IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received March 7, 2022, accepted May 24, 2022, date of publication May 27, 2022, date of current version June 6, 2022.

*Digital Object Identifier 10.1109/ACCESS.2022.3178606*

# Centralized Control in Networks of Underactuated Nonidentical Euler–Lagrange Systems Using a Generalised Multicoordinates Transformation

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**ABSTRACT** Controlling the network of underactuated Euler–Lagrange (EL) systems is challenging because of their coupled inertia matrices and time-variant control input matrices. We present generalized multi-coordinates transformation that renders the network of underactuated Euler-Lagrange dynamics in particular forms, whose mechanical properties should be preserved. The network of **N** nonidentical Euler–Lagrange EL-systems is modeled as a weighted interconnection graph where each EL-system is a node, and the control action at each node is a function of its state and the states of its neighbors. Second, we propose an online optimally centralized control mechanism with the prime objective of energy efficiency. The result is applied to the network of underactuated vertical takeoff and landing aircraft with strong input coupling, including the effect of the weight of the rotors into the dynamical system models. In this regard, we obtain very simple and powerful state-feedback solutions.

**INDEX TERMS** Aerospace, generalized multi-coordinates transformation, network control systems, underactuated Euler–Lagrange systems.

## I. INTRODUCTION

The control of large-scale nonlinear dynamical systems described by Euler–Lagrange (EL) equations is challenging due to the high degree of freedom in such distributed systems [1]. The relevant published works on this domain can roughly be categorized into two major classes: (A) centralized approaches assuming complete information and focusing on precision and efficiency [2], [3] and (B) decentralized approaches assuming only partial observability and focusing on simple reactive and behavior-based control [4], [5]. While both concepts are commonly justified, the centralized method may be unavoidable for specific tasks. Here, we investigate a control problem in underactuated EL systems with low-cost sensors. The reason flows from the fact that, in diverse applications, including aerospace and robotics, safety requirements in combination with low-cost sensors are increasing. These systems have fewer control inputs than degrees of freedom, and hence, they are underactuated mechanical systems. In the case of satellite formation [5], some tasks may not be feasible and able to guarantee the required level of performance relying on a decentralized approach. The advantage of having simple hardware is, in turn, that possibly systems can form a large-scale system with high redundancy. The control mission can be considered of as macroscopic control of a 'cloud of EL systems' defined by a specific distribution [6]. The controller's input can be, for example, the states of all EL systems. The output is a global control that is communicated with a central unit to all agents.

This paper focuses on an optimal energy-efficient control mechanism for the efficiency of EL systems formations where the cost function and constraints couple the motion behavior of individual underactuated systems. In particular, our work starts by showing that it is possible to obtain mathematically a mapping such that underactuated EL equations of motion take partial forms. However, due to the complexity of the dynamics (coupling of the inertia matrix), it is not straightforward to design a nonlinear controller. Another challenge is the control input matrix $G(q) \in \mathbb{R}^{m\times n}$ that is a transformation matrix and time-variant. The

The associate editor coordinating the review of this manuscript and approving it for publication was Nishant Unnikrishnan.

fundamental work of [7]–[9] in this field discusses the input matrix to be in the form of $G = [I_m \, 0_s]^\top$. It should be underscored that, all these previous results deal with simple class of underactuated EL-systems. This manuscript covers the case where $G(q) = [G_u(q) \, G_a(q)]^\top \in \mathbb{R}^{m\times n}$ has a general form. Therefore, we relax this assumption on the input matrix $G$ differently from what is done in [4], [10], [11], and [12]. This paper proposes a new generalized multi-coordinates transformation to decouple the network of EL systems. The proposed multi-decoupling methodology eases the development of an optimal control mechanism with the prime objective of energy efficiency. From a control engineering perspective, several techniques exist to design optimal control laws [13]. Among the existing approaches, the state-dependent Riccati equation (SDRE) [14] does not cancel nonlinear terms, which is promising because canceling such nonlinearities would significantly increase the control effort signals [15]. In addition, SDRE characterizes the system to a state-dependent coefficient (SDC) which is not unique and can be used to enhance performance or effect trade-offs between performance, optimality, stability, and robustness.

The result is applied to the network of a new underactuated vertical take-off and landing (VTOL) aircraft. The VTOL is modeled by (conceptually) breaking it up into its components and then developing a mechanical model as a system of particles considering the effect of the main body and rotors into the dynamical system model. The simulation results show the effectiveness of the controller in keeping the formation flight at a reasonable combustible cost.

*Notation:* $I_n$ is the $n \times n$ identity matrix and $0_{n\times s}$ is an $n \times s$ matrix of zeros, and $0_n$ is an $n$–dimensional column vector of zeros. For any matrix $A \in \mathbb{R}^{n\times n}$, $(A)_i \in \mathbb{R}^n$ denotes the $i$–th column, $(A)^i$ the $i$–th row and $(A)_{ij}$ the $ij$–th element. We denote the weighted–norm $\|x\|_S := x^\top Sx$. The state variables are a function of time, e.g., $x = x(t)$. The first and second derivative with respect to time of a state space variable $x$ are denoted respectively with $\dot{x}$, and $\ddot{x}$. Given $a_i \in \mathbb{R}, i \in \bar{N} := \{1, \ldots, \mathbf{N}\}$, we denote $\mathbf{N}$ is the number of agents. The subscript of functions and agents coordinates are in the set $\bar{N}$, unless stated otherwise, this clarification is omitted for brevity.

## II. NETWORK OF EULER-LAGRANGE DYNAMICS

The considered network is consists of $\mathbf{N}$ nonidentical underactuated EL-systems which can be written as

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + \nabla V_i(q_i) = G_i(q_i)u_i, \tag{1}$$

where $q_i \in \mathbb{R}^n$ are the configuration variables, $u_i \in \mathbb{R}^m$ are the control signals, $M_i(q_i) > 0$ is the generalized inertia matrix, $C_i(q_i, \dot{q}_i)$ represent the Coriolis and centrifugal forces, $V_i(q_i)$ is the systems potential energy, and $G_i(q_i)$ is the input matrix of the $i$-th agent.

Now, the following assumptions hold:

*A 1: There exist invertible mappings $\Phi_i : \mathbb{R}^n \to \mathbb{R}^n$, such that*

$$\nabla_{q_i}\Phi_i(q_i) = T_i^{-1}(q_i). \tag{2}$$

*is invertible for all $q_i$.*

*Lemma 1: Consider mappings $\Phi_i : \mathbb{R}^n \to \mathbb{R}^n$ that satisfies **A.1** and define the generalised multi-coordinates transformation as follows*

$$q_i = \Phi_i(q_i). \tag{3}$$

Therefore, the network of nonidentical EL-systems (1) can be written as follows

$$\mathcal{M}_i(q_i)\ddot{q}_i + \mathcal{C}_i(q_i, \dot{q}_i)\dot{q}_i + \nabla\mathcal{V}_i(q_i) = \mathcal{G}_i(q_i)u_i, \tag{4}$$

where

$$\dot{q}_i := T_i^{-1}(q_i)\dot{q}_i \tag{5}$$

$$\mathcal{M}_i(q_i) := T_i^\top(q_i)M_i(q_i)T_i(q_i)\Big|_{q_i=\Phi_i^{-1}(q_i)} \tag{6}$$

$$\mathcal{V}_i(q_i) := V_i(q_i)\Big|_{q_i=\Phi_i^{-1}(q_i)} \tag{7}$$

$$\mathcal{G}_i := T_i^\top(q_i)G_i(q_i)\Big|_{q_i=\Phi_i^{-1}(q_i)} \tag{8}$$

and $\mathcal{C}_i(q_i, \dot{q}_i)\dot{q}_i$ are the Coriolis and centrifugal forces associated to the inertia matrix $\mathcal{M}_i(q_i)$, and $i \in \bar{N}$, which can be computed as follows

$$\mathcal{C}_i(q_i, \dot{q}_i)\dot{q}_i = \left[\nabla_{q_i}[\mathcal{M}_i(q_i)\dot{q}_i] - \frac{1}{2}\nabla_{q_i^\top}[\mathcal{M}_i(q_i)\dot{q}_i]\right]\dot{q}_i. \tag{9}$$

The Lagrangian in the new generalised multi-coordinates is

$$\mathcal{L}_i(q_i, \dot{q}_i) = \frac{1}{2}\dot{q}_i{}^\top\mathcal{M}_i(q_i)\dot{q}_i - \mathcal{V}_i(q_i). \tag{10}$$

*Proof:* The proof follows from the calculation computing the derivative of the multi-coordinates transformation and using original dynamical system models.

*Remark 1: It should be noted that the matrix $T_i(.)$ can be used to shape the inertia matrix $\mathcal{M}_i(.)$ in the new generalized multi-coordinates. However, we consider all invertible matrices $T_i(.)$ that satisfy the integrability assumption **A.1**. Given an invertible matrix $T_i(.)$, then there exist invertible mappings $\Phi_i : \mathbb{R}^n \to \mathbb{R}^n$ that satisfy*

$$\dot{\Phi}(q_i) = T_i(q_i)\dot{q}_i.$$

Now, we consider a network of $\mathbf{N}$ nonidentical EL-systems of the form (1) with an input matrix of the general form

$$G_i(q_i) = \begin{bmatrix} G_{u_i}(q_i) \\ G_{a_i}(q_i) \end{bmatrix}, \tag{11}$$

where $\operatorname{rank} G_i(q_i) = m < n$, and $G_{a_i}(q_i)$ is an invertible $m \times m$ matrix. $G_{u_i}(q_i)$ and $G_{a_i}(q_i)$ are the underactuated and actuated elements of $G_i(q_i)$, respectively. The network of $\mathbf{N}$ nonidentical EL-systems (1) is coupled when $G_{u_i}(q_i) \not\equiv 0$. For the sake of exposition simplicity, we partition the generalized multi-coordinates and velocity as

$q_i = \mathrm{col}(q_{u_i}, q_{a_i})$, $\dot{q}_i = \mathrm{col}(\dot{q}_{u_i}, \dot{q}_{a_i})$ with $q_{a_i}, \dot{q}_{a_i} \in \mathbb{R}^m$ and $q_{u_i}, \dot{q}_{u_i} \in \mathbb{R}^s$, where $s := n - m$, and partition the inertia and Coriolis matrices as

$$M_i(q_i) = \begin{bmatrix} m_{uu_i}(q_i) & m_{au_i}^\top(q_i) \\ m_{au_i}(q_i) & m_{aa_i}(q_i) \end{bmatrix}, \tag{12}$$

$$C_i(q_i, \dot{q}_i) = \begin{bmatrix} c_{uu_i}(q_i) & c_{ua_i}(q_i) \\ c_{au_i}(q_i) & c_{aa_i}(q_i) \end{bmatrix}, \tag{13}$$

where $m_{aa_i} : \mathbb{R}^n \to \mathbb{R}^{m\times m}$, $m_{au_i} : \mathbb{R}^n \to \mathbb{R}^{s\times m}$, $m_{uu} : \mathbb{R}^n \to \mathbb{R}^{s\times s}$, $c_{aa_i} : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{m\times m}$, $c_{au_i} : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{s\times m}$, $c_{ua_i} : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{m\times s}$, $c_{uu_i} : \mathbb{R}^n \to \mathbb{R}^{s\times s}$.

We now impose some assumptions for each agent to show particular forms of the network of $\mathbf{N}$ nonidentical EL-systems (1) under generalized multi-coordinates transformation.

*A 2: There exist functions $\Phi_{a_i} : \mathbb{R}^m \to \mathbb{R}^s$, such that*

$$\dot{\Phi}_{a_i}(q_{a_i}) = m_{uu_i}^{-1} m_{au_i}^\top \dot{q}_{a_i}. \tag{14}$$

*A 3: The inertia matrix depends only on the actuated variables $q_{a_i}$,* i.e., $M_i(q_i) = M_i(q_{a_i})$.

*A 4: The sub-block matrix $m_{uu_i}$ of the inertia matrix is constant.*

*A 5: The potential energy can be written as*

$$V_i(q_i) = V_{a_i}(q_{a_i}) + V_{u_i}(q_{u_i}).$$

*Proposition 1: The network of $\mathbf{N}$ nonidentical EL-systems* (1), *under assumption **A.2** and using the generalised coordinates $q_i = \mathbf{col}(q_{i_1}, q_{i_2}) = \Phi_i(q_i)$, can be written as follows*

$$\begin{aligned} m_{uu_i}\ddot{q}_{i_1} &+ \left[\nabla_{q_{i_1}}(m_{uu_i}\dot{q}_{i_1}) - \frac{1}{2}\nabla^\top_{q_{i_1}}(m^s_{uu_i}\dot{q}_{i_1})\right]\dot{q}_{i_1} \\ &+ \left[\nabla_{q_{i_2}}(m^s_{uu_i}\dot{q}_{i_2}) - \frac{1}{2}\nabla^\top_{q_{i_1}}(m_{aa_i}\dot{q}_{i_2})\right]\dot{q}_{i_2} \\ &+ \nabla_{q_{i_1}}\mathcal{V}_i(q_i) = G_{u_i}(q_i)u_i \end{aligned} \tag{15}$$

$$\begin{aligned} m^s_{aa_i}\ddot{q}_{i_2} &+ \left[\nabla_{q_{i_1}}(m^s_{aa_i}\dot{q}_{i_2}) - \frac{1}{2}\nabla^\top_{q_{i_2}}(m_{uu_i}\dot{q}_{i_1})\right]\dot{q}_{i_1} \\ &+ \left[\nabla_{q_{i_2}}(m^s_{aa_i}\dot{q}_{i_2}) - \frac{1}{2}\nabla^\top_{q_{i_2}}(m^s_{aa_i}\dot{q}_{i_2})\right]\dot{q}_{i_2} \\ &+ \nabla_{q_{i_2}}\mathcal{V}_i(q) = \left[G_{a_i}(q_i) - G_{u_i}(q_i) m_{au_i} m_{uu_i}^{-1}\right]u_i, \end{aligned} \tag{16}$$

where

$$\begin{bmatrix} q_{i_1} \\ q_{i_2} \end{bmatrix} = \begin{bmatrix} q_{u_i} + \Phi_{a_i}(q_{a_i}) \\ q_{a_i} \end{bmatrix}, \tag{17}$$

$$m^s_{aa_i}(q) = m_{aa_i}(q_i) \tag{18}$$

$$\quad - m_{au_i}(q_i) m_{uu_i}^{-1}(q_i) m_{au_i}^\top(q_i)\Big|_{q_i = \Phi_i^{-1}(q_i)}, \tag{19}$$

$$m_{uu_i}(q_i) = m_{uu_i}(q_i)\Big|_{q_i = \Phi_i^{-1}(q_i)}, \tag{20}$$

$$m_{au_i}(q) = m_{au_i}(q_i)\Big|_{q_i = \Phi_i^{-1}(q_i)}. \tag{21}$$

*Proof:* Under assumption **A.2**, the multi-coordinates transformation (17) satisfies assumption **A.1** with

$$T_i(q_i) = \begin{bmatrix} I_{i_s} & -m_{uu_i}^{-1} m_{au_i}^\top \\ 0_{i_{m\times s}} & I_{i_m} \end{bmatrix}. \tag{22}$$

Then, from Lemma (1) we obtain that the network of $\mathbf{N}$ nonidentical EL-systems can be written in the form (4) with

$$\begin{bmatrix} \dot{q}_{i_1} \\ \dot{q}_{i_2} \end{bmatrix} = \begin{bmatrix} I_{i_s} & m_{uu_i}^{-1} m_{au_i}^\top \\ 0_{i_{m\times s}} & I_{i_m} \end{bmatrix} \begin{bmatrix} \dot{q}_{u_i} \\ \dot{q}_{a_i} \end{bmatrix} \tag{23}$$

and Lagrangian

$$\begin{aligned} \mathcal{L}_i(q_i, \dot{q}_i) = &\frac{1}{2}\begin{bmatrix} \dot{q}_{i_1}^\top & \dot{q}_{i_2}^\top \end{bmatrix} \begin{bmatrix} m_{uu_i} & 0_{i_{s\times m}} \\ 0_{i_{m\times s}} & m^s_{aa_i} \end{bmatrix} \begin{bmatrix} \dot{q}_{i_1} \\ \dot{q}_{i_2} \end{bmatrix} \\ &- \mathcal{V}_i(q_i).. \end{aligned} \tag{24}$$

□

*Corollary 1: The network of nonidentical EL-systems* (1) *satisfying **A.2** can be written as in the EL form as follows*

$$m_{uu_i}(q_{a_i})\ddot{q}_{i_1} + \nabla_{q_{i_1}}\mathcal{V}_i(q_{i_1}, q_{a_i}) = G_{u_i}(q_i)u, \tag{25}$$

$$\begin{aligned} m^s_{aa_i}\ddot{q}_{a_i} &+ \left[\nabla_{q_{a_i}}[m^s_{aa_i}(q_{a_i})\dot{q}_{a_i}] - \frac{1}{2}\nabla^\top_{q_{a_i}}[m^s_{aa_i}(q_{a_i})\dot{q}_{a_i}]\right]\dot{q}_{a_i} \\ &+ \nabla_{q_{a_i}}\mathcal{V}_i(q_{i_1}, q_{a_i}) = \left[G_{a_i}(q_i) - G_{u_i}(q_i) m_{au_i} m_{uu_i}^{-1}\right]u_i, \end{aligned} \tag{26}$$

*Furthermore, if assumption **A.3—A.5** also holds, then the network of EL dynamics* (25)-(26) *can be written as follows*

$$m_{uu_i}\ddot{q}_{i_1} + \nabla_{q_{u_i}} V_{u_i}\Big|_{q_{u_i} = q_{i_1} - \Phi_{a_i}(q_{a_i})} = G_{u_i}(q_i)u_i, \tag{27}$$

$$\begin{aligned} m^s_{aa_i}\ddot{q}_{a_i} &+ \left[\nabla_{q_{a_i}}[m^s_{aa_i}\dot{q}_{a_i}] - \frac{1}{2}\nabla_{q_{a_i}}[m^s_{aa_i}\dot{q}_{a_i}]\right]\dot{q}_{a_i} \\ &+ \nabla_{q_{a_i}} V_{a_i} - m_{au_i} m_{uu_i} \nabla_{q_{u_i}} V_{u_i}\Big|_{q_{u_i} = q_{i_1} - \Phi_{a_i}(q_{a_i})} \\ &= \left[G_{a_i}(q_i) - G_{u_i}(q_i) m_{au_i} m_{uu_i}^{-1}\right]u_i. \end{aligned} \tag{28}$$

*Proof:* The proof follows from Proposition 1 and **A.1-A.3** by setting in (15)-(16) the following conditions: $q_{i_1} = q_{u_i} + \Phi_{a_i}(q_{a_i})$, $q_{i_2} = q_{a_i}$, $m_{uu_i}$ is a constant matrix, and $m^s_{aa_i}(q) = m^s_{aa_i}(q_{a_i})$. The second part follows from the fact that, under assumption **A.5**, the potential function is $\mathcal{V}_i(q_i) = V_{a_i}(q_{a_i}) + V_{u_i}(q_{i_1} - \Phi_{a_i}(q_{a_i}))$. □

*Remark 2: By defining $\bar{\mathbf{x}}_i = (\mathbf{q}_{i_1}, \dot{\mathbf{q}}_{i_1}, q_{a_i}, \dot{q}_{a_i}) \in \mathbb{R}^n$, $u_i \in \mathbb{R}^m$ and $\mathbf{x} = [\bar{\mathbf{x}}_1^\top, \ldots, \bar{\mathbf{x}}_{\mathbf{N}}^\top]^\top \in \mathbb{R}^{n\mathbf{N}}$, $\mathbf{u} = [u_1^\top, \ldots, u_{\mathbf{N}}^\top]^\top \in \mathbb{R}^{m\mathbf{N}}$, the network of underactuated dynamics* (27)-(28) *can be written as an autonomous, nonlinear in the state, and affine in the input, represented in the form*

$$\begin{aligned} \dot{\mathbf{x}} &= A_a(\mathbf{x})\mathbf{x} + B_a(\mathbf{x})\mathbf{u}, \\ \mathbf{x}(0) &= \mathbf{x}_0 \triangleq [\mathbf{x}_{10}^\top, \ldots, \mathbf{x}_{N0}^\top]^\top \end{aligned} \tag{29}$$

*with*

$$A_a(\mathbf{x}) = \begin{bmatrix} A_1(\bar{\mathbf{x}}_1) & 0 & 0 & 0 \\ 0 & A_2(\bar{\mathbf{x}}_2) & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & A_N(\bar{\mathbf{x}}_N) \end{bmatrix}, \tag{30}$$

$$B_a(\mathbf{x}) = \begin{bmatrix} B_1(\bar{\mathbf{x}}_1) & 0 & 0 & 0 \\ 0 & B_2(\bar{\mathbf{x}}_2) & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & B_N(\bar{\mathbf{x}}_N) \end{bmatrix}. \tag{31}$$

## III. CENTRALISED CONTROL PROBLEM

In this section, we present a centralized optimal control for a network of **N** nonidentical underactuated EL-systems for the form (29) where the cost function couples the dynamic behavior of each underactuated agent as (32).

$$J(\mathbf{u}, \mathbf{x}_0) = \int_0^{\infty} \Bigg( \sum_{i=1}^{N} \sum_{i \neq j}^{N} (\bar{\mathbf{x}}_i - \bar{\mathbf{x}}_j)^\top Q_{ij} (\bar{\mathbf{x}}_i - \bar{\mathbf{x}}_j) + \sum_{i=1}^{N} (\bar{\mathbf{x}}_i^\top Q_{ii}(\bar{\mathbf{x}}_i)\bar{\mathbf{x}}_i + u_i^\top R(\bar{\mathbf{x}}_i) u_i) \Bigg) dt, \tag{32}$$

with the design parameters satisfy

$$Q_{ii}(\bar{\mathbf{x}}_i) = Q_{ii}^\top(\bar{\mathbf{x}}_i) \geq 0, R_{ii}(\bar{\mathbf{x}}_i) = R_{ii}^\top(\bar{\mathbf{x}}_i) > 0 \;\; \forall i \tag{33a}$$

$$Q_{ij} = Q_{ij}^\top \geq 0 \;\; \forall i \neq j. \tag{33b}$$

*Remark 3: The considered cost function (32) is non-quadratic in $\bar{\mathbf{x}}_i$ but quadratic in $u_i$. The state and input weighting matrices of each agent are state-dependent.*

In addition, it is possible to write the cost function (32) using the more compact notation as

$$J(\mathbf{u}, \mathbf{x}_0) = \int_0^{\infty} \left( \mathbf{x}^\top \tilde{Q}(\mathbf{x})\mathbf{x} + \mathbf{u}^\top \tilde{R}(\mathbf{x})\mathbf{u} \right) dt \tag{34}$$

where the matrices $\tilde{Q}(\mathbf{x})$ and $\tilde{R}(\mathbf{x})$ have the following structure

$$\tilde{Q}(\mathbf{x}) = \begin{bmatrix} \tilde{Q}_{11}(\bar{\mathbf{x}}_1) & \tilde{Q}_{12} & \dots & \tilde{Q}_{1\mathbf{N}} \\ \vdots & \ddots & \dots & \vdots \\ \tilde{Q}_{\mathbf{N}1} & \dots & \dots & \tilde{Q}_{\mathbf{NN}}(\bar{\mathbf{x}}_\mathbf{N}) \end{bmatrix} \tag{35}$$

$$\tilde{R}(\mathbf{x}) = \begin{bmatrix} R(\bar{\mathbf{x}}_1) & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & \dots & R(\bar{\mathbf{x}}_\mathbf{N}) \end{bmatrix} \tag{36}$$

with

$$\tilde{Q}_{ii}(\bar{\mathbf{x}}_i) = Q_{ii}(\bar{\mathbf{x}}_i) + \sum_{k=1}^{\mathbf{N}} Q_{ik}, \quad i = 1, \dots, \mathbf{N}$$

$$\tilde{Q}_{ij} = -Q_{ij}, \;\; i, j = 1, \dots, \mathbf{N}, \quad i \neq j. \tag{37}$$

$$\tag{38}$$

*Remark 4: The coupled functional (34) is particularly useful in formation flight for underactuated autonomous aerial vehicles like an aircraft [16], or a helicopter with a load stabilizer [17]–[19].*

### A. SDRE CONTROLLER

Consider the following optimization problem:

$$\min_{\mathbf{u}} J(\mathbf{u}, \mathbf{x}_0) \quad \text{subj. to } \dot{\mathbf{x}} = A_a(\mathbf{x})\mathbf{x} + B_a(\mathbf{x})\mathbf{u} \;\; \mathbf{x}(0) = \mathbf{x}_0. \tag{39}$$

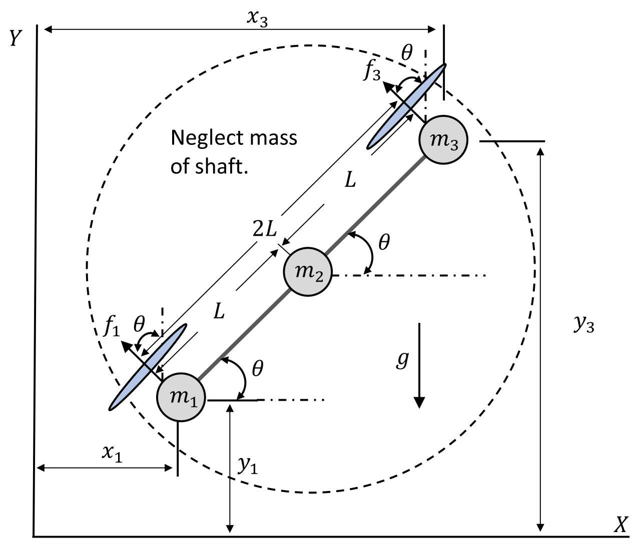


**FIGURE 1.** VTOL aircraft as a system of particles.

We seek a nonlinear state-feedback controller that stabilizes solutions to the problem (39). It is clear that

$$\mathbf{u}(\mathbf{x}) = -\tilde{R}^{-1}(\mathbf{x})B_a^\top(\mathbf{x})P_a(\mathbf{x})\mathbf{x} \tag{40}$$

where $P_a(\mathbf{x})$ is the unique, symmetric, positive-definite solution of the algebraic State-Dependent Riccati Equation (SDRE):

$$P_a(\mathbf{x})A_a(\mathbf{x}) + A_a^\top(\mathbf{x})P_a(\mathbf{x}) - P_a(\mathbf{x})B_a(\mathbf{x})\tilde{R}^{-1}(\mathbf{x})B_a^\top(\mathbf{x})P_a(\mathbf{x}) + \tilde{Q}(\mathbf{x}) = 0. \tag{41}$$

### B. STABILITY ANALYSIS

Throughout the paper, the following conditions are required so that the stabilizing solution that is unique to the problem (39) exists (see [14]):

*Hypotheses 1: Matrices $A_a(\mathbf{x})$, $B_a(\mathbf{x})$, $\tilde{Q}(\mathbf{x})$, and $\tilde{R}(\mathbf{x})$ are $C^1(\mathbb{R}^{n\mathbf{N}\times n\mathbf{N}})$.*

*Hypotheses 2: The pairs $\{A_a(\mathbf{x}), B_a(\mathbf{x})\}$ and $\{A_a(\mathbf{x}), \tilde{Q}^{\frac{1}{2}}(\mathbf{x})\}$ are pointwise stabilizable and detectable of the underactuated network (29) for all $\mathbf{x}$.*

A consequence of Hypotheses 1 is to check that the following controllability matrix

$$\mathcal{C} = \left[ B_a(\mathbf{x}) \;\; | \;\; A_a(\mathbf{x})B_a(\mathbf{x}) \;\; | \;\; \dots \;\; | \;\; A_a^{n\mathbf{N}-1}(\mathbf{x})B_a(\mathbf{x}) \right], \tag{42}$$

has $\text{rank}(\mathcal{C}) = n\mathbf{N} \;\; \forall \mathbf{x} \in \mathbb{R}^{n\mathbf{N}}$. Also, for the observability matrix

$$\mathcal{O} = \left[ \tilde{Q}^{\frac{1}{2}}(\mathbf{x}) \;\; | \;\; \tilde{Q}^{\frac{1}{2}}(\mathbf{x})A_a(\mathbf{x}) \;\; | \;\; \dots \;\; | \;\; \tilde{Q}^{\frac{1}{2}}(\mathbf{x})A_a^{n\mathbf{N}-1}(\mathbf{x}) \right], \tag{43}$$

that has $\text{rank}(\mathcal{O}) = n\mathbf{N} \;\; \forall \mathbf{x} \in \mathbb{R}^{n\mathbf{N}}$. This is true since $\tilde{Q}(\mathbf{x})$ is positive-definite $\forall \mathbf{x} \in \mathbb{R}^{n\mathbf{N}}$.

## IV. APPLICATION: NETWORK OF UNDERACTUATED VTOL AIRCRAFT

In this section, we apply the preceding design methodology to the problem of arbitrary formation flight of the network of underactuated VTOL aircraft with the effect of the weight of the rotors into the dynamical system model of each agent. Each VTOL acts as an independent agent in the formation, and its dynamical system model is considered using Euler-Lagrange equations.

### A. SYSTEM DYNAMICS OF VTOL AIRCRAFT

We consider a VTOL aircraft with masses $m_1$, $m_2$ and $m_3$, as shown in Fig. 1 that are rigidly fastened to the mass-less shaft and are free to fly in the forward flight fashion with the gravity acceleration $g$. We now set up the equation of motion of the VTOL using convenient coordinates $q = [q_1, q_2, q_3]^\top = [x_1, y_1, \theta]^\top$. An external thrust vector $f_1$ is applied to $m_1$ in the direction of $-x_1$ and $y_1$ respectively, and $f_3$ to $m_3$ in the direction of $-x_3$ and $y_3$ respectively. For simplicity, we assume that all representative particle masses are the same (e.g., $m_k = m$ for $k = 1, \ldots, 3$). Applying Euler-Lagrange equations, it follows that

$$\mathcal{L} = \frac{1}{2}\dot{q}^\top \begin{bmatrix} 3m & 0 & -3Lm\sin(q_3) \\ 0 & 3m & 3Lm\cos(q_3) \\ -3Lm\sin(q_3) & 3Lm\cos(q_3) & 5L^2m \end{bmatrix} \dot{q} \tag{44}$$

where $(x_1, y_1)$ is placed at the center of the first mass particle, $L$ is the distance between each mass, and $q_3 = \theta$ is the rotation angle (see Fig. 1). The equations of motion can be written in compact form as

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + \nabla V(q) = G(q)u, \tag{45}$$

where $M(q)$ is the generalized inertia matrix

$$M(q) = \begin{bmatrix} 3m & 0 & -3Lm\sin(q_3) \\ 0 & 3m & 3Lm\cos(q_3) \\ -3Lm\sin(q_3) & 3Lm\cos(q_3) & 5L^2m \end{bmatrix}. \tag{46}$$

$C(q, \dot{q})$ is the Coriolis matrix

$$C(q, \dot{q}) = \begin{bmatrix} 0 & 0 & -3L\dot{\theta}m\cos(q_3) \\ 0 & 0 & -3L\dot{\theta}m\sin(q_3) \\ \frac{3L\dot{q}_3m\cos(q_3)}{2} & \frac{3L\dot{q}_3m\sin(q_3)}{2} & c_\star \end{bmatrix}, \tag{47}$$

with

$$c_\star = -\frac{3Lm(\dot{q}_1\cos(q_3) + \dot{q}_2\sin(q_3))}{2}. \tag{48}$$

and $V(q)$ the systems potential energy

$$V(q) = 3gm(q_2 + L\sin(q_3)). \tag{49}$$

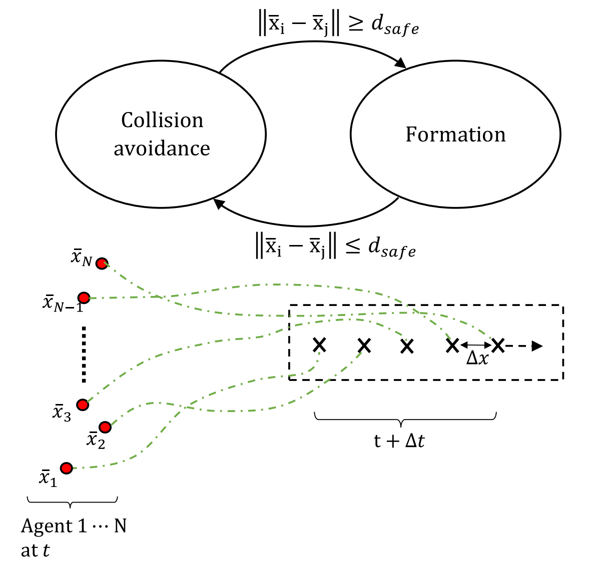


**FIGURE 2.** Hysteresis-based controller of the swarm of VTOLs preserving the formation and collision avoidance.

The elements of the partitioned form of the inertia matrix are given by

$$\begin{aligned} m_{uu} &= 3m, \\ m_{au} &= m_{au}^\top = \begin{bmatrix} 0 & -3Lm\sin(q_3) \end{bmatrix}, \\ m_{aa} &= \begin{bmatrix} 3m & 3Lm\cos(q_3a) \\ 3Lm\cos(q_3) & 5L^2m \end{bmatrix}. \end{aligned} \tag{50}$$

The variation of the work $\delta W$ associated with the applied thrusts $f_1$ and $f_3$, can be computed to be

$$\delta W = \begin{bmatrix} -(f_1 + f_3)\sin(q_3)\delta q_1 \\ +(f_1 + f_3)\cos(q_3)\delta q_2 \\ 2Lf_3\delta q_3 \end{bmatrix}. \tag{51}$$

The derivation of (51) is given in appendix. Finally, $G(q)$ can be written as

$$G(q) = \begin{bmatrix} -sin(q_3) & 0 \\ cos(q_3) & 0 \\ 0 & 1 \end{bmatrix}, \tag{52}$$

with $u = [u_1, u_2]^\top = [f_1 + f_3, 2Lf_3]^\top$.Therefore, $G_u(q) = \begin{bmatrix} -sin(q_3) & 0 \end{bmatrix}$ and $G_a(q) = \begin{bmatrix} \cos(q_3) & 0 \\ 0 & 1 \end{bmatrix}$. Note that $G_a(q)$ is an invertible $2 \times 2$ matrix.

### B. MECHANICAL PROPERTIES OF THE VTOL AIRCRAFT

The VTOL aircraft has several fundamental mechanical properties, in which the preceding design methodology can be used.

*P 1: The inertia matrix $M(q)$ in (45) is a positive definite matrix.*

*P 2: The inertia matrix depends only on the actuated variables $q_a$, i.e., $M(q) = M(q_a)$.*

*P 3: The sub-block matrix $m_{uu}$ of the VTOL is constant.*

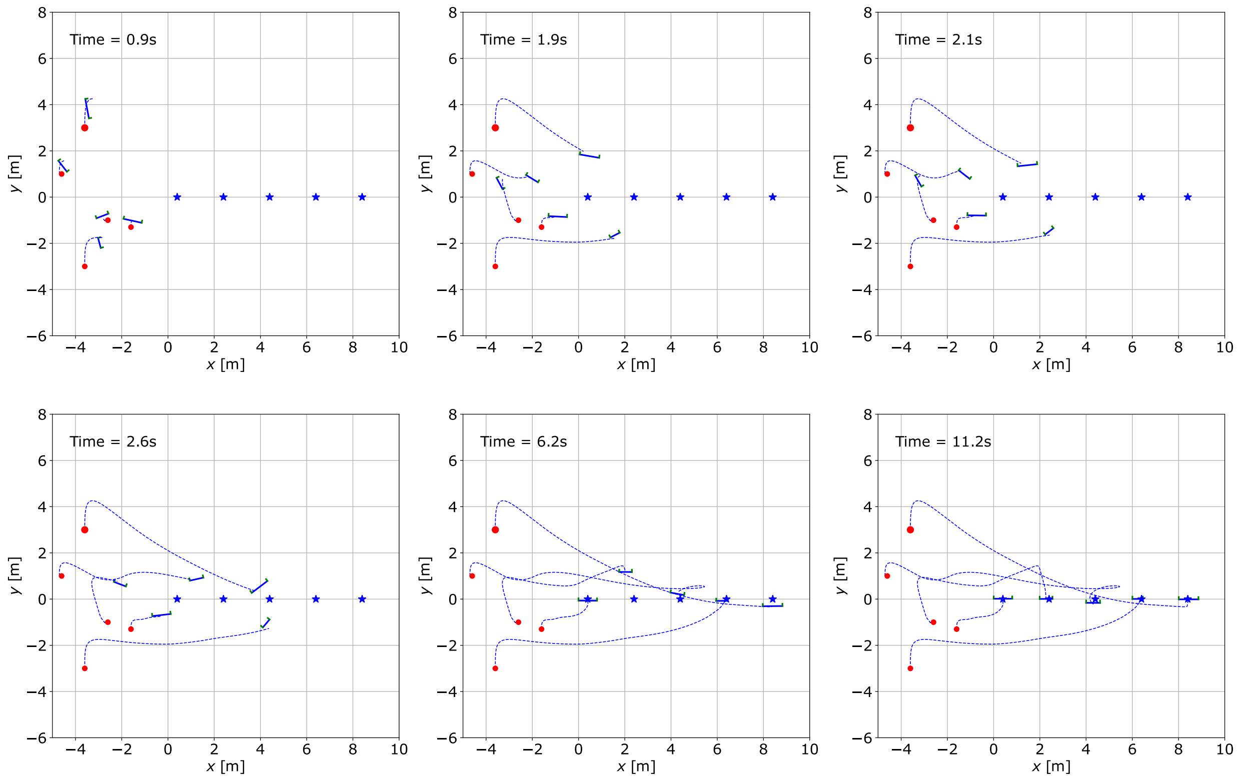


**FIGURE 3. Snapshots of formation flight simulation in the network of N = 5 underactuated VTOL agents where each agent is described by (54)-(56).**

*P 4: $\mathcal{N} = \dot{M}(q) - 2C(q, \dot{q})$ is a skew-symmetric matrix*

$$\mathcal{N} = \begin{bmatrix} 0 & 0 & \frac{9L\dot{q}_3 m\cos(q_3)}{2} \\ 0 & 0 & \frac{9L\dot{q}_3 m\sin(q_3)}{2} \\ -\frac{9L\dot{q}_3 m\cos(q_3)}{2} & -\frac{9L\dot{q}_3 m\sin(q_3)}{2} & 0 \end{bmatrix}, \quad (53)$$

therefore $q^\top \mathcal{N} q = 0 \, \forall \, q \in \mathbb{R}^3$.

*P 5: The potential energy of the VTOL can be written as* $V(q) = V_a(q_a) + V_u(q_u) = 3gmq_2 + 3gmL\sin(q_3)$.

*Remark 5: The VTOL has three degrees of freedom and only two actuators, and therefore, the aircraft is an underactuated mechanical system. The system is a highly nonlinear, constrained multi-variable character. The VTOL translates and rotates by the thrust and torque that make up the movement in the environment. We have nonlinearities because the generalized inertia matrix is off-diagonal, and the input matrix is highly coupled. Due to the lack of actuators, the exact feedback linearization can not be applied.*

### C. NETWORK OF N NONIDENTICAL VTOL SYSTEMS

Given the properties **P. 1—P. 5**, we apply the generalized multi-coordinates transformation based on Proposition 1 to obtain the partial form of the network.

*Proposition 2: Considering the network of* **N** *nonidentical VTOL of the form* (1), *the nonlinear dynamics* (1) *can be written*

$$\ddot{q}_{i_1} = -\frac{0.3333\, u_{i_1}\sin(q_{i_3})}{m_i}, \quad (54)$$

$$\ddot{q}_{i_2} = -1.0002(g - \dot{q}_{i_3}^2 L\sin(q_{i_3})) + 0.8335\frac{u_{i_1}\cos(q_{i_3})}{m_i} - 0.5001\frac{u_{i_2}\cos(q_{i_3})}{L_i m_i}, \quad (55)$$

$$\ddot{q}_{i_3} = -\frac{1}{2L_i^2 m_i}u_{i_1} + \frac{1}{2L_i^2 m_i}u_{i_2}. \quad (56)$$

*Proof:* Applying Proposition 1 the result follows. □

Results presented in Section IV are illustrated through a simulation of nonidentical underactuated multi-agent VTOL aircraft using the centralized SDRE.

Our proposed methodology in total consists of (1) applying the generalized multi-coordinates transformation and (2) proposing and analyzing the centralized optimal control mechanism to steer each agent to the desired position with the stable Euler angle of the network of underactuated systems.

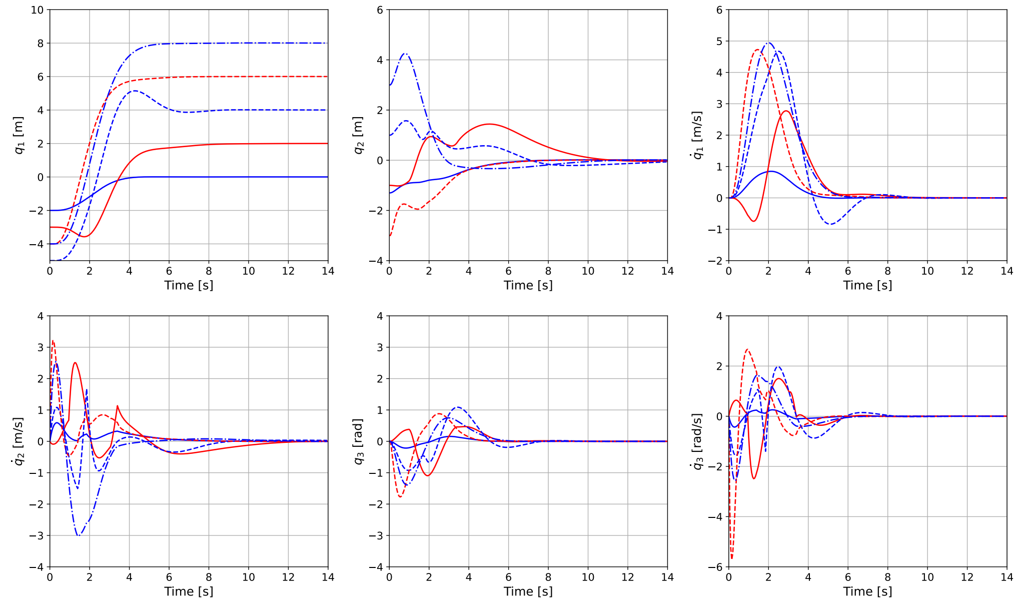


**FIGURE 4.** Evolution of the positions $q_{i_1} = x_i, q_i = y_i$, velocities $\dot{q}_{i_1} = \dot{x}_i, \dot{q}_{i_2} = \dot{y}_i$, Euler angles $q_{i_3} = \theta_i$ and the rate of change of Euler angles $\dot{q}_{i_3} = \dot{\theta}_i$.

**Algorithm 1** Hybrid Formation Flight and Collision Avoidance

**Require:** Partitioned generalized coordinates of the network $q_i = \text{col}(q_{u_i}, q_{a_i})$, $\dot{q}_i = \text{col}(\dot{q}_{u_i}, \dot{q}_{a_i})$. Invertible mappings $\dot{\Phi}(q_i) = T_i(q_i)\dot{q}_i$ and the EL form of the network using Proposition 1. Knowledge of network: initial $\mathbf{x}_0$ and target $\mathbf{x}^*$ states.

**loop**

  **if** $\|\bar{\mathbf{x}}_i - \bar{\mathbf{x}}_j\| \geq d_{safe}$ **then**

    $0 \leftarrow e = \|\bar{\mathbf{x}}_i - \bar{\mathbf{x}}_j\| - d_{safe}$

  **end if**

  Apply the control law (40) to the underactuated network of VTOLs to the desired formation flight

**end loop**

### D. SAFETY OF THE NETWORK

Another challenge concerning the formation flight of the network is accurate navigation with collision avoidance capability in which VTOLs can fulfill and accomplish any given task safely. We adopt the hysteresis-based controller shown in Algorithm. 1 to fulfill the formation flight and collision avoidance. Such a mechanism controls the formation flight but switches to ensuring collision avoidance between any VTOL pair in a formation flight if the safety distance $d_{safe}$ based on the relative distance between the vehicles exceeds. The error $e = \|\bar{\mathbf{x}}_i - \bar{\mathbf{x}}_j\| - d_{safe}$ is reduced to zero, then control again regulates the formation flight and the hysteresis mechanism [20] avoids rapid switching (oscillating) between control modes.

## V. SIMULATIONS

The centralized control designed in the previous sections will be applied to the network of nonidentical VTOLs in which each agent is tilted by an angle $q_{i_3} = \theta_i$ concerning the local $q_1$-axis. The simulation model is a system of particles with three bodies (a central body and two propellers groups), and the simulation results are done in the Python framework. As a proof of concept, two flying scenarios are defined: a lined-up formation and a comparison with the proposed centralized scheme over the decentralized approach.

### A. LINED UP FORMATION

We consider a network of $\mathbf{N} = 5$ whose interconnection structure is represented by the complete graph. The problem setup is defined by assigning the complete graph shown in Fig. 3 to the five VTOLs that are located in different locations in the search space. The automatic control objective is to steer safely each agent to a desired position with the stable Euler angle ( $\lim_{t\to\infty} q_{i_3} = \theta_i = 0$ ) corresponding to its location in the pre-specified lined up formation, which has equal separation distances defined between each neighbor. The network is composed of five different VTOLs. The physical parameters are: $m_1 = 1$ kg, and $L_1 = 0.2$ m, for Agent 1; $m_2 = 1.5$ kg, $L_2 = 0.25$ m, for Agent 2; $m_3 = 2$ kg, $L_3 = 0.3$ m, for Agent 3; $m_4 = 0.5$ kg, $L_4 = 0.1$ m, for Agent 4; $m_5 = 0.8$ kg, $L_5 = 0.18$ m, for Agent 5. The centralized scheme (39)–(41) is applied to the problem using a sampling time of $t_s = 0.05$ s with the total time of flight that is set to $T_f = 14$ s. The design parameters of absolute and relative state information concerning the defined cost function in (32) are in the following

$$Q_{ii}(\bar{\mathbf{x}}_i) = (1 + (\bar{\mathbf{x}}_i - \bar{\mathbf{x}}_i^*)^2)I_6, \quad Q_{ij} = I_6 \; \forall i \neq j, \qquad (57)$$

where $\bar{\mathbf{x}}_i^*$ is the desired state in the considered lined up formation flight. The weight matrix related to the control effort was kept constant for each agent in the simulation

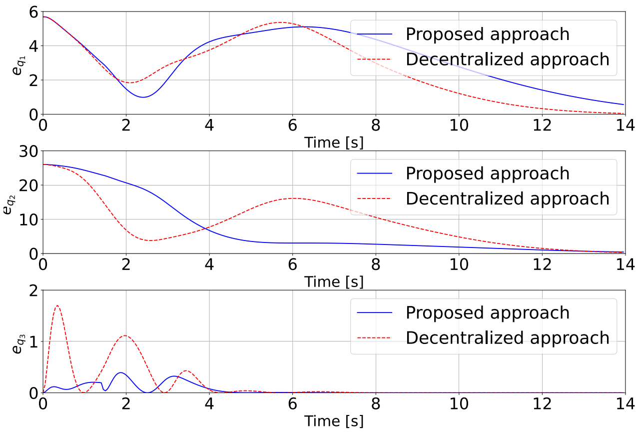


**FIGURE 5. Error performance for the Centralized scheme over the decentralized approach.**

$R_{ii} = I_2$. Fig. 3 and Fig. 4 represent the simulation results for the formation flight scenario under the initial conditions

$$\mathbf{x}_0 = [-2, 0, -1.3, 0, 0, 0, -3, 0, -1, 0, 0, 0, -5, 0, 1, 0, 0, 0, -4, 0, -3, 0, 0, 0, -4, 0, 3, 0, 0, 0]^\top, \tag{58}$$

and the desired states

$$\mathbf{x}^* = [0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 4, 0, 0, 0, 0, 0, 6, 0, 0, 0, 0, 0, 8, 0, 0, 0, 0, 0]^\top. \tag{59}$$

It is worth mentioning that Euler angles $q_{i_3} = \theta_i$ in Fig. 4 represent the inclination that a vectored thrust VTOL would have to reach to guarantee the formation flight. Fig. 4 shows that, after a transient phase, the velocities remain bounded. The simulation demonstrates that the performance of the considered centralized control design in formation flight is independent of the SDRE weighting matrices selection $Q_{ii}(\bar{\mathbf{x}}_i)$, $Q_{ij}$ and $R_{ii}(\bar{\mathbf{x}}_i)$. Therefore, this can be selected freely to achieve the global performance objective of the network. The results indicate that the proposed methodology can be applied successfully and effectively to the network of underactuated VTOL aircraft formation control problem. The hysteresis-based controller for collision avoidance leads to a simplified control law, which enables the utilization of a sophisticated centralized control mechanism and provides excellent performance.

### *B. CENTRALIZED SCHEME OVER THE DECENTRALIZED APPROACH*

To show the potentiality of the proposed method, we analyze the achieved performance over the decentralized approach. We consider two VTOL aircraft systems with the same physical properties, e.g. $m = 1$ kg, and $L = 0.8$ m, for both agents.The formation flight is the same as the previous scenario with the initial and final states of

$$\mathbf{x}_0 = [-2, 0, -1.3, 0, 0, 0, -3, 0, -1, 0, 0, 0]^\top,$$

and the desired states

$$\mathbf{x}^* = [0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0]^\top.$$

Fig. 5 shows the performance of the proposed centralized scheme over the decentralized methodology. Overall it can be seen that our framework provides better performance with smoother trajectories due to the less attenuated oscillations in the rotational angle $q_3 = \theta$.

*Remark 6: The proposed centralized approach can fulfill both optimality and the minimum safety distance of the network during the mission flight due to the access to the hole information of the network.*

## VI. CONCLUSION

We have proposed a simple centralized control for a class of nonidentical underactuated Euler–Lagrange systems based on generalized change of coordinates. We transform the mechanical systems to the partial form that can facilitate the design of the centralized control. The network is modeled as a weighted interconnection graph where each EL-system is a node, and the control action at each node is a function of absolute and relative state information. At the cost of centrally tracking all agents, we gain the benefit of energy-efficient and keeping safety distances between individual VTOLs in the task of formation flight. In particular, we present simulations for the network of five VTOLs aircraft including the effect of the weight of the rotors into their dynamical system models that ensure the convergence performance of the closed-loop system to an arbitrary formation flight for each VTOL with zero Euler angle and zero speed. In future work, we plan to study and analyze keeping minimum and smooth safe distance without oscillation in the transition phase of the network with dynamic obstacle avoidance. Another interesting question involves the robustness of the network navigation based on the available states information, i.e., if the state information of an agent is not available for feedback, how to reconstruct and save an agent based on the available data. This issue leads to the decomposition of state estimation into several local estimators.

## APPENDIX
## DERIVATION OF THE VARIATION OF THE WORK $\delta W$

In this Appendix, the virtual of the work is derived. Let us consider the positions of the mass particles

$$\begin{aligned} x &= x_1 + L\cos(\theta), \\ y &= y_1 + L\sin(\theta), \\ x_3 &= x_1 + 2L\cos(\theta), \\ y_3 &= y_1 + 2L\sin(\theta). \end{aligned}$$

Now talking variations

$$\begin{bmatrix} -f_1 \sin(\theta) \\ f_1 \cos(\theta) \end{bmatrix} \delta \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} -f_1 \sin(\theta)\delta x_1 \\ f_1 \cos(\theta)\delta y_1 \end{bmatrix}, \tag{60}$$

and

$$\begin{bmatrix} -f_3 \sin(\theta) \\ f_3 \cos(\theta) \end{bmatrix} \delta \begin{bmatrix} x_3 \\ y_3 \end{bmatrix} = \begin{bmatrix} -f_3 \sin(\theta)(\delta x_1 - 2L\sin(\theta))\delta\theta \\ f_3 \cos(\theta)(\delta y_1 + 2L\cos(\theta)\delta\theta) \end{bmatrix}.$$

Collecting terms, we have

$$\delta W = \begin{bmatrix} -f_1 \sin(\theta)\delta x_1 - f_3 \sin(\theta)\delta x_1 + f_3 \sin^2(\theta) 2L\delta\theta \\ f_1 \cos(\theta)\delta y_1 + f_3 \cos(\theta)\delta y_1 + f_3 \cos^2(\theta) 2L\delta\theta \end{bmatrix} = \begin{bmatrix} -(f_1 + f_3)\sin(\theta)\delta x_1 \\ +(f_1 + f_3)\cos(\theta)\delta y_1 \\ 2Lf_3\delta\theta \end{bmatrix}. \quad (61)$$

■

## ACKNOWLEDGMENT

Babak Salamat acknowledges Alejandro Donaire for his constructive feedback of the generalized multi-coordinates transformation for the underactuated Euler–Lagrange systems.

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