

RESEARCH ARTICLE

An Improved Synthesis Method Based on ILPP and Colored Petri Net for Liveness Enforcing Controller of Flexible Manufacturing Systems

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ABSTRACT Petri nets are used to design deadlock control strategies for flexible manufacturing systems (FMSs), which typically involve the addition of monitors and the associated arcs to the FMS. The addition of several monitors and associated arcs to the first constructed Petri net model significantly complicates the Petri net controller. This paper develops a two-step method for preventing deadlocks based on a colored Petri net and a structurally minimal approach that significantly reduces the number of monitors. In the first step, a vector covering technique is applied to generate a minimal covered set of first-met bad markings (FBMs) and legal markings that are respectively smaller than the sets of FBMs and legal markings. At one iteration, place invariants (PIs) corresponding to monitors are constructed by solving an integer linear programming problem (ILPP) to prohibit the maximum number of FBMs, while allowing all legal markings in the minimal covering set. The purpose of the ILPP is to maximize the number of FBMs forbidden by the PIs. Then, based on a colored Petri net, all generated monitors are combined into a global control place. Therefore, a supervisor with minimal structural complexity can be constructed. The obtained net model is controlled after the addition of the designed supervisor. Two instances from the literature are considered to illustrate the proposed approach.

INDEX TERMS Colored Petri nets, integer linear programming, flexible manufacturing system, deadlock prevention.

I. INTRODUCTION

A flexible manufacturing system (FMS) executes a variety of tasks through the use of several processes that compete for finite resources including machines, robots, buffers, and fixtures [1], [2]. In an FMS, deadlock can occur as a result of processes competing for system resources [3]. In general, a deadlock causes a system to become inefficient and blocked, and may even result in destructive behavior, which is usually undesirable. As a result, a variety of methods have been developed to address the deadlock problem, including detection

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and recovery of deadlock [4], [5], avoidance of deadlock [6], [7], and prevention of deadlock [1], [2], [8]–[11].

Petri nets (PNs) are efficient mathematical and graphical modeling, analysis, and control tools for FMS deadlocks [2], [12], [13]. It is used to depict the FMSs' properties and behaviors, including conflict, sequencing, and synchronization. Additionally, PNs can be applied to represent characteristics such as liveness and boundedness [8]. The advantage of PNs over other modeling and simulation tools such as Arena [14], [15], queuing network models [16], digraph [17], and automata [18] are that they provide a simple representation of the systems. Petri nets are qualified to represent systems top-down at multiple levels of analysis and complexity, and

they have a strong mathematical foundation that permits both qualitative and quantitative study of such systems [9]. Deadlock prevention approaches are being pursued by a number of researchers, which can work as criteria for liveness-enforcing supervisors. These criteria involve behavioral permissiveness, which improves the system's resource utilization, and structural complexity, which results in a controller with a small number of control places, thus also reducing hardware and software costs, and computational complexity, which permits the implementation of a deadlock control approach to the large-scale systems [9], [10], [19]–[22].

Generally, structural analysis [10], [23], [24] and reachability graph analysis [25]–[27] are used to synthesize deadlock prevention methods based on Petri nets. Structural analysis is a powerful method for overcoming deadlocks in certain types of Petri net structures. In comparison to structural analysis methods, reachability graph-based methods can result in optimal or near-optimal controllers for generalized Petri net systems. Furthermore, these approaches must list all of the system's reachable states [8], [28], [29]. The purpose of this study is to discuss methods for analyzing reachability graphs. All markings (states) on a system can be classified into two groups, legal and illegal, based on their compliance with a control specification. In the deadlock prevention specification, a marking is considered legal if it or one of its successor states may transition back to the original marking; otherwise, it is an illegal state. A monitor is optimal if it prevents all illegal states while enabling legal states. In the studies [30], [31], a reachability graph is classified into a live zone (LZ) and a deadlock zone (DZ), with the LZ including all legal states and the DZ containing all illegal states. Then, a first-met bad marking (FBM) is classified as one that is illegal and indicates the LZ's initial entry into the DZ. First-met bad markings are a subset of illegal states associated with deadlocks, as the system cannot enter the DZ if all of them are prevented. Therefore, if a set of monitors is constructed to prohibit all first-met bad markings, then certain legal states may be forbidden. Such that, the generated supervisor cannot be guaranteed to be behaviorally optimal and also faces structural complexity as a result of a large number of monitors developed. The study [8] proposes a vector covering strategy to solve the above problem by analyzing the relationship between various states. Without considering all legal markings and all FBMs, they first proposed minimal covering sets of legal and FBMs markings be used in designing monitors. However, because a monitor is needed for each first-met bad marking in the minimal covered set of FBMs, a supervisor has an insufficient number of monitors. Chen and Li [28] extend this method by showing how to develop a structurally minimal controller that utilizes the fewest feasible monitors. No redundant monitor exists when this technique is used [21], [28]. Moreover, it guarantees that the supervisor obtained is behaviorally optimal. However, due to the complexity of solving an integer linear program with an excessive number of constraints and variables, it is difficult to construct a maximally permissive

supervisor in an acceptable amount of time using this method for a complicated net model.

In this study, we extend a strategy for supervisory control based on a controller's structural minimization. Without the need for iterations, the structurally minimum method is applied to formulate an integer linear programming problem (ILPP). By solving this ILPP, it is possible to achieve a set of optimal or near-optimal monitors while minimizing the monitors. Consequently, the designed monitors are significantly reduced, and the redundancy test is omitted. Finally, by adding a minimal number of monitors, the final net model becomes live. In comparison to previous work [34], our approach enables the development of an optimal or near-optimal supervisor with fewer monitors and without the need for iterations.

The rest of the paper is structured as follows: Section II presents some of the basic concepts employed in this research, including Petri nets, monitor synthesis using a place invariant, and the structurally minimal method. Section III provides a policy for supervisors with simple structures to prevent deadlocks. Several experimental results obtained using the developed approach are shown in Section IV. Finally, Section V presents conclusions and future research.

II. PRELIMINARIES

A. PETRI NETS

A marked Petri net is represented by $N = (P, T, F, W, M_o)$, where

1. P : Set of places, $P = \{p_1, p_2, \dots, p_m\}$, $m > 1$.
2. T : Set of transitions, $T = \{t_1, t_2, \dots, t_n\}$, $n > 1$.
3. F : $(P \times T) \cup (T \times P)$: Input and output function of a net.
4. W : $(P \times T) \cup (T \times P) \rightarrow \mathbf{IN}$: Mapping function that adds a weight to an arc, $W(p, t) > 0$ if $(p, t) \in F$, otherwise, $W(p, t) = 0$, all $p, t \in P \cup T$, and $\mathbf{IN} = \{0, 1, 2, \dots\}$.
5. $M_o : P \rightarrow \mathbf{IN}$: Initial marking of a net, and the p th element of M_o , represented by $M_o(p)$, is the initial tokens in place p .

A marked Petri net $N = (P, T, F, W, M_o)$ is called

1. an ordinary net if $W(p, t) = 1$, $\forall (p, t) \in F$, $p \in P$, and $t \in T$.
2. a weighted net if $W(p, t) > 1$, $(p, t) \in F$, $\exists p \in P$, and $\exists t \in T$.
3. self-loop free if $W(p, t) > 0$ implies $W(t, p) = 0$ and $\forall (p, t) \in P \cup T$.
4. self-loop if $W(t, p) > 0$ and $\forall (p, t) \in P \cup T$.

Assume that a node $a \in P \cup T$, the preset and postset of a can be respectively represented as ${}^a = \{b \in P \cup T \mid (b, a) \in F\}$ and $a' = \{b \in P \cup T \mid (a, b) \in F\}$. Incidence matrix $[N]$ of net N is a $|P| \times |T|$ integer matrix with $[N](p, t) = W(t, p) - W(p, t)$. A transition $t \in T$ is enabled (can be fired) if $M(p) \geq W(p, t)$, $\forall p \in {}^t$, denoted as $M[t]$, where $M(p)$ is the tokens number in place p . If a transition t fires, it generates a marking M' , represented by $M[t]M'$, where $\forall p \in P$, $M'(p) = M(p) - W(p, t) + W(t, p)$. The set of net N markings that are reachable from the initial marking M_o is represented by $R(N, M_o)$. $R(N, M_o)$ is represented by a reachability graph,

designated as $G(N, M_o)$, which is composed of arcs and nodes; arcs indicate transition firings labeled with t , while nodes contain markings labeled with M_i .

A marked Petri net $N = (P, T, F, W, M_o)$ is

1. live if $\forall t \in T$, t is live at M_o , $\forall M \in R(N, M_o)$, $\exists M \in R(N, M)$ such that $M[t]$.
2. dead at M_o if $t \in T$ such that $M_o[t]$.

A P -vector is a column vector $I: P \rightarrow \mathbf{Z}$ that is indexed by P , where $\mathbf{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$. If $I \neq 0$ and $I^T [N] = \mathbf{0}^T$, P -vector I is said to be a place invariant (PI). $\|I\| = \{p | I(p) \neq 0\}$, $\|I\|^+ = \{p | I(p) > 0\}$, and $\|I\|^- = \{p | I(p) < 0\}$ are said to be respectively the support, positive support, and negative support of place invariant I . If $l_i = I(p_i)$, $\forall p_i \in P$, l_i 's are named the coefficients of place invariant I . Assume that I is a PI of a net N and M is a marking that is reachable from M_o . Then, $I^T M = I^T M_o$.

B. ANALYSIS OF REACHABILITY GRAPH

Consider the reachability graph $R(N, M_o)$ of a net N . For purposes of deadlock control, markings in an $R(N, M_o)$ can be categorized as good, bad, dangerous, and deadlock. A good marking is one that is capable of reaching both the initial and subsequent markings. A bad one has successors, but they cannot achieve the initial marking. A dangerous marking is capable of reaching the initial marking, but at least one of its successors cannot reach the initial marking. A deadlock implies a dead state in a system that has no successor. To ensure optimal supervision, the controlled system should include both dangerous and good markings; these are the legal markings indicated by M_L . The legal markings for a PN system are stated as

$$M_L = \{M | M \in R(N, M_o) \wedge M_o \in R(N, M)\}. \quad (1)$$

A reachability graph is divided into two zones in [30], [31]: a live zone (LZ) and a deadlock zone (DZ), with the live zone containing all legal markings and the deadlock zone containing all illegal markings. An FBM is a specific illegal marking that can be created by firing one transition from the live zone to the deadlock zone. The FBMs are indicated by M_{FBM} and mathematically represented as

$$M_{FBM} = \{M \in DZ | \exists M' \in LZ, \exists t \in T, M'[t]M\}. \quad (2)$$

C. MONITOR FORMULATION METHOD

Yamalidou *et al.* [32] developed a technique based on PI for enforcing algebraic constraints on Petri net elements through the construction of monitors (control places), which includes the initial marking and associated arcs. Let $[N_p]$ with n places and m transitions represent the incidence matrix of an original net that must be controlled. The monitors can be expressed as a matrix containing the arcs connecting the monitors to the original net's transitions, denoted as $[N_c]$. The original net and the monitors are combined into a controlled net with an incidence matrix as

$$[N] = \begin{bmatrix} N_p \\ N_c \end{bmatrix}.$$

The following constraint must be satisfied when there is a control requirement:

$$\sum_{i=1}^n l_i \cdot M(p_i) \leq \beta \quad (3)$$

where β and l_i are positive integer constants, and $M(p_i)$ is the marking of the p_i . Eq. (3) is transformed by the addition of a positive slack variable $M(p_c)$ (the initial marking of a monitor p_c), and Eq. (3) becomes

$$\sum_{i=1}^n l_i \cdot M(p_i) + M(p_c) = \beta. \quad (4)$$

Eq. (4) defines a place invariant that must fulfill the equation $I^T [N] = \mathbf{0}^T$. Therefore, the control place $[N_c]$ can be stated as

$$[N_c] = -\mathcal{L} \cdot [N_p]. \quad (5)$$

At the initial state, the initial marking $M_o(p_c)$ of a monitor p_c can be formulated as

$$M_o(p_c) = \beta - \sum_{i=1}^n l_i \cdot M_o(p_i). \quad (6)$$

D. OPTIMAL MONITOR FORMULATION

Suppose we have an AMS with a net (N, M_o) and its reachability graph $R(N, M_o)$, which comprises of the M_L markings and the M_{FBM} markings. In this study, tokens in operation places (denoted as $P_A, P_A \in P$) are only considered for the purpose of obtaining a PI to prevent an FBM, indicated as $\mathbf{NA} = \{i | p_i \in P_A\}$. To prevent an FBM $M \in M_{FBM}$, the following constraint must be enforced:

$$\sum_{i \in \mathbf{NA}} l_i \cdot M(p_i) \leq \beta \quad (7)$$

where

$$\beta = \sum_{i \in \mathbf{NA}} l_i \cdot M(p_i) - 1. \quad (8)$$

The prohibited condition is denoted by Eq. (7). To ensure the maximally permissive control, after adding a monitor, all legal markings must be kept. To guarantee that no marking $M' \in M_L$ can be prevented, coefficients $l_i (i \in \mathbf{NA})$ should meet the reachability conditions

$$\sum_{i \in \mathbf{NA}} l_i \cdot M'(p_i) \leq \beta, \quad \forall M' \in M_L. \quad (9)$$

By substituting the β in constraint (8) into constraint (9), the legal markings reachability conditions for an FBM can be formulated as

$$\sum_{i \in \mathbf{NA}} l_i \cdot (M'(p_i) - M(p_i)) \leq -1, \quad \forall M' \in M_L. \quad (10)$$

For the coefficients l_i 's, solving constraint (10) generates a set of feasible solutions. Consequently, an optimal PI is calculated to guarantee that no FBM occurs and that all legal markings are reachable.

To decrease the number of legal markings M_L and the number of FBM M_{FBM} , the study [8] introduces a vector covering method for the place invariant control, with the following details:

Definition 1: Let (N, M_0) be a marked Petri net, $R(N, M_0)$ be its reachability markings, and two markings M and M' are in $R(N, M_0)$. If $M(p) \geq M'(p)$, $\forall p \in P_A$ that is represented by $M \geq_A M'$, then M A-covers M' .

Definition 2: Let (N, M_0) be a marked Petri net and M_L^* be a subset of legal markings M_L in N . If the following criteria are fulfilled, then M_L^* is said to be a minimal covered set of M_L :

- 1) $\forall M \in M_L, \exists M' \in M_L^*$, subject to $M' \geq_A M$; and
- 2) $\forall M \in M_L^*, M' \in M_L^*$, subject to $M' \not\geq_A M$ and $M \neq M'$.

Definition 3: Let (N, M_0) be a marked Petri net and M_{FBM}^* be a subset of M_{FBM} in N . If the following criteria are fulfilled, then M_{FBM}^* is called a minimal covered set of M_{FBM} :

- 1) $\forall M \in M_{FBM}, \exists M' \in M_{FBM}^*$, subject to $M \geq_A M'$; and
- 2) $\forall M \in M_{FBM}^*, M' \in M_{FBM}^*$, subject to $M \geq_A M'$ and $M \neq M'$.

M_{FBM}^* and M_L^* are respectively smaller than M_{FBM} and M_L , when the vector covering method is used. There is no FBM that is reachable if PIs prevent all markings in M_{FBM}^* . Meanwhile, if $\forall M \in M_L^*$ are not prohibited by PIs, and then $\forall M \in M_L^*$ are kept. The optimal supervisor is calculated using the markings in the sets M_{FBM}^* and M_L^* . As a result, for a marking $M \in M_{FBM}$, constraint (10) can be reformulated as

$$\sum_{i \in \mathbf{NA}} l_i \cdot (M'(p_i) - M(p_i)) \leq -1, \quad \forall M' \in M_L^*. \quad (11)$$

E. MONITOR FORMULATION FOR FORBIDDING FBMS

This section describes how to construct a place invariant PI, which prohibits the maximum number of FBMs. We can develop a PI to prohibit a certain FBM using an approach described in Section II-D. Indeed, more FBMs may be prohibited by a PI. Next, we design a method to increase the number of FBMs, which a PI prohibits. Initially, we use the notations \mathbf{N}_I^* , \mathbf{N}_{FBM}^* and \mathbf{N}_{LM}^* to indicate the number of PIs, $\{l | M_l \in M_{FBM}^*\}$ and $\{i | M_i \in M_L^*\}$, respectively. Note that $\mathbf{N}_I^* = \mathbf{N}_{FBM}^*$. Let I_j be a PI for the constraint

$$\sum_{k \in \mathbf{NA}} l_{jk} \cdot M_i(p_k) \leq \beta_j \cdot I_j \quad \forall j \in \mathbf{N}_I^*, i \in \mathbf{N}_{LM}^*, M_i \in M_L^* \quad (12)$$

where l_{jk} 's are the coefficients of I_j , I_j ($j \in \mathbf{N}_{FBM}^*$) a set of binary variables, and β_j is a positive integer variable. In constraint (12), if PI I_j is selected to prohibit FBM, then $I_j = 1$, otherwise, $I_j = 0$.

I_j prohibits the marking $M_l \in M_{FBM}^*$ if

$$\sum_{k \in \mathbf{NA}} l_{jk} \cdot M_l(p_k) \quad \forall j \in \mathbf{N}_I^*, l \in \mathbf{N}_{FBM}^*, \geq \beta_j \cdot I_j + 1 \quad M_l \in M_{FBM}^* \quad (13)$$

To represent the relationship between I_j and M_l in M_{FBM}^* , a set of binary variables f_{jl} 's ($j, l \in \mathbf{N}_{FBM}^*$) is introduced. Constraint (13) is modified as

$$\sum_{k \in \mathbf{NA}} l_{jk} \cdot M_l(p_k) \quad \forall j \in \mathbf{N}_I^*, l \in \mathbf{N}_{FBM}^* \geq \beta_j \cdot I_j + 1 - H \cdot (1 - f_{jl}) \quad \mathbf{N}_{FBM}^*, M_l \in M_{FBM}^* \quad (14)$$

where $f_{jl} \in \{0, 1\}$ and H is a sufficiently large positive integer value. In constraint (14), $f_{jl} = 1$ shows that I_j prohibits M_l , while $f_{jl} = 0$ denotes that M_l cannot be prohibited by I_j and it is redundant constraint.

Constraints (15) and (16) guarantees that each FBM can be prohibited by one PI I_j as

$$\sum_{j \in \mathbf{N}_I^*} f_{jl} \leq 1 \quad \forall l \in \mathbf{N}_{FBM}^* \quad (15)$$

$$f_{jl} \leq I_j \quad \forall j \in \mathbf{N}_I^*, l \in \mathbf{N}_{FBM}^* \quad (16)$$

Constraints (17) ensures that at least one FBM can be prohibited by one PI I_j as

$$\sum_{l \in \mathbf{N}_{FBM}^*} f_{jl} \geq 1 - I_j \quad \forall j \in \mathbf{N}_I^* \quad (17)$$

The objective function maximizes the set of FBMs which, a PI prohibits and can be formulated as

$$\text{Max } z = \sum_{j=1}^{\mathbf{N}_I^*} \sum_{l=1}^{\mathbf{N}_{FBM}^*} f_{jl} \quad (18)$$

The coefficients of I_j and β_j must meet the conditions of reachability. Therefore, to design PI, the following ILPP is constructed, namely, an improved maximum number of forbidding FBM problem (IMFFP).

IMFFP:

$$\text{Max } z = \sum_{j=1}^{\mathbf{N}_I^*} \sum_{l=1}^{\mathbf{N}_{FBM}^*} f_{jl}$$

$$\text{subject to } \sum_{k \in \mathbf{NA}} l_{jk} \cdot M_i(p_k) \leq \beta_j \cdot I_j$$

$$\forall j \in \mathbf{N}_I^*, i \in \mathbf{N}_{LM}^*, M_i \in M_L^* \quad (19)$$

$$\sum_{k \in \mathbf{NA}} l_{jk} \cdot M_l(p_k) \geq \beta_j \cdot I_j + 1 - H \cdot (1 - f_{jl})$$

$$\forall j \in \mathbf{N}_I^*, l \in \mathbf{N}_{FBM}^*, M_l \in M_{FBM}^* \quad (20)$$

$$\sum_{l \in \mathbf{N}_{FBM}^*} f_{jl} \leq 1 \quad \forall j \in \mathbf{N}_I^* \quad (21)$$

$$f_{jl} \leq I_j \quad \forall j \in \mathbf{N}_I^*, l \in \mathbf{N}_{FBM}^* \quad (22)$$

$$\sum_{l \in \mathbf{N}_{FBM}^*} f_{jl} \geq 1 - I_j \quad \forall j \in \mathbf{N}_I^* \quad (23)$$

$$l_{jk} = \{0, 1, 2, \dots\} \quad \forall j \in \mathbf{N}_I^*, k \in \mathbf{NA} \quad (24)$$

$$f_{jl} = \{0, 1\} \quad \forall j \in \mathbf{N}_I^*, l \in \mathbf{N}_{FBM}^* \quad (25)$$

$$I_j = \{0, 1\} \quad \forall j \in \mathbf{N}_I^* \quad (26)$$

$$\beta_j = \{0, 1, 2, \dots\} \quad \forall j \in \mathbf{N}_I^* \quad (27)$$

The IMFFP objective function z is employed to maximize the set of FBMs prohibited by PIs and to achieve a structurally minimal and behaviorally optimal supervisor, by ensuring that all markings in M_L^* are reachable and the number of monitors is minimized.

Theorem 1: If $z = 0$, no FBM in M_{FBM}^* has a maximally permissive PI.

Proof: Assume that there is a PI I_j , which can prohibit marking $M_l \in M_{FBM}^*$ by contradiction. Due to the permissive design of I_j , its coefficients l_{11}, l_{12}, \dots , satisfy constraint (19). Given that M_l is prohibited by I_j , we have $\sum_{k \in \mathbf{NA}} l_{jk} \cdot M_l(p_k) \geq \beta_j \cdot I_j + 1$. Thus, $f_{jk} = 1$ satisfies constraints (20-22). We have $z = \sum_{l \in \mathbf{N}_{FBM}^*} f_{jl} \geq 1$, $\forall j \in \mathbf{N}_I^*$. This contradicts $z = 0$. As a result, the conclusion is correct. \square

As known, it is NP-hard to solve an ILPP. The computational time required to solve an IMFFP is strongly influenced by the number of variables (denoted by N_v) and constraints (denoted by N_c) in it. Thus, we can discuss IMFFP in terms of its number of variables and constraints. The number of variables l_{jk} 's ($j \in \mathbf{N}_I^*$, $k \in \mathbf{NA}$) is $|M_{FBM}^*| \cdot |P_A|$, where $|P_A|$ and $|M_{FBM}^*|$ represent respectively the number of the operation places and PIs. The number of variables f_{jl} 's ($j \in \mathbf{N}_I^*$, $l \in M_{FBM}^*$), I_j 's ($j \in \mathbf{N}_I^*$), and β_j 's ($j \in \mathbf{N}_I^*$) are $|M_{FBM}^*| \cdot |M_{FBM}^*|$, $|M_{FBM}^*|$, and $|M_{FBM}^*|$, respectively. As a result, IMFFP has $|M_{FBM}^*| \cdot |P_A| + (|M_{FBM}^*|)^2 + 2|M_{FBM}^*|$ variables in total. Now, we can consider the number of constraints in the IMFFP, the total numbers of constraints (19), (20), (21), (22), and (23) are $|M_L^*| \cdot |M_{FBM}^*|$, $|M_{FBM}^*| \cdot |M_{FBM}^*|$, $|M_{FBM}^*|$, $|M_{FBM}^*| \cdot |M_{FBM}^*|$, and $|M_{FBM}^*|$, respectively. Finally, the total number of the constraints in IMFFP is $|M_L^*| \cdot |M_{FBM}^*| + 2(|M_{FBM}^*|)^2 + 2|M_{FBM}^*|$.

III. DEADLOCK PREVENTION METHODS

A. DEADLOCK PREVENTION METHOD-BASED IMFFP

In this section, we present a structurally minimal method and the deadlock prevention policy to prevent deadlocks by using IMFFP. The structurally minimal method is applied in one iteration to develop a set of maximally permissive monitors and minimize their number. The main advantage is that a few number of monitors are designed and it allows for the development of an optimal or nearly optimal supervisor. Algorithm 1 illustrates the deadlock prevention method based IMFFP.

Consider the FMS example in Figure 1 to demonstrate the proposed Algorithm 1. Figure 2 shows the system's PN model. The model contains 20 reachable markings, 5 of which are FBMs markings M_{FBM} and 15 of which are legal markings M_L . The minimal covered sets of FBMs M_{FBM}^* and legal markings M_L^* are $M_{FBM}^* = \{p_2 + p_5, p_3 + p_5, p_2 + p_6\}$ and $M_L^* = \{p_2 + p_3 + p_4, p_5 + p_6 + p_7\}$, respectively, when using a vector covering method.

Now, Algorithm 1 is considered, we introduce

1. three binary variables I_1, I_2 , and I_3 to be computed.

Algorithm 1 A Deadlock Prevention Algorithm Based IMFFP

Input: A net (N, M_0) .

Output: A controlled net (N_1, M_1) .

1. Calculate the M_L and the M_{FBM} .
 2. Calculate the M_L^* and the M_{FBM}^* .
 3. $VS = \emptyset$. /* The notation VS represents the monitors to be calculated.*/
 4. **for** all M_{FBM}^* **do**
 - i. Build the IMFFP;
 - ii. Solve IMFFP;
 - if** $z \neq 0$ **then** /* Objective function.*/
 - Let l'_{jk} 's and β_j be the solution;
 - else**
 - Exit, because there is no solution;
 - end if**
 - iii. Based on I_j , design a monitor p_c ;
 - iv. $VS = VS \cup \{p_c\}$. /* All M_{FBM}^* is covered */
5. Insert all obtaining monitors in VS to the initial net (N, M_0) .
6. **Output** (N_1, M_1) .
7. **End**.

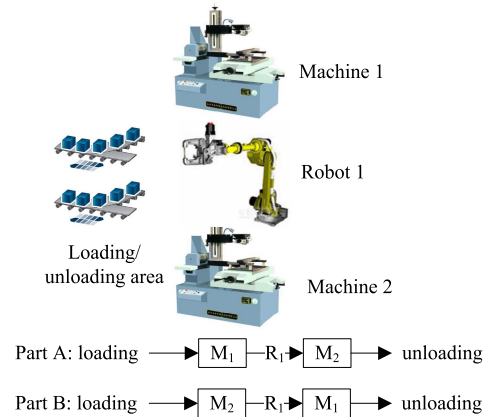


FIGURE 1. FMS example and its process route.

2. nine binary variables $f_{11}, f_{12}, f_{13}, f_{21}, f_{22}, f_{23}, f_{31}, f_{32}$, and f_{33} to indicate if I_1, I_2 , and I_3 prohibit three FBMs in M_{FBM}^* .

Finally, we have IMFFP as follows

$$\begin{aligned} \text{Max } z &= f_{11} + f_{12} + f_{13} + f_{21} + f_{22} + f_{23} + f_{31} + f_{32} + f_{33} \\ \text{subject to } & l_{15} + l_{16} + l_{17} \leq \beta_1 \cdot I_1 \\ & l_{12} + l_{13} + l_{14} \leq \beta_1 \cdot I_1 \\ & l_{25} + l_{26} + l_{27} \leq \beta_2 \cdot I_2 \\ & l_{22} + l_{23} + l_{24} \leq \beta_2 \cdot I_2 \\ & l_{35} + l_{36} + l_{37} \leq \beta_3 \cdot I_3 \\ & l_{32} + l_{33} + l_{34} \leq \beta_3 \cdot I_3 \\ & l_{13} + l_{15} \geq \beta_1 \cdot I_1 + 1 - H \cdot (1 - f_{11}) \end{aligned}$$

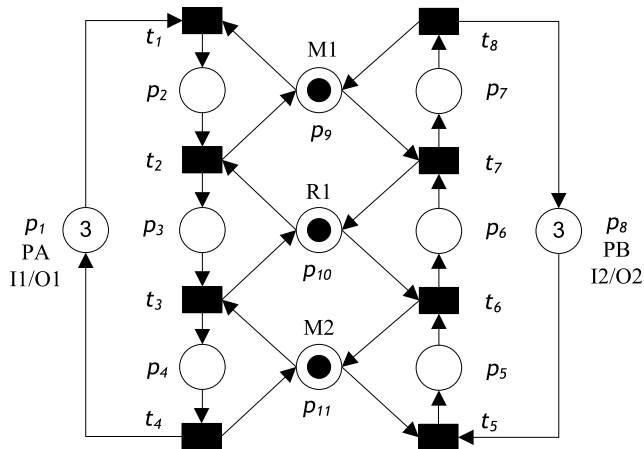


FIGURE 2. A net (N, M_o) of a system presented in Figure 1.

$$\begin{aligned}
 l_{12} + l_{15} &\geq \beta_1 \cdot I_1 + 1 - H \cdot (1 - f_{12}) \\
 l_{12} + l_{16} &\geq \beta_1 \cdot I_1 + 1 - H \cdot (1 - f_{13}) \\
 l_{23} + l_{25} &\geq \beta_2 \cdot I_2 + 1 - H \cdot (1 - f_{21}) \\
 l_{22} + l_{25} &\geq \beta_2 \cdot I_2 + 1 - H \cdot (1 - f_{22}) \\
 l_{22} + l_{26} &\geq \beta_2 \cdot I_2 + 1 - H \cdot (1 - f_{23}) \\
 l_{33} + l_{35} &\geq \beta_3 \cdot I_3 + 1 - H \cdot (1 - f_{31}) \\
 l_{32} + l_{35} &\geq \beta_3 \cdot I_3 + 1 - H \cdot (1 - f_{32}) \\
 l_{32} + l_{36} &\geq \beta_3 \cdot I_3 + 1 - H \cdot (1 - f_{33}) \\
 f_{11} + f_{21} + f_{31} &\leq 1 \\
 f_{12} + f_{22} + f_{32} &\leq 1 \\
 f_{13} + f_{23} + f_{33} &\leq 1 \\
 f_{11} + f_{12} + f_{13} &\geq 1 - I_1 \\
 f_{21} + f_{22} + f_{23} &\geq 1 - I_2 \\
 f_{31} + f_{32} + f_{33} &\geq 1 - I_3 \\
 f_{11} &\leq I_1, f_{12} \leq I_1, f_{13} \leq I_1 \\
 f_{21} &\leq I_2, f_{22} \leq I_2, f_{23} \leq I_2 \\
 f_{31} &\leq I_3, f_{32} \leq I_3, f_{33} \leq I_3 \\
 \beta_j &\in \{0, 1, 2, \dots\}, \quad \forall j \in \{1, 2, 3\} \\
 l_{jk} &\in \{0, 1, 2, \dots\}, \quad \forall j \in \{1, 2, 3\}, \\
 &\quad k \in \{2, 3, 4, 5, 6, 7\} \\
 f_{jl} &\in \{0, 1\}, \quad \forall j \in \{1, 2, 3\}, l \in \{1, 2, 3\} \\
 I_j &\in \{0, 1\}, \quad \forall j \in \{1, 2, 3\}
 \end{aligned}$$

The above IMFFP is solved using the Lingo solver, and the optimal solution is $l_{12} = 2, l_{15} = 1, l_{16} = 1, I_1 = 1, \beta_1 = 2, f_{12} = 1, f_{13} = 1$. Then, a monitor p_{c1} is developed for PI1: $2\mu_2 + \mu_5 + \mu_6 + \mu_{p_{c1}} = 2$. Thus, I_1 prohibits FBM2 and FBM3, and the preset transitions, postset transitions, and initial marking of the monitor p_{c1} are respectively $\cdot p_{c1} = \{2t_2, t_7\}, p'_{c1} = \{2t_1, t_5\}$, and $M_{1o}(p_{c1}) = \beta_1 = 2$. In addition, $l_{33} = 1, l_{35} = 1, I_3 = 1, \beta_3 = 1, f_{31} = 1$. Then, a monitor p_{c2} is developed for PI2: $\mu_3 + \mu_5 + \mu_{p_{c2}} = 1$. Thus, I_3 forbids FBM1, and the preset transitions, postset transitions,

and initial marking of the monitor p_{c2} are respectively $\cdot p_{c2} = \{t_3, t_6\}, p'_{c2} = \{t_2, t_5\}$, and $M_{1o}(p_{c2}) = \beta_3 = 1$. All rest variables are equal zero.

Table 1 presents a summary of the results, with the first column indicating the calculated PI I_j , the second column indicating the number of covered FBMs in M_{FBM}^* that are prohibited by I_j . The third to fifth columns indicating respectively the output transitions p'_{cj} , the input transitions $\cdot p_{cj}$, and initial marking ($M_{1o}(p_{cj})$) of monitor p_{cj} . The sixth and seventh columns indicating respectively the number of variables N_v and the number of constraints N_c in IMFFP. The last column indicating the required computational time (denoted by $\Psi(s)$) to solve the ILPP. Figure 3 illustrates the controlled system after adding two monitors to the initial net model.

TABLE 1. Calculated monitors using Algorithm 1 for the model shown in Figure 2.

PI	Covered M_{FBM}^*	$\cdot p_{cj}$	p'_{cj}	$M_{1o}(p_{cj})$	N_v	N_c	$\Psi(s)$
I_1	2	$2t_1, t_5$	$2t_2, t_7$	2	33	30	< 2
I_3	1	t_2, t_5	t_3, t_6	1			

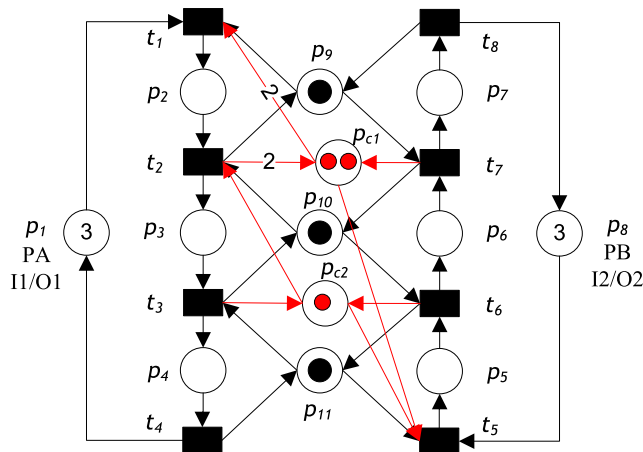


FIGURE 3. A net (N_1, M_1) of the net shown in Figure 2 using Algorithm 1.

B. DEADLOCK PREVENTION METHOD -BASED COLORED PETRI NETS

A colored Petri net (CPN) is represented by $N = (P, T, C, F, K, M_o)$, where

1. P and T are defined in Section 2.1;
2. $C(p)$ and $C(t)$ are respectively represent the sets of colors connected with $p \in P$ and $t \in T$. We let $C(p_i) = \{a_{i1}, a_{i2}, \dots, a_{iui}\}$ and $C(t_i) = \{b_{j1}, b_{j2}, \dots, b_{jvj}\}$ where $u_i = |C(p_i)|$ and $v_j = |C(t_j)|$;
3. $F: I(p, t) \cup O(p, t)$: Input and output function of a net, where the input function is expressed as $I(p, t): C(p) \times C(t) \rightarrow \mathbf{IN}$, and the output function is expressed as $O(p, t): C(p) \times C(t) \rightarrow \mathbf{IN}$;

4. $K: P \rightarrow \mathbf{IN}$: represents the function of capacity that assigns the maximal number of tokens to each place $K(p_i)$;
5. $M_o: P \rightarrow \mathbf{IN}$ is a marking function that allocates tokens to the places. $M_o(p_i)$ denotes the initial number of tokens in p_i , regardless of their color, while $M_o(p_i, a_{ij})$ denotes the initial tokens in p_i , which have the color a_{ij} .

The enabling and firing rules of the transition t_j in a colored Petri net can be stated as below.

1. A transition t_j is said to be a process-resource-enabled if

$$M(p_i, a_{ih}) \geq I(p_i, t_j)(a_{ih}, b_{jk}), \quad \forall p_i \in P, \\ \forall p_i \in t_j, a_{ih} \in C(p_i), b_{ik} \in C(t_j) \quad (28)$$

and

$$K(p_i) \geq M(p_i, a_{ih}) + O(p_i, t_j)(a_{ih}, b_{jk}) \\ - I(p_i, t_j)(a_{ih}, b_{jk}), \quad \forall p_i \in P, \\ \forall p_i \in t_j, a_{ih} \in C(p_i), b_{ik} \in C(t_j). \quad (29)$$

2. At marking M , the transition t_j can fire if the t_j is enabled and the marking M transformed to marking M' as follows.

$$M'(p_i, a_{ih}) = M(p_i, a_{ih}) + O(p_i, t_j)(a_{ih}, b_{jk}) \\ - I(p_i, t_j)(a_{ih}, b_{jk}), \\ \forall p_i \in P, a_{ih} \in C(p_i), b_{ik} \in C(t_j). \quad (30)$$

Definition 4: Let (N, M_o) be a marked Petri net. The deadlock supervisor for (N, M_o) designed in IMFFP is represented as $(V, M_{V_o}) = (P_V, T_V, F_V, M_{V_o})$. Here, (V, M_{V_o}) can be replaced by a common colored subnet that is a net with $N_{DC} = (\{p_{global}\}, \{T_{DCi} \cup T_{DCo}\}, F_{DC}, C_{DC}, M_{DCo})$, where p_{global} is named the combined monitor of all control places $P_V = \{p_{cj} \mid p_{cj} \in VS\}$, $VS = \{p_{c1}, p_{c2}, \dots, p_{cj}\}$. $T_{DCi} = \cup_{i \in VS} \{t \mid t \in p_{ci}\}$. $T_{DCo} = \cup_{i \in VS} \{t \mid t \in p_{ci}\}$. $F_{DC} \subseteq (\{p_{global}\} \times \{T_{DCi} \cup T_{DCo}\}) \cup (\{T_{DCi} \cup T_{DCo}\} \times \{p_{global}\})$ is the set of arrows, which connect the combined monitor with transitions (and vice versa). $C_{DC} = \cup_{i \in VS} \{C_{pci}\}$ is the set of all monitors color, where C_{pci} is the color of the monitor p_{ci} that maps p_{global} into colors. (N_{DC}, M_{DCo}) is named a common colored subnet. For all $p_c \in P_V$, $M_{DCo}(p_{global}) = \sum M_{V_o}(p_c)$, where $M_{DCo}(p_{global})$ is an initial tokens with the colors of the combined control place.

Definition 5: Let (N, M_o) be a marked Petri net and (N_{DC}, M_{DCo}) be a common colored deadlock control subnet with $N_{DC} = (\{p_{global}\}, \{T_{DCi} \cup T_{DCo}\}, F_{DC}, C_{DC}, M_{DCo})$. We call (N_{CN}, M_{CN_o}) a controlled colored Petri net. Furthermore, $(N_{CN}, M_{CN_o}) = (N, M_o) \parallel (N_{DC}, M_{DCo})$, which is the integration of (N, M_o) and (N_{DC}, M_{DCo}) , where $N_{CN} = (P \cup \{p_{global}\}, T \cup T_{DCi} \cup T_{DCo}, F \cup F_{DC}, C_{DC}, M_{CN_o})$, and $R(N_{CN}, M_{CN_o})$ be its reachable graph.

Algorithm 2 illustrates the deadlock prevention method by using IMFFP and CPN. Reconsider the controlled net in Figure 3 to demonstrate the proposed Algorithm 2. Figure 4 depicts the p_{global} place of all control places P_V in Figure 3, as generated by Algorithm 2. The output arcs of p_{global} that

obtained from Algorithm 1 are represented as $p_{c1} = \{2t_1, t_5\}$ and $p_{c2} = \{t_2, t_5\}$. Therefore, T_{DCo} can be represented as $T_{DCo} = \{2t_1, t_2, 2t_5\}$, as depicted in Figure 5. The input arcs of p_{global} that obtained from Algorithm 1 are represented as $p_{c1} = \{2t_2, t_7\}$ and $p_{c2} = \{t_3, t_6\}$. Thus, T_{DCi} be stated as $T_{DCi} = \{2t_2, t_3, t_6, t_7\}$, as displayed in in Figure 6. In addition, $M_{DCo}(p_{global}) = \sum M_{1_o}(V_S) = M_{1_o}(p_{c1}) + M_{1_o}(p_{c2}) = 2 + 1 = 3$. Petri net model in Figure 3 contains two color types: $C_{DC} = \{C_{pc1}, C_{pc2}\}$. Accordingly, as shown in Figure 7, the p_{global} has three colored tokens: two tokens with color C_{pc1} and one token with color C_{pc2} . Finally, the controlled colored Petri net (N_{CN}, M_{CN_o}) of the net shown in Figure 3 using Algorithm 2 is presented in Figure 8.

Algorithm 2 Deadlock Prevention Method Based IMFFP and CPN

Input: A net (N_1, M_1) . /* By using Algorithm 1.*/

Output: A net (N_{CN}, M_{CN_o})

1. Merge all monitors P_V into a single monitor (p_{global}), considering the procedures below:
 - i. Design the output arcs T_{DCo} , then connect them with p_{global} ; /* By Definition 4*/
 - ii. Design the input arcs T_{DCi} , then connect them with p_{global} ; /* By Definition 4*/
 - iii. Define colors C_{pci} for a monitor p_{global} ; /* By Definition 4*/
 - iv. Calculate the initial tokens with colors $M_{DCo}(p_{global}) = \sum M_{V_o}(V_S)$. /* By Definition 4.*/
2. Add the p_{global} into the net (N_1, M_1) .
3. **Output** (N_{CN}, M_{CN_o}) .
4. **End**

Control places by Algorithm 1

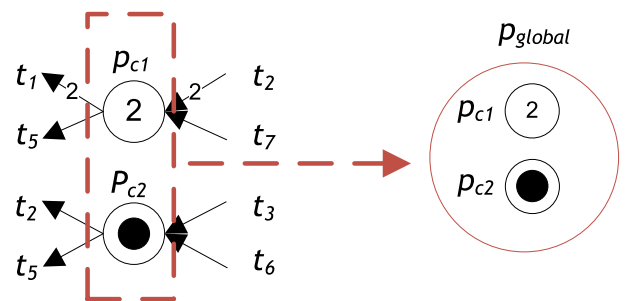


FIGURE 4. p_{global} of all control places P_V of the model presented in Figure 3 using Algorithm 2.

IV. EXPERIMENTAL RESULTS

In this section, we demonstrate the application of the proposed Algorithms 1 and 2 by presenting some FMS examples. C++ programs are applied to generate the minimal covered sets of FBMs and legal markings, as well as to construct IMFFP that can be used in Algorithm 1. Then, the Lingo solver was used to solve IMFFP. In addition, we have coded Algorithm 2 to construct the global control

TABLE 2. Calculated monitors using Algorithms 1 for the net shown in Figure 9.

PI	Covered M_{FBM}^*	p_{cj}^*	$^*p_{cj}$	$M_{1o}(p_{cj})$	N_v	N_c	$\Psi(s)$
I_1	3	$t_1, t_2, t_4, 3t_9$	$2t_7, 3t_7$	9	168	352	< 152
I_2	5	$4t_1, 4t_2, t_4, t_9, 7t_{11}$	$3t_5, 5t_6, t_{12}, 7t_{13}$	14			

TABLE 3. Comparison of Algorithms 1 and 2 performance with some deadlock prevention methods for the net shown in Figure 9.

Parameters	[37]	[38]	[39]	[8]	[21]	Algorithm 1	Algorithm 2
No. states	205	205	205	205	205	205	205
No. control places	6	9	5	8	2	2	1
No. arcs	32	42	23	37	15	15	10
Permissiveness (%)	100	100	100	100	100	100	100

Control places by Algorithm 1

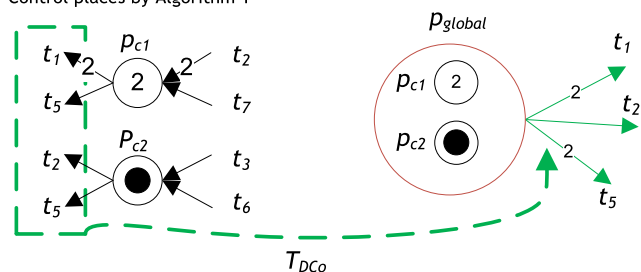


FIGURE 5. Output arcs of the p_{global} of all control places P_V of the model presented in Figure 3 using Algorithm 2.

Control places by Algorithm 1

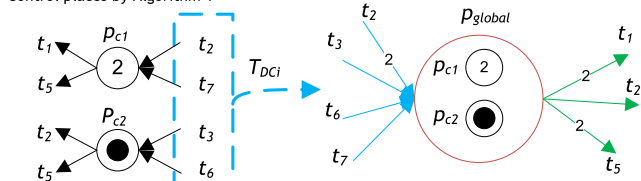


FIGURE 6. Input arcs of the p_{global} of all control places P_V of the model presented in Figure 3 using Algorithm 2.

Control places by Algorithm 1

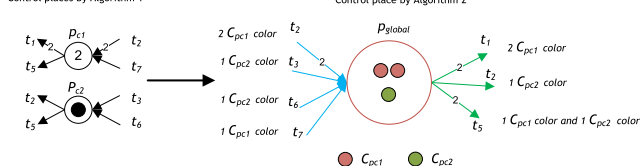


FIGURE 7. Global control place of the net shown in Figure 3 using Algorithm 2.

point using the GPenSIM tool [33]–[36]. Figure 9 illustrates a Petri net model, which has been studied in [8], [10], [21], [28], [37]–[40]. It consists of 19 places and 14 transitions. The model has 282 reachable states with 205 legal markings and 54 FBMs. The minimal covered sets of legal markings M_L^* and FBMs M_{FBM}^* are respectively 26 and 8 markings. The implementations of Algorithm 1 are summarized in Table 2. Next, the two resulting monitors using Algorithm 1 are

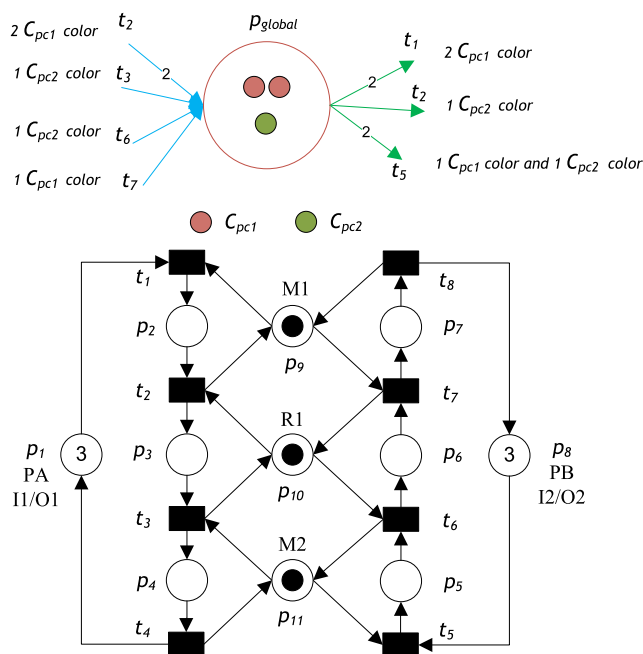


FIGURE 8. Controlled colored Petri net (N_{CN}, M_{CN0}) of the net shown in Figure 2 using Algorithm 2.

combined to form p_{global} using Algorithm 2. The output arcs of p_{global} are represented as $T_{DCo} = \{t_1, 5t_2, 2t_4, 4t_9, 7t_{11}\}$. The input arcs of p_{global} are represented as $T_{DCi} = \{3t_5, 5t_6, 3t_7, t_{12}, 7t_{13}\}$. In addition, $M_{DCo}(p_{global}) = \sum M_{1o}(V_S) = M_{1o}(p_{c1}) + M_{1o}(p_{c2}) = 9 + 14 = 23$. Thus, we have two color types: $C_{DC} = \{C_{pc1}, C_{pc2}\}$. The p_{global} place has 23 colored tokens: 9 tokens with the color C_{pc1} and 14 tokens with the color C_{pc2} . Table 3 shows the comparison the Algorithms 1 and 2 to other existing deadlock control methods in terms of the numbers of added monitors, added arcs, and states of the controlled net. Algorithm 2 yields a supervisor with one monitor and 10 arcs, both of which are minimal in comparison to other methods in [8], [21], [37]–[39].

Next, Figure 10 illustrates a Petri net model, which has been studied in [41], [44], [45]. It consists of 26 places

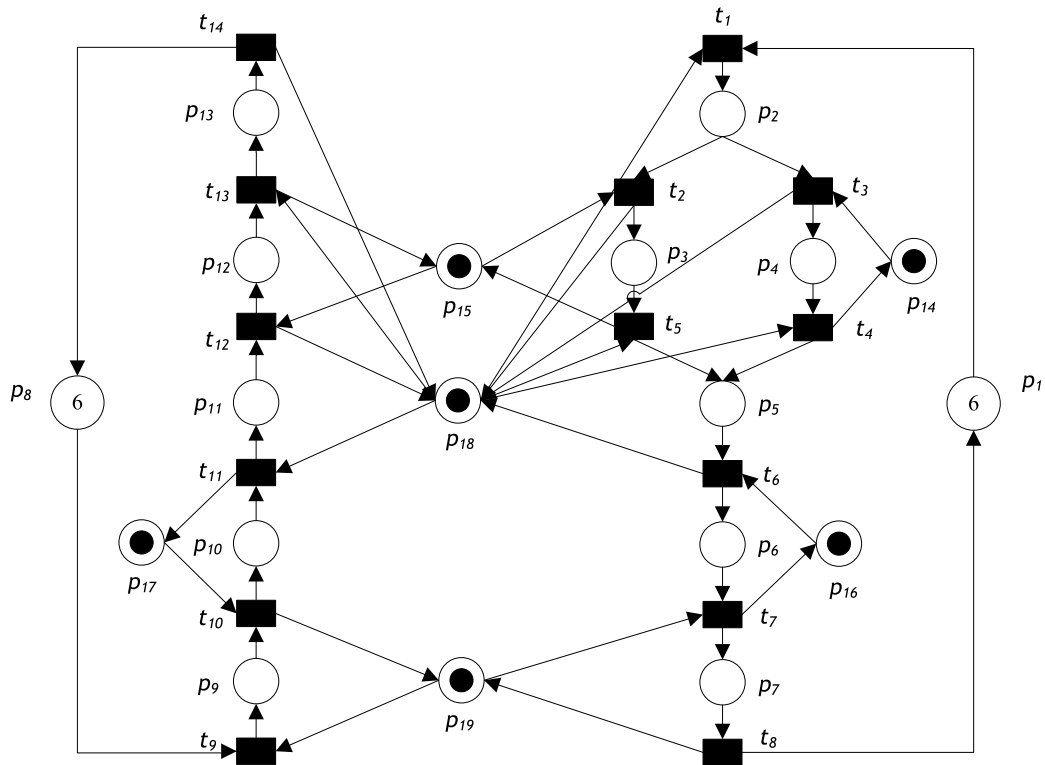


FIGURE 9. A net (N, M_0) of the first FMS.

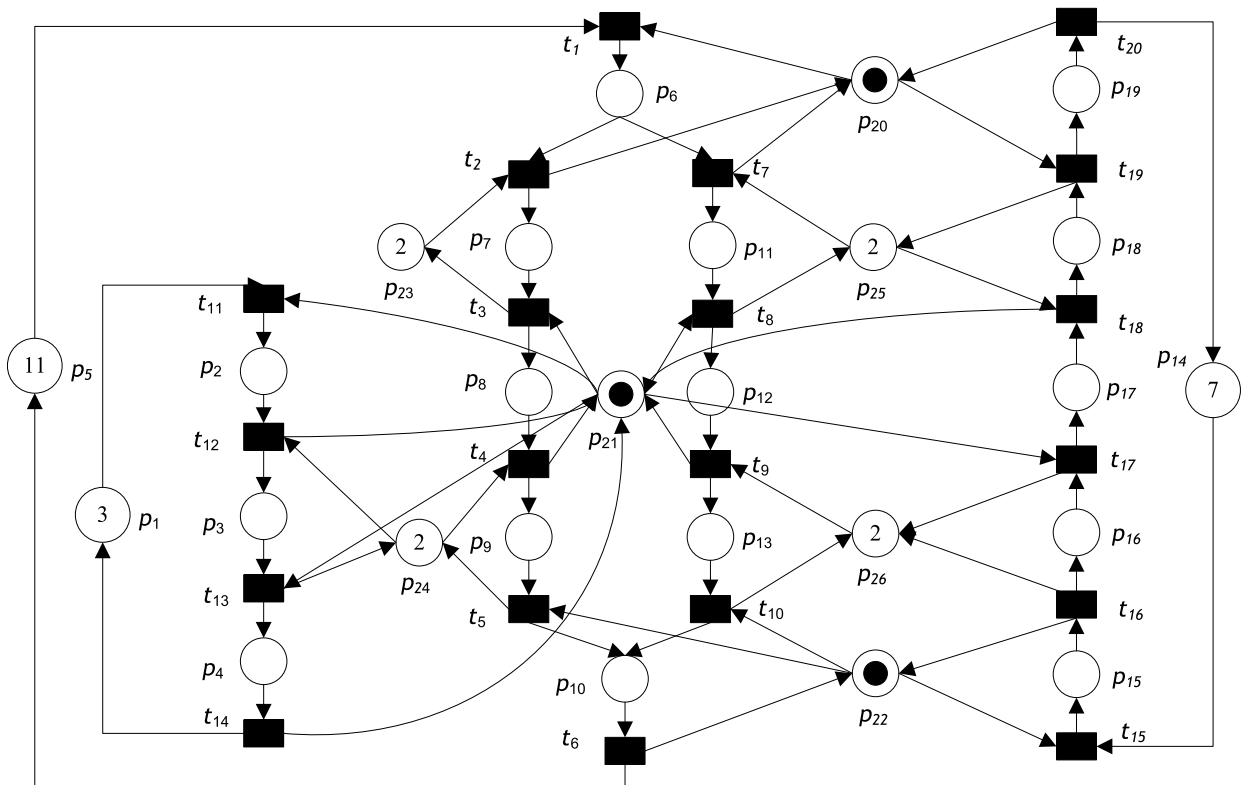


FIGURE 10. A net (N, M_0) of the second FMS presented in Ezpeleta et al. [41].

and 20 transitions. The model has 26750 reachable states with 21581 legal markings and 4211 FBMs. The minimal covered sets of legal markings M_L^* and FBMs M_{FBM}^* are respectively 393 and 3 markings. Table 4 illustrates the

computed monitors for the model presented in Figure 10 using Algorithm 1. Then, the six resulting control places using Algorithm 1 are combined to form p_{global} using Algorithm 2. The output arcs of p_{global} are represented

TABLE 4. Calculated monitors using Algorithm 1 for the net shown in Figure 10.

PI	Covered M^*_{FBM}	p_{ci}^*	p_{cj}^*	$M_{Io}(p_{ci})$	N_v	N_c	$\Psi(s)$
I_1	8	$t_3, 27t_7, t_{11}, 6t_{15},$ $3t_{16}, 9t_{17}$	$t_5, 24t_8, 3t_{10}, t_{13},$ $18t_{18}$	71			
I_2	19	$3t_1, 14t_3, 44t_8, 3t_9,$ $15t_{11}, 50t_{15}, t_{18}$	$17t_5, 50t_{10},$ $15t_{13}, 49t_{17}, 2t_{19}$	196			
I_3	3	$6t_1, t_4, t_{15}, 4t_{17}, t_{18}$	$6t_3, t_5, 5t_8, t_9,$ $6t_{19}$	34	1768	15742	< 300
I_4	1	t_8, t_{16}	t_9, t_{17}	2			
I_5	1	t_9, t_{15}	t_{10}, t_{16}	2			
I_6	2	t_3, t_{11}	t_4, t_{13}	2			

TABLE 5. Comparison of Algorithms 1 and 2 performance with some deadlock prevention methods for the net shown in Figure 10.

Parameters	[41]	[42]	[43]	[30]	[39]	[8]	[21]	Algorithm 1	Algorithm 2
No. states	6287	6287	12656	21562	21581	21581	21581	21581	21581
No. control places	18	6	16	19	13	17	6	6	1
No. arcs	106	32	88	112	82	101	45	45	22
Permissiveness (%)	29.13	29.13	58.64	99.91	100	100	100	100	100

as $T_{DCo} = \{9t_1, 16t_3, t_4, 27t_7, 45t_8, 4t_9, 2t_{11}, 48t_{15}, t_{16}, 13t_{17}, 46t_{18}\}$. The input arcs of p_{global} are represented as $T_{DCi} = \{6t_3, t_4, 18t_5, 29t_8, 2t_9, 54t_{10}, 17t_{13}, t_{16}, 50t_{17}, 18t_{18}, 8t_{19}\}$. In addition, $M_{DCo}(p_{global}) = \sum M_{Io}(V_S) = M_{Io}(p_{c1}) + M_{Io}(p_{c2}) + M_{Io}(p_{c2}) + M_{Io}(p_{c3}) + M_{Io}(p_{c5}) + M_{Io}(p_{c6}) = 71 + 196 + 34 + 2 + 2 + 2 = 341$. Thus, we have six color types: $C_{DC} = \{C_{pc1}, C_{pc2}, C_{pc3}, C_{pc4}, C_{pc5}, C_{pc6}\}$. The p_{global} place has 341 colored tokens: 71 tokens with color C_{pc1} , 196 tokens with the color C_{pc2} , 34 tokens with the color C_{pc3} , 2 tokens with the color C_{pc4} , 2 tokens with the color C_{pc5} , and 2 tokens with the color C_{pc6} . Finally, the comparison of the Algorithms 1 and 2 performance with some deadlock prevention methods in terms of the numbers of added monitors, added arcs, and states of the controlled net is shown in Table 5. Algorithm 2 provides a controller with one control place and 22 arcs, both of which are minimal in comparison to other methods in [8], [21], [30], [39], [41]–[43].

V. CONCLUSION

This paper presents an approach for preventing deadlocks based on colored Petri nets and a structurally minimal method. First, a vector covering technique is applied to calculate a minimal covered set of FBMs and legal markings. By solving an ILPP in one iteration, place invariants corresponding to control places are constructed to prohibit the maximum number of FBMs. The first-step-obtained controlled model makes the Petri net supervisor significantly more complicated. In the second step, colored Petri nets are applied to design the smallest number of monitors by integrating all generated control places into a single global control place. In comparison to previous work [8], [21], [30], [37]–[39], [41]–[43], our approach enables the development of an optimal or near-optimal supervisor with fewer monitors and without the need for iterations to design place invariants

to prohibit the FBMs, while there are no prohibited legal markings.

The main disadvantage of the developed approach is that it is subject to modifications in control requirements and specifications, such as adding new equipment and products or modifying the system's processing routes. In the case that these problems appear, the system must be changed. The proposed model may thus be subject to new deadlock problems. Therefore, our future study will focus on optimizing the efficiency of the proposed method for valid and quick reconfiguration of the FMS [46] and the fault and its security issues [47], [48].

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