re(.)$ depict the roots clusters of $[\Delta(s)]$ and $[\delta(s)]$ [22].

From (9), one can formulate the set of linear inequalities given in (10), as shown at the bottom of the next page, that poses constraints to the controller's coefficients so that (9) can be satisfied.

Then, the robust design problem is summarized in the choice of a parameters vector of the controller to be optimized (x_c), if possible, so that the set of inequalities in (10) is satisfied for all $n_i^- \le n_i \le n_i^+$ and $d_i^- \le d_i \le d_i^+$. The aforementioned robust performance control design problem, given in (10), can be rewritten as the following optimization problem [21], [39]:

$$\begin{aligned} &\min f(x_c) \\ &s.t\ \ a_i x_c \le b_i \end{aligned} \quad (11)$$

The cost function $f(x_c)$ must be built and minimized or maximized according to the control goals and restrictions. The solution of the problem given in (11) can be idealized as a solution to a linear programming (LP) problem, although different techniques solve it.

1) LINEAR PROGRAMMING BASED ON CHEBYSHEV's THEOREM

The Chebyshev's theorem guarantees to be possible to find the largest ball B of center x_c and maximum radius r, which is contained in the politope P, described by the set of linear

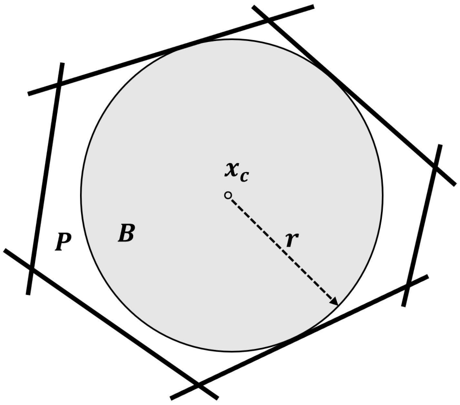


FIGURE 2. **Largest ball B inscribed in P.**

inequalities, whose norm is Euclidean, according to Fig. 2. The ball center x_c is called the Chebyshev Center and is a point inside P that is farthest from the exterior of P [40].

When the set P is convex, the computation of x_c become a convex optimization problem. More specifically, suppose $P \subseteq \mathrm{R}^n$ is defined by a set of convex inequalities, i.e., $P = \{a_i x_c \leq b_i, i = 1, \ldots, n\}$. Then, one can find x_c by solving the LP given in (12) for $r \geq 0$, [40]:

$$\begin{aligned} &\max r \\ &s.t \;\; a_i x_c + r\|a_i\|_2 \leq b_i \end{aligned} \tag{12}$$

Comparing (11) and (12), the robust pole assignment technique reduces to an LP combined with the Chebyshev's theorem as follows:

$$\begin{aligned} &\max f(x) \\ &s.t \;\; a_i x_c + r\|a_i\|_2 \leq b_i \end{aligned} \tag{13}$$

Equivalently,

$$\begin{aligned} &\max f(x) \\ &s.t \;\; A_c x \leq b_i \end{aligned} \tag{14}$$

where

$$A_c = \begin{bmatrix} a_i & \|a_i\|_2 \\ -a_i & \|a_i\|_2 \\ 0_{1\times i} & -1 \end{bmatrix}, \quad b_i = \begin{bmatrix} b_i^+ \\ b_i^- \\ 0 \end{bmatrix} \quad \text{and} \quad x = \begin{bmatrix} x_c \\ r \end{bmatrix}$$

The vector x contains the controller gains and the radius of the Chebyshev sphere, $f(x)$ is a linear cost function that must be built and maximized according to the control goals, $\|a_i\|_2$ is the Euclidean norm of the coefficients of a_i, i.e., $\|a_i\|_2 = \sqrt{(a_i{}^T a_i)}$ [41]. In this study, the cost function $f(x)$ has been chosen to be the sum of the elements of x [21], [38]. Then, when a solution exists, (14) provides the robust parameters of $C(s)$ and ensures a maximum stability region for uncertain systems.

2) KHARITONOV STABILITY TEST

Considering that (6) can be represented by

$$[\Delta(s,p)] = \sum\nolimits_{i=0}^{n} \left[p_i^-, p_i^+\right] s^i \tag{15}$$

the polynomial family $[\Delta(s,p)]$ is stable if and only if all its roots are contained in the left-hand side of the complex plane. Then, $[\Delta(s,p)]$ is robustly stable if and only if all its polynomials are stable for a set of operating points different from the nominal operating point within its minimum and maximum limits. However, it is not necessary to check the stability of all its polynomials to ensure robust stability since this can be verified by using Kharitonov's theorem [20], [25].

The Kharitonov theorem states that the stability of the four Kharitonov polynomials, given by

$$\begin{aligned} K_1(s) &= a_0^- + a_1^- s + a_2^+ s^2 + a_3^+ s^3 + a_4^- s^4 + a_5^- s^5 + \ldots \\ K_2(s) &= a_0^+ + a_1^+ s + a_2^- s^2 + a_3^- s^3 + a_4^+ s^4 + a_5^+ s^5 + \ldots \\ K_3(s) &= a_0^+ + a_1^- s + a_2^- s^2 + a_3^+ s^3 + a_4^+ s^4 + a_5^- s^5 + \ldots \\ K_4(s) &= a_0^- + a_1^+ s + a_2^+ s^2 + a_3^- s^3 + a_4^- s^4 \\ &\quad + a_5^+ s^5 + \ldots \end{aligned} \tag{16}$$

$$\begin{bmatrix} \delta_{l+t}^- \\ \delta_{l+t-1}^- \\ \vdots \\ \vdots \\ \vdots \\ \delta_1^- \\ \delta_0^- \end{bmatrix} \leq \begin{bmatrix} n_m & & & & d_l & & & \\ n_{m-1} & n_m & & & d_{l-1} & d_l & & \\ n_{m-2} & n_{m-1} & \cdot & & d_{l-2} & d_{l-1} & \cdot & \\ n_{m-3} & n_{m-2} & \cdot\cdot & & d_{l-3} & d_{l-2} & \cdot\cdot & \\ \cdot & n_{m-3} & \cdot\cdot\cdot & & \cdot & d_{l-3} & \cdot\cdot\cdot & \\ \cdot & \cdot & \cdot\cdot\cdot & n_m & \cdot & \cdot & \cdot\cdot\cdot & d_l \\ \cdot & \cdot & \cdot\cdot & n_{m-1} & \cdot & \cdot & \cdot\cdot & d_{l-1} \\ n_2 & \cdot & \cdot & n_{m-2} & d_2 & \cdot & \cdot & d_{l-2} \\ n_1 & n_2 & & n_{m-3} & d_1 & d_2 & & d_{l-3} \\ n_0 & n_1 & \cdot & \cdot & d_0 & d_1 & \cdot & \cdot \\ & n_0 & \cdot\cdot & \cdot & & d_0 & \cdot\cdot & \cdot \\ & & \cdot\cdot\cdot & \cdot & & & \cdot\cdot\cdot & \cdot \\ & & \cdot\cdot & n_2 & & & \cdot\cdot & d_2 \\ & & \cdot & n_1 & & & \cdot & d_1 \\ & & & n_0 & & & & d_0 \end{bmatrix} \begin{bmatrix} b_r \\ b_{r-1} \\ \vdots \\ b_0 \\ a_t \\ a_{t-1} \\ \vdots \\ a_0 \end{bmatrix} \leq \begin{bmatrix} \delta_{l+t}^+ \\ \delta_{l+t-1}^+ \\ \vdots \\ \vdots \\ \vdots \\ \delta_1^+ \\ \delta_0^+ \end{bmatrix} \tag{10}$$

is a necessary and sufficient condition to guarantee the stability of the interval polynomial family [42]. Therefore, the closed-loop characteristic polynomial, given in (6), is robustly stable if and only if the four Kharitonov polynomials in (16) are stable [35], [43].

III. INCREMENTAL GMV CONTROL

The GMVC was developed from the Minimum Variance Regulator (MVR) first introduced by Karl Johan Åström in the 1970s. According to [12], its original structure does not guarantee reference tracking and disturbance rejection for plants without integrators [11], [12], being necessary to add a filter $\Delta = 1 - z^{-1}$ at the control weight of the generalized output [44],

$$\phi(k+d) = P(z^{-1})y(k+d) - T(z^{-1})r(k+d) + Q(z^{-1})\Delta u(k) \quad (17)$$

being

$$\begin{aligned} P(z^{-1}) &= p_0 + p_1 q^{-1} + \cdots + p_{n_p} q^{-n_p} \\ T(z^{-1}) &= t_0 + t_1 q^{-1} + \cdots + t_{n_t} q^{-n_t} \\ Q(z^{-1}) &= q_0 + q_1 q^{-1} + \cdots + q_{n_q} q^{-n_q} \end{aligned}$$

The polynomials $P(z^{-1})$, $T(z^{-1})$ and $Q(z^{-1})$ are weighting filters for system's output, reference and control signal, respectively. The generalized output $\phi(k)$ is posed into a stochastic optimization problem of minimizing the GMV cost function [30], [45],

$$J = E[\phi^2(k+d)] \quad (18)$$

with respect to the control signal increment, i.e.,

$$\frac{\partial J}{\partial \Delta u(k)} = 0 \quad (19)$$

where $E[\cdot]$ denotes the mathematical expectation operator.

Let us expand GMVC's equations from ARIMAX (Auto-Regressive Integrated Moving Average with Exogenous Input) to generalized stochastic plant models of the following form:

$$\Delta A(z^{-1})y(k) = z^{-d}B(z^{-1})\Delta u(k) + C_\xi(z^{-1})\xi(k) \quad (20)$$

being

$$\begin{aligned} A(z^{-1}) &= 1 + a_1 z^{-1} + \cdots + a_{n_a} q^{-n_a} \\ B(z^{-1}) &= b_0 + b_1 z^{-1} + \cdots + b_{n_b} q^{-n_b} \\ C_\xi(z^{-1}) &= 1 + c_1 z^{-1} + \cdots + c_{n_c} q^{-n_c} \end{aligned}$$

where $y(k)$ is the system's output, $u(k)$ is the input, d is the discrete time delay and $\xi(k)$ represents a white noise disturbance. By inspection of the right hand side of the generalized output in (17), $y(k+d)$ is a future measurement not available at instant k, so it must be predicted in order to work with $\phi(k+d)$ [11]. Then, shifting (20) d-steps ahead and remembering that $P(z^{-1})$ multiplies the output $y(k+d)$ in (17), one obtains the following:

$$P(z^{-1})\Delta A(z^{-1})y(k+d) = P(z^{-1})B(z^{-1})\Delta u(k) + P(z^{-1})C_\xi(z^{-1})\xi(k+d) \quad (21)$$

The future term of the noise, $\xi(k+d)$, is unknown at present k. Therefore it can be represented by present and future parts, as follows:

$$\frac{P(z^{-1})C_\xi(z^{-1})\xi(k+d)}{\Delta A(z^{-1})} = \underbrace{\frac{F(z^{-1})\xi(k)}{\Delta A(z^{-1})}}_{present} + \underbrace{E(z^{-1})\xi(k+d)}_{future} \quad (22)$$

whereas these two auxiliary polynomials, $E(z^{-1})$ and $F(z^{-1})$, can be determined by solving the following Diophantine equation:

$$P(z^{-1})C_\xi(z^{-1}) = \Delta A(z^{-1})E(z^{-1}) + z^{-d}F(z^{-1}) \quad (23)$$

being

$$\begin{aligned} E\left(z^{-1}\right) &= e_0 z^{-1} + e_1 z^{-1} + \cdots + e_{n_{d-1}} z^{-n_{d-1}} \\ F\left(z^{-1}\right) &= f_0 + f_1 z^{-1} + \cdots + f_{n_f} z^{-n_f} \\ n_f &= \max(n_p + n_c, n_a + n_e) - d \end{aligned}$$

Using only the present data of (22), the predicted output $\hat{y}(k+d|k)$ is

$$P(z^{-1})\hat{y}(k+d|k) = \frac{P(z^{-1})B(z^{-1})\Delta u(k)}{\Delta A(z^{-1})} + \frac{F(z^{-1})\xi(k)}{\Delta A(z^{-1})} \quad (24)$$

By considering the elimination of the part $E(z^{-1})\xi(k+d)$ of (22) it introduces an error in the prediction, given by

$$E(z^{-1})\xi(k+d) = P(z^{-1})[y(k+d) - \hat{y}(k+d|k)] \quad (25)$$

Thus, the present stochastic signal $\xi(k)$, obtained from the prediction error, is

$$\xi(k) = \frac{P(z^{-1})}{E(z^{-1})}[y(k) - \hat{y}(k|k)] \quad (26)$$

Substituting (26) into (24) and after some algebraic manipulations and using (22), the d-steps ahead Minimum Variance Predictor (MVP) turns to,

$$\hat{y}(k+d|k) = \frac{B(z^{-1})E(z^{-1})\Delta u(k) + F(z^{-1})y(k)}{P(z^{-1})C_\xi(z^{-1})} \quad (27)$$

By substitution of (27) into the generalized output in (17), the GMVC control law is thus obtained by solving (19) [9], [45], resulting in

$$\Delta u(k) = \frac{C_\xi(z^{-1})T(z^{-1})y_r(k+d) - F(z^{-1})y(k)}{B(z^{-1})E(z^{-1}) + C_\xi(z^{-1})Q(z^{-1})} \quad (28)$$

which guarantees in steady-state that the variance of the generalized output, σ_ϕ^2, is a minimum.

At last, considering $C_\xi(z^{-1}) = 1$, the closed-loop transfer function is obtained through the substitution of (28) and (23) into (20). After some algebraic manipulations, one finds that

$$y(k) = \frac{z^{-d}B(z^{-1})T(z^{-1})}{B(z^{-1})P(z^{-1}) + \Delta A(z^{-1})Q(z^{-1})}y_r(k) + \frac{B(z^{-1})E(z^{-1}) + Q(z^{-1})}{B(z^{-1})P(z^{-1}) + \Delta A(z^{-1})Q(z^{-1})}\xi(k) \quad (29)$$

which considers the closed-loop dynamics from the reference, $y_r(k)$, to the output, and from the white noise type sequence, $\xi(k)$, to the output.

IV. PRGMV CONTROLLER DESIGN

This section presents the hybridization of the GMVC with PRC, whose design is based on a PID controller with a low-pass filter, resulting in a PRGMV control law that utilizes the predictive nature of the GMVC for parametric uncertainties plant. The controller gains are obtained by solving the convex optimization problem given in (14) and transferred to the GMVC using Tustin's method.

A. GMVC BASED ON PID CONTROLLER

The controller C(s) is represented in the Laplace complex domain by

$$C(s) = \frac{U(s)}{E_r(s)} = \frac{k_d s^2 + k_p s + k_i}{s(s+\alpha)} \tag{30}$$

with $U(s)$, $\mathrm{E}_r(\mathrm{s}) = \mathrm{Y}_r(\mathrm{s}) - \mathrm{Y}(\mathrm{s})$, α, k_p, k_i and k_d being the controller's output signal, error signal, high frequency lag, proportional, integral and derivative gains, respectively.

The low-pass filter is included on the control law to reduce the influence of non-minimum phase (undershoot) and to assess further enhancements on the performance of the control-loop, particularly if the error is measured in a noisy environment [2]. Then, the problem of stabilization comes down to determine the gains of $C(s)$, i.e., $x_c := [k_d, k_p, k_i, \alpha]$, for which the closed-loop characteristic polynomial family $[\Delta(s)]$ in (6) is Hurwitz [46]–[48].

Due to an integral part of the error signal, any controller with this feature already provides asymptotic tracking and disturbance rejection for step-like references [2], [32]. On the other hand, integrating plants might impose more difficulties to be stabilized using PID controllers, and sometimes it might be necessary to exclude an integral part of the controller.

Defining

$$s := \frac{2}{t_s}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) \tag{31}$$

the Tustin's method for converting (30) to its corresponding discrete-time control law, being t_s the sampling time, the discrete PID control law is obtained after some algebraic manipulations, which results in the following RST structure [31]:

$$R_{rst}(z^{-1})\Delta u(k) = T_{rst}(z^{-1})y_r(k) - S_{rst}(z^{-1})y(k) \tag{32}$$

being

$$\begin{aligned} R_{rst}(z^{-1}) &= r_0 + r_1 z^{-1} \\ S_{rst}(z^{-1}) &= s_0 + s_1 z^{-1} + s_2 z^{-2} \\ T_{rst}(z^{-1}) &= s_0 + s_1 z^{-1} + s_2 z^{-2} \end{aligned}$$

so that

$$\begin{aligned} s_0 &= 4k_d + 2k_p t_s + k_i {t_s}^2 \\ s_1 &= -8k_d + 2k_i {t_s}^2 \\ s_2 &= 4k_d - 2k_p t_s + k_i {t_s}^2 \\ r_0 &= 4 + 2\alpha t_s \\ r_1 &= -4 + 2\alpha t_s \end{aligned} \tag{33}$$

Adding $\Delta = 1 - z^{-1}$ to both (28) and (32) will guarantee step-like reference tracking and disturbance rejection for the closed-loop systems. These controllers generate the control increment, which is then applied to the process as follows:

$$u(k) = u(k-1) + \Delta u(k) \tag{34}$$

From (17), when $\phi(k+d) = 0$, we have

$$Q(z^{-1})\Delta u(k) = T(z^{-1})y_r(k+d) - P(z^{-1})y(k+d) \tag{35}$$

By comparing (35) to (32), the polynomials $P(z^{-1})$, $T(z^{-1})$ and $Q(z^{-1})$ become

$$\begin{aligned} P(z^{-1}) &= S_{rst}(z^{-1}) \\ T(z^{-1}) &= T_{rst}(z^{-1}) \\ Q(z^{-1}) &= R_{rst}(z^{-1}) \end{aligned} \tag{36}$$

By looking at (36) and (33), note that the parameters of the polynomials $P(z^{-1})$, $T(z^{-1})$ and $Q(z^{-1})$ strongly depend on the PID controller gains, which will be computed by solving the convex optimization problem given in (14), making them robust. Therefore, since the PID controller gains given in (33) are robust, the GMVC control law in (28) becomes

$$\Delta u(k) = \frac{C_\xi(z^{-1})T_{prgmv}(z^{-1})y_r(k+d) - F(z^{-1})y(k)}{B(z^{-1})E(z^{-1}) + C_\xi(z^{-1})Q_{prgmv}(z^{-1})} \tag{37}$$

which represents the PRGMV controller control law and guarantees in steady-state that the variance of the generalized output, σ_ϕ^2, is a minimum, whereas the Diophantine equation, given by (23), then becomes

$$P_{prgmv}(z^{-1})C_\xi(z^{-1}) = \Delta A(z^{-1})E(z^{-1}) + z^{-d}F(z^{-1}) \tag{38}$$

where

$$\begin{aligned} P_{prgmv}(z^{-1}) &= s_0 + s_1 z^{-1} + s_2 z^{-2} \\ T_{prgmv}(z^{-1}) &= s_0 + s_1 z^{-1} + s_2 z^{-2} \\ Q_{prgmv}(z^{-1}) &= r_0 + r_1 z^{-1} \end{aligned} \tag{39}$$

Note that the PRGMV control law in (37) is in the RST structure, and its control loop is depicted in Fig. 3 [49]. All (z^{-1}) were hidden for the sake of simplicity; $d(k)$ can represent an output disturbance or a noise added to the measured output.

At last, considering $C_\xi(z^{-1}) = 1$, the closed-loop transfer function is given by

$$\begin{aligned} y(k) = &\frac{z^{-d}B(z^{-1})T_{prgmv}(z^{-1})}{B(z^{-1})P_{prgmv}(z^{-1}) + \Delta A(z^{-1})Q_{prgmv}(z^{-1})} y_r(k) \\ &+ \frac{\Delta A(z^{-1})\left[B(z^{-1})E(z^{-1}) + Q_{prgmv}(z^{-1})\right]}{B(z^{-1})P_{prgmv}(z^{-1}) + \Delta A(z^{-1})Q_{prgmv}(z^{-1})} d(k) \end{aligned} \tag{40}$$

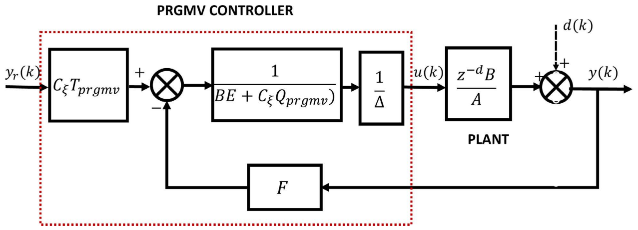


FIGURE 3. Block diagram of the PRGMV control loop.

Note that the transfer function that characterizes the closed-loop dynamic behavior of $y(k)$ from $y_r(k)$ in (40) is explicitly represented by the polynomials that weight the optimization problem in the GMVC, i.e., $P_{prgmv}(z^{-1})$, $T_{prgmv}(z^{-1})$ and $Q_{prgmv}(z^{-1})$. Therefore, these polynomials guarantee robust stability and performance for parametric uncertainties systems, satisfying (9) since its parameters strongly depend on the PID controller gains, which are computed by solving the problem (14), making it robustly stable.

Remark that since the PRGMV is a stochastic controller, the sensitivity function of the controlled systems might differ from the closed-loop sensitivity of the PID case, leading to better results regarding the PRGMV that enhances to a more stable behavior with less sensitivity to the measurement noise. This occurs due to the presence of the $E(z^{-1})$ polynomial in both (29) and (40), that is obtained from the solution of GMV's Diophantine equation and is related to the noise prediction [9].

B. ROBUSTNESS ANALYSIS

This section covers robustness analysis using magnitude plots of the sensitivity functions $S_{sen}(z^{-1})$ and $T_{sen}(z^{-1})$ in the frequency domain. From those functions, important indices, such as gain margin (GM), phase margin (PM) and maximum amplitude ratios, can be obtained to quantify the trade-off between robustness and performance to guarantee a suitable well tuned controller. Thus, according to [50], (40) is rewritten to

$$y(k) = T_{sen}(z^{-1})y_r(k) + S_{sen}(z^{-1})d(k) \qquad (41)$$

where $S_{sen}(z^{-1})$ is the transfer function between the disturbance $d(k)$ and the output $y(k)$ (output sensitivity function) and characterizes the system performance from the point of view of disturbance rejection, and $T_{sen}(z^{-1})$ is the sensitivity function between $y_r(k)$ and $y(k)$ (also called complementary sensitivity function) and characterizes the closed loop system performance [11], [49].

By considering $C_\xi(z^{-1})T_{prgmv}(z^{-1}) = F(z^{-1})$ in (37), since the PRGMV controller is based on the PID structure, the sensitivity functions have the property $S_{sen}(z^{-1}) + T_{sen}(z^{-1}) = 1$ and reflect many interesting properties of the closed-loop system, particularly, robustness to the model's uncertainties. The maximum amplitude ratios of $S_{sen}(z^{-1})$ and $T_{sen}(z^{-1})$, respectively M_S and M_T, are given by

$$\begin{aligned} M_S &= \max_{\omega} \left|S_{sen}(e^{j\omega t_s})\right| \\ M_T &= \max_{\omega} \left|T_{sen}(e^{j\omega_s})\right| \end{aligned} \qquad (42)$$

The equation (42) quantifies the sensibility of the control loop to the excitation signal under consideration, e.g., small M_S values make the system less sensitive to $d(k)$, whereas M_T considers the influence of the reference signal, $y_r(k)$, and is equivalent to the amplitude of the resonant peak as well, that in general, is desirable to be kept small [11]. In [50], it is suggested that M_S and M_T values be respectively in the range of 1.2–2 and 1–1.5 to give a suitable trade-off between robustness and performance.

According to [51], [52], gain and phase margins have been vastly used to characterize robustness of control loops, and it is common sense, that $GM \to \infty$ and $PM \geq 60°$ characterize robust stability. To obtain a softer compromise between robustness and performance, [50] suggests that GM and PM should be kept in the range of 1.7 dB–4dB and 30° to 45°, respectively, and that both indices can be obtained using the following equations:

$$\begin{aligned} GM_S &\geq 20\log 10\left(\frac{M_S}{M_S-1}\right) \\ PM_S &\geq 2\sin^{-1}\left(\frac{1}{2M_S}\right)\left(\frac{180}{\pi}\right) \\ GM_T &\geq 20\log 10\left(1+\frac{1}{M_T}\right) \\ PM_T &\geq 2\sin^{-1}\left(\frac{1}{2M_T}\right)\left(\frac{180}{\pi}\right) \end{aligned} \qquad (43)$$

Therefore, from (43), it is possible to characterize the "stability margin" and "robustness" of the closed-loop system concerning variations of the system parameters (or uncertainties in parameter values).

V. SIMULATIONS RESULTS

In this section, two simulation examples illustrate the effectiveness of the design presented in Section IV. The PRGMV performance is compared to two other controllers, one designed by a robust pole placement (RPP) method, presented in [22] and the other by a classic pole placement (CPP) method, presented in [31]. In addition, the performance of the three controllers is analyzed using the integral of the squared error (ISE) [24].

A. EXAMPLE 1: SECOND ORDER SYSTEM

Consider the following process to represent a DC motor position control problem with intervals parameters [39]:

$$[G(s)] = \frac{[15.43, 16.55]}{s^2 + [10.09, 10.82]s} \qquad (44)$$

Since (44) is an integrating process, it is not considered the integral part of the controller given in (30). So, the design objective is to find $x_c := [k_d, k_p, \alpha]$ by solving the LP problem in (14). For transient response, it was considered an interval overshoot $[M_p] = [7, 13]\%$ centered in $M_{pc} = 10\%$ and a peak time $t_p = 1$ s, resulting in

TABLE 1. PID controller parameters.

	Proposed Method	RPP [22]	CPP [31]
K_d	0.0554	-2.8972	-3.6504
K_p	18.6071	18.5518	21.8723
α	14.4455	14.3902	17.1854

the desired or target closed-loop characteristic polynomial $[\delta(s)] = s^3 + [24.4826, 31.9111]s^2 + [97.2821, 158.3745]s + [286.2859, 450.5124]$. Table 1 presents the PID gains for the PRGMV, RPP and CPP methods. Their digital counterparts were obtained using a sampling time $t_s = 0.05$ s.

1) ROBUST STABILITY ANALYSIS

The robust stability analysis of the three control-loops was performed employing the four Kharitonov polynomials by plotting its roots in the s-plane, as shown in Fig. 4.

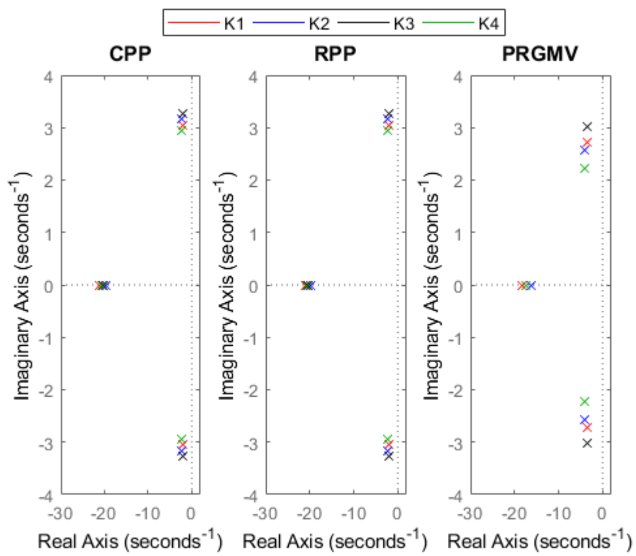


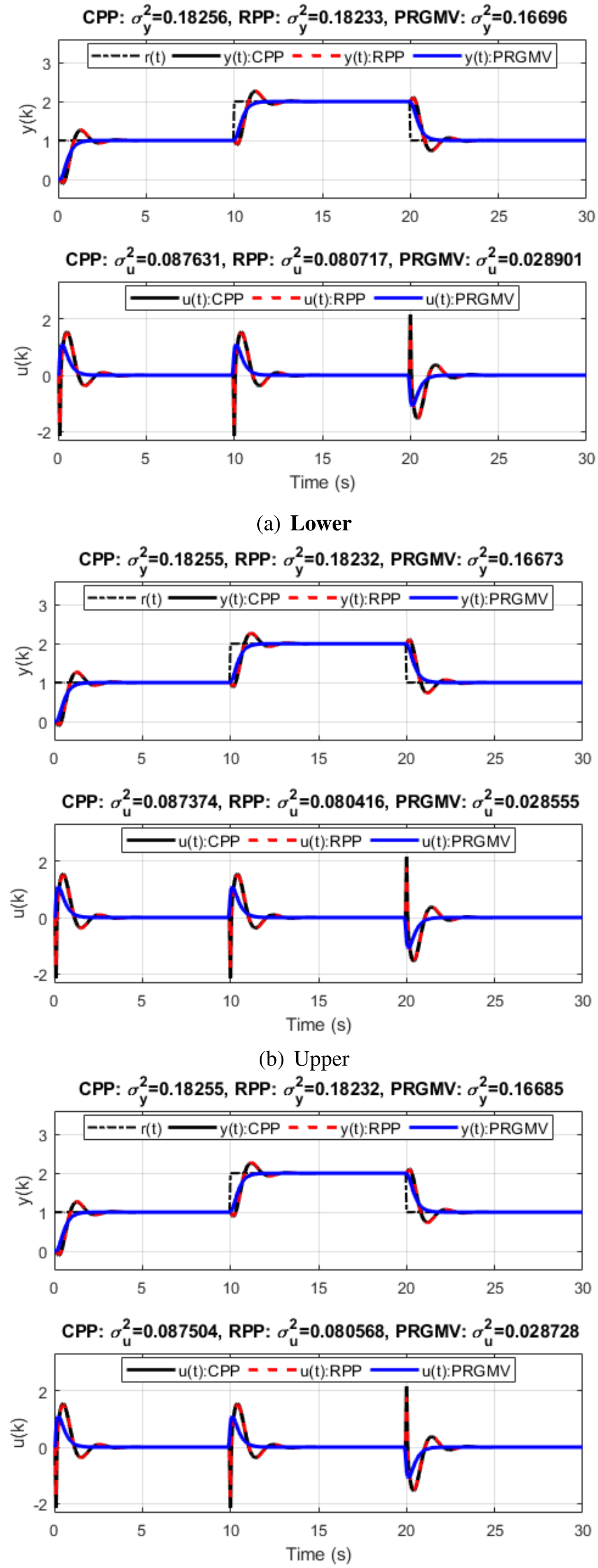
FIGURE 4. Four Kharitonov polynomials pole map in the *s*-plane.

From Fig. 4, note that the four Kharitonov polynomials are stable, which means that the three controllers can stabilize all the interval polynomial family given in (44). Therefore, this feature is transferred to GMV, stabilizing the interval polynomial family.

2) REFERENCE TRACKING

Fig. 5 depicts the simulated output behavior and the control signal using the three controllers. During the tests, a step reference is first applied for three operating points: lower, nominal and upper. After the system achieves the steady-state, the reference is varied from 1 pu to 2 pu at $t = 10$ sec and at $t = 20$ sec it is reduced again to 1 pu.

According to Fig. 5, both controllers can compensate for the reference variation, and the output could asymptotically track the step-like sequence in the face of parameter uncertainty. It can be seen that the CPP and RPP have similar



(a) **Lower**

(b) Upper

(c) Nominal

FIGURE 5. Closed-loop output and control signal.

performance with severe control signal spikes and take a longer time to restore the output values to their respective set points. The PRGMV outperforms them both since it quickly compensates for the oscillations and provides a faster response while requiring less control effort.

Fig. 6 shows the test result when a load disturbance of magnitude 0.5 pu is applied to the nominal system output at $t = 5$ sec. Observe that the PRGMV presents the best performance, outperforming the other approaches by minimizing the disturbance effect and reducing the recovery time of the system output to the desired set point.

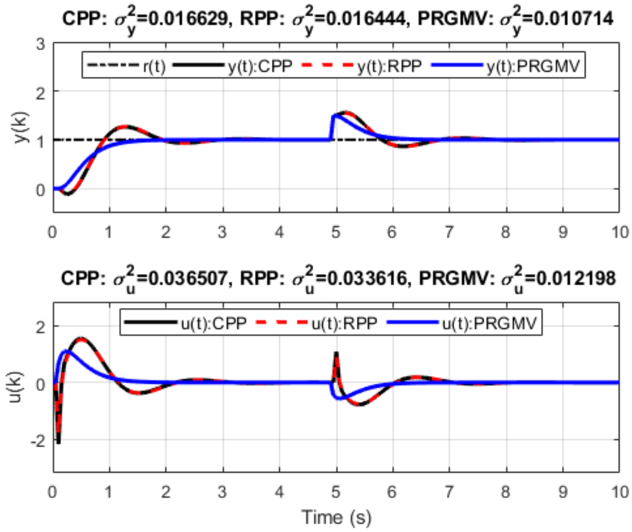


FIGURE 6. Closed-loop output with load disturbance.

Fig. 7 present the results when a white noise signal $\xi(k)$, with a relatively small variance of $\sigma_\xi^2 = 0.01$, is applied to the system output. Once again, the PRGMV presents the best results, outperforming the other approaches by mitigating the influence of the white noise signal on the system and minimizing the variance of the control and the output signals.

Fig. 8 shows a cost comparison between the assessed controllers utilizing the ISE cost function. This index evaluates the impact of the step reference on the controllers' performances and is almost constant regardless of the operating point. The PRGMV presents the smallest ISE, confirming its improved performance with the lowest cost.

Fig. 9 presents the robustness analysis of the three controllers by plotting the sensitivity functions. Gain and phase margins shown in Table 2 ratify the effectiveness of the PRGMV in the three operating points because this controller, at least in this studied example, presented minimal control effort and energy consumption to track the reference signal for the assessed operating points. Note that the PRGMV again outperforms the other controllers because it makes the system less sensitive to the output disturbance $d(k)$ and guarantees a more significant stability margin and consequent closed-loop robustness to disturbances and modeling uncertainties.

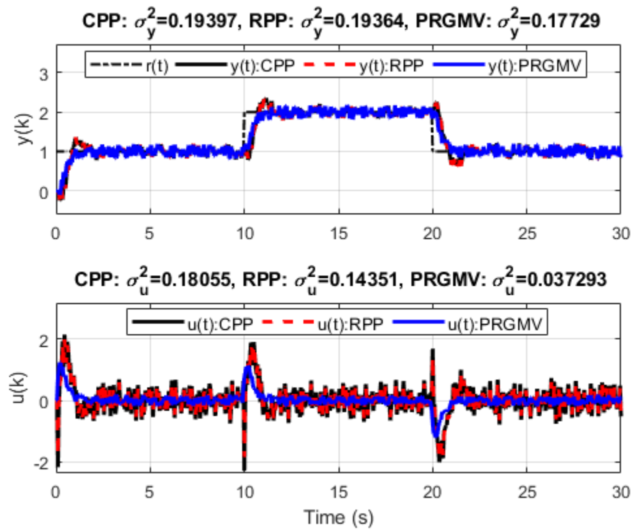


FIGURE 7. Closed-loop output with white noise.

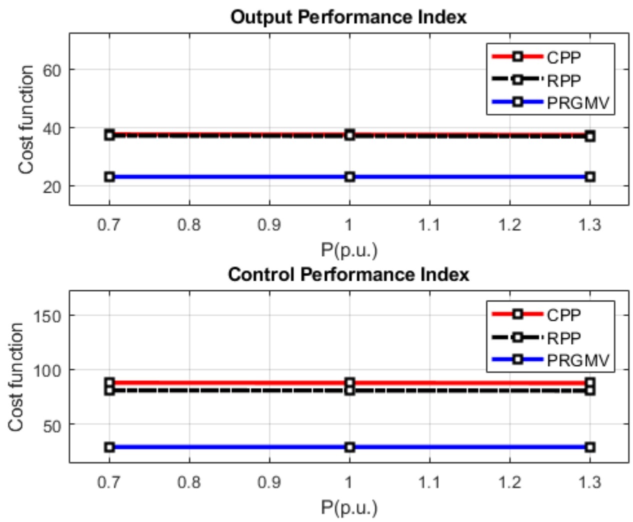


FIGURE 8. Performance index.

TABLE 2. Robustness index and margins.

Sensitivity Functions	Method	maximum amplitude	GM	PM
$T(z^{-1})$	PRGMV	0.9999	2.0003 dB	60.0225°
	RPP	1.2170	1.8217 dB	48.5155°
	CPP	1.2129	1.8244 dB	48.6889°
$S(z^{-1})$	PRGMV	1.3239	4.087 dB	44.3777°
	RPP	1.7635	2.3097 dB	32.9411°
	CPP	1.7489	2.3354 dB	33.2253°

By considering that the three controllers can stabilize the interval system given in (44), the same procedure is repeated for a sinusoidal reference sequence at a frequency $f_o = 0.1$ Hz ≈ 0.628 rad/s, which lies within the lower frequencies region where the complementary sensitivity is still close to 0 dB. However, since integral control action is included in all three design methods investigated, a phase lag of 90° is introduced into the control loop, which is eventually reflected in the

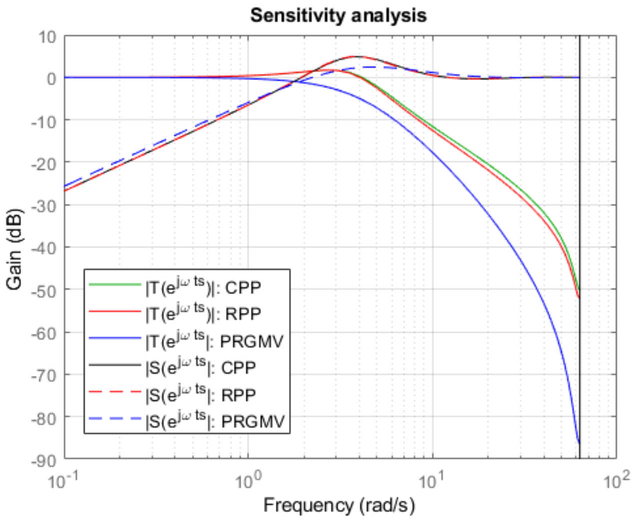


FIGURE 9. Sensitivity analysis.

results shown in Fig. 10. This sinusoidal tracking test was conducted only at the nominal operating point.

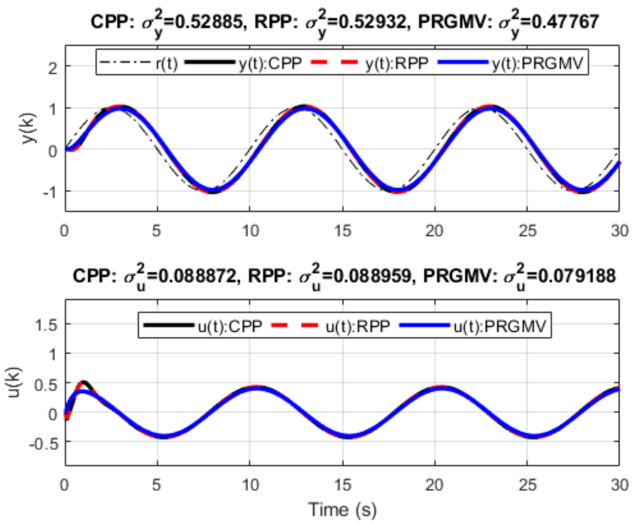


FIGURE 10. Closed-loop output with sinusoidal input.

According to [53], [54], if a stable linear time-invariant system is submitted to a sinusoidal input, it will have a steady-state output sinusoidal with the same frequency as the input. Fig. 10 shows that the steady-state response of the system for the sinusoidal input is also sinusoidal, at the same input frequency and differing slightly only in amplitude and phase angle, as expected. It is also noticeable that the PRGMV outperforms the other controllers by minimizing the variances of the controlled system.

B. EXAMPLE 2: NON-MINIMUM PHASE PROCESS

The following example represents a SISO linear model with a variation of 10% in its parameters: [55].

$$[G(s)] = \frac{n_1 s + n_0}{s^3 + d_2 s^2 + d_1 s + d_0} \quad (45)$$

where $n_1 = [-34.13, -27.92]$, $n_0 = [40.2, 49.14]$, $d_2 = [9.715, 11.87]$, $d_1 = [33.02, 40.35]$ and $d_0 = [35.89, 43.87]$.

For transient response, it was considered an interval overshoot $[M_p] = [15, 25]\%$ centred in $M_{pc} = 20\%$, a peak time $t_p = 10\,s$ and non-dominant poles, which leads to $[\delta(s)] = s^5 + \delta_4 s^4 + \delta_3 s^3 + \delta_2 s^2 + \delta_1 s + \delta_0$, such that $\delta_4 = [10.6745, 14.6078]$, $\delta_3 = [39.0346, 73.0157]$, $\delta_2 = [52.8448, 134.2061]$, $\delta_1 = [15.7907, 49.5679]$ and $\delta_0 = [4.9085, 14.3691]$.

The main design purpose is to obtain $x_c := [k_d, k_p, k_i, \alpha]$ by solving the LP problem given in (14). Table 3 presents the PID gains for PRGMV and RPP methods only, whose values in the discrete domain were obtained using a sampling time $t_s = 0.25$ sec, since for CPP it was not possible to find a solution, because the PID is designed by a classic pole placement that works only with plants of order $n_a \leq 2$ [56]. Then it is clear that the order of a control system considerably limite the design of traditional controllers by pole assignment due to their limited number of tuning parameters [57].

TABLE 3. PID controller gains.

	Proposed Method	RPP [22]
k_d	0.1121	-0.5104
k_p	0.1455	-0.0550
k_i	0.4716	0.3744
α	1.8050	1.7440

1) ROBUST STABILITY ANALYSIS

Fig. 11 shows the robust stability analysis of the closed-loop system employing the four Kharitonov polynomials by plotting its roots in the complex plane. Note that the four Kharitonov polynomials are stable, meaning that the two controllers stabilize the interval polynomial family given in (45). Therefore, this feature will be transferred to GMV, which will also stabilize the interval polynomial family.

2) REFERENCE TRACKING

The reference tracking performance in closed-loop for three operating points (lower, nominal and upper) is presented in Fig. 12. After the system achieves the steady state, the reference is varied from 1 pu to 2 pu at $t = 10$ sec and at $t = 20$ sec it is reduced again to 1 pu.

Fig. 12 shows the simulated output behavior and the control signal using RPP and PRGMV. Observe that both controllers can compensate for the step-like reference change, and the output converges asymptotically in the face of parameter uncertainty. However, the RPP presents output oscillations and control signal spikes and takes a slightly longer time to restore the output values to the desired set points. The PRGMV, on the other hand, quickly tracks the reference without oscillations, providing better performance with faster response and less control effort.

Fig. 13 presents a load disturbance essay. The load magnitude of 0.5 was applied to the nominal system output at $t = 50$ sec. The PRGMV presented the best recovery

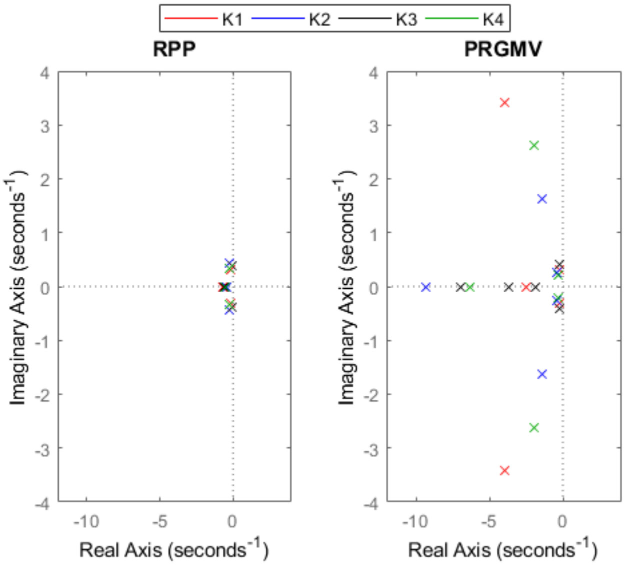


FIGURE 11. Four Kharitonov polynomials pole map.

TABLE 4. Robustness index and margins.

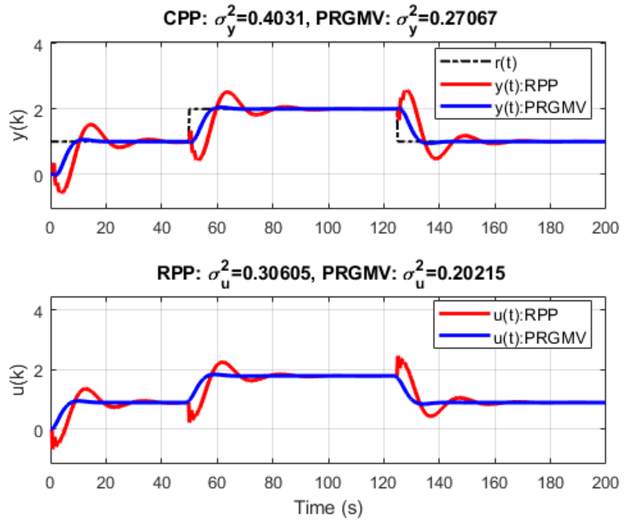
Sensitivity Functions	Method	maximum amplitude	GM	PM
$T(z^{-1})$	PRGMV	1.0122	1.9879 dB	59.2029°
	RPP	2.3696	1.4220 dB	24.3622°
$S(z^{-1})$	PRGMV	1.5869	2.7039 dB	36.7318°
	RPP	2.3018	1.7682 dB	25.0916°

performance, outperforming the RPP by minimizing the load effect and reducing the recovery time.

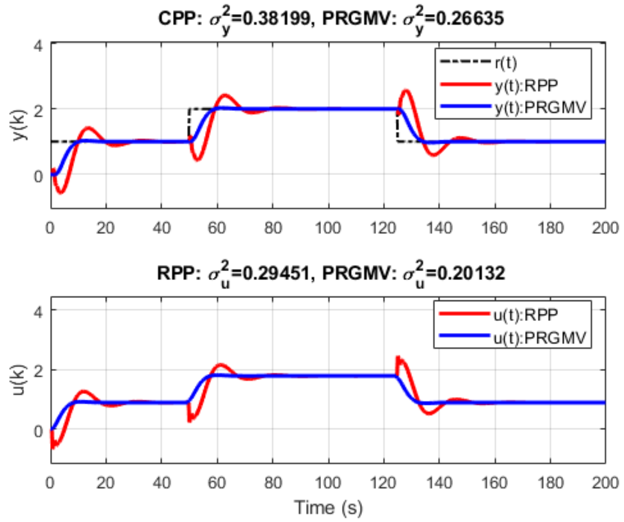
Fig. 14 presents the simulation results when a white noise sequence is applied to the system's output. The noise signal ξ, with a relatively small variance of $\sigma^2 = 0.01$, was assessed. Once again, the PRGMV presented the best performance compared to the RPP since the control and output variances in the PRGMV case were smaller, confirming that PRGMV is more cost-effective than RPP.

Fig. 15 shows the performance comparison using the ISE cost function. This index evaluates the impact of the step-like reference on the control loop. Note that the ISE is almost constant regardless of the operating point value for step input variation. However, for PRGMV, the ISE was smaller than for RPP, confirming PRGMV's performance improvement while requiring less control energy.

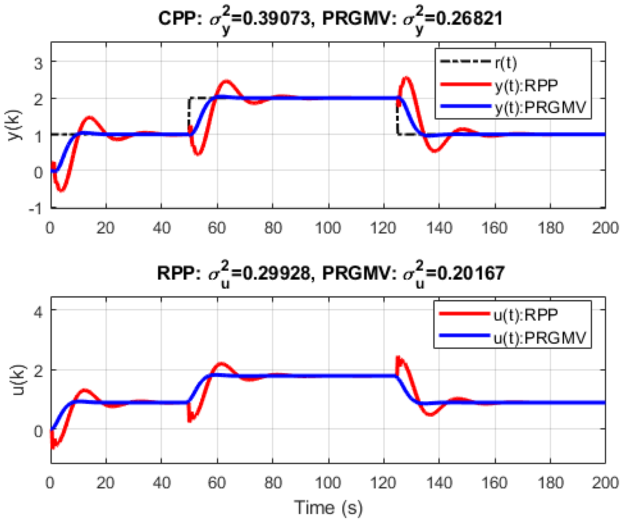
Fig. 16 presents the robustness analysis of the two controllers by plotting the sensitivity functions. The values of GM and PM ratify the effectiveness of the proposed controller for reference tracking and can confirm, at least for this studied example, the excellent performance of the PRGMV for parameter uncertainty plants with minimum control effort and minimum energy for tracking the reference signal. The PRGMV once again outperforms the RPP because it makes the system less sensitive to $d(k)$ and exhibits a more significant stability margin and robustness of the closed-loop system, as shown in Table 4.



(a) Lower



(b) Upper



(c) Nominal

FIGURE 12. Closed-loop output and control signal.

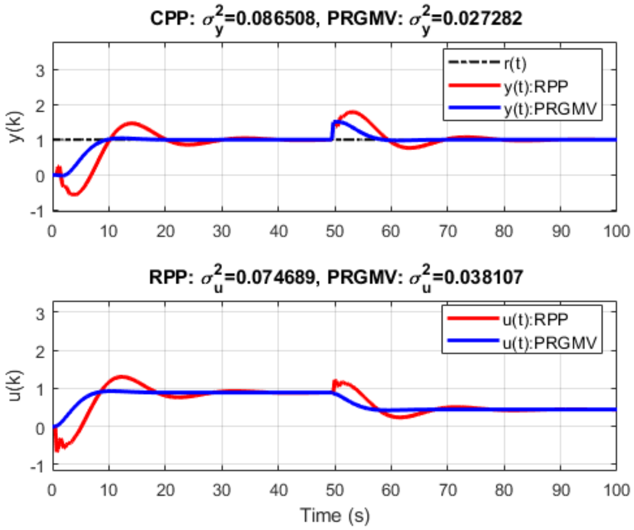


FIGURE 13. Closed-loop output with disturbance.

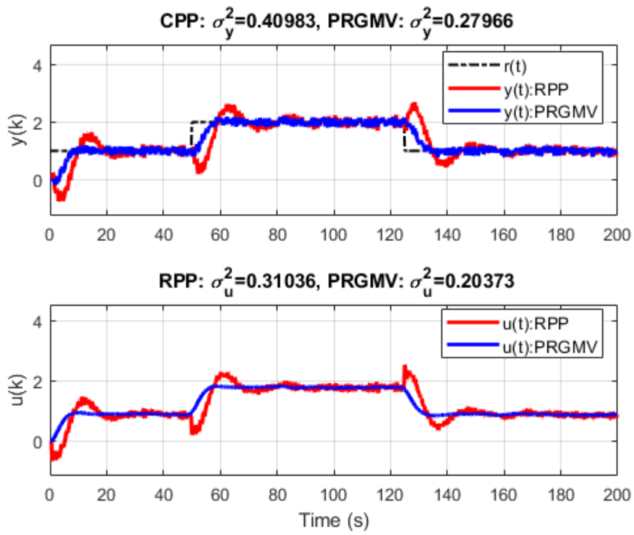


FIGURE 14. Closed-loop output with white noise.

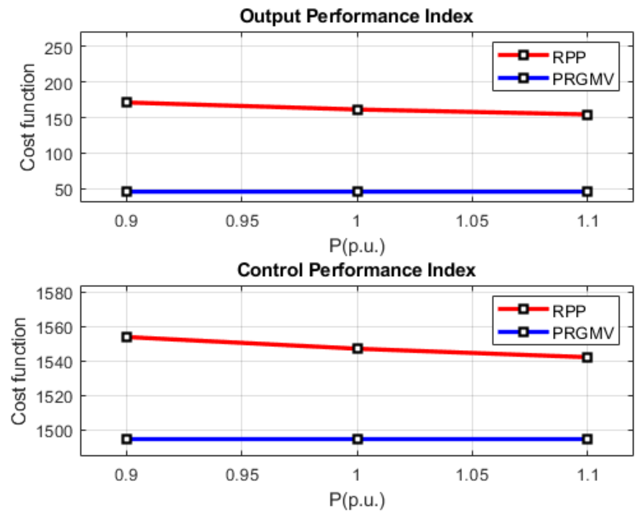


FIGURE 15. Performance index.

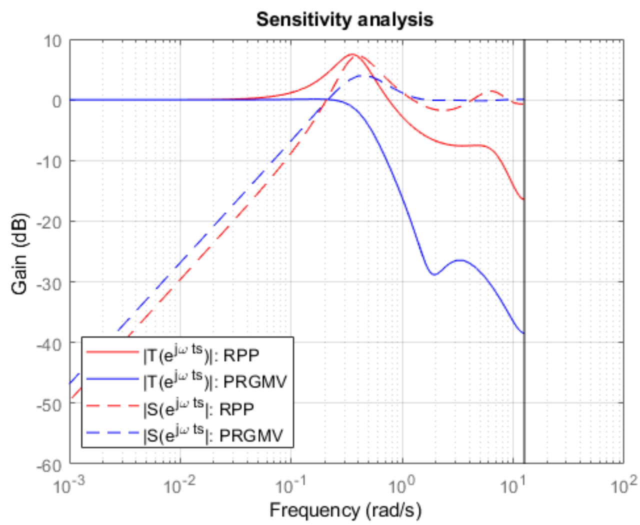


FIGURE 16. Sensitivity analysis.

According to Fig. 11, the RPP and PRGMV stabilize the system given in (45), so the reference tracking performance analysis in closed-loop for a sinusoidal input with frequency $f_o = 0.0159H$ considering only nominal operation point is realized. The results are presented in Fig. 17.

Fig. 17 shows that the steady-state response of the system for the sinusoidal input is also sinusoidal, at the same input frequency and differing in amplitude and phase angle [53], [54]. The results highlight the excellent performance of the PRGMV since it outperforms the other controllers by minimizing the variance of system output and control signal.

C. GENERAL ASSESSMENT OF RESULTS

Based on the presented results of the previous section, note that PRGMV copes better with the requirements of disturbance rejection and asymptotic reference tracking while keeping a smooth control signal and faster response with considerable variance minimization in the output and control signals and therefore provoking little stress on the controlled system whereas the other controllers presented control signal spikes and chattering.

The PRGMV outperforms the other controllers thanks to its formulation structure with polynomial filters $P_{prgmv}(z^{-1})$, $T_{prgmv}(z^{-1})$ and $Q_{prgmv}(z^{-1})$ which robustly weight the output, the reference and the control signals, respectively. The non-scalar polynomial $Q_{prgmv}(z^{-1}) = r_0 + r_1 z^{-1}$ filters out the control signal variations caused by the feedback of noisy measurements or disturbances and reduces the aggressiveness of the controller, leading to a smooth behavior with minimum control effort and energy consumption in the task of tracking the reference signal, according to Fig. 7 and Fig. 14 and confirmed by the variance minimization, i.e.., the linear power minimization.

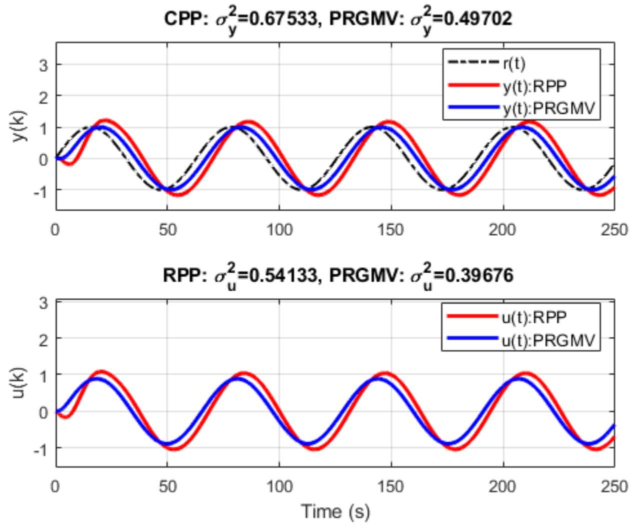


FIGURE 17. Closed-loop output with sinusoidal input.

The weighting polynomials $P_{prgmv}(z^{-1})$ and $T_{prgmv}(z^{-1})$ assign robust performance characteristics for the PRGMV, producing a better desired closed-loop response for reference tracking and disturbance rejection, as depicted in the figures 5, 6, 10, 12, 13 and 17. It also assigns greater stability margins and minor sensitivity to disturbances, as presented in figures 8, 9, 15 and 16, as well as in Table 2 and Table 4. The presented data confirmed, for the studied cases, that PRGMV can guarantee the stability of the system for all the vertices of the uncertain polytopic P domain as depicted in Fig. 2. It also achieves a desired maximum stability region thanks to Chebyshev's theorem that minimizes the overall deviation from the desired performance for the closed-loop system, maintaining an adequate compromise between robustness and performance with lower sensitivity to parametric uncertainties.

VI. CONCLUSION

This paper presented a novel methodology for designing robust predictive controllers to ensure robust stability and performance for a predefined uncertainty region. The design procedure is inspired in [22] and [25]. The proposed method was compared to two control methods, assessing performance in reference tracking and robustness to modeling uncertainties, load and noise. The results clearly showed that the proposed method could maintain the desired performance while compensating for load and stochastic disturbances, outperforming the other controllers.

The performance indicators obtained for the proposed method improved the system performance compared with the other methods that could not adequately compensate for the parametric variations and avoid the performance degradation. The results indicate that the PRGMV is justified and presented relevant improvements for parametric uncertainties systems, offering robust performance and stability.

Therefore, considering the lack of research regarding the hybridization of the two areas mentioned for plants with parametric uncertainty, it glimpses the development of the proposed methodology directly in the discrete frequency domain. In other words, obtaining the PID gains by pole placement and performing the Kharitonov's test directly in the discrete domain is an open topic for an investigation that might pay off in the end. In addition, it glimpses the extension of this work for MIMO system, systems with transport delay, e other predictive controllers, such as GPC.

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