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EntangleNetSat: A Satellite-Based Entanglement Resupply Network

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ABSTRACT In the practical context of quantum networks, quantum teleportation plays a key role in transmitting quantum information. In the process of teleportation, a maximally entangled pair is consumed. Through this paper, an efficient scheme of re-establishing entanglement between different nodes in a quantum network is explored. A hybrid land-satellite network is considered, where the land-based links are used for short-range communication, and the satellite links are used for transmissions between distant nodes. This new scheme explores many different possibilities of resupplying the land nodes with entangled pairs, depending on: the position of the satellites, the number of pairs available and the distance between the nodes themselves. As to make the entire process as efficient as possible, we consider the situations of direct transmissions of entangled photons and also the transmissions making use of entanglement swapping. An analysis is presented for concrete scenarios, sustained by numerical data.

INDEX TERMS Quantum communication, entanglement, teleportation, entanglement swapping, routing scheme, satellite.

I. INTRODUCTION

In Quantum Communication, quantum teleportation [14], [18] is a reliable channel for transmitting information. Since quantum states have a very high potential for granularity, the information embedded within them is also densely packed. As such, two nodes (laboratories Alice and Bob), separated by distance, can use the properties of quantum entanglement to securely transfer vast amounts of information by quantum teleportation. However, in the process, a maximally entangled pair (an ebit) is consumed. For the two spatially separated parties to be able to transfer information again through the means of quantum teleportation, a quantum entanglement resupply schema must be put in place.

While ground-based resupply schemes have been previously studied by [1], [15], [19], [34], an inherent limitation of such protocols is the exponential attenuation caused by the transmission medium: fiber optics or air. Therefore, high fidelity transmissions of entangled pairs is feasible only for relatively short ranges (approximately 421KM [17]). Better efficiency can be achieved by transmitting through less dense mediums, such as the upper layers of the atmosphere, using drones [16] or satellites [8]. Recently, there have been advances in satellite transmissions of entangled photon bursts [21], [23] up to 1200KM [20], [22], [29], using a single satellite to act as relay.

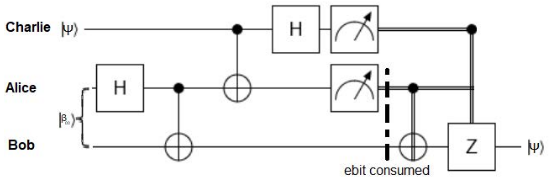

FIGURE 1. **Quantum teleportation circuit. The three qubits are spread around 3 laboratories: Charlie with ψ, Alice and Bob each with one qubit, forming the β_{00} state. After performing the teleportation protocol, the ebit shared between Alice and Bob is consumed and the ψ state is transferred to Bob.**

A new possibility arises to use multiple satellites, mounted with quantum repeaters and quantum entanglement sources, as hops between Alice and Bob. By generating quantum entangled photons in orbit [24], [25] and then routing them through the satellite network, we can minimize interference caused by long range transmissions through the atmosphere.

We present a routing schema using a hybrid ground and satellite network, which aims to provide global coverage for quantum entanglement distribution, by using a satellite mesh to act as the medium for long range resupply queries.

II. ENTANGLEMENT RESUPPLY VIA SATELLITE

Distribution of quantum entanglement via satellites is a novel direction, that has the potential of positively impacting

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aspects of the quantum internet [3], [13] infrastructure, such as Quantum Key Distribution [43]–[45] protocols, as well as enhancing classical communication reliability [12]. While the idea of distributing entangled pairs through a satellite network has been suggested since 2003 [28], only recently, due to advances in quantum entanglement sources and quantum repeaters [27], such a thing became feasible.

The setting in which our protocol operates involves two laboratories, Alice and Bob, spatially separated by a large distance, such that sending an entangled photon between the two cannot be accomplished reliably. The two parties are either equipped with a receiver for satellite communications, or are part of a terrestrial quantum network, where at least one node has the means of receiving entangled photons from satellites. For resupplying entanglement with nodes outside of the local network, a satellite network backbone is used, according to the protocol which we will elaborate in the following sections.

A. NETWORK INFRASTRUCTURE AND REQUIREMENTS

The scenario that we present in this paper assumes an already established land-based quantum network using Free Space Optical(FSO) transmissions for short range entangled pair distribution. Such a network could use various routing schemes for the resupply process, such as the protocols explored in [2], [4] which assume a homogeneous network, where each node is equipped with quantum repeaters.

The composition of these networks consists of a graph for which the nodes are: (1) either various laboratories around the world, which aim to use the maximally entangled pairs for the purpose of quantum teleportation or quantum repeaters; (2) or devices which use entanglement purification along with entanglement swapping to enhance the hop-to-hop fidelity of the entangled particles' beam; (3) or quantum/classical switches. This land-based infrastructure is mostly in place, as classical channels such as fiber optics are widely used for classical high-speed communications. However, due to the optical attenuation caused by these mediums, distribution of entangled pairs becomes lossy, as the further the transmitted resource has to travel through such mediums, the greater the fidelity loss, thus becoming exponentially less than 1 ebit. Experimentally, it has been shown that the maximum distance one could distribute entangled pairs and reconstitute a maximally entangled pair is in the order of hundreds of km through terrestrial FSO mediums.

The primary cause for entanglement loss in ground based mediums (e.g. fiber optics) is the attenuation of the medium, caused by the absorption and scattering of the photons along the path. The attenuation of a fiber optics cable is modeled by the equation:

$$Att(dB) = 10 \times \log_{10} \frac{Input\ power}{Output\ power}$$

However, due to the sensible nature of quantum states, quantum transmitters do not operate at high powers, leading to a high attenuation factor. Even though a pair is successfully transported to a different location, the entanglement purity is less than what it was before the transmission and may lead to communication errors between the nodes.

Regarding deep-space transmissions of quantum information, Ntanos *et al.* [9] has studied the effect of atmospheric conditions for downlink transmissions, as well as other environmental factors in deep space, that have a lesser effect at sea-level, or no effect at all. The team has found, through simulations, that due to solar irradiance, transmissions during daylight hours have a higher loss than the ones during nighttime, but downlink communications are still feasible during daytime. A common loss factor for deep-space and open-air transmissions is due to scattering, which leads to an additional loss modeled by the inverse square law, as the area receiving the photons is larger than the area of the emitter. This effect is more pronounced at sea-level, due to a higher particle density in the atmosphere, but can be observed to some degree in deep-space as well.

Alongside the short-range land network, we consider a constellation of LEO satellites [42] in equally spaced polar orbits, known in literature as a Walker constellation. Experiments have proven that ground-to-satellite transmissions of entangled pairs are possible [11], [28], [30]–[32] over long distances [29]. However, such schemes used a single satellite as a relay for the photons [9], [20]. We assume a grid of satellites equipped with either very high precision mirrors or quantum repeaters, thus enabling satellite-to-satellite quantum transmissions [10]. We only consider entanglement transmissions from the satellites down to terrestrial stations (down-links), as it has been shown [35] that the down-link transmission fidelity is greater than for up-links.

Due to the relative lack of particles in deep space and upper atmosphere, the transmission of entangled pairs from a satellite to a ground station seems to be a better alternative than ground-based FSO alternatives for long range transmissions. Previous works have shown that as long as the transmitting satellite is within approximately 20 degrees deviation from the zenith direction, transmission fidelity remains at about the same transmittance levels as from zenith [33], [35], [36]. Furthermore, the effects of direct sunlight within the medium of transmission have been proven to be greatly reduced when using specific wavelengths for the photon beams, e.g. 1,550 nm [47].

An even better transmission fidelity could be attained in satellite-to-satellite transmissions, given the photons wouldn't pass through the upper layers of the atmosphere. This can be achieved by having fairly short distances between adjacent satellites. The satellites should be able to transmit photon beams on each of the 4 cardinal directions: North, South, East and West and the 4 ordinal directions: North-East, North-West, South-East and South-West.

Among recent research efforts, which focused on performing satellite-based Quantum Key Distribution (QKD), it is worth mentioning the use of a single satellite as a Bell State Measurement (BSM) machine for quantum swapping [5], together with an analysis regarding optimal placement and

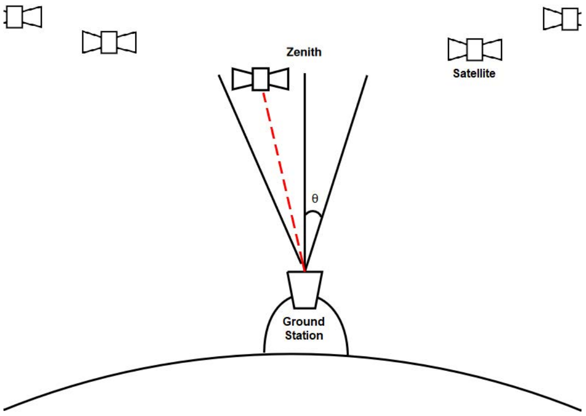

FIGURE 2. Transmittance of the photon beam depends on the deviation from the Zenith direction of the receiver. Ideally, the satellite closest to the zenith position (terminal satellite) should perform the down-link transmission.

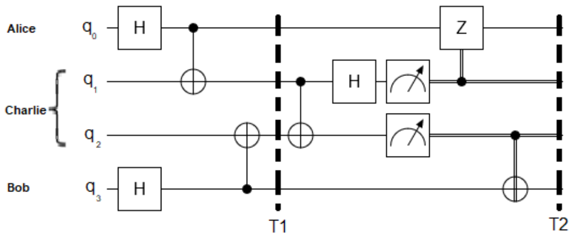

FIGURE 3. Quantum entanglement swapping circuit. This protocol happens between 3 parties: Alice, Bob and Charlie. At the beginning, Alice has the q_0 qubit, Charlies has q_1, q_2 and Bob has q_3. At T1, both Alice and Bob share an entangled pair with Charlie. Afterwards, Charlie will perform a BSM, swapping the two qubits' entanglement. At T2, q_0 and q_3 are entangled, without leaving Alice and Bob's laboratories.

optimal number of satellites for continuous resupply service, across an arbitrarily set of terrestrial nodes [7].

Due to the increasing amount of satellites in Low Earth Orbit, it is becoming infeasible to place a new constellation of classical satellites for the sole purpose of entanglement distribution. However, with the advances in the field of nanosatellites [37]–[39], it is possible to reach a high enough density for the satellite swarm as to have a very high transmission fidelity for entangled pairs. Moreover, given a high density of satellites, the ground stations would experience much less service down-time, therefore one could resupply two random ground stations with entanglement on-demand. Given a dense enough mesh of satellites, it might be feasible to continuously distribute entangled pairs, rather than on-demand, giving the land nodes the option to either ignore the entangled bits, or selectively capture them as the need arises, simillar to the scenario proposed in [6].

III. ROUTING SCHEME

As presented previously, we assume a number of ground stations around the globe, which consume maximally entangled pairs (epairs) with the purpose of communicating via quantum teleportation. Once the epair is consumed, the two nodes need to be resupplied with a number of entangled particles, which can be used in the process of entanglement purification [40] to produce a maximally entangled pair.

We assume that each ground station is equipped with QRAM [41], thus having the possibility of storing entangled bits (ebits) linked to all other ground stations. Each of these ebit storages would have a statically or dynamically determined threshold, so that when the number of ebits remaining for communication with a given station drops below it, the resupply process would get triggered.

We have identified 3 main steps in the resupply process: (1) the selection of terminal satellites, (2) route and entanglement source satellite selections and (3) the transmission of entangled photons. Since the first two steps require route computations, the problem of which node should execute them arises. We consider a total order relationship between the ground stations (e.g. IP), and the station that ranks highest (or lowest) in the relationship should compute the terminal satellites.

Having calculated the terminal satellites, a request of the form (Alice $\longleftarrow S_A$, Bob $\longleftarrow S_B$) is then forwarded by the designated ground station to the nearest node of a Command and Control (C&C) network for the satellites. This C&C network should contain a unified view of the satellites grid's statuses and coordinates. The second step, i.e., the route and source selection, is performed by this C&C network. For the third step, the C&C network commands the satellites to execute the transmission with the parameters computed at the second step.

STEP 1: TERMINAL SATELLITE SELECTION

The main metric of concern when choosing the satellites, which will act as the final relays in the satellite grid, is their angle with respect to the zenith direction of their corresponding ground station. As such, for keeping maximum possible transmission fidelity, the two terminal satellites should be as close to the zenith of the requesting stations as possible. The computations required for choosing the two satellites can be done by either having a precomputed (time $\Longleftrightarrow$ satellite and ground station) mapping, either by calculating the mapping on-demand, or by a hybrid approach between the two. We continue by giving a closer view of each approach.

- **Precomputed Mapping**
  Since for the polar orbit satellites there exists a known system of equations, giving their position at any given moment in time, it is feasible to compute and store all possible mappings, given the periodicity of the rotations of both the Earth and the satellite mesh, while assuming no change in their orbits. However, such a precalculation wouldn't account for small deviations in the trajectory of the satellites caused by solar winds, or other natural occurring phenomena, which can temporarily place the satellite slightly off-orbit. Given the very high accuracy needed for the transmission of the short burst of photons, this approach might not be enough for a high accuracy service.

- **On-Demand Computation**
  The easiest way to account for all the slight deviations of orbit would be to query all satellites for their actual positions and find the closest one, based on their most up to date coordinates. While this approach wouldn't prove computationally difficult, it poses the challenge of acquiring these coordinates for a large number of satellites at once. While the GPS system manages to overcome this [46], given the relatively small amount of satellites involved, it might be infeasible to simultaneously query all the satellites for their positions, or to simultaneously process their positions in the case of a continuous coordinate transmission coming from the mesh.
- **The Hybrid approach**
  While the first approach has a very low computational cost and 0 messages transmitted between Earth and the satellite mesh, it provided low accuracy. The second approach provides maximum accuracy at the cost of computational power and at least $N_{satellites}$ messages processed by the ground station. The best trade-off would be to use the precomputed mappings to minimize the scope of the query for satellites' coordinates to a small and manageable number of satellites, that are nearest to the ground station at a given moment in time. Such a system would require a small number of messages to be transmitted to and from the satellites toward the ground station, and at the same time could have an insignificant computational cost to select the closest satellite to the ground station.

Given a representation of the satellites positions in spherical coordinates, the selection of the closest one in terms of angular offset with respect to the zenith direction would be fairly straight forward (see Algorithm 1).

STEP 2: ROUTE AND SOURCE SATELLITE SELECTION

The main metric which we try to minimize is the amount of satellites involved in the transmission. There is no apparent drawback to transmitting through the 4 ordinal directions, even though the photon beam would travel closer to the Earth's atmosphere. By using the 1550 nm wavelength, part of the Wulf band, the altitude at which these beams are absorbed by the atmosphere is at about 15-40km, which corresponds to the altitude of the ozone layer [48]. Given the high altitudes at which LEO satellites are typically orbiting, interference caused by the ozone layer would happen only when the satellites would almost be out of the line-of-sight. Therefore, it is preferable to do a single transmission in an ordinal direction (e.g. NE), rather than two transmissions along the cardinal directions (e.g. N $\longrightarrow$ E), both in terms of satellites involved, and distance traveled by the entangled particles.

A separate aspect of the arrangement of the satellites in polar orbits is that, while in most of the cases the adjacency of satellites doesn't change, however due to the geometry of Walker Constellations, on one hemisphere (e.g. Eastern Hemisphere) the satellites would be moving North, while on the other they would be moving South. This generates a ring within the orbital shell where the adjacency of satellites is constantly changing, therefore it needs to be continually computed.

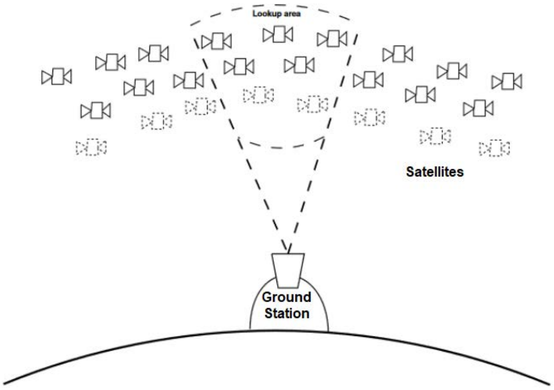

FIGURE 4. The trade-off we suggest, in terms of accuracy and computational efficiency, is a precomputed lookup table for satellites. The corresponding entries for the ground station and the time of the query will then have their positions accurately determined, as to correctly choose the one closest to the Zenith position of the ground station.

We introduce a metric which will measure how long the satellites used in a route will be used for one transmission:

$$\tau_u = \tau_c + QR_d + \tau_t$$

where τ_u is the timeslice for which a satellite cannot be used for another transmission; τ_c is the commutation time, the time a satellite needs to change it's transmission direction to another satellite; QR_d is the delay introduced by a quantum repeater, in the case the satellite is equipped with such a device (otherwise this term is 0); τ_t is the effective transmission time for the photon beam. The first two terms will depend on the specific hardware installed in the satellites, while the transmission time is configurable. Previous works have shown that photon bursts with a frequency of 10Hz with a duty cycle of 3.6% [26] produce satisfactory transmission fidelity. Therefore, we can assume that τ_u would be on the order of magnitude of seconds, possibly even less.

For $\tau_u = 0$, assuming a negligible τ_t, the transmissions could happen at will, asynchronously. However, for a non-zero τ_u, a scheduling scheme must be established. All transmissions should be synchronized, with each clock unit being equal to τ_u. For a given satellite S_i, serving a route S_A, S_B takes exactly one clock unit.

Having a calculated pair of satellites S_A and S_B, the terminal satellites for Alice and Bob respectively, a route through the satellite mesh can be computed. From here on, we will describe the satellite mesh as a graph $G(V, E)$ where the vertices are the constituent satellites, and the edges are the possible transmission FSO routes between adjacent satellites. The topology of the mesh resembles a 2D torus topology, with a periodic change between the outermost lateral edges. The nodes can be equipped with 2 devices that would increase their importance in the mesh: quantum repeaters and quantum

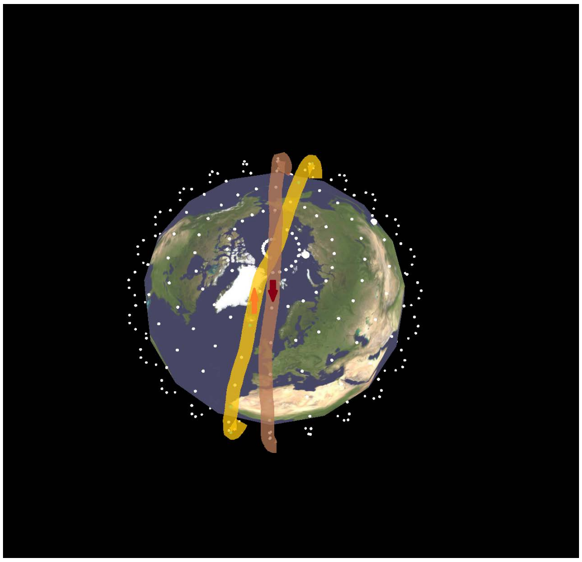

FIGURE 5. Due to the nature of polar orbits, there are two adjacent rings of satellites which move in opposite directions relative to each other. Therefore, the graph representing the satellite network is constantly changing, but in a cyclic fashion.

entanglement sources. The least complex protocol for a possible scenario, is the one for a homogeneous network, where all satellites would have a quantum entanglement source and repeaters. In this case a fairly simple algorithm based upon Bellman-Ford could be applied to determine the shortest route between S_A and S_B (see Algorithm 2), otherwise, optimal routing protocols such as the ones explored by [2], [4] can be deployed, given that they can be computed in a sufficiently short amount of time.

Once a route is established, an entanglement source needs to be determined along the route. Optimally, it should be placed as close to the middle of the route as possible, in the case of a homogeneous network, or it should have an equal amount of repeaters on the routes to S_A and S_B respectively. This would minimize the total resupply time, computed by the formula:

$$\tau_r = QR_d \times \max\{n_{qr}(Source \longrightarrow S_A), \times n_{qr}(Source \longrightarrow S_B)\} \tag{1}$$

where $n_{qr}(\text{Source} \longrightarrow S_*)$ is the number of repeaters along the route from the source to S_*. The above formula doesn't take into account the travel time of the photon beam, which we can assume is negligible in comparison to the delay introduced by the quantum repeaters. Minimizing τ_r would enhance the throughput of the entire network, since it could theoretically enable more transmissions to be done in the same amount of time for that network region.

However, more complexity arises when the satellite mesh is heterogeneous in terms of satellite equipment (i.e. only a number of satellites have quantum entanglement sources, and others are equipped with quantum repeaters). For very long range transmissions, quantum repeaters might need to be chosen along the route to boost the received entanglement, while for a network with sparse quantum sources, each route ought to have at least one source along its path. In the case of at least 2 quantum sources, an entanglement transmission using quantum swapping can be established. The optimal placement of the two sources involved is as terminal satellites, in which case the transmission time τ_r would be identical to the scenario where there is a source in the middle satellite of the route. Transmission with entanglement swapping implies that each satellite is equipped with a BSM (Bell State Measurement) device. The satellite which will perform the entanglement swapping should be in the middle of the route between the two sources, to minimize the total transmission time. Choosing this satellite can be performed in a similar manner as the source satellite in the homogeneous network case. Protocols which account for quantum swapping have been developed for terrestrial networks [1], but can be adapted for the satellite mesh, too. Such routing protocols could be considered as a black box, where the inputs are the two nodes requiring entanglement, and its output is the route and sources chosen. These algorithms should be given the two terminal satellites as inputs, after which the down-link transmission directly to the ground station can take place.

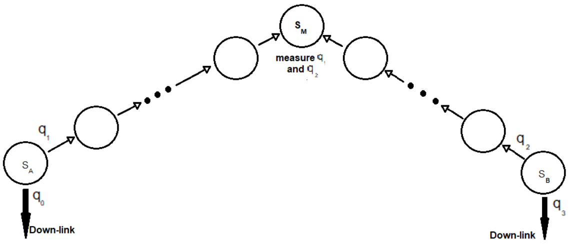

FIGURE 6. An example of quantum swapping used to generate an entangled pair which is then transmitted to the ground stations. In this particular case, the two entanglement sources are at S_A and S_B, and are sending two qubits to the satellite designated to perform the BSM (see fig. 3). After the swapping protocol is complete, the q_0 and q_3 qubits are ready to be transmitted to the ground stations.

STEP 3: TRANSMISSION OF ENTANGLED PHOTONS

As the route is selected and validated with the other nodes in the C&C network, messages must be sent to all the involved satellites, to prepare them for the transmission. Once the time duration for the resupply is reached, the source satellite forwards the two photon beams to their first hops along the routes to S_A and S_B respectively. When the beams reach the terminal satellites S_A and S_B, the beams are directed towards the requesting stations, Alice and Bob, where the process of quantum entanglement purification may proceed. If, however, due to a low fidelity transmission, an upper threshold for ebits isn't reached upon purification, the process of requesting entanglement is restarted as to satisfy the upper threshold.

IV. SATELLITE DENSITY ANALYSIS

In this section, we analyse the effect of satellite density upon two transmission parameters: number of hops and the distance travelled by the photons. We introduce a subunitary parameter α, which controls the density of satellites.

TABLE 1. Samples of α values and the corresponding number of satellites, orbits and total number of satellites (N_o and N_s, for a maximum $number\ of\ orbits = number\ of\ satellites = 200$). The computed distances are the inter-orbital distances at the equator, with a satellite altitude of 500km.

α	N_o, N_s	Total Satellites	Satellite Distances (km)
0.01	2	4	13670
0.1	20	4e+2	2138.4
0.33	66	4.35e+3	650.42
0.66	132	1.74e+4	325.3
0.75	150	2.25e+4	286.27
1	200	4e+4	214.71

The number of satellites is described by the number of orbits and number of satellites per orbit, for which we consider an upper and lower limit, and we consider an equal number of orbits and satellites per orbit, to be able to better represent our results.

The distance between the satellites is calculated as the distance of direct transmission with the following formula:

$$distance = 2 \times altitude \times sin(\theta/2)$$

where θ is the angle between two consecutive satellite belts.

For our simulations, we have considered two laboratories, one in Washington D.C. (ground station A) and one in Bucharest (ground station B). The geographical distance between these two sites is approx. 7972 km. We have simulated a satellite mesh, characterized by an orbital altitude of 500 km, number of orbits and number of satellites per orbit, controlled by α. The two laboratories request entanglement throughout the day, at fixed intervals. We have measured the number of hops each of these resupply transmissions have to traverse, and the combined distance the photon pair has to travel to reach the two laboratories. We have considered a homogeneous network for our simulations, where all satellites can produce entangled pairs. We have approximated an orbital period of 90 minutes, such that when 24 hours pass, the system would be in the same configuration as when it started.

In Figure 7 we are plotting the number of total hops between S_A and S_B. The number of hops directly affects the transmission time τ_r and the entanglement loss, assuming each hop has a negative effect upon the entanglement L_{hop} (such as the effect of a slightly polarized mirror), and is a metric we aim to minimize in the routing algorithms for the satellite network.

It is observable that by increasing the satellite density, the number of hops between two sites increases linearly. We also observe that the time of day has little effect upon this metric, with the values staying roughly the same at low α values and oscillating rapidly at high alpha values. This is explained by the fact that the optimal terminal satellites are changing at a fast pace for a high density satellite mesh.

In Figure 8 we have the effect of α and time of day on the combined distance travelled by the photon pair, calculated as the sum of the distances from S_{source} to the ground station A and distance from S_{source} to ground station B. In the case of transmission via quantum swapping, the distance would be the sum of distances: (1) from the source satellites to the designated BSM satellite plus (2) the distances from the sources to their respective ground stations.

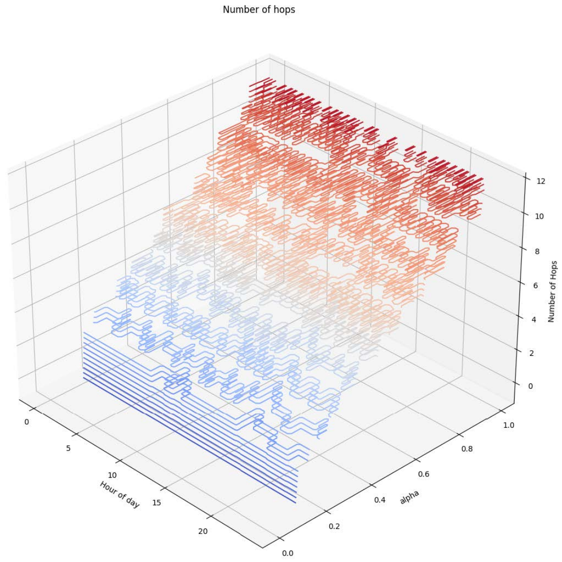

FIGURE 7. Number of hops along the route against satellite density α and time of day (0:00 through 23:59). For values under approx. 0.17, immediate transmission is impossible, since there are no satellites in position to serve both sites at once.

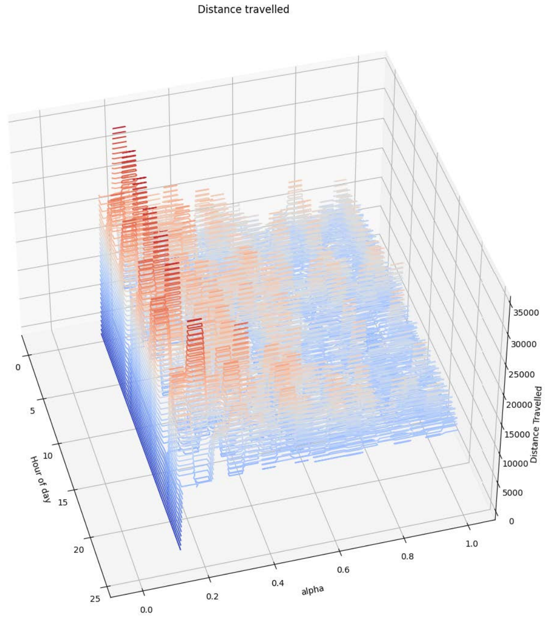

FIGURE 8. The cumulated distance the photon pair has to travel to reach both sites against satellite density α and time of day.

It is observed that for greater satellite densities, the distance the photons have to travel tends to be equal to the geographical distance plus a scaling factor, given the orbital altitude. While for a lower α value, there are configurations where transmission distance is low, there are periodic peaks where the distance reaches almost three times the geographical distance. This is explained by the fact that with a small number of satellites in the network, there are times when the

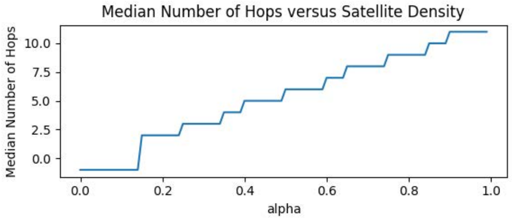

FIGURE 9. **Median number of hops along the route against satellite density.**

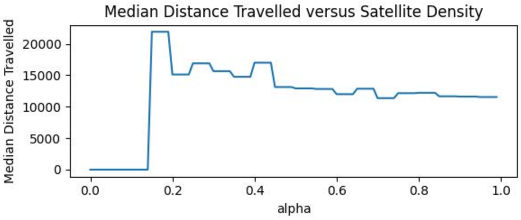

FIGURE 10. **Median cumulated distance the photon pair has to travel against satellite density.**

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Algorithm 1: Selecting the Terminal Satellites
Input  : satellite_candidates, ground_$\varphi$, ground_$\theta$
Output : designated_satellite
begin
    min_deviation ← $\infty$
    for each satellite in satellite_candidates do
        $\theta, \varphi$ ← get_coordinates(satellite)
        $\theta$_offset ← ground_$\theta$ − $\theta$
        $\varphi$_offset ← station_$\varphi$ − $\varphi$
        if $\theta\_offset^2 + \varphi\_offset^2 < min\_deviation^2$ then
            min_deviation ← $\sqrt{\theta\_offset^2 + \varphi\_offset^2}$
            designated_satellite ← satellite
        end
    end
end
```

terminal satellites S_A and S_B are far from the Zenith direction of the ground stations, therefore a significant portion of these distances are represented by the down-link distance. An effect which isn't analysed in these plots is the entanglement loss effect as a function of distance in the upper atmosphere versus the lower parts of the atmosphere, which is traversed by the down-link transmission.

In Figures 9 and 10 we have plotted the median values of both the number of hops and the distance travelled over an entire day. From both of these plots, we can observe the relationship between satellite density and two important parameters for the network and resupply schema: the resupply time τ_r and the entanglement loss throughout the network. τ_r is directly tied to the number of satellites involved in the transmission, while the loss depends on both the number of satellites and the total distance travelled.

```
Algorithm 2: Selecting Shortest Path
Input  : $S_A$, $S_B$, G(V, E)
Output : $S_S$, route neighbour of $S_S$, route backtrace
begin
    push $S_A$, $S_B$ in queue
    visited(A) ← A
    visited(B) ← B
    bt(A) ← None
    bt(B) ← None
    while queue is not empty do
        current ← queue.pop()
        for each neighbour in G(current) do
            if visited(current) = visited(neighbour) then
                continue
            else if visited(neighbour) = 0 then
                visited(neighbour) ← visited(current)
                bt(neighbour) ← bt(current) + {current}
                push neighbour in queue
            else
                return neighbour, neighbour, bt
            end
        end
    end
end
```

From both plots we can observe that there has to be a trade-off between the number of hops for the resupply routes and the total distance travelled for the resupply process. Considering formula 1, the total time the route has exclusive access to the satellites is directly proportional to the number of hops. At the same time, we can describe the total entanglement loss as:

$$\Lambda_r = L_{hop} \times hops + f(distance)$$

where $f(distance)$ is a function which models the entanglement loss for a given distance. For the sake of simplicity only, we consider f to be a function of distance only, although this part of the equation is modeled by many more other variables such as optical medium composition, direct sunlight and random conditions in the upper atmosphere.

Therefore, there is a trade-off for a given satellite density between having a low number of hops, which directly affects both τ_r and Λ_r, and the total distance the photons have to travel. While both the number of hops and distance are coefficients in the total loss of the process, choosing an optimal satellite density requires physical characteristics of both the satellites and the photon beams used.

V. ALGORITHMS

Algorithm 1 performs a lookup in the satellite table, selecting the satellite which has the minimum deviation vector norm, computed with the following formula:

$$deviation = \sqrt{\theta_{offset}^2 + \varphi_{offset}^2}$$

This algorithm has been discussed in section III, step 1, satellite selection.

Algorithm 2 is suitable for the particular case of a homogeneous network, where all satellites are identically equipped. This way, the source satellite selection process is immediate, since the source should be chosen in the middle hop of the route. Therefore, a simple Bellman-Ford based algorithm is enough for this task, returning the source satellite S_S along with the routes from the S_S to S_A and to S_B, respectively.

This algorithm has been discussed in section III, step 2, route and source selection.

VI. CONCLUSION

In this paper we have presented a theoretical approach to a global entanglement resupply infrastructure, using a hybrid land and satellite based network. We have introduced a framework to distribute entangled photons to two distant nodes (laboratories Alice and Bob), by using a constellation of satellites, equipped with the necessary means for relaying a beam of photons, i.e. either by a non-polarizing mirror or by a quantum repeater.

We have described 3 main steps of the resupply process: the terminal satellite selection, the route and source selection, and the effective proper entangled pair transmission. Each of these steps comes with architectural decisions that need to take into account the conditions of an in-place satellite mesh. Choosing the satellites which will perform the transmission from the orbital shell to the ground stations is a conceptually simple process, but there are aspects which may prove difficult in practice. As such, we proposed a hybrid between precomputing the nearest satellites and individual querying of the satellites for their exact coordinates, which may prove to be the best trade-off in terms of performance and accuracy. The next step of the process, which is selecting the route and source satellite, is different, depending on the structure of the network. In the cases of homogeneous networks, where all satellites are identically equipped, the route can be resolved as a shortest-paths problem, from satellite S_A to S_B. However, for the more probable case of a heterogeneous network, the routes must satisfy some conditions, specifically having quantum repeaters after a certain maximum number of hops, while also having at least a quantum entanglement source along the path from source to destination. For this case, there are two possibilities for generating and transmitting the entangled pairs, either by having the source in a place of equilibrium regarding quantum repeaters on both directions of transmission, either by making use of quantum swapping, selecting two sources as close as possible to Alice and Bob, and performing a Bell State measurement at the route's midpoint.

We have performed an analysis[1] for a concrete scenario, resupplying two nodes separated by nearly 8000 km. measuring the effect of the satellite density and time of day. From this analysis, we have concluded that increasing the satellite density decreases the total distance the photons have to travel and creates less fluctuations in the number of hops throughout the day for a given pair of ground nodes. However, decreasing the satellite density decreases the number of hops needed for a given transmission, which in turn decreases the total resupply time and improves the transmission fidelity.

As future work, we suggest studying algorithms for optimal path selection with at least one entanglement source, algorithms for optimal source selection, and the applicability of nanosatellites in the context of inter-satellite quantum entanglement transmissions.

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[1] The source code and used data are available upon request

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