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Electric Circuit-Based Modeling and Analysis of the Translational, Rotational Mechanical and **Electromechanical Systems Dynamics**

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ABSTRACT To cope with the rapid development in technology, engineers are dealing with complex and heterogenous systems composed of blocks that belong to different engineering fields such as electrical, mechanical, chemical, electromechanical, fluid, thermal, etc. Mechanical and electrical systems more often go hand to hand in many industrial systems. For system analyzing and designing purposes, engineers must model and simulate the systems to investigate problems and aim for the best performance before proceeding to the manufacturing stage. In the presence of complex mechanical system blocks, electrical and electronics engineers are often facing difficulties in modeling the mechanical blocks. Although the similarity between individual mechanical and electrical elements is recognized for a long time, it has not drawn deserved attention for its use at the system level. In this paper, we investigate in great detail how enabling electrical and electronics engineers to easily model and analyze complex mechanical and electromechanical systems through a systematic approach. For this objective, thirteen rules are set, established, and elaborated on how to find the electrical circuit equivalent of a given mechanical or electromechanical system in order to be modeled and analyzed. The proposed approach is tested on both complex translational mechanical and electromechanical systems which includes a rotational mechanical system. Findings demonstrate that models generated by the equivalent of electric circuits are matching the models of existing mechanical and electromechanical systems by 100%. The proposed systematic approach is promising and can be widely implemented in several industrial fields.

INDEX TERMS Modeling and simulation, a set of rules, complex mechanical systems, electrical system, analogy.

I. INTRODUCTION

Analogy exists in different physical domains such as mechanical, electrical, fluid, and thermal systems. The concept of analogy is vast and versatile and it has been implemented in different domains. For instance, the analogy between the disk dynamos and the geomagnetic field reversal patterns, where the Rikitake dynamo system has been examined with respect to the earth's magnetic field [1]–[3]. The idea behind

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analogy backs to the differential equations used to model these systems are comparable in the first place. It's worth mentioning that the work of Harrison in [4] was one of the earliest efforts where he solved a mechanical system through the theory of electrical networks. This work was presented in a patent of innovation in 1929. The electromechanical analogy was developed and extended in the 30-40's of the 20th century for purpose of studying vibrations in linear mechanical systems [5]–[9]. Two analogies have been used to translate mechanical systems into electrical systems. The first one is known as the "force-voltage" analogy [6]. This analogy was so constrained and struggled to achieve the translation since the obtained electrical system is not directly came from the mechanical system [5]. To overcome these limitations, a new analogy based on "force-current" was proposed [4], [5]. Additionally, the mobility concept has been introduced [10]. Despite the fact that two perspectives of analogy exist, the use of each analogy was set depending on the system and which analogy type fits the problem more and can be implemented and derived easily [7], [11]. In other cases, there might be a need to use both analogies to represent different parts of the mechanical system. This was achieved using appropriate couplers to derive different electric diagrams before linking them together. [12].

Converting a mechanical system into an electrical system requires deep understanding related to both mechanical and electrical advanced concepts. For the industrial purpose, this translation is required for many applications [13] and hence, modeling mechanical elements into electrical becomes essential. However, electrical engineers would commit serious modeling failures if decent knowledge related to such systems is missed. Furthermore, deriving the governing differential equations by the given mechanical system is challenging, involving high skills in calculus and such analysis could be lengthy and even tedious. Therefore, making this translation as fast as possible was indeed a motivation factor for conducting this study due to its valuable significance brought by this research to the niche area.

In this study, the preliminary approach given in our previous work [15] is expanded to cover the translational as well as rotational mechanical systems and electromechanical systems including DC motors. The developed rules cover all necessary aspects of a complete translation of mechanical and electromechanical systems into an equivalent electrical circuit. With this more comprehensive approach, electrical engineers will be able to model and analyze any complex dynamic translational, rotational, and electromechanical mechanical systems. The results obtained from the electrical circuit system are compared with the simulation results obtained from direct original mechanical modeling and 100% match is achieved between the two sets. It is also possible to do the reverse process and model the electrical circuits as mechanical systems.

The main contributions of this paper are:

- 1) Investigating how enabling electrical engineers to easily model and analyze complex mechanical and electromechanical systems.
- 2) Proposing a set of rules to transfer translational, rotational mechanical, and electromechanical systems into electrical systems.
- 3) The proposed rules are general and can cover any complex mechanical system.
- 4) Our approach is validated in MATLAB and 100% accuracy is obtained. Two design examples have been presented to demonstrate how the proposed systematic approach can be implemented for real systems.

This paper proceeds as follows: Section II presents the related work. Section III introduces the modeling of the proposed translation approach based on the analogy between the mechanical/electromechanical systems and electrical systems. Section IV lists the developed rules for a successful transformation. Applying the translation approach over mechanical and electromechanical systems into electrical systems, deriving the governing equations, and validating the proposed approach via MATLAB are demonstrated in Section V. Finally, the paper concludes itself in Section VI.

II. RELATED WORK

The analogy between mechanical and electrical systems is well-known [5] and proved its efficiency in the mechanical and electrical fields. This Section introduces some recent works which are related to our study. In [15], authors have explored new applications and possibilities for applying these analogues. The proposed work is based on electrostatic coupling which was implemented for planar vehicle models using vertical dynamic analysis. The study shed on the electrical analogue of the half-car model and for the more complicated and relative model, the three-axle vehicle. For this aim, electrostatic capacitor coupling in its force current analogue has been used to identify and model the inertial components of such a system. Rather than using mixing angle and displacement variables, pure displacement coordinates have been implemented. There are two factors that capacitance values of the coupled capacitors depend on which are: the mass and moment of inertia of the vehicle mainframe, and the distance of each wheel to the center of gravity of the mainframe. To reduce the complexity obtained due to the dealing with coupling capacitors, equivalent Π network has been used which simplifies the resulting equations. To make use of KCL, real voltage and current sources have been transformed. The results obtained for the three-axle vehicle model revealed that both electrical equivalent circuit and mechanical model have been equations declared to be identical for the time and frequency domain.

In [16], the study aimed to find the electrical equivalent circuit-based modeling for direct current motors. Electromechanical expressions for some existing DC motors have been analyzed, then an identical model for the electrical equivalent circuit related to mechanical equations was presented. For the armature circuit and output circuit attached to load, both mechanical variables and components have been considered. Furthermore, the same equation structure related to the electrical equivalent circuit compared to the mechanical system is provided. Applying the similarity of the obtained equations, it was found that all generated electrical circuit elements and electrical variables are representing the original mechanical system variables including its components. Hence, the DC motor was completely represented concerning variables and components in the electrical model, although it includes some mechanical components and variables with and without load for transient-state and steady-state analysis. Implementing the same electrical equivalent circuit led to

obtaining transient-state and steady-state analysis. During the analysis for getting the same electrical equivalent circuit and for obtaining a linear model, both excitation current and magnetic flux are assumed to be constant. However, the analysis works fine and might be valid for most of the variable flux applications. Although the analysis of this study is decent, but it is limited to DC motor.

In other previous works, representation of some mechanical elements by electrical elements is introduced in several works [17]–[22]. But this topic generally has remained at a very basic level without exploring the details for application at the system level. Therefore, the topic did not find widespread use before, although its importance. Mechanical engineers often analyze mechanical systems using sophisticated software tools such as MATLAB toolbox SimMechanics [20], but electrical engineers rarely use this software tool. In [14], a step forward has been made to develop a systematic approach to model and analyze complex dynamic mechanical systems as analog to electrical circuits. A set of preliminary rules have been proposed for this purpose. But the aforementioned approach is limited to translational mechanical systems only.

The analogy between mechanical and electrical systems yields many advantages in this domain. The analogy has been implemented in many applications such as suspension modeling and control [24]–[27], vehicle drive trains [28], structural dynamics [29], in designing and controlling of flexible structures [30], optimization and design of inductive power transfer systems [31], piezoelectric vibration energy harvesters [32]. While other studies aimed for vibration applications as the their main of interest [33]–[39]. The variety of applications that benefit from the idea of analogy, is another substantial reason for conducting this study.

Other than a purely mechanical system, the use of analogy is quite useful when a mechanical system and electrical system are linked together forming an electromechanical system. In such multi-domain problems, the mechanical part of the system is translated into its electrical circuit equivalent and the whole system is modeled and analyzed as a unique electrical system [33], [34]. In [40], the use of linear graphs is adopted to propose a model for multi-domain systems. Analogy is useful for the educational field as well, since understanding one system leads to a better and easier understanding of other different systems [41].

Most of the studies discussed above are mostly focused on specific examples using some different forms of analogy between mechanical or electromechanical systems into electrical networks, either with direct analog (force-voltage) or indirect analog (force-current). However, and to the best of the author's knowledge, there are no solid-based rules which electrical engineers could apply over different combinations of inner mechanical elements connections to find the corresponding electric circuit equivalent. The advantage of this study compared to the previously published ones is that this paper is rule-based, i.e., 13 rules are developed to achieve valid transformation for a different combination of

TABLE 1.	Analogy between	the electrical	and mechanical	quantities in
translatio	nal mechanical sy	stems.		-

Symbol	Mechanical	Unit	Symbol	Electrical Analog	Unit
f	Force	Ν	Ε	Voltage	V
x	Displacement	m	q	Electrical Charge	С
ν	Speed	m/s	i	Electric Current	А
М	Mass	kg	L	Inductance	Н
В	Damper (Viscos friction)	N.s/m	R	Electrical Resistance	Ω
K	Spring (Stiffness element)	N/m	$\frac{1}{C}$	Capacitor	F

mechanical or electromechanical system connections into the corresponding equivalent electrical circuit.

III. MODELING OF MECHANICAL SYSTEMS BY EQUIVALENT ELECTRIC CIRCUITS

Mechanical systems can be classified into two major groups, namely, translational mechanical systems and rotational mechanical systems. In translational mechanical systems, there are three main mechanical system elements, which are mass (M), translational stiffness or spring element (K), and viscous friction or damper element (B). Driving signal in translational systems is the force $f_{(t)}$. Additionally, in rotational mechanical systems, there are three corresponding elements which are: the moment of inertia (J), rotational stiffness or rotational spring element (K), and rotational viscous friction or rotational damper element (B). In rotational mechanical systems, there is a gear train element which is widely used to match the rotational loads that must run at a different angular speed than the driving machine. In rotational mechanical systems, the driving signal is the torque $\tau(t)$. Similarly, in electrical systems, there are three well-known elements: inductance (L), capacitance (C), and resistance elements besides the transformer which imitates a gear train in equivalent electric circuits. In electrical systems, the driving signal is the voltage source $E_{(t)}$, which represents the force in translational mechanical systems or the torque in a rotational mechanical system. Each velocity in translational or each angular velocity in a rotational mechanical system corresponds to an electrical current. Each displacement in translational or angular displacement in a rotational mechanical system corresponds to an electrical charge. Therefore, it can be concluded that there is an identical analogy between the matching elements and physical quantities of translational or rotational mechanical systems with electrical systems. The analogy between mechanical, rational mechanical and electrical systems introduced in Table 1, Table 2.

IV. DEVELOPED RULES FOR THE SYSTEM INTERCONNECTIONS

The analogy between the physical elements and their related governing equations presented in Tables 1 to 6, clearly shows a complete correspondence between the two systems. Therefore, both systems virtually represent each other. The

 TABLE 2. Analogy between the electrical and rotational mechanical quantities.

Symbol	Mechanical	Unit	Symbol	Electrical Analog	Unit
τ	Torque	N.m	Е	Voltage	V
θ	Displacement	rad	q	Electrical Charge	С
ω	Angular speed	rad/s	i	Electric Current	А
J	Inertia	Kg.m ²	L	Inductance	Н
В	Rotational damper (Viscous friction)	N.ms	R	Electrical Resistance	Ω
K	Rotational stiffness (Torsional spring)	N.m	$\frac{1}{C}$	Capacitor	F

rules below aim to present the unexplored interconnections between the different blocks of mechanical and electromechanical systems to the corresponding equivalent electrical circuits. The main steps are listed as follows: (a)

- Each mass in a mechanical system corresponds to a loop in the equivalent electrical circuit. Therefore, the number of loops in the equivalent electrical circuit is defined by the number of masses in the mechanical system. The same is valid for each inertia (mass moment of inertia) in rotational mechanical systems.
- Velocity of each mass in a mechanical system corresponds to each loop current in the equivalent electrical circuit. The same is valid for angular velocity in rotational mechanical systems.
- 3) Displacement of a mass in a mechanical system corresponds to the total electrical charge flowing through the branch of an equivalent electric circuit loop, where the inductance that corresponds to the mass is placed in. The same is valid for angular displacement in rotational mechanical systems.
- 4) The interconnected elements in each mass block in mechanical systems correspond to the electrical elements shared between the equivalent electrical circuit loops.
- 5) Series elements carrying the same value of force in a mechanical system, correspond to parallel elements in the equivalent electric circuit. The same is valid for torque in rotational mechanical systems.
- 6) Parallel connected elements in a mechanical system, correspond to series connected elements in equivalent electrical circuits.
- 7) From (c) and (d), it becomes evident that the equivalent of the parallel connected springs in translational systems or rotational stiffness elements in rotational systems corresponds to a single equivalent capacitor obtained by applying the rule of series capacitors. Similarly, the equivalent of series connected springs in translational systems or rotational stiffness elements in rotational systems corresponds to a single equivalent capacitor obtained by applying the rule of parallel capacitors.

- 8) Equivalent of parallel connected viscous friction elements in translational systems or rotational viscous friction elements in rotational systems corresponds to a single equivalent resistor obtained by applying the rule of parallel resistors. Similarly, the equivalent of series connected viscous friction elements in translational systems or viscous friction elements in rotational systems corresponds to a single equivalent resistor obtained by applying the rule obtained by applying the rule of series resistor.
- 9) Force on a viscous friction element in translational systems or rotational viscous friction element in rotational systems which includes a fixed end and movable one on the other side corresponds to a voltage drop across a resistor connected with a loop in a series of the equivalent electric circuit. The velocity of the movable end corresponds to the current passing through that resistor.
- 10) A viscous friction element in a translational system or rotational viscous friction element in a rotational system that both ends are movable corresponds to a resistor placed between the two loops of an equivalent electric circuit. Force or torque acting on such elements corresponds to a voltage drop across that resistor. The difference in velocity or angular velocity between the two ends of such elements corresponds to the difference between the currents of the two loops sharing that resistor.
- 11) Force acting on a spring element or torque acting on a rotational stiffness element that one end is fixed, while the other end is movable, corresponds to a voltage drop across a capacitor series connected in a loop of an equivalent electric circuit. The displacement or angular displacement of the movable end corresponds to an electric charge on that capacitor.
- 12) A spring or rotational stiffness element that both ends are movable, corresponds to a capacitor placed between the two loops of an equivalent electric circuit. Force or torque acting on such elements corresponds to a voltage drop across that capacitor. The displacement or angular displacement difference between the two ends of such elements corresponds to the difference between the electric charge on that capacitor associated with the currents of the two loops sharing that capacitor.
- 13) Total number of state variables is equal to the doubled number of masses (or inertia(s) in rotational mechanical systems) added to the number of the stiffness or the spring elements that one end is connected to a mass element or a fixed body, and the other end is free to move. In the equivalent electrical circuit, it is well known that the number of state variables is equal to the total number of energy storing elements, i.e. inductors and capacitors.

The governing equations for mass, inertia, rotational damper, rest of friction elements and translational elements, have been introduced previously [17]–[22].

V. APPLICATION DESIGN

In this Section, application details for a mechanical system and a rotational electromechanical system are introduced. For each case study, the translation approach is applied, governing equations for the electrical system are discussed. Then, results obtained from solving the governing equations, and results obtained directly from mechanical systems are compared using MATLAB environment to validate the accuracy of the proposed approach.

A. APPLICATION EXAMPLE 1: MODELING AND ANALYSIS OF A COMPLEX TRANSLATIONAL SYSTEM USING ELECTRIC CIRCUIT EQUIVALENT

System parameters for mechanical system given below in Figure 1 are: Mechanical Load Mass ($M_1 = 100 \text{ kg}, M_2 = 40 \text{ kg}, M_3 = 80 \text{ kg}$), Stiffness Element ($K_1 = 500 \text{ N/m}, K_2 = 250 \text{ N/m}, K_3 = 150 \text{ N/m}, K_4 = 300 \text{ N/m}, K_5 = 200 \text{ N/m}, K_6 = 180 \text{ N/m}, K_{eq} = k_1 + k_2$) and Viscous Friction ($B_1 = 80 \text{ N.s/m}, B_2 = 30 \text{ N.s/m}, B_3 = 50 \text{ N.s/m}, B_4 = 40 \text{ N.s/m},$

iz

 $B_5 = 10 \text{ N.s/m}, B_6 = 30 \text{ N.s/m}, B_7 = 20 \text{ N.s/m}).$ The applied force is taken as $F(t) = 400 \sin(4t)e^{(-0.1 t)}$.

Governing equations from electrical circuit equivalent (1)–(3), as shown at the bottom of the page, where $R_A = R_1+R_2$ and $R_B = R_4+R_6 +R_7$. State-space model equations are executed in MATLAB environment and solved using Runge-Kutta 4th order method [42] The governing equations directly from the original mechanical system are (4), as shown at the bottom of the page, where $K_A = K_1 + K_{eq} + K_4, K_B = K_4 + K_5, K_C = K_5 + K_6, B_A = B_1 + B_2, B_B = B_4 + B_5, and B_C = B_4 + B_6 + B_7.$

$$\frac{dx_1}{dt} = v_1, \frac{dx_2}{dt} = v_2, \frac{dx_4}{dt} = v_4, \frac{dx_5}{dt} = v_5$$
(5)

$$v_1 = \frac{F(t)}{B_1} - \frac{K_1}{B_1} x_1 + \frac{K_1}{B_1} x_2 + v_2 \tag{6}$$

$$v_3 = \frac{K_{eq}}{B_3} \left(x_2 - x_3 \right) + v_5 \tag{7}$$

Substituting system parameters, solving the governing equations using Runge-Kutta 4th order method, and implementing them in MATLAB environment would produce the

$$i_1 = \frac{u(t)}{R_1} - \frac{q_1}{R_1 C_1} + i_2 \tag{1}$$

$$=\frac{q_{eq}}{R_3 C_{eq}} + i_5 \tag{2}$$

$$\begin{bmatrix} \frac{di_2}{dt} \\ \frac{di_4}{dt} \\ \frac{di_4}{dt} \\ \frac{di_5}{dt} \\ \frac{dq_4}{dt} \\ \frac{dq_4}{dt} \\ \frac{dq_5}{dt} \\ \frac{dq_5}{dt} \\ \frac{dq_5}{dt} \\ \frac{dq_6}{dt} \\ \frac{dq_6}{dt}$$

$$\begin{bmatrix} \frac{dv_2}{dt} \\ \frac{dv_4}{dt} \\ \frac{dv_5}{dt} \\ \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \\ \frac{dx_3}{dt} \\ \frac{dx_3}{dt} \\ \frac{dx_5}{dt} \\ \frac{dx_5}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{B_A}{M_1} & 0 & 0 & 0 & -\frac{K_A}{M_1} & \frac{K_{eq}}{M_1} & \frac{K_4}{M_1} & 0 \\ 0 & -\frac{B_B}{M_2} & \frac{B_4}{M_2} & 0 & \frac{K_4}{M_2} & 0 & -\frac{K_B}{M_2} & \frac{K_5}{M_2} \\ 0 & \frac{B_4}{M_3} & -\frac{B_C}{M_3} & 0 & \frac{K_{eq}}{M_3} & -\frac{K_{eq}}{M_3} & \frac{K_5}{M_3} & -\frac{K_C}{M_3} \\ 1 & 0 & 0 & -\frac{K_1}{M_1} & \frac{K_1}{M_1} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -\frac{K_{eq}}{B_3} & \frac{K_{eq}}{B_3} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{bmatrix} \begin{bmatrix} v_2 \\ v_4 \\ v_5 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} \frac{1}{M_1} \\ 0 \\ 0 \\ \frac{1}{B_1} \\ 0 \\ 0 \\ 0 \end{bmatrix} F(t)$$
(4)



TABLE 3. Analogy between governing equations of the translational mechanical and electrical elements (Part I).

TABLE 4. Analogy between governing equations of the translational mechanical and electrical elements (Part II).





TABLE 5. Analogy between governing equations of the translational mechanical and electrical elements (Part III).







FIGURE 1. A translational mechanical system.



FIGURE 2. Electric circuit equivalent of the translational mechanical system given in Figure 1.



FIGURE 3. Comparison of the velocity 1 obtained from the electrical and mechanical models.



FIGURE 4. Comparison of velocity 4 obtained from the electrical and mechanical models.



FIGURE 5. Comparison of the velocity 5 obtained from the electrical and mechanical models.

required comparison to validate this study. Results obtained from both models are compared in Figures. 3, 4, 5, 6, 7 and 8



FIGURE 6. Comparison of the displacement x1 obtained from the electrical and mechanical models.



FIGURE 7. Comparison of the displacement x4 obtained from the electrical and mechanical models.



FIGURE 8. Comparison of the displacement x5 obtained from the electrical and mechanical models.



FIGURE 9. Motor supply circuit.

introduced above. The analogy between the two models is conclusive and 100% match between the two sets is glaring.

B. APPLICATION EXAMPLE 2: A ROTATIONAL ELECTROMECHANICAL SYSTEM

System parameters for mechanical system given in Figure 10 are: Motor Inductance ($L_a = 0.0645$ H), Resistance ($R_a = 0.203 \Omega$), Rotational Stiffness Element ($K_1 = K_2 = 80$ N.m), Motor Torque Constant ($K_m = 1.297$ N.m/A) Load Torque Constant ($K_L = 0.0372$ N.m.s²) Inertia ($J_1 = 0.0048$ Kg.m², $J_2 = 0.0094$ Kg.m²), Load Inertia ($J_L = 0.026$ Kg.m²), Motor Inertia ($J_m = 0.0423$ Kg.m²) Viscous Friction ($B_m = 0.0012$ N.s.m), Gear Ratio ($n_g = 0.5$), Load Torque ($\tau_L = K_L \omega_2^2$), $J_{eq} = J_1 + n_g^2 J_2$ and $K_{eq} = K_1 + n_g^2 K_2$.



FIGURE 10. A rotational electromechanical system.



FIGURE 11. Equivalent electric circuit of the electromechanical system shown in Figure 10.



FIGURE 12. Duty cycle versus time at motor starting time.



FIGURE 13. Voltage across the motor terminals for duty cycle D=0.65.

The dynamic model of a DC motor using controlled voltage power source such as PWM control is widely elaborated in literature [43]–[45]. Therefore, details will not be mentioned here. The mechanical system and its electrical equivalent system are presented in Figures 10 and 11 respectively, while Figure 13 shows the voltage across the DC motor. Voltage across the motor terminals is modeled as:

$$1V_{a} = 250 |\sin(\omega_{s}t)| \left(D + \frac{1}{\pi} \times \sum_{n=1}^{251} \frac{(\sin(\pi nD) - \sin(\pi n(2 - D)))}{n} \cos(n\omega_{c}t) \right)$$
(8)

where ω_s is the angular frequency of the supply voltage $(\omega_s = 2.\pi \cdot f_s, f_s = 50 \text{ Hz})$, D is the duty cycle and ω_c is the angular frequency of the trigger signal applied to the gate of IGBT respectively. $\omega_c = 2.\pi \cdot f_c$, where f_c is taken as 1kHz.

Governing equation from electrical circuit equivalent introduced as follows. At first step E_1 and E_2 must be eliminated from the state-space model. From Table 5, we have:

$$E_2 = \frac{E_1}{n_a} \tag{9}$$

$$i_2 = n_g i_1 \tag{10}$$

From Figure 11 we have:

$$E_1 = \frac{q_1}{C_1} - L_1 \frac{di_1}{d_t}$$
(11)

$$E_2 = \frac{E_1}{n_g} = L_2 \frac{di_2}{d_t} + \frac{q_2}{C_2}$$
(12)

From Equations 9,10,11 and 12, we obtain:

$$\frac{di_1}{d_t} = \frac{q_1}{L_{eq}C_1} - \frac{q_2}{L_{eq}C_2}$$
(13)

where $L_{eq} = L_1 + n_g^2 L_2$. The complete state-space model is given in Equation 14, as shown at the bottom of the next page, where $R_d = R_{mB} + R_1$ and $L_d = L_{mJ} + L_1$. The statespace equation set are solved using Runge-Kutta 4th order method and coded in MATLAB environment. Governing equations are given directly from original mechanical system in Equation 15, as shown at the bottom of the next page.



FIGURE 14. Comparison of the motor currents obtained from the electrical and mechanical models.



FIGURE 15. Comparison of the motor speeds obtained from the electrical and mechanical models.

Comparison between Figures 15 and 16 reveals that, although the amplitudes of the motor speed and primary gear speed are the same, but a ripple with a very small amplitude



FIGURE 16. Comparison of the primary gear speeds obtained from the electrical and mechanical models.



FIGURE 17. Comparison of the load speeds obtained from the electrical and mechanical models.

in the primary gear speed exists. This ripple is due to the shaft connecting the gear to the motor is not 100% rigid, it has a finite stiffness (K_1). The presence of this ripple is a solid indication of the accuracy of the presented model. As in V-A,

$$\begin{bmatrix} \frac{di_{a}}{dt} \\ \frac{di_{m}}{dt} \\ \frac{di_{1}}{dt} \\ \frac{di_{1}}{dt} \\ \frac{di_{1}}{dt} \\ \frac{di_{1}}{dt} \\ \frac{di_{1}}{dt} \\ \frac{dq_{2}}{dt} \\ \frac{dq_{2}}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_{a}}{L_{a}} - \frac{K_{m}}{L_{a}} & 0 & 0 & -\frac{1}{L_{d}C_{1}} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{L_{eq}C_{1}} - \frac{n_{g}}{L_{eq}C_{2}} \\ 0 & 0 & 0 & -\frac{R_{2}}{L_{4}} & 0 & \frac{1}{L_{eq}C_{2}} \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & n_{g} & -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{a} \\ i_{1} \\ i_{3} \\ q_{1} \\ q_{2} \end{bmatrix} + \begin{bmatrix} \frac{1}{L_{a}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} V_{a} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\frac{K_{L}}{L_{4}} \\ 0 \\ 0 \\ 0 \end{bmatrix} i^{2}$$

$$\begin{bmatrix} \frac{di_{a}}{dt} \\ \frac{d\omega_{1}}{dt} \\$$



FIGURE 18. Comparison of the load torgues obtained from the electrical and mechanical models.

the similarity between mechanical and electrical models shown in V-B, presented in Figures 14 to 18 is decisive.

VI. CONCLUSION AND FUTURE WORK

In this paper, a systematic approach is developed for modeling and analysis of complex translational, rotational and electromechanical mechanical systems using the analogy to obtain the equivalent electrical circuit. With the help of the developed 13 rules, concerning the unexplored interconnections, electrical and electronics engineers will be able to model and analyze complex mechanical systems and electromechanical systems, by utilizing the rich, widely, and readily available software packages used in the electrical and electronics engineering. The developed systematic approach is applied to a relatively complex translational mechanical system and a rotational electromechanical system. Findings demonstrate that results obtained from the equivalent electric circuit modeling are coinciding with results obtained from the direct mechanical models.

The proposed approach is semi-directional, as it performs the translation from mechanical system to electrical system completely, it might perform the reverse process, i.e., from electrical system to mechanical system. However, when non-linearity is remarkably involved in some mechanical elements, such as translational or rotational springs, that may require special treatment. Therefore, enabling the proposed approach to carry out the translation in a reverse process will further enrich the proposed approach. Moreover, electromechanical systems including AC motors such as induction motors, have more complicated models as compared to DC motors. This could be our future work as well, empowering the current rules to fully support the AC models.

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