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An Intelligent Heap-Based Technique With Enhanced Discriminatory Attribute for Large-Scale Combined Heat and Power Economic Dispatch

ABDULLAH M. SHAHEEN[1], ABDALLAH M. ELSAYED[2], (Member, IEEE), EHAB E. ELATTAR[3], (Senior Member, IEEE), RAGAB A. EL-SEHIEMY[4], (Senior Member, IEEE), AND AHMED R. GINIDI[1]

[1]Electrical Engineering Department, Faculty of Engineering, Suez University, Suez 43533, Egypt
[2]Electrical Engineering Department, Faculty of Engineering, Damietta University, Damietta 34517, Egypt
[3]Electrical Engineering Department, College of Engineering, Taif University, Taif 21944, Saudi Arabia
[4]Electrical Engineering Department, Faculty of Engineering, Kafrelsheikh University, Kafrelsheikh 33516, Egypt

Corresponding author: Ragab A. El-Sehiemy (elsehiemy@eng.kfs.edu.eg)

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ABSTRACT The basic goal of Combined Heat and Power Economic Dispatch (CHPED) is to find the best value for heat obtained from heat generators, power obtained from power generators, and both power and heat obtained from co-generators such that fuel costs are kept minimum while heat and power demands and constraints are met precisely. Based on enhanced discriminatory attribute, a newly Improved version of the Heap-based Technique (IHT) is to increase the searching capacity around the leader position and avoid trapping in a local optimum. Additionally, an adaptive parameter is used linearly to half of the iteration to select an effective operation for creating the new solutions. On 25 benchmark optimizing functions of unimodal or multimodal properties, the efficacy of the proposed IHT in contrast to the traditional HT is tested. Additionally, the proposed IHT in contrast to the traditional HT are employed for CHPED with small scale (seven units), medium scale (twenty-four) and two large-scale (eighty-four and ninety-six) systems with consideration of valve point loading and transmission losses constraints. According to comparisons of results obtained by the IHT with existing approaches, it is shown that the proposed IHT is particularly effective and resilient for finding optimal solutions for the CHPED.

INDEX TERMS Heap-based technique, enhanced discriminatory attribute, combined heat and power, benchmark functions.

I. INTRODUCTION

The development of the management patterns and efficient energy consumption has been promoted in response to the rising prominence of environmental pollution and the energy crisis. Integrated energy systems, in which various energy sectors are incorporated to achieve energy complementation and cascade utilization, have received a lot of attention in recent years. To meet various demands, combined heat and power (CHP) plants are used as the basic and core energy conversion sector in a variety of integrated energy systems. They can supply both electricity and heat energy at the same time, where it provides recycling and utilizing waste heat.

Moreover, the CHPED challenge necessitates the dispatch of units to meet the heat and power demands, while achieving the goal of lowering system operation costs and satisfying system constraints. The rapid expansion of today's society has resulted in a massive increase in load demand for economically reliable power energy. The CHPED issue, under these conditions, plays a pivotal role in the modern power system's operation. Because the complexity and scope of the CHPED problem are unavoidable, tackling large-scale CHPED has become a challenging task. Deterministic methods have been employed, in the early years, to obtain minimum system

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cost of solving the CHPED problem including branch and bound algorithms (BB) [1], dual and quadratic programming [2], benders decomposition (BD) [3], and Lagrangian relaxation (LR) [4]. Nevertheless, due to valve-point effects, these methods have had difficulty in handling non-convex fuel cost functions of the CHPED problem. Metaheuristic algorithms (MHAs) have been demonstrated later to solve the CHPED problem which include particle swarm optimization (PSO) [5], [6], genetic algorithm (GA) [7], and differential evolution (DE) [8], [9]. Because of their ability to deal with the non-linear and non-convex CHPED problem, these MHAs have become highly popular. Accordingly, to address the CHPED issue, a number of MHAs have been elaborated. These methods are applied to CHPED systems of various scales, and the number of units have been categorized into three groups: small-scale instances are with less than 10 units, medium-scale instances are with 10 to 50 units, and large-scale instances are with more than 50 units.

Genetic Algorithm (GA) may be thought of as a broad searching technique, optimization tool relying on Darwinian concepts of evolutionary biology, reproducing, and ''survivability of the strongest'' [10]. GA keeps a population of possible solutions and updates it on a regular basis. At each stage, the GA chooses people from the present population to be parents and utilizes them to generate offspring for the following generations. Generally, the fittest people in any community reproduced and survived to the following generation, therefore enhancing subsequent generations. Inadequate people, on the other hand, can live and reproduce by random. Particle swarm optimizer (PSO) is motivated by the capacity of bird flocks, groups of fishes, and animals to adjust to local surroundings, discover abundant food sources, and escape predators via the use of information exchange. The PSO approach was developed of a social cognition investigation into the concept of collaborative intelligence in natural groups [11]. Through PSO, a collection of completely random individuals adapts in the searching design region toward optimal solution across a number of iterations depending on a significant quantity of space data gathered and exchanged by all individuals in the swarm. PSO and GA are analogous where they are both population-based searching algorithms that seek the best solution by adjusting iterations. Because the two techniques are designed to come up with a solution to a particular target function yet use various tactics and computing effort, comparing their efficiency is adequate.

The Heap-based technique (HT), which was recently published [12], is motivated by the hierarchical structure of organisations. This may be observed when a group of people works together to achieve a mission and organize themselves in a hierarchy, which is known as a corporate rank hierarchy (CRH). Therefore, the notion of CRH is to arrange searching individuals in a hierarchy depending on the objective score, and the heap data architecture is used to represent this idea. HT is built on three basic principles. The first component is teamwork between the assistants and their respective employer. The communication among coworkers is the second component. The third component is employee self-contribution. The HT has been effectively employed for several engineering optimization problems such as the optimal power flow [13], photovoltaic cell parameter estimation [14], distributed generation allocations in power systems [15], [16], economic dispatch with N-1 Unit outages [17].

DE illustrated by Storn and Price in 1995 [18] is considered as one of the population-based MHA. DE has been effectively applied to a variety of real-world issues, including the CHPED problem, because to its few control parameters and ease of implementation. In [19], a canonical coordinates method (CCM) optimization with improving the searching process has been applied to CHPED but with small number of constraints and small system application. In [20], one small CHPED instance with 7 units has been solved using the original DE, while [9] presented an improved DE with Gaussian mutation (DEGM) for 4 units CHPED problem. In [21], stochastic fractal search (SFS) algorithm has been used to solve the bi-objective CHPED problem with many local minima and bounded feasible operating regions. However, only small unit systems have been considered in that article. In [22], a social group entropy (SGE) has been applied to CHPED with solar and wind power uncertainty. However, small unit system has been considered. However, these methods have had good results when applied on numerous small-scale (less than 10) CHPED problems only.

There are numerous typical instances as following. The teaching learning based optimization (TLBO) has been illustrated in [23] for solving 7, 24 and 48 unit systems of CHPED. Grey wolf optimization (GWO) has been applied in [24] to solve 4, 7, 11, 24 and 48 unit systems of CHPED problems with new limitations such as as ramp-rate limits, power losses, and spinning reserve constraints. A real coded genetic algorithm has been emerged with improved Mühlenbein mutation (RCGA-IMM) in [25] for solving 4, 5, 7, and 24 unit systems of CHPED with consideration of power losses. Particle swarm optimization has been combined with time varying acceleration coefficients (TVAC-PSO) in [26] to tackle small and medium-scale CHPED system with 4, 5, 7, 12 and 48-unit system. A novel Kho-Kho optimization has been manifested in [27] for 4, 5, 7, 24 and 48 unit systems of CHPED with consideration of environmental emissions and power losses. In [28], an upgraded whale optimizer was used for the economic dispatch optimization issue, and an adaptive exploratory hunting strategy was developed to improve whale swarm populations variety. In [29], an enhanced version of differential evolution with an adaptive Gaussian–Cauchy mutation was used to solve a large-scale CHPED task. In this work, a constraint repair approach is also used to cope with complicated operational restrictions.

On the large-scale CHPED problem, only a few MHAs have been utilized. For instance, in [30], the CHPED problem has been solved using the crisscross optimization approach. The whale optimization approach (WOA) has been illustrated to solve a large-scale CHPED system with 24 units, 84 units, and 96 units as depicted in [31]. Additionally, in [32], a novel multi-player harmony search method (MPHS) has been characterized to deal the large-scale CHPED problem with 24-unit and 84-unit system.

Numerous techniques for dealing with CHPED constraints have been presented and adopted to operate well on the CHPED problem, including penalty function and constraint repair. However, the large-scale CHPED problem requires immediate attention and represents a significant challenge. As a result, the goal of this study is to develop a robust approach that can handle the small, medium, large-scale CHPED problem. Thus, an IHT is proposed to increase the performance of the Heap-based technique (HT) which was recently published [12]. The HT is based on the institution hierarchy, such as the institution rank hierarchy (IRH), in which a group of people works together to achieve a common goal by arranging themselves in a hierarchy based on their fitness in a hierarchy. The performance of HT is improved by using an enhanced discriminatory attribute to strengthen the searching around the leader position in order to prevent becoming locked in a local optimum and improve its global search capabilities. An assessment is developed to illustrate the quality of the traditional HT and the proposed IHT. The following are the main contributions of this article:

- An improved meta-heuristic IHT is proposed for the first time.
- A novel effective exploitation feature is demonstrated for HT.
- The proposed IHT is tested on 25 benchmark optimizing functions with either unimodal or multimodal properties. The comparison with the traditional HT clearly validates the proposed IHT.
- The proposed IHT is implemented on small scale, medium scale and two large-scale systems with consideration of valve point loading and transmission losses constraints.
- The proposed IHT reveals better performance compared with the traditional HT and most reported approaches in addressing the CHPED.

II. PROPOSED IHT WITH ENHANCED DISCRIMINATORY ATTRIBUTE

A. CONVENTIONAL HT

HT is based on the institution hierarchy [12], in which a group works toward a common objective by arranging themselves in a hierarchy, such as the IRH, to build search agents utilizing the heap data structure in line with their fitness in a hierarchy. The HT is made up of three aspects: the first aspect characterizes the interaction between subordinates and their immediate bosses, whereas the second and third aspect characterize the interaction between colleagues and the employees' self-contribution, respectively [33].

The IRH is modelled using a heap data structure, which is comparable to a tree-shaped data structure, with the whole IRH representing the population and a heap node representing the search agent. Furthermore, the master of the heap node represents the search agent's fitness, whilst the value of the heap node is shown by the search agent's population index.

The top management of the central institution creates the plans and regulations, whilst immediate supervisors supply the instruction, that must be carried out by subordinates. The conundrum can be mathematically stated in each search by changing the agent position as follows:

$$x_i^k(t+1) = B^k + \gamma(2r-1)\left|B^k - x_i^k(t)\right| \tag{1}$$

where k denotes the search agent's k^{th} vector component; t denotes the current iteration and B is the parent node.

As indicated in Eq. (2), the term $(2r-1)$ is generated at random and exemplifies the k^{th} component of the vector λ, whereas γ can be assessed as shown in Eq (3).

$$\lambda^k = 2r - 1 \tag{2}$$

$$\gamma = \left|2 - \frac{t * mod(T^{max}/C)}{T^{max}/4C}\right| \tag{3}$$

where r is a random value in the range [0,1], while T^{max} is the total number of iterations. Furthermore, the parameter (C) can control the variation in the term $\gamma(2r-1)$, completed in T^{max} iterations as follows:

$$C = T^{max}/25 \tag{4}$$

Also, the nodes depict colleagues on the same level wheras each agent position x_i could be modified based on its randomly assigned colleague (S_r):

$$x_i^k(t+1) = \begin{cases} S_r^k + \gamma\lambda^k \left|S_r^k - x_i^k(t)\right|, & f(\overrightarrow{S_r}) < f(\overrightarrow{x_i}(t)) \\ x_i^k + \gamma\lambda^k \left|S_r^k - x_i^k(t)\right|, & f(\overrightarrow{S_r}) \geq f(\overrightarrow{x_i}(t)) \end{cases} \tag{5}$$

where f represents the search agent's fitness.

In addition, an employee's self-contribution is written out as the following equation:

$$x_i^k(t+1) = x_i^k(t) \tag{6}$$

The position updating equations are combined by dividing the proportions into p_1, p_2, and p_3, the probabilities of selection may be obtained using a roulette wheel to balance exploitation and exploration. Therefore, the choice of proportion (p_1, p_2, and p_3) illustrated in (7), (8), and (9) allow a search agent to update its position using (6), (1), and (5), respectively [34].

$$p_1 = 1 - \frac{t}{T^{max}} \tag{7}$$

$$p_2 = p_1 + \frac{1-p_1}{2} \tag{8}$$

$$p_3 = p_2 + \frac{1-p_1}{2} = 1 \tag{9}$$

Thus, Eq. (10), as shown at the bottom of the next page, denotes a process for updating HT's overall position:

The key steps of the conventional HT are depicted in Figure 1.

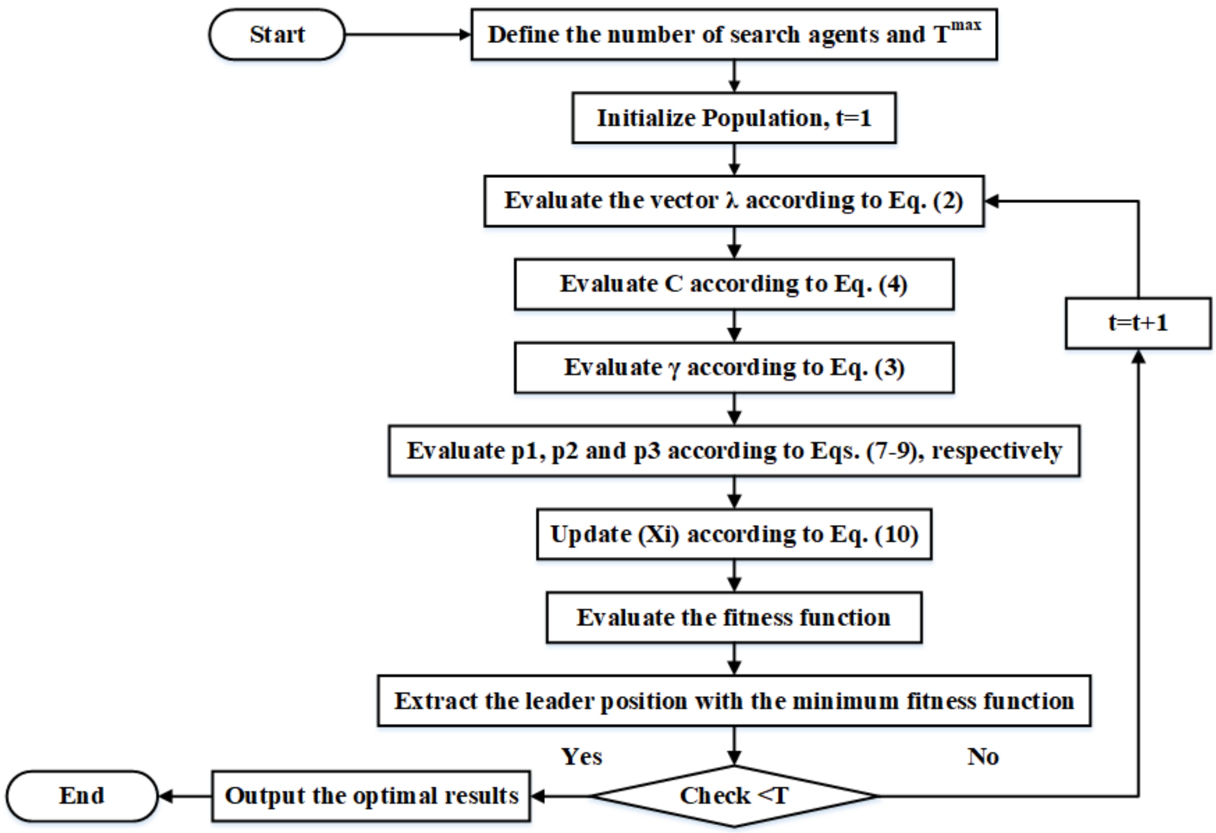


FIGURE 1. key steps of the conventional HT.

B. PROPOSED IHT WITH ENHANCED DISCRIMINATORY ATTRIBUTE

To increase the performance of the HT, two adjustments have been illustrated to improve the performance of HT. Firstly, an adaptive variable (α) which is increased linearly with increasing iterations' number until it reaches to 0.5 at the maximum number of iterations and this parameter can be expressed using this formula [35]:

$$\alpha = \frac{t}{2 * T^{\max}} \tag{11}$$

Secondly, an enhanced discriminatory attribute is merged to increase the searching process for the leader position. As a result, the conventional HT's updating process has been adjusted, and as depicted in (12), the positions of several search agents can be modified.

$$x_i^k(t+1) = x_i^k(t) + \gamma\lambda^k \left|Leader^k - x_i^k(t)\right| \tag{12}$$

where Leader refers to the position of the search agents who attain the lowest fitness value.

The proposed IHT's key steps are depicted in Figure 2. As indicated, the suggested update process of Eq. (12) is not engaged till 75 percent of the iterations have been completed.

This condition preserves the HT's great diversifying potential in exploring new potential directions. At this condition, the enhanced discriminatory attribute is merged to increase the searching process. The more increasing the iterations, the more increasing the value of the adaptive variable (α). Therefore, there are increasing in the production of the positions of new search agents in the surrounding region of the Leader position. The adaptive variable is limited to 0.5 where it doesn't allow activating the enhanced discriminatory attribute to exceed the 50% of search agents. If the adaptive variable extends to 100% at the end of the iterations, the new positions are prone to be produced around the area of the Leader

$$x_i^k(t+1) = \begin{cases} x_i^k(t), & p \le p_1 \\ B^k + \gamma\lambda^k \left|B^k - x_i^k(t)\right|, & p_1 < p < p_2 \\ S_r^k + \gamma\lambda^k \left|S_r^k - x_i^k(t)\right|, & p_2 < p \le p_3 \text{ and } f(\overrightarrow{S_r}) < f(\overrightarrow{x_i}(t)) \\ x_r^k + \gamma\lambda^k \left|S_r^k - x_i^k(t)\right| & p_2 < p \le p_3 \text{ and } f(\overrightarrow{S_r}) \ge f(\overrightarrow{x_i}(t)) \end{cases} \tag{10}$$

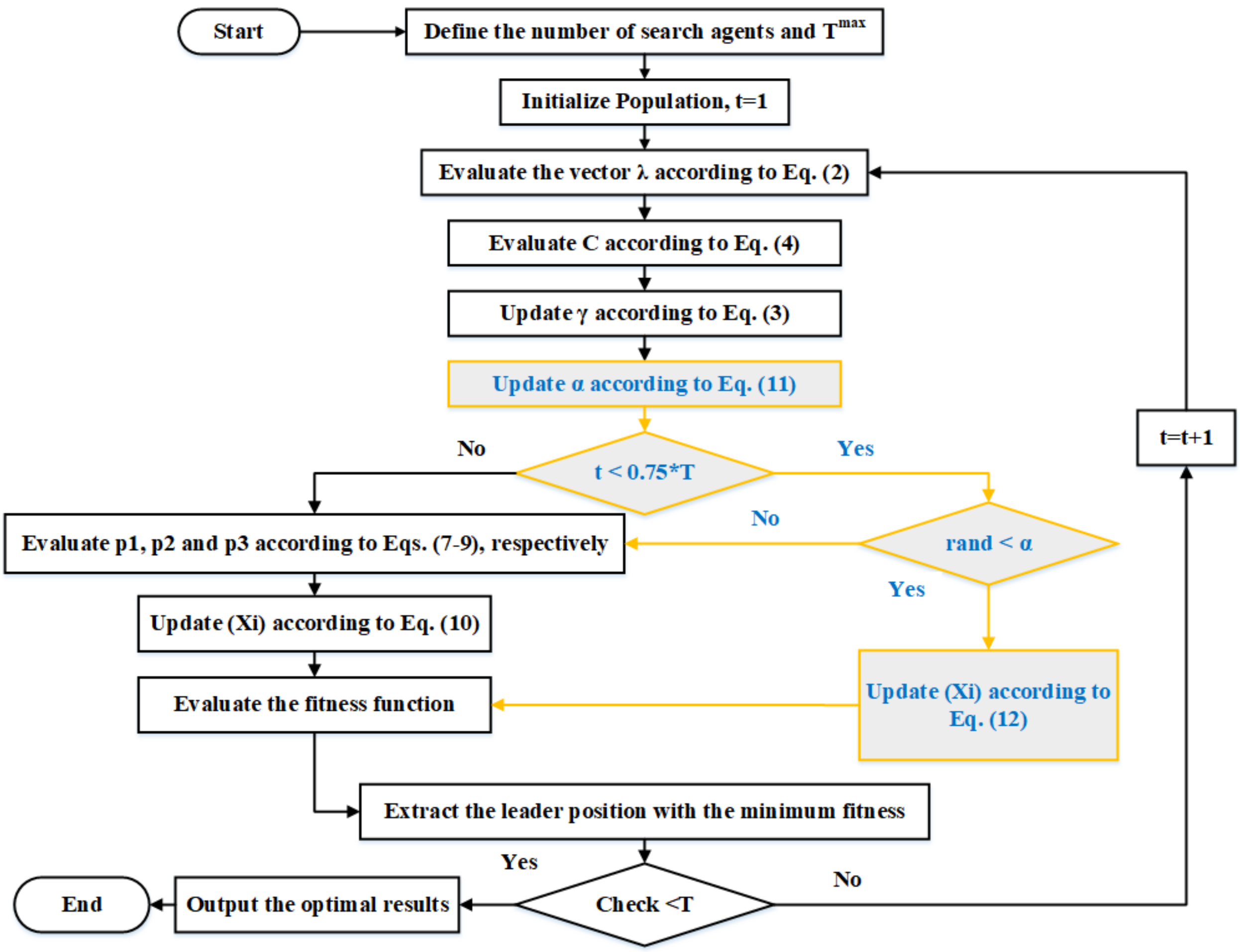


FIGURE 2. Key steps of the proposed IHT.

position. Therefore, this limitation guarantees avoiding the stagnation problem if all the search agents focus on the surrounding region of the Leader position.

III. IMPLEMENTATION OF THE PROPOSED IHT FOR CHPED

The purpose of solving the CHPED challenge is to reduce the cost of system manufacture while meeting all the CHPED requirements. The CHPED problem's objective cost function (OCF) and constraints are provided [31]:

$$\text{Min } F_C = \sum_{j=1}^{N_h} C_j H_j^h + \sum_{i=1}^{N_p} C_i P_i^p + \sum_{k=1}^{N_C} C_k(P_k^C, H_K^C)(\$/\text{h}) \tag{13}$$

The cost functions of this system are the following:

1) OCF of i^{th} CHP units

$$C_i(P_i^c, H_i^c) = a_i(P_i^c)^2 + b_i P_i^p + c_i + d_i(H_i^c)^2 + e_i H_i^c + f_i H_i^c P_i^c(\$/\text{h}) \tag{14}$$

2) OCF of j^{th} Heat only units

$$C_j(H_j^h) = a_j(H_j^h)^2 + b_j P_j^p + c_j(\$/\text{h}) \tag{15}$$

3) OCF of k^{th} Power only units

$$C_k(P_k^p) = a_k(P_k^p)^2 + b_k P_k^p + c_k + \left|\lambda_k \sin(\rho_k(P_k^{p\min} - P_k^p))\right| (\$/h) \tag{16}$$

The cost of i^{th} CHP, j^{th} heat only, and k^{th} power only units are established by $C_i(P_i^c, H_i^c)$, $C_j(H_j^h)$, and $C_k(P_k^p)$. The symbols a_i, b_i, c_i, d_i, e_i and f_i describe the i^{th} CHP unit cost coefficients, whereas the symbols a_j, b_j, and c_j characterize the cost coefficients of j^{th} heat-only plant $a_i b_i c_i$ and a_k, b_k, and c_k express the cost coefficients of k^{th} power-only plant. The non-convexity and non-differentiability of the problem are noticed from the sinusoidal term of the valve-point impacts indicated in the power only units as described in Eq. (16) [36], [37]. Besides, Eq. (14) which represent OCF of CHP contains power output (P^c) and heat output (H^c). There are diverse number or equality and inequality constraints as described in the following equations:

1. *Heat balance constraint*

$$\sum_{i=1}^{N_c} H_i^c + \sum_{j=1}^{N_h} H_j^h = H_d, \tag{17}$$

2. *Bounds of heat only units' Generation*

$$H_j^{h\min} \leq H_j^h \leq H_j^{h\max} \quad j = 1, \ldots, N_h, \tag{18}$$

3. *Power balance constraint*

$$\sum_{k=1}^{N_p} P_k^p + \sum_{i=1}^{N_c} P_i^c = P_d \tag{19}$$

4. *Bounds of power only units' capacity*

$$P_k^{p\min} \leq P_k^p \leq P_k^{p\max} \quad i = 1, \ldots, N_p, \tag{20}$$

5. *Bounds of CHP Capacity*

$$P_i^{c\min}(H_i^c) \leq P_i^c \leq P_i^{c\max}(H_i^c) \quad i = 1, \ldots, N_c, \tag{21}$$
$$H_i^{c\min}(P_i^c) \leq H_i^c \leq H_i^{c\max}(P_i^c) \quad i = 1, \ldots, N_c, \tag{22}$$

where the number of CHP units, heat-only units, and power only units can be indicated by N_{c}, N_{h}, and N_{p}, respectively, whereas H_d and P_d demonstrate the system heat demand and the electric power demand of the system.

Therefore, the solution of CHPED problem can be affected by the mutual dependency among the bounds of CHP units as manifested in Fig. 3. This figure shows that:

- For the setpoint (A), the CHP system is running within its restrictions, hence this operating option is practical. As a result, no penalty would be imposed.
- For the setpoint (B), the CHP system would run over its permitted limits. Regardless of the fact that such operating option is infeasible, the spacing between itself and the closest border is not very great. As a result, a minor penalty value is being designated and applied to the fitness performance.
- For the setpoint (C), the CHP system would run over its permitted limits. The operating option is infeasible in such situation, and the spacing between itself and the closest border is so great. Consequently, a severe punishment period will be imposed.

As a conclusion, for the infeasible operational locations, the greater the distances between itself and the closest border, the greater the extra penalty length, and conversely.

6. *Transmission losses consideration*

Another reason for problem's non-convexity is transmission losses integration, which can be expressed as a function of the power units' output power as manifested in Eq. (23):

$$P_{Loss} = \sum_{i=1}^{N_p}\sum_{m=1}^{N_p} B_{im} P_i^p P_m^p + \sum_{i=1}^{N_p}\sum_{j=1}^{N_c} B_{ij} P_i^p P_j^c + \sum_{j=1}^{N_c}\sum_{n=1}^{N_c} B_{in} P_j^c P_n^c \tag{23}$$

Consequently, Eq. (19) can be rewritten as follows:

$$\sum_{i=1}^{N_{\mathrm{p}}} P_i^{\mathrm{p}} + \sum_{j=1}^{N_{\mathrm{c}}} P_j^{\mathrm{c}} = P_d + P_{Loss} \tag{24}$$

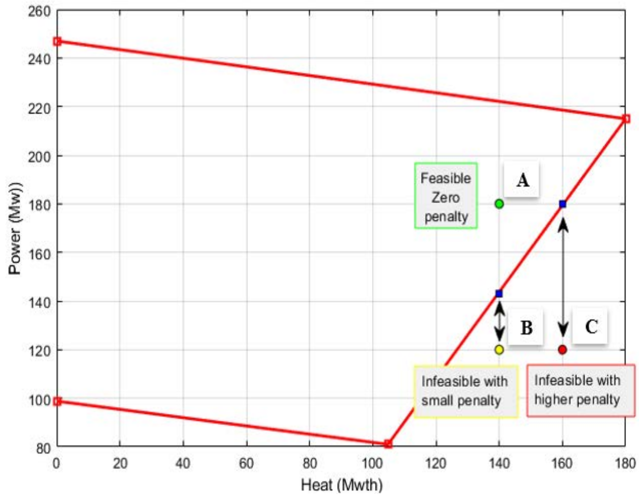


FIGURE 3. Dependency between power and heat for CHP unit.

IV. APPLICATION OF THE HT AND THE PROPOSED IHT TO BENCHMARK MATHEMATICAL FUNCTIONS

To evaluate the proposed IHT's search capacity, 25 benchmark optimizing functions with varying features are run. The testing functions under discussion are distinguished of unimodal and multimodal optimization functions (F1-F25). The functions of testing are Brent, Schaffer No. 4, Wayburn Seader 3, Leon, Zettl, Ackley N.3, Adjiman, Bird, Camel 6 Hump, Goldstien Price, Hartman 3, Hartman 6, Cross-in-tray, Carrom Table, Chichinadze, Cross function, Cross leg Table, Crowned cross, Giunta, Helical Valley, Himmelblau, Holder, Test Tube Holder, Shubert and Shekel. Complete data of these functions are detailed in Table 1.

For equitable assessments, the HT and suggested IHT use 50,000 function evaluations as a maximum number, while the population is set at 40. Figs. 4 and 5 display comparison of the performance of the HT and the suggested IHT using the mean and the standard deviation, respectively. As shown, even though there is no significant difference in the mean goal, the suggested IHT outperforms the traditional HT in the majority of benchmark mathematical functions with a lower standard deviation. Full results are detailed in Table 1 [12].

As indicated, the suggested IHT finds better results than the traditional HT for 17 benchmark functions which are F1, F3-F5, F8, F10-F12, F14-F20, F22, F24 and F25. Added to that, similar performance is declared between the suggested IHT and the traditional HT for 4 benchmark functions which are F6, F7, F13 and F23. On the other side, the suggested IHT finds worse results than the traditional HT for 3 benchmark functions which are F2, F9 and F21.

V. APPLICATION OF THE HT AND THE PROPOSED IHT TO COMBINED HEAT AND POWER ECONOMIC DISPATCH

A series of tests are conducted based on the CHPED problem to assess the performance of the proposed IHT in solving small scale (seven units), medium scale (twenty-four) and two large-scale (eighty-four and ninety-six) systems.

TABLE 1. Comparisons between the HT and IHT for mathematical testing functions.

Function		Vars	Range	HT		IHT		**Comparison**
No.	Name			Mean	St_{dev}	Mean	St_{dev}	**Sign**
F1	Brent	2	[-10,10]	1.38E-87	6.8E-103	1.38E-87	4.56E-103	√
F2	Schaffer No. 4	2	[-100,100]	0.292579	7.14E-17	0.292579	8.40E-17	**X**
F3	Wayburn Seader 3	2	[-500,500]	19.10588	1.45E-14	19.10588	7.57E-15	√
F4	Leon	2	[-1.2,1.2]	3.59E-13	1.29E-12	1.96E-31	6.43E-31	√
F5	Zettl	2	[-5,10]	-0.00379	1.76E-18	-0.00379	1.33E-18	√
F6	Ackley N.3	2	[-32,32]	-195.629	5.8E-14	-195.629	5.8E-14	=
F7	Adjiman	2	[-1,2]	-2.02181	1.36E-15	-2.02181	1.36E-15	=
F8	Bird	2	[-2pi,2pi]	-106.765	2.95E-14	-106.765	2.9E-15	√
F9	Camel 6 Hump	2	[-5,5]	-1.03163	6.71E-16	-1.03163	6.8E-16	**X**
F10	Goldstien Price	2	[-2,2]	3	1.04E-15	3	6.54E-16	√
F11	Hartman 3	3	[0,1]	-3.86278	2.71E-15	-3.86278	2.27E-15	√
F12	Hartman 6	6	[0,1]	-3.322	1.36E-15	-3.322	4.53E-16	√
F13	Cross-in-tray	2	[-10,10]	-2.06261	9.03E-16	-2.06261	9.06E-16	=
F14	Carrom Table	2	[-10,10]	-24.1568	8.9E-15	-24.1568	8.73E-15	√
F15	Chichinadze	2	[-30,30]	-42.9444	3.61E-14	-42.9444	2.9E-14	√
F16	Cross function	2	[-10,10]	4.85E-05	1.38E-20	4.85E-05	6.92E-21	√
F17	Cross leg table	2	[-10,10]	-0.08479	0.000356	-0.08479	4.29E-05	√
F18	Crowned cross	2	[-10,10]	0.001179	6.56E-07	0.001179	4.3E-07	√
F19	Giunta	2	[-1,1]	0.06447	4.82E-17	0.06447	4.25E-17	√
F20	Helical Valley	3	[-10,10]	7.49E-29	2.77E-25	7.49E-29	2.21E-28	√
F21	Himmelblau	2	[-5,5]	5.05E-31	2.41E-31	5.05E-31	1.42E-30	**X**
F22	Holder	2	[-10,10]	-19.2085	8.47E-15	-19.2085	5.08E-15	√
F23	Test Tube Holder	2	[-10,10]	-10.8723	3.63E-15	-10.8723	3.63E-15	=
F24	Shubert	2	[-10,10]	-186.731	2.64E-14	-186.731	1.83E-14	√
F25	Shekel	4	[0,10]	-10.5364	1.81E-15	-10.5364	1.66E-15	√

√, X and = refer to better, worse and equal

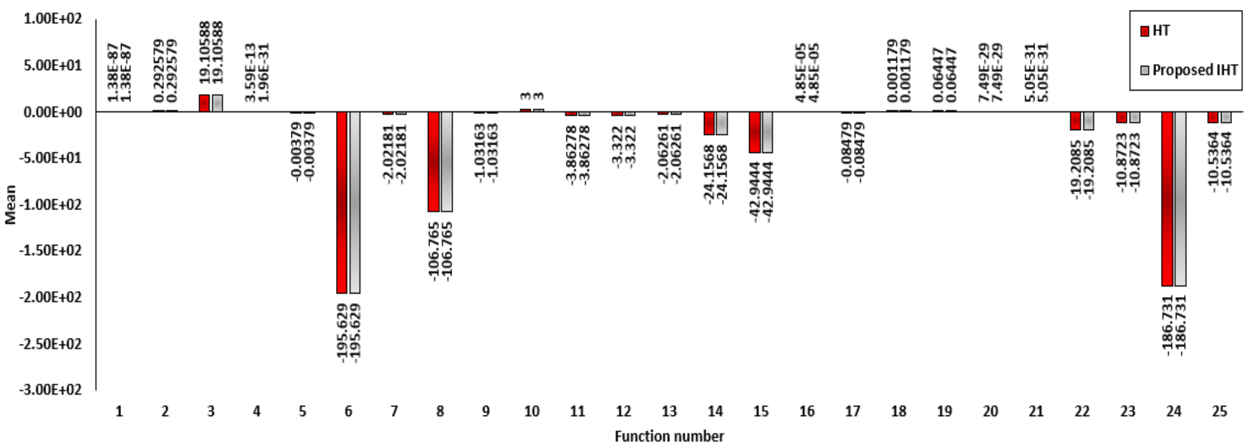


FIGURE 4. Comparison of the performance of HT and IHT using the mean objective of benchmark functions (F1-F25).

The experimental context considers myriads of CHPED test systems with 7, 24, 84, and 96 units. Setting parameters: the maximum number of iterations is 300 for small scale system and 3000 for other systems. The proposed IHT and the traditional HT have a population size of 100 in each experiment and runs 30 times. All of the most recent efficient algorithms have been created and implemented in the MATLAB R2017b 64-bit platform. Tests are performed on a DELL Inspiron computer with an Intel Core i7-4510U CPU running at 2 GHz and 8GB of RAM.

A. THE 7-UNIT SYSTEM

The heat and power demands for the 7-unit CHPED system are 150 MWth and 600 MW, respectively, where it includes 2 CHP units, 1 heat-only units, and 4 conventional thermal units. Literature [7] contains data for systems with 7 units. Two considered cases are investigated as:

Case 1: considers of the valve constraints without the transmission losses Case 2: Considers the transmission losses and valve constraints.

For this system, the proposed IHT and traditional HT are applied on the two cases of the 7-unit test system. Table 2 illustrates the detailed results of the control variables for both cases. As shown, the minimum cost value of the proposed IHT and traditional HT for case 1 are 10091.9034 $ and 10091.9966$, respectively, whilst they achieve 10094.4188$ and 10094.5077$, respectively, for case 2.

For both cases, the convergence curves of the proposed IHT and traditional HT are manifested in figures 6 and 7, respectively. The proposed IHT's curve, in the early, stages

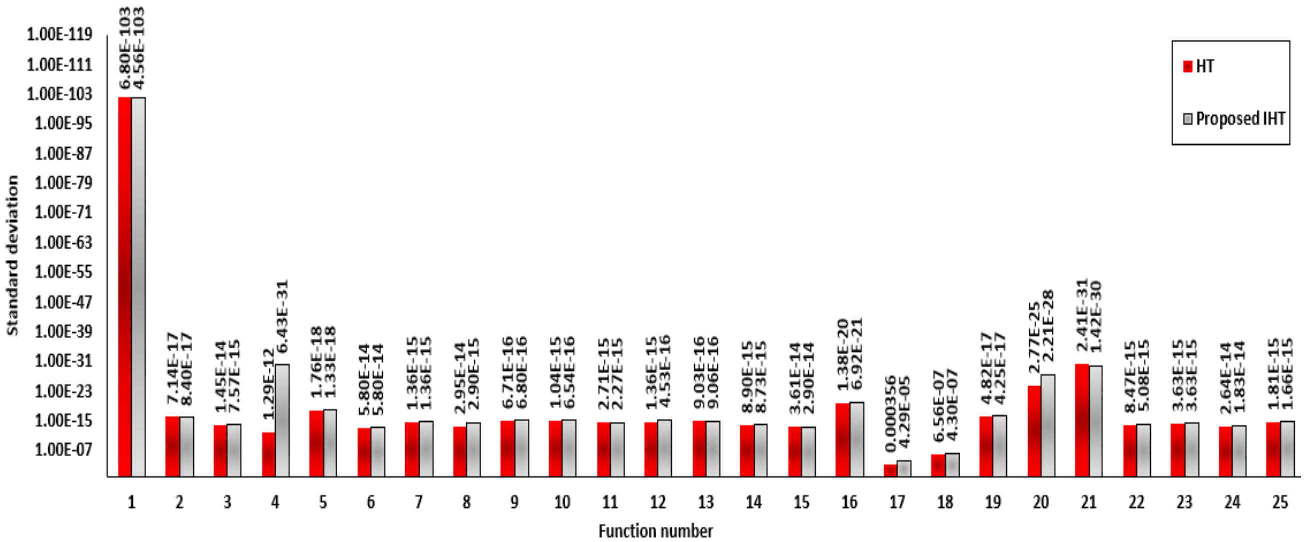


FIGURE 5. Comparison of the performance of HT and IHT using the standard deviation of benchmark functions (F1-F25).

TABLE 2. Simulation results based on traditional HT and the proposed IHT for 7-unit system.

Units		Without transmission losses (Case 1)		Including transmission losses (Case 2)	
		HT	IHT	HT	IHT
Power only units	Pg1	44.6656	44.8644	45.9285	45.7445
	Pg2	98.5398	98.5408	98.5399	98.5400
	Pg3	112.6735	112.6739	112.6732	112.6736
	Pg4	209.8160	209.8159	209.8157	209.8158
CHP	Pg5	94.3051	94.1050	93.8574	94.0409
	Pg6	40.0000	40.0000	40.0000	40.0000
	Hg5	26.4586	27.6373	29.0945	28.0139
	Hg6	74.9921	74.9998	74.9996	74.9987
Heat only unit	Hg7	48.5493	47.3629	45.9059	46.9875
Sum (Pg)		600	600	600.8147	600.8149
Sum (Hg)		150	150	150	150
Total Cost		10091.9966	10091.9034	10094.5077	10094.4188
mean		10093.8554	10093.4080	10096.1738	10095.9123
max		10097.4711	10095.8955	10099.3046	10098.7839
Std		1.1603	1.1503	1.1681	1.2020

clearly have the slowest descent speed, but swiftly converges in the later stages. The number of iterations of the proposed IHT when the curve approaches the optimal solution in cases 1 and 2 is roughly 150 and 200, respectively. That is, the curse of case 1 converges to the optimal solution faster than the curse of case 2 for the proposed IHT. For this tiny CHPED issue with complex constraints, the findings show that the proposed IHT has poor convergence performance. It is worth noting that the maximum number of iterations for each experiment is set high enough to ensure that the best solution attained by each method is a feasible one. To illustrate, the proposed IHT and traditional HT optimal solutions meet all CHPED constraints and have fitness values equal to the objective function values. In addition, a comparison of the obtained OCF($) with several recent techniques are tabulated in Table 3 considering case 1. As shown, the proposed IHT derives the best performance compared to the others since it achieves the

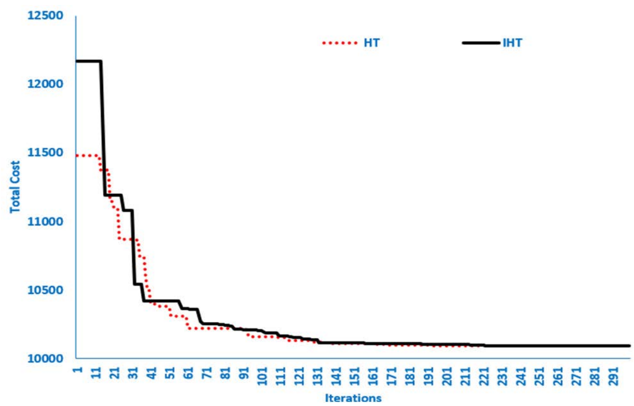


FIGURE 6. Convergence characteristics for the HT and the proposed IHT for case 1 of 7-unit test system.

minimum cost among these techniques. As indicated, the proposed IHT acquires the least costs value with 10091.9034 $. The other algorithms HT, DE [9], Bee Colony Optimization (BCO) [38], CPSO [23], RCGA-IMM [25], TVAC-PSO [26], CSO [30], TVAC-PSO [39], AIS [40], TLBO [23], WVO-PSO [41], MRF [42], LCA [43], CSO&PPS [44], IGA-NCM [7] and ECSA [45] obtain costs of 10091.9966, 10317, 10317, 10325.33, 10094.0552, 10100.32, 10094.1267, 10244.002, 10355, 10094.84, 10372.015, 10092.33, 10104.38, 10111, 10107.9071 and 10121.9466 $, respectively.

B. THE 24-UNIT SYSTEM

The heat and power demands for the 24-unit CHPED system are 1250 MWth and 2350 MW, respectively, where it includes 6 CHP units, 5 heat-only units, and 13 conventional thermal units. The 13 power-only units was derived from a 13-unit standard economic dispatch test instance with a large number of local optima. As a result, the 24-unit system is a multimodal challenge. Literature [26] contains data for this system. For this system, the proposed

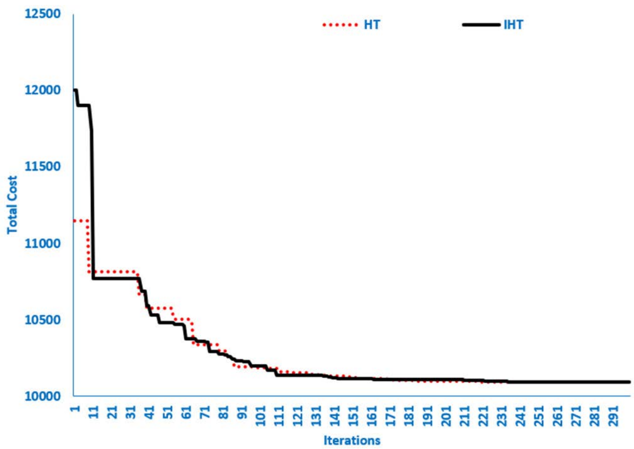


FIGURE 7. Convergence characteristics for the HT and the proposed IHT for case 2 of 7-unit test system.

TABLE 3. Comparison of the proposed IHT, HT and reported techniques for case 1 of 7-unit system.

Optimizer	OCF ($)
IHT	10091.9034
HT	10091.9966
DE[9]	10317
BCO[38]	10317
CPSO[23]	10325.33
RCGA-IMM[25]	10094.0552
TVAC-PSO[26]	10100.32
CSO[30]	10094.1267
TVAC-PSO[39]	10244.002
AIS[40]	10355*
TLBO[23]	10094.84
WVO-PSO[41]	10372.015*
MRF[42]	10092.33
LCA[43]	10104.38*
CSO&PPS[44]	10111
IGA-NCM[7]	10107.9071
ECSA[45]	10121.9466

TABLE 4. Values of heat and power generated Using HT and the proposed IHT for the 24-unit CHPED system.

Unit	HT	IHT	Unit	HT	IHT
Pg1	538.5587	628.3185	Pg18	11.0825	10.0000
Pg2	300.2175	299.3238	Pg19	35.0440	35.0302
Pg3	301.0826	299.2016	Hg14	108.6973	108.2668
Pg4	159.7779	159.7337	Hg15	76.0927	79.7162
Pg5	63.2174	60.0094	Hg16	106.4763	116.7630
Pg6	60.6889	60.0000	Hg17	77.7146	78.1025
Pg7	160.2065	60.0498	Hg18	40.4643	40.0004
Pg8	111.5383	159.7377	Hg19	20.0205	20.0142
Pg9	161.2540	109.9019	Hg20	460.5378	447.1369
Pg10	40.0000	40.0643	Hg21	60.0000	60.0000
Pg11	40.0003	40.0001	Hg22	60.0000	60.0000
Pg12	55.6579	55.0809	Hg23	119.9964	120.0000
Pg13	55.2845	55.0001	Hg24	120.0000	120.0000
Pg14	87.9442	87.1765	Sum(Pg)	2350.0000	2350.0000
Pg15	41.2663	45.4624	Sum(Hg)	1250.0000	1250.0000
Pg16	84.0349	102.3160	Overall FC	57994.515	57953.5263

IHT and traditional HT are applied. Table 4 illustrates the detailed results of the control variables. In this table, the minimum cost value of the proposed IHT and traditional HT for case 1 are 57953.5263 and 57994.515 $, respectively. Additionally, as manifested in Table 5, a comparison for this system with other reported techniques in the

TABLE 5. Comparison the proposed IHT, HT and reported techniques for the 24-unit system.

Optimizer	OCF ($)	Average	Worst	Std
IHT	57953.5263	58056.1326	58192.2204	77.7688
HT	57994.5150	58111.3012	58309.4164	98.6919
HHTJFSO[46]	57968.54	58103.9553	58293.6058	102.9704
JFSO[46]	58739.5241	58949.7824	59125.3301	145.5823
SDO[46]	58061.4768	-	-	-
BSDE[46]	58208.0267	-	-	-
MRFT[46]	58173.93	-	-	-
GSA[47]	58114.9800	-	-	-
GSO[48]	58122.7100	-	-	-
GSO[49]	58225.745	58706.12	58763.915	-
IGSO[49]	58049.01	58311.8439	58545.4748	-
TLBO[23]	58007.00	-	-	-
PSO[26]	59736.26	59853.478	60076.6903	-
TVAC-PSO[26]	58122.746	58198.3106	58359.552	-
CPSO[26]	59736.2635	-	-	-

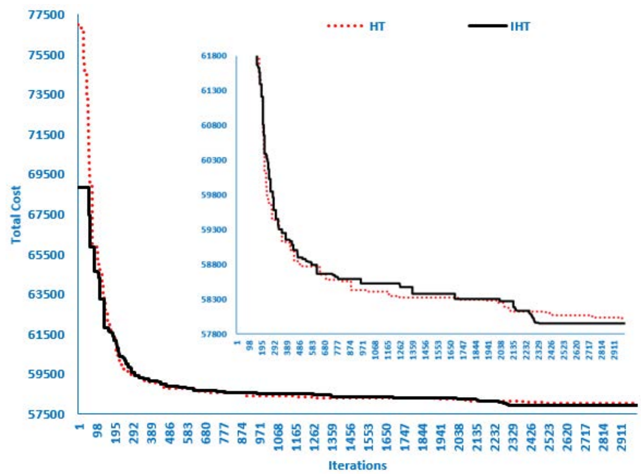


FIGURE 8. Convergence characteristics for the HT and the proposed IHT for of 24-unit test system.

literature which are hybrid HT with Jellyfish search optimization (HHTJFSO) [46], JFSO [46], Supply demand optimization (SDO) [46], gravitational search algorithm (GSA) [47], GSO-based algorithm with modified scrounger and ranger operators (GSO)[48], group search optimization (GSO) [49], Improved GSO (IGSO) [49], TLBO [23], PSO [26], TVAC-PSO [26], CPSO [26]. As shown, the proposed IHT shows the best performance. It achieves the minimum cost among these techniques where the minimum cost value of the proposed IHT and traditional HT are 57953.52 and 57994.52 $, respectively.

Also, in Table 5, the proposed IHT provides the lowest minimum, average, worst and standard deviation of 57953.53, 58056.13, 58192.22 and 77.77, respectively. Therefore, the proposed IHT derives superior robustness compared to the others.The convergence curve of the proposed IHT and traditional HT in this system is manifested in figure 8. The number of iterations of the proposed IHT when the curve approaches the optimal solution is roughly 2200, respectively. For this medium CHPED issue with complex constraints, the findings show that the proposed IHT has excellent convergence performance.

TABLE 6. Values of heat and power generated Using HT and the proposed IHT for the 84-unit CHPED system.

Unit	HT	IHT	Unit	HT	IHT
Pg1	114.0000	108.8230	Pg57	12.9849	17.2881
Pg2	113.1156	113.8396	Pg58	29.5008	10.1858
Pg3	103.8372	103.0866	Pg59	10.2246	16.4866
Pg4	184.8070	184.9292	Pg60	13.3187	21.5248
Pg5	89.5052	96.9991	Pg61	37.7312	35.0222
Pg6	106.6483	140.0000	Pg62	55.4190	41.0626
Pg7	256.2555	300.0000	Pg63	38.5247	39.5772
Pg8	297.0513	291.8100	Pg64	58.3696	53.5076
Pg9	299.9954	286.1507	Hg41	130.3605	122.8731
Pg10	130.0000	204.8212	Hg42	130.2202	125.6308
Pg11	169.3090	94.9266	Hg43	123.5432	119.3404
Pg12	306.0941	243.8372	Hg44	134.1353	118.8916
Pg13	394.5008	394.2794	Hg45	77.3053	97.1438
Pg14	393.7356	394.2794	Hg46	78.0218	92.3829
Pg15	305.5367	394.2799	Hg47	107.1827	93.6937
Pg16	394.4501	394.7165	Hg48	105.0563	76.3861
Pg17	500.0000	498.6131	Hg49	115.1930	122.6936
Pg18	490.8920	491.2098	Hg50	124.3738	130.8235
Pg19	514.6259	511.2890	Hg51	120.6934	120.5675
Pg20	525.3543	518.2541	Hg52	118.7926	132.6720
Pg21	550.0000	524.8894	Hg53	92.8899	96.4150
Pg22	548.5299	524.3433	Hg54	85.7914	89.0431
Pg23	550.0000	537.0902	Hg55	78.1867	94.0979
Pg24	521.6137	544.7989	Hg56	88.9740	86.8481
Pg25	522.5635	532.0267	Hg57	41.2797	43.1238
Pg26	549.3197	527.0546	Hg58	48.3575	40.0797
Pg27	14.5402	10.4116	Hg59	40.0962	42.7803
Pg28	10.0983	10.2187	Hg60	41.2298	44.9396
Pg29	10.9099	12.3562	Hg61	21.1841	20.0105
Pg30	96.9999	91.5958	Hg62	27.1689	22.7562
Pg31	180.3915	190.0000	Hg63	21.6026	20.1271
Pg32	189.8298	190.0000	Hg64	25.8471	28.4104
Pg33	181.7205	189.7575	Hg65	397.9644	395.2297
Pg34	200.0000	177.9203	Hg66	394.0861	398.0446
Pg35	182.9160	200.0000	Hg67	400.2040	395.5937
Pg36	200.0000	200.0000	Hg68	401.4452	391.8615
Pg37	109.9994	110.0000	Hg69	60.0000	59.9352
Pg38	110.0000	109.9760	Hg70	59.3632	59.8413
Pg39	89.8400	110.0000	Hg71	59.8612	60.0000
Pg40	550.0000	516.4148	Hg72	60.0000	59.9875
Pg41	126.9117	113.2038	Hg73	58.8840	60.0000
Pg42	126.6515	118.1178	Hg74	59.5417	60.0000
Pg43	115.3838	106.9109	Hg75	59.8127	60.0000
Pg44	133.2788	106.1210	Hg76	60.0000	60.0000
Pg45	42.8020	65.6507	Hg77	120.0000	119.4271
Pg46	43.6791	60.1357	Hg78	120.0000	120.0000
Pg47	77.2802	61.6550	Hg79	119.9529	120.0000
Pg48	74.8184	41.6090	Hg80	120.0000	120.0000
Pg49	99.5193	112.8840	Hg81	120.0000	118.8970
Pg50	116.0936	127.3709	Hg82	119.4424	119.9989
Pg51	109.3200	109.0954	Hg83	119.9999	119.4527
Pg52	106.0198	130.6645	Hg84	111.9565	120.0000
Pg53	60.7230	64.8147	Sum (Pg)	12700.0000	12700.0000
Pg54	52.5809	56.2668	Sum (Hg)	5000.0000	5000.0000
Pg55	43.6913	62.1223	Total Cost ($)	289822.3922	288368.9610
Pg56	56.1868	53.7240			

C. THE 84-UNIT SYSTEM

The heat and power demands for the 84-unit CHPED system are 5000 MWth and 12700 MW, respectively, where it includes 24 CHP units, 20 heat-only units, and 40 conventional thermal units. Literature [31] contains the detailed data for this system. For this system, the proposed IHT and traditional HT are applied. Table 6 illustrates the detailed results of the control variables. From this table, the minimum cost value of the proposed IHT and traditional HT for case 1 are 288368.9610 and 289822.3922$, respectively.

Table 7 displays a comparison with other reported techniques in the literature to minimize the OCF which are WOA [31], SDO [33], MPA [33], IMPA [33], MRF [42], HT [33], JFSO [46], and HHTJFSO [46]. As shown, the proposed IHT achieves the minimum cost among these techniques where the minimum cost value by it is 288369 $ where the traditional HT obtains 289822.4 $. Additionally, as manifested in Table 7, the proposed IHT provides the lowest minimum, average, worst and standard deviation of 288369, 289101.7, 290282 and 553.47, respectively. Therefore, the proposed IHT derives superior robustness compared to the others.

TABLE 7. Statistical analysis of the proposed IHT, HT and reported techniques for the 84-unit system.

Optimizer	OCF ($)	Average	Worst	Std
IHT	288369	289101.7	290282	553.47
HT	289822.4	290891	292342.5	886.4399
HHTJFSO[46]	288820.7	289813.8	291251.7	688.7185
JFSO[46]	290323.8	292366.9	293747.4	988.0994
WOA[31]	290123.97	-	-	-
SDO[33]	292788.5	-	-	-
MPA[33]	294717.7	-	-	-
IMPA[33]	289903.8	-	-	-
MRF[42]	291225.6	-	-	-

D. THE 96-UNIT SYSTEM

The heat and power demands for the 96-unit CHPED system are 9400 MWth and 5000 MW, respectively, where it includes 24 CHP units, 20 heat-only units, and 52 conventional thermal units. Literature [31] contains the studied data for this system. For this system, the proposed IHT and traditional HT are applied Table 8 illustrates the detailed results of the control variables. Table 9 depicts a comparison of the obtained OCF($) with several recent techniques. As shown, the proposed IHT derives the best performance compared to the others since it achieves the minimum cost among the other techniques. The minimum cost value of the proposed IHT is 234090.7241 $ where the traditional HT attains 235102.65 $. Additionally, as manifested in Table 9, the proposed IHT provides the lowest minimum, average, worst and standard deviation of 234090.72, 234952.84, 236243.85 and 690.33, respectively. Therefore, the proposed IHT derives superior robustness compared to the others. The convergence curve of the proposed IHT and traditional HT for the 84-unit and 96-unit systems are manifested in figures 9 and 10.

The number of iterations of the proposed IHT when the curve approaches the optimal solution is roughly 2300, respectively. For this large CHPED issue with complex constraints, the findings show that the proposed IHT has excellent convergence performance. To recapitulate, the proposed IHT's convergence ability is comparable to that of the traditional HT in systems with small-scale units, but clearly superior to that of the traditional HT in medium and large-scale CHPED systems. That is, the proposed IHT can swiftly identify a global optimal point in a high-dimensional search space, and its optimal solution is superior to that of the comparable methods which demonstrate that the proposed IHT has stability and great convergence in solving the medium and large-scale CHPED problem.

TABLE 8. Values of heat and power generated using HT and the proposed IHT for the 96-unit CHPED system.

Unit	HT	IHT	Unit	HT	IHT
Pg1	537.2547	538.5612	Pg63	18.2728	11.0982
Pg2	341.5738	299.5902	Pg64	45.1559	50.5700
Pg3	151.1409	223.2796	Pg65	88.8294	96.7902
Pg4	109.0978	109.8632	Pg66	43.7499	42.1776
Pg5	64.1087	120.8684	Pg67	95.3407	117.0744
Pg6	110.0481	109.8669	Pg68	67.2978	47.0716
Pg7	94.3364	109.9116	Pg69	11.6382	15.2461
Pg8	60.0000	110.5270	Pg70	35.0287	39.3426
Pg9	108.2875	60.0041	Pg71	87.3841	93.7262
Pg10	115.4310	46.3592	Pg72	53.5237	41.5690
Pg11	49.0134	77.4914	Pg73	106.2847	132.2673
Pg12	92.0266	92.4924	Pg74	65.9161	57.1729
Pg13	55.1884	92.4131	Pg75	12.4492	13.6880
Pg14	360.9754	448.8039	Pg76	35.0023	41.7135
Pg15	299.3861	299.2422	Hg53	117.8187	111.3162
Pg16	359.9222	199.8587	Hg54	81.1879	87.3881
Pg17	159.7120	109.8882	Hg55	108.9226	107.8386
Pg18	109.3603	60.0003	Hg56	83.0247	88.7991
Pg19	110.4137	160.2747	Hg57	40.4907	44.0761
Pg20	101.8946	60.0056	Hg58	24.6589	20.0027
Pg21	109.8649	110.9402	Hg59	109.9476	106.8974
Pg22	179.4592	60.2590	Hg60	83.1405	85.3385
Pg23	40.1303	40.0077	Hg61	120.7707	108.7493
Pg24	77.2296	120.0000	Hg62	81.2269	89.1240
Pg25	66.6128	92.4555	Hg63	39.6766	40.4703
Pg26	91.0228	58.7538	Hg64	23.9844	27.0768
Pg27	359.4498	628.3271	Hg65	108.2052	113.6597
Pg28	299.4260	149.6183	Hg66	77.9651	76.8804
Pg29	289.8301	360.0000	Hg67	112.7832	125.0449
Pg30	161.9249	109.8920	Hg68	98.4691	81.1048
Pg31	107.7670	159.7514	Hg69	40.6643	42.2485
Pg32	159.4641	109.8685	Hg70	18.7219	21.9730
Pg33	162.5783	109.8747	Hg71	106.5922	111.9419
Pg34	159.9002	60.2864	Hg72	86.4916	76.3553
Pg35	60.0376	60.0046	Hg73	118.0987	133.5712
Pg36	113.7052	115.0176	Hg74	97.0143	89.8243
Pg37	114.3820	77.4790	Hg75	40.8904	41.5809
Pg38	94.2927	60.2306	Hg76	18.3538	23.0520
Pg39	92.6546	92.3663	Hg77	385.5746	423.6192
Pg40	448.3943	542.6084	Hg78	59.9977	60.0000
Pg41	297.8196	299.2255	Hg79	60.0000	60.0000
Pg42	146.2330	299.3596	Hg80	118.8203	119.1495
Pg43	110.6346	60.0000	Hg81	119.9989	119.8073
Pg44	161.7768	145.6162	Hg82	438.2205	424.0899
Pg45	61.6652	109.9134	Hg83	60.0000	60.0000
Pg46	108.9738	109.9643	Hg84	59.8580	59.9069
Pg47	110.4948	60.0847	Hg85	119.9515	119.9999
Pg48	109.9051	160.3954	Hg86	118.4601	120.0000
Pg49	42.0283	40.0299	Hg87	450.5034	436.6636
Pg50	72.2518	78.5902	Hg88	59.7841	60.0000
Pg51	93.0609	92.4596	Hg89	59.8306	59.9999
Pg52	92.5732	72.8238	Hg90	119.9995	120.0000
Pg53	104.4403	92.6115	Hg91	118.3277	120.0000
Pg54	47.6036	54.3518	Hg92	451.8177	424.1123
Pg55	88.4450	86.4135	Hg93	60.0000	60.0000
Pg56	50.1439	55.9842	Hg94	59.9164	59.9711
Pg57	12.0340	19.5100	Hg95	119.9999	118.3666
Pg58	45.6270	35.0050	Hg96	119.8391	120.0000
Pg59	91.4130	84.7386	Sum(Pg)	9400.0000	9400.0000
Pg60	51.8627	51.9753	Sum(Hg)	5000.0000	5000.0000
Pg61	110.2162	88.0364	Cost ($)	235102.6529	234090.7241

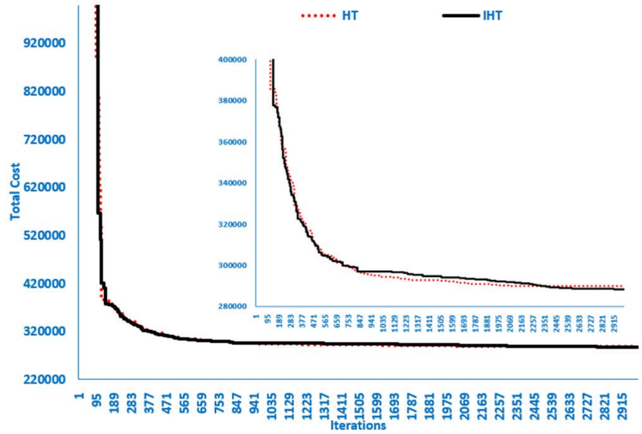


FIGURE 9. Convergence characteristics for the HT and the proposed IHT for of 84-unit test system.

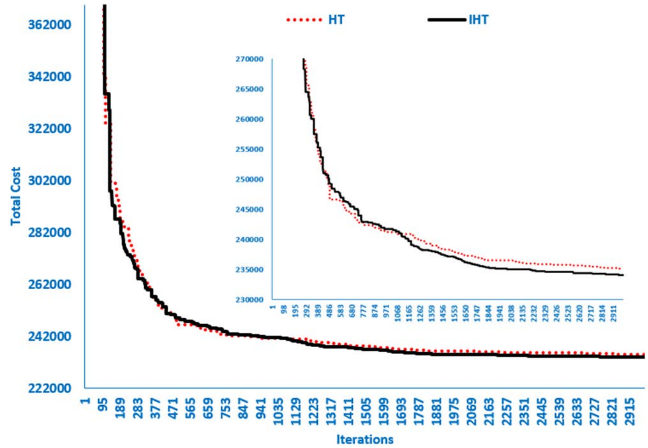


FIGURE 10. Convergence characteristics for the HT and the proposed IHT for of 96-unit test system.

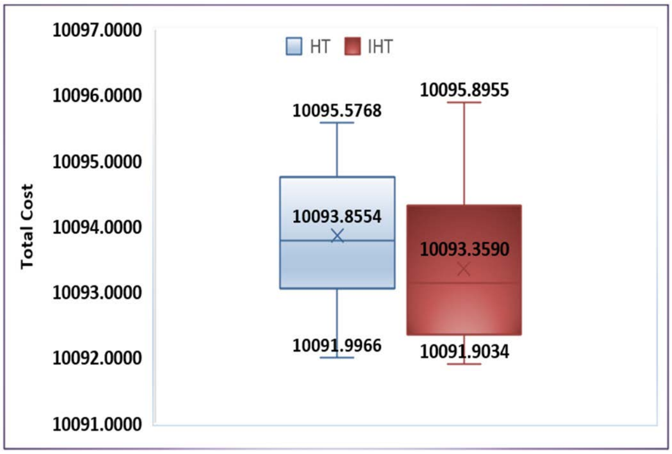
TABLE 9. Statistical analysis of the proposed IHT, HT and reported techniques for the 96-unit system.

Optimizer	OCF ($)	Average	Worst	Std
IHT	234090.7241	234952.8379	236243.8455	690.3345
HT	235102.65	236853.3030	239119.459	1594.7970
HHTJFSO[46]	234836.04	235646.1289	236967.064	764.9310
JFSO[46]	235277.05	236688.7625	237940.189	859.1088
WOA[31]	236699.15	237431.4678	238877.049	971.5473
WVO-PSO[41]	238005.79	-	-	-
PSO-TVAC[26]	239139.5018	-	-	-
WVOPSO[41]	235789.2014	-	-	-
WVO[41]	240861.3210	-	-	-
SDO[33]	236185.18	-	-	-
MRF[50]	235541.4	-	-	-
MPA[51]	236283.1	-	-	-
IMPA[51]	235260.3	-	-	-

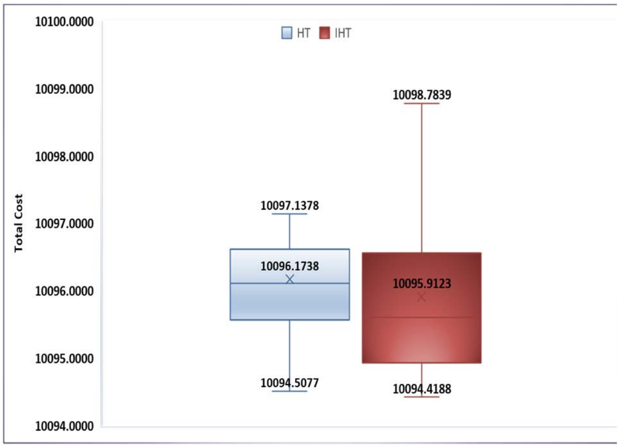
E. STATISTICAL ASSESSMENT OF PROPOSED IHT AND HT FOR CHPED

For all studied systems, the proposed IHT and HT are 30 run times, and the corresponding whiskers box plots are drawn in Fig. 11. As illustrated in Fig. 11 (A), the proposed IHT outperforms HT, for the 7-unit system (case: 1), in attaining the statistical calculations of OCF value. The proposed IHT provides the lowest minimum, average, maximum and standard deviation of OCF values of 10091.9$, 10093.41$, 10095.9$, and 1.15$, respectively, whilst the HT provides 10091.99$, 10093.86$, 10097.47$ and 1.16$, respectively. Additionally, the proposed IHT outperforms HT for the 7-unit system (case: 2) as illustrated in Fig. 11 (B). The proposed IHT provides the lowest minimum, average, maximum and standard deviation of OCF values of 10094.42$, 10095.91$, 10098.78$, and 1.202$, respectively, whilst the HT provides 10094.51$, 10096.17$, 10099.3$ and 1.168$, respectively.

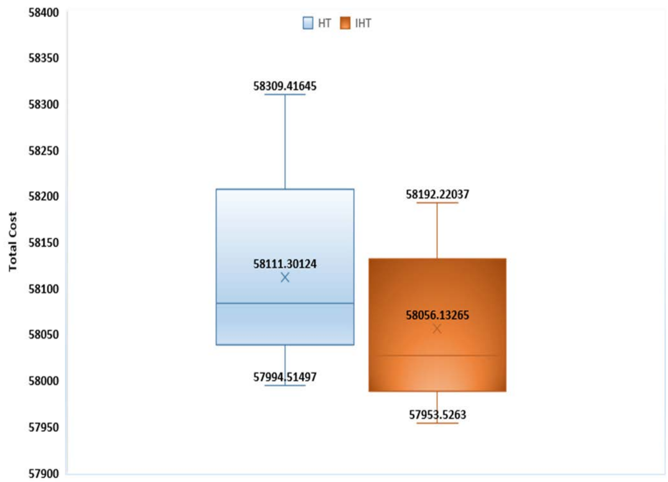
For the 24-unit system, as shown in Fig. 11 (C), the proposed IHT beats HT in statistical computations of OCF value. The suggested IHT gives the smallest minimum,



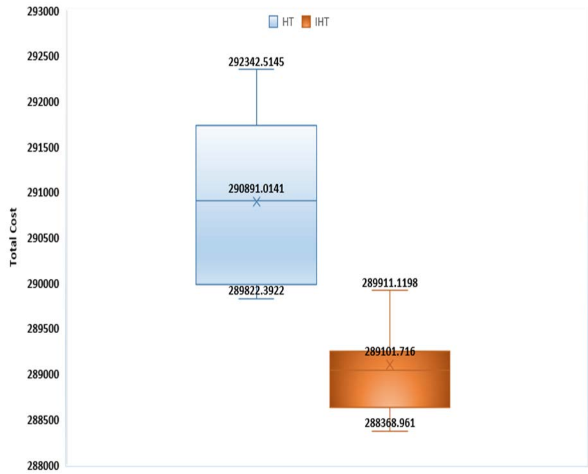
A) 7-unit system: (Case 1)



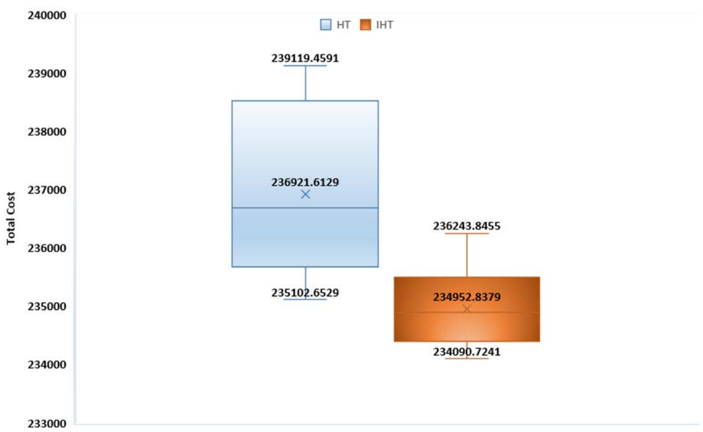
B) 7-unit system: (Case 1)



C) 24-unit system



D) 84-unit system



E) 96-unit system

FIGURE 11. **Whiskers box plot for the total cost obtained by the HT and the proposed IHT of CHPED.**

average, maximum, and standard deviation of OCF values of 57953.526$, 58056.13$, 58192.22$, and 77.77$, respectively, whereas the HT provides 57994.515$, 58111.3$, 58309.42$, and 98.69$, respectively. Similar findings are acquired for the 84-unit system and the 96-unit system as illustrated in Fig. 11 (D) and (E), respectively. Therefore, it is concluded that the proposed IHT has high stability and robustness in attaining the minimum, average, maximum and standard deviation of OCF values with respect to the HT. In addition, Table 10 shows the average computing time for the standard HT and suggested IHT methods. the computational time for both algorithms is stated for the 25 benchmark

TABLE 10. Computational time (Sec) of the proposed IHT and the original HT.

Test function		Computational Time (Sec)	
		IHT	HT
Benchmark models	F1	0.2851	0.3043
	F2	0.2955	0.3133
	F3	0.2622	0.2902
	F4	0.3329	0.3735
	F5	0.2598	0.2706
	F6	0.2716	0.2798
	F7	0.3454	0.3323
	F8	0.2649	0.2674
	F9	0.3023	0.3130
	F10	0.3421	0.3463
	F11	0.4117	0.4223
	F12	0.4004	0.4196
	F13	0.3206	0.3127
	F14	0.2714	0.3027
	F15	0.2345	0.2337
	F16	0.2501	0.2593
	F17	0.2378	0.2309
	F18	0.2407	0.2422
	F19	0.3121	0.3036
	F20	0.2302	0.2323
	F21	0.2794	0.3218
	F22	0.2411	0.2333
	F23	0.2320	0.2267
	F24	0.2379	0.2463
	F25	0.6573	0.6774
CHPED Problem	7-unit system	32.1632	33.0200
	24-unit system	327.0552	325.40316
	84-unit system	383.7011	337.6322
	96-unit system	376.8924	331.38756

functions and the four studied CHPED systems. As shown by below table, the proposed IHT method requires somewhat less computing time than the traditional HT method.

VI. CONCLUSION

In this paper, an improved Heap-based Technique (IHT) is proposed and implemented successfully to 25 benchmark optimizing functions and to solve the CHPED problem. The CHPED has become a challenging task that seeks to find the best value for heat generated by heat generators, power generated by power generators, and both power and heat generated by co-generators, so that fuel costs are kept to a minimum while heat and power demands and constraints are met precisely. In the CHPED problem, the valve point loading and transmission losses constraints are taken into consideration. The IHT is developed in the article by performing two modifications on the conventional HT. Firstly, developing adaptive parameter is incorporated which increases linearly to half of the iteration to select an effective operation for creating the new solutions. Secondly, an enhanced discriminatory attribute is merged to improve the global search capabilities and avoid trapping in a local optimum. The IHT is employed on different small scale, medium scale and large-scale systems and from the simplest to the most intricate. The proposed technique's robustness and effectiveness are investigated in each system by comparing it with other well-known approaches. The comparisons revealed that the proposed IHT is quite promising in terms of tackling the CHPED problem. The proposed IHT is more efficient than the traditional HT and most reported approaches. It provides superior performance compared to the traditional HT and other recent techniques. It also derives the best robustness indices in terms of the statistical terms of minimum, average, maximum and standard deviation OCF values.

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