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# An Observer-Based Composite Nonlinear Feedback Controller for Robust Tracking of Uncertain Nonlinear Singular Systems With Input Saturation

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**ABSTRACT** This study proposes an observer-based Composite Nonlinear Feedback (CNF) controller for the robust tracking of uncertain singular systems with input saturation, nonlinear function, time-delay, and disturbances. The suggested control law is designed based states reconstructed using a singular observer so as to increase steady-state accuracy and improve robustness. The CNF controller is developed based on Generalized Riccati Equations (GRE) and Lyapunov–Krasovskii functional. Additionally, the proposed theorem verifies the stability conditions of the system in the presence of uncertainties and disturbances. Among the advantages of this method, are its fewer restrictive assumptions, transient and high-speed performance improvement and steady-state precision. The uniform boundedness of the tracking error in the presence of the input saturation and external disturbancesis also a prominent feature of this method. The performance of the proposed approach is assessed using a simulation study.

**INDEX TERMS** Generalized Riccati equation, singular systems, observer-based composite nonlinear feedback, robust tracking, nonlinear functions.

## I. INTRODUCTION

Singular systems (also called descriptive systems, low-cost systems, implicit systems, semi-state systems, generalized state-space systems, or differential-algebraic systems) are systems which dynamics are governed by a combination of algebraic and differential equations. In singular systems, algebraic equations along with differential equations create different properties than in ordinary systems. For example, singular systems may not have unique answer. In addition, they may have impulse terms in their response. Such properties in descriptive systems require problems such as different types of controllability, observability, impulse elimination, etc. in these systems. Algebraic limitations in singular systems can considerably entangle the observer and controller design, also the saturation limitation and external disturbances increase the complexity of the tracking problem. Singular systems have many applications in various theoretical and practical fields such as aircraft dynamics, neutral delay systems, chemical, thermal, diffusion processes, large-scale systems, interconnected systems, optimization problems, feedback control systems, robotics, electrical networks, power systems, aerospace engineering, social systems, economic systems, biological systems, network analysis, time-series analysis, etc. [1]–[7]. The complex nature of such systems, however, makes their control a

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challenging problem [8]–[11]. For example, in addition to stability, other issues such as regularity and impulse-free are also addressed in the control of singular systems. In the last two decades, many researches have been done on singular control systems. Most articles in this field are based on the generalization and extension of the theory of non-singular systems to singular systems. One of the important factors that cause instability and weak performance in control systems is the delay. Therefore, time-delay control systems are one of the issues that researchers have addressed recently. Due to the nature of these systems, the problem of controlling singular systems with time-delay is highly complex. On the other hand, there are often uncertainties in systems due to errors in modeling and changes in environmental and operating conditions. Since uncertainties affect systems' optimal performance it is important to design control laws that are robust against them. The problem of robust stability analysis, as well as robust stabilization of indefinite singular systems and indefinite delayed descriptive systems, has been studied in [12]–[14]. Albeit the existence of several approaches in the literature for descriptive systems, model uncertainties and time delays were overlooked in those designs. Stability is another important feature of the dynamical systems that express the system's responsive behavior to disturbances and primary conditions. Lyapunov stability theory is appropriate for the stability analysis of singular systems. Unlike conventional systems, descriptive system response may include impact expressions that can cause saturation in the plant input and even damage the system. So eliminating impulsive behavior through specified feedback control is a very basic issue in singular systems theory. In system design, not all variables can be directly available from measurements. In these cases, the estimation of the immeasurable variables is required for state feedback control implementation. Designing observer-based controllers is very desirable for stabilizing systems [15]–[17]. Observer designs for descriptive systems have attracted significant attention in the past decades [18]–[22]. Input saturation constraint is a common constraint in practical systems that does not allow the control inputs to exceed one specified limit. Ignoring the input saturation limitation in the process of controller design for systems in many cases can lead to undesirable system behavior and even instability. Therefore, designing control laws that take into consideration input saturation is essential. Robustness and optimum performance are two desired features in control systems.

In [23], the stability analysis and design of robust dynamic output feedback controller and controller based on observer for uncertain continuous singular systems with time-delay have been investigated but the nonlinearity and disturbances are not considered. In [24], a new controller and a fault-tolerant observer for a class of nonlinear continuous singular control systems are discussed regardless of uncertainty and time delay. In [25], a $H_\infty$ control based on observer for uncertain descriptive systems with time-delay and actuator saturation is investigated but system nonlinearities were overlooked. In [26], the observer design problem for one-sided Lipschitz nonlinear continuous-time singular systems with unknown input is considered but the nonlinearity and time delay are not investigated. In [27], observer design for a class of nonlinear singular systems with multi-outputs are considered without the nonlinearity, disturbance, and time delay. In [28], the problem of robust passive control based on observer is applied for uncertain singular time-delay systems with actuator saturation without considering the nonlinearities. A new functional observers design method is applied for descriptor systems via LMI in [29]. In [30], a method of designing full order observers is studied for time-delay descriptive systems with Lipschitz nonlinearities, then the Lyapunov–Krasovskii functional and the convexity principle are applied to investigate the stability of the singular systems. In [31], a robust adaptive observer is offered for a class of singular nonlinear non-autonomous uncertain systems with unstructured unknown system and derivative matrices, and unknown bounded nonlinearities and no strong assumption such as Lipschitz condition is applied on the recommended system. In [32], the linear multivariable feedback control is used for multi-input multi-output (MIMO), linear time-invariant (LTI) singular systems to improve the transient response to descriptor systems and follow a step reference with zero over-shoot. In [33], the modified composite nonlinear feedback method is considered for output tracking of non-step signals in singular systems with actuator saturation and external disturbances. In this article, the composite nonlinear feedback control law is not applied for the tracking of reference signals in singular systems. In [34], a robust composite nonlinear feedback controller is considered for descriptor systems with input saturation, this method can ensure general reference tracking for the singular systems with input saturation. In [35], a composite nonlinear feedback control method for tracking control problems is developed for the output regulation problem of singular linear systems with input saturation. In [36], an output-feedback sliding mode control is designed for a class of nonlinear singular systems with time delay and uncertainties, but input saturation was not considered. In [37], the problem of stability and stabilization is examined for singular networked control systems with short time-varying delay. In [38], the non-step tracking control problem is investigated for MIMO linear discrete-time descriptive systems with input saturation. In [39], the robust stability of uncertain fractional-order singular systems with neutral and time-varying delays is studied. In [40], an observer-based controller is considered for a class of singular nonlinear systems with state and exogenous disturbance-dependent noise. In [41], a finite-time observer-based controller is proposed for time-delay descriptor systems with time-varying disturbances, model uncertainties, and one-sided Lipschitz nonlinearities. In [42], the problem of adaptive output-feedback neural tracking control is investigated for a class of uncertain switched MIMO nonstrict-feedback nonlinear systems with time delays. The adaptive intelligent asymptotic tracking control is studied for a class of stochastic nonlinear systems with unknown control gains and full state constraints in [43]. A low-conservative composite nonlinear feedback controller is studied for singular time-delay systems with time-varying delay in [44]. For intermittent control, its control signal is updated in a continuous manner on control time intervals. To overcome the limitation, time-triggered intermittent control (TIC) is considered in [45]. In [46], the TIC is offered to examine the exponential synchronization issue of chaotic Lur'e systems.

Designing control approaches that ensure fast response with reasonable transient dynamics is essential in many applications. Composite nonlinear feedback (CNF) has recently emerged as a good solution for improving the transient performance of tracking control problems [47]–[48]. CNF combines a tracking control law that ensures a quick tracking performance with a nonlinear feedback law designed to smoothly change the damping ratio of the closed-loop system as this latter approaches the reference input so as to reduce the overshoots caused by the tracking law. The CNF method requires fewer limiting assumptions and as a result, more optimal design conditions are achieved. However, we cannot use the CNF control technique directly for singular systems. To overcome the problems of impulse terms and input derivatives in singular systems, we adopt state feedback to make the singular system free of impulses. For this purpose, a singular observer will be designed, based on the state variables, to estimate the states of the singular system. Since the problem is time-delays output tracking, the reference signal is generated by a reference generator and the tracking problem becomes a stabilization problem. The error vector and stability analysis will be carried over based on the Lyapunov's approach. Obviously, in the CNF combination, the CNF control law leads to a linear controller when the nonlinear phrase tends to zero. As a result, the added nonlinear expression allows modifying the linear control law to recover system transient performance and then the error converges to zero. In all the mentioned works, nonlinearity, disturbance, uncertainty, and even time delay are not considered together. To the best of our knowledge, no research has been performed to improve system performance for nonlinear and uncertain singular systems with input saturation, time delay, and external disturbances using the observer-based CNF control method.

This paper proposes an observer-based CNF control law for nonlinear singular systems with time delay, disturbances and input saturation. Its main contributions are as follows:

- An observer-based CNF control design for singular systems that takes into consideration time-delays, nonlinear dynamics and control input saturation.
- A design that yields improved transient performance and steady-state precision in the presence of time-delays, nonlinearities and disturbances.
- A control scheme that guarantees the robustness and stability of uncertain nonlinear singular systems.

The remainder of this paper is organized as follows. The problem formulation and required assumptions are provided in section II. The proposed observer-based composite nonlinear feedback controller is derived in section III. Simulation results are provided in section IV. Finally, some conclusions are presented in section V.

## II. PROBLEM FORMULATION AND PRELIMINARIES

Consider the uncertain nonlinear singular system with input saturation and time delay defined by:

$$\begin{aligned} E\dot{x}(t) &= f(x) + (A + \Delta A(r(t)))x(t) \\ &\quad + \sum_{i=1}^{N} (A_{d_i} + \Delta A_{d_i}(\nu(t)))x(t-\tau_i(t)) \\ &\quad + Bsat(u(t)) + d(t), \\ y(t) &= Cx(t), \end{aligned} \tag{1}$$

where $t \in [t_0, \infty)$, $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, $A, A_{d_i}, B, C$, and constant singular matrix E are matrices of proper dimensions, where rank(E) =r < n and $\tau_i \in \mathbb{R}^+$ is the time-delay. In addition, $d(t) \in \mathbb{R}^n$ is the unknown external disturbance vector. The saturation function is described by:

$$\begin{aligned} sat\,(u\,(t)) &= \begin{bmatrix} sat\,(u_1\,(t)) \\ sat\,(u_2\,(t)) \\ \vdots \\ sat\,(u_m\,(t)) \end{bmatrix} \\ sat(u_i(t)) &= sign(u_i(t))min(|u_i\,(t)|\,, \bar{u}_i\,(t)) \end{aligned} \tag{2}$$

where $\bar{u}_i(t)$ is the maximum value of the $i^{\text{th}}$ control input.

### A. PRELIMINARIES

- Singular system (E,A,B,C) is calledregular, if there exists a scalar s ∈ C, so that $det(sE - A) \neq 0$.
- Singular system (E, A, B, C) is called stable, if $\sigma(E, A) \subset C^-$, where $\sigma(E, A) = \{\lambda | \lambda \in C$, where $\det(\lambda E - A) = 0\}$, and $C^- = \{r | r \in C, Re(r) < 0\}$.
- Singular system (E, A, B, C) is impulse-free, if its solution does not have impulse terms and
$$rank\left(\begin{bmatrix} E & 0 \\ A & E \end{bmatrix}\right) = n + rank\,(E)\,.$$
- Singular system (E, A, B, C) is called admissible if it is stable and impulse-free.
- Singular system (E, A, B, C) is called C-controllable, if rank $\begin{bmatrix} sE - A & B \end{bmatrix} = n$ for all $s \in \bar{C}^+$, $s$ finite, where $\bar{C}^+ = \{s | s \in C, Re(s) \geq 0$.
- Singular system (E, A, B, C) is called C-observable, if rank $\begin{bmatrix} sE - A \\ C \end{bmatrix} = n$ for all $s \in C$, s finite.
- Singular system (E, A, B, C) is said to impulse controllable, if rank $\begin{bmatrix} E & 0 & 0 \\ A & E & B \end{bmatrix} = n + rank(E)$.
- Singular system (E, A, B, C) is called impulse observable, if rank $\begin{bmatrix} E & A \\ 0 & E \\ 0 & C \end{bmatrix} = n$.

Adequate conditions for a singular system to be stabilizable and detectable are C-controllability and C-observability.

*Lemma 1: Let $V : [0, \infty) \times D \to R$ be a continuously differentiable Lyapunov functionfor the singular system (1) so that:*

$$\eta_1(\|x(t)\|) \le V(t, x(t)) \le \eta_2(\|x(t)\|),\quad \forall (t, x) \in [0,\infty) \times D \tag{3}$$

$$\dot{V}(t, x(t)) \le -\Phi,\ \text{whenever}\quad V(t+\theta, x(t+\theta)) \le V(t, x(t)) \tag{4}$$

*where $\eta_1(\cdot)$ and $\eta_2(\cdot)$ are class K functions, $\Phi$ is a continuous positive definite function on the open set D. Then the solution $x(t)$ of the system (1) for $\theta \in [-\tau, 0]$ is uniformly ultimately bounded.*

*Lemma 2: Suppose the singular system is regular and impulse-free, then the response of such a system will be on $[0, \infty)$ unique and the impulse-free.*

*Definition 1: System $(E, A, B, C)$ is called invertible with no zeros at $s = 0$.*

*Definition 2: System $(E, A, B)$ is called stabilizable if there is a matrix F, so that the pair $(E, A + BF)$ is stable.*

*Definition 3: System $(E, A, C)$ is called detectible if there is a matrix L, so that the pair $(E, A + LC)$ is stable.*

*Definition 4: System $(E, A, B)$ is called impulse-controllable if there is a matrix F, so that the pair (E, A+ BF ) is impulse-free.*

*Definition 5: System $(E, A, C)$ is called impulse-observable if there is a matrix L, so that the pair $(E, A+LC)$ is impulse-free.*

*Assumption 1:The external disturbance vector is bounded with $\| d(t) \| \le d_{max}$, where $d_{max}$ is a known positive real constant.*

*Assumption 2: The following inequality holds for the nonlinear function $f(\zeta)$ :*

$$\|f(\zeta_1) - f(\zeta_2)\| \le \|M(\zeta_1 - \zeta_2)\| \tag{5}$$

where $M = diag\{M_0, M_1, \ldots, M_N\}$ and $M_i \in R^{n\times n}$, $i = 0, 1, \ldots, N$ are some known matrices.

*Fact 1: Let $F \in R^{n1\times n2}, G \in R^{n2\times n1}$, for any $\rho > 0$ the following inequality holds :*

$$F^TG + G^TF \le \rho F^TF + \frac{1}{\rho}G^TG. \tag{6}$$

*Remark 1:* Assumption 1 is a reasonable condition considered in practical cases and indicates that the norm of the disturbance vector does not exceed the actuator saturation level in each input channel.

## III. MAIN RESULTS

In this section, the observer-based control law is first designed then its stability and accuracy are proven using the Lyapunov stability analysis for three different cases of input saturation.

### A. TIME VARYING REFERENCE GENERATION

The purpose of this study is to design an observed-based CNF lawfor the uncertain singular system so that the output $y(t)$ can follow the reference $y_m(t)$. The reference signal may be created by a source generator system. The reference signal could be described based on the basic system matrices (E, A, B, C) as follows:

$$\begin{aligned} E\dot{x}_m(t) &= Ax_m(t) + Bu_m(t) + f(x_m) \\ y_m(t) &= Cx_m(t), \end{aligned} \tag{7}$$

where $x_m(t) \in R^n, u_m(t) \in R^m, y_m(t) \in R^l$ are the state, the control input, and the output vectors of the reference signal, respectively. Consider the control input defined by $u_m(t) = F_mx_m(t) + r_s(t)$, where $F_m$ is a static feedback gain that is chosen so that the pair $(E, A + BF_m)$ is stable and impulse-free. Moreover, $r_s(t)$ is an adjustable signal elected by the designer. As a result, the reference model is described as follows

$$\begin{aligned} E\dot{x}_m(t) &= (A + BF_m)x_m(t) + Br_s(t) + f(x_m) \\ y_m(t) &= Cx_m(t), \end{aligned} \tag{8}$$

The auxiliary state vector of the output tracking error is specified as $x_e(t) = \int_{t_0}^{t} e(t)dt$ and the output tracking error as $e(t) = y(t) - y_m(t)$. The dynamic equation of the auxiliary state vector is determined as below

$$\dot{x}_e(t) = y(t) - y_m(t) = Cx(t) - y_m(t) \tag{9}$$

where $x_e(t) \in R^l$. Thus, the augmented system is achieved as follows:

$$\begin{aligned} E^*\dot{x}^*(t) &= (A + \Delta A(r(t)))^*x^*(t) \\ &\quad + \sum_{i=1}^{N}(A_{d_i}+\Delta A_{d_i}(\nu(t)))^*x^*(t-\tau_i(t)) + f^*(x^*) \\ &\quad + B^*sat(u(t)) + B_r^*y_m(t) + d^*(t), \\ y^*(t) &= C^*x^*(t), \end{aligned} \tag{10}$$

where

$$x^*(t) = \begin{bmatrix} x_e(t) \\ x(t) \end{bmatrix} \in R^{l+n},\ B^* = \begin{bmatrix} 0 \\ B \end{bmatrix},$$

$$E^* = \begin{bmatrix} I & 0 \\ 0 & E \end{bmatrix},$$

$$d^*(t) = \begin{bmatrix} 0 \\ d(t) \end{bmatrix},$$

$$x^*(t-\tau_i(t)) = \begin{bmatrix} x_e(t-\tau_i(t)) \\ x(t-\tau_i(t)) \end{bmatrix},$$

$$(A_{d_i} + \Delta A_{d_i}(\nu(t)))^* = \begin{bmatrix} 0 & 0 \\ 0 & A_{d_i} + \Delta A_{d_i}(\nu(t)) \end{bmatrix},$$

$$C^* = \begin{bmatrix} I & 0 \\ 0 & C \end{bmatrix},\ y^*(t) = \begin{bmatrix} x_e(t) \\ y(t) \end{bmatrix},$$

$$B_r^* = \begin{bmatrix} -I \\ 0 \end{bmatrix},$$

$$f^*(x) = \begin{bmatrix} 0 \\ f(x) \end{bmatrix},\ A^* = \begin{bmatrix} 0 & C \\ 0 & A \end{bmatrix},$$

$$(A + \Delta A(r(t)))^* = \begin{bmatrix} 0 & C \\ 0 & A + \Delta A(r(t)) \end{bmatrix}$$

The reference model can also be written as follows:

$$\begin{aligned} E^*\dot{x}_m^*(t) &= A^*x_m^*(t) + B^*u_m(t) + B_r^*y_m(t) + f^*\left(x_m^*\right) \\ u_m(t) &= [0\ F_m]x_m^*(t) + r_s(t) \\ y_m^*(t) &= C^*x_m^*(t), \end{aligned} \tag{11}$$

where $x_m^*(t) = \begin{bmatrix} 0 \\ x_m(t) \end{bmatrix} \in R^{l+n}$, and tracking error vector is defined as $\tilde{x}(t) = x^*(t) - x_m^*(t)$, therefore the dynamical equations of tracking error are obtained as the following equation

$$\begin{aligned} E^*\dot{\tilde{x}}(t) &= (A + \Delta A(r(t)))^*\tilde{x}(t) \\ &\quad + \sum\nolimits_{i=1}^{N} (A_{d_i} + \Delta A_{d_i}(\nu(t)))^*\tilde{x}(t-\tau_i(t)) \\ &\quad + \sum\nolimits_{i=1}^{N} (A_{d_i} + \Delta A_{d_i}(\nu(t)))^*x_m(t-\tau_i(t)) \\ &\quad + (\Delta A(r(t)))^*x_m(t) + f^*(x^*) - f^*(x_m^*) \\ &\quad + B^*\left(sat\left(u\left(t\right)\right) - u_m\left(t\right)\right) + d^*\left(t\right) \\ y^*(t) &= C^*\tilde{x}(t) + y_m^*(t) \end{aligned} \tag{12}$$

### B. SINGULAR OBSERVER DESIGN

Since full state availability is not always possible in control system design, state estimation is often required to implement state feedback control. That's why the singular system must be C-observable or detectable. The singular system must also be impulse-observable to assure the lack of impulse terms in the observer response. The singular observer can be constructed for the system (10) as follows:

$$\begin{aligned} E^*\dot{x}_v(t) &= f^*(x_v) + \left(A^* + LC^*\right)x_v(t) \\ &\quad + \sum_{i=1}^{N} A_{d_i}^* x_v(t - \tau_i(t)) - \mathrm{L}y^* \\ &\quad + B^*sat(u(t)) + B_r^*y_m(t) \end{aligned} \tag{13}$$

where $x_v(t) \in R^{n+l}$ is the state observer vector, and L is the observer gain and should be constructed so that the pair $(E^*, A^* + LC^*)$ is stable and impulse-free. The observer error is determined as $\tilde{x}_v(t) = x_v(t) - x^*(t)$ and the dynamical equations of observer error are created as the following equation

$$\begin{aligned} E^*\dot{\tilde{x}}_v(t) &= \left(A^* + LC^*\right)\tilde{x}_v(t) + \sum\nolimits_{i=1}^{N} A_{d_i}^*\tilde{x}_v(t - \tau_i(t)) \\ &\quad - \sum\nolimits_{i=1}^{N} (\Delta A_{d_i})^*(\nu(t))\,x^*(t-\tau_i(t)) \\ &\quad - (\Delta A)^*(r(t))x^*(t) - d^*(t) + f^*(x_v) - f^*(x^*) \end{aligned} \tag{14}$$

### C. OBSERVER-BASED CNF CONTROL LAW

The CNF includes two linear and nonlinear components. The linear feedback part is constructed in such a way that it can cause a slight damping ratio in the system in order to obtain a faster response. The nonlinear feedback control is applied to increment the damping ratio of the closed-loop system as the system output follows the reference model to lower the overshoot created by the linear portion. The linear part of the observer-based CNF control law is described as:

$$\begin{aligned} u_L(t) &= F(x_v(t) - x_m^*(t)) + u_m(t) \\ &= \begin{bmatrix} F & F \end{bmatrix}\begin{bmatrix} \tilde{x}(t) \\ \tilde{x}_v(t) \end{bmatrix} + u_m(t) \end{aligned} \tag{15}$$

where F is determined so that the pair $(E, A^* + B^*F)$ is stable and impulse-free. Also, the nonlinear part of the CNF law is displayed as

$$u_N(t) = \phi(y_m(t), y(t))B^{*^T}PE^*(x_v(t) - x_m^*(t)) \tag{16}$$

where P is determinedfrom the following Generalized Riccati Equations (GRE) for any $Q > 0$:

$$\begin{aligned} &(A^* + B^*F)^T PE^* + E^{*^T}P(A^* + B^*F) \\ &\quad + E^{*^T}\sum_{i=1}^{N} R_iE^* + \sum_{i=1}^{N}\frac{1}{\rho_{1i}}E^{*^T}P^2E^* \\ &\quad + \sum_{i=1}^{N}\frac{1}{\rho_{2i}}E^{*^T}P^2E^* + \frac{1}{\mu_1}E^{*^T}P^2E^* \\ &\quad + \sum_{i=1}^{N}\frac{1}{\rho_{6i}}E^{*^T}P^2E^* + \sum_{i=1}^{N}\frac{1}{\rho_{7i}}E^{*^T}P^2E^* \\ &\quad + \mu_7E^{*^T}P^2E^* + \mu_1\alpha^2I + \mu_2\alpha^2I + \frac{1}{\mu_4}E^{*^T}P^2E^* \\ &\quad + \mu_5E^{*^T}P^2E^* + \frac{1}{\mu_7}M^2 + \mu_9E^{*^T}P^2E^* + E^{*^T}QE^* = 0 \end{aligned} \tag{17}$$

where $\mu_1, \mu_2, \mu_4, \mu_5, \mu_7, \mu_9, \rho_{1i}, \rho_{2i}, \rho_{6i}, \rho_{7i}$ are positive constants, and $\phi(y_m(t), y(t))$ is defined as

$$\begin{aligned} \phi\left(y_m(t), y(t)\right) = diag[&\phi_1(y_{m1}(t), y_1(t)), \phi_2(y_{m2}(t), y_2(t)) \cdots, \\ &\phi_m(y_{ml}(t)(y_{ml}(t), y_l(t))] \end{aligned} \tag{18}$$

where $\phi_i$ are negative constant functions locally Lipschitz in $y(t)$. The CNF law based on observer can be expressed as follows:

$$\begin{aligned} u(t) &= u_L(t) + u_N(t) \\ &= F\left(x_v(t) - x_m^*(t)\right) + u_m(t) \\ &\quad + \phi(y_m(t), y(t))B^{*^T}PE^*(x_v(t) - x_m^*(t)) \end{aligned} \tag{19}$$

*Remark 2.* The non-unique function $\phi(.)$ can be effective in the transient performance improvement. Some examples and criteria for selecting appropriate $\phi(.)$ are presented in [47].

*Theorem 1:* Consider the system (12) and assume that assumptions 1-2 hold. let $\Gamma > 0$ be a solution of the GRE as follows for any $H > 0$

$$\begin{aligned} &(A^* + LC^*)^T\Gamma E^* + E^{*^T}\Gamma(A^* + LC^*) \\ &\quad + E^{*^T}\sum\nolimits_{i=1}^{N} D_iE^* + \sum\nolimits_{i=1}^{N}\frac{1}{\rho_{3i}}E^{*^T}\Gamma^2E^* \\ &\quad + \sum\nolimits_{i=1}^{N}\frac{1}{\rho_{4i}}E^{*^T}\Gamma^2E^* + \frac{1}{\mu_2}E^{*^T}\Gamma^2E^* \\ &\quad + \sum\nolimits_{i=1}^{N}\frac{1}{\rho_{5i}}E^{*^T}\Gamma^2E^* + \frac{1}{\mu_3}E^{*^T}\Gamma^2E^* \end{aligned}$$

$$+\mu_6 E^{*^T}\Gamma^2 E^* + \mu_8 E^{*^T}\Gamma^2 E^* + \frac{1}{\mu_8}M^2 + \frac{1}{\mu_9}F^T F B^T B + E^{*^T} H E^* = 0 \quad (20)$$

where $\mu_2, \mu_3, \mu_6, \mu_8, \mu_9, \rho_{3i}, \rho_{4i}, \rho_{5i}$ are positive constants, and for any $\delta \in (0, 1)$, let $d_\delta$ be the largest positive constant so that:

$$(a)\left|\begin{bmatrix} f_i & f_i \end{bmatrix}\begin{bmatrix}\tilde{x}(t)\\ \tilde{x}_v(t)\end{bmatrix}\right| \le (1-\delta)\bar{u}_i, \forall \begin{bmatrix}\tilde{x}(t)\\ \tilde{x}_v(t)\end{bmatrix} \in \theta_\delta, \quad (21)$$

$$(b)|u_{m_i}(t)| \le \delta\bar{u}_i, \begin{bmatrix}\tilde{x}(0)\\ \tilde{x}_v(0)\end{bmatrix} \in \theta_\delta \quad (22)$$

where

$$\theta_\delta := \left\{\begin{bmatrix}E^*\tilde{x}(t)\\ E^*\tilde{x}_v(t)\end{bmatrix} \mid \begin{bmatrix}E^*\tilde{x}(t)\\ E^*\tilde{x}_v(t)\end{bmatrix}^T \begin{bmatrix}P & 0\\ 0 & \Gamma\end{bmatrix}\begin{bmatrix}E^*\tilde{x}(t)\\ E^*\tilde{x}_v(t)\end{bmatrix} \le d_\delta\right\}, \quad (23)$$

matrices $P$ and $\Gamma$ satisfy the GRE (17) and (20), $f_i$ is the $i^{\text{th}}$ raw of matrix F, and $u_{m_i}$ is the $i^{\text{th}}$ component of the control input $u_m$. Thus, there exist scalars $\phi_i^*$, for $i = 1, 2, \ldots, m$ such that for any $\phi(y_m(t), C^*x_v(t))$ locally Lipschitz in $C^*x_v(t)$, and $|\phi(y_m^*(t), C^*x_v(t))| \le \phi_i^*$, the CNF lawbasedon observer (19) ensures the following:

- By applying the control law, the controlled output $y(t)$ can track asymptotically the reference input $y_m(t)$ in the presence as well as the absence of the external disturbances and uncertainties with a tracking error that is bounded and limited.
- $\theta_\delta$ is a positively invariant set for the closed-loop singular system.

*Proof:* Due to conditions (21) and (22), the linear part of the control law in all input channels does not exceed the saturation bound. The closed-loop singular system can be attained using the control law (19) and the system's equations that are deduced from Eqs. (12) and (14) as

$$\begin{bmatrix}E^*\dot{\tilde{x}}(t)\\ E^*\dot{\tilde{x}}_v(t)\end{bmatrix} = \begin{bmatrix}(A+\Delta A(r(t)))^* + B^*F & B^*F\\ 0 & A^*+LC^*\end{bmatrix}\begin{bmatrix}\tilde{x}(t)\\ \tilde{x}_v(t)\end{bmatrix} + \begin{bmatrix}\sum_{i=1}^{N}(A_{d_i}+\Delta A_{d_i}(v(t)))^*\tilde{x}(t-\tau_i(t))\\ \sum_{i=1}^{N}(A^*_{d_i}\tilde{x}_v(t-\tau_i(t))\end{bmatrix} + \begin{bmatrix}0\\ -\sum_{i=1}^{N}(\Delta A_{d_i})^*(v(t))x^*(t-\tau_i(t)) - (\Delta A)^*x^*(t)\end{bmatrix} + \begin{bmatrix}\sum_{i=1}^{N}(A_{d_i}+\Delta A_{d_i})^*x_m(t-\tau_i(t)) + (\Delta A)^*x_m(t)\\ 0\end{bmatrix} + \begin{bmatrix}d^*(t)\\ -d^*(t)\end{bmatrix} + \begin{bmatrix}B^*\\ 0\end{bmatrix}\omega(t) + \begin{bmatrix}f^*(x^*)-f^*(x_m^*)\\ f^*(x_v)-f^*(x^*)\end{bmatrix} \quad (24)$$

where

$$\omega(t) = sat\left\{\begin{bmatrix}F & F\end{bmatrix}\begin{bmatrix}\tilde{x}(t)\\ \tilde{x}_v(t)\end{bmatrix} + u_m(t) + \phi B^{*^T}PE^*\left(x_v(t) - x_m^*(t)\right)\right\} - \begin{bmatrix}F & F\end{bmatrix}\begin{bmatrix}\tilde{x}(t)\\ \tilde{x}_v(t)\end{bmatrix} - u_m(t) \quad (25)$$

To examine the performance of the suggested control law in the presence of external disturbances, uncertainties, and nonlinear functions, we present the following Lyapunov function as follows:

$$V(\tilde{x}(t), \tilde{x}_v(t)) = \tilde{x}^T(t)E^{*^T}PE^*\tilde{x}(t) + \tilde{x}_v^T(t)E^{*^T}\Gamma E^*\tilde{x}_v(t) + \sum_{i=1}^{N}\int_{t-\tau_i}^{t}\tilde{x}^T(s)E^{*^T}R_iE^*\tilde{x}(s)ds + \sum_{i=1}^{N}\int_{t-\tau_i}^{t}\tilde{x}_v^T(s)E^{*^T}D_iE^*\tilde{x}_v(s)ds \quad (26)$$

Deriving the Lyapunov function along with the directions of the closed-loop system in (24), yields:

$$\begin{aligned}
&\dot{V}(\tilde{x}(t),\tilde{x}_v(t))\\
&= (\begin{bmatrix}\tilde{x}(t)\\ \tilde{x}_v(t)\end{bmatrix}^T\begin{bmatrix}A^*+B^*F & B^*F\\ 0 & A^*+LC^*\end{bmatrix}^T + \omega^T(t)\begin{bmatrix}B^*\\ 0\end{bmatrix}^T\\
&\quad + \begin{bmatrix}d^*(t)\\ -d^*(t)\end{bmatrix}^T + \begin{bmatrix}f^*(x^*)-f^*(x_m^*)\\ f^*(x_v)-f^*(x^*)\end{bmatrix}^T)\\
&\quad \times \begin{bmatrix}P & 0\\ 0 & \Gamma\end{bmatrix}\begin{bmatrix}E^*\tilde{x}(t)\\ E^*\tilde{x}_v(t)\end{bmatrix}\\
&\quad + \begin{bmatrix}E^*\tilde{x}(t)\\ E^*\tilde{x}_v(t)\end{bmatrix}^T\begin{bmatrix}P & 0\\ 0 & \Gamma\end{bmatrix}(\begin{bmatrix}A^*+B^*F & B^*F\\ 0 & A^*+LC^*\end{bmatrix}\begin{bmatrix}\tilde{x}(t)\\ \tilde{x}_v(t)\end{bmatrix}\\
&\quad + \begin{bmatrix}f^*(x^*)-f^*(x_m^*)\\ f^*(x_v)-f^*(x^*)\end{bmatrix} + \begin{bmatrix}B^*\\ 0\end{bmatrix}\omega(t) + \begin{bmatrix}d^*(t)\\ -d^*(t)\end{bmatrix})\\
&\quad + \tilde{x}^T(t)(\Delta A)^{*^T}PE^*\tilde{x}(t) + \tilde{x}^T(t)E^{*^T}P(\Delta A)^*\tilde{x}(t)\\
&\quad + \sum_{i=1}^{N}\tilde{x}^T(t-\tau_i)(A_{d_i}+\Delta A_{d_i})^{*^T}PE^*\tilde{x}(t)\\
&\quad + \tilde{x}^T(t)E^{*^T}P\sum_{i=1}^{N}(A_{d_i}+\Delta A_{d_i})^*\tilde{x}(t-\tau_i)\\
&\quad + \sum_{i=1}^{N}\tilde{x}_v^T(t-\tau_i)A_{d_i}^{*^T}\Gamma E^*\tilde{x}_v(t)\\
&\quad + \tilde{x}_v^T(t)E^{*^T}\Gamma\sum_{i=1}^{N}A^*_{d_i}\tilde{x}_v(t-\tau_i) + \sum_{i=1}^{N}\tilde{x}^T(t)E^{*^T}R_iE^*\tilde{x}(t)\\
&\quad - \sum_{i=1}^{N}\tilde{x}^T(t-\tau_i)E^{*^T}R_iE^*\tilde{x}(t-\tau_i)\\
&\quad + \sum_{i=1}^{N}\tilde{x}_v^T(t)E^{*^T}D_iE^*\tilde{x}_v(t)\\
&\quad - \sum_{i=1}^{N}\tilde{x}_v^T(t-\tau_i)E^{*^T}D_iE^*\tilde{x}_v(t-\tau_i)
\end{aligned}$$

$$
\begin{aligned}
&-\sum_{i=1}^{N} x^{*^T}(t-\tau_i)(\Delta A_{d_i})^{*^T}\Gamma E^*\tilde{x}_v(t)\\
&-\tilde{x}_v^T(t)E^{*^T}\Gamma\sum_{i=1}^{N}(\Delta A_{d_i})^* x^*(t-\tau_i)\\
&-x^{*^T}(t)(\Delta A)^{*^T}\Gamma E^*\tilde{x}_v(t)\\
&-\tilde{x}_v^T(t)E^{*^T}\Gamma(\Delta A)^* x^*(t)+\tilde{x}^T(t)E^{*^T}P\sum_{i=1}^{N}(A_{d_i}\\
&+\Delta A_{d_i})^* x_m(t-\tau_i(t))\\
&+\sum_{i=1}^{N} x_m^T(t-\tau_i(t))(A_{d_i}+\Delta A_{d_i})^{*^T}PE^*\tilde{x}(t)\\
&+\tilde{x}^T(t)E^{*^T}P(\Delta A)^* x_m(t)+x_m^T(t)(\Delta A)^{*^T}PE^*\tilde{x}(t) \qquad (27)
\end{aligned}
$$

Then, we obtain

$$
\begin{aligned}
&\dot{V}(\tilde{x}(t),\tilde{x}_v(t))\\
&=\begin{bmatrix}\tilde{x}(t)\\ \tilde{x}_v(t)\end{bmatrix}^T\begin{bmatrix} a_{11} & E^{*^T}PB^*F\\ F^TB^{*^T}PE^* & a_{22}\end{bmatrix}\begin{bmatrix}\tilde{x}(t)\\ \tilde{x}_v(t)\end{bmatrix}\\
&\quad+\bar{\chi}^T(t)\mathrm{A}\bar{\chi}(t)+\bar{\chi}_v^T(t)\mathrm{B}\bar{\chi}_v(t)+\tilde{x}^T(t)E^{*^T}P(f^*(x^*)\\
&\quad-f^*(x_m^*))+(f^*(x^*)-f^*(x_m^*))^TPE^*\tilde{x}(t)\\
&\quad+(f^*(x_v)-f^*(x^*))^T\Gamma E^*\tilde{x}_v(t)+\tilde{x}_v^T(t)E^{*^T}\Gamma(f^*(x_v)\\
&\quad-f^*(x^*))+\omega^T(t)B^{*^T}PE^*\tilde{x}(t)+\tilde{x}^T(t)E^{*^T}PB^*\omega(t)\\
&\quad+\tilde{x}^T(t)E^{*^T}Pd^*(t)\\
&\quad+d^{*^T}(t)PE^*\tilde{x}(t)-\tilde{x}_v^T(t)E^{*^T}\Gamma d^*(t)-d^{*^T}(t)\Gamma E^*\tilde{x}_v(t)\\
&\quad-\tilde{x}^T(t)(\Delta A)^{*^T}\Gamma E^*\tilde{x}_v(t)-\tilde{x}_v^T(t)E^{*^T}\Gamma(\Delta A)^*\tilde{x}(t)\\
&\quad-\tilde{x}_v^T(t)E^{*^T}\Gamma\sum_{i=1}^{N}(\Delta A_{d_i})^*\tilde{x}(t-\tau_i)\\
&\quad-\sum_{i=1}^{N}\tilde{x}^T(t-\tau_i)(\Delta A_{d_i})^{*^T}\Gamma E^*\tilde{x}_v(t)\\
&\quad-\tilde{x}_v^T(t)E^{*^T}\Gamma(\Delta A)^* x_m^*(t)-x_m^{*^T}(t)(\Delta A)^{*^T}\Gamma E^*\tilde{x}_v(t)\\
&\quad-\sum_{i=1}^{N} x_m^{*^T}(t-\tau_i)(\Delta A_{d_i})^{*^T}\Gamma E^*\tilde{x}_v(t)\\
&\quad-\tilde{x}_v^T(t)E^{*^T}\Gamma\sum_{i=1}^{N}(\Delta A_{d_i})^* x_m^*(t-\tau_i)\\
&\quad+\tilde{x}^T(t)E^{*^T}P\sum_{i=1}^{N}(A_{d_i}+\Delta A_{d_i})^* x_m(t-\tau_i(t))\\
&\quad+\sum_{i=1}^{N} x_m^T(t-\tau_i)(A_{d_i}+\Delta A_{d_i})^{*^T}PE^*\tilde{x}(t)\\
&\quad+\tilde{x}^T(t)E^{*^T}P(\Delta A)^* x_m(t)+x_m^T(t)(\Delta A)^{*^T}PE^*\tilde{x}(t) \qquad (28)
\end{aligned}
$$

where $a_{11}=(A^*+B^*F)^TPE^*+E^{*^T}P(A^*+B^*F)+E^{*^T}P(\Delta A)^*+(\Delta A)^{*^T}PE^*+E^{*^T}\sum_{i=1}^{N}R_iE^*$ and

$$a_{22}=(A^*+LC^*)^T\Gamma E^*+E^{*^T}\Gamma(A^*+LC^*)+E^{*^T}\sum_{i=1}^{N}D_iE^*$$

$$
\mathrm{A}=\begin{bmatrix}0 & E^{*^T}P(A_{d_1}+\Delta A_{d_1}) & \cdots & E^{*^T}P(A_{d_N}+\Delta A_{d_N})\\ * & -E^{*^T}R_1E^* & 0 & 0\\ * & * & \ddots & 0\\ * & * & * & -E^{*^T}R_NE^*\end{bmatrix} \qquad (29)
$$

$$
\mathrm{B}=\begin{bmatrix}0 & E^{*^T}\Gamma A_{d_1}^* & \cdots & E^{*^T}\Gamma A_{d_N}^*\\ * & -E^{*^T}D_1E^* & 0 & 0\\ * & * & \ddots & 0\\ * & * & * & -E^{*^T}D_NE^*\end{bmatrix} \qquad (30)
$$

$$
\bar{\chi}(t)=\begin{bmatrix}\tilde{x}(t)\\ \tilde{x}(t-\tau_1)\\ \tilde{x}(t-\tau_2)\\ \vdots\\ \tilde{x}(t-\tau_N)\end{bmatrix},\ \bar{\chi}_v(t)=\begin{bmatrix}\tilde{x}_v(t)\\ \tilde{x}_v(t-\tau_1)\\ \tilde{x}_v(t-\tau_2)\\ \vdots\\ \tilde{x}_v(t-\tau_N)\end{bmatrix} \qquad (31)
$$

According to $F^TG+G^TF\le\rho F^TF+\frac{1}{\rho}G^TG$, $||\Delta A_{d_i}||\le\alpha_i$ and $||\Delta A||\le\alpha$, $||x_m^*(t)||\le\beta$ and using GRE (17) and (20), we have:

$$
\begin{aligned}
&\dot{V}(\tilde{x}(t),\tilde{x}_v(t))\\
&\le\begin{bmatrix}\tilde{x}(t)\\ \tilde{x}_v(t)\end{bmatrix}^T\begin{bmatrix} b_{11} & E^{*^T}PB^*F\\ F^TB^{*^T}PE^* & b_{22}\end{bmatrix}\begin{bmatrix}\tilde{x}(t)\\ \tilde{x}_v(t)\end{bmatrix}\\
&\quad+\chi^T(t)\mathrm{M}\chi(t)+\chi_v^T(t)\mathrm{N}\chi_v(t)\\
&\quad+\omega^T(t)B^{*^T}PE^*\tilde{x}(t)+\tilde{x}^T(t)E^{*^T}PB^*\omega(t)\\
&\quad+\mu_4\beta^2\alpha^2+\sum_{i=1}^{N}\rho_{5i}\beta^2\alpha_i^2+\mu_3\beta^2\alpha^2\\
&\quad+\sum_{i=1}^{N}\rho_{6i}\beta^2||A_{d_i}||^2+\mu_3\beta^2\alpha^2\\
&\quad+\sum_{i=1}^{N}\rho_{7i}\beta^2\alpha^2+\frac{1}{\mu_5}d_{max}^2+\frac{1}{\mu_6}d_{max}^2 \qquad (32)
\end{aligned}
$$

where

$$
\begin{aligned}
b_{11}&=(A^*+B^*F)^TPE^*+E^{*^T}P(A^*+B^*F)\\
&\quad+E^{*^T}\sum_{i=1}^{N}R_iE^*+\sum_{i=1}^{N}\frac{1}{\rho_{1i}}E^{*^T}P^2E^*\\
&\quad+\frac{1}{\mu_1}E^{*^T}P^2E^*+\sum_{i=1}^{N}\frac{1}{\rho_{2i}}E^{*^T}P^2E^*+\mu_2\alpha^2I\\
&\quad+\frac{1}{\mu_4}E^{*^T}P^2E^*+\mu_1\alpha^2I\\
&\quad+\sum_{i=1}^{N}\frac{1}{\rho_{6i}}E^{*^T}P^2E^*+\sum_{i=1}^{N}\frac{1}{\rho_{7i}}E^{*^T}P^2E^*+\frac{1}{\mu_7}M^2\\
&\quad+\mu_5E^{*^T}P^2E^*+\mu_7E^{*^T}P^2E^*+\mu_9E^{*^T}P^2E^*
\end{aligned}
$$

and

$$
\begin{aligned}
b_{22}&=(A^*+LC^*)^T\Gamma E^*+E^{*^T}\Gamma(A^*+LC^*)\\
&\quad+E^{*^T}\sum_{i=1}^{N}D_iE^*+\sum_{i=1}^{N}\frac{1}{\rho_{3i}}E^{*^T}\Gamma^2E^*
\end{aligned}
$$

$$+\frac{1}{\mu_2}E^{*^T}\Gamma^2E^*+\frac{1}{\mu_3}E^{*^T}\Gamma^2E^*$$
$$+\sum_{i=1}^{N}\frac{1}{\rho_{4i}}E^{*^T}\Gamma^2E^*$$
$$+\sum_{i=1}^{N}\frac{1}{\rho_{5i}}E^{*^T}\Gamma^2E^*$$
$$+\mu_6E^{*^T}\Gamma^2E^*+\mu_8E^{*^T}\Gamma^2E^*$$
$$+\frac{1}{\mu_8}M^2+\frac{1}{\mu_9}F^TFB^TB$$

$$\begin{aligned}\mathrm{M}=diag\Big(&-E^{*^T}R_1E^*+(\rho_{11}||A_{d_1}||^2+\rho_{21}\alpha_1^2+\rho_{41}\alpha_1^2)\\&\times I,\cdots,-E^{*^T}R_NE^*\\&+(\rho_{1N}||A_{d_N}||^2+\rho_{2N}\alpha_N^2+\rho_{4N}\alpha_N^2)I\Big)\end{aligned}\tag{33}$$

$$\begin{aligned}\mathrm{N}=diag\Big(&-E^{*^T}D_1E^*\\&+\rho_{31}||A_{d_1}||^2I,\cdots,-E^{*^T}D_NE^*+\rho_{3N}||A_{d_N}||^2I\Big)\end{aligned}\tag{34}$$

$$\chi(t)=\begin{bmatrix}\tilde{x}(t-\tau_1)\\\tilde{x}(t-\tau_2)\\\vdots\\\tilde{x}(t-\tau_N)\end{bmatrix},\ \chi_v(t)=\begin{bmatrix}\tilde{x}_v(t-\tau_1)\\\tilde{x}_v(t-\tau_2)\\\vdots\\\tilde{x}_v(t-\tau_N)\end{bmatrix}\tag{35}$$

Since $\tilde{x}^T(t)E^{*^T}PB^*\omega(t)$ and $\omega^T(t)B^{*^T}PE^*\tilde{x}(t)$ are scalars, we have

$$\begin{aligned}\dot{V}(\tilde{x}(t),\tilde{x}_v(t))\le{}&\chi^T(t)\mathrm{M}\chi(t)+\chi_v^T(t)\mathrm{N}\chi_v(t)\\&+2\tilde{x}^T(t)E^{*^T}PB^*\omega(t)+\mu_4\beta^2\alpha^2\\&+\mu_3\beta^2\alpha^2+\sum_{i=1}^{N}\rho_{5i}\beta^2\alpha_i^2\\&+\sum_{i=1}^{N}\rho_{6i}\beta^2||A_{d_i}||^2\\&+\sum_{i=1}^{N}\rho_{7i}\beta^2\alpha_i^2+\frac{1}{\mu_5}d_{max}^2+\frac{1}{\mu_6}d_{max}^2\end{aligned}\tag{36}$$

In what follows, we investigate three different states of the saturation function.

**Case 1:** If all input factors are not saturated, i.e. $|u_i|\le\bar{u}_i$, it can be easily shown that:

$$\omega_i(t)=\phi\begin{bmatrix}b_i^{*^T}PE^* & b_i^{*^T}PE^*\end{bmatrix}\begin{bmatrix}\tilde{x}\\\tilde{x}_v\end{bmatrix}=u_{N_i}\tag{37}$$

where $b_i^*$ is the $i^{\text{th}}$ column of $B^*$ and $u_{N_i}$ is the $i^{\text{th}}$ component of $u_N$.

**Case 2:** In this case, all input channels are higher than the upper saturation bound, i.e. $u_i\ge\bar{u}_i$ so we have

$$u_i\ge\bar{u}_i\Rightarrow u_{L_i}+u_{N_i}\ge\bar{u}_i\tag{38}$$

where $u_{L_i}$ is the $i^{\text{th}}$ component of $u_L$. According to the (25), it is obvious that $\omega_i(t)=\bar{u}_i-u_{L_i}$, and thus $\omega_i(t)\ge\bar{u}_i-|u_{L_i}|\ge0$. Moreover, according to (38) as $u_{N_i}\ge\bar{u}_i-u_{L_i}=\omega_i(t)$, the following inequality is achieved

$$0\le\omega_i(t)\le u_{N_i}\tag{39}$$

**Case 3:** In this case, all input channels are smaller than their lower saturation bound, so we have

$$u_i\le-\bar{u}_i\Rightarrow u_{L_i}+u_{N_i}\le-\bar{u}_i\Rightarrow u_{N_i}\le-\bar{u}_i-u_{L_i}\le0\tag{40}$$

Considering $\omega_i(t)=-\bar{u}_i-u_{L_i}$, one has

$$u_{N_i}\le\omega_i(t)\le0\tag{41}$$

According to (37), (39) and (41), $\omega_i(t)$ can be considered as $\omega_i(t)=\xi_iu_{N_i}$ where $\xi_i\in[0,1]$. Hence, $\omega(t)$ can be achieved as follows:

$$\omega_i(t)=\bar{\phi}\begin{bmatrix}B^{*^T}PE^* & B^{*^T}PE^*\end{bmatrix}\begin{bmatrix}\tilde{x}\\\tilde{x}_v\end{bmatrix}\tag{42}$$

where $\bar{\phi}=\xi\phi$ and $\xi=diag[\xi_1,\xi_2,\dots,\xi_m]$. Because $\phi$ is negative and according to Razumikhin Theorem $\|\tilde{x}(t-\tau_i)\|\le L_i\|\tilde{x}(t)\|$, $L_i>1(i=1,\dots,\mathrm{N})$, the relation (36) is as follows:

$$\dot{V}\le\chi^T(t)\mathrm{M}\chi(t)+\chi_v^T(t)\mathrm{N}\chi_v(t)+\epsilon\tag{43}$$

where$\epsilon=\mu_4\beta^2\alpha^2+\sum_{i=1}^{N}\rho_{5i}\beta^2\alpha_i^2+\mu_3\beta^2\alpha^2+\sum_{i=1}^{N}\rho_{6i}\beta^2||A_{d_i}||^2+\sum_{i=1}^{N}\rho_{7i}\beta^2\alpha_i^2+\frac{1}{\mu_5}d_{max}^2+\frac{1}{\mu_6}d_{max}^2$ As a result, we will have the following form

$$\dot{V}\le\begin{bmatrix}E^*\chi(t)\\E^*\chi_v(t)\end{bmatrix}^T\underbrace{\begin{bmatrix}-\Lambda_1 & 0\\0 & -\Lambda_2\end{bmatrix}}\begin{bmatrix}E^*\chi(t)\\E^*\chi_v(t)\end{bmatrix}+\epsilon\tag{44}$$

where $-\Lambda_1=diag(-R_1+\frac{1}{||E^*||^2}(\rho_{11}||A_{d_1}||^2+\rho_{21}\alpha_1^2+\rho_{41}\alpha_1^2)I,\dots,-R_N+\frac{1}{||E^*||^2}(\rho_{1N}||A_{d_N}||^2+\rho_{2N}\alpha_N^2+\rho_{4N}\alpha_N^2)I)$ and $-\Lambda_2=diag(-D_1+\frac{1}{||E^*||^2}\rho_{31}||A_{d_1}||^2I,\dots,-D_N+\frac{1}{||E^*||^2}\rho_{3N}||A_{d_N}||^2I)$, it can be concluded that $\Lambda>0$ and for convenience one can write

$$\dot{V}\le-\upsilon^TE^{*^T}\Lambda E^*\upsilon+\epsilon\tag{45}$$

where $\upsilon=\begin{bmatrix}\chi(t)\\\chi_v(t)\end{bmatrix}$, and finally, it can be written as

$$\dot{V}\le-\lambda_{\min}(\Lambda)||E^*\upsilon||_2^2+\epsilon,\forall||E^*\upsilon||_2^2>\frac{\epsilon}{\lambda_{\min}(\Lambda)}\tag{46}$$

By introducing a new positive invariant set as $\theta_\mu:=\left\{E^*\upsilon(t)\,|\,||E^*\upsilon||_2^2\le\frac{\epsilon}{\lambda_{\min}(\Lambda)}\right\}\subset\theta_\delta$ and $\Psi:=\theta_\delta-\theta_\mu$. According to Eqs. (45) and (46) and the Lemma 1, it get

$$\dot{V}\le-\lambda_{\min}(\Lambda)||E^*\upsilon||_2^2+\epsilon:=-\Phi,\forall(E^*\upsilon)\in\Psi\tag{47}$$

where $\Phi$ is a positive-definite function so $\dot{V}$ is bounded and negative in $\Psi$. Because of the structure of the Lyapunov function, decrease V leads to a decrease in the norm of the closed-loop singular system's states. So, it can be concluded that $\left\|\begin{bmatrix}\tilde{x}\\\tilde{x}_v\end{bmatrix}\right\|\le\gamma$; thus, ultimately bounded tracking error and observer error is obtained. Considering Lemma 1, Eq. (47) ensures that the tracking error will be bounded (even with saturation and uncertainty). If the system is not affected by the disturbance, that is $d(t)=0$, the region $\theta_\mu$ is removed. Then, the observer-based control lawwill cause the states of the closed-loop singular system (12) to be bounded, and the output of the system (1) will follow thereference signal $y_m(t)$ with a tracking error that is bounded and limited in the presence of disturbance and input saturation.

## IV. SIMULATION RESULTS

The effectiveness and performance of the proposed approach is examined in this section using two different examples. The first considers a singular system with time delay, perturbation, input saturation, and uncertainties. The second example, considers a DC motor modeled using a singular state- space representation with nonlinearity.

*Example 1:* Consider the nonlinear singular system with time delays (1), whose parameters are given as:

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A = \begin{bmatrix} 1 & 1 & 0 \\ 4 & -2 & 0 \\ 0 & 1 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix},$$
$$C = \begin{bmatrix} 2 & 1 & 1 \end{bmatrix}, F = \begin{bmatrix} 1 & 1 & 3 & 2 \end{bmatrix}$$

where $f(x) = -5x_1^2 + 11.4x_1 - 0.21(|x_1 + 1| - |x_1 - 1|)$ and $A_{di}, i= 1,2$ are fixed parameters, $r_1(t)$,$r_2(t)$ and $\Delta A_{di}(\nu(t))$,$i= 1,2$ are the uncertain parameters and the external disturbance is as

$$q(t) = [00.2\, sin(2t)\; 0.02\, sin(t)],$$
$$\Delta A_{d1}(\nu(t)) = \begin{bmatrix} 0.1\cos(t) & 0.2\sin(t) & 0.2\sin(t) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$
$$\Delta A_{d2}(\nu(t)) = \begin{bmatrix} 0.1\sin(t) & -0.2\cos(t) & 0.3\sin(t) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Considering $Q = I_4$ and using (17) and (20 ), the $P$ and the descriptor observer $L$ can be determined as

$$P = \begin{bmatrix} 1.26 & -0.7645 & -0.1647 & 0.1745 \\ -0.6745 & 0.8902 & 0.0686 & -1.1059 \\ -0.1647 & 0.0786 & 0.6098 & 0.1375 \\ 0.1745 & -1.1059 & 0.1175 & 2.5 \end{bmatrix}$$

and $L = \begin{bmatrix} -3 & -2 \\ 2 & -3 \\ -5 & -6 \\ -3 & -2 \end{bmatrix}$. For this simulation study, we consider:

$$A_{d1} = \begin{bmatrix} -1 & -1 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}, A_{d2} = \begin{bmatrix} -1 & -1 & 0.1 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.1 \end{bmatrix},$$

$Fm = \begin{bmatrix} 4 & 2 & 5 \end{bmatrix}$. The constant quantities are considered as $\alpha= 0.01, \beta= 35$, $r_s(t) = 3sin(0.4t)$, and the saturation limit is $\bar{u} = 2$. The primary values are considered as $x(0) = [0.10.3 - 0.3]^T and \tau_1 = 0.2, \tau_2 = 0.4$.

The nonlinear function $\phi(y_m(t), y(t))$ is selected as follows:

$$\phi(y_m(t), y(t)) = -\beta e^{-\alpha_0 \alpha |y(t) - y_m(t)|}, \quad (48)$$

where

$$\alpha_0 = \begin{cases} \dfrac{1}{|y(t_0) - y_m(t)|}, & y(t_0) \neq y_m(t) \\ 1, & y(t_0) = y_m(t) \end{cases} \quad (49)$$

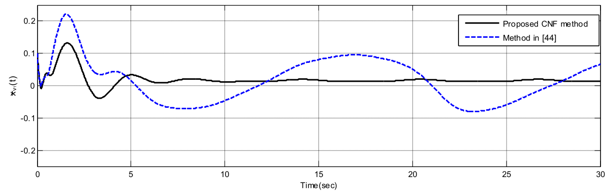


**FIGURE 1.** State response of the observer error ($\tilde{x}_{v1}$).

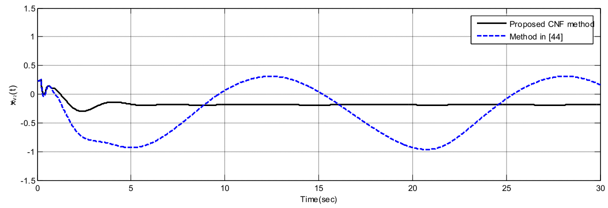


**FIGURE 2.** State response of the observer error ($\tilde{x}_{v2}$).

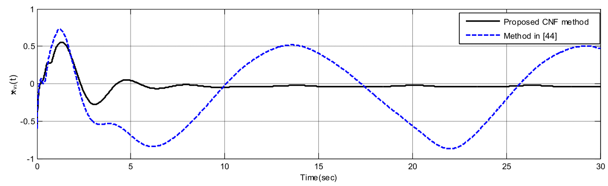


**FIGURE 3.** State response of the observer error ($\tilde{x}_{v3}$).

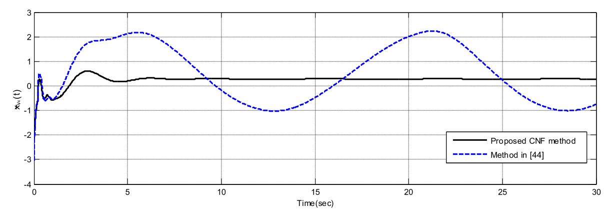


**FIGURE 4.** State response of the observer error ($\tilde{x}_{v4}$).

For comparison purposes, we consider the approach proposed in [44]. Fig. 1-4 depicts the dynamics of the states estimated by the designed singular observer. The trajectories of the tracking error are illustrated in Fig. 5. Also Fig. 6 displays the output responses of $y_m(t)$ and y(t). The dynamics of the nonlinear state-feedback controller are depicted in Fig. 7. The obtained results show that the system is robust to time delays and disturbances compared to those of [44], and also the proposed controller has good convergence rate. The purpose here is not to get zero tracking error, but rather ensure the is error bounded. The simulation results show that the system output tracks the reference signal with a tracking error that is bounded and limited by using the proposed observer-based controller.

*Example 2:* Consider a DC motor with a singular state-space representation in the form of system (1) with the following information [50]:

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, A = \begin{bmatrix} -2 & 3 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix},$$
$$F = \begin{bmatrix} -2 & -7 & -1 \end{bmatrix}$$

where $f(x) = 0.5cos(x_1) - 0.5$ and $A_{di}, i= 1,2$ are fixed parameters, and $\Delta A_{di}(\nu(t))$,$i= 1,2$ are the uncertain

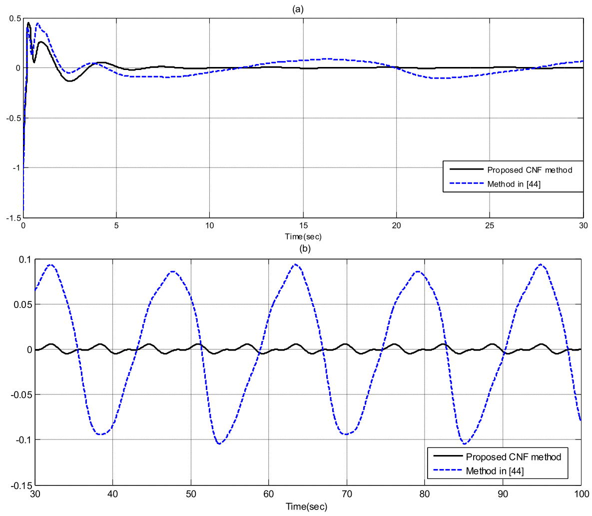


**FIGURE 5.** The tracking errors; (a) time range 0-30(s), (b) time range 30-100(s).

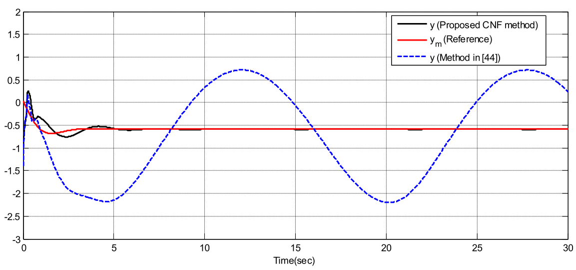


**FIGURE 6.** Output responses of $y_m$(t) and y(t).

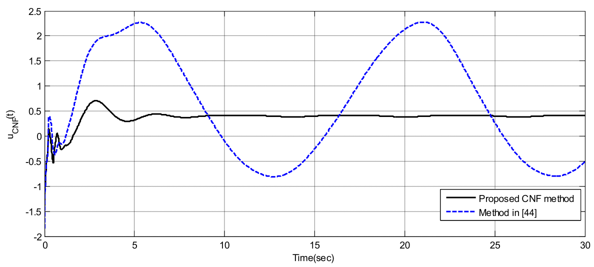


**FIGURE 7.** Control input.

parameters and $q(t)$ is the disturbance, and

$$\Delta A_{d1}(\nu(t)) = \begin{bmatrix} \sin(2t) & \sin(3t) \\ 0 & 0 \end{bmatrix},$$

$$\Delta A_{d2}(\nu(t)) = \begin{bmatrix} \sin(2t) & 0 \\ 0 & \sin(3t) \end{bmatrix}.$$

Using (17) and (20 ), the $P$ and the descriptor observer $L$ can be determined as

$$P = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.3 & 0 \\ 0 & 0 & 0.4 \end{bmatrix} \text{ and } L = \begin{bmatrix} -2.5 & 4.5 \\ -3.5 & 4.5 \\ -1.5 & -3.5 \end{bmatrix}.$$

For simulation use, take $q(t) = 3\sin(0.01t)$, $A_{d1} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, $A_{d2} = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}$, $F_m = [-2 - 4]$, the saturation limit of the system is taken to be $\bar{u} = 5$ and $r_s(t)$ is also approximated by a step function. The constant quantities are considered as $\alpha = 0.01, \beta = 100$. The primary values are supposed as $x(0) = [0.01 - 0.01]^T, \tau_1 = \tau_2 = 0.5$.

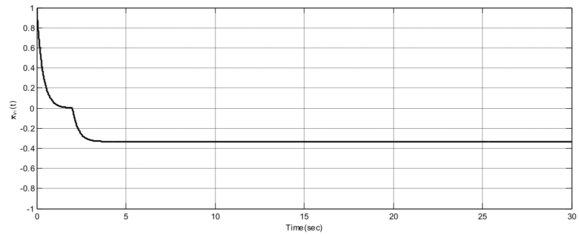


**FIGURE 8.** State response of the observer error ($\tilde{x}_{v1}$).

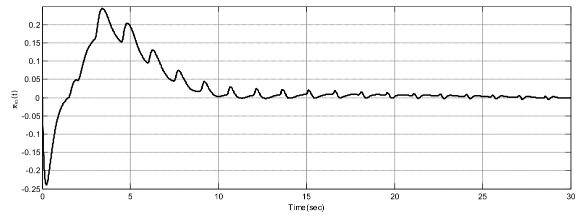


**FIGURE 9.** State response of the observer error ($\tilde{x}_{v2}$).

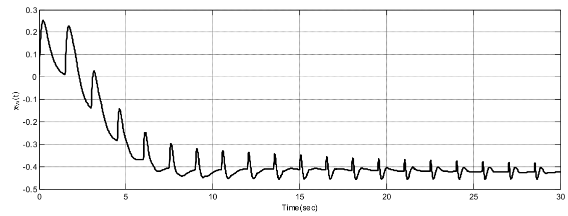


**FIGURE 10.** State response of the observer error ($\tilde{x}_{v3}$).

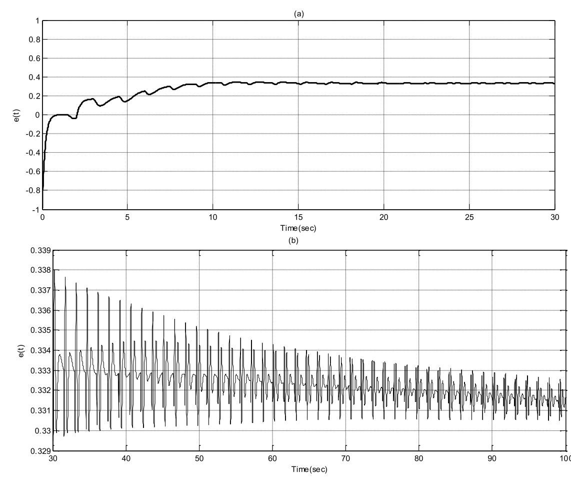


**FIGURE 11.** Dynamics of the tracking error; (a) time range 0-30(s), (b) time range 30-100(s).

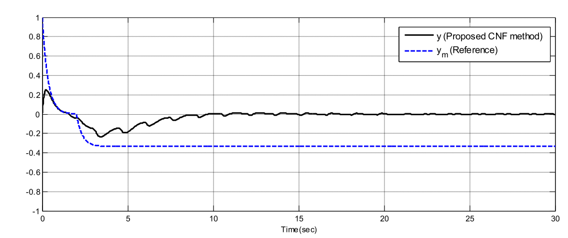


**FIGURE 12.** Output responses of $y_m$(t) and y(t).

Simulation results are represented in Figs. 8-13. Fig. 8-10 shows the vectors of the observer error. Fig. 11 demonstrates the tracking error at two different time intervals. Fig. 12 illustrates the output tracking by the suggested CNF control law based on observer. Fig.13 displays the response of the suggested control law.

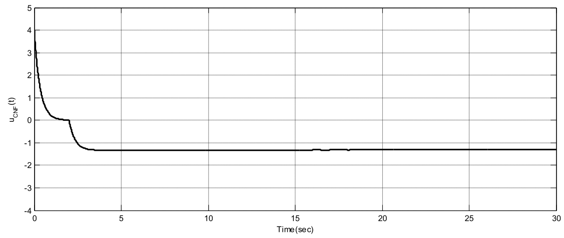


**FIGURE 13.** Control input.

## V. CONCLUSION

This paper proposed an observer-based CNF controller for the robust tracking of uncertain singular systems subject to input saturation, nonlinear functions, time-delay, and disturbances. The control law was designed based on states reconstructed using a singular observer. A theorem was proposed to prove the uniform boundedness of the tracking error albeit the presence of external disturbances and nonlinear dynamics. Implementation of the proposed approach to two case studies confirmed the accuracy and effectiveness of the proposed approach in controlling uncertain nonlinear singular systems with input saturation.

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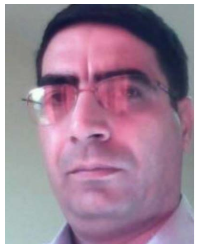

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