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Optimal Disturbance Suppression of Disturbed Underwater Vehicle With State Delay

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ABSTRACT In this paper, the optimal disturbance suppression problem of disturbed underwater vehicle with state delay is studied. Firstly, we establish the mathematical model of the motion of the disturbed underwater vehicle and linearize it. Secondly, the mathematical model with state delay is transformed into a formal mathematical model without time-delay by using the no time-delay transformation method. Then, according to the optimal control theory, the optimal disturbance suppression control law of the disturbed underwater vehicle with state delay is designed based on the quadratic performance index. This control law can be obtained by solving the Riccati matrix equation and Sylvester matrix equation. Finally, the effectiveness of the method is verified by an AUV system simulation example.

INDEX TERMS Underwater vehicle, state delay, optimal disturbance suppression.

I. INTRODUCTION

Autonomous Underwater Vehicle (AUV) is an unmanned underwater vehicle that can move or work autonomously. It can complete specific tasks such as underwater geological survey, marine biological information collection, underwater military operations and marine resources development, and can help human beings complete tasks that human beings cannot directly participate in. Therefore, the application prospect of autonomous underwater vehicle is very broad [1]–[3]. When the underwater vehicle works in the marine environment, it will be affected by various uncertain factors [4], such as marine organisms, ocean current disturbances, uncertain obstacles, etc., resulting in poor stability and dynamic characteristics of the underwater vehicle [5]–[7]. Guerrero *et al.* [8] introduced a second-order sliding mode controller (SMC) named ‘‘generalized super-twisting algorithm (GSTA)’’ for automatic gain adjustment to cope with external disturbances along with uncertain dynamic errors. Yan *et al.* [9] and Li *et al.* [10] respectively proposed a ‘‘real-time reaction obstacle avoidance algorithm (RRA)’’ and a ‘‘predictive guidance obstacle avoidance algorithm (PGOA)’’ to deal with complicated terrain structure in the unpredictable oceanic environment based on information provided by ‘‘forward-looking sonar (FLS)’’. Different path planning controllers such as ‘‘active disturbance rejection controller (ADRC)’’ [11], ‘‘SMC’’ [12] and high precision PD controller [13] have also been proposed in the literature. An adaptive SMC controller to cope with speed changes when a high-speed AUV surfaces to strike air targets has been proposed by Xiao [14]. Jung *et al.* [15] used ‘‘line of sight (LOS)’’ navigation and proposed a methodology for PP of AUVs. It assumed that AUVs track an LOS path as per required formation motion. It was intended to address the dynamics of shallow water effectively without missing targets. In the working process of underwater vehicles, there are state time-delays generally.

It is very meaningful to study the optimal disturbance suppression problem of this kind of disturbed underwater vehicle with state delay. However, there are few studies on these aspects. On the basis of previous studies, the disturbance suppression of this kind of underwater vehicle is studied for the first time in this paper. The no time-delay transformation method is also used to deal with the AUV model for the first time.

II. MATHEMATICAL MODEL OF AUV MOTION

Fossen first proposed the idea of establishing AUV six degrees of freedom model in literature [7]. The kinematic model and dynamic model of underwater vehicle systems are described in detail, but it is a very strong coupling nonlinear system. In this paper, on the premise of making appropriate

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idealization, we appropriately simplify the model so that we can get a mathematical model convenient for us to study. The research object of this paper is a small autonomous underwater vehicle with a total length of about three meters and a three-blade propeller at the tail to provide thrust for AUV. In addition, there are a pair of horizontal rudders and a pair of vertical rudders to control the navigation direction, diving depth and attitude of the AUV. This kind of AUV can be used for seabed cruise, marine resource survey, marine scientific research, search and other purposes. It relies on the propeller installed on the driving carrier to make three-dimensional motion underwater. In addition, when working underwater, this kind of AUV has the phenomena of variable communication time-delay, limited communication distance, information attenuation and distortion, as well as the defect of limited energy storage. The working state is mainly slow navigation. In this case, the impact of ocean currents and other external disturbances on underwater vehicles should be fully considered.

Different from robots moving on land, AUV has poor dynamic and steady-state performance when working underwater, so the driving dynamic model of a single AUV should be fully considered. In this paper, we consider the influence of state delay on AUV system. Due to the progress of computer technology, the problem of control delay has been greatly improved, so we ignore the control delay in AUV control system. The quality of information transmission depends on the parameters of camera and sonar and the accuracy of optical instruments. Communication is often affected by seawater temperature, salinity and pressure. Therefore, the disturbance of AUV during underwater operation can not be ignored.

We set two coordinate systems: ground coordinate system $O - XYZ$ and body coordinate system $O - X_0Y_0Z_0$ of AUV. The X-axis and Y-axis of ground coordinate system $O - XYZ$ are on the horizontal plane and perpendicular to each other, and the Z-axis is perpendicular to the ground and vertically downward. The origin O of the AUV's body coordinate system $O - X_0Y_0Z_0$ is at the center of gravity of the AUV. The X_0-axis is along the AUV forward direction, the Y_0-axis is perpendicular to the X_0-axis and points to the right side of the AUV forward direction, and the Z_0-axis is perpendicular to the plane composed of the X_0-axis and Y_0-axis. The direction of Z_0-axis is determined by the right-hand rule.

As shown in Fig-2, u, v, w respectively represent the speed of AUV during forward and backward, side shift and lifting movement; p, q, r respectively represent the angular velocity of AUV during roll, pitch and yaw; x, y, z respectively represent the displacement of AUV during forward and backward, lateral movement and lifting movement. In addition, we use m to represent the mass of AUV and $I_{(\cdot)}$ to represent the inertia moment of the system.

According to the literature [7], the dynamic equation of AUV can be written as:

$$
\begin{aligned}
& m[\dot{u} - vr + wq - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})] \\
& = X \\
& m[\dot{v} - wp + ur - y_G(p^2 + r^2) + z_G(pr - \dot{p}) + x_G(qp + \dot{r})] \\
& = Y \\
& m[\dot{w} - uq + vp - z_G(p^2 + q^2) + x_G(rp - \dot{q}) + y_G(rq + \dot{q})] \\
& = Z \\
& I_x\dot{p} + (I_z - I_y)qr - (\dot{r} + pq)I_{xz} + (r^2 - q^2)I_{yz} + (pr - \dot{q})I_{xy} \\
& \quad + m[y_G(\dot{w} - uq + vp) - z_G(\dot{v} - wp + ur)] = K \\
& I_y\dot{q} + (I_x - I_z)rp - (\dot{p} + qr)I_{xy} + (p^2 - r^2)I_{zx} + (qp - \dot{r})I_{yz} \\
& \quad + m[z_G(\dot{u} - vr + wq) - x_G(\dot{w} - up + vp)] = M \\
& I_z\dot{r} + (I_y - I_x)pq - (\dot{q} + rp)I_{yz} + (p^2 - q^2)I_{xy} + (rq - \dot{p})I_{zx} \\
& \quad + m[x_G(\dot{v} - wp + ur) - y_G(\dot{u} - vr + wq)] = N
\end{aligned} \tag{1}
$$

FIGURE 1. Remus-6000 underwater vehicle.

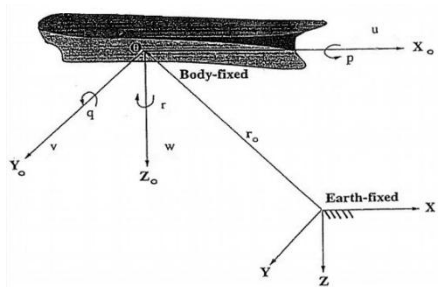

FIGURE 2. Ground coordinate system $O - XYZ$ and body coordinate system $O - X_0Y_0Z_0$.

Here, we let $\upsilon = [u, v, w, p, q, r]^{\mathrm{T}}$ be the vector composed of linear velocity and angular velocity, and let $\tau = [X, Y, Z, K, M, N]^{\mathrm{T}}$ be the vector of the magnitude of external force and moment. In this way, according to Fossen's description, the motion model of the six degrees of freedom AUV can be described as:

$$M\dot{v} + C(v)v + D(v)v + g(\eta) = \tau \tag{2}$$

Taking the AUV introduced in document [7] as the controlled object, its hydrodynamic and inertial parameters are shown in TABLE 1:

We substitute the parameters in TABLE 1 into (1) and (2), and order:

$$x = [x_i]^{\mathrm{T}} = [\bar{x}, \bar{y}, \bar{z}, \psi, \bar{u}, \bar{v}, \bar{\omega}, r]^{\mathrm{T}}$$

TABLE 1. Hydrodynamic and inertial parameters of AUV.

$m = 5454\text{kg}$	$\rho = 1000\text{kg/m}^3$	$L = 5.3\text{m}$	$I_z = 13587\text{Nms}^2$
$X_{\dot{\bar{u}}} = 7.6\text{e}-3$	$Y_{\dot{\bar{v}}} = -5.5\text{e}-2$	$Z_{\dot{\bar{\varpi}}} = -2.4\text{e}-1$	$N_{\dot{\bar{v}}} = 1.2\text{e}-3$
$X_{\bar{v}\bar{v}} = 5.3\text{e}-2$	$Y_{\dot{r}} = 1.2\text{e}-3$	$Z_{\bar{v}\bar{v}} = -6.8\text{e}-2$	$N_{\dot{r}} = -3.4\text{e}-3$
$X_{\bar{\varpi}\bar{\varpi}} = 1.7\text{e}-1$	$Y_{\bar{u}\bar{v}} = -1.0\text{e}-1$	$Z_{rr} = -7.4\text{e}-3$	$N_{\bar{u}\bar{v}} = -7.4\text{e}-3$
$X_{rr} = 4.0\text{e}-3$	$Y_{\bar{u}r} = 3.0\text{e}-2$	$Z_{\bar{u}\bar{\varpi}} = -3.0\text{e}-1$	$N_{\bar{v}\bar{\varpi}} = -2.7\text{e}-2$
$X_{\bar{v}r} = 2.0\text{e}-2$	$Y_{\bar{v}\bar{\varpi}} = 6.8\text{e}-2$	$Z_{\bar{v}r} = 4.5\text{e}-2$	$N_{\bar{u}r} = -1.6\text{e}-3$
	$Y_{\bar{\varpi}r} = -1.9\text{e}-2$		$N_{\bar{\varpi}r} = 7.4\text{e}-3$

$$\begin{aligned} i &= 1, 2, \ldots, 8 \\ u &= [u_1, u_2, u_3, u_4]^{\mathrm{T}} \end{aligned} \tag{3}$$

Then we can get:

$$\begin{bmatrix} T_x \\ T_y \\ T_z \\ M_z \end{bmatrix} = \begin{bmatrix} 6.021 \times 10^3 & 0 & 0 & 0 \\ 0 & 9.549 \times 10^3 & 0 & -4.734 \times 10^2 \\ 0 & 0 & 2.332 \times 10^4 & 0 \\ 0 & -4.735 \times 10^2 & 0 & 2.069 \times 10^4 \end{bmatrix} u \tag{4}$$

T_x, T_y and T_z respectively represent the converted component of the propeller force on each axis, and $M_{\bar{z}}$ is the torque around the $O_L - z$ axis.

The kinematic dynamic equation of AUV can be written as a nonlinear system in the following general form:

$$\dot{x}(t) = \bar{f}(x) + Bu(t) \tag{5}$$

where $x \in R$ represents the state vector of AUV, $u \in R^p$ represents the control vector of AUV, and the nonlinear term is

$$\bar{f}(x) = \begin{bmatrix} x_5 \cos x_4 - x_6 \sin x_4 \\ x_5 \sin x_4 + x_6 \cos x_4 \\ x_7 \\ x_8 \\ 0.127x_6^2 + 0.397x_7^2 + 0.262x_8^2 + 1.153x_6x_8 \\ -0.353x_5x_8 - 0.142x_7x_8 - 0.149x_5x_6 + 0.095x_6x_7 \\ -0.041x_6^2 - 0.125x_8^2 - 0.181x_5x_7 + 0.144x_6x_8 \\ -0.313x_5x_8 + 0.138x_7x_8 - 0.031x_5x_6 - 0.095x_6x_7 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ I_4 \end{bmatrix} \in R^{8\times 4}. \tag{6}$$

The first two components can be obtained by series expansion of $\bar{f}(x)$ at $x = 0$:

$$\begin{aligned} f_1(x) &= x_5 \cos x_4 - x_6 \sin x_4 = x_5 - x_4x_6 - O(x_i^3) \\ f_2(x) &= x_5 \sin x_4 + x_6 \cos x_4 = x_6 + x_4x_5 - O(x_i^3) \end{aligned} \tag{7}$$

Ignoring the higher-order term, the nonlinear term can be rewritten as

$$\begin{aligned} \bar{f}(x) = & \begin{bmatrix} 0 & I_4 \\ 0 & 0 \end{bmatrix}_{8\times 8} x \\ & + \begin{bmatrix} -x_4x_6 \\ x_4x_5 \\ 0 \\ 0 \\ 0.127x_6^2 + 0.397x_7^2 + 0.262x_8^2 + 1.153x_6x_8 \\ -0.357x_5x_8 - 0.142x_7x_8 - 0.149x_5x_6 + 0.095x_6x_7 \\ -0.041x_6^2 - 0.125x_8^2 - 0.181x_5x_7 + 0.144x_6x_8 \\ -0.313x_5x_8 + 0.138x_7x_8 - 0.031x_5x_6 - 0.095x_6x_7 \end{bmatrix} \\ \triangleq \; & Ax + f(x) \end{aligned} \tag{8}$$

where Ax is the linear part and $f(x)$ is the nonlinear part. The series expansion of its elements does not contain terms lower than the power of 2.

So far, the following nonlinear AUV control system with the separation of linear and nonlinear terms is obtained:

$$\dot{x} = Ax(t) + Bu(t) + f(x(t)) \tag{9}$$

If the influence of ocean currents on the AUV system is considered, the AUV control system (1) can be described as:

$$\dot{x} = \bar{f}(x(t), w(t)) + Bu(t) \tag{10}$$

$w(t)$ is the external disturbing force acting on the AUV. Similar to the linearization process, the system (10) can be described as:

$$\dot{x} = Ax(t) + Bu(t) + Dw(t) + f(x(t), w(t)) \tag{11}$$

Considering that there is time-delay in the state transition of AUV system, the AUV control system (5) can be described as:

$$\dot{x}(t) = \bar{f}\,(x(t), x(t-\tau), w(t)) + Bu(t) \tag{12}$$

τ is the state delay. Similar to the previous linearization process, the system (12) can be described as:

$$\begin{aligned} \dot{x} = & Ax(t) + A_1x(t-\tau) + B(u) + Dw(t) \\ & + f(x(t), x(t-\tau), w(t)) \end{aligned} \tag{13}$$

So far, the mathematical model of underwater vehicle with state delay under continuous disturbance has been established, which lays a foundation for the following research.

III. DISTURBED UNDERWATER VEHICLE MODEL

Consider the following class of AUV time-delay systems with disturbances. After linearization, the coefficients of the nonlinear terms are very small and can be ignored without affecting the control accuracy [13]. Assuming that the disturbance term $w(t)$ and the state delay term τ are linear, the mathematical model can be described as:

$$\begin{aligned} \dot{x}(t) &= A_0x(t) + A_1x(t-\tau) + Bu(t) + Dw(t), t > 0, \\ x(t) &= x_0, t \in [-\tau, 0], \\ y(t) &= Cx(t) \end{aligned} \tag{14}$$

$x \in R$ is the state vector, $u \in R^p$ is the control vector, $w(t)$ is the external continuous disturbance, $y \in R^q$ is the output vector of AUV pose, A_0, A_1, B, C, D is a constant matrix of appropriate dimension. We assume that (A_0, B) is controllable, (A_0, C) is fully observable and $\tau > 0$ is the state delay of known size.

AUV is often disturbed by ocean waves in the process of near surface navigation. Ocean waves are extremely complex and irregular random waves. For the convenience of research, the irregular long wind wave is simplified into the following fixed-point long peak wave in practical application [16]:

$$\vartheta(t) = \sum_{j=1}^{l} \vartheta_j(t) = \sum_{j=1}^{l} L_j \cos\theta_j \tag{15}$$

l is the number of constituent waves; $L_j = \sqrt{2S_\vartheta(\omega_j)\Delta\omega_j}$; $\theta_j = -\omega_j t + \varepsilon_j$.

ε_j is a random variable; according to the wave theory, it is evenly distributed between $0 - 2\pi$. ω_j is the frequency of the j-th constituent wave, and $S_\vartheta(\cdot)$ is the spectral density function of the wave. Next, a system model will be constructed to describe the irregular wave force acting on AUV in a two-dimensional horizontal plane [17], [18]. From $\ddot{v} = -\omega_j^2 v_j, j = 1, 2, \cdots l$, we can get:

$$\ddot{v} = -\Omega v \tag{16}$$

$\Omega = diag\{\omega_1^2, \omega_2^2, \cdots, \omega_l^2\}$.

We order $\omega(t) = [v(t)^T, \dot{v}(t)^T]^T$, and then the dynamic characteristics of external disturbance $w(t)$ are described by the following external systems:

$$\dot{\omega}(t) = \begin{bmatrix} 0 & I \\ -\Omega & 0 \end{bmatrix} \omega(t) G\omega(t), \quad t \geq 0 \tag{17}$$

where I is a l-dimensional unit matrix; 0 is l-dimensional zero matrix; G is a constant matrix of appropriate dimension. We assume

$$Re(\lambda_i(G)) \leq 0, \quad i = 1, 2, \cdots n \tag{18}$$

We transform the system with state delay (10) into a system without delay in form. We order

$$\tilde{x}(t) = x(t) + \int_{t-\tau}^{t} e^{A(t-\tau-h)} A_1 x(h) dh \tag{19}$$

Substituting (13) into (10) can get the system:

$$\begin{aligned} \dot{\tilde{x}}(t) &= A\tilde{x}(t) + Bu(t) + Dw(t), \\ \tilde{y}(t) &= C\tilde{x}(t), \\ \tilde{x}(0) &= x(0). \\ x(t) &= \tilde{x}(t) - \int_{t-\tau}^{t} e^{A(t-\tau-h)} A_1 x(h) dh \end{aligned} \tag{20}$$

A is determined by $A = A_0 + e^{-\tau A} A_1$. On account of$(A_0, B)$ is controllable and (A_0, C) is fully observable, we can know (A, B) is controllable and (A, C) is fully observable.

Different quadratic performance indexes can be selected according to the different dynamic characteristics of the external system (17):

I. If the external system (17) is asymptotically stable, we can choose the infinite time-domain quadratic performance index:

$$J = \int_0^\infty \left[\tilde{x}^T(t) Q\tilde{x}(t) + u^T(t) Ru(t)\right] dt \tag{21}$$

II. If the external system (11) is stable but not asymptotically stable, we can choose the average quadratic performance index:

$$J = \lim_{T\to\infty} \frac{1}{\mathrm{T}} \int_0^\infty \left[\tilde{x}^T(t) Q\tilde{x}(t) + u^T(t) Ru(t)\right] dt \tag{22}$$

In (21) and (22), Q is a positive semidefinite matrix and R is a positive definite matrix.

For the quadratic performance index described by (21) and (22), the design method of the optimal disturbance control law is completely consistent.

In the following discussion, our purpose is to find an optimal disturbance suppression control law $u^*(t)$ to minimize the quadratic performance index (21) of the system (14) under the action of disturbance (17).

IV. DESIGNING OF OPTIMAL DISTURBANCE SUPPRESSION CONTROL LAW

Theorem The optimal disturbance suppression control law for the linear system (14) under the disturbance of the external system (17) to minimize the performance index (21) exists and is unique. At the same time:

$$u^*(t) = -R^{-1}B^T \left\{ P\left[x(t) + \int_{t-\tau}^{t} e^{A(t-\tau-h)} A_1 x(h) dh\right] + P_1 w(t) \right\} \tag{23}$$

P is the only positive definite solution of Riccati equation:

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{24}$$

P_1 is the unique solution of Sylvester equation:

$$(A - BR^{-1}B^T P)^T P_1 + P_1 G + PD = 0 \tag{25}$$

Proof: We let Hamiltonian function

$$H(x, u, \lambda) = \frac{1}{2}\left[\tilde{x}^T(t) Q\tilde{x}(t) + u^T(t) Ru(t)\right] + \lambda^T(t)\left[A\tilde{x}(t) + Bu(t) + Dw(t)\right] \tag{26}$$

$\tilde{x}(t)$ and $\lambda(t)$ satisfy the following canonical equations:

$$\begin{aligned} \frac{\partial H}{\partial \tilde{x}} &= -\dot{\lambda}(t) = Q\tilde{x}(t) + A^T\lambda(t), \\ \frac{\partial H}{\partial \lambda} &= \dot{\tilde{x}}(t) = A\tilde{x}(t) + Bu(t) + Dw(t), \\ \frac{\partial H}{\partial u} &= 0 = Ru(t) + B^T\lambda(t), \\ \tilde{x}(0) &= x_0, \\ \lambda(\infty &== 0 \end{aligned} \tag{27}$$

From the third equation of (27), we can get:

$$u(t) = -R^{-1}B^T\lambda(t) \tag{28}$$

By substituting (28) into the second equation and sorting out (27), the following two-point boundary value problems can be obtained:

$$\begin{aligned} -\dot{\lambda}(t) &= Q\tilde{x}(t) + A^T\lambda(t) \ , \\ \dot{\tilde{x}}(t) &= A\tilde{x}(t) - BR^{-1}B^T\lambda(t) + Dw(t) \\ \tilde{x}(0) &= x_0 \ , \\ \lambda(\infty) &= 0 \end{aligned} \tag{29}$$

In order to solve the two-point boundary value problem (29), we let

$$\lambda(t) = P\tilde{x}(t) + P_1 w(t) \tag{30}$$

$P \in R^{n\times n}$, $P_1 \in R^{m\times m}$ are undetermined matrices.

We take the derivative on both sides of (30) and substitute (17), the second equation of (29) into it. We can get:

$$\begin{aligned} \dot{\lambda}(t) &= P\dot{\tilde{x}}(t) + P_1\dot{w}(t) \\ &= P\left[A\tilde{x}(t) - BR^{-1}B^T\lambda(t) + Dw(t)\right] + P_1 Gw(t) \\ &= P\left\{A\tilde{x}(t) - BR^{-1}B^T\left[P\tilde{x}(t) + P_1 w(t)\right] + Dw(t)\right\} \\ &\quad + P_1 Gw(t) \\ &= (PA - PBR^{-1}B^T P)\tilde{x}(t) + (PD + P_1 G \\ &\quad - PBR^{-1}B^T P_1)w(t) \end{aligned} \tag{31}$$

Substituting (30) into the first formula of (27) can obtain:

$$\begin{aligned} \dot{\lambda}(t) &= -Q\tilde{x}(t) - A^T\lambda(t) \\ &= \left(-Q - A^T P\right)\tilde{x}(t) - A^T P_1 w(t) \end{aligned} \tag{32}$$

Comparing the coefficients of (31) and (32), we can get:

$$\begin{aligned} PA - PBR^{-1}B^T P + Q + A^T P &= 0 \\ PD + P_1 G - PBR^{-1}B^T P_1 + A^T P_1 &= 0 \end{aligned} \tag{33}$$

On account of (A_0, B) is controllable and (A_0, C) is fully observable, we can know (A, B) is controllable and (A, C) is fully observable. Therefore, P is the only positive definite solution of Riccati equation (24). We can get the unique P and P_1 by solving (24) and (25). In this way, we can obtain the optimal disturbance suppression control law (23) by substituting equations (30) and (19) into (28). The theorem is proved.

V. SIMULATION EXAMPLE

Considering the mathematical model of a continuously disturbed underwater vehicle with state delay described in (14), we take

$$A_0 = \begin{bmatrix} -0.5940 & 1.0597 \\ -3.9403 & -0.3960 \end{bmatrix}, \ A_1 = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 3 \end{bmatrix},$$
$$D = \begin{bmatrix} 0.2 & 0.3 \\ 0.5 & 1 \end{bmatrix}, \ \tau = 0.1, C = \begin{bmatrix} 1 & 0 \end{bmatrix}, R = 1,$$
$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

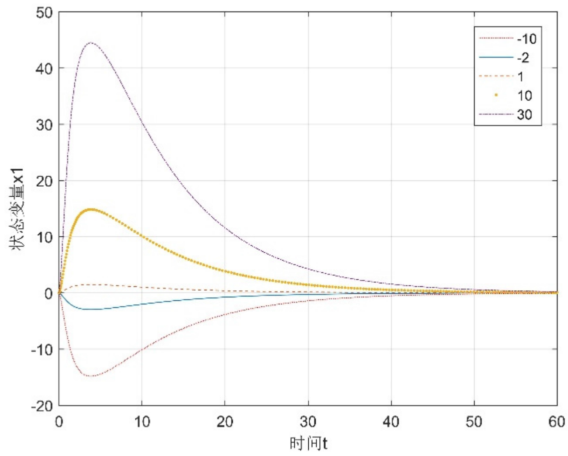

FIGURE 3. Simulation curve of state variable $x_1(t)$.

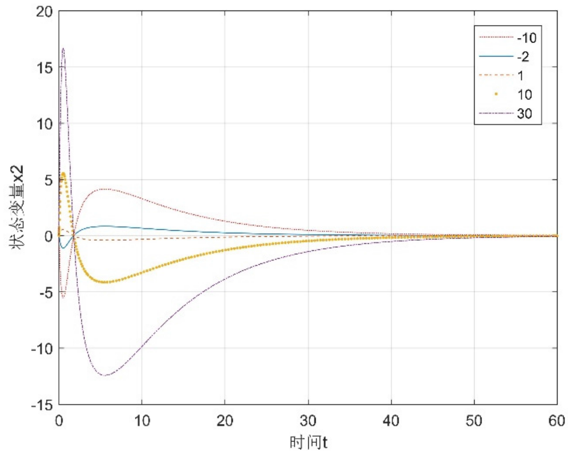

FIGURE 4. Simulation curve of state variable $x_2(t)$.

If we take $G = \begin{bmatrix} -0.5 & 0.4 \\ -0.1 & 0 \end{bmatrix}$, the external system (17) is an asymptotically stable disturbance.

According to (24) and (25), the following results can be obtained:

$$P = \begin{bmatrix} 1.2826 & 0.2403 \\ 0.2403 & 0.4056 \end{bmatrix}, \quad P_1 = \begin{bmatrix} \text{-}0.1384 & -0.2723 \\ -0.0886 & -0.2152 \end{bmatrix}$$

The simulation curve of system state variable x_1, state variable x_2, disturbance suppression control law $u(t)$ and output $y(t)$ are shown in Fig. 1-Fig. 4.

"...", "—", "- -", "......", "–·" is respectively represent the simulation curves with the initial values of −10, −2, 1, 10 and 30 of the reference input external system:

From the simulation results, it can be seen that after the underwater vehicle is subjected to external disturbances of different sizes, the control law $u(t)$ is quickly started to suppress the disturbance, which can quickly achieve gradual stability of states $x_1(t)$ and $x_2(t)$, and then achieve gradual stability of state output $y(t)$. Experiments show that the optimal disturbance suppression control law is effective

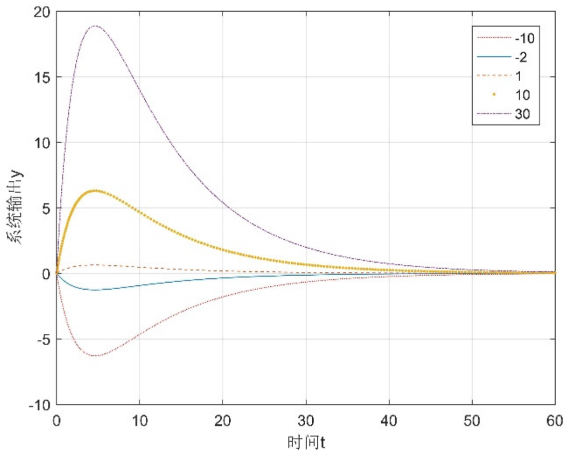

FIGURE 5. Simulation curve of output $y(t)$.

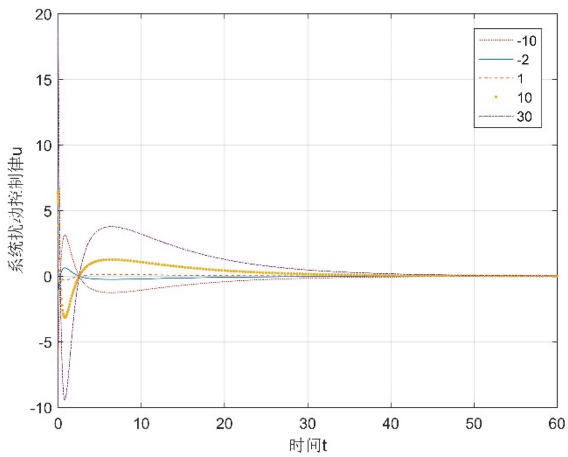

FIGURE 6. Simulation curve of disturbance suppression control law $u(t)$.

for external disturbance of different sizes. For the beauty of the drawing, we only selected such five groups of data as simulation examples. This control law makes the AUV system have good robustness and anti-interference ability.

VI. CONCLUSION

In this paper, the optimal disturbance suppression problem of such an underwater vehicle is studied. The coefficient of the nonlinear term $f(x)$ is very small and can be ignored without affecting the accuracy. The optimal disturbance suppression problem with state delay is transformed into the optimal disturbance suppression problem without time-delay through model transformation. We propose a control law for optimal disturbance suppression and the simulation results show that this control law is effective.

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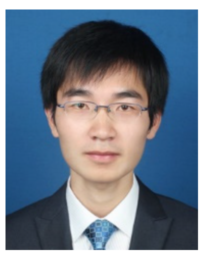

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