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Model Order Diminution of Discrete Interval Systems Using Kharitonov Polynomials

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ABSTRACT In this research proposal, diminution of higher order (HO) discrete interval system (DIS) is accomplished by utilizing Kharitonov polynomials. The DIS is firstly, transformed into continuous interval system (CIS). The Markov-parameters (MPs) and time-moments (TMs) are exploited for determination of approximated models. The ascertainment of model order diminution (MOD) of DISs is done by Routh-Padé approximation. The Routh table is utilized to obtain the denominator of approximated model. The unknown numerator coefficients of desired approximated model are determined by matching MPs and TMs of DISs and desired model. This whole procedure of MOD is elucidated with the help of one test illustration in which third order system is reduced to first order model as well as second order model. To prove applicability of the proposed method, impulse, step and Bode responses are plotted for both system and model. For relative comparison, time-domain specifications of proposed model are tabulated for both upper and lower limits. Further, performance indices are specified for dissimilarities between responses of system and model. The obtained results depict the effectiveness and efficacy for the proposed method.

INDEX TERMS Discrete interval systems, interval system, padé-approximation, Routh approximation, model order diminution.

I. INTRODUCTION

In many real-time engineering and industrial applications, systems are represented by higher order (HO) transfer functions. These higher order systems (HOSs) are very complex for study and complicated for control design. Due to limitations associated with HOSs, order diminution is necessary to obtain the comparatively lower-order model (LOM) without eliminating its important characteristics. In the diminution process of stable HOS, it is also desired to obtain stable LOM. But, in some cases, it is found that stability of LOM is difficult to retain. In addition, matching of steady-state of existing system with its LOM often fails in some cases. In literature [1]–[10], various model order diminution (MOD) techniques are available for determination of stable LOMs. The steady state matching of these obtained LOMs are also ensured to acquire better LOM.

The MOD techniques are employed for diminution of large-scale systems in frequency-domain as well as in time-domain, both. Some of the predominant techniques are Padé approximation method [11], biased factor division method [12], Routh approximation (RA) method [13]–[15], stable approximation method [9], direct series and dominant pole method [2], [3], stability equation method [16], etc. Further, some other order diminution techniques utilized are mixed methods [4]–[9]. In mixed approaches, two order diminution methods are combined in order to achieve better stable LOM. Generally, implementation of mixed methods are limited to continuous non-interval systems.

In literature [2], [17]–[20], several methods are also available for DIS. Some of the methods in this category are direct series and dominant pole retention [2], Padé and multipoint Padé approximation [17], [18], Gamma-delta approximation [19], etc. The prospective two new approaches of order diminution for DISs are differentiation calculus [21] and Mikhailov stability criterion [20]. In some articles [1], [22]–[24], relevant works on DIS are presented

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by exploiting different order diminution methods. Along with these available MOD techniques in literature, Kharitonov polynomials technique of MOD is also available and implemented in several areas, related to control applications [25]–[27], microgrid systems [28], [29], power systems [30], [31], etc. This technique is oriented towards the solutions which are relevant to expletive of dimensionality in HOSs. The advantages of this technique lies not only in the matching of steady-state but also in its preservation of stability, transient state matching, efficiency of approximation, and simpler computation. The salient features of this method can be listed as simple to understand and ease of implementation.

In this proposed work, MOD of DIS is done by employing mathematical procedures entitled as Kharitonov polynomials technique. Diminution of DIS is accomplished by utilizing Routh-Padé approximation and matching of Markov-parameters (MPs) along with TMs for transformed continuous interval system (CIS) from DIS. The denominator of desired LOM is determined using Routh table. However, Padé approximation is employed for deriving the numerator of desired LOM. In Padé approximation, the matching of expansion coefficients about $\omega = 0$ are utilized. The parameters calculated by this expansion are commonly termed as TMs. The matching parameters derived by expanding HOS about $\omega = \infty$ are also utilized. The parameters derived from this expansion about $\omega = \infty$ are known as MPs [4]. The MPs and TMs are achieved in a manner where inversion of denominator of system transfer function is not required. In addition, there is no necessity to solve a set of linear interval equations to achieve MPs and TMs. With the help of MOD of DISs, the superiority and effectiveness of derived MPs and TMs are assured. A third order DIS as a test system is considered in support of proposed methodology. Moreover, the derived LOM from MOD preserves the stability of system. The results are compared utilizing time responses. The performance indices of proposed model are compared with those of the recently proposed order reduction methods. The illustrations show the significant improvement in the system approximation through the proposed method in comparison with the other conventional approaches.

This paper is organized in five different sections. Section I includes introduction of proposed method and motivation behind the proposed work with the help of available literature. A brief discussion of the previous works on MOD for CISs and DISs is also provided. The description of problem statement is specified in Section II. The proposed methodology for determination of denominator coefficients and numerator coefficients of desired LOM using diminution process of HOSs is given in Section III. In this section, flow chart of proposed method is also provided. The numerical experiments and results are depicted in Section IV, The obtained results are compared with the results already available in literature in this section, to validate the proposed method. Finally, Section V comprises conclusion by incorporating future scope of the proposed work.

II. THE PROBLEM FORMULATION

Suppose, the transfer function of higher order (HO) discrete interval system (DIS) of order h is represented as

$$\mathrm{G_h}(z) = \frac{[\xi_{h-1}^-, \xi_{h-1}^+]z^{h-1} + \cdots + [\xi_1^-, \xi_1^+]z + [\xi_0^-, \xi_0^+]}{[\varphi_h^-, \varphi_h^+]z^h + \cdots + [\varphi_1^-, \varphi_1^+]z + [\varphi_0^-, \varphi_0^+]} \tag{1}$$

where, $[\xi_0^-, \xi_0^+]$, $[\xi_1^-, \xi_1^+]$, $\cdots$, $[\xi_{h-1}^-, \xi_{h-1}^+]$ and $[\varphi_0^-, \varphi_0^+]$, $[\varphi_1^-, \varphi_1^+]$, $\cdots$, $[\varphi_{h-1}^-, \varphi_{h-1}^+]$ are coefficients of numerator and denominator of HO system in discrete domain, respectively. The discrete interval transfer function given in (1) is transformed into transfer function in ω-domain by using bi-linear transformation approach *i.e* $z \rightarrow (1+\omega)$. Thus, higher order interval system (HOIS) in ω-domain changes to

$$\mathrm{V_h}(\omega) = \frac{[u_{h-1}^-, u_{h-1}^+]\omega^{h-1} + \cdots + [u_1^-, u_1^+]\omega + [u_0^-, u_0^+]}{[v_h^-, v_h^+]\omega^h + \cdots + [v_1^-, v_1^+]\omega + [v_0^-, v_0^+]} \tag{2}$$

The expansions of system given in (2) about $\omega = \infty$ and $\omega = 0$ are obtained as

$$V_h(\omega) = \Phi_1\omega^{-1} + \Phi_2\omega^{-2} + \cdots + \Phi_k\omega^{-k} + \cdots \tag{3}$$

$$V_h(\omega) = \tau_0 + \tau_1\omega + \tau_2\omega^2 + \cdots + \tau_k\omega^k + \cdots \tag{4}$$

where, $M_k = \Phi_k$ for $k = 1, 2, \cdots$ are MPs and $T_k = \tau_k$ for $k = 0, 1, \cdots$ are TMs of (2). An l^{th} order model of HO system can be given as

$$\widehat{V}_l(\omega) = \frac{[\widehat{u_{l-1}^-}, \widehat{u_{l-1}^+}]\omega^{l-1} + \cdots + [\widehat{u_1^-}, \widehat{u_1^+}]\omega + [\widehat{u_0^-}, \widehat{u_0^+}]}{[\widehat{v_l^-}, \widehat{v_l^+}]\omega^l + \cdots + [\widehat{v_1^-}, \widehat{v_1^+}]\omega + [\widehat{v_0^-}, \widehat{v_0^+}]} \tag{5}$$

where, $l < h$.

The expansions of (5) about $\omega = \infty$ and $\omega = 0$ are given by

$$\widehat{V}_l(\omega) = \widehat{\Phi_1}\omega^{-1} + \widehat{\Phi_2}\omega^{-2} + \cdots + \widehat{\Phi_l}\omega^{-l} + \cdots \tag{6}$$

$$\widehat{V}_l(\omega) = \widehat{\tau_0} + \widehat{\tau_1}\omega + \widehat{\tau_2}\omega^2 + \cdots + \widehat{\tau_l}\omega^l + \cdots \tag{7}$$

where, $\widehat{\Phi_l}$ for $l = 1, 2, \cdots$ are MPs of (6) and $\widehat{\tau_l}$ for $l = 0, 1, \cdots$ are TMs of (7).

The required equivalent model of l^{th} order model ($l < h$) is calculated using the inverse bi-linear conversion method *i.e.* $\omega \rightarrow$ (z- 1), as follows

$$\hat{g}_l(z) = \frac{[\hat{\alpha}_{l-1}^-, \hat{\alpha}_{l-1}^+]z^{l-1} + \cdots + [\hat{\alpha}_1^-, \hat{\alpha}_1^+]z + [\hat{\alpha}_0^-, \hat{\alpha}_0^+]}{[\hat{\beta}_l^-, \hat{\beta}_l^+]z^l + \cdots + [\hat{\beta}_1^-, \hat{\beta}_1^+]z + [\hat{\beta}_0^-, \hat{\beta}_0^+]} \tag{8}$$

where, $[\hat{\alpha}_0^-, \hat{\alpha}_0^+]$, $[\hat{\alpha}_1^-, \hat{\alpha}_1^+]$, $\cdots$, $[\hat{\alpha}_{l-1}^-, \hat{\alpha}_{l-1}^+]$ and $[\hat{\beta}_0^-, \hat{\beta}_0^+]$, $[\hat{\beta}_1^-, \hat{\beta}_1^+]$, $\cdots$, $[\hat{\beta}_l^-, \hat{\beta}_l^+]$ are the coefficients of numerator and denominator of lower-order model (LOM) in discrete domain, respectively.

TABLE 1. Routh table for the denominator of CIS.

$\varepsilon_{1,1}=[v_h^-,v_h^+]$	$\varepsilon_{1,2}=[v_{h-2}^-,v_{h-2}^+]$	$\varepsilon_{1,3}=[v_{h-4}^-,v_{h-4}^+]$	$\cdots$
$\varepsilon_{2,1}=[v_{h-1}^-,v_{h-1}^+]$	$\varepsilon_{2,2}=[v_{h-3}^-,v_{h-3}^+]$	$\cdots$	
$\varepsilon_{3,1}$	$\varepsilon_{3,2}$	$\cdots$	
$\vdots$	$\vdots$	$\ddots$	
$\varepsilon_{h,1}$	$\varepsilon_{h,2}$		
$\varepsilon_{h+1,1}$			

III. PROPOSED METHODOLOGY

A. THE STEPS FOR OBTAINING DENOMINATOR OF MODEL

Using proposed method, denominator of continuous interval system (CIS) is obtained with the help of Routh table. The Routh table for denominator $D_h(\omega)$ of CIS is constructed as shown in Table 1.

The elements of Routh table are derived using

$$\varepsilon_{i,j}=\frac{\varepsilon_{i-2,j+1}\varepsilon_{i-1,1}-\varepsilon_{i-2,1}\varepsilon_{i-1,j+1}}{\varepsilon_{i-1,1}} \tag{9}$$

In (9), $i\geq 3$ and $1\leq j\leq (h-i+3)/2$. The denominator, $\widehat{D}_l(\omega)$, of l^{th} order model is calculated using (10) by utilizing $(h+1-l)^{th}$ and $(h+2-l)^{th}$ rows of Routh Table 1.

$$\begin{aligned}\widehat{D}_l(\omega)&=D_{h+1-l,1}\omega^l+D_{h+2-l,1}\omega^{l-1}\\&\quad+D_{h+1-l,2}\omega^{l-2}+\cdots\end{aligned} \tag{10}$$

B. MARKOV-PARAMETERS (MPs) AND TIME-MOMENTS (TMs)

The denominator of system shown in (2) can be expressed in interval form as

$$\begin{aligned}D_h(\omega)&=\wp_0+\wp_1\omega+\wp_2\omega^2+\wp_3\omega^3+\cdots+\wp_h\omega^h\\&=[\wp_0^-,\wp_0^+]+[\wp_1^-,\wp_1^+]\omega+[\wp_2^-,\wp_2^+]\omega^2\\&\quad+[\wp_3^-,\wp_3^+]\omega^3+\cdots+[\wp_h^-,\wp_h^+]\omega^h\end{aligned} \tag{11}$$

where, $[\wp_h^-,\wp_h^+]$ are coefficients of ω^h with $\wp_h^-$ as lower bound (LB) and $\wp_h^+$ as upper bound (UB) of interval $[\wp_h^-,\wp_h^+]$. For (11), the Kharitonov polynomials derived are

$$D_{h1}(\omega)=\underline{\wp_0^-}+\underline{\wp_1^-}\omega+\underline{\wp_2^+}\omega^2+\underline{\wp_3^+}\omega^3+\cdots \tag{12}$$

$$D_{h2}(\omega)=\underline{\wp_0^+}+\underline{\wp_1^-}\omega+\underline{\wp_2^-}\omega^2+\underline{\wp_3^+}\omega^3+\cdots \tag{13}$$

$$D_{h3}(\omega)=\underline{\wp_0^+}+\underline{\wp_1^+}\omega+\underline{\wp_2^-}\omega^2+\underline{\wp_3^-}\omega^3+\cdots \tag{14}$$

$$D_{h4}(\omega)=\underline{\wp_0^-}+\underline{\wp_1^+}\omega+\underline{\wp_2^+}\omega^2+\underline{\wp_3^-}\omega^3+\cdots \tag{15}$$

The stability of the interval polynomial (11) can be determined using the four Kharitonov polynomials listed in (12)-(15). In general, the Kharitonov polynomials represented in (12)-(15) can be written as

$$D_{hk}(\omega)=\kappa_0+\kappa_1\omega+\kappa_2\omega^2+\kappa_3\omega^3+\cdots \tag{16}$$

Taking (16) into account in the place of (11), interval system represented in (2) tends to be

$$V_h(\omega)=\frac{[u_{h-1}^-,u_{h-1}^+]\omega^{h-1}+\cdots+[u_1^-,u_1^+]\omega+[u_0^-,u_0^+]}{\kappa_0+\kappa_1\omega+\kappa_2\omega^2+\kappa_3\omega^3+\cdots+\kappa_h\omega^h} \tag{17}$$

Now, MPs are computed by expansion of (17) about $\omega=\infty$. The MPs are given as

$$\begin{aligned}\Phi_1&=\frac{[u_{h-1}^-,u_{h-1}^+]}{\kappa_h}\\\Phi_2&=\frac{[u_{h-2}^-,u_{h-2}^+]-\Phi_1\kappa_{h-1}}{\kappa_h}\\\Phi_3&=\frac{[u_{h-3}^-,u_{h-3}^+]-\Phi_1\kappa_{h-2}-\Phi_2\kappa_{h-1}}{\kappa_h}\\&\;\;\vdots\end{aligned} \tag{18}$$

Similarly, expansion of (17) about $\omega=0$ provide the TMs as

$$\begin{aligned}\tau_0&=\frac{[u_0^-,u_0^+]}{\kappa_0}\\\tau_1&=\frac{[u_1^-,u_1^+]-\tau_0\kappa_1}{\kappa_0}\\\tau_2&=\frac{[u_2^-,u_2^+]-\tau_0\kappa_2-\tau_1\kappa_1}{\kappa_0}\\&\;\;\vdots\end{aligned} \tag{19}$$

In general, MPs as obtained in (18) are written in (20).

$$\Phi_p=\frac{u_{h-p}}{\kappa_h}-\sum_{i=1}^{p-1}\frac{\kappa_{h-p+i}\Phi_i}{\kappa_h},\quad p=1,2,3,\cdots \tag{20}$$

Similarly, the generalized expression for determination of TMs can be given as

$$\tau_p=\frac{u_p}{\kappa_0}-\sum_{i=0}^{p-1}\frac{\kappa_{p-i}\tau_i}{\kappa_0},\quad p=0,1,2,\cdots \tag{21}$$

In similar fashion, expressions for MPs and TMs of model given in (5) are denoted by

$$\widehat{\Phi}_p=\frac{\widehat{u}_{l-p}}{\widehat{\kappa}_l}-\sum_{i=1}^{p-1}\frac{\widehat{\kappa}_{l-p+i}\widehat{\Phi}_i}{\widehat{\kappa}_l},\quad p=1,2,3,\cdots \tag{22}$$

$$\widehat{\tau}_p=\frac{\widehat{u}_p}{\widehat{\kappa}_0}-\sum_{i=0}^{p-1}\frac{\widehat{\kappa}_{k-i}\widehat{\tau}_i}{\widehat{\kappa}_0},\quad p=0,1,2,\cdots \tag{23}$$

C. THE STEPS FOR OBTAINING NUMERATOR OF THE MODEL

After attaining denominator, $\widehat{D}_l(\omega)$ of desired model using (10), numerator, $\widehat{N}_l(\omega)$ can be also obtained by matching first

TABLE 2. Routh table for denominator $D_{h1}(\omega)$.

ω^3	6	40.9
ω^2	27.5	20.7
ω	36.38	
ω^0	20.7	

l MPs and TMs of system and model. The matching is done as

$$\widehat{\tau_k} = \tau_k, \quad k = 0, 1, \cdots, (u-1)$$
$$\widehat{\Phi_k} = \Phi_k, \quad k = 1, 2, \cdots, v; \; v = l - u \tag{24}$$

It can be concluded from (24), that l MPs and TMs are required for matching to derive the desired model in total.

D. IMPLEMENTATION OF PROPOSED METHOD

The steps of implementing the proposed method are described in the flowchart as shown in Fig. 1.

IV. TEST CASES AND RESULTS

Let, a third order DIS [9], [19], [22], [23], [32] be

$$G(z) = \frac{[8, 10] + [3, 4]\, z + [1, 2]\, z^2}{[0.8, 0.85] + [4.9, 5]\, z + [9, 9.5]\, z^2 + [6, 6]\, z^3} \tag{25}$$

The discrete system represented in (25) is transformed by replacing $z = 1 + \omega$ into continuous system as

$$V_h(\omega) = \frac{[12, 16] + [5, 8]\, \omega + [1, 2]\, \omega^2}{[20.7, 21.35] + [40.9, 42]\, \omega + [27, 27.5]\, \omega^2 + [6, 6]\, \omega^3} \tag{26}$$

Now, for this test case, Kharitonov polynomials obtained in (12)-(15) turn out to be

$$D_{h1}(\omega) = 20.7 + 40.9\omega + 27.5\omega^2 + 6\omega^3 \tag{27}$$
$$D_{h2}(\omega) = 21.35 + 40.9\omega + 27\omega^2 + 6\omega^3 \tag{28}$$
$$D_{h3}(\omega) = 21.35 + 42\omega + 27\omega^2 + 6\omega^3 \tag{29}$$
$$D_{h4}(\omega) = 20.7 + 42\omega + 27.5\omega^2 + 6\omega^3 \tag{30}$$

Taking (27) in account, the transfer function given in (17) becomes

$$V_{h1}(\omega) = \frac{[12, 16] + [5, 8]\, \omega + [1, 2]\, \omega^2}{20.7 + 40.9\omega + 27.5\omega^2 + 6\omega^3} = \frac{N_h(\omega)}{D_{h1}(\omega)} \tag{31}$$

The Routh Table 1 for denominator of (31) is modified to Table 2.

Using (10), first order and second order denominator polynomials as derived from Table 2 are

$$\widehat{D}_{l,11}(\omega) = 20.7 + 36.38\omega \tag{32}$$
$$\widehat{D}_{l,12}(\omega) = 20.7 + 36.38\omega + 27.5\omega^2 \tag{33}$$

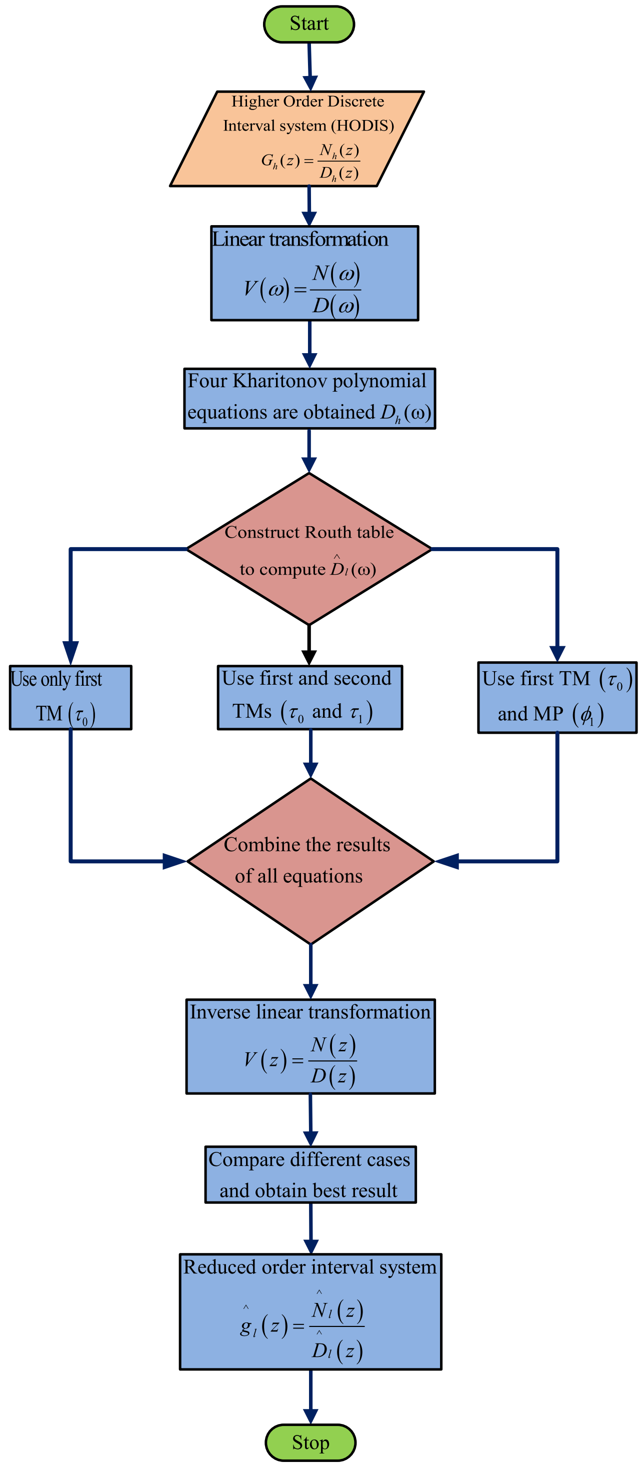

FIGURE 1. Flowchart of proposed method.

The first MP and first two TMs of (31) are calculated using (20) and (21). These are obtained as

$$\left.\begin{aligned} \Phi_1 &= [0.167, 0.333] \\ \tau_0 &= [0.5797, 0.7729] \\ \tau_1 &= [-1.285, -0.7589] \end{aligned}\right\} \tag{34}$$

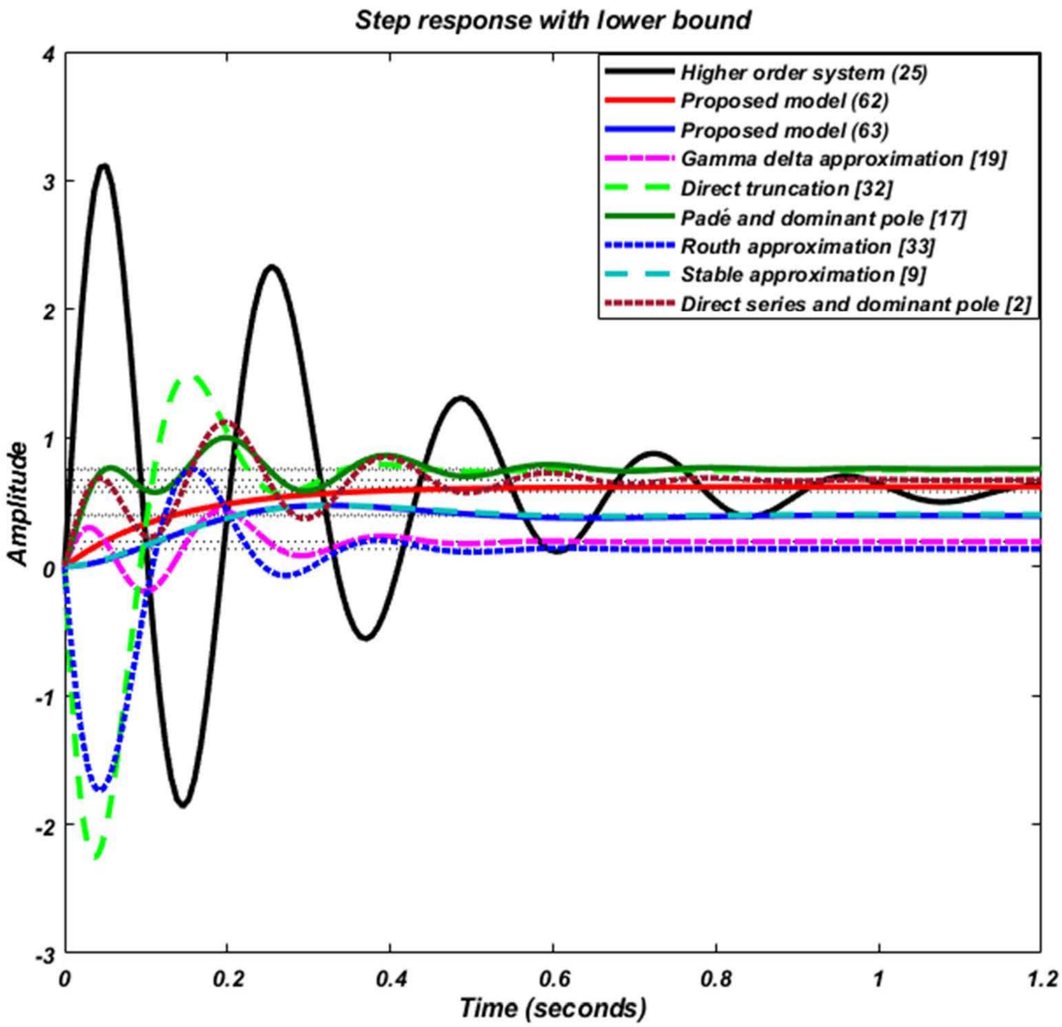

FIGURE 2. Step response of the original system and LOM for lower limit.

By matching of first TM of system and model, $\tau_0 = \widehat{\tau}_0$, the first order model is attained as

$$V_{\tau 0}^{1p}(\omega) = \frac{[12, 16]}{20.7 + 36.38\omega} \tag{35}$$

The second order model is attained by matching first MP and first TM as $\Phi_1 = \widehat{\Phi}_1$ and $\tau_0 = \widehat{\tau}_0$, respectively. The second order model obtained is

$$V_{\tau \Phi 1}^{1p}(\omega) = \frac{[12, 16] + [4.4, 9.07]\omega}{20.7 + 36.38\omega + 27.5\omega^2} \tag{36}$$

While, the model attained by matching first and second TMs as $\tau_0 = \widehat{\tau}_0$ and $\tau_1 = \widehat{\tau}_1$, is given by

$$V_{\tau 01}^{1p}(\omega) = \frac{[12, 16] + [-5.5103, 12.4096]\omega}{20.7 + 36.38\omega + 27.5\omega^2} \tag{37}$$

Now, taking (28) into consideration, transfer function of (17) replaces to

$$V_{h2}(\omega) = \frac{[12, 16] + [5, 8]\,\omega + [1, 2]\,\omega^2}{21.35 + 40.9\omega + 27\omega^2 + 6\omega^3} = \frac{N_h(\omega)}{D_{h2}(\omega)} \tag{38}$$

The Routh table represented in Table 1 for denominator of (38) is reconstructed into Table 3.

TABLE 3. Routh table for denominator $D_{h2}(\omega)$.

ω^3	6	40.9
ω^2	27	21.35
ω	36.16	
ω^0	21.35	

From (10), first order and second order denominator polynomials are obtained by utilizing Table 3 as

$$\widehat{D}_{l,21}(\omega) = 21.35 + 36.16\omega \tag{39}$$

$$\widehat{D}_{l,22}(\omega) = 21.35 + 36.16\omega + 27\omega^2 \tag{40}$$

The first MP and first two TMs of (38) are calculated with the help of (20) and (21) as

$$\left.\begin{aligned} \Phi_1 &= [0.167, 0.333] \\ \tau_0 &= [0.562, 0.7494] \\ \tau_1 &= [-1.20, -0.7019] \end{aligned}\right\} \tag{41}$$

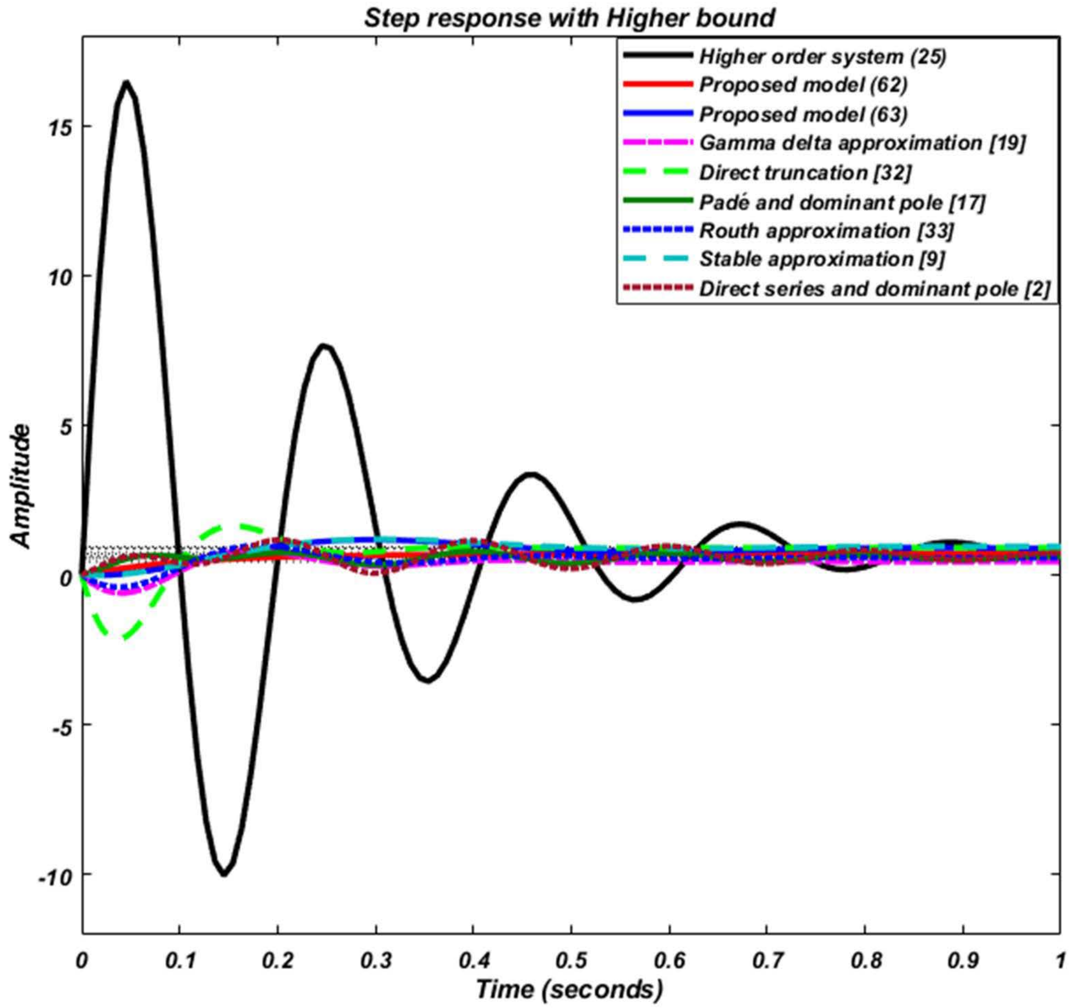

FIGURE 3. Step response of the original system and LOM for higher limit.

By equating the first TM as $\tau_0 = \widehat{\tau}_0$, first order model is attained as

$$V^{2p}_{\tau 0}(\omega) = \frac{[12, 16]}{21.35 + 36.16\omega} \tag{42}$$

The second order model is calculated by matching first MP and TM, respectively, as $\Phi_1 = \widehat{\Phi}_1$ and $\tau_0 = \widehat{\tau}_0$. The obtained second order model is given as

$$V^{2p}_{\tau\Phi 1}(\omega) = \frac{[12, 16] + [4.3, 8.91]\omega}{21.35 + 36.16\omega + 27\omega^2} \tag{43}$$

However, by matching the first and second TMs, respectively as $\tau_0 = \widehat{\tau}_0$ and $\tau_1 = \widehat{\tau}_1$, the second order model derived is given as

$$V^{2p}_{\tau 01}(\omega) = \frac{[12, 16] + [-5.2991, 12.1097]\omega}{21.35 + 36.16\omega + 27\omega^2} \tag{44}$$

Now, by considering (29), the transfer function given in (17) modifies to

$$V_{h3}(\omega) = \frac{[12, 16] + [5, 8]\,\omega + [1, 2]\,\omega^2}{21.35 + 27\omega + 27\omega^2 + 6\omega^3} = \frac{N_h(\omega)}{D_{h3}(\omega)} \tag{45}$$

For (45), Table 1 changes to Table 4.

TABLE 4. Routh table for denominator $D_{h3}(\omega)$.

ω^3	6	42
ω^2	27	21.35
ω	37.26	
ω^0	21.35	

The Table 4 is used to determine the denominator of LOM for (45). By utilizing (10), first order and second order denominator polynomials as calculated from Table 4 are

$$\widehat{D}_{l,31}(\omega) = 21.35 + 37.26\omega \tag{46}$$

$$\widehat{D}_{l,32}(\omega) = 21.35 + 37.26\omega + 27\omega^2 \tag{47}$$

The first MP and first two TMs of (45), are calculated using (20) and (21). These are

$$\left.\begin{aligned} \Phi_1 &= [0.167, 0.333] \\ \tau_0 &= [0.562, 0.7494] \\ \tau_1 &= [-1.24, -0.7308] \end{aligned}\right\} \tag{48}$$

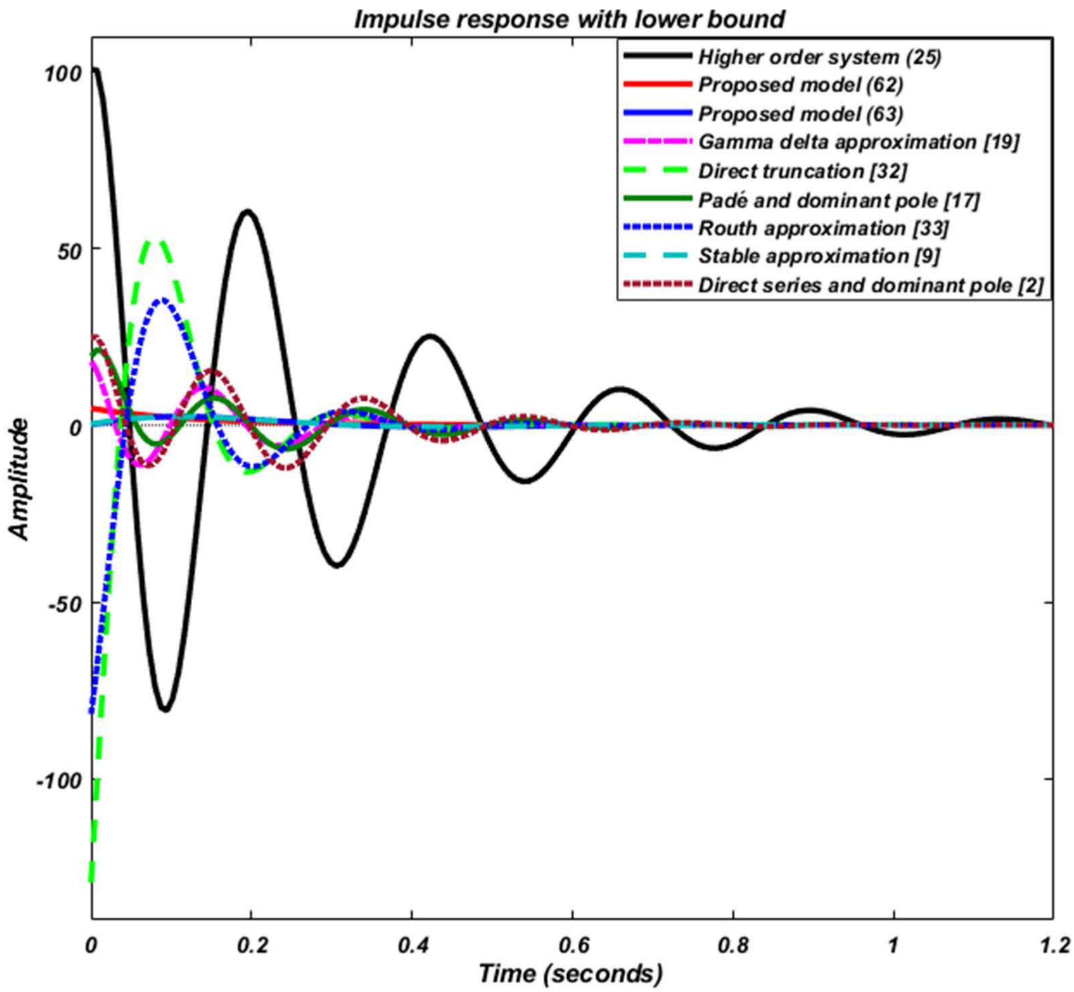

FIGURE 4. Impulse response of the original system and LOM for lower limit.

By matching first TM of system and model as $\tau_0 = \widehat{\tau}_0$, the first order model calculated is

$$V^{3p}_{\tau 0}(\omega) = \frac{[12, 16]}{21.35 + 37.26\omega} \tag{49}$$

While, the second order model is obtained by equating first MP, $\Phi_1 = \widehat{\Phi}_1$ and first TM, $\tau_0 = \widehat{\tau}_0$ of the system and desired model. The second order model becomes

$$V^{3p}_{\tau\Phi 1}(\omega) = \frac{[12, 16] + [4.3, 8.91]\omega}{21.35 + 37.26\omega + 27\omega^2} \tag{50}$$

The LOM model attained by matching first and second TMs, $\tau_0 = \widehat{\tau}_0$ and $\tau_1 = \widehat{\tau}_1$, is provided as follows

$$V^{3p}_{\tau 01}(\omega) = \frac{[12, 16] + [-5.5339, 12.3381]\omega}{21.35 + 37.26\omega + 27\omega^2} \tag{51}$$

Using (30), transfer function given in (17) changes to

$$V_{h4}(\omega) = \frac{[12, 16] + [5, 8]\,\omega + [1, 2]\,\omega^2}{20.7 + 42\omega + 27.5\omega^2 + 6\omega^3} = \frac{N_h(\omega)}{D_{h4}(\omega)} \tag{52}$$

To obtain the desired LOM models, Routh Table 1 turns out to be Table 5, for denominator of (52).

TABLE 5. Routh table for denominator $D_{h4}(\omega)$.

ω^3	6	42
ω^2	27.5	20.7
ω	37.48	
ω^0	20.7	

By utilizing (10), first order and second order denominator polynomials are calculated as depicted in (53) and (54), respectively.

$$\widehat{D}_{l,41}(\omega) = 20.7 + 37.48\omega \tag{53}$$
$$\widehat{D}_{l,42}(\omega) = 20.7 + 37.48\omega + 27.5\omega^2 \tag{54}$$

The first MP and first two TMs of (52), computed using (20) and (21), are

$$\left.\begin{aligned} \Phi_1 &= [0.167, 0.333] \\ \tau_0 &= [0.5797, 0.7729] \\ \tau_1 &= [-1.33, -0.7898] \end{aligned}\right\} \tag{55}$$

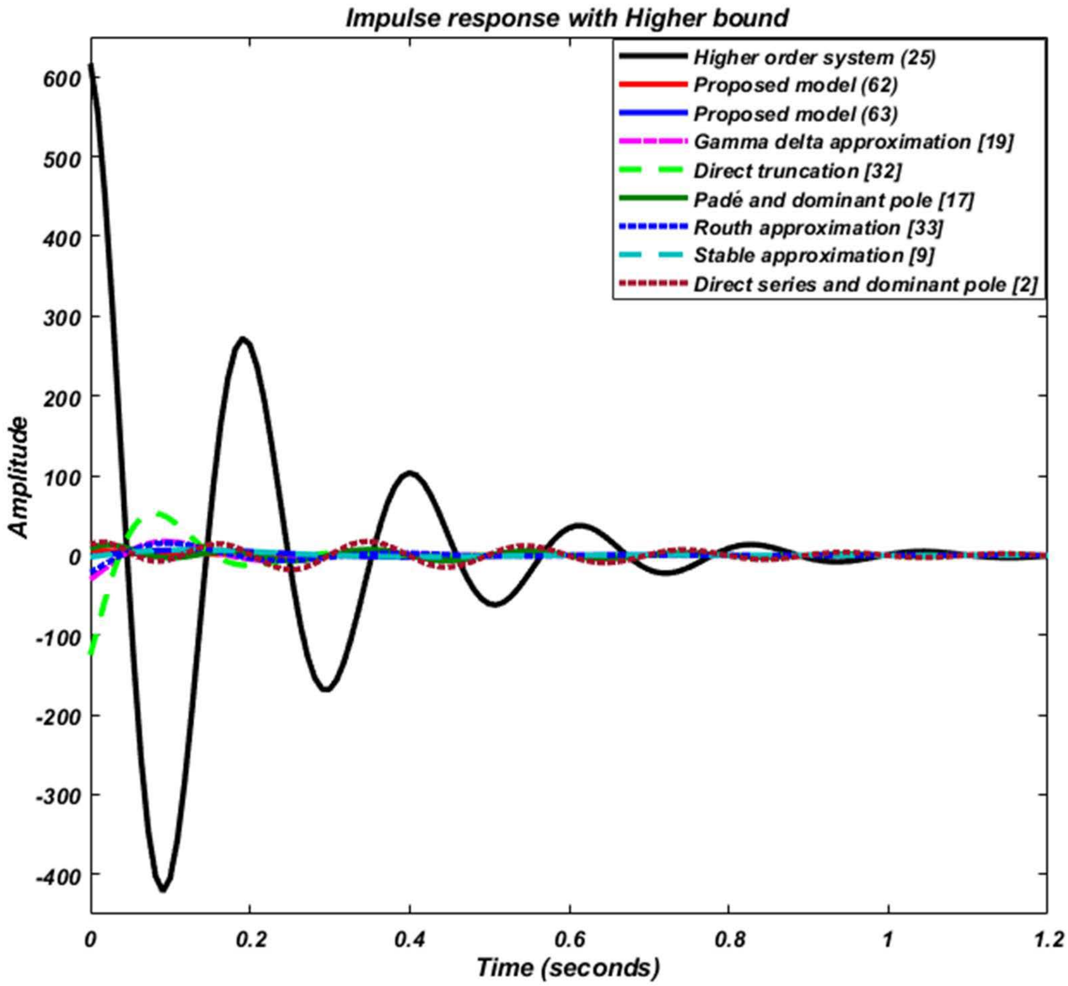

FIGURE 5. Impulse response of the original system and LOM for higher limit.

The first order model derived by matching first TM of system and model, $\tau_0 = \widehat{\tau}_0$ is given as

$$V_{\tau 0}^{4p}(\omega) = \frac{[12, 16]}{20.7 + 37.48\omega} \tag{56}$$

The second order model obtained by matching first MP and first TM, $\Phi_1 = \widehat{\Phi}_1$ and $\tau_0 = \widehat{\tau}_0$ is written as

$$V_{\tau\Phi 1}^{4p}(\omega) = \frac{[12, 16] + [4.4, 9.07]\omega}{20.7 + 37.48\omega + 27.5\omega^2} \tag{57}$$

By matching first and second TMs, $\tau_0 = \widehat{\tau}_0$ and $\tau_1 = \widehat{\tau}_1$, respectively, second order model obtained is given as

$$V_{\tau 01}^{4p}(\omega) = \frac{[12, 16] + [-5.8042, 12.6187]\omega}{20.7 + 37.48\omega + 27.5\omega^2} \tag{58}$$

By combining the models given in (35), (42), (49) and (56), first order interval model of original system modifies to

$$V_1^p(\omega) = \frac{[12, 16]}{[20.7, 21.35] + [36.16, 37.48]\,\omega} \tag{59}$$

Similarly, second order interval models, attained by considering models given in (36), (43), (50) and (57), and (37), (44), (51) and (58) are provided in (60) and (61), respectively.

$$V_{\tau\Phi}^p(\omega) = \frac{[12, 16] + [4.3, 9.07]\,\omega}{[20.7, 21.35] + [36.16, 37.48]\,\omega + [27, 27.5]\,\omega^2} \tag{60}$$

$$V_{\tau}^p(\omega) = \frac{[12, 16] + [-5.8042, 12.6187]\,\omega}{[20.7, 21.35] + [36.16, 37.48]\,\omega + [27, 27.5]\,\omega^2} \tag{61}$$

The models proposed in (59), (60) and (61) can be converted into the continuous system by replacing ω with $(z-1)$. After this replacement, models obtained are

$$V_1^p(z) = \frac{[12, 16]}{[-16.78, -14.81] + [36.16, 37.48]\,z} \tag{62}$$

$$V_{\tau\Phi}^p(z) = \frac{[2.93, 11.7] + [4.3, 9.07]\,z}{[10.22, 12.69] + [-18.84, 16.52]\,z + [27, 27.5]\,z^2} \tag{63}$$

$$V_{\tau}^p(z) = \frac{[-0.61, 21.8042] + [-5.8042, 12.6187]\,z}{[10.22, 12.69] + [-18.84, 16.52]\,z + [27, 27.5]\,z^2} \tag{64}$$

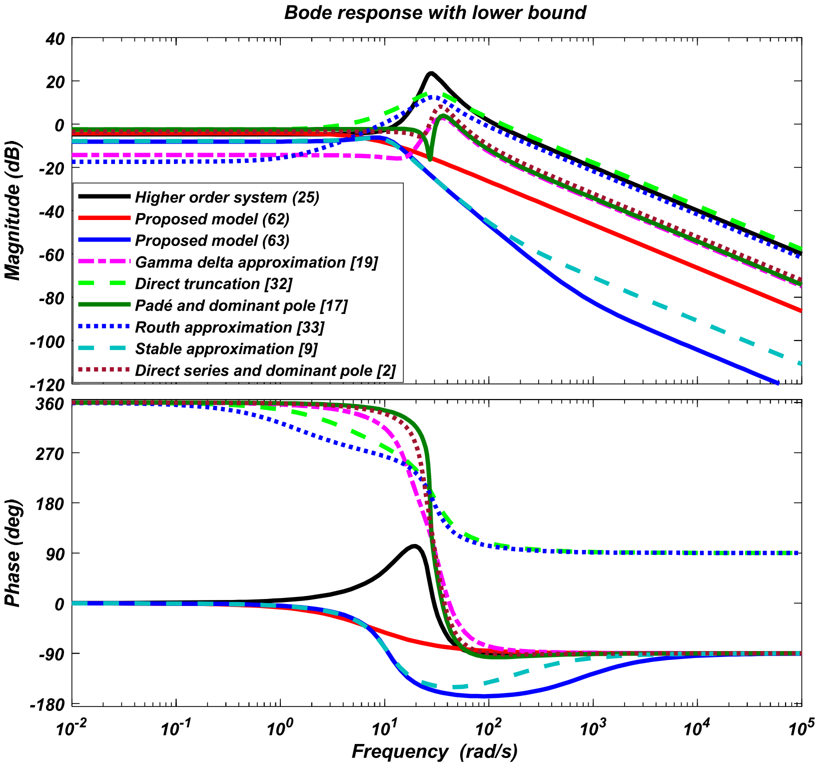

FIGURE 6. Bode response of the original system and LOM for lower limit.

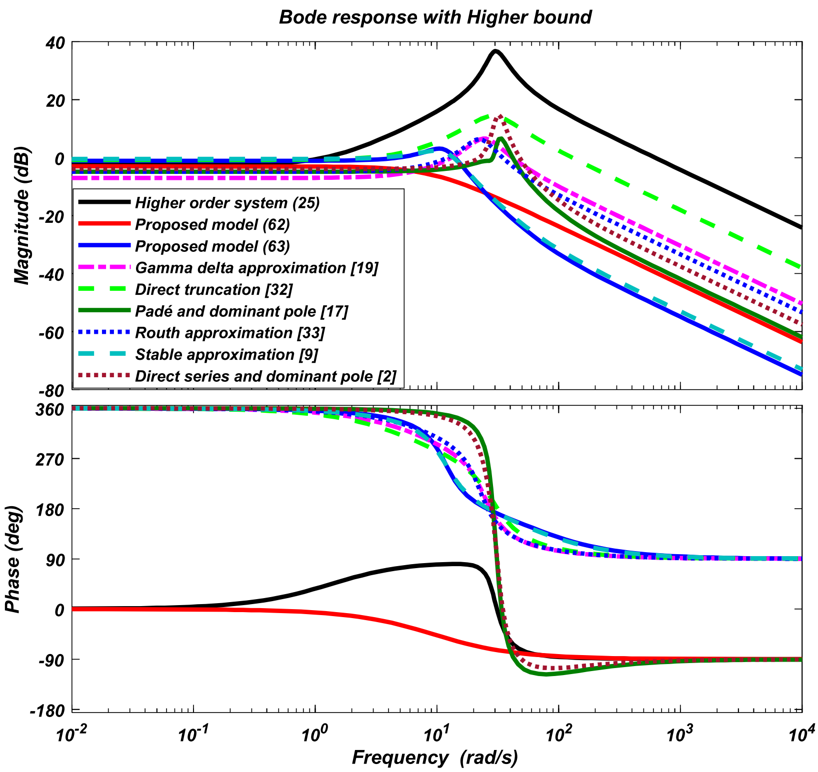

FIGURE 7. Bode response of the original system and LOM for higher limit.

A comparative study of step, impulse and Bode responses of HOS and LOM are presented for proposed model as depicted in (62) and proposed model as provided in (63), along with the models obtained by existing methods [2], [9], [17], [19], [32], [33]. The step responses are shown in Fig(s). 2 and 3, and Fig(s). 4 and 5 depict the impulse

TABLE 6. Comparison of time-domain specification for lower limits.

Methods	Rise time (s)	Peak	Settling time (s)	Peak time (s)
Higher order system (25)	0.0046	3.1172	1.5711	0.0509
Proposed model (62)	**0.2862**	**0.6192**	**0.5095**	**1.3736**
Proposed model (63)	**0.1428**	**0.4747**	**0.7696**	**0.3223**
Stable approximation [9]	0.1512	0.4737	0.7667	0.3319
Direct series and dominant pole [2]	0.0265	1.1218	0.8121	0.1959
Gamma delta approximation [19]	0.0099	0.4341	0.6167	0.1951
Direct Truncation [32]	0.0136	2.2541	0.4346	0.0381
Routh approximation [33]	0.0037	1.7388	0.6619	0.0428
Padé and dominant pole [17]	0.0348	1.0002	0.7109	0.1959

TABLE 7. Comparison of time-domain specification for upper limits.

Methods	Rise time (s)	Peak	Settling time (s)	Peak time (s)
Higher order system (25)	0.0010	16.4698	1.4000	0.0454
Proposed model (62)	**0.2366**	**0.7058**	**0.5000**	**1.1358**
Proposed model (63)	**0.1133**	**1.1761**	**1.0000**	**0.2977**
Stable approximation [9]	0.1196	1.1977	0.8000	0.3061
Direct series and dominant pole [2]	0.0353	1.6920	1.6108	0.1988
Gamma delta approximation [19]	0.0217	0.8280	0.6223	0.1709
Direct Truncation [32]	0.0166	2.1404	0.4390	0.0382
Routh approximation [33]	0.0305	0.9821	0.7773	0.1831
Padé and dominant pole [17]	0.0382	0.7969	1.4126	0.3977

TABLE 8. Comparison of performance indices of LOM for lower limit.

Methods	ITSE	ISE	ITAE	IAE
Higher order system (25)	-	-	-	-
Proposed model (62)	**0.0667**	**0.0572**	**0.1260**	**0.1060**
Proposed model (63)	**0.0827**	**0.0693**	**0.1662**	**0.1340**
Stable approximation [9]	0.0812	0.0681	0.1630	0.1316
Direct series and dominant pole [2]	0.1167	0.1009	0.1781	0.1497
Gamma delta approximation [19]	0.1611	0.1320	0.2569	0.2039
Direct Truncation [32]	0.0972	0.0828	0.1843	0.1518
Routh approximation [33]	0.1930	0.1574	0.2823	0.2232
Padé and dominant pole [17]	0.1121	0.0957	0.2015	0.1667

TABLE 9. Comparison of performance indices of LOM for higher limit.

Methods	ITSE	ISE	ITAE	IAE
Higher order system (25)	-	-	-	-
Proposed model (62)	**0.1773**	**0.1495**	**0.2267**	**0.1877**
Proposed model (63)	**0.1854**	**0.1557**	**0.2387**	**0.1960**
Stable approximation [9]	0.1971	0.1648	0.2549	0.2074
Direct series and dominant pole [2]	0.3904	0.3309	0.3493	0.2898
Gamma delta approximation [19]	0.2413	0.2011	0.2991	0.2422
Direct Truncation [32]	0.2141	0.1810	0.2705	0.2231
Routh approximation [33]	0.2036	0.1716	0.2641	0.2169
Padé and dominant pole [17]	0.2734	0.2301	0.3029	0.2491

responses. Bode responses are provided in Fig(s). 6 and 7. From the figures presented, it is clear that LOMs derived for original system employing proposed technique are much closer than other techniques available in literature. Further, the obtained LOMs from proposed method are compared with already available LOMs in terms of time-domain specifications and performance indices.

The Table 6 and Table 7 provide the information and comparative analysis of proposed LOMs and other available LOMs on the basis of time-domain specifications. Table 6 and Table 7 are presenting the time-domain specifications for upper and lower limits of first order model and second order model, respectively. The tabulated time-domain specifications are peak-time, rise-time, peak-value, and settling-time. It is clear from bold values of the tables, that the values of proposed models are closer to system in comparison with others LOMs.

To further demonstrate appropriate comparisons of proposed models, various performance indices are also obtained in Tables 8 and 9. The performance indices incorporated in tables are integral-time-multiplied-absolute-error (ITAE), integral-square-error (ISE), integral-time-multiplied-square-error (ITSE), and integral-absolute-error (IAE). These performance indices are tabulated for both, upper and lower limits.

Performance error indices values, presented in Tables 8 and 9, also prove the superiority of proposed method.

V. CONCLUSION

In this paper, Markov-parameters (MPs) and time-moments (TMs) of discrete interval systems (DISs) are utilized for model order diminution (MOD). The Routh-Padé approximation technique is utilized to obtain lower order model (LOM) of DIS. The Routh-Padé approximation method guarantees the stability of obtained LOM. The presented technique is validated with the help of a test system. To describe the superiority of proposed method, impulse, step, and Bode responses are plotted. The results are compared with other well-known and recently published works available in the literature on the basis of performance indices and time-domain specifications. The future work of this contribution lies in extension of proposed technique for multi-input-multi-output (MIMO) system. This work can also be extended to design the controller for DIS in future. Additionally, other approximation techniques should be investigated for order diminution of single-input-single-output and MIMO DISs.

APPENDIX

Appendix A: Interval Arithmetic

$$
\begin{aligned}
[\Re, \Im] + [\mu, \sigma] &= [\Re + \mu, \Im + \sigma],\\
[\Re, \Im] - [\mu, \sigma] &= [\Re - \sigma, \Im - \mu],\\
[\Re, \Im] - [\Re, \Im] &= 0,\\
[\Re, \Im] \times [\mu, \sigma] &= [\min(\Re\mu, \Re\sigma, \Im\mu, \Im\sigma),\\
&\quad \max(\Re\mu, \Re\sigma, \Im\mu, \Im\sigma)],\\
[\Re, \Im]/[\mu, \sigma] &= [\Re, \Im]\left[\frac{1}{\sigma}, \frac{1}{\mu}\right], \quad \mu \neq 0,\ \sigma \neq 0,\\
[\Re, \Im]/[\Re, \Im] &= 1, \quad \Re \neq 0,\ \Im \neq 0.
\end{aligned}
$$

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