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Vendor Managed Inventory Coordination Under Contractual Storage Agreement and Carbon Regulation Policies

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ABSTRACT In essence, the extent of collaboration among supply chain (SC) members, via practical mechanisms such as Vendor Managed Inventory (VMI), shall look beyond economic gains to strive for curbing the environmental impact of their logistics activities. This aligns well with the recently imposed sustainability related targets set by many legislative authorities. In this paper, we address a realistic SC problem wherein a single vendor serves multiple retailers under a VMI partnership. In order to deter the vendor from shipping larger batches downstream, the contractual agreement stipulates an upper limit on the inventory level held at each retailer's location where the vendor is penalized upon exceeding this level. In this context, economic and environmental as well as combined models are developed, with the latter employing carbon cap and carbon tax policies. Upon exploiting the structure of the developed models, computationally-efficient exact solution algorithms are devised. The conducted numerical analysis suggest that minor operational adjustments could lead to substantial carbon footprint reduction at the expense of a slight increase in the operational cost. Based on a novel use of the Lagrange multiplier, this work provides decision makers with a useful tool that aids with the selection of the most-suited carbon policy to embrace.

INDEX TERMS Sustainable supply chain, vendor managed inventory, carbon policies, exact solution procedures, single vendor, multiple retailers.

I. INTRODUCTION

To survive in today's business environment where fierce competition is on the rise, firms can no longer operate as individual and autonomous entities but rather as an integral part of the supply chain (SC) wherein each partner's objective is aligned to that of the SC as a whole. In essence, a high level of coordination and cooperation among SC members is vital to efficiently manage inventories, deliver products to the final customer, mitigate the bullwhip effect, and respond in a timely fashion to sudden changes in demand and customers preferences. To attain elevated levels of coordination and to enhance the effectiveness of distribution and inventory management practices across the SC, collaborative initiatives/mechanisms have been devised and successfully implemented by many firms over the years, including Vendor Managed Inventory (VMI), Continuous Replenishment (CR), and Consignment Stock (CS) among others. As pointed out by De Giovanni [1], the presence of an integrated SC system is the natural outcome of the emerging Industry 4.0 technologies which widely justifies the existence of VMI. At its core, VMI presents a collaborative scheme where the vendor is responsible for managing the inventory of the buyers including initiating orders on their behalf. The vendor in return is granted access to the buyers' real-time demand data via information sharing platforms where such improved visibility allows the vendor to better plan production, schedule deliveries, and manage stock levels at the buyers' facilities. Under such type of partnership, the vendor may be tempted to push more inventories to the buyers who are responsible for inventory holding cost. To guard against the vendor's opportunistic practice and to avoid any conflicts between SC members, the adjusted VMI contract stipulates an upper limit on the inventory level at the buyers' premises, where the vendor is penalized on a per-unit basis if this limit is exceeded (Fry *et al.* [2]; Darwish and Odah [3]; Hariga *et al.* [4]; Verma and Chetterjee [5]). In addition, the cost penalty can be considered as a transfer payment to encourage the buyers

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to accommodate larger shipment sizes at their locations. Needless to say, open communication and good partnership that is built on trust, cross-company information sharing, and transparency are the building blocks for a successful VMI implementation.

The industry use of VMI has gained more eminence over time where the successful VMI-based partnership between Procter & Gamble and Walmart in 1985 greatly stimulated the wide-spread popularity of this practice [6]. Since then, several other leading companies, such as HP and Shell, have successfully adopted VMI [7]. The implementation of VMI partnership has provided the food company Barilla with a competitive advantage as the largest pasta vendor in the world, with 22% of the pasta sold in Europe [8]. Nowadays, around 60% of the consumer processed goods retail stores in the United States operate through VMI [9]. This is mainly attributed to the proven compelling benefits that all parties are likely to attain upon effectively embracing such partnership. Several studies have reported substantial benefits of VMI implementation including improved service levels, reduced lead times and increased inventory turnovers, reduced stock-outs, improved control of the bullwhip effect due to better visibility and lower demand uncertainty, and chain-wide cost savings among others (e.g. Disney and Towill [10]; Angulo *et al.* [11]; Hariga *et al.* [4], Kim and Shin [12]). It is noted that the benefits of the VMI agreement are better assessed once the vendor considers not only his own costs but also those of the retailers [13]. In the context of single-vendor multi-retailer supply chains, Anand *et al.* [14] pointed out that the main impediment to successful VMI implementation is the unequal benefits distribution where, in many instances, the retailers tend to avail higher profits. For interested readers, a list of the advantages as well as the limitations of a VMI arrangement to all parties involved is also provided therein.

In principle, considering solely the potential operational and financial benefits associated with VMI contractual agreements might sound appealing enough for SC members to embrace such scheme. Yet, establishing close collaboration has become an inevitable necessity and a more pressing issue than ever given the negative environmental impact of SC operations and the accompanying stringent governmental regulations that have been put in place. Indeed, SC activities (procurement, production, distribution and warehousing) are responsible for an average of 75% of all industrial sector carbon emissions [15]. In addition, over 90% of emissions can be found in the SC of firms [16]. Accordingly, companies ought to continuously rethink their strategies such that environmental considerations and sustainable practices are at the forefront of their day-to-day operations. Several carbon regulatory policies have been established, which include carbon tax, carbon cap, carbon offset, cap-and-trade, and eco-friendly technology standards. The efficacy of these emissions control policies across a variety of industries and operating conditions have been extensively analyzed in the literature. Following the carbon tax policy, a firm is charged a fixed tax rate for every ton of carbon emitted where the rate is determined by the policymaker or the regulatory body. The advantage of the carbon tax is that it is relatively easy to implement as it can be incorporated within existing taxation systems and caters for price stability [17]. Park *et al.* [18] pointed out that a carbon tax is more effective towards balancing the tradeoff between economic development, environmental protection, and social benefits. In a study conducted in the UK, the adoption of carbon tax has helped achieve 18% reduction in energy intensity and 22% reduction in electricity consumption [19]. In their recent review paper, Zhou *et al.* [15] reported that there has been a wider implementation of the carbon tax as compared to cap-and-trade policies, with the former being implemented in 21 countries or regions versus only 8 countries for the latter. On the other hand, the quantity, rather than price, based carbon cap policy imposes a limit on the maximum footprint generated over a certain period of time which is exogenously imposed by the legislative authorities. Several studies have adopted the carbon cap policy where they established the necessary operational adjustments that need to be carried out in adherence to various levels of the preset cap (see for instance Absi *et al.* [20], [21] and As'ad *et al.* [22]). A detailed discussion of these policies seeking to curb carbon emissions can be found in the works of Hariga *et al.* [23] and Mohammed *et al.* [24].

It is evident that the escalating pressure from customers in conjunction with the plentiful environmental regulations is tapping on the long-standing economic-based only measures and is calling upon SC members to jointly instill environmental concerns as an integral part of their decision making process. As such, this work presents integrated economic and environmental models for a two-stage single-vendor multi-buyer SC operating under the VMI contractual agreement, while assuming synchronized replenishment cycles for all retailers and limited storage space at the retailers' premises. Given their proven effectiveness and wide acceptance nowadays, we adopt the carbon cap and carbon tax policies in the development of the combined economic and environmental models. The results indicate that each of the two carbon policies yields a distinct lot sizing strategy, where it turns out that embracing this sort of partnership subject to the two carbon regulation policies affects not only the economic but also the environmental performance of the whole SC. Furthermore, this work utilizes the concept of Lagrange multiplier to aid practitioners with the selection of the best-fit carbon regulatory policy to be adopted. To the authors' best knowledge, this work is the first to develop sustainable VMI models for a SC comprising a single vendor and multi buyers with storage contractual agreements. Efficient solution algorithms that exploit the structure of the developed models and guarantee convergence to the optimal solutions are also proposed herein which is another novelty of this work.

In essence, the research questions addressed in this work are:

(1) What is the impact of incorporating environmental considerations on the operational and lot sizing policy

of a single-vendor multi-retailer SC operating under a centralized VMI agreement?

(2) What are the chain-wide cost implications associated with considering environmental – rather than purely economic – related performance measures?

(3) What problem/parameter settings would render the carbon tax or carbon cap regulatory policy a more effective strategy to use?

The remainder of this paper is organized as follows. A review of the state-of-the-art literature pertaining to the problem at hand is provided in Section 2. In Section 3, the proposed economic, environmental, and the integrated carbon cap and carbon tax based VMI models are presented along with efficient solution algorithms yielding optimal solutions. Illustrative examples that illustrate the simplicity and effectiveness of the proposed algorithms are also provided along with a discussion on the role of the Lagrange multiplier and the obtained differences in the lot sizing strategy under the two carbon policies. Section 4 summarizes the paper, presents concluding remarks and highlights future research avenues.

II. RELEVANT LITERATURE

The modeling of integrated production and inventory decisions has been a topic of interest for many researchers since such models can help in the assessment of the benefits leveraged via SC coordination. The integrated single-vendor single-buyer (SV-SB) problem, which constitutes the building block for any SC, has been an active area of research for many years. Goyal [25] first suggested a lot-for-lot policy for a single vendor, with an infinite production rate, serving a single retailer. This work was later extended by several researchers addressing different problem settings and forming a stream of research commonly referred to as the joint economic lot sizing problem. This body of literature is closely related to the topic of this paper but does not necessarily address VMI-based collaboration where interested readers are referred to the thorough review papers of Ben-Daya *et al.* [26], Glock [27] and Utama *et al.* [28]. In the remainder of this section, we first focus on research works dealing with the analytical modeling of VMI relationships and then survey some relevant works addressing the modeling of sustainable inventory systems accounting for environmental aspects.

One of the earliest models to explicitly consider VMI contractual agreements is that of Fry *et al.* [2] who tackled a SV-SB case under a specific arrangement where the supplier is penalized if the retailer's inventory level is not maintained within a certain range. Contractual agreement was also explicitly addressed in Shah and Goh [29] who modeled a VMI problem in the context of a supply hub with a single retailer. Other VMI coordination issues were also considered in Mishra and Raghunathan [30], Choi *et al.* [31], Lee and Chu [32], Yao *et al.* [33], Nagarajan and Rajagopalan [34], Lee and Ren [35], Van den Bogaert and Van Jaarsveld [36] among many others. For comprehensive works addressing various aspects of VMI contractual agreements, readers are referred to Govindan [37], Lee *et al.* [38] and Nimmy *et al.* [39].

CS is another common collaboration mechanism that is closely related to VMI practice, whereby goods are owned by the vendor until they are sold by the retailer. In fact, goods are typically stored at the retailer's premises who only pays for them upon use. While considering an upper bound on the inventory level acts as a remedy against vendor's opportunistic practice of overstocking products at the retailers' premises, combining VMI with CS (VMI-CS) serves the same purpose as now the vendor bears the financial component of holding cost for the stock kept downstream [40]. Several authors looked at combined VMI-CS policies under different problem settings. In the context of SV-SB partnerships, Gümüş [41] provided some general conditions under which CS creates benefits for the vendor, for the retailer, and for the two parties together, and then considered similar issues for the combined VMI-CS approach. For the single-vendor multi-buyer (SV-MB) case, Ben-Daya *et al.* [40] modeled a VMI-CS policy involving three vendor-buyers partnerships. Analytical and numerical examples were presented including insights on when such agreements are more attractive. Other works addressing combined VMI-CS policy include Zanoni *et al.* [42], Bazan *et al.* [43], Khan *et al.* [44], Bazan *et al.* [45], Hariga *et al.* [23], Mateen *et al.* [46] and Bieniek [47]. An alternative approach to enhancing VMI collaboration schemes is via the use of technology where Omar *et al.* [48] proposed utilizing a blockchain-based approach with smart contracts to transform VMI SC operations. Along the same lines, Weißhuhn and Hoberg [49] applied Internet-of-Things technology for VMI implementation at end consumers. They analyzed a smart replenishment system that leverages point-of-consumption information at end consumers to decide on deliveries, while assuming a constrained vendor's dispatch capacity.

As pointed out before, this paper addresses SV-MB system where such SC configuration was investigated by many authors, including Viswanathan and Piplani [50]; Cheung and Lee [51], Lu [52], Siajadi *et al.* [53], Hoque [54] among many others. In the presence of multi buyers/retailers, the coordination of the replenishments and the scheduling of the shipments to the downstream retailers further complicate the modeling of the problem. In the extant literature, the two commonly adopted approaches are equal and unequal cycle policies, where the retailers are replenished at the same time or at different points in time, respectively. Following the equal/synchronized cycle policy, Darwish and Odah [3] were among the first to propose a model for a SC involving a single vendor and multiple retailers operating under a VMI collaborative agreement. Similar to this work, they also explicitly take into consideration upper bounds on inventory levels at the retailers' premises, where the vendor is penalized if these upper inventory levels are exceeded. They developed a solution procedure based on finding the optimal solution among all Karush-Kuhn-Tucker (KKT) points for $(m + 1)$ optimization problems, where m is the number of retailers.

Braide *et al.* [55] tackled the case of volume discounts and price sensitive demand. Mateen and Chatterjee [56] explored various approaches via which a SV-MB system may be coordinated through VMI and analyzed their analytical models. Mateen *et al.* [7] analyzed the problem under stochastic demand. Zhang *et al.* [57] developed an integrated VMI model for a SV-MB following the more generalized unequal cycle policy, but the VMI contract is not explicitly included in the model. Hariga *et al.* [4] addressed the problem of synchronizing the vendor's cycle time with the buyers' unequal ordering cycles, which they formulated as a mixed integer non-linear program. Models that allow unequal shipment frequencies to the retailers were also considered by Hariga *et al.* [13]. Verma *et al.* [58] also proposed an alternative replenishment scheme allowing for different replenishment cycles for each retailer.

In another work, Son and Ghosh [59] investigated SC coordination that includes retailer incentives and cross-docking at the vendor. They studied the impact of SC design parameters (number of retailers and retailer heterogeneity) and a policy parameter (shipment frequency) on the effectiveness of the VMI system. Retailer heterogeneity was also considered by Verma and Chatterjee [5]. Tarhini *et al.* [60] looked at the benefits of transshipment between buyers and showed that it can be a tool for decreasing the total cost faced by both buyers and their suppliers. While both the penalty and the threshold are exogenous in most of previous studies, Chakraborty *et al.* [61] developed a model to determine both of them endogenously. Kumar *et al.* [62] discussed how a vendor manages multiple products within its multiple non-identical retailers operating under a VMI contract. They formulated the problem as a mixed-integer nonlinear program that allows unequal shipment frequencies to the retailers. It shall be noted that all of the aforementioned SV-MB works focused solely on economic measures while overlooking environmental related concerns.

The incorporation of sustainability considerations into the modeling literature of inventory management and SC operations has seen a noticeable surge over the past decade. Broadly speaking, this body of literature suggests that substantial reduction in carbon emissions can be attained via better coordinated lot sizing and shipping decisions among SC members. However, the majority of these models address single stage and single product settings [22]. A pioneering work in this area is that of Chen *et al.* [63] who adopted the four carbon policies (carbon tax, carbon cap, carbon offset, and cap-and-trade) in the context of the infamous single stage Economic Order Quantity (EOQ) model. Benjaafar *et al.* [64] also considered the same four carbon policies but for a product exhibiting a time-varying demand over a finite planning horizon. Recent review papers that provide a comprehensive discussion on sustainable inventory models incorporating carbon emissions aspects include Das and Jharkharia [65], Shaharudin *et al.* [66], Chelly *et al.* [67], and Zhou *et al.* [15] with the latter addressing only carbon tax policy. As for VMI contractual agreements, few works have incorporated environmental concerns into the modeling aspects of this collaborative framework. These studies suggest that VMI-based coordination not only leads to cost savings, but is also a very useful tool for enhancing sustainability efforts such as reducing greenhouse gas emissions. While adopting the equal cycle policy, Karimi and Niknamfar [68] considered a multi-product SV-MB VMI system that takes into consideration the generated carbon footprint and utilizes the carbon tax and carbon cap policies. To increase the system reliability, the authors also consider the manufacturer's redundancy allocation problem where the developed profit-maximization model is solved using an off-the-shelf optimization software to maximize the worst (minimum) value of the objective function. In a related work, Mokhtari and Rezvan [69] also addressed a multi-product SV-MB system employing VMI partnership while allowing for shortages that are partially backordered. They imposed a constraint on the total emissions and solved the resulting model to optimality using a decomposition based analytical approach. Nugroho and Wee [70] incorporated imperfect quality deteriorating items in their proposed model of a SV-SB system. Mateen *et al.* [46] proposed models that include both operational and emission related costs in the context of a SV-SB VMI system. They analytically showed that the benefits of VMI-based SC coordination extend beyond cost savings to include carbon emissions reduction as well.

The above review of the literature clearly indicates that the incorporation of environmental considerations in the modeling and analysis of VMI SC partnership is still in its infancy stage. This work brings the following contributions to the existing SC collaboration literature. First, purely economic and environmental as well as integrated economic and environmental models under the carbon cap and carbon tax policies are proposed for a two stage SV-MB system operating under VMI partnership with contractual storage agreement. The developed models provide valuable insights pertaining to the chain-wide economic and environmental impact of adopting this partnership under the two carbon policies. For instance, the conducted numerical experiments reveal that substantial reduction in carbon emissions can be attained via operational adjustments with only a minor increase in the total operational cost. To the authors' best knowledge, this last result is being established for the first time in the literature in the context of SV-MB VMI-based systems even under the equal replenishment cycle policy. In essence, this work may be viewed as an extension of the works of Darwish and Odah [3] and Hariga *et al.* [23] to present sustainable VMI practices for SV-MB systems. Secondly, upon exploiting the structure of the proposed models, computationally-efficient solution algorithms are proposed which are guaranteed to yield the optimal solutions. Lastly, through utilizing the optimal value of the Lagrange multiplier, this work provides decision makers with a novel idea to aid with the selection of which of the two carbon policies shall be adopted.

III. VMI MODELS WITH ECONOMIC AND ENVIRONMENTAL CONSIDERATIONS

Consider a SC system comprising a vendor who is supplying a single product to m retailers under VMI agreement. The vendor manages the inventory of the retailers at their premises and takes the responsibility to decide about the size of the orders and their timings. However, each retailer owns his/her inventory and incurs its holding cost. As such, the vendor is charged for the cost of initiating orders on behalf of the retailers besides his/her own ordering and holding costs. Towards reducing his/her holding costs along with the number of shipments dispatched to the retailers, the vendor is keen on transferring much of his/her inventory to the retailers' storage facilities. However, to avoid storing excess inventory resulting from the vendor's delivery policy, each retailer bounds its on-hand stock to a maximum level that should not be exceeded by the vendor, otherwise the latter will be penalized. The maximum limit on the retailers' inventory and the associated penalty costs are included in the contractual agreement between the vendor and the retailers. In addition, environmental considerations are integrated into the SC partnership in order to reduce the total carbon footprint generated by the chain-wide ordering and storage activities.

For the sake of completeness, below is a list of the assumptions adopted in the paper.

1. Demand is deterministic
2. Shortages are not allowed
3. The vendor orders a single product from an outside supplier with unlimited capacity.
4. Holding cost rates of the retailers are larger than that of the vendor
5. The same number of shipments is made to each retailer.
6. Each retailer charges the vendor a penalty cost for each unit of the product exceeding the maximum allowed inventory level. The vendor incurs such cost as long as the inventory is above the upper stock level.
7. A profit sharing mechanism, such as transfer payment, is stated in the VMI contract to ensure savings for all SC members in case the VMI generates system-wide savings as compared to the decentralized/locally optimized strategy. This ensures all parties eagerness to engage in such contractual agreement (see Birim and Sofyalioglu [71] for a detailed discussion).

Throughout this work, it is assumed that the vendor delivers the same number of shipments to all retailers where the latter share equally-spaced common replenishment cycle. Such a replenishment policy, also called complete aggregation policy [72], is adopted in practice as a coordination mechanism across several SC systems. It allows the vendor to aggregate retailers' orders and, consequently, reduces his/her ordering processing costs. From an environmental standpoint, it also contributes significantly to lower levels of carbon emissions as the truck, or several of them, would make one trip with multiple stops at the different retailers' premises rather than a single round trip per delivery. In essence, complete aggregation policies are also simple to implement and are commonly used in practice. Indeed, there are many practical situations where vendors adopt common replenishment policies. For instance, in the cases of dairy, soft drinks, and bottled water products, it is a common practice for the distributors to replenish all their retailers (supermarkets, mini-marts, grocery stores, etc.) within the same city simultaneously on the same day. Moreover, there are several published SV-MB works in the SC literature adopting the assumption of equal replenishment cycles (e.g., see Banerjee and Banerjee [73], Viswanathan and Piplani [50], Woo *et al.* [74], Mishra [75], Hoque [54], Darwish and Odah [3], Ben-Daya *et al.* [40], Mateen *et al.* [7], Karimi and Niknamfar [68], and Pramudyo and Luong [76], among many others). In the development of the mathematical models, the following notation is used.

General Parameters:

m	Number of retailers
U_j	Maximum allowed inventory level at the jth retailer facility, $j = 1, 2, \ldots, m$
D_j	Annual demand rate for retailer $j, j = 1, 2, \ldots, m$

Cost Parameters:

A_j	Cost of initiating and releasing an order to retailer $j, j = 1, 2, \ldots, m$
h_j	Annual cost of holding one unit of the item in the jth retailer facility, $j = 1, 2, \ldots, m$
π_j	Penalty cost charged by the jth retailer if the vendor exceeds the maximum allowed inventory, $j = 1, 2, \ldots, m$
h_v	Annual cost of holding one unit of the item in the vendor's facility
A_v	Vendor's ordering cost
h_{vj}	$= h_j - h_v, j = 1, 2, \ldots, m$

Environmental Parameters:

$\hat{A}_j$	Carbon quantity emitted by one order to retailer $j, j = 1, 2, \ldots, m$
$\hat{A}_v$	Carbon quantity emitted by one order placed by the vendor
$\hat{h}_j$	Amount of carbon generated due to the storage of one unit of the item in the jth retailer facility, $j = 1, 2, \ldots, m$
$\hat{h}_v$	Amount of carbon generated because of the storage of one unit of the item in the vendor facility.
C	Maximum amount of carbon that can be generated by all SC members
t_x	Tax rate charged against each carbon ton emitted
$\hat{h}_{vj}$	$= \hat{h}_j - \hat{h}_v, j = 1, 2, \ldots, m$

Decision Variables:

n	Number of shipments made to each retailer (integer variable)
T	Vendor's replenishment cycle
q_j	Shipment lot size to the jth retailer, $j = 1, 2, \ldots, m$
T_j	Replenishment cycle for the retailer, $j = 1, 2, \ldots, m$

In the following sub-sections, we develop VMI models with economic and environmental considerations for a SC system comprised of a single vendor and multiple buyers. Numerical examples are also provided to illustrate the developed models.

A. ECONOMIC VMI MODEL

The objective of the economic VMI model is the minimization of chain-wide related costs with complete disregard to the environmental impact. The model's objective function is given by (Hariga *et al.* [4]):

$$\frac{A_v}{T} + 0.5h_vDT + \sum_{j=1}^{m}\frac{A_j}{T_j} + 0.5\sum_{j=1}^{m}\left(h_j - h_v\right)D_jT_j$$
$$+ Vendor's\ penalty\ cost$$

where $D = \sum_{j=1}^{m} D_j$.

The first term is the vendor's ordering cost per unit of time; the second term is the vendor's holding cost per unit of time based on its echelon stock; the third term is the sum of the retailers' ordering cost per unit of time, and the fourth term is the total retailers' holding costs per unit of time expressed in terms of their echelon stocks, $(h_j - h_v)$. The echelon stock at a particular location is defined as the on-hand inventory at the location and all of its upstream locations (Clark and Schwarz [77]).

Next, given that n shipments are delivered to each retailer during the vendor's delivery cycle, the retailers' replenishment cycles are also equal. Therefore, we have

$$T_i = T_j \quad \text{and } q_i/D_i = q_j/D_j \tag{1}$$

Consequently, the number of decision variables related to the shipment sizes reduces to only one variable, q_1. In the sequel, we let $q = q_1$.

The over-stock penalty cost incurred by the vendor when the inventory level exceeds the maximum inventory allowed by the jth retailer is (Hariga *et al.* [4]):

$$\pi_j n\frac{(q_j - U_j)^2}{2D_j} \quad \text{if } q_j > U_j,$$

which, after using (1), can be rewritten as

$$\pi_j n\frac{(\frac{D_j}{D_1}q - U_j)^2}{2D_j} \quad \text{if } q_j > U_j,$$

Now, let the over-stock quantity at the jth retailer be denoted by

$$z_j = Max\left(0, \frac{D_j}{D_1}q - U_j\right).$$

Then, the total penalty cost incurred by the vendor per cycle is

$$0.5n\frac{\pi_j}{D_j}\sum_{j=1}^{m} z_j^2.$$

Substituting $T = n.T_j = nq/D_1$, the total operational cost (TOC) per unit of time for the integrated supply chain is given in a simple mathematical expression as follows:

$$\begin{aligned} TOC(n, q, z_j) = {} & \frac{A_v + nA}{nq}D_1 + 0.5\frac{(n-1)\,Dh_vq}{D_1} \\ & + 0.5\frac{D_1}{q}\sum_{j=1}^{m}\frac{\pi_j}{D_j}z_j^2 \\ & + 0.5\frac{q}{D_1}\sum_{j=1}^{m}h_jD_j, \end{aligned} \tag{2}$$

where $A = \sum_{j=1}^{m} A_j$.

After rearranging the terms in (2), this cost function can be rewritten as

$$\begin{aligned} TOC\left(n, q, z_j\right) = {} & \frac{K(n)}{q}D_1 + 0.5\frac{G(n)}{D_1}q \\ & + 0.5\frac{D_1}{q}\sum_{j=1}^{m}\frac{\pi_j}{D_j}z_j^2, \end{aligned} \tag{3}$$

where

$$\begin{aligned} K(n) &= \frac{A_v}{n} + A, \\ G(n) &= nDh_v + H, \quad \text{and} \\ H &= \sum_{j=1}^{m}\left(h_j - h_v\right)D_j \end{aligned} \tag{4}$$

The optimization problem to be solved is a constrained mixed-integer nonlinear problem with a convex objective function subject to m linear constraints.

VMI-EC:
$$\begin{aligned} Min\ TOC\left(n, q, z_j\right) = {} & \frac{K(n)}{q}D_1 + 0.5\frac{G(n)}{D_1}q \\ & + 0.5\frac{D_1}{q}\sum_{j=1}^{m}\frac{\pi_j}{D_j}z_j^2 \\ s.t.\ & \frac{D_j}{D_1}q - U_j - z_j \le 0 \quad \text{for } j = 1, 2, \ldots, m, \\ & q \ge 0 \text{ and } n \text{ integer} \end{aligned}$$

Let's now consider the space unconstrained problem. That is, the optimization problem that minimizes

$$\begin{aligned} TOC_u\left(n, q\right) = {} & \frac{K(n)}{q}D_1 + 0.5\frac{G(n)}{D_1}q \\ & s.t.\ q \ge 0 \text{ and } n \text{ integer} \end{aligned} \tag{5}$$

For a given n, the minimum unconstrained operational cost and replenishment quantity are:

$$TOC_u\left(n, q_u\right) = \sqrt{2K(n)\,G(n)}, \tag{6}$$

and

$$q_u = \sqrt{\frac{2K(n)}{G(n)}D_1}, \tag{7}$$

respectively. Note that minimizing the unconstrained operational cost in (6) with respect to n is equivalent to minimizing

$$\frac{A_vH}{n} + nAh_vD.$$

The optimal number of replenishments can then be found by applying the first difference approach to the last cost function. In this case, the optimal n_u is the first integer satisfying

$$n\left(n-1\right) \leq \frac{A_vH}{Ah_vD} \leq \left(n+1\right).$$

The two inequalities can be rewritten as

$$0.5\left(-1+\sqrt{1+4\frac{A_vH}{Ah_vD}}\right) \leq n \leq 0.5\left(1+\sqrt{1+4\frac{A_vH}{Ah_vD}}\right).$$

Given that there is only one integer in the above interval for n, then n_u is given by

$$n_u = \left\lceil 0.5\left(-1+\sqrt{1+4\frac{A_vH}{Ah_vD}}\right)\right\rceil \tag{8}$$

where $\lceil x \rceil$ is the first integer larger than or equal to x.

The optimal $q_{1u} = q_u$ is then obtained after substituting n_u into (7). The replenishment quantities for the remaining retailers are determined using (1); that is $q_{ju} = q_{1u}D_j/D_1$. In the case where

$$\frac{D_j}{D_1}q_{1u} - U_j \leq 0 \quad \text{for } j = 1, 2, \ldots, m,$$

then the unconstrained operating policy is the optimal solution to VMI-EC. In the following, it is assumed that this is not the case.

For a given n, we next develop a simple and efficient algorithm to generate an optimal solution to VMI-EC optimization problem. VMI-EC has a unique global minimum since it has a convex objective function and a convex feasible region. Therefore, the Karush-Kuhn-Tucker (KKT) equations are necessary and sufficient conditions to achieve optimality (Bazaraa *et al.* [78]). Let $\lambda_j\,(j = 1, 2, ..m)$ and μ be the Lagrangian multipliers corresponding to the constraints in VMI-EC. Therefore, $(q, z_j : j = 1, 2, \ldots, m)$ is an optimal solution if and only if there exists non-negative $\lambda_j\,(j = 1, 2, ..m)$ and μ satisfying the following KKT conditions:

$$-\frac{K(n)+0.5\sum_{j=1}^{m}\frac{\pi_j}{D_j}z_j^2}{q^2}D_1 + \frac{\sum_{j=1}^{m}\lambda_jD_j+0.5G(n)}{D_1} = 0, \tag{9}$$

$$\frac{D_1}{q}\frac{\pi_j}{D_j}z_j - \lambda_j = 0, \quad for\ j = 1, 2, \ldots, m \tag{10}$$

$$\lambda_j\left(\frac{D_j}{D_1}q - U_j - z_j\right) = 0, \quad for\ j = 1, 2, \ldots, m \tag{11}$$

$$\mu q = 0, \tag{12}$$

$$\frac{D_j}{D_1}q - U_j - z_j \leq 0 \quad for\ j = 1, 2, \ldots, m$$
$$\lambda_j \geq 0, \quad for\ j = 1, 2, \ldots, m,$$
$$\mu \geq 0$$

Based on the mathematical form of the objective function, q should be positive otherwise the total cost per unit of time will go to infinity. Therefore, $\mu = 0$.

Next, let the set $J^+(n)$ be defined as the set of retailers with positive Lagrangian multipliers. Based on (10), the set $J^+(n)$ is also the set of retailers with positive over-stock quantities. The over-stock quantity for each retailer j belonging to the set $J^+(n)$ is given by:

$$z_j = \frac{D_j}{D_1}q - U_j \text{ for every } j \in J^+(n) \tag{13}$$

Moreover, using (10) we have

$$\frac{\lambda_jD_j}{D_1} = \frac{\pi_j}{q}z_j \tag{14}$$

After substituting (13) and (14) into (9) and carrying out some algebraic simplification, the latter equation gives:

$$q^2\left(n\right) = \frac{2K(n)+\sum_{j\epsilon J^+}\frac{\pi_jU_j^2}{D_j}}{G(n)+\sum_{j\epsilon J^+}\pi_jD_j}D_1^2 \tag{15}$$

Next, note that the requirement that λ_j be positive for all j in the set $J^+(n)$ implies that z_j be also positive, which, based on (13), requires that

$$q^2 \geq \left(\frac{U_j}{D_j}D_1\right)^2 \qquad \text{for all } j\epsilon J^+(n) \tag{16}$$

Suppose that the retailers are sorted so that

$$\frac{U_1}{D_1} \leq \frac{U_2}{D_2} \leq \ldots \leq \frac{U_m}{D_m}. \tag{17}$$

Then, by using (15) to (16), a retailer j belongs to the set $J^+(n)$ if it satisfies

$$\frac{2K(n)+\sum_{k=1}^{j}\frac{\pi_kU_k^2}{D_k}}{G(n)+\sum_{k=1}^{j}\pi_kD_k} \geq \frac{U_j^2}{D_j^2} \tag{18}$$

Note that if $(2K(n)+\pi_1U_1^2/D_1)/(G(n)+\pi_1D_1) < U_1^2/D_1^2$, then the set $J^+(n)$ is empty corresponding to the case where all shipments are of size below the upper space limit.

Based on the above analysis, the following algorithm generates an optimal solution to VMI-EC problem for a given n. A mathematical proof to this statement is provided in the theorem below.

VMI-EC(n) Algorithm:

```
0- Renumber the retailers such that the inequalities in (17) are satisfied.
1- Find the largest value of $j$ satisfying (18), called $j(n)$.
   If (18) is not satisfied for $j = 1$, then set $j(n) = 0$.
2- If $j(n) \geq 1$, set:
```

$$q_{ec}(n) = \sqrt{\frac{2K(n) + \sum_{k=1}^{j(n)} \frac{\pi_k U_k^2}{D_k}}{G(n) + \sum_{k=1}^{j(n)} \pi_k D_k} D_1} \tag{19}$$

$$z_{ec,j}(n) = \frac{D_j}{D_1} q_{ec}(n) - U_j \quad \text{for } j = 1, 2, \ldots, j(n) \tag{20}$$

$$z_{ec,j}(n) = 0 \quad \text{for } j = j(n) + 1, \ldots, m. \tag{21}$$

If $j(n) = 0$, set:

$$q_{ec}(n) = \sqrt{\frac{2K(n)}{G(n)} D_1} \tag{22}$$

$$z_{ec,j}(n) = 0 \quad \text{for } j = 1, \ldots, m. \tag{23}$$

3- Compute $TOC(n, q_{ec}(n), z_{ec,j}(n))$ using (3)

Theorem 1: Algorithm VMI-EC(n) provides an optimal solution to VMI-EC optimization problem for a given n.

Proof: One needs to prove that for any retailer $j > j(n)$, condition (18) is not satisfied and, consequently, $z_j = 0$. In this case, the solution generated by the algorithm satisfies all KKT condition, and therefore it is optimal. To that end, assume that:

$$\frac{2K(n) + \sum_{k=1}^{j} \frac{\pi_k U_k^2}{D_k}}{G(n) + \sum_{k=1}^{j} \pi_k D_k} < \frac{U_j^2}{D_j^2} \quad \text{for a given retailer } j \geq j(n) + 1. \tag{24}$$

To simplify the notation, let:

$$NUM\,(j) = 2K(n) + \sum_{k=1}^{j} \frac{\pi_k U_k^2}{D_k},$$

$$DEN\,(j) = G(n) + \sum_{k=1}^{j} \pi_k D_k,$$

$num\,(j) = \frac{\pi_j U_j^2}{D_j}$, and $den\,(j) = \pi_j D_j$. Then,

$$\begin{aligned} &\frac{NUM\,(j+1)}{DEN\,(j+1)} - \frac{num\,(j+1)}{den\,(j+1)} \\ &= \frac{NUM\,(j) + num\,(j+1)}{DEN\,(j) + den\,(j+1)} - \frac{num\,(j+1)}{den\,(j+1)} \\ &= \frac{NUM\,(j)\,den\,(j+1) - DEN\,(j)\,num\,(j+1)}{[DEN\,(j) + den\,(j+1)]den\,(j+1)}. \end{aligned}$$

The numerator of the last equation can be rewritten as

$$DEN\,(j)\,den\,(j+1)\left[\frac{NUM\,(j)}{DEN\,(j)} - \frac{num\,(j+1)}{den\,(j+1)}\right].$$

Next, given (21), we have

$$\frac{NUM\,(j)}{DEN\,(j)} < \frac{num\,(j)}{den\,(j)}.$$

Therefore,

$$\frac{NUM\,(j)}{DEN\,(j)} < \frac{num\,(j+1)}{den\,(j+1)} \text{ given that } \frac{num\,(j)}{den\,(j)} < \frac{num\,(j+1)}{den\,(j+1)}.$$

Therefore,

$$\frac{NUM\,(j+1)}{DEN\,(j+1)} < \frac{num\,(j+1)}{den\,(j+1)}$$

implying that retailer $j+1$ does not belong to the set $J^+(n)$, which completes the proof.

In order to find the optimal solution to the VMI-EC optimization problem, we propose the following iterative algorithm based on the convexity of its objective function with respect to n.

VMI-EC Algorithm:

1- Find the space unconstrained optimal solution. If $\frac{D_j}{D_1} q_{1u} \leq U_j$ for all retailers, then it is optimal. Stop.

2- Set $n = 1$ and $TOC\,(n_{ec}, q_{ec}, z_{ec}) = \infty$.

3- Find $TOC(n, q_{ec}(n), z_{ec,j}(n))$ using *VMI-EC*(n) *Algorithm.*

4- If $TOC(n, q_{ec}(n), z_{ec,j}(n)) < TOC\,(n_{ec}, q_{ec}, z_{ec})$, then set $n_{ec} = n, q_{ec} = q_{ec}\,(n), z_{ec,j} = z_{ec,j}\,(n)$, and $TOC\,(n_{ec}, q_{ec}, z_{ec}) = TOC(n, q_{ec}(n), z_{ec,j}(n))$ $n \leftarrow n + 1$ and go to step 3.

5- If $TOC(n, q_{ec}(n), z_{ec,j}(n)) \geq TOC\,(n_{ec}, q_{ec}, z_{ec})$, stop. The optimal number of replenishments, ordering quantity of the first retailer, and total costs of the integrated SC are n_{ec}, q_{ec}, and $TOC\,(n_{ec}, q_{ec}, z_{ec})$, respectively.

The total amount of carbon footprint emitted by the activities of the vendor and the m retailers assumes its maximum value upon following the optimal operational policy that minimizes the total SC costs, which is given by:

$$\begin{aligned} E(n_{ec}, q_{ec}) &= E_{ec}^{max}(n_{ec}, q_{ec}) \\ &= \frac{\hat{K}(n_{ec})}{q_{ec}} D_1 + 0.5 \frac{\hat{G}(n_{ec})}{D_1} q_{ec} \end{aligned} \tag{25}$$

where

$$\hat{K}(n) = \frac{\hat{A}_v}{n} + \hat{A}, \quad \hat{G}(n) = nD\hat{h}_v + \hat{H},$$
$$\text{and } \hat{H} = \sum_{j=1}^{m} \left(\hat{h}_j - \hat{h}_v\right) D_j. \tag{26}$$

Numerical Example: Consider the VMI-EC problem with one vendor and 5 retailers having the model parameters shown in Table 1, along with $A_v = 300$, $h_v = 0.75$, $\hat{A}_v = 50$ and $\hat{h}_v = 4$.

The space unconstrained solution ($n_u = 1$, $q_u = 327.7$) is infeasible. Table 2 depicts the results of the different iterations of *VMI-EC algorithm* and Figure 1 shows the variation of the total operational costs as a function of the number of shipments.

It can be seen in Table 2 that $n_{ec} = 4$ and $q_{ec} = q_1 = 90.56$. The lot sizes for the remaining retailers can be determined using (1). The third column shows the over-stock quantities which are computed using (20) and (21). The resulting total cost per unit of time is 7020.02 and the maximum carbon emissions by the integrated SC, $E_{ec}^{max} = 7347.47$ tons. It can also be noticed form the table that the shipment size to the first retailer is decreasing in n, which is an expected result as

TABLE 1. Example data.

Retailer	D_j	A_j	h_j	π_j	U_j	$\hat{A}_j$	$\hat{h}_j$
1	1200	30	8.5	4.5	60	18	5
2	800	25	9	3.5	50	16	5
3	2300	45	7.5	4	170	25	4.5
4	1800	35	8	4	140	20	5
5	3000	60	7	2.5	240	30	4.5

TABLE 2. Results of the different iterations of *VMI-EC ALGORITHM*.

n	$q_{ec}(n)$	$z_{ec}(n)$	TOC	$\mathrm{E}_{\mathrm{ec}}^{\max}$
1	127.67	67.67, 35.11, 74.70, 51.50, 79.17	8582.9	3773.9
2	106.37	46.37, 20.91, 33.87, 19.55, 25.95	7374.72	5024.13
3	96.98	36.98, 14.65, 15.87, 5.46, 2.34	7073.11	6228.03
4	**90.56**	**30.56, 10.38, 3.58, 0, 0**	**7020.02**	**7347.47**
5	85.39	25.39, 6.93, 0, 0, 0	7065.65	8377.15
6	81.19	21.19, 4.12, 0, 0, 0	7158.28	9340.83
7	77.73	17.73, 1.82, 0, 0, 0	7276.28	10254.34
8	74.78	14.78, 0, 0, 0, 0	7408.75	11123.49
9	72.15	12.15, 0, 0, 0, 0	7549.53	11947.96

the vendor would deliver smaller batches for more frequent orders during his/her delivery cycle. On the other hand, the carbon footprint generated by the SC increases in n. This is also an expected outcome which is attributed to the increase in the number of shipments.

B. ENVIRONMENTAL VMI MODEL

In the environmental model, the vendor and retailers coordinate their operations through a VMI partnership in order to reduce their total carbon footprint. Therefore, the VMI model with environmental considerations can be mathematically stated as follows:

$$\textbf{VMI-EV:}\quad E_{ev}^{min} = Min\ E(n,q) = \frac{\hat{K}(n)}{q}D_1 + 0.5\frac{\hat{G}(n)}{D_1}q$$
$$s.t.\ q \geq 0 \text{ and } n \text{ integer} \tag{27}$$

where $\hat{K}(n)$ and $\hat{G}(n)$ are given by (26).

The objective function of VMI-EV has the same functional form as $TOC_u(n,q)$ given in (5). Therefore, the size of the first shipment and the number of shipments that minimize the total carbon emissions are given by:

$$q_{ev} = \sqrt{\frac{2\hat{K}(n_{ev})}{\hat{G}(n_{ev})}D_1}, \tag{28}$$

and

$$n_{ev} = \left\lceil 0.5\left(-1+\sqrt{1+4\frac{\hat{A}_v\hat{H}}{\hat{A}\hat{h}_vD}}\right)\right\rceil. \tag{29}$$

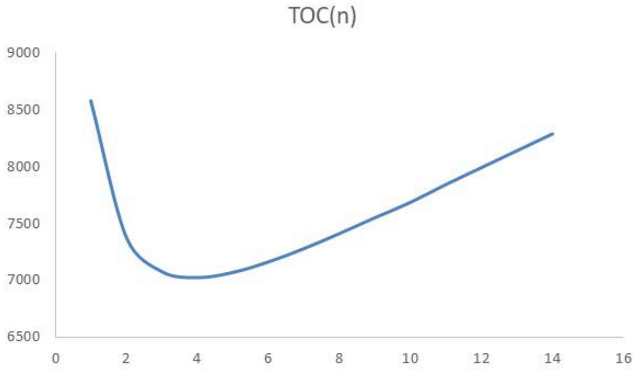


FIGURE 1. Variation of total operational cost as function of *n*.

Numerical Example: Consider the same illustrative example of Section 3.1. The operating policy that minimizes the SC carbon footprint calls for sending only one shipment to each retailer ($n_{ev} = 1$) with a lot size of $q_{ev} = 103.38$ to the first retailer. The resulting minimum carbon footprint is $E_{ev}^{min} = 3691.38$ tons.

C. ECONOMIC VMI MODEL UNDER CARBON CAP CONSTRAINT

The goal of the SC members collaborating under VMI partnership is now to determine the least-cost operational policy subject to a carbon cap constraint on the total emissions. In such situation, the total carbon emitted by the vendor and retailers should not exceed an agreed upon/stipulated level of C tons. The carbon constrained economic VMI model is then formulated as:

$$\textbf{VMI-CC:}\quad Min\ TOC(n,q,z_j) = \frac{K(n)}{q}D_1 + 0.5\frac{G(n)}{D_1}q + 0.5\frac{D_1}{q}\sum_{j=1}^{m}\frac{\pi_j}{D_j}z_j^2$$
$$s.t\ E(n,q) = \frac{\hat{K}(n)}{q}D_1 + 0.5\frac{\hat{G}(n)}{D_1}q \leq C \tag{30}$$
$$\frac{D_j}{D_1}q - U_j - z_j \leq 0 \text{ for } j = 1,2,\ldots,m$$
$$q \geq 0,\ \text{ and } n \text{ integer}$$

Given that E_{ev}^{min} is the minimum total carbon emitted by the vendor and the buyers, then the VMI-CC optimization problem is feasible only when

$$E(q_{ev}) = E_{ev}^{min} \leq C.$$

In what follows, it is assumed that the carbon cap satisfies this condition to ensure the feasibility of VMI-CC problem. In addition, for a given n value, if $C > E(n, q_{ec}(n)) = E_{ec}^{max}(n, q(n))$, then $q_{ec}(n)$, given by (19), is feasible and optimal replenishment quantity to VMI-CC.

It is noted that VMI-CC has a unique optimal solution given the convexity of its objective function and constraints. For a given n, let $\theta(n)$ represent the Lagrangian multiplier associated with the carbon cap constraint. Then, following the same analysis as in Section 3.1, it can be shown that:

$$q^2\left(n,\theta(n)\right) = \frac{2\left(K(n)+\theta(n)\hat{K}(n)\right)+\sum\limits_{j\in J^+(\theta(n))}\frac{\pi_j U_j^2}{D_j}}{G(n)+\theta(n)\hat{G}(n)+\sum\limits_{j\in J^+(\theta(n))}\pi_j D_j}D_1^2 \tag{31}$$

$$g\left(n,\theta(n)\right) = \frac{\hat{K}(n)D_1}{q(\theta(n))}+0.5\frac{q\left(\theta\left(n\right)\right)\hat{G}(n)}{D_1}-C=0 \tag{32}$$

where $J^+(\theta\left(n\right))$ is the set of retailers satisfying

$$q^2\left(n,\theta(n)\right) \geq U_j^2 \big/ D_j^2 \tag{33}$$

$$z_j\left(n,\theta\left(n\right)\right) = D_j\frac{q\left(n,\theta\left(n\right)\right)}{D_1}-U_j \quad \text{for } j=1,2,\ldots,j\left(\theta(n)\right) \tag{34}$$

$$z_j\left(n,\theta\right)=0 \quad \text{for } j>j\left(\theta(n)\right) \tag{35}$$

It can be observed from (31) that $q\left(n,\infty\right) = q_{ev}$ and $g\left(n,\infty\right) = E_{ev}^{min}-C < 0$ for large θ. In addition, when $\theta(n)$ is equal to zero, then $q\left(n,0\right) = q_{ec}\left(n\right)$. In this case, if $g\left(n,0\right)=E_{ec}^{max}\left(n,q_{ec}\left(n\right)\right)-C<0$, then $q_{ec}\left(n\right)$ is feasible and optimal. On the other hand, if $E_{ec}^{max}\left(n,q_{ec}\left(n\right)\right)-C>0$, then by the uniqueness of the optimal solution to VMI-CC, then there exists a unique solution to equation (32). It is difficult to prove analytically that $g\left(n,\theta(n)\right)$ is a strictly decreasing function of $\theta(n)$ because of the set $J^+\left(\theta(n)\right)$ of retailers with overstock. However, based on several numerical examples, we observed that the function $g\left(n,\theta(n)\right)$ strictly decreases when $\theta(n)$ increases as shown in Figure 2. Solving the VMI-CC optimization problem using Lagrangian multiplier method could be time consuming as the *VMI-EC algorithm* in Section 3.1 has to be used for each value $\theta(n)$ to find the optimal one satisfying (32). Instead of this numerical method, we propose in Theorems 2 and 3 below closed form equations to find the optimal replenishment quantity, $q_{ec}\left(n\right)$, and optimal Lagrangian multiplier, $\theta(n)$.

Theorem 2: The optimal replenishment quantity, $q_{cc}(n)$, to VMI-CC problem is given by:

$$q_{cc}\left(n\right) = \begin{cases} q_{ec}\left(n\right) & \textit{if } q_{ccl}\left(n\right)\leq q_{ec}\left(n\right)\leq q_{ccu}(n) \\ q_{ccl}\left(n\right)=\dfrac{C-\sqrt{C^2-2\hat{K}(n)\hat{G}(n)}}{\hat{G}}D_1 & \textit{if } q_{ec}(n)\leq q_{ccl}(n) \\ q_{ccu}(n)=\dfrac{C+\sqrt{C^2-2\hat{K}(n)\hat{G}(n)}}{\hat{G}}D_1 & \textit{if } q_{ec}(n)\geq q_{ccu}(n) \end{cases} \tag{36}$$

Proof: Equation (30) can be rewritten as:

$$q^2\hat{G}(n)-2CD_1q+2\hat{K}(n)D_1^2\leq 0$$

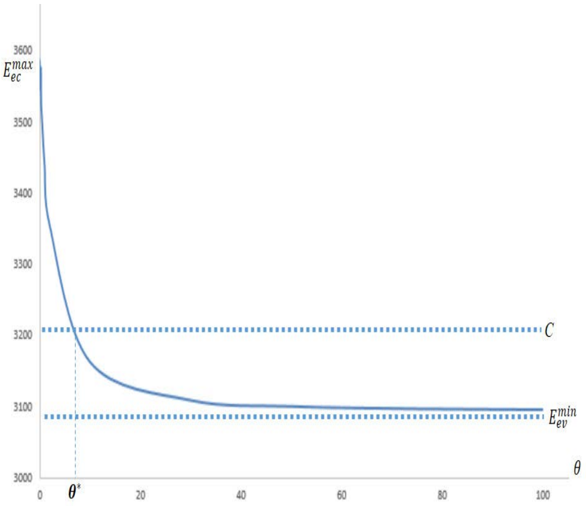


FIGURE 2. Variations of carbon emissions as function of θ.

Given the quadratic form of the left hand side of the last inequality, the carbon constraint (30) is satisfied as long as $q_{ccl}(n) \leq q \leq q_{ccu}(n)$. It then follows from the convexity of the VMI-CC objective function that the optimal replenishment quantity to VMI-CC, $q_{cc}(n)$, is given by (36). As it can be observed from (37), the lower and upper bounds for the batch size cannot be calculated when $C^2 < 2\hat{K}(n)\hat{G}\left(n\right)$. Therefore, the largest number of shipments that can be delivered to the retailers is the largest number, n^{max}, such that $C^2 \geq 2\hat{K}(n)\hat{G}(n)$. In addition, the VMI-CC optimization problem is feasible when $C^2 \geq 2\hat{K}(1)\hat{G}(1)$.

The optimal value of the Lagrangian multiplier is important to assess the cost and carbon footprint performance of the carbon cap regulatory policy in comparison to the carbon tax policy. Finding such value numerically using the *VMI-EC algorithm* in Section 3.1 could be computationally expensive. Instead, we propose a closed form equation in Theorem 3 to find the optimal Lagrangian multiplier. The results of the following proposition, which can be easily shown with some basic algebra, will be useful in proving Theorem 3.

Proposition: For any real positive numbers a, b, c, and d, if $a\big/b < c\big/d$, then $a\big/b < (a+c)\big/(b+d) < c\big/d$.

Theorem 3: Let $q = q_{cc}\left(n\right)$ given by (36). Then, the Lagrangian multiplier $\theta(n)$ is given by:

$$\theta\left(n\right) = -\frac{G\left(n\right)+\sum\limits_{j\in J^+(\theta(n))}\pi_j D_j}{\hat{G}\left(n\right)}\frac{q_{ec}^2\left(\theta\left(n\right)\right)-q^2}{q_{ev}^2\left(n\right)-q^2} \tag{37}$$

where

$$q_{ec}^2\left(\theta\left(n\right)\right) = \frac{2K\left(n\right)+\sum_{j\in J^+(\theta(n))}\frac{\pi_j U_j^2}{D_j}}{G(n)+\sum_{j\in J^+(\theta(n))}\pi_j D_j}D_1^2 \tag{38}$$

$$q_{ev}^2\left(n\right) = \frac{2\hat{K}(n)}{\hat{G}(n)}D_1^2 \tag{39}$$

Proof: First, when $q = q_{ec}\left(n\right)$, then $\theta\left(n\right) = 0$ as $q_{ec}\left(n\right)$ is feasible. Assume now that it is not the case. Then, knowing q, the set $J^{+}(\theta(n))$ satisfying (16) can be determined by finding the retailers with overstock quantities. That is, the set of retailers such that $z_j = \frac{D_j}{D_1}q - U_j > 0$.

Next, solving the following equation for $\theta(n)$

$$\frac{2\left(K(n)+\theta(n)\hat{K}(n)\right)+\sum_{j\epsilon J^{+}(\theta(n))}\frac{\pi_j U_j^2}{D_j}}{G(n)+\theta(n)\hat{G}(n)+\sum_{j\epsilon J^{+}(\theta(n))}\pi_j D_j}D_1^2 = q^2,$$

it can be shown after some algebraic manipulations that $\theta(n)$ is given by (37). It remains to show that $\theta(n) > 0$, using the above proposition.

Let $a = (2K(n) + \sum_{j\in J^{+}(\theta(n))}\frac{\pi_j U_j^2}{D_j})D_1^2$, $b = G(n) + \sum_{j\in J^{+}(\theta(n))}\pi_j D_j$, $c = 2\hat{K}(n)D_1^2$, and $d = \hat{G}(n)$

$$\text{If}\left(\frac{a}{b} = \frac{2K(n)+\sum_{j\in J^{+}(\theta(n))}\frac{\pi_j U_j^2}{D_j}}{G(n)+\sum_{j\in J^{+}(\theta(n))}\pi_j D_j}D_1^2 = q_{ec}^2(\theta(n))\right)$$

$$< \left(\frac{c}{d} = \frac{\theta(n)}{\theta(n)}\frac{2\hat{K}(n)}{\hat{G}(n)}D_1^2 = q_{ev}^2(n)\right),$$

then, $q_{ec}^2(\theta(n))$

$$< \left(\frac{2\left(K(n)+\theta(n)\hat{K}(n)\right)+\sum_{j\epsilon J^{+}(\theta(n))}\frac{\pi_j U_j^2}{D_j}}{G(n)+\theta(n)\hat{G}(n)+\sum_{j\epsilon J^{+}(\theta(n))}\pi_j D_j}D_1^2 = q^2\right)$$

$$< q_{ev}^2(n).$$

Therefore, $\frac{q_{ec}^2(\theta(n))-q^2}{q_{ev}^2(n)-q^2} < 0$ and $\theta(n) > 0$.

Similarly, if

$$\left(\frac{a}{b} = \frac{2K(n)+\sum_{j\in J^{+}(\theta(n))}\frac{\pi_j U_j^2}{D_j}}{G(n)+\sum_{j\in J^{+}(\theta(n))}\pi_j D_j}D_1^2 = q_{ec}^2(\theta(n))\right)$$

$$> \left(\frac{c}{d} = \frac{\theta(n)}{\theta(n)}\frac{2\hat{K}(n)}{\hat{G}(n)}D_1^2 = q_{ev}^2(n)\right)$$

Then, $q_{ev}^2(n) < q^2 < q_{ec}^2(\theta(n))$, which implies that $\frac{q_{ec}^2(\theta(n))-q^2}{q_{ev}^2(n)-q^2} < 0$ and $\theta(n) > 0$.

Based on the above, the following algorithm can be used to find the optimal operating policy for VMI-CC optimization problem.

VMI-CC Algorithm:

```
0- If $C > E_{ec}^{max}(n_{ec}, q_{ec}(n_{ec}))$, then $q_{ec}(n_{ec})$ is feasible and optimal. Stop.
1- Find $n^{max}$, the largest $n$ such that $C^2 \geq 2\hat{K}(n)\hat{G}(n)$.
2- Set $n = 1$ and $TOC(n_{cc}, q_{cc}, z_{cc}) = \infty$.
3- Find $q_{ec}(n)$ using VMI-EC algorithm, $q_{cc}(n)$ using (36), set $J^{+}(\theta(n))$, $\theta(n)$ using (37), and
   $TOC(n, q_{cc}(n), z_{ec,j}(n))$.
4- If $TOC(n, q_{cc}(n), z_{ec,j}(n)) < TOC(n_{cc}, q_{cc}, z_{cc})$, then set
   $n_{cc} = n, q_{cc} = q_{cc}(n), \theta = \theta(n)$,
   $z_{ec,j} = z_{cc,j}(n)$,
   $TOC(n_{cc}, q_{cc}, z_{cc}) = TOC(n, q_{cc}(n), z_{cc,j}(n))$
   $n \leftarrow n + 1$.
5- If $n \leq n^{max}$, go to step 3. Otherwise, set $n \leftarrow n - 1$ and go to step 7.
6- If $TOC(n, q_{cc}(n), z_{cc,j}(n)) \geq TOC(n_{cc}, q_{cc}, z_{cc})$, go to step 7.
7- The optimal number of replenishments, ordering quantity of the first retailer, Lagrangian multiplier, and total costs of the integrated SC are $n_{cc}, q_{cc}, \theta$, and $TOC(n_{cc}, q_{cc}, z_{cc})$, respectively.
```

Numerical Example: Let's consider the same illustrative example presented in Section 3.1. Table 3 shows the results of using *VMI-CC algorithm* for different carbon cap values. The optimal solution for each carbon cap is shown in boldface.

As can be noted from Table 3, the total operational cost decreases as the carbon cap increases. When the carbon emission is limited to 5400 tons, the optimal operating policy is to send 2 shipments to each retailer with the first one receiving 106.37 units at a total cost of 7374.72. In this case, the carbon emission is reduced from 7347.47 tons to 5024 tons representing a 46.24% reduction at the expense of an increase in the total operating cost of only 5% (an increase from 70720.02 to 7374.72). Therefore, the minor increase in total operational cost is far outweighed by the reduction in carbon emissions.

Given that carbon cap is an exogenous parameter, we next assess its impact on the total operational cost and carbon emissions. To that end, different values are examined with an increment of 100 tons between the minimum (3691.38 tons) and maximum (7347.48 ton) carbon quantities that can be emitted by the SC based on the same illustrative example. The results of this sensitivity analysis are shown in Figure 3.

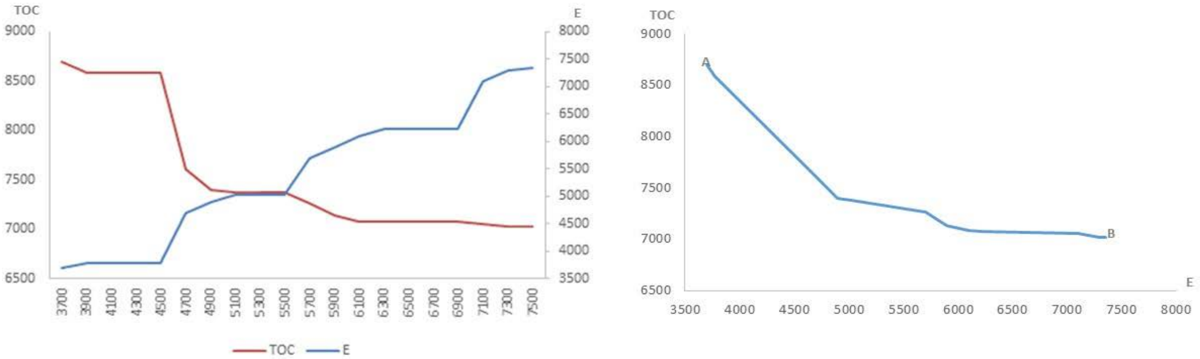
Looking at Figure 3.a, one can notice that when VMI-CC becomes less tight via higher values of the carbon cap, a stepwise reduction (increase) in the operational cost (carbon emission) is realized. Such stepwise pattern in the variation of the operational costs and carbon footprint is due to the fact that the Lagrangian multiplier remains unchanged for a range of carbon cap values. It can also be seen from Figure 3.b that curbing carbon emission can be achieved without substantially impacting the operational cost. In fact, by moving from the operational policy A to operational policy B, the carbon emission is reduced by 99% while the total operational cost is increased by 23.8%. Therefore, it is possible to significantly reduce SC carbon emission without compromising the total operational cost through an adjustment of its operational policy (number and sizes of the shipments to the buyers). Such result was also observed in the literature but within different SC settings (Chen *et al.* [63] and Hariga *et al.* [23]).

D. ECONOMIC VMI MODEL UNDER CARBON CAP TAX

In this sub-section, the SC is assumed to operate under the carbon tax regulation wherein it is charged *tx* for each ton

TABLE 3. Optimal VMI-EC policies for different carbon caps.

C	n^{max}	n	$q_{ec}(n)$	$q_{ccl}(n)$	$q_{ccu}(n)$	$q_{cc}(n)$	$J^+(\theta(n))$	$q_{ev}(n)$	$q_{ec}(\theta(n))$	$\theta(n)$	$TOC(n)$
3700	1	**1**	**127.67**	**96.55**	**110.69**	**110.69**	**1,2,3,4,5**	**103.38**	**127.38**	**6.162**	**8693.77**
5400	3	1	127.67	40.85	261.60	127.67	-	-	-	0	8582.9
		2	**106.37**	**39.15**	**124.38**	**106.37**	**-**	**-**	**-**	**0**	**7374.72**
		3	96.98	52.86	59.20	52.86	∅	55.94	100.75	12.82	8044.33
6500	4	1	127.67	32.20	331.86	127.67	-	-	-	**0**	8582.9
		2	106.37	29.02	167.83	106.37	-	-	-	0	7374.72
		3	**96.98**	**29.77**	**105.12**	**96.98**	**-**	**-**	**-**	**0**	**7073.11**
		4	90.56	33.13	69.47	69.47	1	47.97	91.15	0.87	7287.11
7000	5	1	127.67	29.47	362.59	127.67	-	-	-	0	8582.9
		2	106.37	26.21	185.77	106.37	-	-	-	0	7374.72
		3	96.98	26.31	118.96	96.98	-	-	-	0	7073.11
		4	**90.56**	**27.85**	**82.64**	**82.64**	**1,2**	**47.97**	**90.74**	**0.2**	**7052.45**
		5	85.39	31.60	57.54	57.54	∅	42.65	86.86	1.47	7652.0



a. Operational cost and carbon emission as function of carbon cap level

b. Relationship between total operational cost and total carbon emission

FIGURE 3. Impact of the carbon cap on the SC operational costs and carbon emissions.

of carbon emitted. The objective function of the VMI model under carbon tax is then given by:

$$\frac{K(n)}{q}D_1 + 0.5\frac{G(n)}{D_1}q + 0.5\frac{D_1}{q}\sum_{j=1}^{m}\frac{\pi_j}{D_j}z_j^2 + tx\left(\frac{\hat{K}(n)}{q}D_1 + 0.5\frac{\hat{G}(n)}{D_1}q\right)$$

which can be rewritten as

$$TOC_{tx}\left(n, q, z_j\right) = \frac{K(n) + tx\hat{K}(n)}{q}D_1 + 0.5\frac{G(n) + tx\hat{G}(n)}{D_1}q + 0.5\frac{D_1}{q}\sum_{j=1}^{m}\frac{\pi_j}{D_j}z_j^2$$

Defining $K_{tx} = K(n) + tx\hat{K}(n)$ and $G_{tx} = G(n) + tx\hat{G}(n)$, the VMI optimization problem under carbon tax policy can be stated as

$$\textbf{VMI-TX:} \quad Min\ TOC_{tx}\left(n, q, z_j\right) = \frac{K_{tx}(n)}{q}D_1 + 0.5\frac{G_{tx}(n)}{D_1}q + 0.5\frac{D_1}{q}\sum_{j=1}^{m}\frac{\pi_j}{D_j}z_j^2$$

$$s.t. \ \frac{D_j}{D_1}q - U_j - z_j \le 0 \quad \text{for } j = 1, 2, \ldots, m$$

$$q \ge 0 \text{ and } n \text{ integer}$$

As VMI-TX has the same structure as VMI-EC, the same *VMI-EC algorithm* can be used to generate the optimal operating policy under carbon tax regulation. The following Lemma provides the relationship between the solutions to

TABLE 4. VMI-TX results under different tax values.

tx	q	n	TOC	E
1	92.57	2	7437.37	4793.85
6	110.82	1	8594.47	3700.32
11	108.1	1	8650.06	3695.07
16	106.67	1	8682.18	3693.11
21	106.11	1	8695.11	3692.63
26	105.53	1	8706.44	3692.24
31	105.29	1	8714.55	3692.00

VMI-CC and VMI-TX based on the values of the Lagrangian multiplier and the tax level.

Lemma:

1- If $tx < \theta(C)$, where $\theta(C)$ is the optimal Lagrangian multiplier of VMI-CC with C as the carbon cap, then:
  a- Carbon tax policy generates larger amount of carbon emission than the carbon cap policy.
  b- The total operational costs is smaller under carbon cap tax policy than under the carbon cap policy.
2- If $tx > \theta(C)$, where $\theta(C)$ is the optimal Lagrangian multiplier of VMI-CC with C as the carbon cap, then:
  c- Carbon tax policy generates smaller amount of carbon emission than the carbon cap policy.
  d- The total operational costs is larger under carbon cap tax policy than under the carbon cap policy.

Proof: 1.a. Given that $tx < \theta(C)$, then $g(n, tx) > g(n, \theta(C))$ where n is the optimal number shipments under a carbon cap C and the function $g(.,.)$ is given by (32) and displayed in Figure 2. Therefore, $E(n, q(n, tx)) > E(n, q(n, \theta(C))) = C$.

1.b. The proof follows from 1.a. since the carbon cap policy is generating less carbon emission than the carbon tax policy.

The proof for the second part of the lemma is similar to 1.a and 1.b.

Note that if $\theta(C) = 0$ for a given carbon cap C, then the carbon cap policy will generate more carbon emission than the carbon tax policy but will result in less total operational cost.

Numerical Example: Consider again the same illustrative example. Table 4 reports the operational policy of VMI-TX under different tax values with an increment of 5. Note that the carbon tax policy converges to the VMI-EV policy as the tax rate increases. This can easily be shown by replacing $\theta(n)$ by tx in equation (31) and letting tx tend to infinity.

IV. CONCLUSION AND FUTURE RESEARCH DIRECTIONS

The integration of sustainability dimensions into the various aspects of SC operations has emerged as an inevitable necessity in order to align with the stringent governmental regulations put in place and the international climate change initiatives seeking to combat global warming. To that end, this work embraces the widely implemented VMI collaborative scheme in the context of a SC comprising a single vendor and multi retailers while accounting for the carbon footprint generated by all parties involved. For a better resemblance to reality, an upper bound on the stock levels at the retailers' premises is explicitly noted in the contractual agreement with the vendor incurring a penalty upon exceeding those limits. While pure economic and environmental mathematical models are presented, integrated models that incorporate both performance measures under the carbon cap and carbon tax policies are also devised. The results clearly highlight the impact of these two policies on the chain-wide total cost and carbon footprint and suggest different operational strategies, depending on the values set for the tax and the cap. It is noted that via minor adjustments in the adopted operational and lot sizing strategy, significant reductions in carbon emissions are attainable at the expense of a minor increase in the total cost. Furthermore, this tool suggests a novel approach that utilizes Lagrange multiplier to aid the policy makers with the best best-fit carbon regulatory policy to adopt.

An interesting venue for future research is to relax the assumption of equal replenishment cycles. Then, the problem to be addressed in this case is more challenging as it necessitates the introduction of additional integer variables pertaining to the replenishment frequency for each retailer. Furthermore, extending the same modeling and solution approach to the combined VMI and CS (VMI-CS) framework poses as another promising future research direction. Another interesting extension is to consider the demand at the retailers as being stochastic rather than deterministic.

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