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A Novel Efficacious PID Controller for Processes With Inverse Response and Time Delay

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ABSTRACT Design of controller for the inverse response processes has been a challenge for researchers. Water level control in a steam boiler is one of the best examples, where the time delay and inverse response are inherent. Proportional Integral Derivative(PID) controller is the extensively employed regulator in industries. The present work introduces a new form of PID for the processes which are having time delay and inverse response simultaneously. The proposed PID is associated with a higher order filter. The controller and filter parameters are computed by using polynomial approach. Maximum sensitivity of the control loop is used to determine the tuning parameter. A set-point filter is utilized to diminish the settling time and overshoot in servo response. The suggested method is evaluated by considering several performance indices and bench marking examples. The proposed method is evaluated against the existing methods and tested in real-time scenario also.

INDEX TERMS Integrating processes, inverse response, maximum sensitivity, Pade-2, PID controller.

NOMENCLATURE

k	Process gain.
τ	Process time constant.
θ	Time delay.
k_p	Proportional gain.
K_i	Integral gain.
k_d	Derivative gain.
G_p	Process.
G_c	Controller.
y	Process output.
r	Set point.
d	Input disturbance.
F	Setpoint filter.
α,β	PID filter coefficients.
λ	Tuning parameter.
MS	Maximum Sensitivity.
γ,δ	Setpoint filter coefficients.

I. INTRODUCTION

The dynamic behaviour of certain processes exhibit a typical behaviour such that the initial direction of response is antithesis to that of the steady state response. This phenomena is termed as inverse response [1]. The typical examples of the inverse response processes are: The change in tray composition of a distillation column with respect to changes in vapour flow rate, change of water level of a boiler drum with respect to the variations in flow rate of heating medium etc. [2]. Inverse response is characterized by the presence of a zero on right hand side s-plane in the transfer function of a process [3]. Designing the controller for non-self-regulating processes is a challenging task for the control engineers. In addition, if there exists a time delay also,it gets further complicated in terms of control of processes. If the process includes inverse response as well along with time delay, it becomes a massive challenge. The presence of Right-Half-Plain (RHP) zero results in high gain instability [4].

Majority of the previously reported approaches for inverse response processes can be broadly divided into two categories. The methods that have employed conventional PID controller with different tuning techniques and the methods that have employed inverse response compensation techniques. Various techniques that have employed conventional PID are briefly explained below.

Waller and Nygard [3] have suggested a PID controller which is based on Ziegler, J.G. and Nichols(Z-N) [5] method of tuning. Luyben [6] has suggested traditional PI controller in which the controller parameters are derived as a function of RHP zero and time delay. However, this method's PI settings are empirically estimated and results significant oscillations and overshoot [7]. Chen *et al.* [7] have merged the right hand side zero into time delay using Pade's first

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order approximation to estimate the controller parameters. Internal Model Control(IMC) based control schemes have been developed by several researches to deal with inverse response characteristics ([8]–[11]). Also, Direct synthesis based methods have been reported by some of the researchers for controlling inverse response processes([12], [13]).

Kumar and Narayana [14] have proposed a controller as combination of PID and First-order lead/lag filter. This method is applicable for different kinds of integrating process with inverse response and time delay. Controller parameters are derived by using polynomial approach and tuning parameter is computed by MS. This method's performance is proved to be better than the method of Pai *et al.* [13]. Begum *et al.* [11] have proposed a $IMC - H_2$ controller where the controller parameters are estimated by H_2 optimization. This method is applicable for integrating and double integrating processes with inverse response and time delay. The Maclaurin series is utilised to derive PID from a higher order controller in this technique. Kumar and Manimozhi [15] have extended the work of [14] and proposed a PID controller with second order lead-lag filter based on polynomial approach.This method's tuning guidelines are based on MS and this method has obtained superior performance than Begum *et al.* [11] method. Irasad and Ali [16] have used minimization of integral performance criteria and Ozyetkin *et al.* [17] have suggested boundary locus approach for PI controller. Siddiqui *et al.* [18] have proposed a PID controller with filter for integrating processes with time delay and inverse response in Parallel Control Structure(PCS) and the tuning parameters are calculated based on direct synthesis method. Lloyds *et al.* [19] have proposed PI-PD controller for integrating and unstable processes with inverse response. Recently Patil *et al.* [20] have proposed PID controller for second order plus time delay(SOPTD) processes using multi-objective optimization problem. This method is applicable for non-minimum phase SOPTD.

In the second category, Methodologies have been developed to counteract inverse response with the use of compensators. Majority of them are based on Smith-Predictor concept. Iinoya and Altpeter [1] have applied the Smith predictor idea for controlling the inverse response processes. Later Zhang *et al.* [21] have developed inverse response compensators along with optimal controllers to enhance the performance. Based on Direct Synthesis method, Uma *et al.* [22] have modified the smith predictor design by employing two controllers, one for set-point tracking and other for disturbance rejection. Multi loop controllers are usually designed to handle servo and regulatory responses independently. But, multi-loop controllers are difficult to analyze, necessitate longer computation times and offer greater tuning complexity.

The goal of the present study is to develop a controller for the kind of processes shown in Equations 1 and 2. Equation 1 represnts stable First Order Process with Time Delay and Inverse response(FOPTD-IR) and Equation 2 represents Integrating unstable/stable First Order Process with Time Delay and Inverse Response (IFOPTD-IR).

$$G_p = \frac{k(-sz+1)e^{-s\theta}}{(\tau s+1)} \tag{1}$$

$$G_p = \frac{k(-sz+1)e^{-s\theta}}{s(\tau s \pm 1)} \tag{2}$$

where k and z represent process gain and zero located at right half of s-plane respectively. Time constant and delay of the process are represented by τ and θ respectively. Some of the salient features of proposed work are:

- It is customary to adopt some kind of approximation for time delay in the process of deriving controller parameters. Many existing research works have employed lower order time delay approximation to derive the PID settings.The proposed work has employed higher order time delay approximation for improved accuracy.
- Most of the existing proposed PID controllers are associated with lower order filters or no filters. The proposed PID controller is associated with higher order (4^{th}) lead-lag filter. Higher order filters are renounced for precise filtering and several researchers have proved that PID when augmented with a filter is able to produce enhanced performance([23]–[25]).
- Tuning parameter(λ) is selected analytically based on Maximum sensitivity.

The present paper is organised in several sections and brief description of sections is as follows: The mathematical description of the proposed methodology is illustrated in section 2, section 3 presents the simulation study of various bench marking examples and analysis, section 4 is about the validation of proposed controller for a real time process, and section 5 presents the conclusion.

II. PROPOSED CONTROLLER

The proposed control scheme is depicted in Figure 1. G_p represents process, G_c represents the controller, y is process output, d is input disturbance, F is set point filter and r is set point. Equations 3 and 4 represent servo and regulatory transfer functions([13]–[15]) respectively.

$$\frac{y}{r} = \frac{FG_cG_p}{1+G_cG_p} \tag{3}$$

$$\frac{y}{d} = \frac{G_p}{1+G_cG_p} \tag{4}$$

A. DESIGN OF CONTROLLER FOR IFOPDT-IR

The proposed controller is considered to be a combination of conventional PID and fourth order filter as shown in Equation 5.

$$G_c = \frac{q}{p} = \left(k_p + \frac{k_i}{s} + k_d s\right) \frac{(\beta_2 s^2 + \beta_1 s + 1)^2}{(\alpha_2 s^2 + \alpha_1 s + 1)(\alpha_4 s^2 + \alpha_3 s + 1)} \tag{5a}$$

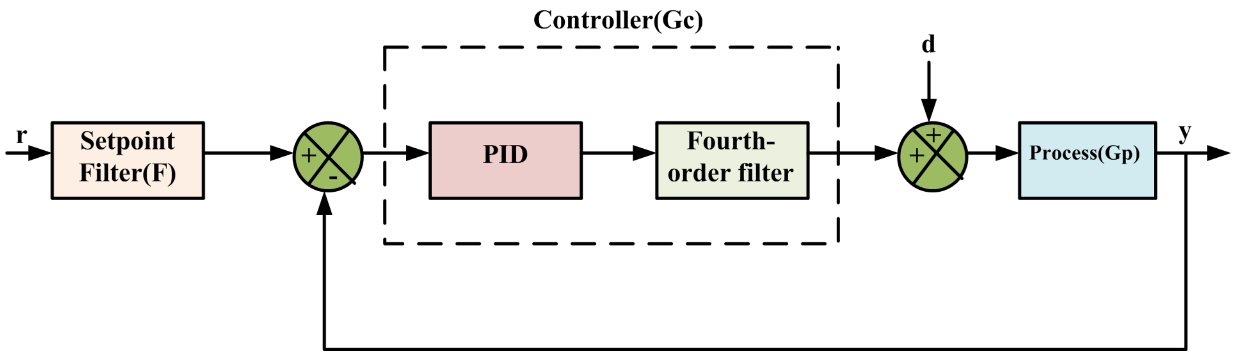

FIGURE 1. **Proposed control structure.**

where,

$$q = (k_d s^2 + k_p s + k_i)(\beta_2 s^2 + \beta_1 s + 1)^2 \quad (5b)$$

$$p = s(\alpha_2 s^2 + \alpha_1 s + 1)(\alpha_4 s^2 + \alpha_3 s + 1) \quad (5c)$$

Equation 6 represents the stable and unstable IFOPTD-IR as ratio of two polynomials.

$$G_p = \frac{k(1-sz)e^{-s\theta}}{s(\tau s \pm 1)} = \frac{u}{v}e^{-s\theta} \quad (6a)$$

where,

$$u = k(1-sz) \quad (6b)$$

$$v = s(\tau s \pm 1) \quad (6c)$$

The servo and regulatory transfer functions are obtained by substituting Equations 5 and 6 into Equations 3 and 4. They are presented in Equation 7 and Equation 8 respectively. (Where ever $\pm$ or $\mp$ is present in equations, the upper symbol relates to stable IFOPTD-IR and the lower symbol relates to unstable IFOPTD-IR.)

$$\frac{y}{r} = \frac{F(k_d s^2 + k_p s + ki)(\beta_2 s^2 + \beta_1 s + 1)^2 k(1-sz)e^{-s\theta}}{\left(\begin{array}{c} s^2(\alpha_2 s^2 + \alpha_1 s + 1)(\alpha_4 s^2 + \alpha_3 s + 1)(\tau s \pm 1) \\ +k(\beta_2 s^2 + \beta_1 s + 1)^2(k_d s^2 + k_p s + ki)(1-sz)e^{-s\theta} \end{array}\right)} \quad (7)$$

$$\frac{y}{d} = \frac{ks(1-sz)(\alpha_2 s^2 + \alpha_1 s + 1)(\alpha_4 s^2 + \alpha_3 s + 1)e^{-s\theta}}{\left(\begin{array}{c} s^2(\alpha_2 s^2 + \alpha_1 s + 1)(\alpha_4 s^2 + \alpha_3 s + 1)(\tau s \pm 1) \\ +k(\beta_2 s^2 + \beta_1 s + 1)^2(k_d s^2 + k_p s + ki)(1-sz)e^{-s\theta} \end{array}\right)} \quad (8)$$

The Characteristic Equation (CE) of the control loop is presented in Equation 9. The roots of the CE determine stability dynamics of the control loop.

$$\begin{aligned} &s^2(\alpha_2 s^2 + \alpha_1 s + 1)(\alpha_4 s^2 + \alpha_3 s + 1)(\tau s \pm 1) \\ &\quad +k(k_d s^2 + k_p s + ki)(\beta_2 s^2 + \beta_1 s + 1)^2(1-sz)e^{-s\theta} = 0 \end{aligned} \quad (9)$$

To obtain the unknown controller parameters, the CE should be solved against a desired CE. In this process, many authors have used either Maclaurin series approximation ([4], [11]) or Pade's approximation ([14], [15], [22]) in their studies. Pade's first order approximation is the most widely used approximation among the all. However, the present study used a higher order approximation i.e Pade's shift(Pade-2) approximation in order to obtain higher accuracy. Pade's shift approximation is presented in Equation 10.

$$e^{-s\theta} = \frac{(1-(\theta s/2n) + \frac{1}{3}(\theta s/2n)^2)^n}{(1+(\theta s/2n) + \frac{1}{3}(\theta s/2n)^2)^n} \quad (10)$$

Pade's shift approximation becomes equal to Pade's second order approximation when n=1.The authors have designed controller using n=2.

$$e^{-\theta s} = \frac{(1-(\theta s/4) + (\theta^2 s^2/48))^2}{(1+(\theta s/4) + (\theta^2 s^2/48))^2} \quad (11)$$

Equation 11 is substituted in Equation 9 which results CE as indicated in Equation 12. It is considered that $\beta_2 = \frac{\theta^2}{48}$ and $\beta_1 = \frac{\theta}{4}$ for mathematical convenience([14], [15]).

$$\begin{aligned} &s^2(\alpha_2 s^2 + \alpha_1 s + 1)(\alpha_4 s^2 + \alpha_3 s + 1)(\tau s \pm 1) \\ &\quad +k(k_d s^2 + k_p s + ki)(1-sz)(1-(\theta s/4) + (\theta^2 s^2/48))^2 = 0 \end{aligned} \quad (12)$$

Further simplification leads to a simplified CE shown in Equation 13, at the bottom of the next page. The unknown control parameters are calculated by solving Equation 13 against a desired CE. Equation 14 represents the desired CE.Various combinations of Desired CE are tested and the authors have arrived at the present desired CE which is giving better response characteristics relatively. The Desired CE is selected so as to compensate overshoot in servo response by cancelling some of the controller introduced zeros in the servo response.

$$(\lambda s + 1)^4(\beta_2 s^2 + \beta_1 s + 1)(\alpha_4 s + 1) = 0 \quad (14)$$

The proposed desired CE eliminates few zeros in the responses of servo response and it locates the remaining poles at $s = -\frac{1}{\lambda}$ and $s = -\frac{1}{\alpha_4}$.

B. CONTROLLER DESIGN FOR STABLE FOPTD-IR

The stable FOPTD-IR is presented in Equation 15.

$$G_p = \frac{k(1-sz)}{(\tau s+1)}e^{-s\theta} \tag{15}$$

The process is rearranged as shown in Equation 16.

$$G_p = \frac{k(1-sz)}{\tau s+1}e^{-s\theta} = \frac{k(1+sz)(1-sz)}{(1+sz)(\tau s+1)}e^{-s\theta} \tag{16a}$$

According to Pade's approximation,

$$\frac{(1-sz)}{(1+sz)} = e^{-s2z} \tag{16b}$$

$$G_p = \frac{k(1+sz)}{(\tau s+1)}e^{-s\theta'} \tag{16c}$$

where,

$$\theta' = \theta + 2z \tag{16d}$$

The proposed controller form for this process is same as the controller form for the previous process (Equation 5). The servo and regulatory transfer functions are derived by substituting Equations 5 and 16 into Equations 3 and 4. The servo and regulatory responses are represented in Equation 17 and the CE is represented in Equation 18.

$$\frac{y}{r} = \frac{F(k_d s^2+k_p s+ki)(\beta_2 s^2+\beta_1 s+1)^2 k(1+sz)e^{-s\theta'}}{\begin{pmatrix} s(\alpha_2 s^2+\alpha_1 s+1)(\alpha_4 s^2+\alpha_3 s+1)(\tau s+1) \\ +k(\beta_2 s^2+\beta_1 s+1)^2(k_d s^2+k_p s+ki)(1+sz)e^{-s\theta'} \end{pmatrix}} \tag{17a}$$

$$\frac{y}{d} = \frac{ks(1+sz)(\alpha_2 s^2+\alpha_1 s+1)(\alpha_4 s^2+\alpha_3 s+1)e^{-s\theta'}}{\begin{pmatrix} s(\alpha_2 s^2+\alpha_1 s+1)(\alpha_4 s^2+\alpha_3 s+1)(\tau s+1) \\ +k(\beta_2 s^2+\beta_1 s+1)^2(k_d s^2+k_p s+ki)(1+sz)e^{-s\theta'} \end{pmatrix}} \tag{17b}$$

$$\begin{aligned} &s(\alpha_2 s^2+\alpha_1 s+1)(\alpha_4 s^2+\alpha_3 s+1)(\tau s+1) \\ &+k(k_d s^2+k_p s+ki)(\beta_2 s^2+\beta_1 s+1)^2(1+sz)e^{-s\theta'} = 0 \end{aligned} \tag{18}$$

After adopting second order Pade-2 approximation for time delay and considering $\beta_2 = \frac{\theta'^2}{48}$ and $\beta_1 = \frac{\theta'}{4}$, the CE attains the following form as shown in Equation 19.

$$\begin{aligned} &s(\alpha_2 s^2+\alpha_1 s+1)(\alpha_4 s^2+\alpha_3 s+1)(\tau s+1) \\ &+k(k_d s^2+k_p s+ki)(1+sz)(1-(\theta' s/4)+(\theta'^2 s^2/48))^2=0 \end{aligned} \tag{19}$$

The simplified CE is represented in Equation 20, as shown at the bottom of the next page. The CE in Equation 20 can be solved against the desired CE (Equation 14) in order to get the controller parameters.

C. SELECTION OF λ

Many authors have employed MS based selection of tuning parameter([4], [14], [15], [19], [24]). The MS can be defined as the inverse of the shortest distance from the Nyquist plot of the loop transfer function to the critical point [15]. Higher MS values provide good speed of response at the cost of loss of robustness. Lower MS values provide good robustness but at the loss of speed. As a result, the choice of MS is a trade-off between robustness and speed. The mathematical description of MS is illustrated in Equation 21.

$$MS = max\left(\left|\frac{1}{1+G_cG_p}\right|\right) \tag{21}$$

where G_cG_p is loop transfer function. The range of MS value for satisfactory operation of a controller output is considered to be between 1.2 to 2.0 [15]. However, researchers have been using higher values of MS for integrating and unstable processes as it is difficult to obtain MS within the prescribed

$$kk_i(c_7 s^7+c_6 s^6+c_5 s^5+c_4 s^4+c_3 s^3+c_2 s^2+c_1 s+1) = 0 \tag{13a}$$

$$c_7 = \frac{\alpha_2\alpha_4\tau - \left(\frac{kk_d\theta^4 z}{2304}\right)}{kk_i} \tag{13b}$$

$$c_6 = \frac{\frac{\theta^4(kk_d-kk_pz)}{2304} \pm \alpha_4(\alpha_2 \pm \alpha_1\tau) + \alpha_2\alpha_3\tau + \frac{kk_d\theta^3 z}{96}}{kk_i} \tag{13c}$$

$$c_5 = \frac{\alpha_2\tau - \frac{\theta^3(kk_d-kk_pz)}{96} + \frac{\theta^4(kk_p-kk_i)z}{2304} \pm \alpha_3(\alpha_2 \pm \alpha_1\tau) \pm \alpha_4(\alpha_1 \pm \tau) - \frac{5kk_d\theta^2 z}{48}}{kk_i} \tag{13d}$$

$$c_4 = \frac{\pm\alpha_2 \pm \alpha_4 + \alpha_1\tau + \frac{5\theta^2(kk_d-kk_pz)}{48} - \frac{\theta^3(kk_p-kk_iz)}{96} \pm \alpha_3(\alpha_1 \pm \tau) + \frac{kk_i\theta^4}{2304} + \frac{kk_d\theta z}{2}}{kk_i} \tag{13e}$$

$$c_3 = \frac{\pm\alpha_1 \pm \alpha_3 + \tau - \frac{\theta(kk_d-kk_pz)}{2} + \frac{5\theta^2(kk_p-kk_iz)}{48} - \frac{kk_i\theta^3}{96} - kk_dz}{kk_i} \tag{13f}$$

$$c_2 = \frac{kk_d - \frac{\theta(kk_p-kk_iz)}{2} + \frac{5kk_i\theta^2}{48} - kk_pz \pm 1}{kk_i} \tag{13g}$$

$$c_1 = \frac{\left(-\frac{kk_i\theta}{2}\right) \mp kk_p \pm kk_iz}{kk_i} \tag{13h}$$

band. The dependency between MS value and the lower limits of GM(Gain Margin) and PM(Phase Margin) are shown in Equations 22 and 23 respectively.

$$GM \geq \frac{Ms}{Ms-1} \tag{22}$$

$$PM \geq 2\sin^{-1}\left(\frac{1}{2Ms}\right) \tag{23}$$

D. SET POINT FILTERING

From Equation 7, it is observed that controller introduces zeros in servo response. As a result, overshoot, oscillations and longer settling periods can be resulted in the servo response. In order to reduce the effect of zeros, the desired CE is assumed accordingly to cancel some of the controller introduced zeros. To further counteract, the present method uses a set point filter. For instance, servo response of IFOPTD-IR is shown in Equation 24(using Equations 7 and 13 and 14).

$$\frac{y}{r} = \frac{F((k_d/k_i)s^2+(k_p/k_i)s+1)(\beta_2 s^2+\beta_1 s+1)^2(1-sz)e^{-s\theta}}{(\lambda s+1)^4(\beta_2 s^2+\beta_1 s+1)(\alpha_4 s+1)} \tag{24}$$

The selected setpoint filter is represented in Equation 25

$$F = \frac{\gamma s+1}{((\frac{kd}{ki})s^2+(\frac{kp}{ki})s+1)(\beta_2 s^2+\beta_1 s+1)(\delta s+1)} \tag{25}$$

If the response is showing overshoot, δ can be manipulated to mitigate the overshoot. If the response is slower, γ can be manipulated to get satisfactory speed of response However, as γ and δ lie outside the closed loop, they do not affect the stability characteristics of the closed loop system. So selection of γ and δ is trivial in perspective of stability.

III. SIMULATION ANALYSIS

This section assesses the performance of the proposed controller by comparing it to different control mechanisms found in the literature. Standard performance indices are used to conduct simulation analysis of various bench marking examples. Equations 26 to 28 give a mathematical description of various performance indices.

$$Integral\ Absolute\ Error(IAE) = \int_0^\infty |e(\tau)|d\tau \tag{26}$$

$$Integral\ Square\ Error(ISE) = \int_0^\infty e^2(\tau)d\tau \tag{27}$$

$$Integral\ Time\ Absolute\ Error(ITAE) = \int_0^\infty \tau|e(\tau)|d\tau \tag{28}$$

where, e represents the error. IAE measures the ability of the controller to penalize the oscillations quickly. ISE is measure of the ability of controller to mitigate the large errors quickly. ITAE is a measure of faster settling times. Total variance (TV) measures the smoothness of manipulated variable and its mathematical representation is shown in Equation 29.

$$Total\ Variance(TV) = \sum_{i=0}^{\infty} |u_{i+1}-u_i| \tag{29}$$

The input to the process at the i^{th} and $(i+1)^{th}$ instants are u_i and u_{i+1} respectively. Minute TV emphasises smoother variations in manipulated variable which thus accomplishes the safety and long life of final control elements.

Researchers commonly incorporate filter in derivative mode to ensure not to have kick-in response of controller output due to noise. Begum *et al.* [11] has chosen the derivative filter coefficient as $0.01\tau_d$ and Kumar and Manimozhi [15] has considered $0.01k_d$ as derivative filter coefficient in their

$$kk_i(c_7s^7+c_6s^6+c_5s^5+c_4s^4+c_3s^3+c_2s^2+c_1s+1)=0 \tag{20a}$$

$$c_7 = \frac{k_d\theta'^4 z}{2304k_i} \tag{20b}$$

$$c_6 = \frac{\frac{\theta'^4(kk_d+kk_pz)}{2304}+\alpha_2\alpha_4\tau-\frac{kk_d\theta'^3z}{96}}{kk_i} \tag{20c}$$

$$c_5 = \frac{\frac{\theta'^4(kk_p+kk_iz)}{2304}-\frac{\theta'^3(kk_d+kk_pz)}{96}+\alpha_4(\alpha_2+\alpha_1\tau)+\alpha_2\alpha_3\tau+\frac{5kk_d\theta'^2z}{48}}{kk_i} \tag{20d}$$

$$c_4 = \frac{\alpha_2\tau+\frac{5\theta'^2(kk_d+kk_pz)}{48}-\frac{\theta'^3(kk_p+kk_iz)}{96}+\alpha_3(\alpha_2+\alpha_1\tau)+\alpha_4(\alpha_1+\tau)+\frac{kk_i\theta'^4}{2304}-\frac{kk_d\theta'z}{2}}{kk_i} \tag{20e}$$

$$c_3 = \frac{\alpha_2+\alpha_4-\frac{\theta'(kk_d+kk_pz)}{2}+\alpha_1\tau+\frac{5\theta'^2(kk_p+kk_iz)}{48}+\alpha_3(\alpha_1+\tau)-\frac{kk_i\theta'^3}{96}+kk_dz}{kk_i} \tag{20f}$$

$$c_2 = \frac{\alpha_1+\alpha_3+\tau-\frac{\theta'(kk_p+kk_iz)}{2}+kk_d+\frac{5kk_i\theta'^2}{48}+kk_pz}{kk_i} \tag{20g}$$

$$c_1 = \frac{kk_p-\frac{kk_i\theta'}{2}+kk_iz+1}{kk_i} \tag{20h}$$

TABLE 1. Controller settings.

Process	Method	PID Parameters			Filter	Ms
		k_p	k_i	k_d		
$\frac{0.547(1-0.418s)}{s(1.06s+1)}e^{-0.1s}$	$Proposed^a$	3.779	1.4319	2.4609	$\frac{(0.0002s^2+0.025s+1)^2}{(0.00002s^2+0.004s+1)(0.0021s^2+0.0399s+1)}$	2.94
	$Begum\ et.al^b$ [11]	3.230	0.9215	2.578	-	2.94
$\frac{(1-0.2s)}{s(s-1)}e^{-0.2s}$	$Proposed^c$	0.7513	0.2162	1.952	$\frac{(0.0008s^2+0.05s+1)^2}{(0.0001s^2+0.0065s+1)(0.0054s^2+0.037s+1)}$	7.239
	$Kumar\ et.al^d$ [15]	0.5635	0.1467	1.8559	$\frac{(0.0225s^2+0.3s+1)}{0.0097s^2+0.2485s+1}$	7.23
	$Begum\ et.al^e$ [11]	0.4451	0.0853	1.9272	-	7.23
$\frac{0.5(1-0.5s)}{s(0.5s+1)(0.4s+1)(0.1s+1)}e^{-0.7s}$	$Proposed^f$	0.964	0.134	0.938	$\frac{(0.013s^2+0.202s+1)^2}{(0.004s^2+0.086s+1)(0.095s^2+0.2857s+1)}$	2.019
	$Begum\ et.al^g$ [11]	0.994	0.119	1.231	-	2.8
Quanser qube servo motor	*Proposed*	0.0283	0.1485	0.0005	$\frac{(0.0003s^2+0.0321s+1)^2}{(0.0018s^2+0.0718s+1)(0.0004s^2+0.0352s+1)}$	-
	R.Padma sree et.al [2]	0.0083	0.0485	-	$\frac{1}{0.0264s+1}$	-

$f_R : Setpoint filter$

$a: \ f_R = \frac{0.2s+1}{0.0004s^4+0.043s^3+1.784s^2+2.664s+1}$ $b: \ f_R = \frac{0.85s+1}{2.5018s^2+3.4186s+1}$, $c: \ f_R = \frac{0.1s+1}{0.0075s^4+0.4542s^3+9.195s^2+3.358s+1}$

$d: \ f_R = \frac{1}{12.65s^2+3.84s+1}$ $e: \ f_R = \frac{1.3s+1}{21.9s^2+5.0833s+1}$ $f: \ f_R = \frac{2s+1}{0.0954s^4+1.5121s^3+8.4489s^2+7.375s+1}$

$g: \ f_R = \frac{1.984s+1}{8.3863s^2+8.0975s+1}$

studies. The proposed method also has chosen derivative coefficient as $0.01k_d$. The proposed PID controller with derivative filter is illustrated in Equation 30.

$$G_c = \left(k_p + \frac{k_i}{s} + \frac{k_d s}{0.01k_d s + 1}\right) \times \left(\frac{(\beta_2 s^2 + \beta_1 s + 1)^2}{(\alpha_2 s^2 + \alpha_1 s + 1)(\alpha_4 s^2 + \alpha_3 s + 1)}\right) \quad (30)$$

Example 1:

A well-known industrial example of IFOPTD is the regulation of level of a boiler steam drum by adjusting boiler feed water to the drum [11]. The process model of this example is presented in Equation 31. Several studies [11], [13], [22] have investigated this process in the literature.The proposed technique is compared with Begum *et al.* [11] method, which outperforms the other methods([13], [22]) in the literature.

$$G_p = \frac{0.547(1-0.418s)}{s(1.06s+1)}e^{-0.1s} \quad (31)$$

Both approaches are tuned at the same MS value of 2.94 for the purpose of fair comparison. The present method has achieved this MS value at $\lambda = 0.536$. Figure 2 depicts the relationship between the MS and λ for the suggested method. Table 1 summarises the controller parameters for both methods. A step input of magnitude 0.2 units applied at t = 0 s and at time t = 15 s, a negative disturbance of 0.5 units is considered. The response under nominal condition is presented in Figure 3. Performance comparison of the both the methods is shown in Table 2. It can be understood that the proposed method has shown overall better performance compared to the other method in all integral error performance indices. However the other method is noticed to be marginally good at producing low TV values though lagging in terms of integral error performance indices.

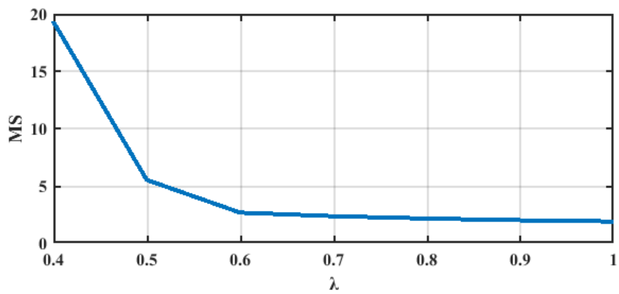

FIGURE 2. Relation between λ and MS for example 1.

A 10% increment in θ, 5% decrement in τ, 10% decrement in z and 10% increment in k are applied to analyse the robust performance. Figure 4 depicts the perturbed response. Table 3 presents the results of the performance study under perturbed conditions. Table 3 reveals that the present strategy is superior in terms of all performance indices except for TV. The other method is marginally better in terms of TV in regulatory response conditions.

Example 2:

In this example, an unstable IFOPTD-IR as described in Equation 32 is considered. This process has been considered

TABLE 2. Performance comparison under nominal condition.

		Servo					Regulatory				
Process	**Method**	**IAE**	**ISE**	**ITAE**	**TV**	**ts(s)**	**IAE**	**ISE**	**ITAE**	**TV**	**ts(s)**
$\frac{0.547(1-0.418s)}{s(1.06s+1)}e^{-0.1s}$	Proposed	0.4931	0.077	0.7028	0.6007	5.12	0.374	0.04	6.553	2.44	6.27
	Begum et.al [11]	0.514	0.073	0.862	0.754	6.48	0.568	0.05	10.43	2.28	9.35
$\frac{(1-0.2s)}{s(s-1)}e^{-0.2s}$	Proposed	0.653	0.098	1.302	0.2136	6.99	0.139	0.004	4.662	0.244	8.57
	Kumar et.al [15]	0.769	0.115	1.811	0.157	8.303	0.205	0.007	6.965	0.247	10.07
	Begum et.al [11]	0.798	0.105	2.47	0.246	9.08	0.359	0.014	12.76	0.30	14.54
$\frac{0.5(1-0.5s)}{s(0.5s+1)(0.4s+1)(0.1s+1)}e^{-0.7s}$	Proposed	1.104	0.1666	3.82	0.208	11.65	0.7589	0.0572	35.96	0.326	17.96
	Begum et.al [11]	1.232	0.1705	5.141	0.2143	15.53	0.845	0.056	40.93	0.3602	22.21

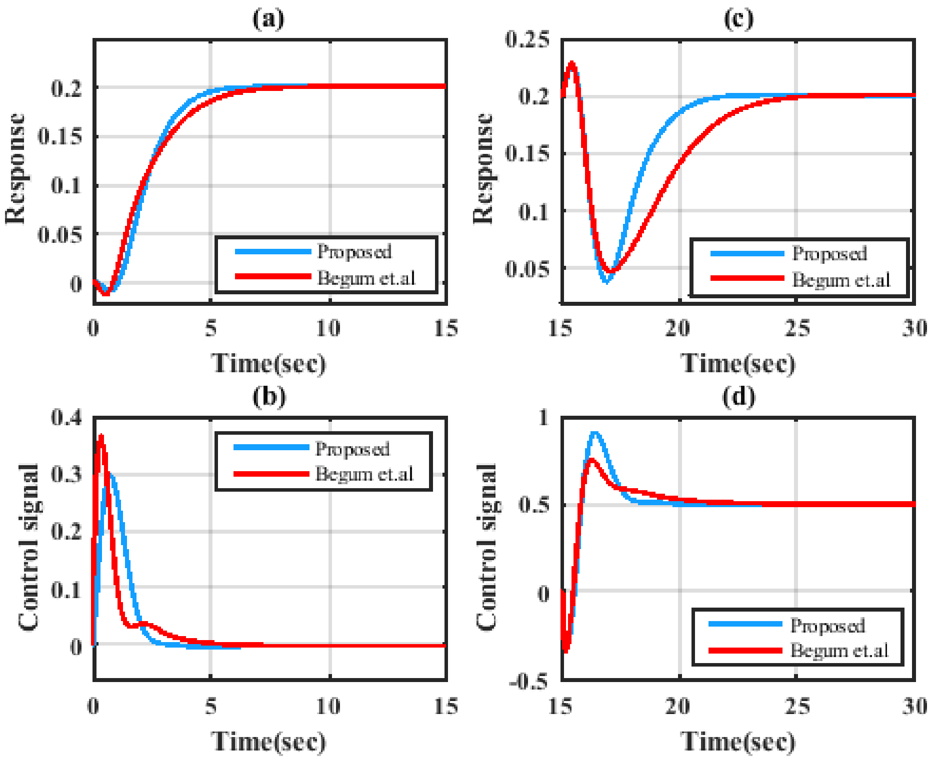

FIGURE 3. Responses along with control signal of example 1 under nominal condition. (a) Servo response, (b) Control signal for servo response, (c) Regulatory response, (d) Control signal for regulatory response.

previously by Begum *et al.* [11], Kumar and Manimozhi [15] and Pai *et al.* [13]. Kumar and Manimozhi [15] method has shown superior performance than the other methods.

$$G_p = \frac{(1-0.2s)}{s(s-1)}e^{-0.2s} \tag{32}$$

In this example, the present method is compared with the method of Begum *et al,* [11] and Kumar and Manimozhi [15]. The variation of MS with λ of the suggested method is depicted in Figure 5. For the uniform comparison, all the methods are tuned to MS value equal to 7.23. The proposed method has chosen λ value as 0.738 for obtaining this MS value. The controller settings of the respective methods are indicated in Table 1. At t = 0 s, a 0.2 units step input is used to analyze the servo response. A disturbance with a negative magnitude of 0.03 units is applied at t = 30 s for the regulatory response analysis. Figure 6 depicts the responses obtained under nominal condition. Table 2 shows the performance comparison. From Table 2, it is evident that present method has delivered overall better performance compared to other two methods. In case of TV, method of Kumar and Manimozhi [15] has produced marginally better values than the proposed method but lagging in all other indices.

For analysing perturbed conditions, a 10% increment in θ, 10% increment in z and 5% decrement in τ are considered. Negative disturbance is applied at t = 50 s for the perturbed regulatory response analysis. The response is shown in Figure 7. From the comparison analysis presented in Table 3, it is clear that the present method has shown overall superior performance.

Example 3

Many industrial processes are actually of higher order. However, these processes are approximated to a lower order process in order to obtain the controller parameters. A higher order process is considered in this example as indicated

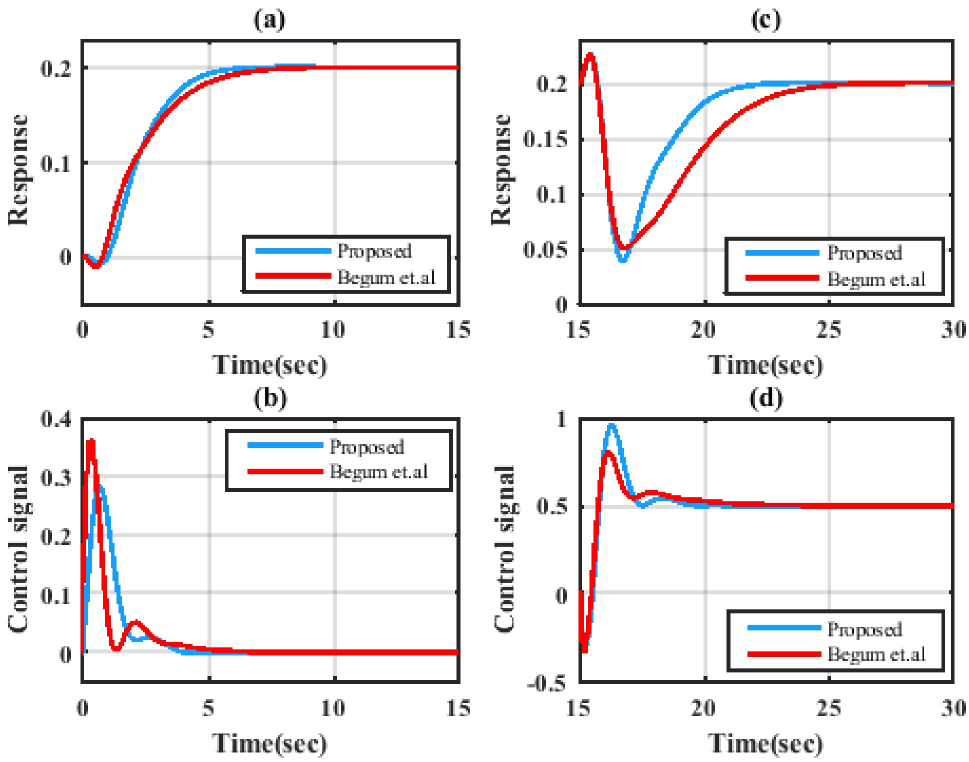

FIGURE 4. Responses along with control signal of example 1 under perturbed condition. (a) Servo response, (b) Control signal for servo response, (c) Regulatory response, (d) Control signal for regulatory response.

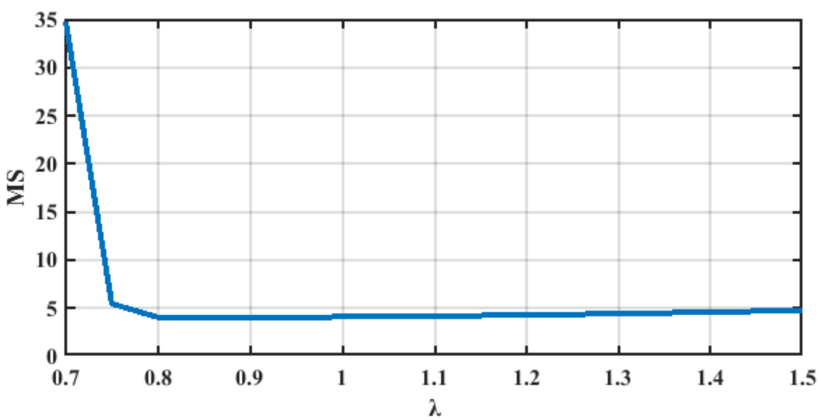

FIGURE 5. Relation between λ and MS for example 2.

in Equation 33.

$$G_p = \frac{0.5(1-0.5s)}{s(0.4s+1)(0.5s+1)(0.1s+1)}e^{-0.7s} \qquad (33)$$

This model is previously studied by various researchers [11], [13], [26]. Begum *et al.* [11] method shown the superior performance than Pai *et al.* method [13] and Ali and Majhi method [26].This model can be approximated to a reduced order model [13] as presented in Equation 34 using step response data and non linear least squares fit.

$$G_p = \frac{0.5183(1-0.4699s)}{s(1.1609s+1)}e^{-0.81s} \qquad (34)$$

To analyze the performance, the present method is compared with Begum et. al [11] method. Table 1 shows the controller settings for both the methods. The present method is tuned to $\lambda = 1.5$ and the corresponding MS value is 2.019. The relationship between MS and λ is depicted in Figure 8. Begum *et al.* [11] method has considered MS value equal to 2.8. The two methods are not compared for similar MS

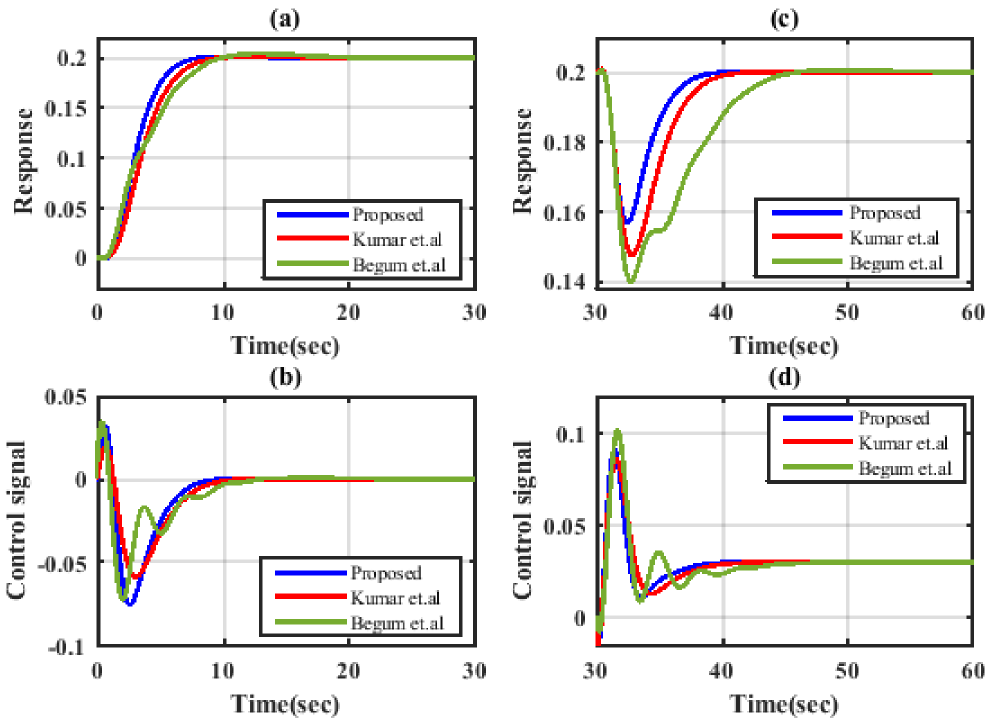

FIGURE 6. **Responses along with control signal of example 2 under nominal condition. (a) Servo response, (b) Control signal for servo response, (c) Regulatory response, (d) Control signal for regulatory response.**

TABLE 3. **Performance comparison under perturbed condition.**

		Servo					Regulatory				
Process	**Method**	**IAE**	**ISE**	**ITAE**	**TV**	**ts(s)**	**IAE**	**ISE**	**ITAE**	**TV**	**ts(s)**
$\frac{0.6017(1-0.3762s)}{s(1.007s+1)}e^{-0.11s}$	Proposed	0.493	0.075	0.731	0.581	5.42	0.371	0.037	6.51	2.60	6.612
	Begum et.al [11]	0.513	0.0707	0.896	0.839	6.81	0.563	0.05	10.37	2.49	9.66
$\frac{(1-0.22s)}{s(0.95s-1)}e^{-0.22s}$	Proposed	0.658	0.097	1.35	0.231	7.13	0.1407	0.004	7.516	0.311	8.78
	Kumar et.al [15]	0.775	0.113	1.888	0.173	8.49	0.206	0.007	11.17	0.332	10.33
	Begum et.al [11]	1.144	0.165	4.37	0.591	16.50	0.39	0.014	22.8	1.18	29.53
$\frac{0.525(1-0.55s)}{s(0.5s+1)(0.4s+1)(0.1s+1)}e^{-0.84s}$	Proposed	1.082	0.168	3.54	0.254	12.22	0.753	0.063	61.97	0.40	17.06
	Begum et.al [11]	1.385	0.169	10.84	0.803	36.83	1.113	0.079	98.58	1.323	51.53

values as it is not possible to achieve a common MS value. In general, Higher MS values corresponds to faster response and less robustness and lower MS values correspond to good robustness and slower response. To prove the efficacy of the present method in this scenario, it should be proved that the proposed method is capable of delivering faster response and yet being more stable with relatively lower MS values than other methods. A unit step of magnitude 0.2 is considered as a set point at t = 0 s and a negative disturbance of magnitude 0.1 is applied at t = 40 s. The comparison for nominal case is depicted in Figure 9. The performance matrix is presented in Table 2. From the Table 2, it can be concluded that the suggested method has shown better performance both in servo and regulatory conditions. It is to be noted here that, the suggested method is able to compete with other method in terms speed even with a lower MS value.

For the assessment of robustness, a 5% increase in k, 10% increase in z and 20% increase in θ is applied. Negative disturbance is applied at t = 75 s.The perturbed case output wave forms are depicted in Figure 10 and Table 3 provides performance analysis. The proposed method has shown overall superior performance. In comparison to the proposed method, the other method is more oscillatory.

IV. REAL TIME IMPLEMENTATION

To verify the applicability of proposed methodology on a real time scenario, the suggested controller is applied to regulate the speed of the Quanser QUBE-servo motor. The Matlab-Simulink environment that is interfaced with data acquisition system (DAQ) provides a real time execution of the proposed controller. Schematic diagram of the armature circuit of DC servo motor is depicted in Figure 11. The electrical and mechanical parameters of the Qube servo process are presented in Table 4.

Equation 35 represents the estimated transfer function between voltage and speed.

$$G_p = \frac{24.7523}{(0.1708s + 1)} \tag{35}$$

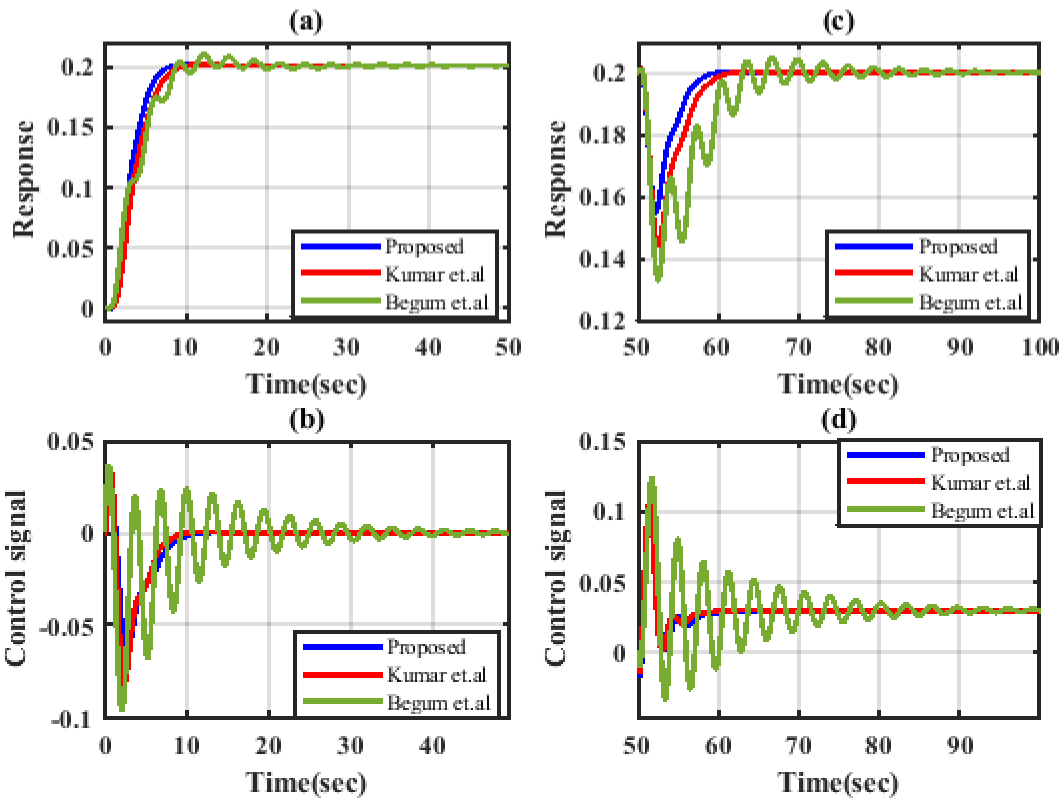

FIGURE 7. Responses along with control signal of example 2 under perturbed condition. (a) Servo response, (b) Control signal for servo response, (c) Regulatory response, (d) Control signal for regulatory response.

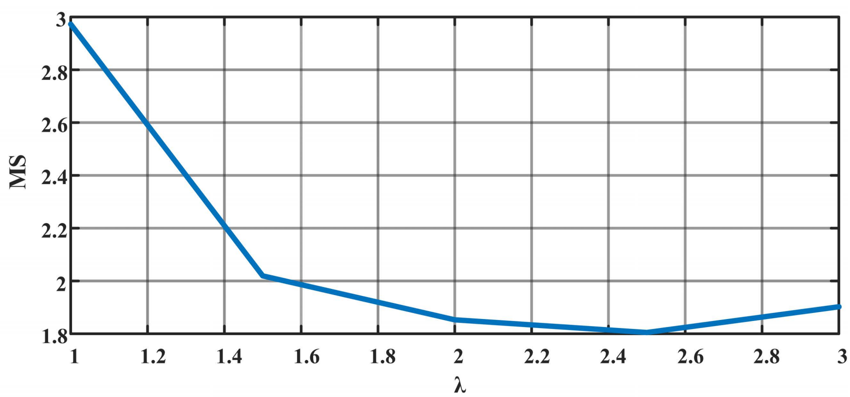

FIGURE 8. Relation between the λ with MS for example 3.

However, to validate the proposed method the authors have passed the measured output through additionally included transfer function with positive zero and time delay and then fed back to the controller. This results the open loop transfer function as represented in Equation 36.

$$G_p = \frac{24.7523(1 - s(0.03416))}{(0.1708s + 1)} e^{-0.06s} \tag{36}$$

Here the value of z is taken as 20% of time constant of the process. The test bench kit of Qube DC servo motor process is shown in Figure 12. The PID parameters are obtained for the proposed method at tuning parameter $\lambda = 0.1$ and the corresponding MS value is 2.83. The proposed method is compared with Sree and Chidambaram [2] technique. As the other method is implemented without set point filter, proposed method is also implemented without set point filter for fair comparison. Table 1 displays the resulting controller parameters for both methods. Sree and Chidambaram [2] method's control parameters are obtained at the tuning parameter(λ) value is equal to 0.7 where the MS value equal to 1.1386. As both the MS values are different and the proposed method is having higher MS value, to prove the efficacy of the

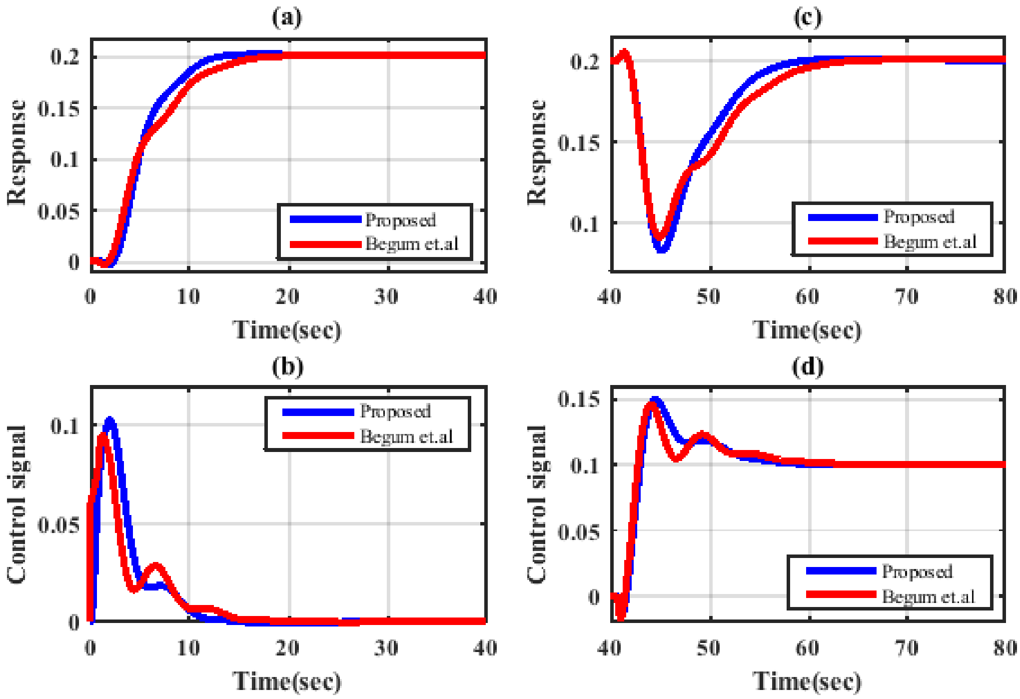

FIGURE 9. Responses along with control signal of example 3 under nominal condition. (a) Servo response, (b) Control signal for servo response, (c) Regulatory response, (d) Control signal for regulatory response.

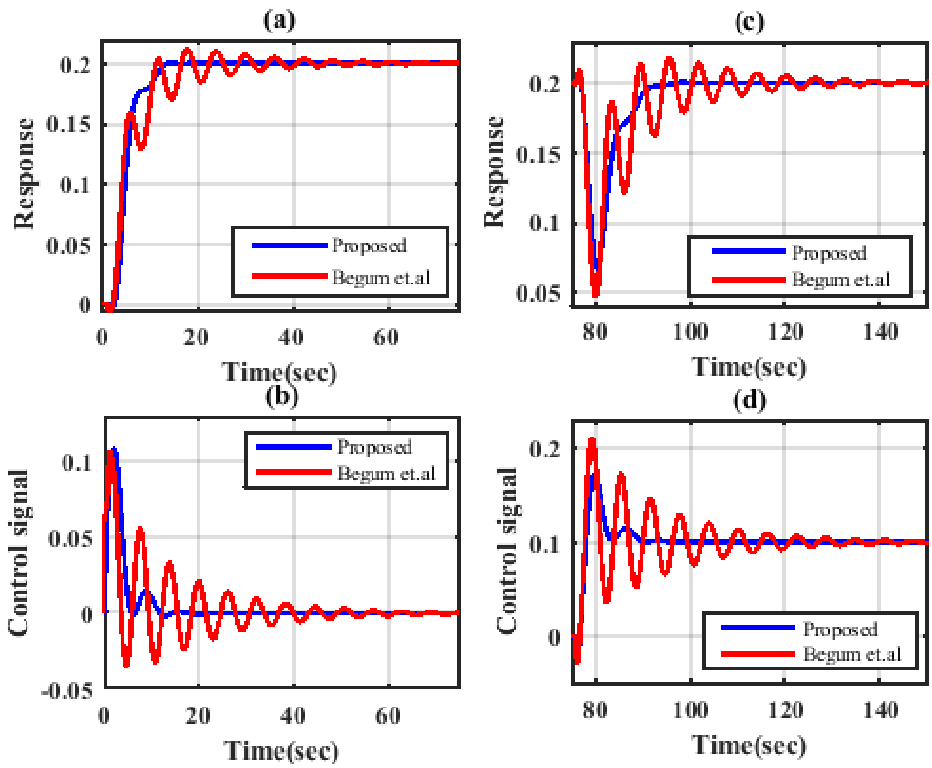

FIGURE 10. Responses along with control signal of example 3 under perturbed condition. (a) Servo response, (b) Control signal for servo response, (c) Regulatory response, (d) Control signal for regulatory response.

proposed method, it should be proved that the proposed method offers better robust performance i.e. stable response.

A set point in terms of speed having the magnitude of 20 rad/sec at t=0 s is applied and in order to study the

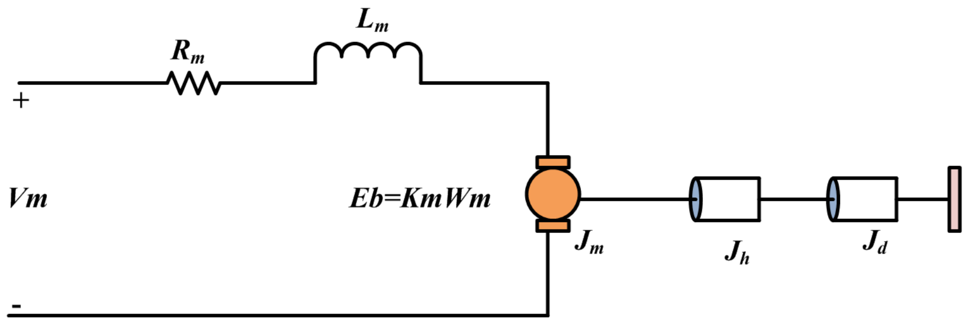

FIGURE 11. Schematic diagram of qube-servo motor and load.

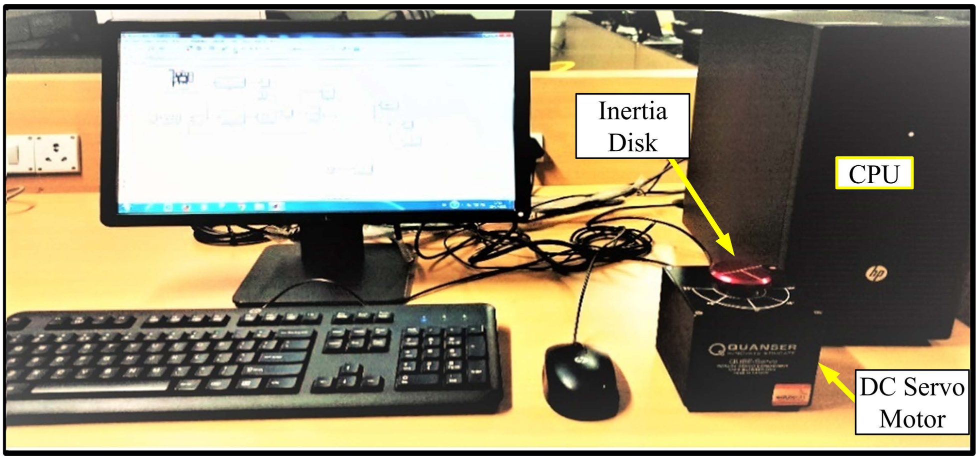

FIGURE 12. Experimental test bench of the qube DC servo process.

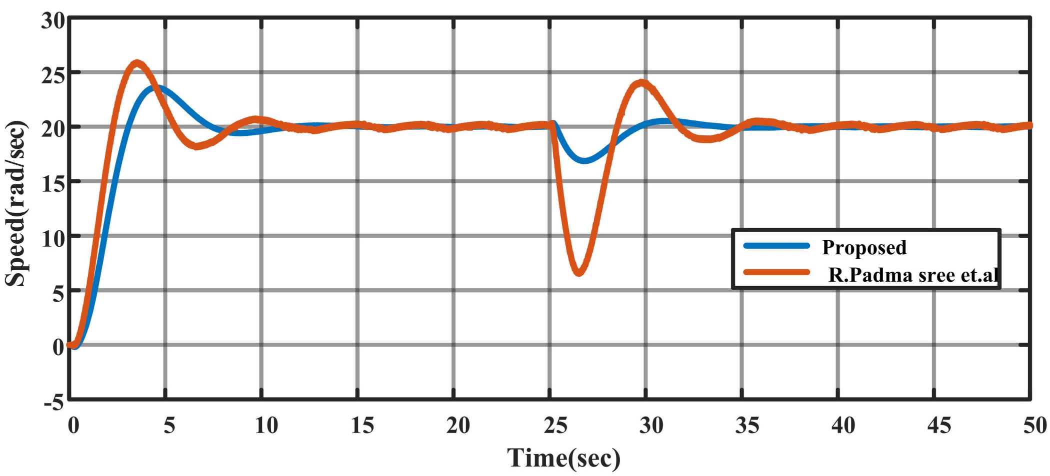

FIGURE 13. Step response of qube servo motor.

regulatory response a negative disturbance of magnitude 1 is applied at t=25 s. The response of DC servo motor is depicted in Figure 13 for both of the methods. In order to know the set point tracking capabilities, a square wave as a set point with amplitude changing between 0 and 20 rad/sec is applied. A pulse width of 50% is considered. The resulting responses are depicted in Figure 14. It is significant from the graphs that the proposed method offers lesser peak overshoot, oscillations and settling times in servo response and the other method offers better raise time. Importantly, the proposed method performs relatively very well in rejecting disturbances which is evident from regulatory response. Even

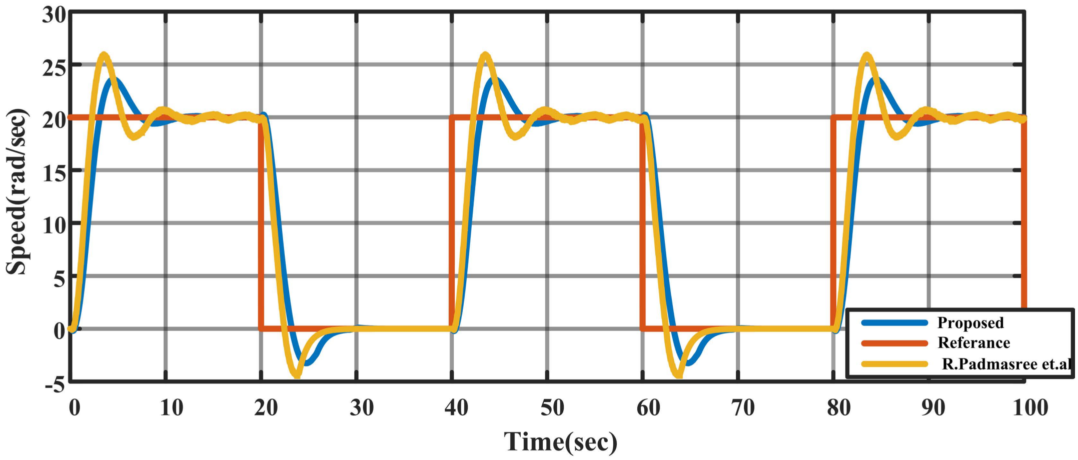

FIGURE 14. Square wave set point tracking response of qube DC servo motor.

TABLE 4. Qube Servo motor parameters.

Description	Symbol	Value
DC Motor		
Terminal Resistance	R_m	$8.4\,\Omega$
Motor back emf constant	K_m	0.042 V/(rad/s)
Torque Constant	k_t	0.042N.m/A
Rotor Inductance	L_m	1.16mH
Rotor Inertia	J_m	$4.0 \times 10^{-6} kgm^2$
Load disk		
Load disk radius	r_d	0.0248m
Load disk mass	m_d	0.053 kg
Moment of inertia of disk load	J_d	-
Moment of inertia of hub	J_h	-

with the higher MS value, the proposed method is able to produce relatively stable response when compared to the other method.

V. CONCLUSION

A novel PID controller augmented with fourth order filter is proposed for stable FOPTD-IR and stable/unstable IFOPTD-IR. The proposed method has adopted higher order time delay approximation of time delay for improved accuracy which resulted a higher order lead/lag filter in conjunction with the controller. Selection of tuning parameter is carried out with reference to maximum sensitivity. The presence of overshoot in the servo response is addressed by analytically designed set point filter. Simulation studies on several processes have been conducted to verify the efficacy of the proposed method. Examination of several performance criterion reveals that the proposed method offers better response characteristics. The proposed method is able to produce faster and less oscillatory responses while maintaining the smoothness of manipulated which is evident from the numerical values presented in performance comparison tables. Also, the proposed method is validated for real time application. In future work, the authors would like to enhance the proposed work by incorporating multi loop control structure, fractional order filters etc.

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