IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received May 19, 2022, accepted June 5, 2022, date of publication June 14, 2022, date of current version June 20, 2022.

Digital Object Identifier 10.1109/ACCESS.2022.3183100

Design of Decimation Filter With Improved Magnitude Characteristic and Low Complexity

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ABSTRACT This paper presents a novel multiplierless decimation filter with low complexity, increased aliasing rejection, and a compensated passband. Simple multiplierless filters derived from symmetric polynomials, which introduce four additional zeros into certain comb folding bands, were introduced. As a result, the comb-folding bands are wider and provide improved aliasing rejection. Polyphase decomposition is applied to move this filtering to a low rate, thus decreasing the complexity expressed in the number of adders per output sample (APOS). The improvement in the passband characteristic is achieved with the compensator, which operates at a low rate. The compensator parameters were obtained using particle swarm optimization (PSO), and presented in a signed power of two (SPT) form to obtain a multiplierless design. The compensator design provides flexibility in the trade-off between the required number of adders and the quality of compensation. The superiority of the proposed filter is verified by comparison with the methods recently proposed in the literature, regarding complexity, aliasing attenuation, and passband compensation.

INDEX TERMS CIC structure, compensator, decimation, optimization, passband.

I. INTRODUCTION

Decimation is the process of decreasing the sampling rate in the digital domain using, an integer, called the decimation factor. This process has applications, in sigma-delta analog-digital-converters (SD ADC), software radio, and communications, [1], [2]. A decrease in the sampling rate introduces aliasing, which may deteriorate the decimated signal. To prevent aliasing, it is necessary to filter the input signal using a decimation filter. The comb filter is the simplest decimation filter, because all its coefficients are equal to unity. The cascaded integrator-comb (CIC) is a popular comb structure made of integrator and comb sections, separated by a block that decreases the sampling rate by the decimation factor. The comb filter attenuates aliasing, which occurs in the bands around the comb zeros, called folding bands. However, the attenuation in folding bands is insufficient for many applications, and must be increased. The most efficient method to increase the aliasing rejection is obtained by distributing the zeros in the comb folding bands, since the former results in an increase in the widths of the folding bands. This method was first proposed in [3] by, introducing a rotated recursive sinc filter, which introduces two additional zeros into each comb-folding band. However, this method requires two multipliers, and introduces a possible instability. Different methods have been proposed to further improve the method in [3], considering a multiplierless design, and a nonrecursive rotation term, thus avoiding possible instability. Sharpening using Chebyshev polynomials was proposed in [4]. The authors of [5] presented methods based on combs of different lengths. In [6], a simple two-stage comb-based filter was proposed for even decimation factors, in which a simple modified cosine filter is introduced in the second stage, to improve aliasing rejection in all odd folding bands.

The methods for the simultaneous improving the comb passband and stopband, based on sharpening technique, are proposed in [7]–[9]. Recently, in [10] is presented method for design of decimator with a minimum complexity expressed in the number of adders per output sample.

The methods in [11]–[15] used simple filters derived from certain symmetric polynomials to introduce additional zeros into comb-folding bands. The method in [11] uses a simple filter to separate the double-comb zeros. Theoretical background for choice of this simple filter, and the two-stage structure, with the compensated passband, are proposed in [12]. Similarly, in [13], the cascade of two simple filters derived from symmetrical polynomials was presented to introduce additional zeros into comb-folding bands. This approach was improved in a two-stage structure with a compensated passband, as proposed in [14]. The method in [15] combines

The associate editor coordinating the review of this manuscript and approving it for publication was Qiang Li.

the simple filters from [11] and, [13] to increase the widths and attenuation of the folding bands, resulting in increased complexity.

Our goals here are the following:

- Increase the attenuation in the comb folding bands while maintaining a low complexity expressed in the number of adders per output sample (APOS), [16], in comparison with the state of the art.
- Compensate for the passband droop, providing a trade-off between the required number of adders and the quality of compensation, expressed as the absolute value of the maximum passband deviation in dB.

The novelty of this work is the proposed decimation structure with four design parameters that ensure a trade-off between an improved aliasing rejection and a low complexity expressed in the number of APOS, and the proposed structure compensator design, providing flexibility between the maximum passband deviation in dB, and the number of compensator adders.

The remainder of this paper is organized as follows. The next section presents the proposed method for increasing aliasing rejection, which is illustrated using examples. Section III presents the design of the compensation filter, as well as examples. Comparisons with state-of-the-art methods are presented in Section IV. Finally, conclusions are presented in Section V.

II. INCREASING ALIASING REJECTION WITH A LOW NUMBER OF APOS

A. INTRODUCING FOUR ZEROS INTO CERTAIN COMB FOLDING BANDS

We consider that the decimation factor M can be presented as a product of two factors, M_1 and M_2, $M = M_1M_2$. Initially, we propose a two-stage structure in which the first stage is decimated by M_1 and the second stage by M_2.

The transfer functions of the comb filters in the first, and second stages are as follows:

$$H_1(z) = \left[\frac{1}{M_1}\frac{1-z^{-M_1}}{1-z^{-1}}\right]^{K_1}, H_2(z) = \left[\frac{1}{M_2}\frac{1-z^{-M_2}}{1-z^{-1}}\right]^{K_2}, \tag{1}$$

where K_1 and K_2 are the orders of the combs in the first and second stages, respectively.

In the second stage, the multiplierless filters $Q_1(z)$, and $Q_2(z)$ are derived from the symmetric polynomials in [11] and [13], respectively:

$$Q_1(z) = \left[\frac{1}{M_2}\frac{1-z^{-M_2}}{1-z^{-1}}\right]^2 - 2^{-1}z^{-(M_2-1)}. \tag{2}$$

$$Q_2(z) = \left[1 + 2^{-1}\sum_{i=1}^{M_2-2} z^{-i} + z^{-(M_2-1)}\right] \times \left[1 + (1+2^{-1})\sum_{i=1}^{M_2-2} z^{-i} + z^{-(M_2-1)}\right]. \tag{3}$$

The cascade of filters (2) and (3) is denoted as $Q(z)$ and is called inserting zeros (IZ) filter, because this filter introduces additional four zeros in certain comb-folding bands with the aim of increasing width and attenuation in the comb folding bands.

$$Q(z) = Q_1(z) \times Q_2(z), \tag{4}$$

The transfer function of the proposed filter is given as:

$$H_Q(z) = H_1(z)H_2\left(z^{M_1}\right)Q\left(z^{M_1}\right), \tag{5}$$

The benefit of introducing the IZ filter (4) in the second stage is illustrated in the following example:

Example 1: We consider $M = 8$ and $M_1 = 2$, $M_2 = 4$, $K_1 = K_2 = 1$. Fig. 1(a) presents a pole-zero plot of the comb filter $H(z)$, $M = 8$, $K = 1$. Similarly, Figs. 1(b) and 1(c) present pole-zero plots of the filter $Q(z^{M1})$, and the cascade of comb $H(z)$ and filter $Q(z^{M1})$, respectively, at a high input rate (expanded by M_1).

From Fig. 1 we can conclude the following:

- Filter $Q(z^{M1})$ introduces 4 additional zeros into all comb folding bands except in the fourth folding band, ($M_2 = 4$). Generally, the additional zeros are not introduced in the folding bands that are multiples of M_2.
- The zeros in these folding bands are introduced by the comb filter, which works at a high input rate. Because filter $H_1(z)$ in a two-stage structure, works at a high input rate, the parameter K_1 in (1) may be used to improve the attenuation in those folding bands.
- The former is confirmed in the corresponding magnitude responses shown in Fig. 2, which presents the magnitude responses of the filter from Fig. 1(c) with orders of combs, $K_1 = K_2 = 1$, and $K_1 = 5$, $K_2 = 1$.

B. STRUCTURE AND COMPLEXITY

The complexity of the proposed filter was expressed as the number of adders per output sample (APOS).

To decrease the complexity of filter $Q(z)$, polyphase decomposition was applied:

$$Q(z) = \sum_{k=0}^{M_2-1} z^{-k}P_k\left(z^{M_2}\right), \tag{6}$$

where $P_k\left(z^{M_2}\right)$ are the polyphase components.

Using the multirate identity, $P_k\left(z^{M_2}\right)$ can be moved after decimation by M_2 to become $P_k(z)$. Similarly, the numerator of the comb filter $H_1(z)$ can be moved after decimation by M_1, and the numerator of the comb filter $H_2(z)$ can be moved after decimation by M_2. The final structure is shown in Fig. 3. Next we find the number of APOS for the structure in Fig. 3. The number of APOS for both combs in the proposed structure is equal to:

$$APOS_{\text{combs}} = K_1M + (K_1 - K_2)M_2 + K_2. \tag{7}$$

Denoting the number of APOS of filter $Q(z)$ as APOS_{Q}, we have:

$$APOS_{\text{Q}} = M_2 - 1 + N_P, \tag{8}$$

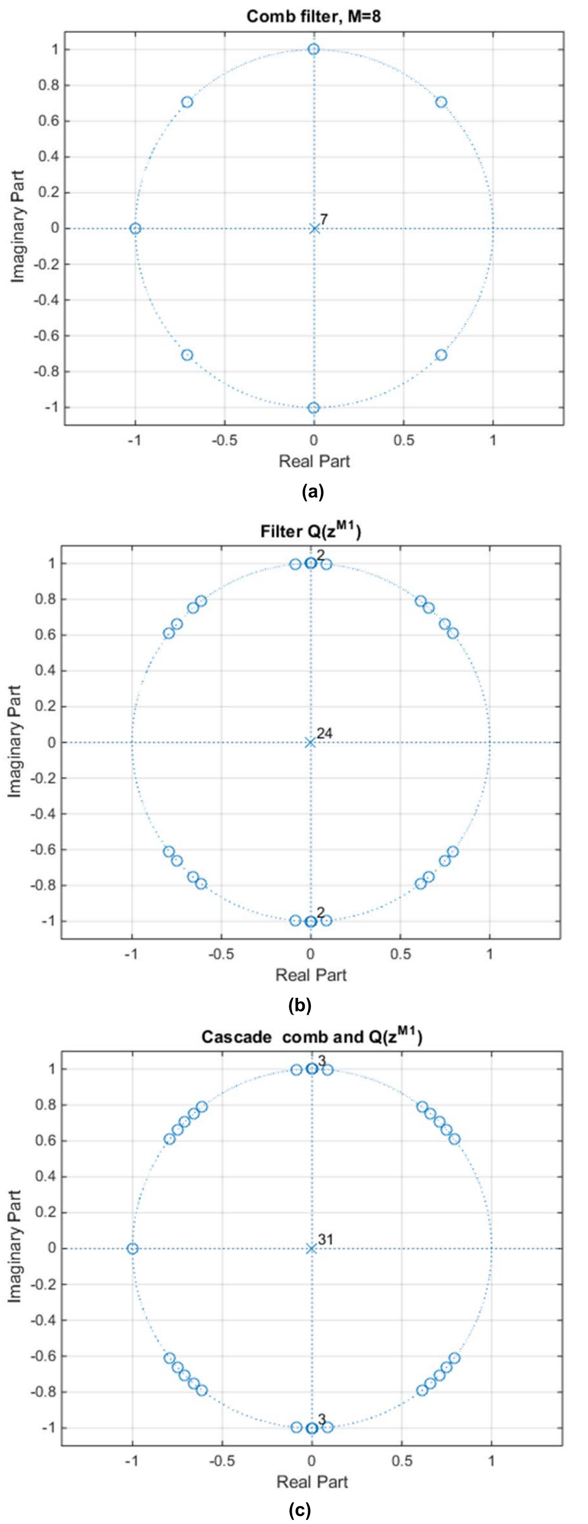


FIGURE 1. Pole-zero plots of comb H(z), filter Q(z^{M1}), and cascade of comb H(z) and Q(z^{M1}). (a) Comb filter H(z), M=8, K=1, working at high input rate. All zeros are single and in the center of folding bands. (b) Filter Q(z^{M1}) introduces four zeros in all folding bands except in the fourth one. (c) Cascade of filter H(z) and Qz^{M1}) introduces five zeros into all folding bands except in the fourth one which gets only one zero from comb filter H(z).

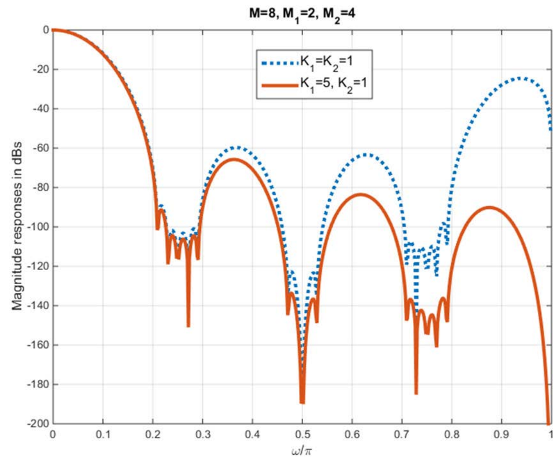


FIGURE 2. Magnitude responses of proposed filter for $K_1 = K_2 = 1$, and $K_1 = 5$, $K_2 = 1$ illustrate how the increasing of K_1 improves aliasing rejection, especially in the fourth folding band.

where N_P is the number of adders in the polyphase components in Eq. (6).

From (7) and (8), we obtain the total number of APOS for the proposed filter as follows:

$$\begin{aligned} APOS &= APOS_Q + APOS_{\text{combs}} \\ &= M_2 - 1 + N_P + K_1 M + (K_1 - K_2) M_2 + K_2 \,. \end{aligned} \quad (9)$$

C. DESIGN PARAMETERS

There are four design parameters: the decimation factors M_1 and M_2, and the comb filter orders K_1 and K_2. Generally, the choice of parameters is a trade-off between the increase in complexity and aliasing rejection in the folding bands. Additionally, it is worth mentioning that, according to (3), the decimation factor M_2 must be, $M_2 \geq 3$.

The choice of M_1 and M_2 is a trade-off between the increase in complexity and aliasing rejection in the folding bands. For example, as a result of an increase in M_2 (decreasing M_1), more folding bands will result in additional zeros, and thus, an increase in the overall aliasing rejection. However, from (9), an increase in M_2 leads to an increase in APOS.

As mentioned before, the choice of K_1 improves aliasing rejection in the folding bands, which does not obtain additional zeros from the IZ filter $Q(z)$. However, from (9), we observe that the choice $K_1 = K_2$ may decrease the number of APOS and improve aliasing rejection. However, this choice results in an increased passband droop. Next example illustrates the choice of the parameters.

Example 2: We consider the choice of the decimation factors M_1 and M_2 taking $M = 12$. In the first case, the priority is to improve the aliasing rejection in folding bands, as much as possible. Therefore, we chose $M_1 = 3$, $M_2 = 4$, $K_1 = 3$, $K_2 = 1$, so that only the fourth folding band will not obtain additional zeros.

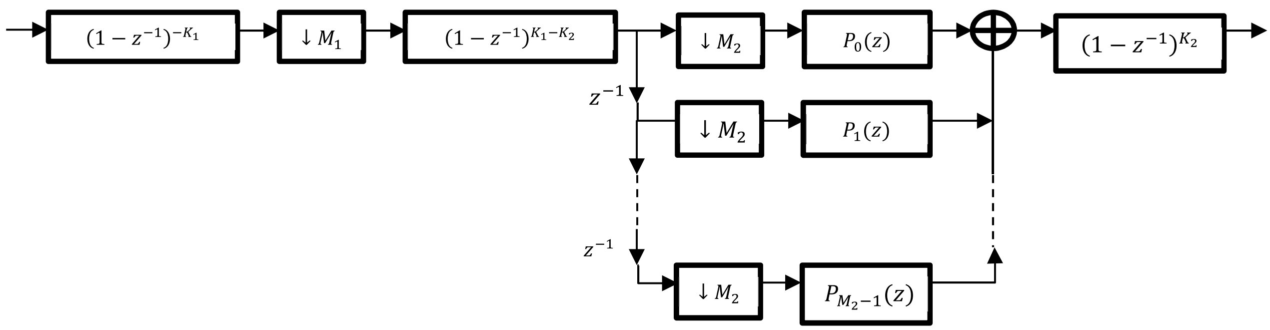


FIGURE 3. Proposed structure with polyphase decomposition.

TABLE 1. SPT Coefficients of $Q_1(z)$ for $M_2 = 4$. $N_p = 29$.

P_k	SPT coefficients of $Q_1(z)$	N_{pk}
P_0	$2^0, 2^5-2^2, 2^5-2^2, 2^0$	4
P_1	$2^2, 2^5+2^1+2^0+2^{-1}+2^{-3}, 2^4+2^1+2^{-1}$	8
P_2	$2^3+2^1+2^{-1}+2^{-2}, 2^5+2^2+2^1+2^{-1}+2^{-2}, 2^3+2^1+2^{-1}+2^{-2}$	9
P_3	$2^4+2^1+2^{-1}, 2^5+2^1+2^0+2^{-1}+2^{-3}, 2^2$	8

TABLE 2. SPT Coefficients of.

P_k	SPT coefficients of $Q_2(z)$	N_{pk}
P_0	$2^0, 2^4-2^1+2^{-1}, 2^3+2^1+2^{-2}$	6
P_1	$2^2, 2^4+2^{-1}+2^{-2}+2^{-3}, 2^2$	5
P_2	$2^3+2^1+2^{-2}, 2^4-2^1+2^{-1}, 2^2$	6

Table 1 presents the SPT coefficients of the polyphase components of the IZ filter for the first case, that is, $Q_1(z)$. The number of adders for the polyphase components is given in the third column, considering the symmetry in P_0 and P_2. From Table 1, $N_p = 29$, and from Eq. (8), the number of $\mathrm{APOS}_{Q1} = 3 + 29 = 32$. We obtain the total number of $\mathrm{APOS}_{\mathrm{case1}}$ from (9), as follows:

$$\begin{aligned} APOS_{\mathrm{case1}} &= APOS_{\mathrm{Q1}} + APOS_{\mathrm{combs1}} \\ &= 32 + 36 + 8 + 1 = 77. \end{aligned}$$

In the second case, the priority is low complexity, so we chose $M_1 = 4$, $M_2 = 3$, $K_1 = 3$, $K_2 = 1$. The SPT coefficients of the polyphase components of filter $Q_2(z)$ are listed in Table 2, yielding $\mathrm{APOS}_{Q2} = M_2\text{-}1+N_p = 2 + 17 = 19$.

From (9) we have:

$$\begin{aligned} APOS_{\mathrm{case2}} &= APOS_{\mathrm{Q2}} + APOS_{\mathrm{combs2}} \\ &= 19 + 36 + 6 + 1 = 62. \end{aligned}$$

As expected, the number of APOS is higher in the first case. The magnitude responses are contrasted in Fig. 4, showing better aliasing rejection in the first case than in the second. (The third and sixth folding bands did not have any additional zeros).

However, from (9), we can observe that using $K_1 = K_2$ will decrease the number of APOS in both cases and will improve the aliasing rejection in the folding bands with the additional zeros at the price of increasing the passband droop. If $K_1 = K_2 = 3$, the number of APOS is **70**for the first case and **58** for the second case. Figure 4(b) presents the overall magnitude responses for $M_1 = 3$, $M_2 = 4$, and $K_1 = K_2 = 3$ (first case) and $M_1 = 4$, $M_2 = 3$, and $K_1 = K_2 = 3$ (second case). It can be observed that the magnitude responses are improved in certain folding bands and that the passband droops are increased, requiring a more complex compensator.

III. COMPENSATOR DESIGN

A. MAGNITUDE CHARACTERISTIC

Usually, the compensator design is presented for comb filters [17]–[21], where the design parameters are the decimation factor M and order of comb K. In [15] a comb compensator designed for an equivalent comb order was used. However, we have four design parameters, and cannot directly use a comb compensator.

We consider the magnitude response of the comb compensator [17] to be a product of two sinusoidal functions:

$$\left|G\left(e^{jM\omega}\right)\right| = \left[1 + Asin^4\left(\omega M/2\right)\right] \times \left[1 + Bsin^2\left(\omega M/2\right)\right], \quad (10)$$

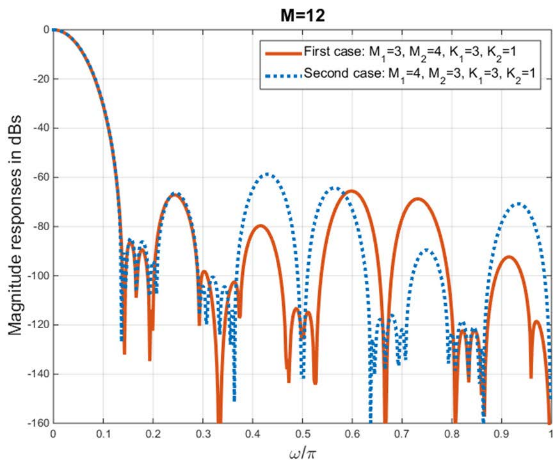
where A and B are the amplitudes of the sinusoidal functions that depend on comb parameters M and K.

The compensator coefficients were obtained from parameters A and B, as presented in [17]. Note that the magnitude response of the compensator is expanded by M, because the compensator works at a low rate, that is, after decimation by M.

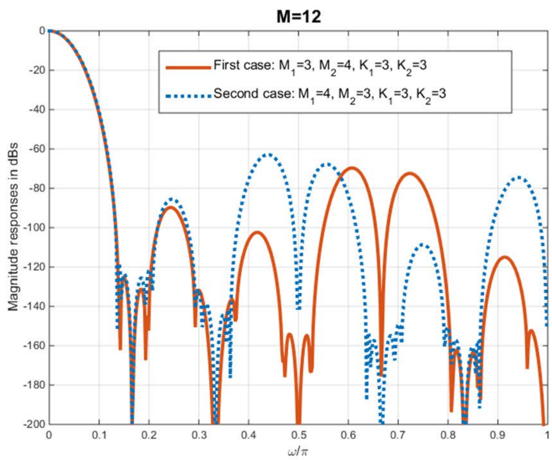
Because the compensator (10) may perform good compensation even for high values of passband droop, under the condition that the values of the compensator parameters A and B are chosen properly, we adopted the magnitude characteristic of the compensator as a product of the sinusoidal functions:

$$\left|C\left(e^{jM\omega}\right)\right| = \left[1 + A_1 sin^4\left(\omega M/2\right)\right] \times \left[1 + A_2 sin^2\left(\omega M/2\right)\right], \quad (11)$$

where the parameters A_1 and A_2 depend on the parameters of the proposed filter M_1, M_2, K_1, and K_2.



(a)



(b)

FIGURE 4. Magnitude responses of the proposed filter for different design parameters. (a) This figure shows the choice of decimations factors M_1 and M_2. In the case 1, $M_2 > M_1$, the aliasing rejection is higher than in the case 2, $M_2 < M_1$, while in both cases $K_1 = 3$, $K_2 = 1$. (b) This figure illustrates how the choice $K_1 = K_2$ affects the magnitude characteristic. The aliasing rejection is increased, while the passband droop is also increased.

From (5) the magnitude response of the proposed compensated filter is equal:

$$H_p\left(e^{j\omega}\right) = H_1\left(e^{j\omega}\right) H_2\left(e^{j\omega M_1}\right) Q\left(e^{j\omega M_1}\right) C\left(e^{j\omega M}\right), \tag{12}$$

where $C\left(e^{j\omega M}\right)$ is given in (11), and $H_1\left(e^{j\omega}\right)$, $H_2\left(e^{j\omega M_1}\right)$, and $Q\left(e^{j\omega M_1}\right)$, are the frequency characteristics of the filters given in (1)-(4), respectively.

B. SELECTING COMPENSATOR PARAMETERS

Following good results for comb compensation design based on particle swarm optimization (PSO) in [18]–[21], and the possibility of solving problems using MATLAB, we also propose PSO to find the compensator parameters.

We consider wideband passband compensation in the range of $[0, \omega_p]$, where

$$\omega_p = \pi / 2M, \tag{13}$$

and M is the overall decimation factor.

For given values M_1, M_2, K_1, and K_2, the optimum values of parameters A_1 and A_2 are obtained, satisfying the local minimum of the function:

$$\left|H_1\left(e^{j\omega}\right) H_2\left(e^{j\omega M_1}\right) Q\left(z^{M_1}\right) C\left(e^{j\omega M}\right)\right|, for 0 \leq \omega \leq \omega_p. \tag{14}$$

Next, the obtained parameters A_{1opt} and A_{2opt} are presented in signed power of two (SPT) forms.

The total number of adders N_C for the compensator is given as [17]

$$N_C = 9 + N_{A1} + N_{A2}, \tag{15}$$

where N_{A1} and N_{A2} are the numbers of adders in the SPT parameters A_1 and A_2, respectively.

From (9) and (15), the total number of APOS for the compensated proposed filter is:

$$APOS_{\mathrm{p}} = APOS + N_{\mathrm{C}} = M_2 - 1 + N_P + K_1 M + (K_1 - K_2)M_2 + K_2 + 9 + N_{A1} + N_{A2}. \tag{16}$$

The optimum values of A_{1opt} and A_{2opt} were obtained using the MATLAB function `particleswarm.m` [22].

Using (14), we generated the MATLAB function `psoiz.m`, which calls `particleswarm.m`.

In the following example, we illustrate the steps for determining the parameters of the compensator using MATLAB.

Example 3: We consider $M = 15$, $M_1 = 3$, $M_2 = 5$, $K_1 = 5$, and $K_2 = 1$.

First step:

We determine the optimal values for the parameters A_1 and A_2 and the corresponding passband deviation δ_{opt}.

$$A_{1\mathrm{opt}} = 0.8891; A_{2\mathrm{opt}} = 0.8048; \delta_{\mathrm{opt}} = 0.0249.$$

We compare the magnitude responses of the proposed filter with and without the compensator in Fig. 5. We observe that the compensation does not affect the attenuation in the folding bands.

Second step:

We present the obtained optimal values in the SPT form by adapting the MATLAB program given in [18] and [19], choosing $N_{A1} = N_{A2} = 2$.

According to (15), the compensator requires 13 adders, and the compensator parameters are

$A_1 = 2^0$-2^{-3}-2^{-10}, $A_2 = 2^1$-2^{-2}+2^{-4}, resulting in passband deviation of $\delta = 0.0281$dB.

As a result of the SPT presentation, the maximum passband deviation δ was slightly increased in comparison with δ_{opt}. Taking the SPT compensator parameters $A_1 = 2^0$-2^{-3}-2^{-10} and $A_2 = 2^1$-2^{-2}+2^{-4}, we plot the magnitude responses

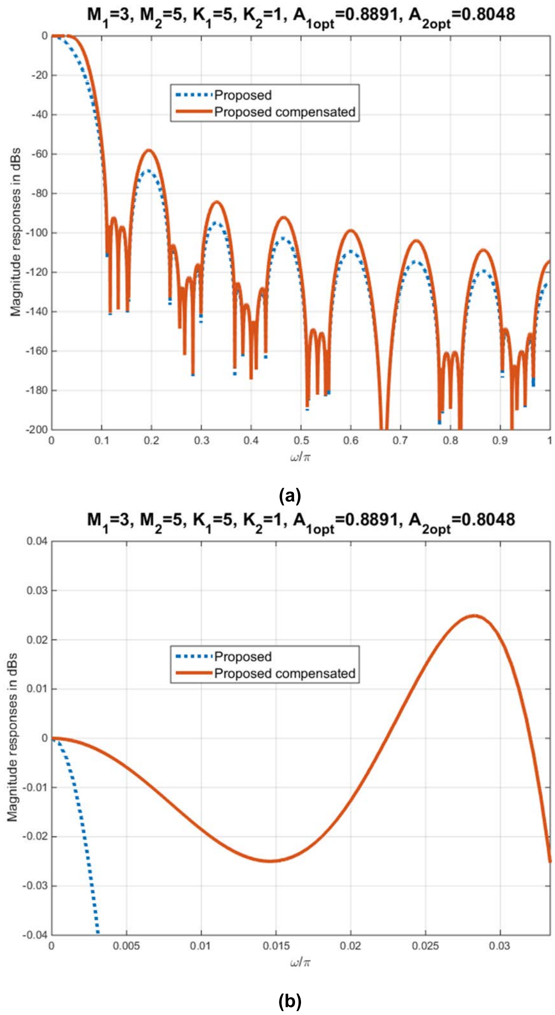


FIGURE 5. Magnitude responses of the non-compensated filter, and the compensated filter with the optimum compensator parameters. (a) Overall magnitude responses. This figure demonstrates that the compensation filter does not affect the attenuation in the folding bands. (b) Passband zoom. This graphic shows that compensator compensates passband droop resulting in the maximum deviation of 0.0249 dB.

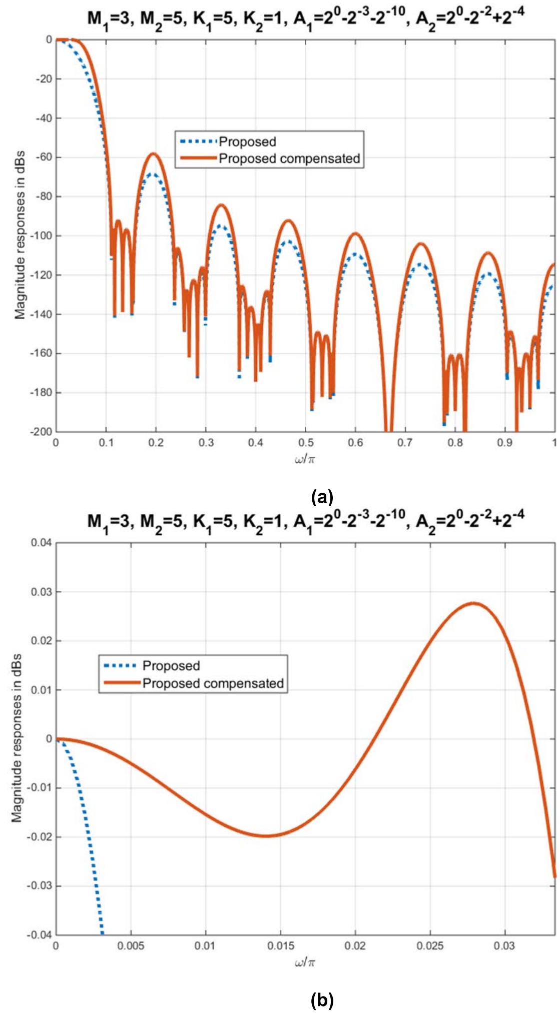


FIGURE 6. Magnitude responses of the non-compensated filter and the compensated filter with SPT compensator parameters. (a) Overall magnitude responses. The compensator does not affect the attenuation in the folding bands. (b) Passband zoom. In comparison with Fig. 5(b) the maximum passband deviation is slightly increased and is equal to 0.0281dB.

of the proposed compensated and non-compensated filters, as shown in Fig. 6.

The flexibility of the compensator design is presented in Table 3, which shows how an increased number of adders of the compensator results in a decreased maximum passband deviation.

The first column presents the total number of adders of the compensator, whereas the second column shows the maximum values of the passband deviation, δ, in dB. Similarly, the third column presents the number of adders N_{A_1}, and N_{A_2}, while the last two columns show the parameters A_1 and A_2 in the SPT forms.

The compensator with 15 adders has a passband deviation of **0.0252 dB** which is close to the optimal value of **0.0249 dB**.

TABLE 3. SPT coefficients of compensator parameters.

N_C	DEV.[DB]	N_{A1}; N_{A2}	A_1	A_2
11	0.0686	1; 1	2^0-2^{-5};	2^0-2^{-2}
12	0.0287	1;2	2^0-2^{-3}	2^0-2^{-2}+2^{-4}
13	0.0278	1;3	2^0-2^{-3}	2^0-2^{-2}+2^{-4}-2^{-11}
14	0.0263	2; 3	2^0-2^{-3}+2^{-6}	2^0-2^{-2}+2^{-4}-2^{-7}
15	0.0252	3;3	2^0-$2^{-3}2^{-6}$-2^{-10}	2^0-2^{-2}+2^{-4}-2^{-7}

IV. DISCUSSION

To demonstrate the benefits of our proposal, we compared the proposed method with similar methods recently proposed in the literature with respect to overall aliasing rejection, worst-case attenuation γ in dB, overall passband characteristic, maximum passband deviation δ in dB, and the complexity expressed in the number of APOS.

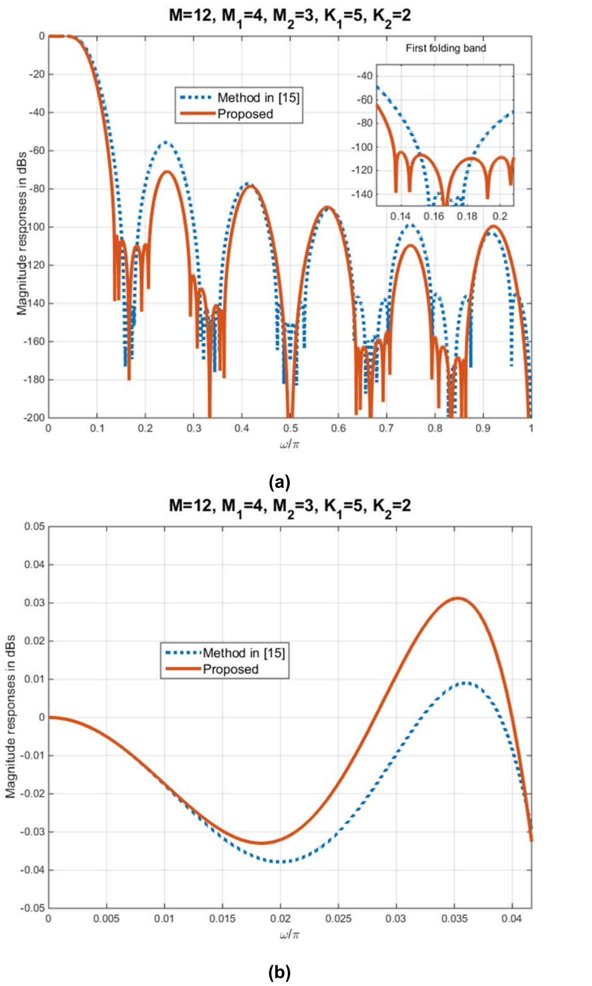


FIGURE 7. **Comparison with method in [15]. (a) Overall magnitude responses and the first folding band zoom. Filter in [15] provides slightly better aliasing rejection in the third and seventh folding bands, while the proposed filter provides much better aliasing rejection in all other folding bands. The zoom in the first folding band shows that the worst case attenuation in [15] is 48.4 dB, while in the proposed filter is 64 dB. (b) Passband zoom. The proposed filter has better passband characteristic with the maximum deviation of 0.0330 dB, while the filter in [15] has the maximum deviation of 0.0373 dB.**

A. COMPARISON WITH METHOD IN [15]

In [15], a comb decimation filter was proposed in which the aliasing rejection was improved by introducing simple filters at a high input rate, while the passband droop was decreased by the comb compensator working at a low rate, taking as a parameter an equivalent comb order K. In this comparison, we used the filter in Example 2 in [15]: $M = 12, K = 5, a_1 = 2^{-1}, a_2 = 2^{-4}$. The compensator is designed as in Example 3, [15] requiring 12 adders, $A = 2^0\text{-}2^{-3}, B = 2^0\text{-}2^{-2}+ 2^{-6}$. The total number of $\text{APOS}_{[15]}$ =**397**.

In the proposed design $M_1 = 4$, $M_2 = 3$, $K_1 = 5$, $K_2 = 2$. The compensator parameters were $A_1 = 2^0+ 2^{-3}\text{-}2^{-7}$ and $A_2 = 2^0\text{-}2^{-4}\text{-}2^{-8}$ requiring 13 adders.

According to (9) and Table 2, the total number of $\text{APOS}_\text{p} = 5 \times 12 + (5\text{-}2)3 + 2 + 19 + 13 =$ **103**. The magnitude responses are shown in Fig.7.

The filter in [15] provides slightly better aliasing rejection in the third and sixth folding bands, whereas the proposed method provides a better aliasing rejection in all other folding bands, including the first folding band, where the minimum attenuation occurs, which is equal to $\gamma_{[15]}$ = **48.4 dB**, and γ_p = **64 dB** in [15] and the proposed method, respectively. The proposed method also provides better passband characteristic, with the maximum deviation δp = **0.0330 dB**. However, in the filter [15] the maximum deviation is equal to $\delta_{[15]}$ = **0.0373 dB**.

B. COMPARISON WITH METHOD IN [12]

A two-stage structure, in which a separation zero (SZ) filter is inserted in the second stage and the compensator is added at a low rate, was proposed in [12]. For comparison, we selected Example 4 in [12] with the following parameters: $M_1 = 3$, $M_2 = 5$, $K = 3$. The compensator with the parameters $A = 1\text{-}2^{-4}$, and $B = 2^{-1}+ 2^{-4}$, requires eight adders. The total number of APOS in [12] is $\text{APOS}_{[12]}$ = **85**.

In the proposed method, the design parameters were $M_1 = 5$, $M_2 = 3$, $K_1 = 3$, $K_2 = 3$. The parameters of the compensator are as follows: $A_1 = 2^0+2^{-2}+2^{-4}$, $A_2 = 2^0+2^{-5}+2^{-7}$, requiring 13 adders. The total number of APOS, according to (9) and Table 2, was equal to $\text{APOS}_\text{p} = 3\times 15+3+19+13 =$ **80**. The magnitude responses are shown in Fig. 8.

Note that the proposed method provides better aliasing rejection in all folding bands except in the third and sixth folding bands. The minimum attenuation of the filter [12] is in the first folding band and is equal to $\gamma_{[12]}$ = **30 dB**, whereas in the proposed filter, it is in the third folding band and is equal to γ_p = **65.6 dB**.

The proposed method also has a better passband characteristic with a maximum passband deviation of

δ_p = **0.041 dB** in comparison with that of the filter [12], which is equal to $\delta_{[12]}$ = **0.0429 dB.**

C. COMPARISON WITH METHOD IN [9]

A two-stage comb filter, where the sharpening coefficients were obtained by linear programming, was proposed in [9]. The filter improves aliasing rejection and compensates for passband droop. For comparison, we take the values of $M = 32$, $M_1 = 4$, and $M_2 = 8$ from [9]. The comb order in the first stage was $Q = 2$. The design parameters of the second stage are listed in Table 1 in [9].

In the proposed method, $M_1 = 8$, $M_2 = 4$, $K_1 = 3$, $K_2 = 1$, and the parameters of the compensator are $A_1 = 2^0\text{-}2^{-3}+2^{-8}$, $A_2 = 2^0\text{-}2^{-4}$, requiring 12 adders. The magnitude responses are compared in Fig. 9. The proposed method provides better aliasing rejection for all folding bands. The minimum attenuation in the filter [9] is $\gamma_{[9]}$ = **30 dB**, whereas in the proposed filter is γ_p = **55 dB**.

Additionally, the proposed method has a better passband characteristic with a maximum passband deviation of δ_p = **0.029 dB** in comparison with that of the filter [9], which is equal to $\delta_{[9]}$ = **0.16 dB.**

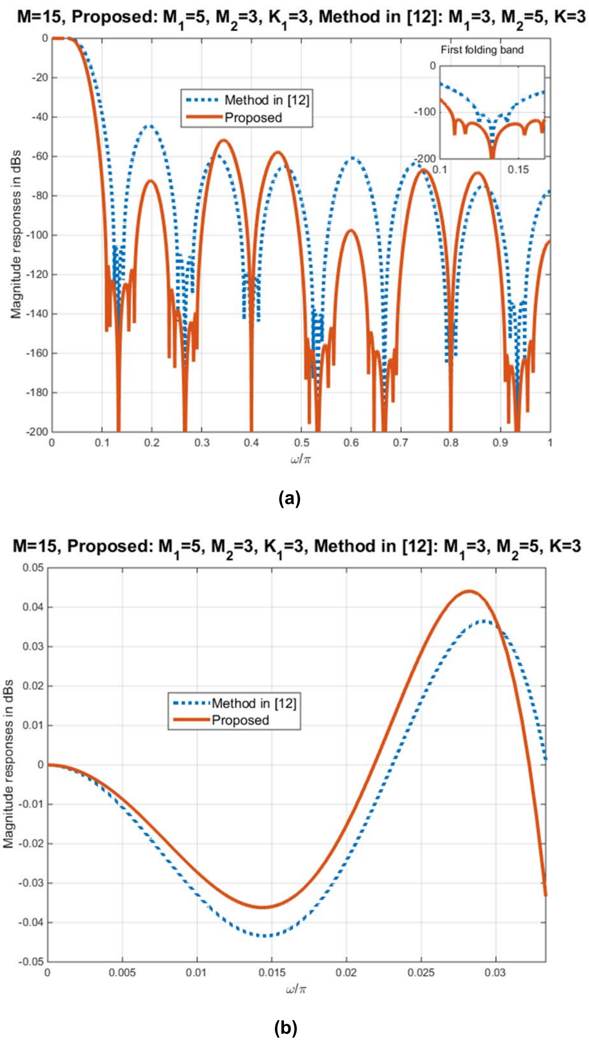


FIGURE 8. Comparison with method in [12]. (a) Overall magnitude responses and first folding band zoom. The proposed method provides higher aliasing rejection in all folding bands except in third and sixth folding bands, where filter [12] provides slightly better aliasing rejection. Minimum attenuations in filter [12] and the proposed filter are 40 dB (first folding band) and 61 dB, (third folding band), respectively. (b) Passband zoom. The proposed filter provides better passband characteristic with the maximum deviation equal to 0.041 dB, in comparison with that in [12], which is equal to 0.0429 dB.

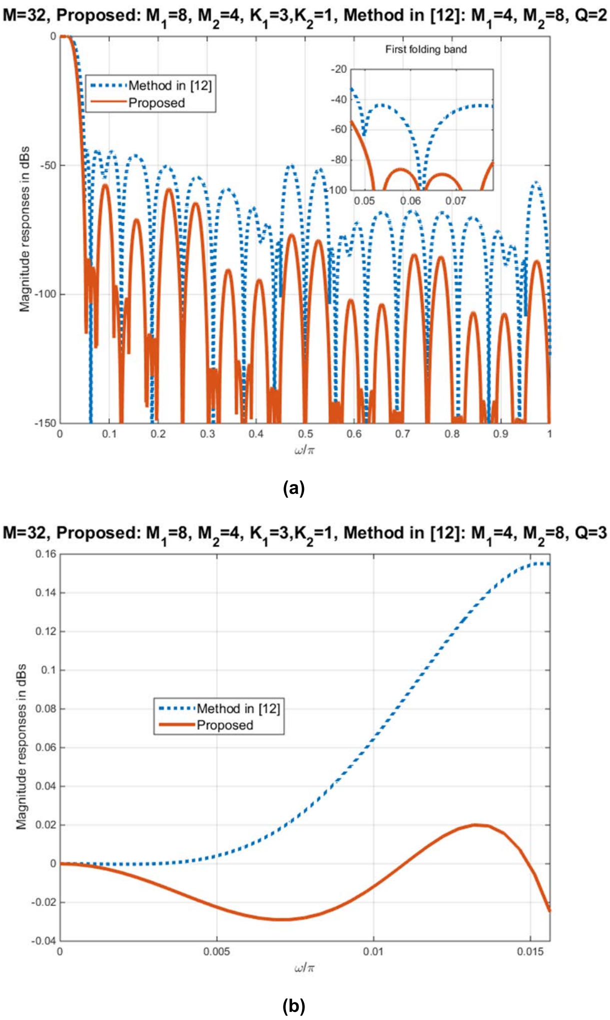


FIGURE 9. Comparison with method in [9]. (a) Overall magnitude responses and zooms in the first and fourth folding bands. The zoom in the fourth folding band is presented to show that it provides higher minimum attenuation than the first folding band. The minimum attenuations in the first folding band are equal to 30 dB, and 55 dB for filter in [9], and the proposed filter, respectively. (b) Passband zoom. The proposed filter provides better passband characteristic with the maximum deviation of 0.029 dB, while the maximum deviation in filter [9] is equal to 0.16 dB.

The number of APOS reported in [9] in the first stage, $Q(M_1+1) = 10$, is incorrect and should be $Q(M + M_2) = 80$. The number of APOS for the sharpening structure in the second stage was obtained using equation (18) in [9], resulting in $\mathrm{APOS}_{\mathrm{Sh}} = 83$. Thus, the total number of APOS in [9] is equal to $\mathrm{APOS}_{[9]}$=$Q(M + M_2)$+ 83 = 2(32+8) + 83 = **163.** The number of APOS in the proposed filter was obtained from (9) and Table 1 as $\mathrm{APOS}_{\mathrm{p}} = 3 \times 32$+(3-1)4+1+ 32 = **149.**

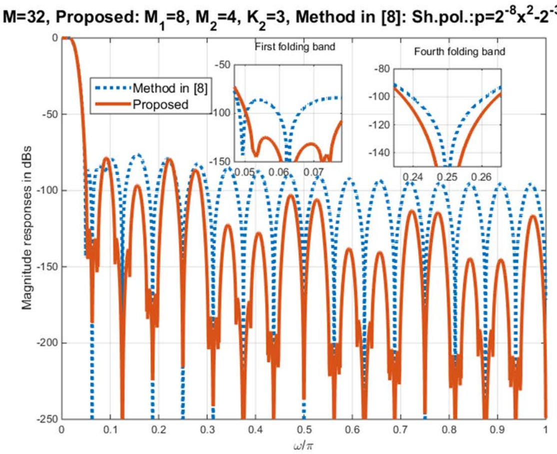
D. COMPARISON WITH METHOD IN [8]

Optimum compensators for sharpened CIC filters were proposed in [8]. For comparison, we choose the sharpening polynomial $p = 2^{-8}x^2\text{-}2^{-3}x^4 + x^6$, where x is a comb filter of order 1, and $M = 32$, from Table 1 in [8]. The coefficients of the compensator are given in the table, and the compensator requires seven adders, whereas $\delta_{[8]=}$ **0.13 dB**.

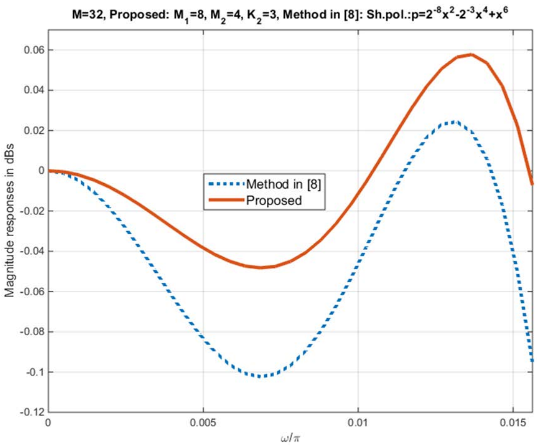
The number of APOS in [8] was calculated using (17) presented in [9] for the APOS of sharpened CIC filters. To this end, we get $\mathrm{APOS}_{[8]} = 6(32+1)+(3\text{-}1)+7 =$ **207**.

The design parameters in the proposed method were $M_1 = 8$, $M_2 = 4$, $K_1 = 4$, $K_2 = 3$. The parameters of the compensator are $A_1 = 2^1\text{-}2^{-1}\text{-}2^{-4}$ and $A_2 = 2^0 + 2^{-4}$, requiring 12 adders and resulting in $\delta_p =$ **0.0578 dB**. From (9) and Table 1, the total number of APOS is equal to $\mathrm{APOS}_{\mathrm{p}} = 4 \times 32 + 1 \times 4 + 3 + 32 + 12 =$ **179.**

The magnitude responses are shown in Fig. 10. The proposed method provides better aliasing rejection for all folding bands. The minimum attenuation in the proposed filter is



(a)



(b)

FIGURE 10. Comparison with method in [8]. (a) Overall magnitude responses and first and fourth folding bands. Proposed method provides better aliasing rejection in all folding bands. Minimum attenuation provided by proposed method is equal to 75 dB, while minimum attenuation provided by method [8] is equal to 76.4 dB. (b) Passband zoom. The proposed method provides better passband characteristic in all passband. The maximum passband deviation in proposed filter is 0.0578 dB, while in filter [8] is equal to 1.25 dB.

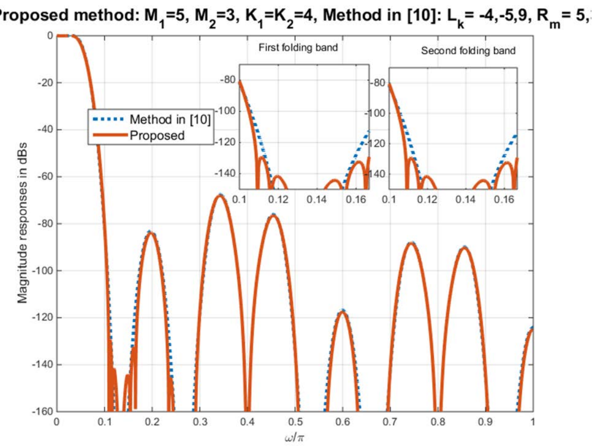
equal to $\gamma_{\rm p} = \mathbf{75\ dB}$, whereas that in method [8] is equal to $\gamma_{[8]} = \mathbf{76.4\ dB}$.

E. COMPARISON WITH METHOD IN [10]

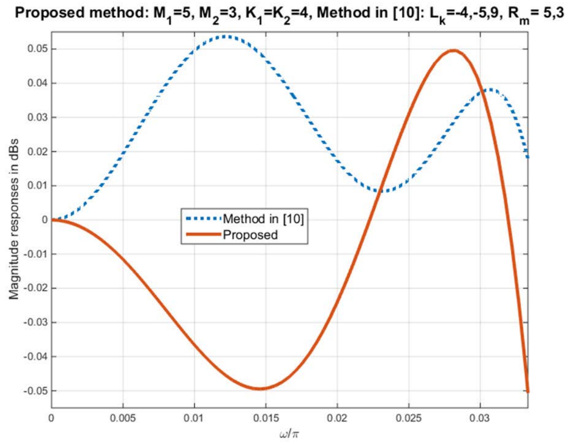
The design of a multistage decimation filter with a minimum number of APOS for a given decimation factor and minimum worst-case attenuation was proposed in [10].

For comparison, we take an example for $M = 15$, as given in Table 4 in [10]. The distribution of integrators and combs along the stages are $L_k = -4, -5, 9$; the decimation factors are $R_m = 5, 3$. The minimum attenuation is $\gamma_{\rm NC[10]} = \mathbf{89.4\ dB}$, and the total number of $\text{APOS}_{\rm NC[10]} = \mathbf{84}$.

In the proposed method, the design parameters were $M_1 = 5, M_2 = 3, K_1 = K_2 = 4$. The minimum attenuation is



(a)



(b)

FIGURE 11. Comparison with method in [10]. (a) Overall magnitude responses and zooms in first and second folding bands. Both methods provide equal minimum attenuation of 81.4 dB. Proposed method provides slightly better aliasing rejection in the first and second folding bands, while attenuations in all other folding bands are equal. (b) Passband zoom. Proposed method provides slightly better passband characteristic with maximum deviation of 0.051 dB, while maximum deviation in filter [10] is equal to 0.053 dB.

$\gamma_{NCp} = \mathbf{88\ dB}$, while the number of APOS$= 15 \times 4 + 4 + 2 + 17 = \mathbf{83}$. The proposed filter has one APOS less, and slight better attenuation in the first folding band, while the method in [10] provides slightly increased the minimum attenuation.

The compensator from [8] was applied in [10] with the reported coefficients: [511, -240, 72, -12] requiring 10 adders and resulting in maximum deviation of $\delta_{[10]} = \mathbf{0.053\ dB}$. It is worth mentioning that this compensator, unlike the proposed compensator, does not force compensators-unity gain and should be normalized. The total number of APOS is equal to $\text{APOS}_{[10]} = \text{APOS}_{\rm NC[10]} + 10 = \mathbf{94}$.The proposed compensator has the following parameters: $A_1 = 2^1$-2^{-1}+ 2^{-3}, $A_2 = 2^0$+ 2^{-2}-2^{-4}-2^{-7}, and requires 14 adders. The maximum passband deviation is $\delta_p = \mathbf{0.051\ dB}$. The total number of APOS in the proposed filter is $\text{APOS}_{\rm p} = \text{APOS}+14 = \mathbf{97}$, i.e. three more APOS than in work from 10]. The magnitude

TABLE 4. Summary of comparisons.

Method and design parameters	Numb. of APOS	Min. atten.in dB	Max. passb. dev. in dB	Comment
[15] M=12, K=5	397	48.4	0.0373	This filter has a high number of APOS since all its filters work at high input rate. The filter provides slightly better aliasing rejection in third and sixth folding bands.
Proposed M_1=4, M_2=3, K_1=5, K_2=2	**103**	**64**	**0.0330**	The number of APOS in proposed filter is approximately 25.9% of the APOS$_{[15]}$, while has an increased minimum attenuation and a decreased passband deviation. Additionally, the proposed filter provides better aliasing rejection in the first, second, fourth, and fifth folding bands.
[12] M_1=3, M_2=5, K=3	85	30	0.0429	The filter provides better aliasing rejection in third and fifth folding bands.
Proposed M_1=5, M_2=3,	**80**	**65.6**	**0.041**	Proposed filter provides better aliasing rejection in the first, second, fourth,
K_1=K_2=3				and sixth folding bands, while requires less number of APOS, higher minimum attenuation, and reduced passband deviation.
[9] M_1=4, M_2=8, Q=2	163	30	0.16	The number of APOS for the second stage with the sharpening combs is obtained using (18) in [9].
Proposed M_1=8, M_2=4, K_1=3, K_2=1	**149**	**55**	**0.029**	Proposed filter provides better aliasing rejection in all folding bands, while requires fewer number of APOS, higher minimum attenuation and smaller passband deviation.
[8] M=32 $p=2^{-8}x^2-2^{-3}x^4+x^6$	207	**76.4**	0.13	The authors did not report the number of APOS. In order to get fair comparison, the number of APOS is calculated using (18) for sharpening combs from [9].
Proposed M_1=8, M_2=4, K_1=4, K_2=3	**179**	75	**0.0578**	Proposed filter provides better aliasing rejection in all folding bands, while requires less number of APOS, and less passband deviation.

TABLE 4. *(Continued.)* Summary of comparisons.

[10] L_k=-4, -5, 9; R_m=5, 3	84*	**89.4***		* without compensator. Non-compensated filter requires 84 APOS, while the minimum attenuation is slightly better than in the proposed method.
	94**	81.4**	0.053	** with compensator. The compensator used from [8], does not force unity gain and should be normalized. It requires 10 adders, and the compensated filter needs 94 APOS, i.e. three APOS less than the proposed method.
Proposed M_1=5, M_2=3, K_1=K_2=4	**83***	88*		* without compensator. The number of APOS without compensator is equal to 83, i.e. less than in [10], having slightly better aliasing rejection in the first folding band, while the minimum attenuation is better in [10].
	97**	81.4**	**0.051****	. ** with compensator. The proposed compensator requires 14 adders resulting in a less passband deviation than in [10]. As a result, the compensated filter requires three APOS more than filter in [10].The minimum attenuation is equal in both filters, while the proposed filter provides slightly better attenuation in the first and second folding bands

responses are compared in Fig. 11. The aliasing rejections in both methods are similar, with a slight improvement in the first and second folding bands in the proposed method, whereas the minimum attenuations are equal. Generally, the method in [10] with the compensator requires a slightly smaller number of APOS, whereas the proposed filter has a slightly better passband characteristic and slightly better attenuation in the first and second folding bands.

F. SUMMARY OF COMPARISONS

Comparisons with recently proposed methods from the literature, presented in previous sections, are summarized in Table 4. The first column presents the design methods and parameters used in the comparison. The second column presents the number of APOS, whereas the third and fourth columns present the minimum attenuation in dB and the maximum passband deviation in dB, respectively. The last column contains specific comments related to the corresponding method. The favorable values are presented in bold.

V. CONCLUSION

Comparisons with methods from the literature show that the proposed method generally provides better aliasing rejection and passband compensation, while requiring fewer APOS. It is worth mentioning that the method in [10] is dedicated to the design of multistage decimation filters for minimum stopband attenuation with the minimum number of APOS. However, the proposed method decreases aliasing rejection while maintaining a low number of APOS. Nevertheless, the comparison of non-compensated filters shows that the proposed method requires less APOS while providing better aliasing rejection in the first folding band. However, the compensated filter in [10], with the compensator from [8] requires three APOS less, while the proposed compensated filter provides less passband deviation. The minimum attenuations are equal while the proposed filter provides slightly better aliasing rejection in the first and second folding bands.

This work is an algorithmic and simulation–based work that is in the same scope as all the references used for the comparison, including the work published in IEEE Access [9].

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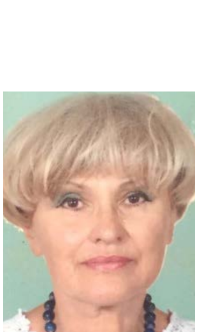

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