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Rotated Golden Codewords Based Space-Time Line Code Systems

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ABSTRACT The space-time line code (STLC) has been recently proposed in the literature. In STLC systems, the channel state information is assumed to be known at the transmitter, but not at the receiver. In the conventional STLC systems, the inputs of the STLC encoder are either M-ary phase shift keying symbols or M-ary quadrature amplitude modulation symbols and can achieve diversity order with very low detection complexity. In order to further improve the error performance of the STLC systems, in this paper, rotated Golden codewords based STLC (RGC-STLC) systems are proposed. We extend the 1×2 and 2×2 transmit-receive antenna structure in the conventional STLC into 1×4 and 2×4 transmit-receive antenna structures in the proposed RGC-STLC system. The inputs of the proposed RGC-STLC encoder are the rotated Golden codewords. Compared to the conventional 1×2 and 2×2 STLC systems, the 1×4 and 2×4 RGC-STLC systems achieve a higher diversity order. Based on the feature of the rotated Golden codewords, low complexity detection schemes for the 1×4 and 2×4 RGC-STLC systems are further proposed. Finally, the union bounds on the average symbol error probability for the 1×4 and 2×4 RGC-STLC systems are derived and shown to match the simulation results very well at high signal-to-noise ratios.

INDEX TERMS In-phase and quadrature maximum likelihood detection, reduced complexity detection, space-time block code, space-time line code (STLC), threshold based signal detection.

I. INTRODUCTION

There exists an ever-growing demand to increase data transmission rate and improve communication reliability in 5th and beyond-5th generation wireless communication systems. In general, data transmission rate can be increased by employing multiplexing techniques and the reliability of a wireless communication system can be improved by employing diversity techniques. Multiple-input multiple-output (MIMO) techniques can be used to increase data transmission rate and/or improve the reliability of a wireless communication system. Actually, there is a tradeoff between data transmission rate and communication reliability in terms of the diversity-multiplexing gain [1]. In this paper, the main focus is to improve on the reliability of communications in a single-input multiple-output (SIMO) or MIMO system setting.

In the literature, the encoding approach of a MIMO system enables the system to increase the data transmission rate and/or improve the reliability of a wireless communication system. For example, the space-time block code (STBC) is one type of MIMO system. A well-known STBC system with two transmit antennas is the Alamouti STBC which achieves full diversity at the receiver [2]. The encoding of the Alamouti STBC makes the codeword matrix orthogonal. The orthogonal codeword matrix further requires linearly combining the received signal to detect the transmitted symbol. Another example of an STBC is the Golden code [3]. The encoding of the conventional Golden code enables the Golden code system to simultaneously achieve diversity order and multiplexing gain. However, the maximum likelihood (ML) detection complexity of the Golden code is proportional to $O(M^4)$, which is very high.

In the above two examples, the channel state information (CSI) is assumed to be known or fully estimated at the receiver. When the CSI is fully known at the transmitter, not the receiver, the diversity order can also be achieved through the encoding of the space-time line code (STLC), which is recently proposed in [4]. In all STLC systems, the receiver only needs to linearly combine the received signals. No CSI or effective channel gain is required for detecting M-ary phase

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shift keying (M-PSK) symbols at the receiver. However, it is required to estimate the effective channel gain for detecting M-ary quadrature amplitude modulation (M-QAM) symbols. If the effective channel gain is fully estimated at the receiver, the STLC systems achieve diversity order with very low detection complexity. This is the key feature of the STLC systems.

Since the STLC was proposed, the technique has been applied in different communication systems. For example, STLC has been applied in massive MIMO and multiuser systems with antenna allocations [5]. STLC has also been applied in the uplink non-orthogonal multiple access (NOMA) random access system [6]. An STLC coded regenerative two-way relay system has been studied in [7]. Very recently, double STLC and generalized STLC have been proposed in [8] and [9], respectively. Compared to the conventional STLC, double STLC and generalized STLC provides the multiplexing gain and spatial diversity.

Since the CSI is known at the transmitter, transmit antenna selection is an easy way to achieve a higher diversity order, and consequently improve error performance in an STLC system. On this note, transmit antenna selection for STLC systems and multiuser STLC has been discussed in [10] and [11], respectively.

In the literature, there are other approaches to achieve an increased diversity order. The simple way to achieve receive diversity order is that the receiver has more independent receive antennas. However, only two receive antennas are available in the STLC systems because of the special encoding of the STLC at the transmitter. Signal space diversity (SSD) is another technique to improve diversity order [12]. In SSD systems, the signal diversity is achieved by transmitting the in-phase and quadrature of the rotated multi-dimensional signal in independent fading channel. While no additional bandwidth, transmit power and receive antennas are required in SSD systems, ML detection is required to achieve optimal error performance. Recently, Golden codeword based SIMO (GC-SIMO) modulation has been proposed in [13]. In [13], an increased diversity order can be achieved through the transmission of the Golden codewords. The ML detection complexity of GC-SIMO is proportional to $O(M^2)$, where M is the order of the modulation. Very recently, an alternative encoding of the Golden code has been proposed in [14]. The rotated Golden codeword is also proposed in the alternative encoding of the Golden code. Compared to the conventional Golden codeword, the rotated Golden codewords are able to convert one two-dimensional ML detection into two one-dimensional ML detections. Hence, in this paper, we propose to apply the rotated Golden codewords in the STLC system. We refer to the proposed system as a rotated Golden codewords based STLC (RGC-STLC) system. We extend 1×2 and 2×2 in the conventional STLC into 1×4 and 2×4 in the proposed RGC-STLC system. In order to achieve the optimal error performance, the in-phase and quadrature ML (IQML) detection is proposed in this paper. The IQML detection complexity of the RGC-STLC is only proportional to $O(2M)$.

The encoding of the rotated Golden codewords can be regarded as superposition encoding in the NOMA system. The threshold based signal detection (TSD) can also be applied to detect the transmitted symbols in the proposed RGC-STLC system. By aid of the TSD, in this paper, a reduced complexity IQML (RC-IQML) is further proposed. Compared to the IQML, the RC-IQML further reduces detection complexity. Typically, the detection complexity of the proposed RC-IQML is very low at high signal-to-noise ratios (SNRs).

The error performance of the conventional STLC with and without transmit antenna selection has been analyzed in [10]. In order to derive a closed-form error performance expression, the union bounds on the average symbol error probability (ASEP) for the proposed 1×4 and 2×4 RGC-STLC systems are also derived in this paper.

The main contributions of this paper are summarized as:

- Both 1×4 and 2×4 RGC-STLC systems are proposed. The proposed RGC-STLC systems also achieve an increased diversity order, and have a low complexity signal detection.
- The union bound of the ASEP of the proposed 1×4 and 2×4 RGC-STLC systems are derived;
- By aid of the TSD, the RC-IQML is proposed to detect the transmitted symbols for both 1×4 and 2×4 RGC-STLC systems.

The remainder of this paper is organized as follows: in Section II, the system models of the proposed 1×4 and 2×4 RGC-STLC systems are presented. The signal detection schemes of the proposed 1×4 and 2×4 RGC-STLC systems are presented in Section III. In Section IV, the union bounds of ASEP for the 1×4 RGC-STLC systems are derived. The simulation results for the 1×4 and 2×4 RGC-STLC systems are demonstrated in Section V. Finally, the paper is concluded in Section VI.

Notation: Bold lowercase and uppercase letters are used for vectors and matrices, respectively. $[\cdot]^T$ and $[\cdot]^*$ represent the transpose and conjugate operations, respectively. $(\cdot)^H$, $|\cdot|$ represents Hermitian and magnitude operations, respectively. $\mathcal{D}(\cdot)$ denotes the constellation demodulator function. Let x be a complex valued symbol. $Re\{x\}$ denotes the operation to select the in-phase or real part of x. $Q(\cdot)$ is the Gaussian Q-function. $(\cdot)^I$, $(\cdot)^Q$ denotes the in-phase and quadrature components, respectively.

II. SYSTEM MODEL

In this paper, we mainly focus on the proposed 1×4 and 2×4 RGC-STLC systems. We will present the rotated Golden codewords in Subsection A, reveal the encoding and decoding/detection of the conventional STLC systems in Subsection B, and then describe the 1×4 and 2×4 RGC-STLC systems in Subsection C and Subsection D, respectively.

A. THE GOLDEN CODE

The Golden code is a linear dispersion STBC (LD-STBC) with two transmit antennas and two or more receive antennas [3]. The Golden code achieves full-rate and full-diversity. Its encoder takes four complex-valued symbols and generates four super-symbols. Let x_i be an MQAM symbol with $E\{|x_i|^2\} = \varepsilon$, $x_i \in \Omega$, $i \in [1:4]$, where Ω is the signal set of MQAM. The Golden code transmission matrix is given by [3]:

$$\boldsymbol{X} = [\boldsymbol{X}_1\ \boldsymbol{X}_2] = \begin{bmatrix} x_{11} & x_{21} \\ x_{12} & x_{22} \end{bmatrix}, \tag{1}$$

where $x_{11} = \frac{1}{\sqrt{5}}\alpha(x_1 + x_2\theta)$, $x_{22} = \frac{1}{\sqrt{5}}\bar{\alpha}(x_1 + x_2\bar{\theta})$, $x_{12} = \frac{1}{\sqrt{5}}\alpha(x_3 + x_4\theta)$ and $x_{21} = \frac{1}{\sqrt{5}}\gamma\bar{\alpha}(x_3 + x_4\bar{\theta})$, with $\theta = \frac{1+\sqrt{5}}{2}$, $\bar{\theta} = 1 - \theta$, $\alpha = 1 + j\bar{\theta}$, $\bar{\alpha} = 1 + j(1 - \bar{\theta})$ and $\gamma = j$. Since $E\{|x_i|^2\} = \varepsilon$, we have $E\{|x_{ik}|^2\} = \varepsilon$, $i, k \in [1:2]$.

In (1), there are four super-symbols, $\frac{1}{\sqrt{5}}\alpha(x_1 + x_2\theta)$, $\frac{1}{\sqrt{5}}\bar{\alpha}(x_1 + x_2\bar{\theta})$, $\frac{1}{\sqrt{5}}\alpha(x_3 + x_4\theta)$ and $\frac{1}{\sqrt{5}}\gamma\bar{\alpha}(x_3 + x_4\bar{\theta})$. In this paper, we refer to these super-symbols as the Golden codewords. These super-symbols form two pairs of Golden codewords $\left\{\frac{1}{\sqrt{5}}\alpha(x_1 + x_2\theta), \frac{1}{\sqrt{5}}\bar{\alpha}(x_1 + x_2\bar{\theta})\right\}$ and $\left\{\frac{1}{\sqrt{5}}\alpha(x_3 + x_4\theta), \frac{1}{\sqrt{5}}\gamma\bar{\alpha}(x_3 + x_4\bar{\theta})\right\}$ in the conventional Golden code. In this paper, only the pair of Golden codewords $\left\{\frac{1}{\sqrt{5}}\alpha(x_1 + x_2\theta), \frac{1}{\sqrt{5}}\bar{\alpha}(x_1 + x_2\bar{\theta})\right\}$ is applied in the proposed STLC system.

In the pair of Golden codewords, both α and $\bar{\alpha}$ are complex. Let $\alpha = |\alpha|e^{j\varphi_1}$ and $\bar{\alpha} = |\bar{\alpha}|e^{j\varphi_2}$. For convenient discussion, we further let $\beta_1 = \frac{1}{\sqrt{5}}|\alpha|$ and $\beta_2 = \frac{1}{\sqrt{5}}|\bar{\alpha}| = \frac{1}{\sqrt{5}}|\alpha|\theta$. It is noted that $(\beta_1)^2 + (\beta_2)^2 = 1$. It is also easy to derive that $\frac{1}{\sqrt{5}}|\bar{\alpha}|\bar{\theta} = \beta_2\bar{\theta} = -\beta_1$.

If we rotate $x_{11} = \frac{1}{\sqrt{5}}\alpha(x_1 + x_2\theta)$ by angle $-\varphi_1$ and $x_{22} = \frac{1}{\sqrt{5}}\bar{\alpha}(x_1 + x_2\bar{\theta})$ by angle $-\varphi_2$, we then have:

$$g_1 = e^{-j\varphi_1}x_{11} = \beta_1 x_1 + \beta_2 x_2, \tag{2.1}$$

$$g_2 = e^{-j\varphi_2}x_{22} = \beta_2 x_1 - \beta_1 x_2. \tag{2.2}$$

Both g_1 and g_2 in (2.1) and (2.2) are referred to as rotated Golden codewords (RGC), which will be applied in the proposed STLC system.

The $N_t \times N_r$ STLC has been documented in [4], where $N_t = 1$ or $N_t = 2$ and $N_r = 2$ are the numbers of transmit antennas and the number of receive antennas, respectively. The encoding and linear combination detection (LCD) of the conventional STLC have been presented in [4]. However, [4] did not explain how the encoding and the LCD of the STLC are derived from theory. In the following subsection, we reveal how the encoding and the LCD may be derived from theory.

B. EXTENDING ALAMOUTI ENCODING INTO STLC ENCODING AND DECODING

Consider an $N_t \times N_r$ Alamouti scheme with $N_t = 2$ and $N_r = 1$. Let $h_{n_t}, n_t \in [1:2]$, be the channel fading coefficient between transmit antenna n_t and the receive antenna. Let $x_i, i \in [1:2]$ be MQAM symbols with $E\{|x_i|^2\} = \varepsilon$, where $x_i \in \Omega$. In terms of transmitting x_1^* and x_2, the Alamouti codeword matrix is given by [2]:

$$\boldsymbol{X} = \begin{bmatrix} x_1^* & x_2 \\ -x_2^* & x_1 \end{bmatrix}. \tag{3}$$

At the receiver, the received signals are given by:

$$\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} x_1^* & x_2 \\ -x_2^* & x_1 \end{bmatrix}\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}, \tag{4}$$

where $n_i, i \in [1:2]$ represents the additive white Gaussian noise (AWGN) elements.

Ignoring the noise terms in (4), then (4) may be rewritten as:

$$\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} h_1 x_1^* + h_2 x_2 \\ -h_1 x_2^* + h_2 x_1 \end{bmatrix}. \tag{5}$$

(5) is further rewritten as:

$$\begin{bmatrix} s_1^* \\ s_2 \end{bmatrix} = \begin{bmatrix} h_1^* & h_2^* \\ h_2 & -h_1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2^* \end{bmatrix}. \tag{6}$$

In (6), let $\boldsymbol{H} = \begin{bmatrix} h_1^* & h_2^* \\ h_2 & -h_1 \end{bmatrix}$. $\boldsymbol{H}$ is an orthogonal matrix. We further have $\boldsymbol{H}^H\boldsymbol{H} = (|h_1|^2 + |h_2|^2)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Either (5) or (6) can be regarded as the encoding of the STLC system.

Suppose there is another $N_t \times N_r$ system with $N_t = 1$ and $N_r = 2$. The channel fading coefficient between the transmit antenna and receive antenna n_r, $n_r \in [1:2]$, is denoted as h_{n_r} which is known at the transmitter. (6) can be regarded as the encoding of the 1×2 system assuming that the CSI is known at the transmitter.

Multiplying $\begin{bmatrix} h_1^* & h_2^* \\ h_2 & -h_1 \end{bmatrix}^H$ in both sides of (6), we have:

$$\begin{bmatrix} h_1^* & h_2^* \\ h_2 & -h_1 \end{bmatrix}^H \begin{bmatrix} s_1^* \\ s_2 \end{bmatrix} = \begin{bmatrix} h_1^* & h_2^* \\ h_2 & -h_1 \end{bmatrix}^H \begin{bmatrix} h_1^* & h_2^* \\ h_2 & -h_1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2^* \end{bmatrix}. \tag{7}$$

(7) is further rewritten as:

$$\begin{bmatrix} h_1 s_1^* + h_2^* s_2 \\ h_2 s_1^* - h_1^* s_2 \end{bmatrix} = \delta_2 \begin{bmatrix} x_1 \\ x_2^* \end{bmatrix}, \tag{8}$$

where $\delta_2 = |h_1|^2 + |h_2|^2$.

(8) can be regarded as the decoding or detection for the above scheme, which will be discussed in detail below.

Suppose s_1^* and s_2^* are transmitted in the above 1×2 system, the received signal r_{n_r, s_i^*} in terms of the receive antenna n_r and the transmitted symbol s_i^*, $i \in [1:2]$ are given by:

$$r_{1,s_1^*} = h_1 s_1^* + n_{1,s_1^*}, \tag{9.1}$$

$$r_{1,s_2^*} = h_1 s_2^* + n_{1,s_2^*}, \tag{9.2}$$

$$r_{2,s_1^*} = h_2 s_1^* + n_{2,s_1^*}, \tag{9.3}$$

$$r_{2,s_2^*} = h_2 s_2^* + n_{2,s_2^*}. \tag{9.4}$$

If noise terms in (9.1) to (9.4) are ignored, then substituting (9.1) to (9.4) into (8), we have:

$$r_{1,s_1^*} + r^*_{2,s_2^*} = \delta_2 x_1, \quad (10.1)$$
$$r^*_{2,s_1^*} - r_{1,s_2^*} = \delta_2 x_2. \quad (10.2)$$

Either $r_{1,s_1^*} + r^*_{2,s_2^*}$ in (10.1) or $r^*_{2,s_1^*} - r_{1,s_2^*}$ in (10.2) represents LCD. The LCD of x_1 and x_2 in (10.1) and (10.2) is the same as the ones in (16a) and (16b) in [4]. Hence, from the Alamouti orthogonal codeword matrix we have derived the encoding and decoding or detection of the STLC. In the following two subsections, we will describe in detail the proposed 1 × 4 RGC-STLC and 2 × 4 RGC-STLC systems.

C. 1 × 4 ROTATED GOLDEN CODEWORDS BASED SPACE-TIME LINE CODE SYSTEM

In this subsection, we only consider the 1×4 RGC-STLC system. Based on (2.1) and (2.2), the rotated Golden codewords g_{2i-1} and g_{2i} are given by:

$$g_{2i-1} = \beta_1 x_{2i-1} + \beta_2 x_{2i}, \quad (11.1)$$
$$g_{2i} = \beta_2 x_{2i-1} - \beta_1 x_{2i}, \quad (11.2)$$

where $i \in [1:2]$. $x_k, k \in [1:4]$ are the squared MQAM symbols with $E[|x_k|^2] = \varepsilon$. The modulation order of the squared MQAM is M.

The proposed 1 × 4 RGC-STLC system needs two block transmissions, where each block contains two time slots. Let $\boldsymbol{h}_i = [h_{1,2i-1}\ h_{1,2i}]^T \in \mathbb{C}^{2\times1}$, be the i^{th} block channel fading vector, where h_{n_t,n_r} is defined as the channel fading coefficient between the transmit antenna n_t and the receive antenna n_r. h_{n_t,n_r} are independent and identically distributed (i.i.d) complex Gaussian random variables (RVs) distributed as $CN(0,1)$. Every h_{n_t,n_r} lasts one block transmission and takes on another independent value in the next one block transmission. In the proposed 1 × 4 RGC-STLC system $n_t = 1$. Define $\delta_{i,2} = |h_{1,2i-1}|^2 + |h_{1,2i}|^2$, which is the sum of the channel gains in the i^{th} block transmission. Similar to the definition in [4], $\sqrt{\delta_{i,2}}$ is defined as the effective channel gain.

Based on (6), the encoder of the proposed 1×4 RGC-STLC takes the rotated Golden codewords g_{2i-1}, g_{2i} and the channel coefficients $h_{1,2i-1}$ and $h_{1,2i}$ as inputs, and outputs RGC-STLC symbols, s_1^i and s_2^i, which are given by:

$$s_1^1 = [h^*_{1,1} g_1 + h^*_{1,2} g^*_3]\frac{1}{\sqrt{\delta_{1,2}}}, \quad (12.1)$$
$$s_2^1 = [h^*_{1,2} g^*_1 - h^*_{1,1} g_3]\frac{1}{\sqrt{\delta_{1,2}}}, \quad (12.2)$$
$$s_1^2 = [h^*_{1,3} g_2 + h^*_{1,4} g^*_4]\frac{1}{\sqrt{\delta_{2,2}}}, \quad (12.3)$$
$$s_2^2 = [h^*_{1,4} g^*_2 - h^*_{1,3} g_4]\frac{1}{\sqrt{\delta_{2,2}}}. \quad (12.4)$$

It takes two block transmissions to transmit RGC-STLC symbols in the proposed 1 × 4 RGC-STLC system. As discussed earlier, each block transmission contains two time slots. If s_k^i is transmitted in the k^{th} time slot of the i^{th} block transmission, then at receive antenna $2i-1$ and $2i$ the received signals $z_{k,2i-1}$ and $z_{k,2i}$ are given by:

$$z_{k,2i-1} = h_{1,2i-1} s_k^i + n_{k,2i-1}, \quad (13.1)$$
$$z_{k,2i-0} = h_{1,2i-0} s_k^i + n_{k,2i-0}, \quad (13.2)$$

where $n_{k,2i-1}$ and $n_{k,2i}$ are distributed as $CN(0, \frac{\varepsilon}{\rho})$, where ρ is the SNR at each receive antenna in the proposed 1 × 4 RGC-STLC system.

D. 2 × 4 ROTATED GOLDEN CODEWORDS BASED SPACE-TIME LINE CODE SYSTEM

In this subsection, we consider the 2 × 4 RGC-STLC system. The definitions of the channel fading vector, the effective channel gain and the rotated Golden codewords g_{2i-1} and g_{2i} are exactly the same as those in the 1 × 4 RGC-STLC system. Again, define $\delta_{i,4} = \sum_{n_t=1}^{2}\sum_{n_r=(i-1)\times2+1}^{(i-1)\times2+2} |h_{n_t,n_r}|^2$, which is the sum of the channel gains in the i^{th} block transmission.

Then, based on (6), the encoder of the proposed 2 × 4 RGC-STLC takes the rotated Golden codewords g_{2i-1}, g_{2i} and the channel coefficients $h_{n_t,(i-1)\times2+n_r}$, $n_t \in [1:2]$, $n_r \in [1:2]$ as inputs, and outputs two pairs of RGC-STLC symbols, $s^i_{n_t,1}$ and $s^i_{n_t,2}$, which are given by:

$$s^1_{n_t,1} = [h^*_{n_t,1} g_1 + h^*_{n_t,2} g^*_3]\frac{1}{\sqrt{\delta_{1,4}}}, \quad (14.1)$$
$$s^1_{n_t,2} = [h^*_{n_t,2} g^*_1 - h^*_{n_t,1} g_3]\frac{1}{\sqrt{\delta_{1,4}}}. \quad (14.2)$$
$$s^2_{n_t,1} = [h^*_{n_t,3} g_2 + h^*_{n_t,4} g^*_4]\frac{1}{\sqrt{\delta_{2,4}}}, \quad (14.3)$$
$$s^2_{n_t,2} = [h^*_{n_t,4} g^*_2 - h^*_{n_t,3} g_4]\frac{1}{\sqrt{\delta_{2,4}}}. \quad (14.4)$$

Transmit antenna n_t transmits $s^i_{n_t,1}$ and $s^i_{n_t,2}$ during time slot 1 and time slot 2 of the i^{th} block transmission, respectively.

In the proposed 2 × 4 RGC-STLC system, it takes four time slots to transmit two pairs of RGC-STLC symbols for each transmit antenna. During time slot k of the i^{th} block transmission if transmit antenna $n_t = 1$ transmits $s^i_{1,k}$ and transmit antenna $n_t = 2$ transmits $s^i_{2,k}$ then at receive antennas $2i-1$ and $2i$ the received signals $z_{k,2i-1}$ and $z_{k,2i}$ are given by:

$$z_{1,2i-1} = h_{1,rx+1} s^i_{1,1} + h_{2,rx+1} s^i_{2,1} + n_{1,2i-1}, \quad (15.1)$$
$$z_{1,2i-0} = h_{1,rx+2} s^i_{1,1} + h_{2,rx+2} s^i_{2,1} + n_{1,2i-0}, \quad (15.2)$$
$$z_{2,2i-1} = h_{1,rx+1} s^i_{1,2} + h_{2,rx+1} s^i_{2,2} + n_{2,2i-1}, \quad (15.3)$$
$$z_{2,2i-0} = h_{1,rx+2} s^i_{1,2} + h_{2,rx+2} s^i_{2,2} + n_{2,2i-0}, \quad (15.4)$$

where $rx = (i-1)\times 2$, $n_{n_t,2i-1}$ and $n^i_{n_t,2i}$, $n_t \in [1:2]$, $i \in [1:2]$ are exactly the same as those in (13.1) and (13.2) in the 1 × 4 RGC-STLC system.

III. SIGNAL DETECTION SCHEMES FOR THE 1 × 4 AND 2 × 4 RGC-STLC SYSTEMS

Similar to the discussion in [4], at the receiver the effective channel gain is required to be estimated for detecting MQAM symbols. In this paper, we assume that the effective channel gain is perfectly estimated at the receiver. The signal detection schemes of the 2 × 4 RGC-STLC systems are the same as the ones of 1 × 4 RGC-STLC systems except for different effective channel gain. The effective channel gain is $\delta_{k,2}$, $k \in [1:2]$ in the 1 × 4 RGC-STLC systems, while the effective channel gain is $\delta_{k,4}$, $k \in [1:2]$ in 2 × 4 RGC-STLC systems. Thus, in this section, we mainly focus on the signal detections for the 1 × 4 RGC-STLC systems.

A. LINEAR COMBINATION DETECTION FOR 1 × 4 RGC-STLC SYSTEM

LCD for the conventional STLC has been discussed in [4]. In Subsection II.A, we revealed how the LCD is derived in (10.1) and (10.2) from theory. In this subsection, we will apply (10.1) and (10.2) to detect the transmitted symbols in the proposed 1 × 4 RGC-STLC system. The received signals in (13.1) and (13.2) in the 1 × 4 RGC-STLC system can be further rewritten as:

$$z_{k,1} = h_{1,1}s_k^1 + n_{k,1}, \quad (16.1)$$
$$z_{k,2} = h_{1,2}s_k^1 + n_{k,2}, \quad (16.2)$$
$$z_{k,3} = h_{1,3}s_k^2 + n_{k,3}, \quad (16.3)$$
$$z_{k,4} = h_{1,4}s_k^2 + n_{k,4}, \quad (16.4)$$

where $k \in [1:2]$.

Based on the LCD, we further have:

$$r_1 = \sqrt{\delta_{1,2}}g_1 + n_{1,1} + n_{2,2}^*, \quad (17.1)$$
$$r_2 = \sqrt{\delta_{2,2}}g_2 + n_{1,3} + n_{2,4}^*, \quad (17.2)$$
$$r_3 = \sqrt{\delta_{1,2}}g_3 + n_{1,2}^* - n_{2,1}, \quad (17.3)$$
$$r_4 = \sqrt{\delta_{2,2}}g_4 + n_{1,4}^* - n_{2,3}, \quad (17.4)$$

where $r_1 = z_{1,1} + z_{2,2}^*$, $r_2 = z_{1,3} + z_{2,4}^*$, $r_3 = z_{1,2}^* - z_{2,1}$ and $r_4 = z_{1,4}^* - z_{2,3}$.

Let $w_1 = n_{1,1}+n_{2,2}^*$, $w_2 = n_{1,3}+n_{2,4}^*$, $w_3 = n_{1,2}^*-n_{2,1}$ and $w_4 = n_{1,4}^* - n_{2,3}$. w_i, $i \in [1:4]$, are distributed as $CN(0, \frac{2\varepsilon}{\rho})$.

Let $rg_1 = r_1/\sqrt{\delta_{1,2}}$, $rg_2 = r_2/\sqrt{\delta_{2,2}}$, $rg_3 = r_3/\sqrt{\delta_{2,1}}$ and $rg_4 = r_4/\sqrt{\delta_{2,2}}$. Based on the encoding of rotated Golden codewords in (11.1) and (11.2), the linearly combined signals are expressed as:

$$rx_1 = x_1 + \frac{\beta_1}{\sqrt{\delta_{1,2}}}w_1 + \frac{\beta_2}{\sqrt{\delta_{2,2}}}w_2, \quad (18.1)$$
$$rx_2 = x_2 + \frac{\beta_2}{\sqrt{\delta_{1,2}}}w_1 - \frac{\beta_1}{\sqrt{\delta_{2,2}}}w_2, \quad (18.2)$$
$$rx_3 = x_3 + \frac{\beta_1}{\sqrt{\delta_{1,2}}}w_3 + \frac{\beta_2}{\sqrt{\delta_{2,2}}}w_4, \quad (18.3)$$
$$rx_4 = x_4 + \frac{\beta_2}{\sqrt{\delta_{1,2}}}w_3 - \frac{\beta_1}{\sqrt{\delta_{2,2}}}w_4, \quad (18.4)$$

where $rx_1 = \beta_1 rg_1 + \beta_2 rg_2$, $rx_2 = \beta_2 rg_1 - \beta_1 rg_2$, $rx_3 = \beta_1 rg_3 + \beta_2 rg_4$ and $rx_4 = \beta_2 rg_3 - \beta_1 rg_4$. In the derivation of (18.1) to (18.4), we apply $\beta_1^2 + \beta_2^2 = 1$.

Finally, the estimation of the transmitted symbols are given by:

$$\hat{x}_i = \mathcal{D}(rx_i). \quad (19)$$

B. IN-PHASE AND QUADRATURE MAXIMUM LIKELIHOOD DETECTION

The above LCD algorithm also has very low detection complexity. However the above detection scheme cannot achieve full diversity from $\delta_{1,2}$ and $\delta_{2,2}$ in the RGC-STLC system. In this subsection, we propose a detection scheme, IQML to achieve full diversity from $\delta_{1,2}$ and $\delta_{2,2}$.

Since β_1 and β_2 are real, (11.1) and (11.2) can be rewritten as:

$$g_{2i-1} = (\beta_1 x_{2i-1}^I + \beta_2 x_{2i}^I) + j(\beta_1 x_{2i-1}^Q + \beta_2 x_{2i}^Q), \quad (20.1)$$
$$g_{2i} = (\beta_2 x_{2i-1}^I - \beta_1 x_{2i}^I) + j(\beta_2 x_{2i-1}^Q - \beta_1 x_{2i}^Q). \quad (20.2)$$

Let $g_i = g_i^I + jg_i^Q$, $x_i = x_i^I + jx_i^Q$, $i \in [1:4]$. (20.1) and (20.2) can be further rewritten as:

$$g_{2i-1}^p = \beta_1 x_{2i-1}^p + \beta_2 x_{2i}^p, \quad (21.1)$$
$$g_{2i}^p = \beta_2 x_{2i-1}^p - \beta_1 x_{2i}^p, \quad (21.2)$$

where $p \in [I, Q]$.

Since only squared MQAM is taken into account in this paper, we have $K = \sqrt{M}$. Let Ω_K^p be the signal set of x_i^p and Ω_M^p be the signal set of g_i^p. The cardinalities of Ω_K^p and Ω_M^p are K and M, respectively. Then in (21.1) and (21.2), $x_i^p \in \Omega_K^p$ and $g_i^p \in \Omega_M^p$.

The linearly combined signals in (17.1) to (17.4) can be rewritten as:

$$r_1^p = \sqrt{\delta_{1,2}}(\beta_1 x_1^p + \beta_2 x_2^p) + w_1^p, \quad (22.1)$$
$$r_2^p = \sqrt{\delta_{2,2}}(\beta_2 x_1^p - \beta_1 x_2^p) + w_2^p, \quad (22.2)$$
$$r_3^p = \sqrt{\delta_{1,2}}(\beta_1 x_3^p + \beta_2 x_4^p) + w_3^p, \quad (22.3)$$
$$r_4^p = \sqrt{\delta_{2,2}}(\beta_2 x_3^p - \beta_1 x_4^p) + w_4^p. \quad (22.4)$$

The IQML detection of the transmitted x_{2i-1} and x_{2i} can be expressed as:

$$\left[\hat{x}_{2i-1}\ \hat{x}_{2i}\right] = \min_{x_{2i-1}^p, x_{2i}^p \in \Omega_K^p} (d_{2i-1}^p + d_{2i}^p), \quad (23)$$

where

$$d_{2i-1}^p = |r_{2i-1}^p - \sqrt{\delta_{1,2}}(\beta_1 x_{2i-1}^p + \beta_2 x_{2i}^p)|^2, \quad (24.1)$$
$$d_{2i}^p = |r_{2i}^p - \sqrt{\delta_{2,2}}(\beta_2 x_{2i-1}^p - \beta_1 x_{2i}^p)|^2. \quad (24.2)$$

The above IQML detection does achieve full diversity from $\delta_{1,2}$ and $\delta_{2,2}$, however the detection complexity in terms of two absolute operations is proportional to $\mathcal{O}(M)$. In order to meet the aim of STLC, low complexity detection, we propose the RC-IQML detection in the following subsections.

C. THRESHOLD BASED SIGNAL DETECTION

Both g^p_{2i-1} and g^p_{2i} in (11.1) and (11.2) can be regarded as a pair of superposition encodings in NOMA systems. The TSD has been proposed to detect the transmitted symbols [15]. In this subsection, we apply the TSD to detect x_{2i-1} and x_{2i}. Let $i = 1$, then (11.1) and (11.2) can be rewritten as:

$$g^p_1 = \beta_1 x^p_1 + \beta_2 x^p_2, \quad (25.1)$$
$$g^p_2 = \beta_2 x^p_1 - \beta_1 x^p_2, \quad (25.2)$$

where $x^p_i \in \Omega^p_K$ and $g^p_i \in \Omega^p_M$.
Given $x^p_i \in \Omega^p_K, i \in [1 : 2]$, based on either g^p_1 in (25.1) or g^p_2 in (25.2), Ω^p_M can be constructed. Suppose $\Omega^p_K = \Omega^p_4 = [-3, -1, 1, 3]$ then we have $\Omega^p_M = \Omega^p_{16}$ =[−4.129, −3.078, −2.428, −2.026, −1.376, −0.975, −0.727, −0.325, 0.325, 0.727, 0.975, 1.376, 2.026, 2.428, 3.078, 4.129]. For convenient discussion, we let $z^1_x = -3, \cdots, z^4_x = 3$ in the signal set Ω^p_4 and $z^1_g = -4.129, \cdots, z^{16}_g = 4.129$ in the signal set Ω^p_{16}. In Ω^p_{16}, it is easily seen that $z^{k_1}_g \neq z^{k_2}_g$ for $k_1 \neq k_2$.

Suppose z^k_g is transmitted over an AWGN channel, which is given by:

$$y = z^k_g + n, \quad (26)$$

where n is distributed as $CN(0, \sigma^2)$.

At the receiver, the transmitted symbol in (26) can be estimated by using the TSD. The thresholds play a key role in the TSD. Let t^k_{th}, $k \in [1 : M - 1]$, represent these thresholds. These thresholds t^k_{th} are easily derived as:

$$t^k_{th} = z^k_g + \frac{z^{k+1}_g - z^k_g}{2}. \quad (27)$$

For convenient discussion, we set $t^0_{th} = -\infty$ and $t^M_{th} = +\infty$. Now we apply the TSD in (27) to estimate the transmitted symbol. Suppose $t^{k_1-1}_{th} < y < t^{k_1}_{th}$. Then the TSD estimates the transmitted signal as $z^{k_1}_g$. As discussed in [15], the superposition coding of (25.1) and (25.2) can be alternatively expressed as encoding functions f_1 and f_2. Given $x^p_i = z^{k_i}_x$, $i \in [1 : 2]$ then $g^p_1 = z^{s_1}_g$ and $g^p_2 = z^{s_2}_g$ are expressed as:

$$z^{s_1}_g = f_1\left(z^{k_1}_1, z^{k_2}_2\right), \quad (28.1)$$
$$z^{s_2}_g = f_2\left(z^{k_1}_1, z^{k_2}_2\right), \quad (28.2)$$

where $z^{s_1}_g, z^{s_2}_g \in \Omega^p_M$.

The signal detection at the receiver can be regarded as two inverse functions f_1^{-1} and f_2^{-1}. Suppose that the g^p_1 and g^p_2 are estimated as $\hat{g}^p_1 = z^{s_1}_g$ and $\hat{g}^p_2 = z^{s_2}_g$ at the receiver, then the estimation of x^p_i, $i \in [1 : 2]$ can be expressed as two inverse functions f_1^{-1} and f_2^{-1}, which are given by:

$$\left[\hat{x}^{g1}_1, \hat{x}^{g1}_2\right] = f_1^{-1}\left(z^{s_1}_g\right), \quad (29.1)$$
$$\left[\hat{x}^{g2}_1, \hat{x}^{g2}_2\right] = f_2^{-1}\left(z^{s_2}_g\right). \quad (29.2)$$

D. REDUCED COMPLEXITY IQML DETECTION

In this subsection, we propose a RC-IQML algorithm. From the previous subsection, it is easily seen that the transmitted x^p_i can be estimated based on either f_1^{-1} in (29.1) or f_2^{-1} in (29.2). If $\hat{x}^{g1}_i = \hat{x}^{g2}_i$, the probability of transmitting $x^p_i = \hat{x}^{g2}_i$ is very large. If both $\hat{x}^{g1}_1 = \hat{x}^{g2}_1$ and $\hat{x}^{g1}_2 = \hat{x}^{g2}_2$, we directly output the estimation of the transmitted x^p_1 as $\hat{x}^{g2}_1$ and x^p_2 as $\hat{x}^{g2}_2$. If $\hat{x}^{g1}_i \neq \hat{x}^{g2}_i$, we perform IQML to estimate the transmitted x^p_i. The implementation of the RC-IQML is shown in Algorithm 1. In Algorithm 1, we only detect the transmitted symbols x_1 and x_2 by use of the RC-IQML. Similarly, we can also detect the transmitted symbols x_3 and x_4 by use of the RC-IQML.

Algorithm 1: Implementation of the RC-IQML

Input:
(1): r^p_1 in (22.1) and r^p_2 in (22.2);
(2): β_1 and β_2;
(3): Estimation of transmitted x_i: $\hat{x}^{g1}_i$, $\hat{x}^{g2}_i$ and $\hat{x}_i$, $i \in [1 : 2]$, and Ω^p_K.
Output: Finally estimated $\hat{x}_i$, $i \in [1 : 2]$.

Clear Ω^1_x, Ω^2_x;
for $i \leftarrow 1$ **to** 2 **do**
  if $\hat{x}^{g1}_i = \hat{x}^{g2}_i$ **then**
    $\Omega^i_x = \hat{x}^{t_1}_i$;
  else
    $\Omega^i_x = \Omega^p_K$;
  end if
end for
if $\hat{x}^{g1}_1 = \hat{x}^{g2}_1$ AND $\hat{x}^{g1}_2 = \hat{x}^{g2}_2$ **then**
  $\hat{x}_1 = \hat{x}^{g1}_1$;
  $\hat{x}_2 = \hat{x}^{g1}_2$.
else
  $d^p_1 = |r^p_1 - \sqrt{\delta_{1,2}}(\beta_1 x^p_1 + \beta_2 x^p_2)|^2$;
  $d^p_2 = |r^p_2 - \sqrt{\delta_{2,2}}(\beta_2 x^p_1 - \beta_1 x^p_2)|^2$;
  $\left[\hat{x}_1\ \hat{x}_2\right] = \min_{x^p_1 \in \Omega^1_x, x^p_2 \in \Omega^2_x}(d^p_1 + d^p_2)$.
end if

IV. UNION BOUND ON THE AVERAGE SYMBOL ERROR PROBABILITY OF THE 1 × 4 AND 2 × 4 RGC-STLC SYSTEMS

In this section, we focus on the derivation of the union bound on the ASEP of the 1 × 4 and 2 × 4 RGC-STLC systems. We will derive the ASEP union bound for 1 × 4 and 2 × 4 RGC-STLC systems in Subsection IV.A and Subsection IV.B, separately.

A. THE ASEP UNION BOUND OF THE 1 × 4 RGC-STLC SYSTEM

The derivation of the ASEP union bound for the 1 × 4 RGC-STLC system is based on one pair of received signals, either (17.1) and (17.2) or (17.3) and (17.4). In this section, (17.1) and (17.2) will be used to derive the ASEP union bound. Both (17.1) and (17.2) are rewritten as:

$$r_1 = \sqrt{\delta_{1,2}}(\beta_1 x_1 + \beta_2 x_2) + w_1, \tag{30.1}$$

$$r_2 = \sqrt{\delta_{2,2}}(\beta_2 x_1 - \beta_1 x_2) + w_2, \tag{30.2}$$

where w_i, $i \in [1:4]$, are distributed as $CN(0, \frac{2\varepsilon}{\rho})$.

Using (30.1) and (30.2), either x_1 or x_2 may be taken into account in the following derivation. We use only x_1 to derive the union bound of the symbol error probability. Let $p_x(e)$ be the ASEP of x_1 during the transmission. Since there are M signal values of x_1, $p_x(e)$ may be expressed as:

$$p_x(e) = \sum_{k_1=1}^{M} p(e|x_1 = z_x^{k_1})p(x_1 = z_x^{k_1}), \tag{31}$$

where $p(e|x_1 = z_x^{k_1})$ is the conditional error probability when $x_1 = z_x^{k_1}$ is transmitted. $p(x_1 = z_x^{k_1})$ is the probability for $x_1 = z_x^{k_1}$.

If $x_1 = z_x^k$ were directly transmitted, $p(e|x_1 = z_x^k)$ would be easily derived. However the proposed RGC-STLC system transmits either $g_1 = \beta_1 x_1 + \beta_2 x_2$ or $g_2 = \beta_2 x_1 - \beta_1 x_2$, not x_1. At the receiver only linearly combined received signals $\sqrt{\delta_{1,2}}g_1 + w_1$ and $\sqrt{\delta_{2,2}}g_2 + w_2$ are known, and $x_1 + n_1$ is not known. Thus, we need to derive the symbol error performance of the proposed RGC-STLC system based on the linearly combined received signals $\sqrt{\delta_{1,2}}g_1 + w_1$ and $\sqrt{\delta_{2,2}}g_2 + w_2$.

Based on the encoding of the rotated Golden codewords in (11.1) and (11.2), transmitting $x_1 = z_x^{k_1}$ is equivalent to transmitting one pair of M rotated Golden codewords (g_1, g_2), where $g_1 = \beta_1 x_1 + \beta_2 x_2$, $g_2 = \beta_2 x_1 - \beta_1 x_2$, $x_1 = Z_{k_1}$, $x_2 \in \Omega$. Then $p(e|x_1 = z_x^{k_1})$ in (31), can be expressed as:

$$p(e|x_1 = z_x^{k_1}) = \sum_{k_2=1}^{M} p(e|g_1, g_2)p(x_2 = z_x^{k_2}). \tag{32}$$

Let $\boldsymbol{g} = (g_1, g_2)$ and $\hat{\boldsymbol{g}} = (\hat{g}_1, \hat{g}_2)$. The pairwise error probability (PEP) $P(\boldsymbol{g} \to \hat{\boldsymbol{g}})$ is defined as $\boldsymbol{g}$ which is detected as $\hat{\boldsymbol{g}}$ at the receiver. Then the $p(e|g_1, g_2)$ in (32) is the PEP $P(\boldsymbol{g} \to \hat{\boldsymbol{g}})$. For $p(x_1 = z_x^{k_1}) = \frac{1}{M}$ and $p(x_2 = z_x^{k_2}) = \frac{1}{M}$, $p_x(e)$ may be further expressed as:

$$p_x(e) = \frac{1}{M^2} \sum_{k_1=1}^{M} \sum_{k_2=1}^{M} P(\boldsymbol{g} \to \hat{\boldsymbol{g}}). \tag{33}$$

The conditional PEP $P(\boldsymbol{g} \to \hat{\boldsymbol{g}}|\boldsymbol{h}_1, \boldsymbol{h}_2)$ is given by:

$$P(\boldsymbol{g} \to \hat{\boldsymbol{g}}|\boldsymbol{h}_1, \boldsymbol{h}_2) = P\,(\,A \geq B\,)\,, \tag{34}$$

where
$A = |r_1 - \sqrt{\delta_{1,2}}g_1|^2 + |r_2 - \sqrt{\delta_{2,2}}g_2|^2$;
$B = |r_1 - \sqrt{\delta_{1,2}}\hat{g}_1|^2 + |r_2 - \sqrt{\delta_{2,2}}\hat{g}_2|^2$.
Substituting (30.1) and (30.2) into A and B we further have:
$A = |w_1|^2 + |w_2|^2$;
$B = |\sqrt{\delta_{1,2}}d_1 + w_1|^2 + |\sqrt{\delta_{2,2}}d_2 + w_2|^2$.
where $d_1 = g_1 - \hat{g}_1$ and $d_2 = g_2 - \hat{g}_2$.
$P\,(\,A \geq B\,)$ in (34) is further derived as:

$$P\,(\,A \geq B\,) = P\,(C \geq D)\,, \tag{35}$$

where
$C = Re\{\sqrt{\delta_{1,2}}d_1^* w_1\} + Re\{\sqrt{\delta_{2,2}}d_2^* w_2\}$;
$D = \frac{1}{2}\{\delta_{1,2}|d_1|^2 + \delta_{2,2}|d_2|^2\}$.
C is the equivalent AWGN component distributed as $N(0, \{\delta_{1,2}|d_1|^2 + \delta_{2,2}|d_2|^2\}\frac{\varepsilon}{\rho})$.
Finally, we have:

$$\begin{aligned} P(\boldsymbol{g} \to \hat{\boldsymbol{g}}|\boldsymbol{h}_1, \boldsymbol{h}_2) &= P\,(\,A \geq B\,) \\ &= Q\left(\sqrt{\frac{\rho}{4\varepsilon}\{\delta_{1,2}|d_1|^2 + \delta_{2,2}|d_2|^2\}}\right). \end{aligned} \tag{36}$$

Define the instantaneous SNR at the receive antenna l during time slot i as $\gamma_{i,l} = \{|h_{i,l}|^2\}\rho$. Since $h_{i,l}$ are i.i.d complex Gaussian RVs distributed as $CN(0, 1)$ we have $\bar{\gamma}_{i,l} = \rho$. Let $\gamma_i = \gamma_{i,1} + \gamma_{i,2}$. Then the PDF of γ_i is given by [15]:

$$f_{\gamma_i}(\gamma_i) = \frac{1}{\bar{\gamma}_i^2}\gamma_i e^{-\frac{\gamma_i}{\bar{\gamma}_i}}, \tag{37}$$

where $\bar{\gamma}_i = \bar{\gamma}_{i,l} = \rho$.
Finally, the PEP $P(\boldsymbol{g} \to \hat{\boldsymbol{g}})$ is given by:

$$\begin{aligned} &P(\boldsymbol{g} \to \hat{\boldsymbol{g}}) \\ &= \int_0^{+\infty}\int_0^{+\infty} P(\boldsymbol{g} \to \hat{\boldsymbol{g}}|\boldsymbol{h}_1, \boldsymbol{h}_2) f_{\gamma_1}(\gamma_1) f_{\gamma_2}(\gamma_2) d\gamma_1 d\gamma_2. \end{aligned} \tag{38}$$

In order to derive a closed-form expression for PEP $P(\boldsymbol{g} \to \hat{\boldsymbol{g}})$, we need to approximate the Gaussian Q-function in (38) for integration. Taking into account conciseness of the ASEP expressions and approximation accuracy, we approximate $Q(\cdot)$ in (36) using the trapezoidal rule as:

$$Q(x) \approx \frac{1}{2c}\left[\frac{1}{2}e^{-\frac{x^2}{2}} + \sum_{k=1}^{c-1} e^{-\frac{x^2}{2s_k}}\right], \tag{39}$$

where $s_k = \sin^2(k\pi/(2c))$ and c is the number of partitions for the integration in the Q-function.
Substituting (37) and (39) into (38), $P(\boldsymbol{g} \to \hat{\boldsymbol{g}})$ may be derived as:

$$\begin{aligned} &P(\boldsymbol{g} \to \hat{\boldsymbol{g}}) \\ &= \frac{1}{4c}\left[\prod_{k=1}^{2}\left(\frac{1}{1+\frac{\rho d_k}{8\varepsilon}}\right)^2 + 2\sum_{k=1}^{c-1}\prod_{l=1}^{2}\left(\frac{1}{1+\frac{\rho d_k}{8\varepsilon s_k}}\right)^2\right]. \end{aligned} \tag{40}$$

B. THE ASEP UNION BOUND OF THE 2 × 4 RGC-STLC SYSTEM

(30.1) and (30.2) are the equivalent received signal models for the 1×4 RGC-STLC ASEP union bound analysis. Since the signal detection schemes of the 2×4 RGC-STLC systems are the same as the ones of the 1×4 RGC-STLC systems except for different effective channel gain based on (30.1) and (30.2), the equivalent received signal models for the 2×4 RGC-STLC ASEP union bound analysis is given by:

$$r_1 = \sqrt{\delta_{1,4}}(\beta_1 x_1 + \beta_2 x_2) + w_1, \tag{41.1}$$

$$r_2 = \sqrt{\delta_{2,4}}(\beta_2 x_1 - \beta_1 x_2) + w_2. \tag{41.2}$$

In (30.1) and (30.2), $\delta_{i,2} = |h_{1,2i-1}|^2 + |h_{1,2i}|^2$, while in (41.1) and (41.2), $\delta_{i,4} = \sum_{n_t=1}^{2} \sum_{n_r=(i-1)\times 2+1}^{(i-1)\times 2+2} |h_{n_t,n_r}|^2$. It is easily seen that the diversity order of $\delta_{i,2}$ is 2, while the diversity order of $\delta_{i,4}$ is 4. Similar to the error performance analysis for the 1×4 RGC-STLC system, (33) is also the union bound on the ASEP for the 2×4 RGC-STLC system. But the $P(\boldsymbol{g} \to \hat{\boldsymbol{g}})$ for 2×4 RGC-STLC system is given by:

$$P(\boldsymbol{g} \to \hat{\boldsymbol{g}}) = \frac{1}{4c}\Big[\prod_{k=1}^{2}\Big(\frac{1}{1+\frac{\rho d_k}{8\varepsilon}}\Big)^4 + 2\sum_{k=1}^{c-1}\prod_{l=1}^{2}\Big(\frac{1}{1+\frac{\rho d_k}{8\varepsilon s_k}}\Big)^4\Big]. \quad (42)$$

where the definition of ρ is the same as earlier.

V. SIMULATION RESULTS

In this section, we present the simulation results for the proposed 1×4 and 2×4 RGC-STLC systems for 16QAM and 64QAM. As discussed in Section II system model, it is assumed that the CSI is fully known at the transmitter. It is also assumed that the effective channel gains are known at the receiver. Rayleigh frequency-flat fading channels with AWGN, as described in (13.1) and (13.2) are considered in all simulations. For comparison, we also simulate the conventional 1×2 and 2×2 STLC systems.

In all figures, the legend, "$N_t \times 2$ STLC MQAM stands for conventional $N_t \times 2$ STLC MQAM; the legend, "$N_t \times 4$ RGC-STLC MQAM", stands for the proposed $N_t \times 4$ RGC-STLC MQAM.

In this section, we firstly discuss the detection complexity of the IQML and RC-IQML, then analyze the error performance of the proposed RGC-STLC systems, and finally compare the error performance between the proposed RGC-STLC system and the conventional STLC system.

A. DETECTION COMPLEXITY ANALYSIS

As discussed in Section III.C, the IQML achieves optimal error performance with detection complexity M, where detection complexity is in terms of the number of absolute operations per symbol in either (24.1) or (24.2). However, it is difficult to quantify the number of absolute operations in the RC-IQML detection. We analyze the detection complexity for the RC-IQML detection through simulations.

FIGUREs 1 and 2 show the simulated detection complexity of the 1×4 and 2×4 RGC-STLC 16QAM and 64QAM with IQML and RC-IQML detection schemes, respectively. For the proposed 1×4 and 2×4 RGC-STLC MQAM, it is observed from FIGUREs 1 and 2 that:

1) The detection complexity of the IQML scheme is M, which is constant;
2) The detection complexity of the RC-IQML scheme decreases as the SNR increases.
3) The detection complexity of the RC-IQML for the RGC-STLC decreases as the number of transmit antennas increases.

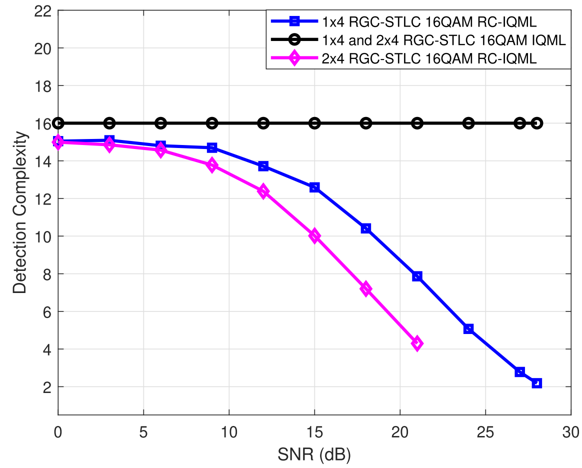

FIGURE 1. Detection complexity of RGC-STLC 16QAM vs SNR.

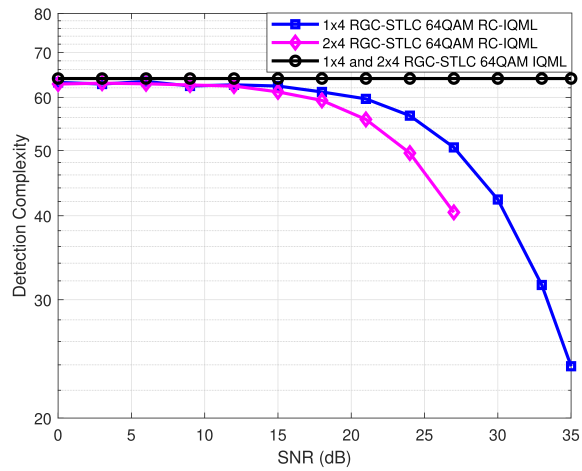

FIGURE 2. Detection complexity of RGC-STLC 64QAM vs SNR.

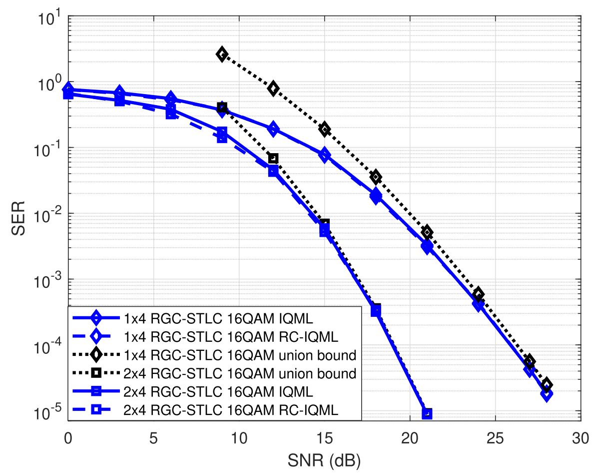

FIGURE 3. Error performance of the RGC-STLC 16QAM vs SNR.

B. ERROR PERFORMANCE ANALYSIS

In this subsection, we analyze the symbol error performance of the proposed RGC-STLC systems. FIGUREs 3 and 4 show

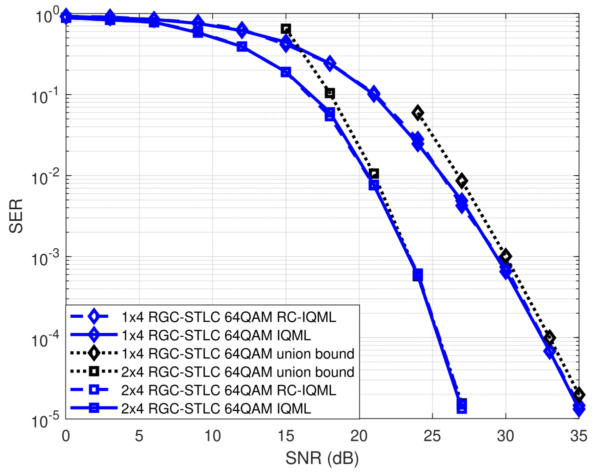

FIGURE 4. Error performance of the RGC-STLC 64QAM vs SNR.

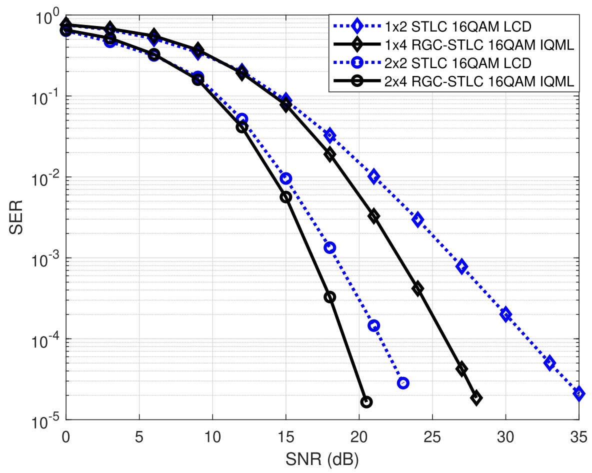

FIGURE 5. Error performance comparison between RGC-STLC and STLC 16QAM.

the error performance of the $N_t \times 4$ RGC-STLC with 16QAM and 64QAM, respectively. The ASEP union bounds of the proposed $N_t \times 4$ RGC-STLC MQAM in (33), are also shown in FIGUREs 3 and 4, respectively. From FIGUREs 3 and 4, it is observed that:

1) Compared to the 1×4 RGC-STLC MQAM systems, 2×4 RGC-STLC MQAM systems achieves at least 7 dB at the symbol error rate (SER) of 2×10^{-5};
2) Until the SER of 2×10^{-5}, the RC-IQML achieves the error performance of IQML;
3) The ASEP union bounds match the simulated SER at high SNRs. Typically, the ASEP union bounds of 2×4 RGC-STLC MQAM systems overlap the simulated SER at high SNRs.

C. RGC-STLC MQAM VS CONVENTIONAL STLC MQAM

In this subsection, we compare the symbol error performance between the proposed $N_t \times 4$ RGC-STLC MQAM systems and the conventional $N_t \times 2$ STLC MQAM systems. FIGUREs 5 and 6 show the error performance comparison

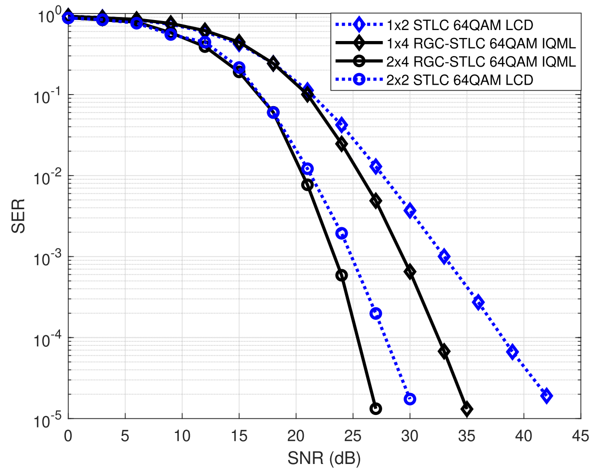

FIGURE 6. Error performance comparison between RGC-STLC and STLC 64QAM.

between the proposed RGC-STLC and the conventional STLC with 16QAM and 64QAM, respectively. In the following discussion, all comparisons are at the SER of 2×10^{-5}. From FIGUREs 5 and 6 it is observed that:

1) Compared to the 1×2 STLC MQAM systems, 1×4 RGC-STLC MQAM systems achieves at least 7 dB. However, 2×4 RGC-STLC MQAM systems achieves only around 3 dB compared to 2×2 STLC MQAM systems;
2) Simulation results in FIGUREs 5 and 6 validate that both 2×2 STLC MQAM systems and 1×4 RGC-STLC MQAM systems achieve the same diversity order.

VI. CONCLUSION

In order to further improve the error performance of the STLC system, in this paper, the RGC-STLC system was proposed. The configuration of two receive antennas in the conventional STLC system was extended to the configuration of four receive antennas in the proposed RGC-STLC system. A reduced complexity detection scheme, RC-IQML, was proposed to detect the transmitted symbols in the RGC-STLC systems. In order to validate our simulations, the ASEP union bounds of the proposed RGC-STLC systems were derived. Simulation results and union bounds showed that 1×4 RGC-STLC MQAM systems achieves at least 7 dB compared to 1×2 STLC MQAM systems. However, 2×4 RGC-STLC MQAM systems only achieves around 3 dB compared to 2×2 STLC MQAM systems.

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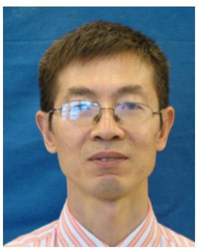

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