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An Inhomogeneous Grid-Based Evolutionary Algorithm for Many-Objective Optimization

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ABSTRACT Today, many-objective optimization problems have attracted widespread attention. There are significant advantages of the grid-based algorithm in solving multi-objective problems. Grid-based algorithm could offer a transformation of objectives and further distinguish the non-dominated solutions. However, the advantages of grid have not been fully exploited. For example, the traditional homogeneous grid divisions can't sufficiently reveal the similarity of adjacent solutions. And overemphasizing the selection pressure may cause the diversity decline of grid. To exploit the potentialities of grid, an inhomogeneous grid-based evolutionary algorithm (named IGEA) is proposed. IGEA applies a dynamic inhomogeneous grid division approach and redefining the coordinate assignment of individuals, which makes the dominance relationship more obvious. IGEA also applied the shift-based density estimation (SDE) strategy in discriminating the non-dominated solutions in grid coordinate. SDE can provide a good balance of convergence and diversity. The IGEA compares with several state-of-the-art evolutionary algorithms against the regular and irregular many-objective optimization problems. The experimental results demonstrate that IGEA is very competitive against the peer algorithms in terms of providing a good balance between convergence and diversity.

INDEX TERMS Many-objective optimization, grid-based, inhomogeneous grid, shift-based density estimation.

I. INTRODUCTION

A many-objective optimization problems (MaOPs) could be defined as follows:

$$\begin{aligned} &\text{minimize } \boldsymbol{F}(\boldsymbol{x}) = F_1(\boldsymbol{x}), F_2(\boldsymbol{x}), \ldots, F_M(\boldsymbol{x}) \\ &\text{subject to } \boldsymbol{x} \in \Omega \end{aligned} \tag{1}$$

where $\vec{x}$ denotes a solution in the feasible solution space Ω, $F_i(i = 1, 2, \ldots, M)$ is ith objective, where $M \geq 3$.

The individual $\vec{x}$ dominates individual $\vec{y}$ is expressed as follows:

$$\begin{aligned} &\forall i \in (1, 2, \ldots, M) : F_i(x) \leq F_i(y) \\ &\exists j \in (1, 2, \ldots, M) : F_j(x) < F_j(y) \end{aligned} \tag{2}$$

Individual $\vec{x}$ is called non-dominated, individual $\vec{y}$ is called dominated.

For MaOPs, the set of all obtained Pareto solutions is called Pareto Set (PS) and the set of all Pareto optimal objective vectors is called Pareto Front (PF). Any algorithms should optimize MaOPs with two limits: converging towards true PF as close as possible and distributing PS as widely as possible [1], [2].

It is difficult to solve MaOPs by the traditional gradient-based mathematical method. Evolutionary optimization methods give a way to handle MaOPs. Nowadays, many-objective evolutionary algorithms (MOEAs) are proposed to handle MaOPs [3].

At present, intelligent genetic optimization algorithms, as a class of heuristic search algorithms, have been successfully applied to the field of multi-objective optimization, and some popular research directions have emerged, such as evolutionary multi-objective optimization, while the research on the application of multi-objective intelligent optimization algorithms in power systems, manufacturing systems and control systems has also made great progress.

MOEAs can be generally into three categories: decomposition-based MOEA, indicator-based MOEA and domination-based MOEA. Each type of MOEAs has

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its own advantages and disadvantages in dealing with different MaOPs.

Decomposition-based MOEA includes MOEA/D [4] and its improved versions [5], [6], [7], [8]. The decomposition-based MOEA decomposes a multi-objective problem into a set of single-objective problems. Every individual assigning to a single-objective problem benefits from the progress of its neighbors. There are three commonly aggregation functions in decomposition strategies: weighted sum approach, Tchebycheff approach and penalty-based boundary intersection approach. Weighted sum approach can well deal with the minimization problem when the real Pareto front surface is convex. Tchebycheff approach can deal with the problem with convex or concave Pareto surface, but it deals with the discontinuous problems non-ideally. Penalty-based boundary intersection approach has a great advantage in dealing with high-dimensional objective problems. It can adjust the balance between diversity and convergence by controlling parameters, which may be also a potential disadvantage for solving MaOPs without any prior knowledge.

Indicator-based MOEAs use one or some indicators [9], [10], [11], [12] to distinguish the acceptable solutions from the current population. The indicator is a quantitative tool to evaluate the performance of different solutions. It can be divided into three classes: convergence indicator, distribution indicator, and comprehensive indicator. This kind of MOEAs employs indicators to guide the search process and the solution selection process. The famous indicator-based MOEAs include IBEA [13], indicator-based CMOEA [14], MaOEA-IBP [15], H-RVEA [16], IMIA [17], CDG-MOEA [18]. Theoretically, any metric can be integrated into MOEAs in some specific ways. However, when some evaluation indicators (SP, IGD, GD) are integrated into MOEAs, the complication of MOEAs may emerge and the operating efficiency of MOEA may decline.

Domination-based MOEAs includes non-dominated ranking algorithm (NSGA-II) [19], (NSGA-III) [20], Pareto dominance [21], ε-domination [1], SPEA2 [22] and so on. Domination-based MOEAs usually use the dominance relationship to assign a fitness to individuals. Individuals are sorted by fitness, and the best individuals are selected through the elite retention strategy. Domination-based MOEAs has many advantages: First, the computation complexity of non-dominated sorting is low. Second, the domination method does not have many additional parameters that need to be set. Third, domination method is flexible and can be combined with any diversity or convergence indicators.

The dominance-based algorithms have to face some difficulties [23], [24]: (1) Domination-based MOEAs performs poorly in dealing with MaOPs with some many objectives. The non-dominated relationship occupies among most of the individuals, which makes the elite retention mechanism difficult. (2) In a high-dimensional space, the diversity indicators and identifying neighbors cost lots of computations. To make the computations faster, any simplification in diversity estimate may lead to a poor distribution of PS. (3) When dealing with high-dimension problems, the huge difference between each two non-dominated parents cause that the offspring solutions produced by recombination operator are far from their parents. (4) Inappropriate density estimation deteriorates the search performance. Some well-converged non-dominated individuals can't be distinguished from the inferior offspring with poor convergence to PF.

There are two kinds of attempts to solve these problems:

First, to increase the Pareto selection pressure, the Pareto dominance relationship should be changed or enhanced [25]. The strengthened dominance relation (SDR) and controlled SDR (CSDR) are representative of this way [26], [27], [28]. One way is to partition objective space. In the space partitioning selection and angle-based truncation (SPSAT) [29], the normalized objective space is divided into many subspaces. One way is to reduce dimensionality [30], [31]. Many-objective space can be changed into one or two-objective space for clarifying the Pareto dominance relationship. Pareto corner search evolutionary algorithm (PCSEA) [32] iteratively eliminates the objectives according to the principal components analysis. The originally obtained non-dominated solutions can be compared with dimensionality reduction process. Another way is to build grid to deal with MaOPs. Grid-based algorithms [33] can maintain wide distribution of solutions towards PF under the appropriate selection pressure.

Second, the indicator method is to design a metric to estimate either or both of the convergence and diversity of individuals. e.g., GD/MaOEA [34], DIR [35] metric and Pareto dominance-based MOEA (PDMOEA) [36]. Shift-based density estimation (SDE) [23] could estimate both diversity and convergence of individuals in the population. The main idea of SDE is to give the preference of density estimators for individuals in sparse regions. SDE attempts to drive individuals with poor convergence towards crowded areas through coordinate transformation.

Although the existing methods can solve the above problems, there is still some space of improvements. For example, the advantages of the grid are not fully exploited. Traditional grid divisions are homogeneous and can't sufficiently reveal the similarity of adjacent solutions. It is possible to take advantage of the characteristics of the population to divide the grid inhomogeneously. Some novel diversity and convergence conservation mechanisms may be also beneficial for the grid-base selection.

This paper proposes a novel dynamical grid-based evolutionary algorithm (named IGEA) to solve MaOPs. In IGEA, the grid is dynamically divided inhomogeneously by the clustering algorithm, and the coordinates of individuals is assigned by a normalization method. To maintain the convergence and diversity, IGEA also introduces the SDE strategy that can distinguish the poorly converged or crowded solutions in the population. To demonstrate the characteristics of IGEA for solving MaOPs, it has been compared extensively with four famous algorithms on a series of regular and irregular issues.

The remainder of this paper is organized as follows. Section II discusses the current mainstream grid-based algorithms and their shortcomings. Section III gives the improvement plan towards some problems of GrEA. Section IV shows some details of the proposed algorithm. Section V gives some details of the experimental design. The experimental results are given in Section VI. Finally, Section VII provides some summary of the algorithm and gives some suggestions for the future.

II. REVIEW OF GRID-BASED MOEAS

The grid strategy has been introduced in MOEAs for solving MaOPs [37]. The number of solutions sharing grid places is to determine a solution's crowding degree. When a non-dominated solution joins a full archive, it replaces the most crowded solution if its own congestion is low. Different from PESA [38] and PESA-II [21], the grid is viewed as a tool of storing convergence and diversity of individuals. According to the solution's position in grid, each individual is assigned with a rank level and a density value [39]. In [40], a dynamic grid adjustment strategy is presented, which makes the size of hyper-box change as needed. Also, the DIR [35] metric is used to assess the diversity of individuals in the grid.

Although traditional grid-based algorithms have good performance on MaOPs, they also have some drawbacks. The reasons of these drawbacks can be outlined as follows:

1) The objective space is inappropriately divided, and the number of non-dominated solutions grows geometrically as the number of objectives increases [41].
2) The traditional grid algorithm lacks an effective dominant evaluation method [33], because the existing indicators cannot discriminate individuals effectively.
3) Many grid-based algorithms are biased towards convergence or diversity, but it is a challenge to balance convergence and diversity [23].
4) Traditional homogeneous-grid-based algorithms have limitations in solving irregular multi-objective problems, especially discontinuous problems.

In recent years, several grid-based evolutionary algorithms have been designed to solve MaOPs. In [42], one single solution of the cell is used to create the non-dominated area in grid. Moreover, the method utilizes the smallest number of virtual solutions to determine whether a solution is a non-dominated solution. The selection pressure, diversity of solutions, time and memory consumption are taken into account. As the most famous grid-based MOEA, GrEA adopts homogeneous grid division and proposes a new punish strategy based on neighborhood and grid dominance relationship. GrEA can provide higher selection pressure and wide and uniform distribution of PS towards PF. GrDE [43] propose a novel grid-based differential evolution (DE) algorithm. The method of dividing grid is similar to GrEA, and GrDE also uses indicators of GrEA to select individuals for local mutation. In [44], a new mechanism for maintaining diversity is designed. The vision of grid-based crowding distance in decision space is brought in. In addition, the algorithm maintains diversity of solutions in not only decision spaces but also objective spaces. Penalty strategy [45], Pareto dominance and weighted sum [46], grid-based archiving [47] approaches are also used to deal with grid-based on many-objectives. These approaches can't make a significant improvement of grid-based multi-objective algorithms on overcoming the drawbacks mentioned before. Similar to GrEA, most of these strategies act on neighbors and lack a global evaluation for crowding distance and fitness.

In [47], the Pareto Archived Evolution Strategy employs an adaptive crowding mechanism to recursively divide the objective space into grid segments. Meantime, adaptive grid archiving (AGA) system is introduced in grid. The main concept is to prevent premature aggregation of dominant solutions in the optimization process. Such common optimization solutions usually lead to genetic drift and result in a final set of approximations gathering around these elite solutions. These non-elite solutions appear early in the optimization process, and they may contain potential information that would is helpful in searching undiscovered areas of the objective space later. In [18], the grid system is introduced in to the decomposition-based algorithm. When one objective is optimized, other objectives are regarded as its constraints. The constrained decomposition is to sort the objectives of each solution. This algorithm offers a well performance on multi-objective optimization but not many-objective optimization. In the above grid-based algorithms, the grid is still divided homogeneously, and the potential of the grid are not fully utilized.

In summary, although the grid-based methods would enhance the performance of MOEAs on solving MaOPs, some deficiencies still limit the performance, which is worth deeply researching on grid division methods.

III. MOTIVATION

GrEA uses the grid's potential to solve MaOPs. The solutions can be compared by three grid-based indicators: Grid Rank (GR), Grid Crowding Distance (GCD) and Grid Coordinate Point Distance (GCPD). However, there are some drawbacks in handling the multi-objective problems with grid. As shown in Figure 1, the GR of individual A in the grid is better than that of B. However, actually, B is closer to the Pareto Front than A. The reason is that the homogeneous grid can't reveal the dominance relationship or makes dominance relationship biased toward A.

The inhomogeneous grid division has more potential in solving the above problem. Each objective is divided independently by certain method, such as cluster algorithm [48], [49], [50]. The individuals with high similarity are located in the same cell, and individuals are assigned to coordinates by a normalization method. An illustration of the inhomogeneous grid division is given in Figure 2. Even though A and B locate in the same cell, their grid coordinates are not consistent. In comparison with the homogeneous grid division, the inhomogeneous grid division can offer a more obvious dominance relationship.

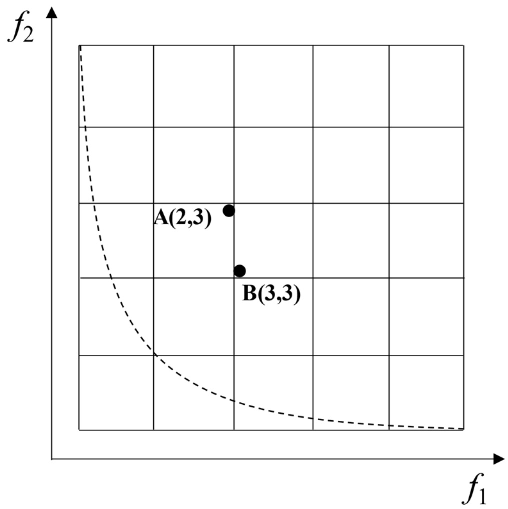

FIGURE 1. Disadvantages of GrEA.

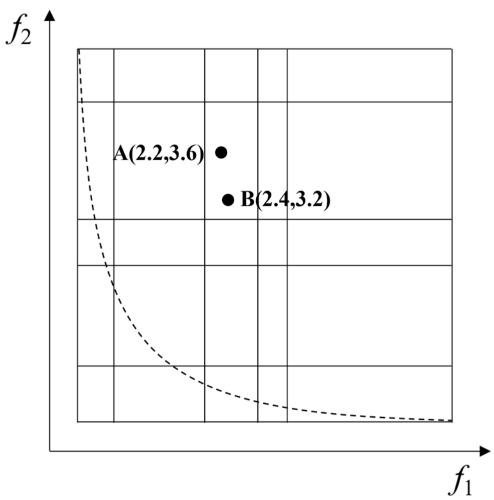

FIGURE 2. Distribution of similar individuals in the inhomogeneous grid.

With the help of grid, the convergence could reflect the evolutionary state of the solution. Grid coordinates consider not only the similarity between individuals but also the differences between them. As an illustration shown in Figure 3, the difference between A and B in the objective f_2 is greater than the difference in the objective f_1. Grid difference enables to provide higher selection pressures.

This paper proposes IGEA to solve MaOPs. The inhomogeneous grid division implemented by K-means clustering method could appropriately and simultaneously maintain the diversity and convergence. Offspring are generated based on the novel Grid Dominance, Grid Difference (GD) and Grid Crowding Distance (GCD). In the process of environment selection, the SDE method that covers both the distribution and convergence information of individuals is used to reflect the relative closeness of the individual to the PF.

Moreover, most grid-based multi- or many-objective optimization algorithms perform really well on regular problems but fail in dealing with irregular problems. Nevertheless,

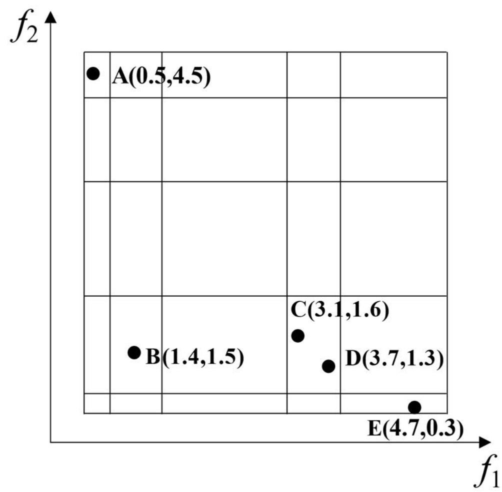

FIGURE 3. Illustration of individuals in two-objective space.

the proposed IGEA with inhomogeneous grid division own an outstanding performance on solving irregular problems, because the inhomogeneous grid division can make not only a qualitative comparison of the dominant relationship between the solutions but also a quantitative calculation of the differences among them.

The original design intentions of designing IGEA and GrEA is similar, but there still are some differences between them:

(1) The inhomogeneous grid division implemented by K-means clustering is one of the main differences between IGEA and GrEA.
(2) In GrEA, the normalization process used in calculating GCDP is only to distinguish the solutions in the same cell. In IGEA, the normalization process is to assign the coordinates to all solutions in different cell.
(3) In GrEA, the individuals are selected based on an adaptive penalty mechanism that is mainly related to neighbors. In IGEA, the individuals are selected based on the fitness calculation that is related to global individuals.

Here, the inhomogeneous grid is important to be able to distinguish individuals better. The SDE strategy provides an auxiliary function in environment selection, which prevents individuals from overcrowding on PF compared to the GrEA penalty strategy.

IV. THE PROPOSED ALGORITHM

The main steps of IGEA involves population initialization, inhomogeneous grid division and coordinate assignment, assignment of Grid Difference (GD) and Grid Crowding Distance (GCD) metrics, mating selection, and environment selection. The flow chart of the algorithm is in Figure 4. The detailed will be given in the following chapters.

A. MAIN FRAMEWORK

IGEA is similar to most common MOEAs, whose main steps include: population initialization, mating selection, and environment selection. The main framework of IGEA is

Algorithm 1 Algorithmic Framework of IGEA

Require: *P (parent population), N (archive size), i (Max number of iterations)*

1: $P \leftarrow Initialize(P)$
2: **While** *iterations* $\leq i$ **do**
3: *Inhomogeneous Grid Division(P)*
4: *GD/GCD _assignment(P)*
5: *P' ← Matting_selection(P)*
6: *P'' ← Variation(P')*
7: $P \leftarrow Environmental\_selection(N = P \cup P'')$
8: **end while**
9: **return** *P*

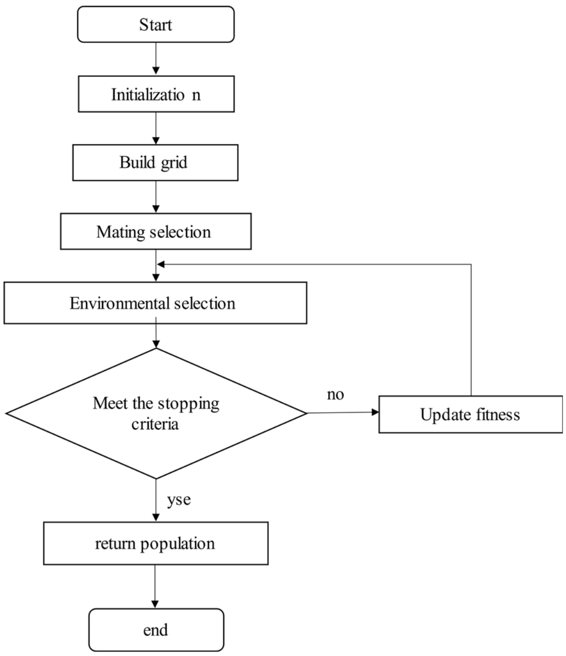

FIGURE 4. The flow chart of the algorithm.

demonstrated in Algorithm 1. Here, the grid is dynamically divided by K-means clustering method. Every individual is assigned to a new coordinate according its grid location. And the optimal individuals are selected based on grid dominance and two grid indicators (GD and GCD) in mating selection, which are similar to GrEA. The environment selection uses the SDE strategy to pick out the individuals with well convergence and distribution. The details of above-mentioned strategies are given in the following sections.

B. INHOMOGEENOUS GRID DIVISION

In IGEA, the k-means method is adopted to construct the inhomogeneous grid. The minimum value $\min(P_l)$ and maximum value $\max(P_l)$ in the lth objective are found. K classes are obtained by k-means clustering in the lth objective, and the distances among these classes are inhomogeneous. These classes are used to partition grid. The inhomogeneity grid division in the lth objective is displayed in Figure 5. The same is used if the objective space is a single, when the whale algorithm can be applied to optimize that objective very well.

In Figure 5, after clustering, K classes are obtained, and the minimum and maximum values in each class are used as the left and right boundaries of the class, respectively. On its left side of the ith class, the middle value of its the min and max value of (i-1) classes are regarded as the dividing grid line. On the right side of the ith class, the middle value of its maximum value and the minimum value of $(i + 1)$ classes is regarded as the line dividing the grid. On this way, the inhomogeneous grid is built.

After dividing the grid, a local normalization method is used to determine the coordinates in the lth objective. First, the class that the lth objective of an individual locates at is found, and then the normalized coordinate of the lth objective value of this individual's location based on the ratio of the distance between the individual and the left boundary to the cell length. The normalized coordinate of certain individual in the ith class is calculated as follow:

$$GL_l(x) = (i-1) + \left|\frac{F_l(x) - CL_l(i)}{CL_l(i+1) - CL_l(i)}\right| \tag{3}$$

where $GL_l(x)$ represents the coordinates of this individual in the lth objective, $F_l(x)$ represents the lth objective value of the individual, $CL_l(i)$ represents the left boundary of the ith cell in the lth objective, and $CL_l(i+1)$ - $CL_l(i)$ represents the length of ith cell in the lth objective, i denotes the ith class $(1 < i \leq K-1)$. For the 1st class, $CL_l(1)$ is equal to $\min(p_l)$.

The normalized coordinate of certain individual in the 1st class is calculated as follow:

$$GL_l(x) = \left|\frac{F_l(x) - \min(P_l)}{CL_l(2) - \min(P_l)}\right| \tag{4}$$

Similarly, the normalized coordinate of certain individual in the last class is calculated as follow:

$$GL_l(x) = (K-1) + \left|\frac{F_l(x) - CL_l(K)}{\max(P_l) - CL_l(K)}\right| \tag{5}$$

Definition (*Grid Dominance*): Let $x, y \in \Omega$, $x \prec_{grid} y$: ⇔

$$\begin{aligned} &\forall i \in (1, 2, \ldots, M) : G_i(x) \leq G_i(y) \\ &\exists j \in (1, 2, \ldots, M) : G_j(x) < G_j(y) \end{aligned} \tag{6}$$

where $\boldsymbol{x} \prec_{\boldsymbol{grid}} \boldsymbol{y}$ denotes that the $\boldsymbol{x}$ grid-dominates $\boldsymbol{y}$ and M is the objective number.

The idea of grid dominance and Pareto dominance is basically the same. The dominance relationship between two individuals is judged by the coordinate. Through grid dominance, the individuals (such as B and E in Figure 1) that can't be distinguished based on the original Pareto domination can be compared.

As slack forms of the Pareto dominance relation, there are some differences between the proposed grid dominance and the ε-dominance [1]. In ε-dominance, the objective space is divided into several boxes. For grid dominance, after each cycle, the grid is dynamical inhomogeneous re-divided. And a normalized coordinate is assigned to the individual. Even if two individuals are in the same grid, their coordinates may be different after normalization, which makes the dominance relationship between them clearer.

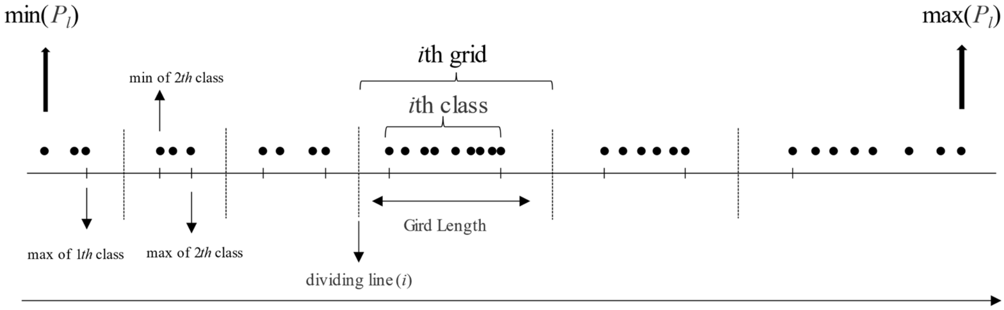

FIGURE 5. **Division of the grid in lth objective.**

C. GD/GCD_ASSIGNMENT

Definition (*Grid Difference*): Let $x, y \in P$, the grid differences as follows:

$$GD(x, y) = \sum_{k=1}^{M} |G_k(x) - G_k(y)| \tag{7}$$

where G_k represents the cell coordinates on the kth objective. M is the number of objectives. The grid difference is related to the number of classes. The more classes there are, the finer the division between population, which results in a larger difference between their coordinates in each dimension of the grid.

Definition (*Grid Crowding Distance*): Grid Crowding Distance (*GCD*) of individual x as follows:

$$GCD(x) = \sum_{y \in N(x)} (M - GD(x, y)) \tag{8}$$

where $N(x)$ represents the set of neighbors of x. GCD represents the crowd of solution $\vec{x}$ in the objective space. Here, the density is estimated only in certain area. The solution $\vec{x}$ neighboring to the solution $\vec{y}$ meets $GD(x, y) < M$. For example, in Figure 6, as the neighbors of F are E and G, so the GCD of F is $(2 - (|4.4 - 3.5| + |0.6 - 0.9|)) + (2 - (|4.4 - 4.6| + |0.6 - 0.3|)) = 2.3$. The density could reflect the distribution quantitatively.

GCD of the solution depends not only on difference between individuals but also on the range of the neighborhoods. The number of objectives M determines the size of neighborhoods. As the number of objectives increases, the number of cells in the hyperbox space becomes larger. The finite number of individuals distributed in grid will be sparse, so the range of the neighborhood that varies with M may be a good scheme. On the other hand, grid differences are also important for the measurement of GCD. The distance among neighbors is larger, so that the grid differences will be larger, which makes a smaller contribution to the value of GCD. For example, in Figure 6, individuals C and F have the same number of neighbors, but the GCD value of C is smaller than that of F(1.6 vs. 2.3).

D. ENVIRONMENT SELECTION

The purpose of environmental selection is to pick the optimal solutions from the parents and offspring. In traditional MOEAs, convergence indicators and distribution indicators are usually introduced to environment selection. Therefore, SDE strategy is utilized in our proposed IGEA. After inhomogeneously dividing the grid by the clustering method, the normalized coordinate of the individual is assigned. Then, the individuals are assigned to a fitness level by SDE. When density of individual p is calculated, the density of p is not directly calculated based on Euclidean distance. If a solution better than p in a certain objective, then the individual will move to the same coordinate in this objective of p. On the contrary, it will not move. Assuming a minimization MaOPs is considered, the new density of individual p in population P can be defined as follows:

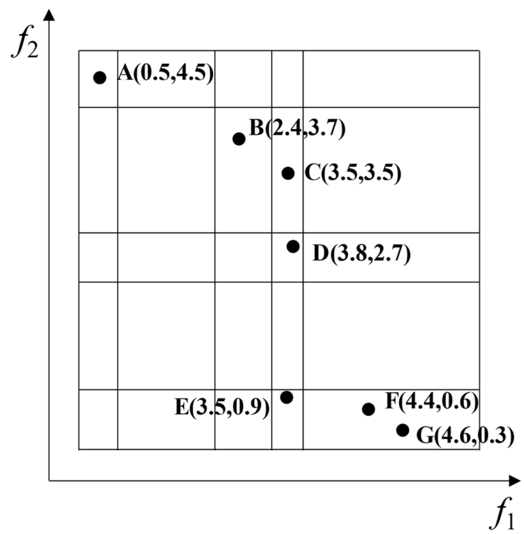

FIGURE 6. **Distribution of individuals in the grid after clustering.**

$$D'(p, P) = D(dist(p, q'_1), dist(p, q'_2), \ldots, dist(p, q'_{N-1})) \tag{9}$$

where N represents the size of the population P, $dist(p,q'_i)$ represents the degree of similarity between individuals p and q'_i, and q'_i is the shifted version of $q_i (q_i \in P, q_i \neq p)$, which is defined as follows :

$$q'_i(j) = \begin{cases} p(j), & q_i(j) < p_i(j) \\ q_i(j), & otherwise, \end{cases} \quad j \in (1, 2, \ldots, M) \tag{10}$$

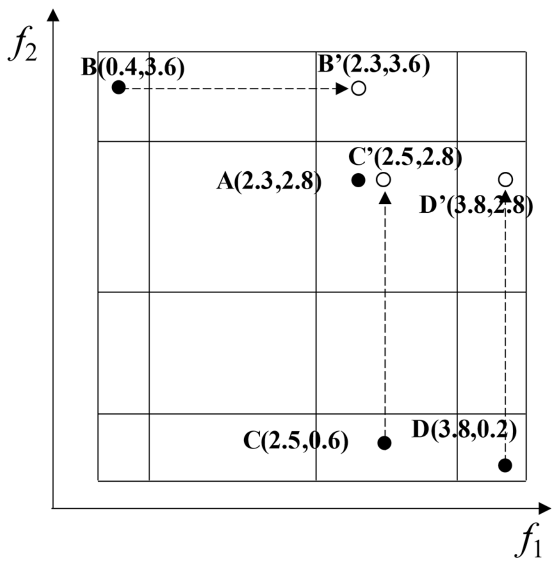

FIGURE 7. **Individuals *B*, *C* and *D* are shifted to *A*′, *B*′ and *C*′.**

where $p(j)$, $q_i(j)$ and $q_i'(j)$ represent the jth objective value of individual p, q_i, q_i', respectively. M is number of objectives. The shifted distance density can be defined as:

$$Dis(y) = \sum_{i=1}^{M} (\sum_{x,y \in P'} (G_i(x) - G_i(y))) \quad (11)$$

where $Dis(x)$ denotes the shifted distance density of individual y. P' is the population after SDE operation and x, $y \in$ population P', the solutions $\vec{x}$ and $\vec{y}$ in population P' meet $G_i(x) > G_i(y)$ in ith objective.

Individuals with poor convergence are shifted to a crowded region and assigned to a large density value. Individuals with larger density values may be removed during the selection process. Figure 7 illustrates the shift-based density operate in two objectives. In order to estimate the density of individuals, A. B, C, D are shifted to B', C', and D', respectively. Therefore, $B(1) = 0.4 < A(1) = 2.3$, $C(2) = 0.6 < A(2) = 2.8$ and $D(2) = 0.2 < A(2) = 2.8$.

After moving, A with three neighbors B', C' and D' will be assigned to a larger density value than before. Because three original individuals B, C and D are better than A. This means A has not significant advantage against individuals B, C and D.

The SDE strategy does not perform very well in a homogeneous grid, because in the same cell the individuals with different positions have the same coordinates. Since the density may be the same after moving with SDE, and the worse solutions cannot be removed from the archive. This problem is clearly explained in [23]. This problem can be solved with the inhomogeneous grid division by clustering, because the coordinates of individuals in the same cell can be discriminated after the normalized coordinate assignment.

Algorithm 2 gives the processing flow of environment selection. First, the fitness of the parents and offspring is calculated, then sort by fitness, and algorithm chose

Algorithm 2 Environment Selection

Require: N (archive size), Original Population(P_1), Offspring(P_2)

```
Generate an empty set Q for archive
Calculate fintness([P1,P2])
Q = sort (Fitness<1)
if Q < N
   Q1 = sort (Fitness(2N-Q))
   Q ← Q ∪ {Find_best(N-Q) from Q1}
else:
   Re-Calculate Fitness(Q)
   Q ← Q\ {Select_Worst(Q-N)}
end if
return Q
```

individuals Q with fitness less than 1. If the size of Q is less than archive size, individuals Q from parents and offspring were removed, then (N-Q) individuals will be recalculated fitness and be chosen with the best value of fitness until the size of Q equal archive size. If the size of Q is more than archive size, individuals Q will be recalculated fitness alone, and individuals (Q-N) with the worst value of fitness are deleted until the size of Q equal archive size. The fitness can be expressed as:

$$Fitness(i) = R(i) + \frac{1}{Dis(i)} \quad (12)$$

where $Fitness(i)$ represents the fitness of the ith individual, $R(i)$ represents the number of individuals dominating the ith individual in the population, and $Dis(i)$ represents thehifted distance density of the ith individual. The fitness includes not only the shifted distance but also the dominance relationship.

In IGEA, it applies K-means to establish a new coordinate system, and its algorithmic complexity is $O(MNKT)$. M is the number of objectives, N is the size of population, K represent the number of clusters, T is the number of iterations. Some metrics of IGEA are also calculated under this coordinate system. The complexity when calculating GD is $O(N^2)$, and the computational complexity of GCD is less than $O(N)$. In mating selection, IGEA and GrEA both apply binary tournament selection, and their complexity is the same in this stage. In environment selection, the worst case of the algorithm is considered, the algorithmic complexity of the penalty strategy in GrEA is $O(MN_q log(2N))$, N_q is the number of neighbors. The algorithmic complexity of SDE in IGEA is $O(MN^2)$. Overall, Since IGEA introduces K-means, the algorithm complexity of IGEA is a little higher than that of GrEA, but it brings about an improvement in performance.

V. EXPERIMENTAL DESIGN

This section will depict the IGEA's performance. First, the test problem and performance evaluation metrics will be introduced. Then, several state-of-the-art MOEAs are briefly introduced: MSOPS-II [51], MOEA/D [4], GrEA [33], SPEA2+SDE [23], DGEA and TiGE2. And the

TABLE 1. Parameter settings for regular problems.

Problem	M=4		M=5		M=6		M=8		M=10	
	div	*K*	*div*	*K*	*div*	*K*	*div*	*K*	*div*	*K*
WFG1	4	8	5	10	5	10	5	10	4	8
WFG3	5	10	4	7	4	7	4	7	4	8
WFG4	5	10	4	7	4	7	4	7	4	8
WFG5	5	10	5	10	4	7	4	7	4	8
WFG6	5	10	5	10	4	7	4	7	4	8
WFG7	5	10	4	7	4	7	4	7	4	8
WFG8	5	10	5	10	4	7	4	7	4	8
WFG9	5	10	4	7	4	7	4	7	4	8

TABLE 2. Parameter settings for irregular problems.

Problem	M=4		M=5		M=6		M=8		M=10	
	div	*K*	*div*	*K*	*div*	*K*	*div*	*K*	*div*	*K*
DTLZ7	5	10	5	9	5	9	4	8	4	8
WFG2	5	10	4	7	4	7	4	7	4	8
ZDT3	$M = 2, div = 5, K = 10$									
UF6	$M = 2, div = 5, K = 10$									
UF9	$M = 3, div = 5, K = 10$									

parameter settings are described. Finally, the experiments of parameters will be given.

A. TEST PROBLEMS

The irregular and regular test suites are used to verify the performance of IGEA. For irregular test suites, it includes DTLZ7, WFG2, ZDT3, UF6 and UF9. Irregular problems have many local PFs. It is difficult for general algorithms to maintain diversity and convergence in irregular functions. Irregular problems challenge algorithms' ability of locating all disconnected PF segments and handling nonseparable variable dependencies. These test suites with challenges of varying shapes and locations are used to verify algorithm's performance.

It contains WFG1, WFG3 to WFG9 for regular test suites. The WFG problems could test whether the algorithm is suitable for solving bias and mixed PF forms or not. WFGs have degenerated and linear PF forms and their variables are also nonseparable. Some WFGs with multimodal assess the capacity of escaping from local optima. WFGs have a considerable nonseparable reduction, and they all incorporate some bias to increase the challenge of diversity.

B. PARAMETERS SETTING

The parameter settings of GrEA and IGEA for regular and irregular problems are given in TABLE 1 and TABLE 2, respectively, where *div* belongs to GrEA and *K* belongs to IGEA.

For regular problems WFG1&3-8, the parameter k is set to $2(M\text{-}1)$, where M is the number of objectives. For irregular problem WFG2, the parameter k of WFG2 is set to $(M\text{-}1)$, where M is the number of objectives.

In this experiment, the simulated binary crossover (SBX) and polynomial mutation are used in IGEA, SPEA2+SDE, GrEA and DGEA. More precisely, the crossover operator produces an offspring, which is then modified by the mutation operator. The distribution index in both SBX and polynomial mutation was set to 15. The crossover rate is 0.94 and mutation rate is $1/n$, where n is the number of decision variable; In TiGE2, rand/1 differential evolution (DE) operator and polynomial mutation are used.

C. EXPERIMENTAL DESIGN

1) STOPPING CONDITIONS AND NUMBER OF RUNS

To prevent chance errors, each problem is tested 30 times independently. And then the average values of the final results are calculated. The criterion for stopping is the predefined number of evaluations. For the regular and irregular test sets, the number of evaluations is set to 30,000 [33].

2) POPULATION AND ARCHIVE SIZE

For the compared MOEAs and IGEA, the population size and the archive size are the same. The population size is not arbitrarily specified. Here, the population sizes are set to 120,

TABLE 3. The results of seven algorithm's IGD value on regular problems, the best mean of IGD is shown with gray background.

Prob.	obj.	DGEA		MOEA/D		MSOPS-II		SPEA2+SDE		TiGE2		GrEA		IGEA
WFG1	4	1.6811e+0 (5.32e-2)	+	7.9516e-1(8.22e-2)	+	3.5904e-1 (4.82e-2)	+	**3.0713e-1 (2.44e-2)**	-	4.2649e-1 (2.11e-2)	+	6.3415e-1 (8.00e-2)	+	3.1667e-1 (3.43e-2)
	5	1.8917e+0 (1.10e-1)	+	1.1164e+0(1.75e-1)	+	6.2247e-1 (2.45e-1)	+	4.4696e-1 (2.36e-2)	+	7.2424e-1 (6.56e-2)	+	6.9479e-1 (7.69e-2)	+	**4.4243e-1 (1.60e-2)**
	6	1.9951e+0 (9.79e-2)	+	1.4775e+0(2.54e-1)	+	7.6651e-1 (1.55e-1)	+	5.8258e-1 (2.28e-2)	+	8.8381e-1 (6.01e-2)	+	8.2583e-1 (7.48e-2)	+	**5.8031e-1 (2.56e-2)**
	8	1.9821e+0 (2.01e-1)	+	2.2490e+0(3.47e-1)	+	1.2409e+0 (1.69e-1)	+	9.1921e-1 (2.37e-2)	+	1.2599e+0 (3.89e-2)	+	1.0013e+0 (2.96e-2)	+	**9.1375e-1 (3.45e-2)**
	10	1.5287e+0 (1.05e-1)	+	2.5872e+0(4.24e-1)	+	1.7697e+0 (1.16e-1)	+	1.6442e+0(5.63e-1)	+	1.5492e+0 (7.88e-2)	+	1.6632e+0 (4.10e-1)	+	**1.3671e+0 (2.36e-1)**
WFG3	4	4.2601e-1 (1.07e-1)	=	5.0001e-1(1.21e-1)	+	**1.7373e-1 (5.33e-2)**	-	4.4539e-1 (1.43e-1)	=	1.8117e-1 (3.35e-2)	-	3.5218e-1 (6.04e-2)	=	4.2307e-1 (1.54e-1)
	5	6.2981e-1 (1.62e-1)	-	1.8759e+0(6.60e-2)	+	**1.7201e-1 (8.67e-2)**	-	7.3814e-1 (2.38e-1)	=	2.5461e-1 (4.16e-2)	-	5.9053e-1 (5.85e-2)	-	7.0089e-1 (1.83e-1)
	6	9.4978e-1 (1.05e-1)	=	2.8735e+0(6.62e-2)	+	**1.7949e-1 (8.18e-2)**	-	9.8865e-1 (2.81e-1)	=	3.6750e-1 (5.75e-2)	-	8.1014e-1 (2.09e-1)	-	1.0110e+0 (2.36e-1)
	8	1.1346e+0 (9.02e-2)	=	4.1975e+0(1.16e-1)	+	**2.0800e-1 (7.52e-2)**	-	1.5722e+0(3.75e-1)	=	6.4303e-1 (1.42e-1)	-	1.2687e+0 (3.19e-1)	-	1.6869e+0 (5.09e-1)
	10	1.5149e+0 (1.96e-1)	-	5.1131e+0(8.09e-1)	+	**5.7129e-1 (1.65e-1)**	-	1.8957e+0 (8.10e-1)	-	5.7210e+0 (2.10e+0)	+	2.1973e+0 (7.32e-1)	-	2.5797e+0 (1.10e+0)
WFG4	4	7.5263e-1 (3.31e-2)	+	1.4560e+0(8.50e-2)	+	**6.5952e-1 (1.28e-2)**	-	7.3687e-1 (2.28e-2)	=	8.1350e-1 (2.51e-2)	+	9.6424e-1 (2.09e-2)	+	7.3656e-1 (1.74e-2)
	5	1.3228e+0 (5.29e-2)	+	2.8717e+0(2.40e-1)	+	1.2632e+0 (2.23e-2)	+	1.2857e+0 (2.57e-2)	+	1.4382e+0 (5.81e-2)	+	1.4931e+0 (5.39e-2)	+	**1.2579e+0 (2.72e-2)**
	6	1.9957e+0 (5.38e-2)	+	3.7712e+0(1.68e-1)	+	2.0756e+0 (3.67e-1)	+	1.9042e+0 (3.57e-2)	+	2.1589e+0 (6.98e-2)	+	2.2878e+0 (7.97e-2)	+	**1.8788e+0 (2.88e-2)**
	8	3.6088e+0 (1.10e-1)	+	6.1832e+0(6.41e-1)	+	5.8152e+0(1.11e+0)	+	3.3760e+0 (3.93e-2)	+	4.0753e+0 (2.68e-1)	+	3.4188e+0 (4.16e-2)	+	**3.3269e+0 (3.85e-2)**
	10	6.0621e+0 (2.25e-1)	-	9.1246e+0(1.01e+0)	+	1.1232e+1(1.91e+0)	+	6.2352e+0(2.26e-1)	=	8.3305e+0 (4.57e-1)	+	**5.7588e+0 (1.41e-1)**	-	6.2065e+0 (2.07e-1)
WFG5	4	7.1063e-1 (2.69e-2)	-	1.4177e+0(5.91e-2)	+	**6.4861e-1 (1.38e-2)**	-	7.3813e-1 (1.73e-2)	+	8.0735e-1 (2.39e-2)	+	9.8930e-1 (2.27e-2)	+	7.2730e-1 (2.00e-2)
	5	1.3065e+0 (5.33e-2)	+	2.8342e+0(1.82e-1)	+	1.2583e+0 (2.34e-2)	+	1.2829e+0 (2.26e-2)	+	1.4310e+0 (4.78e-2)	+	1.4327e+0 (3.53e-2)	+	**1.2145e+0 (2.46e-2)**
	6	2.0142e+0 (7.75e-2)	+	3.8782e+0(2.06e-1)	+	**1.8582e+0 (3.18e-2)**	=	1.8757e+0 (2.52e-2)	+	2.1567e+0 (7.27e-2)	+	2.4164e+0 (8.63e-2)	+	1.8606e+0 (2.33e-2)
	8	3.5481e+0 (1.28e-1)	+	7.0714e+0(3.69e-1)	+	3.3734e+0 (1.48e-1)	=	3.3448e+0 (5.72e-2)	=	4.1706e+0 (3.54e-1)	+	3.4379e+0 (6.43e-2)	+	**3.3187e+0 (3.16e-2)**
	10	5.4887e+0 (8.09e-2)	-	1.1718e+1(6.93e-1)	+	6.5544e+0 (7.54e-1)	=	6.5049e+0(2.32e-1)	+	8.7167e+0 (4.19e-1)	+	**5.8184e+0 (1.67e-1)**	-	6.3908e+0 (1.84e-1)
WFG6	4	8.1738e-1 (3.25e-2)	+	1.4515e+0(5.60e-2)	+	**7.1378e-1 (1.81e-2)**	-	7.7070e-1 (1.80e-2)	=	8.3433e-1 (3.59e-2)	+	1.0037e+0 (3.86e-2)	+	7.6277e-1 (2.00e-2)
	5	1.3767e+0 (6.05e-2)	=	2.4076e+0(3.10e-1)	+	1.3593e+0 (3.36e-2)	+	1.3346e+0 (2.21e-2)	+	1.4582e+0 (3.89e-2)	+	1.4471e+0 (3.29e-2)	+	**1.3157e+0 (2.49e-2)**
	6	2.0398e+0 (7.61e-2)	+	3.3463e+0(2.66e-1)	+	2.0780e+0 (6.81e-2)	+	1.9843e+0 (2.50e-2)	+	2.1623e+0 (4.70e-2)	+	2.5758e+0 (1.53e-1)	+	**1.9583e+0 (3.40e-2)**
	8	3.6390e+0 (8.88e-2)	+	5.6853e+0(3.10e-1)	+	3.6752e+0 (1.28e-1)	+	3.4915e+0 (5.44e-2)	+	3.8532e+0 (1.24e-1)	+	3.5829e+0 (7.46e-2)	+	**3.4091e+0 (6.37e-2)**
	10	5.8622e+0 (2.07e-1)	-	9.0036e+0(6.50e-1)	+	6.6504e+0 (2.35e-1)	+	6.3787e+0 (1.25e-1)	=	1.3592e+1 (2.57e+0)	+	**5.7449e+0 (1.09e-1)**	-	6.3944e+0 (1.52e-1)
WFG7	4	7.8134e-1 (4.06e-2)	+	1.3931e+0(6.30e-2)	+	**6.9235e-1 (1.56e-2)**	-	7.5963e-1 (1.84e-2)	+	8.3249e-1 (3.07e-2)	+	1.1144e+0 (7.83e-2)	+	7.4434e-1 (1.52e-2)
	5	1.3938e+0 (5.75e-2)	+	2.3689e+0(1.18e-1)	+	1.2998e+0 (3.81e-2)	+	1.2908e+0 (2.79e-2)	+	1.4762e+0 (5.27e-2)	+	1.5712e+0 (7.04e-2)	+	**1.2767e+0 (2.35e-2)**
	6	2.0359e+0 (8.26e-2)	+	3.3575e+0(2.56e-1)	+	1.9327e+0 (4.79e-2)	+	1.8940e+0 (3.51e-2)	=	2.2040e+0 (6.38e-2)	+	2.4628e+0 (8.88e-2)	+	**1.8854e+0 (2.52e-2)**
	8	3.4554e+0 (1.06e-1)	+	6.3396e+0(4.83e-1)	+	4.2445e+0 (4.34e-1)	+	3.3895e+0 (4.94e-2)	+	3.9394e+0 (1.82e-1)	+	3.4590e+0 (5.76e-2)	+	**3.3182e+0 (3.52e-2)**
	10	**5.5128e+0 (6.69e-2)**	-	1.0520e+1(8.40e-1)	+	8.9454e+0(1.14e+0)	+	5.8690e+0(8.15e-2)	+	1.4570e+1 (2.20e+0)	+	5.7948e+0 (8.29e-2)	+	5.7382e+0 (1.07e-1)
WFG8	4	1.0245e+0 (9.49e-2)	+	1.5313e+0(4.77e-2)	+	1.0020e+0 (3.65e-2)	+	8.3803e-1 (3.91e-2)	+	9.9838e-1 (3.85e-2)	+	9.9723e-1 (4.14e-2)	+	**8.0871e-1 (5.01e-2)**
	5	1.6153e+0 (9.29e-2)	+	3.3388e+0(1.51e-1)	+	1.6419e+0 (2.75e-2)	+	1.3992e+0 (3.94e-2)	+	1.5422e+0 (4.53e-2)	+	1.4707e+0 (4.85e-2)	+	**1.3522e+0 (3.77e-2)**
	6	2.2652e+0 (1.21e-1)	+	4.4886e+0(8.41e-2)	+	2.3916e+0 (5.50e-2)	+	2.0391e+0 (3.68e-2)	+	2.2530e+0 (7.91e-2)	+	2.4113e+0 (1.10e-1)	+	**1.9985e+0 (4.67e-2)**
	8	3.9668e+0 (2.83e-1)	+	7.8422e+0(3.79e-1)	+	4.4612e+0 (2.70e-1)	+	3.5215e+0(7.17e-2)	+	4.5141e+0 (3.18e-1)	+	3.5594e+0 (7.00e-2)	+	**3.2896e+0 (7.51e-2)**
	10	6.6938e+0 (3.06e-1)	+	1.2468e+1(6.55e-1)	+	7.7909e+0 (7.30e-1)	+	6.1122e+0 (2.17e-1)	+	1.0801e+1 (2.66e+0)	+	**5.8166e+0 (1.24e-1)**	-	5.9883e+0 (1.48e-1)
WFG9	4	7.7191e-1 (5.94e-2)	-	1.3966e+0(3.73e-2)	+	**6.3005e-1 (1.23e-2)**	-	6.7667e-1 (1.85e-2)	=	7.9123e-1 (3.40e-2)	+	8.9584e-1 (2.16e-2)	+	6.7640e-1 (1.83e-2)
	5	1.3612e+0 (7.68e-2)	+	2.9685e+0(7.62e-2)	+	1.1821e+0 (2.54e-2)	+	1.1910e+0 (1.98e-2)	+	1.3547e+0 (5.35e-2)	+	2.1029e+0 (1.65e-1)	+	**1.1584e+0 (1.84e-2)**
	6	1.9861e+0 (1.22e-1)	+	4.0959e+0(1.38e-1)	+	1.8056e+0 (3.03e-2)	+	1.7998e+0 (2.70e-2)	+	2.0201e+0 (5.32e-2)	+	2.2327e+0 (7.12e-2)	+	**1.7846e+0 (2.10e-2)**
	8	3.5904e+0 (2.51e-1)	+	7.0311e+0(5.39e-1)	+	3.8063e+0 (2.49e-1)	+	3.2905e+0 (3.35e-2)	=	3.6487e+0 (1.45e-1)	+	3.4876e+0 (4.06e-2)	+	**3.2782e+0 (3.92e-2)**
	10	**5.4291e+0 (1.34e-1)**	-	1.1742e+1(8.02e-1)	+	7.7059e+0 (6.25e-1)	+	5.7060e+0 (1.60e-1)	-	8.3379e+0 (3.89e-1)	+	5.8363e+0 (1.14e-1)	=	5.8076e+0 (2.24e-1)
+/=/-		27/4/9		40/0/0		27/3/10		25/12/3		36/0/4		30/2/8		/

'+', '=' and '-' indicate IGEA performs significantly better than, equivalently to and worse than the corresponding compared algorithm, respectively.

126, 126, 120, 55 for 4, 5, 6, 8, and 10 objectives, respectively. In particular, in the irregular test sets (ZDT3, UF6, UF9), the population sizes are set to: 120, 120 and 150, respectively.

3) OTHER PARAMETER SETTINGS IN MOEA/D, GREA, TIGE2 AND IGEA

MOEA/D uses the Chebyshev function as the scalarization function. In GrEA and IGEA, the parameter *div* is set to about half of the number of classes in a certain objective. In TiGE2, the number of reference vector offspring generation is set 10;

D. PERFORMANCE METRICS

Inverted Generational Distance (IGD) focuses on calculating the sum of the distances between true PF and the obtained PS. It can be expressed as:

$$IGD(P, Q) = \frac{\sum_{v \in P} d(v, Q)}{|P|} \tag{13}$$

where P is the points of PF, $|P|$ is the number of P. Q is the obtained PS. And $d(v, Q)$ is the minimum Euclidean distance from v in P to Q. An ideal set of points is uniformly and widely distributed in PF, and these points are very close to PF.

Hypervolume (HV) is used to measure the volume of a target space that is dominated by at least one solution from a non-dominated solution set. HV provides a good measure of the convergence and diversity of the algorithm. The specific calculation is as follows:

$$HV(X, P) = \bigcup_{x \in X}^{X} v(x, P) \tag{14}$$

where $v\,(x, P)$ denotes the hypervolume of the space formed between the solution $\vec{x}$ and the reference point P in the non-dominated solution set X, i.e., the volume of the hypercube constructed by taking the line between the solution $\vec{x}$ and the reference point P as the diagonal.

TABLE 4. The results of seven algorithm's HV value on regular problems, the best mean of HV is shown with gray background.

Prob.	obj.	DGEA	MOEA/D	MSOPS-II	SPEA2+SDE	TiGE2	GrEA	IGEA
WFG1	4	3.6266e-1 (1.62e-2)	9.3988e-1 (5.09e-2)	9.7035e-1 (1.73e-2)	**9.7363e-1 (3.17e-2)**	9.7468e-1 (2.01e-2)	8.9363e-1 (6.81e-2)	9.2980e-1 (5.25e-2
	5	3.5686e-1 (2.27e-2)	8.6030e-1 (1.35e-1)	9.7224e-1 (5.05e-2)	9.8645e-1 (3.19e-2)	9.8184e-1 (2.69e-2)	9.2237e-1 (6.09e-2)	**9.8702e-1 (3.02e-2)**
	6	3.6507e-1 (2.89e-2)	7.5198e-1 (1.63e-1)	9.7779e-1 (5.11e-2)	9.7942e-1 (5.23e-2)	9.9174e-1 (2.75e-3)	9.7285e-1 (4.83e-2)	**9.9695e-1 (8.33e-4)**
	8	4.6450e-1 (8.87e-2)	5.6120e-1 (1.92e-1)	9.9638e-1 (3.33e-3)	9.7750e-1 (7.42e-2)	9.9225e-1 (2.20e-3)	9.9310e-1 (1.91e-3)	**9.9766e-1 (7.76e-4)**
	10	**9.9928e-1 (7.16e-4)**	4.5539e-1 (3.00e-1)	9.9296e-1 (5.37e-3)	7.8369e-1 (3.25e-1)	9.8500e-1 (5.77e-3)	9.8588e-1 (5.26e-3)	9.7068e-1 (1.27e-1)
WFG3	4	1.1767e-1 (5.12e-2)	2.0087e-1 (2.99e-2)	2.9512e-1 (1.80e-2)	1.8045e-1 (3.45e-2)	**2.9536e-1 (9.68e-3)**	2.6466e-1 (1.70e-2)	1.7676e-1 (3.57e-2)
	5	3.1962e-2 (5.03e-2)	9.1727e-2 (1.06e-3)	**2.4712e-1 (2.17e-2)**	6.5949e-2 (3.56e-2)	2.3908e-1 (1.31e-2)	2.1064e-1 (2.07e-2)	7.4836e-2 (3.44e-2)
	6	4.5429e-4 (2.49e-3)	8.7105e-2 (3.45e-3)	**2.1851e-1 (1.32e-2)**	2.2199e-2 (2.27e-2)	1.9535e-1 (1.05e-2)	1.7940e-1 (2.86e-2)	1.3772e-2 (1.61e-2)
	8	0.0000e+0 (0.00e+0)	8.2513e-2 (7.51e-3)	**1.6639e-1 (2.07e-2)**	6.5349e-4 (2.06e-3)	1.3810e-1 (1.60e-2)	6.6968e-2 (3.66e-2)	0.0000e+0 (0.00e+0)
	10	3.7007e-5 (1.56e-4)	8.2716e-2 (1.15e-2)	**1.3691e-1 (1.85e-2)**	5.4028e-4 (1.70e-3)	1.0608e-1 (2.10e-2)	7.9761e-2 (1.98e-2)	9.5235e-5 (3.63e-4)
WFG4	4	5.0724e-1 (2.47e-2)	5.0538e-1 (2.67e-2)	6.7029e-1 (5.12e-3)	6.7465e-1 (4.29e-3)	6.6580e-1 (5.34e-3)	6.4376e-1 (3.43e-3)	**6.8118e-1 (5.29e-3)**
	5	5.1717e-1 (3.38e-2)	5.0394e-1 (5.04e-2)	7.5794e-1 (4.16e-3)	7.4076e-1 (6.60e-3)	7.5698e-1 (5.11e-3)	7.4619e-1 (4.83e-3)	**7.6858e-1 (6.52e-3)**
	6	5.2473e-1 (3.55e-2)	5.1688e-1 (3.93e-2)	7.4530e-1 (8.81e-2)	7.8560e-1 (7.84e-3)	**8.1098e-1 (1.59e-2)**	7.4458e-1 (2.18e-2)	7.7533e-1 (9.40e-3)
	8	5.4009e-1 (4.64e-2)	5.3678e-1 (4.03e-2)	6.0427e-1 (7.94e-2)	8.3210e-1 (1.06e-2)	8.1329e-1 (2.68e-2)	7.3689e-1 (1.74e-2)	**8.5111e-1 (1.21e-2)**
	10	5.1589e-1 (4.36e-2)	4.5960e-1 (4.94e-2)	4.6511e-1 (9.42e-2)	**8.1528e-1 (2.19e-2)**	6.8460e-1 (3.66e-2)	7.6356e-1 (1.38e-2)	8.0056e-1 (1.91e-2)
WFG5	4	5.3441e-1 (1.92e-2)	4.5539e-1 (1.97e-2)	6.1504e-1 (4.90e-3)	**6.2489e-1 (3.35e-3)**	6.1823e-1 (5.27e-3)	4.9952e-1 (4.03e-3)	6.2273e-1 (3.81e-3)
	5	5.3834e-1 (2.51e-2)	4.8450e-1 (4.02e-2)	6.8572e-1 (5.81e-3)	6.9468e-1 (4.91e-3)	6.9971e-1 (6.06e-3)	6.0774e-1 (3.80e-3)	**7.0697e-1 (4.77e-3)**
	6	5.0377e-1 (3.23e-2)	5.0080e-1 (3.72e-2)	7.2650e-1 (4.89e-3)	7.2833e-1 (5.64e-3)	7.5122e-1 (8.06e-3)	7.5966e-1 (2.31e-3)	**7.3051e-1 (6.03e-3)**
	8	4.7935e-1 (4.68e-2)	4.0374e-1 (2.93e-2)	7.2259e-1 (3.26e-2)	7.4708e-1 (1.24e-2)	7.4394e-1 (4.76e-2)	7.1223e-1 (7.92e-3)	**7.6566e-1 (1.09e-2)**
	10	5.5486e-1 (2.55e-2)	3.5755e-1 (3.28e-2)	5.9810e-1 (5.15e-2)	6.9675e-1 (2.53e-2)	5.9169e-1 (2.41e-2)	4.1326e-1 (4.11e-2)	**7.0223e-1 (1.93e-2)**
WFG6	4	4.4804e-1 (2.14e-2)	4.1831e-1 (4.55e-2)	5.9863e-1 (1.98e-2)	**6.3103e-1 (1.76e-2)**	6.0734e-1 (2.26e-2)	5.8496e-1 (2.53e-2)	6.2956e-1 (2.55e-2)
	5	4.6586e-1 (3.04e-2)	4.6091e-1 (8.11e-2)	6.8087e-1 (2.95e-2)	6.9995e-1 (2.52e-2)	6.8115e-1 (3.33e-2)	6.6913e-1 (2.16e-2)	**7.1079e-1 (3.29e-2)**
	6	4.6270e-1 (2.44e-2)	4.7038e-1 (4.86e-2)	7.4407e-1 (3.31e-2)	7.6120e-1 (2.87e-2)	7.4339e-1 (2.54e-2)	6.4344e-1 (3.73e-2)	**7.6607e-1 (3.30e-2)**
	8	4.6953e-1 (7.19e-2)	4.7988e-1 (5.62e-2)	8.4011e-1 (1.17e-2)	**8.4625e-1 (2.14e-2)**	8.3995e-1 (2.73e-2)	7.6321e-1 (2.98e-2)	8.2835e-1 (2.36e-2)
	10	5.7740e-1 (9.24e-2)	3.6481e-1 (4.48e-2)	7.5543e-1 (3.67e-2)	8.2491e-1 (1.37e-2)	2.5478e-1 (1.53e-1)	6.3712e-1 (2.79e-2)	**8.3289e-1 (1.91e-2)**
WFG7	4	4.7966e-1 (3.21e-2)	5.4533e-1 (2.02e-2)	6.6830e-1 (4.91e-3)	6.8498e-1 (2.51e-3)	6.7028e-1 (3.54e-3)	6.2245e-1 (1.07e-2)	**6.8536e-1 (3.51e-3)**
	5	4.5909e-1 (3.55e-2)	5.9508e-1 (2.43e-2)	7.5745e-1 (4.53e-3)	**7.6711e-1 (3.76e-3)**	7.5942e-1 (5.48e-3)	7.4762e-1 (4.54e-3)	7.6514e-1 (3.04e-3)
	6	4.5603e-1 (4.59e-2)	5.9243e-1 (3.56e-2)	8.0667e-1 (1.76e-2))	8.1454e-1 (4.81e-3)	8.2157e-1 (6.68e-3)	8.1342e-1 (6.11e-3)	**8.2645e-1 (4.08e-3)**
	8	5.1403e-1 (6.62e-2)	5.1595e-1 (5.10e-2)	6.7242e-1 (6.09e-2)	8.5959e-1 (7.99e-3)	8.2695e-1 (2.23e-2)	7.8377e-1 (1.95e-2)	**8.5973e-1 (8.76e-3)**
	10	7.9195e-1 (1.74e-2)	4.4195e-1 (3.48e-2)	5.2371e-1 (6.00e-2)	**8.4611e-1 (1.09e-2)**	2.4567e-1 (1.28e-1)	7.9676e-1 (1.65e-2)	8.3492e-1 (1.45e-2)
WFG8	4	3.7161e-1 (4.18e-2)	3.3269e-1 (3.50e-2)	4.5931e-1 (1.36e-2)	5.5166e-1 (2.28e-2)	4.9309e-1 (1.34e-2)	5.3002e-1 (4.84e-2)	**5.7397e-1 (3.63e-2)**
	5	3.9380e-1 (3.76e-2)	2.7793e-1 (5.74e-2)	4.9827e-1 (1.50e-2)	6.4601e-1 (2.76e-2)	5.7276e-1 (2.26e-2)	6.6772e-1 (4.09e-2)	**6.7269e-1 (2.61e-2)**
	6	4.1419e-1 (4.86e-2)	2.6805e-1 (2.62e-2)	5.3041e-1 (2.21e-2)	7.2536e-1 (2.44e-2)	6.5693e-1 (2.62e-2)	7.1648e-1 (2.37e-2)	**7.2716e-1 (1.35e-2)**
	8	4.5458e-1 (4.71e-2)	2.8412e-1 (2.92e-2)	5.6410e-1 (3.04e-2)	**8.0400e-1 (2.00e-2)**	7.3497e-1 (3.03e-2)	6.7052e-1 (3.74e-2)	7.9470e-1 (1.58e-2)
	10	3.6713e-1 (4.58e-2)	2.5098e-1 (4.05e-2)	4.6582e-1 (4.15e-2)	8.1638e-1 (2.15e-2)	4.7246e-1 (1.91e-1)	5.6586e-1 (2.39e-2)	**8.2093e-1 (1.82e-2)**
WFG9	4	4.7107e-1 (4.78e-2)	4.0181e-1 (7.34e-2)	6.0857e-1 (1.24e-2)	6.3318e-1 (6.03e-3)	6.0746e-1 (3.26e-2)	5.9417e-1 (1.84e-2)	**6.3341e-1 (5.36e-3)**
	5	4.7477e-1 (5.69e-2)	3.4262e-1 (1.01e-1)	6.6008e-1 (1.88e-2)	**6.9270e-1 (7.10e-3)**	6.8725e-1 (9.36e-3)	6.8298e-1 (6.33e-3)	6.8733e-1 (6.77e-3)
	6	5.1243e-1 (7.12e-2)	3.6432e-1 (5.55e-2)	6.9111e-1 (1.33e-2)	7.2266e-1 (6.62e-3)	7.2903e-1 (9.25e-3)	6.9950e-1 (1.53e-2)	**7.3584e-1 (6.75e-3)**
	8	5.5223e-1 (9.06e-2)	3.3154e-1 (4.35e-2)	6.6983e-1 (2.40e-2)	**7.4686e-1 (1.03e-2)**	7.4567e-1 (1.68e-2)	7.1352e-1 (1.01e-2)	7.3668e-1 (8.60e-3)
	10	6.0859e-1 (4.06e-2)	3.5873e-1 (2.47e-2)	5.5160e-1 (2.42e-2)	7.2738e-1 (1.75e-2)	6.0817e-1 (2.07e-2)	5.0508e-1 (2.76e-2)	**7.2855e-1 (1.63e-2)**

The regular problems (WFG1, WFG3 to WFG9) will reveal some characteristics of the proposed IGEA. Moreover, to verify the performance for the irregular problems, a set of irregular solution sets (DTLZ7, ZDT3, WFG2, UF6, UF9) be utilized. These questions are used to validate whether the irregular solution sets still maintain good convergence and distribution.

VI. EXPERIMENTAL RESULT AND DISCUSSION

In this section, the characteristics of IGEA are revealed. The performance of IGEA will be compared with other algorithms on both regular and irregular test suites.

A. PERFORMANCE COMPARISON OF REGULAR MAOPS

TABLE 3 gives the IGD experimental results of MSOPS-II, SPEA2+SDE, MOEA/D, GrEA and IGEA against the regular test problems (WFG1&3-9). TABLE 4 shows HV of these algorithms, IGEA was able to get 23 best out of 40 test cases. HV provides a good measure of the convergence and diversity of the algorithm. From TABLE 3, the proposed IGEA outperforms the compared MOEAs. The optimal results for each problem are signed with gray background. The overall performance of the IGEA is the ideal. And IGEA obtains 22 best IGD values of 40 test instances, especially on WFG1, WFG7, WFG8.

On WFG1, IGEA is the best among seven algorithms. Although the performance of IGEA is slightly worse than SPEA2+SDE in terms of mean on 4-objective, though the Wilcoxon rank sum test with p-value $= 0.05$, the performance of IGEA is equal to the performance of SPEA2+SDE. However, on WFG3, IGEA's performance is not ideal. WFG3 has a degenerated and linear Pareto-optimal front (POF) form, and its variable in the decision space is also non-separable. For maintaining the diversity, IGEA attempts to disperse population throughout the whole objective space, which results in a relatively small number of points on the degraded POF. On WFG 4, IGEA's IGD value is 10% worse than the best IGD value obtained by MSOPS-II on 4-objective. And on 10-objective, IGEA is competitive and its IGD value is 6% worse than the best IG value obtained by GrEA. On 5,

TABLE 5. The results of seven algorithm's IGD value on irregular problems, the best mean of IGD is shown with gray background.

Prob.	obj.	DGEA		MOEA/D		MSOPS-II		SPEA2+SDE		TiGE2		GrEA		IGEA
DTLZ7	4	4.4177e-1 (2.15e-1)	+	5.0210e-1(2.99e-2)	+	2.8190e-1 (1.49e-2)	+	1.5859e-1 (9.22e-2)	+	9.8152e-1 (3.10e-1)	+	2.4656e-1 (2.00e-2)	+	**1.3798e-1 (7.23e-3)**
	5	5.7671e-1 (2.10e-1)	+	6.6361e-1(1.06e-1)	+	4.3532e-1 (1.16e-2)	+	3.0315e-1 (3.77e-2)	+	1.4222e+0 (3.78e-1)	+	3.7950e-1 (2.17e-2)	+	**2.8896e-1 (1.02e-2)**
	6	8.7683e-1 (2.39e-1)	+	6.8993e-1(1.07e-1)	+	5.0528e-1 (3.74e-2)	+	4.0578e-1 (1.62e-2)	=	1.9451e+0 (5.63e-1)	+	4.7862e-1 (2.38e-2)	-	**4.0291e-1 (4.79e-3)**
	8	1.2755e+0 (1.75e-1)	+	1.0680e+0(1.34e-1)	+	1.2796e+0 (2.11e-1)	+	8.1069e-1 (5.36e-2)	=	3.1405e+0 (5.82e-1)	+	**7.9095e-1 (3.83e-2)**	-	8.1379e-1 (3.71e-2)
	10	1.5000e+0 (2.19e-1)	+	1.9771e+0(5.46e-1)	+	1.8085e+0 (2.80e-1)	+	1.1518e+0 (7.08e-2)	+	4.8175e+0 (7.80e-1)	+	1.1986e+0 (5.28e-2)	+	**1.1112e+0 (7.97e-2)**
ZDT3	2	3.9485e-1 (1.76e-1)	+	2.3149e-2 (2.06e-2)	+	1.7544e-1 (8.96e-2)	+	2.4857e-2 (3.23e-2)	+	6.0720e-2 (7.23e-2)	+	9.7884e-2 (5.86e-2)	+	**1.6417e-2 (2.81e-2)**
WFG2	4	5.2695e-1 (6.51e-2)	-	9.3396e-1 (4.08e-2)	+	**3.4869e-1 (9.60e-3)**	-	4.2651e-1 (1.77e-2)	+	4.6767e-1 (1.14e-2)	+	5.6147e-1 (2.45e-2)	+	4.0903e-1 (2.53e-2)
	5	6.5473e-1 (7.45e-2)	-	1.3791e+0(1.68e-1)	+	**5.1219e-1 (1.62e-2)**	-	5.5882e-1 (2.88e-2)	+	6.4054e-1 (2.63e-2)	+	6.9158e-1 (3.48e-2)	+	5.4510e-1 (2.43e-2)
	6	7.8662e-1 (7.83e-2)	=	1.6403e+0(3.14e-1)	+	**6.9172e-1 (3.90e-2)**	=	7.2280e-1 (3.58e-2)	+	8.6272e-1 (5.08e-2)	+	7.5294e-1 (3.04e-2)	+	7.0325e-1 (3.44e-2)
	8	1.1154e+0 (7.15e-2)	+	2.7882e+0(1.34e+0)	+	1.4000e+0 (2.61e-1)	+	1.1565e+0 (4.49e-2)	+	1.2770e+0 (5.43e-2)	+	1.2158e+0 (7.20e-2)	+	**1.1120e+0 (3.69e-2)**
	10	**1.3708e+0 (5.33e-2)**	+	3.2869e+0(1.68e+0)	+	3.0130e+0 (7.38e-1)	+	1.4803e+0 (5.53e-2)	+	1.8032e+0 (5.45e-1)	+	1.6221e+0 (9.03e-2)	+	1.4771e+0 (5.49e-2)
UF6	2	7.6164e-1 (2.51e-1)	=	4.4273e-1 (1.94e-1)	+	2.2043e-1 (1.30e-1)	=	**1.7696e-1 (9.08e-2)**	-	4.7487e-1 (8.58e-2)	+	1.9509e-1 (9.38e-2)	=	2.3843e-1 (1.06e-1)
UF9	3	4.9064e-1 (6.80e-2)	+	4.0862e-1 (8.01e-2)	+	3.7334e-1 (7.79e-2)	+	3.2092e-1 (1.08e-1)	+	4.4189e-1 (1.35e-1)	+	3.0162e-1 (8.47e-2)	+	**2.7274e-1 (8.86e-2)**
+/=/-		9/2/2		13/0/0		9/2/2		10/2/1		13/0/0		10/1/2		/

'+', '=' and '-' indicate IGEA performs significantly better than, equivalently to and worse than the corresponding compared algorithm, respectively.

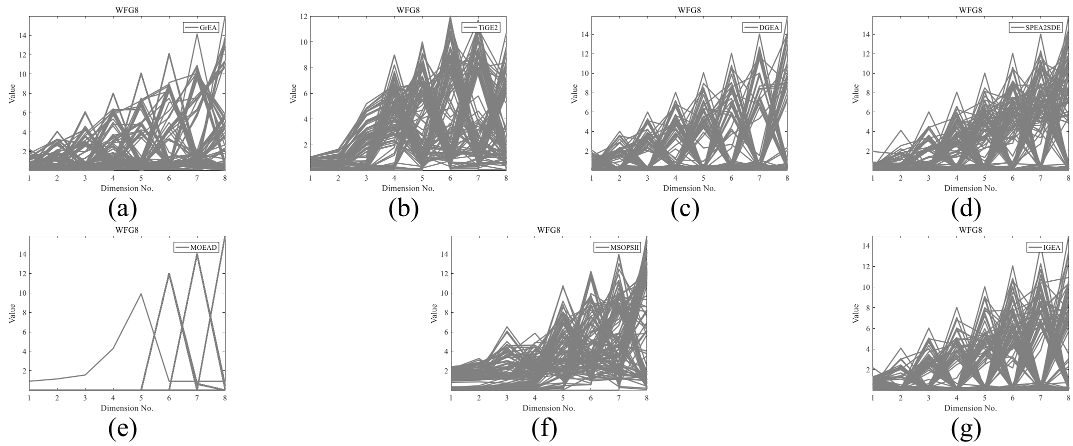

FIGURE 8. The distribution of solutions based on one run on the 8-objective WGF8. (a) GrEA. (b) TiGE2. (c) DGEA. (d) SPEA2+SDE. (e) MOEA/D. (f) MSOPSII. (g) IGEA.

6 & 8-objective, the IGEA are outstanding. On WFG5, IGEA's performance is not ideal, and the IGD value of IGEA is 12% worse than the best IGD value obtained by MSOPS-II on 4-objectives. The IGD value of IGEA is 0.5% worse than the best IGD value obtained by MSOPS-II on 6-objective. Though the Wilcoxon rank sum test with p-value = 0.05, the performance of IGEA is equal to the performance of MSOPS-II. And on 10-objective, the IGD of IGEA is 9% worse than the best IGD value obtained by GrEA. It should be noted that although the IGD value of IGEA is the best on 8-objective, though the Wilcoxon rank sum test with p-value = 0.05, the performance of IGEA is equal to the performance of MSOPS-II and SPEA2+SDE. On WFG6, WFG7 and WFG8, overall, the performance of IGEA is steadily ideal in both low and high dimensions. Except on the 10-objective WFG6, the IGD value of IGEA is 11% worse than the best IGD obtained by GrEA. In other test instances, IGEA's overall performance is competitive than the other four algorithms. On 5, 6 & 8-objectives WFG9, the performance of IGEA is idea. On other objectives, the difference between the performance of IGEA algorithm and other algorithms is not very obvious. DGEA does not perform well on all test sets, TiGE2 performs very well on WFG3, and the performance on other test sets is not excellent.

Meanwhile, MSOPS-II performs well on WFG3 and WFG5. Intuitively, SPEA2+SDE and the IGEA are similar in terms of SDE strategy, but the SPEA2+SDE does not perform as well as the IGEA. When SPEA2+SDE and IGEA calculate the distance, the former uses the true values of the individuals to calculate the shifted distance, and the latter with cluster method calculate the shifted distance based on the inhomogeneous grid division that can assign the similar individuals into the same grid. Through the grid characteristic, the individuals selected in the mating selection are closer to the Pareto Front. Above phenomenon caused by the inhomogeneous grid division make IGEA outperforms SPEA2+SDE. MOEA/D perform unsatisfactorily on these test suites. The weakness of MOEA/D is the diversity. Although the individuals obtained by MOEA/D are fairly near to the PF, these individuals are too concentrated in some narrow regions of the PF, which results

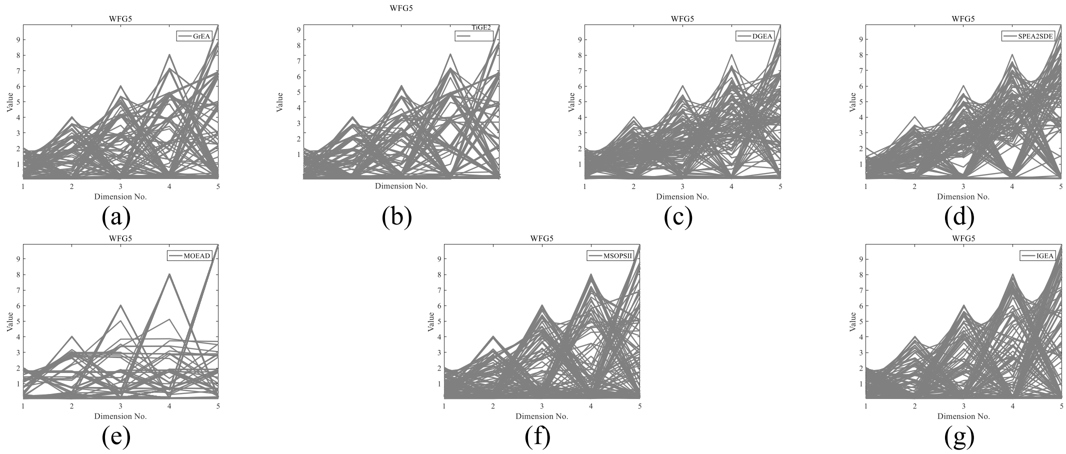

FIGURE 9. The distribution of solutions based on one run on the 8-objective WGF8. (a) GrEA. (b) TiGE2. (c) DGEA. (d) SPEA2+SDE. (e) MOEA/D. (f) MSOPSII. (g) IGEA.

in a poor diversity. Although GrEA with the parameter div set to about 6 [33] can obtain the extremely competitive results, the overall performances of GrEA are still not as well as that of IGEA.

The performance of IGEA is satisfactory on the regular test problems, especially on the five to eight objectives. Figure 8 displays the distributions of solution on the 8-objective WFG8 based on the one run. The performance of GrEA is indeed good, but it is slightly worse in terms of distribution. The convergence and diversity of MOEA/D is not ideal, because the Chebyshev scalar function is not a tool of guaranteeing convergence and diversity. The comprehensive performances of MSOPS-II and SPEA2+SDE are not as well as IGEA.

One distinguishing feature of WFG issues is that each objective has a different range. MOEA/D scores badly in terms of diversity for WFG issues, which implies that adding restrictions to subproblems is insufficient for dealing with difficult-to-converge and diversity-resistant problems. On the other hand, other algorithms retain diversity well on WFG, even if it does not fully converge to the PF in certain runs.

On low-dimensional objectives, IGEA still has an outstanding performance. In Figure 9, the distribution of solutions on the 5-objective WFG5 based on the one run is shown. From Figure 9, intuitively, IGEA outperforms the compared algorithms. GrEA and SPEA2+SDE also performed very competitively. Since SPEA2-SDE also uses the SDE method, when solving the problem of hyper-ellipse surface type WFG5 [52], it has a better performance. MSOPS-II uses the automatic objective vector generation strategy. When solving such problems, it can also guarantee convergence, but the diversity is insufficient. In terms of diversity, IGEA performed best among it and compared algorithms. The experimental results prove the effectiveness of dynamic inhomogeneous grid division in dealing with regular problems. Compared with other algorithms, with the benefit of dynamic inhomogeneous grid division, IGEA keeps the diversity well in dealing with regular problems. With the help of SDE strategy, IGEA enhances the diversity and convergence simultaneously. Overall, on the regular problems, the modification of IGEA achieves the anticipative enhancement.

B. PERFORMANCE COMPARISON OF IRREGULAR MAOPS

TABLE 5 shows the experimental IGD results of the state-of-the-art MOEAs and the proposed IGEA against some irregular problems (DTLZ7, ZDT3, WFG2, UF6 and UF9). TABLE 6 shows HV of these algorithms, IGEA was able to get 6 best out of 13 test cases. HV provides a good measure of the convergence and diversity of the algorithm. On 4, 5, 6 & 10-objectives DTLZ7, the performances of IGEA are ideal. But on 8-objective DTLZ7, the IGD of IGEA is 2.8% worse than the best IGD value obtained by GrEA. DGEA and TiGE2 not ideal on DTLZ7;

To illustrate the performance of IGEA intuitively, the distributions of the final solutions obtained by it and other MOEAs against ZDT3 are compared in Figure 10. On ZDT3, the performance of IGEA is outstanding. GrEA is not very effective in maintaining diversity. Any preference toward single aim (convergence or diversity) will inevitably exacerbate the performance. In MaOPs, the balance between convergence and diversity should be considered.

MSOPS-II is biased towards convergence. The diversity of MOEA/D and SPEA2+SDE are not ideal. IGEA can maintain a good balance between convergence and diversity.

On WFG2, the performance of IGEA is not ideal on 4, 5, 6-objective. However, IGEA's performance is excellent on 8 and 10-objective. On UF6, the IGD value of IGEA is 34% worse than the best IGD value obtained by SPEA2+SDE. UF6 has many local PFs, and its Pareto Front is not smooth curve. IGEA doesn't have ability of competently dealing with cliff PF. On UF9, IGEA's performance is best among these algorithms. DGEA performance well on

TABLE 6. The results of seven algorithm's HV value on irregular problems, the best mean of HV is shown with gray background.

Prob.	obj.	DGEA	MOEA/D	MSOPS-II	SPEA2+SDE	TiGE2	GrEA	IGEA
DTLZ7	4	1.6514e-1 (6.71e-2)	1.5857e-1 (1.56e-2)	2.4104e-1 (3.79e-3)	2.7786e-1 (7.82e-3)	2.0117e-1 (2.00e-2)	2.6540e-1 (2.89e-3)	**2.7866e-1 (1.72e-3)**
	5	1.6653e-1 (4.33e-2)	1.6717e-1 (2.12e-2)	2.2513e-1 (5.26e-3)	2.5688e-1 (4.36e-3)	1.9378e-1 (1.85e-2)	2.5612e-1 (2.99e-3)	**2.5919e-1 (3.49e-3)**
	6	9.7298e-2 (6.99e-2)	1.4639e-1 (3.40e-2)	2.0658e-1 (7.79e-3)	2.3056e-1 (5.78e-3)	1.8040e-1 (1.96e-2)	**2.2473e-1 (9.61e-3)**	2.3381e-1 (6.15e-3)
	8	3.5098e-2 (5.12e-2)	4.7190e-2 (4.23e-2)	**1.6741e-1 (9.82e-3)**	6.8343e-2 (3.96e-2)	1.6356e-1 (1.22e-2)	1.4649e-1 (1.26e-2)	5.7247e-2 (2.58e-2)
	10	5.4392e-2 (5.82e-2)	2.7567e-3 (5.15e-3)	1.5139e-2 (9.74e-3)	2.2184e-2 (2.04e-2)	1.2601e-2 (7.83e-3)	7.5975e-3 (9.85e-3)	**3.7438e-2 (3.08e-2)**
ZDT3	2	3.3240e-1 (1.32e-1)	5.9115e-1 (4.39e-2)	6.2606e-1 (1.36e-1)	6.3448e-1 (6.56e-2)	6.1083e-1 (4.62e-2)	**6.3598e-1 (6.59e-2)**	6.1202e-1 (5.73e-2)
WFG2	4	8.3933e-1 (3.52e-2)	8.9095e-1 (1.68e-2)	9.7722e-1 (1.89e-3)	9.6872e-1 (3.63e-3)	9.6763e-1 (5.18e-3)	9.4848e-1 (7.04e-3)	**9.8032e-1 (3.67e-3)**
	5	8.7011e-1 (4.13e-2)	9.2141e-1 (4.91e-2)	**9.9227e-1 (2.04e-3)**	9.7560e-1 (3.75e-3)	9.6260e-1 (3.16e-3)	9.6606e-1 (5.12e-3)	9.7610e-1 (4.77e-3)
	6	8.5472e-1 (6.08e-2)	9.2975e-1 (7.28e-2)	9.9478e-1 (1.46e-3)	9.7557e-1 (3.77e-3)	**9.8667e-1 (2.57e-3)**	9.6861e-1 (4.75e-3)	9.7538e-1 (5.13e-3)
	8	8.8176e-1 (6.83e-2)	9.2259e-1 (7.73e-2)	9.8650e-1 (1.44e-2)	9.7963e-1 (3.18e-3)	9.9037e-1 (3.06e-3)	9.7202e-1 (8.00e-3)	**9.9862e-1 (3.69e-3)**
	10	**9.7554e-1 (1.12e-2)**	8.8510e-1 (6.89e-2)	9.4178e-1 (3.25e-2)	9.7218e-1 (4.69e-3)	9.6067e-1 (4.25e-2)	9.6842e-1 (1.15e-2)	9.6988e-1 (7.46e-3)
UF6	2	2.9266e-2 (3.94e-2)	1.7997e-1 (9.53e-2)	2.9397e-1 (5.57e-2)	**3.0734e-1 (7.28e-2)**	6.0409e-2 (3.97e-2)	2.9849e-1 (8.97e-2)	2.4882e-1 (8.97e-2)
UF9	3	2.7098e-1 (5.77e-2)	4.1739e-1 (5.47e-2)	3.9873e-1 (6.05e-2)	4.8776e-1 (7.64e-2)	2.8698e-1 (1.07e-1)	4.2949e-1 (5.43e-2)	**5.2638e-1 (7.05e-2)**

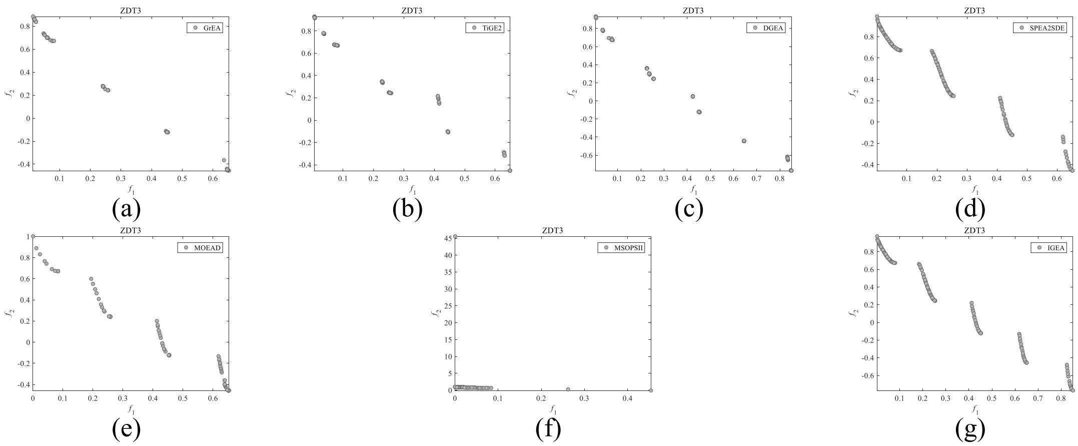

FIGURE 10. The distribution of solutions based on one run on the 8-objective WGF8. (a) GrEA. (b) TiGE2. (c) DGEA. (d) SPEA2+SDE. (e) MOEA/D. (f) MSOPSII. (g) IGEA.

10-objective WFG2, but not well on other objectives. TiGE2 performance not ideal on WFG2.

MOEA/D does not perform well on irregular problems. In fact, for the above test problems, MOEA/D is located fairly near to the PF, but it does not cover the PF very well, which results in dissatisfactory IGD values. Its poor performance on irregular problems mainly results from the aggregation-based selection operation in MOEA/D. A collection of uniformly distributed weight vectors fails in providing a uniform distribution of crossing sites on the irregular PF. The Chebyshev-based scalarization function is not a good tool of guaranteeing diversity. GrEA, SPEA2+SDE and MSOPS-II are very competitive, but these algorithms does not perform well in dealing with irregular problems. Toward irregular problems, clustering methods are a good choice.

On the whole, IGEA has succeeded in maintaining diversity and convergence for solving MaOPs with different characteristics. The IGEA outperformed the other algorithms.

VII. CONCLUSION

Through systematic experiments on the proposed IGEA, several state-of-the-art MOEAs have been extensively compared. Regular and irregular test problems were chosen to verify the capabilities of IGEA. IGEA is highly competitive in terms of IGD value. The IGEA is able to reach a good balance between diversity and convergence. Inhomogeneous grid division makes the differences between individuals more obvious, and it making the point to PF more convergent. The SDE strategy prevents the solution from converging locally on the target space and allows the solution to be distributed more uniformly and widely on the PF. The proposed IGEA can mainly be characterized as:

1) IGEA dynamically and inhomogeneously divides the grid through the clustering algorithm, and assigns coordinates to individuals through a normalization method.

2) The comparisons and domination analysis are bases on the normalized grid coordinates rather than the objectives of solutions.

3) SDE strategy is introduced to calculate the fitness of the solution according to not only neighbors but also the global solutions in the normalized grid coordinates, which is helpful for evaluating the diversity and convergence.

A number of future directions were identified from the current work. IGEA can't offer an acceptable performance on a degenerated and linear Pareto-optimal front (WFG3). In the following work, the optimization process of IGEA on

WFG3 should be tracked and the reason of failure should be clarified. Some modifications of IGEA should be made for improving the universality. IGEA uses a clustering algorithm to inhomogeneously divide the grid, which leads to a higher complexity than before. In the future, some efficient methods that can reflect the characteristics of the population could be adopted to divide the grid. In addition, the inhomogeneous grid division method also be used in combining with the decomposed-based MOEAs.

REFERENCES

[1] M. Laumanns, L. Thiele, K. Deb, and E. Zitzler, "Combining convergence and diversity in evolutionary multiobjective optimization," *Evol. Comput.* vol. 10, no. 3, pp. 263–282, Sep. 2002.

[2] J. G. Falcon-Cardona, H. Ishibuchi, and C. A. C. Coello, "Exploiting the trade-off between convergence and diversity indicators," in *Proc. IEEE Symp. Ser. Comput. Intell. (SSCI)*, Canberra, ACT, Australia, Dec. 2020, pp. 141–148.

[3] L. Ma, S. Cheng, and Y. Shi, "Enhancing learning efficiency of brain storm optimization via orthogonal learning design," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 51, no. 11, pp. 6723–6742, Nov. 2021.

[4] Q. Zhang and H. Li, "MOEA/D: A multiobjective evolutionary algorithm based on decomposition," *IEEE Trans. Evol. Comput.*, vol. 11, no. 6, pp. 712–731, Dec. 2007.

[5] W.-X. Wang, K.-S. Li, X.-Z. Tao, and F.-H. Gu, "An improved MOEA/D algorithm with an adaptive evolutionary strategy," *Inf. Sci.*, vol. 539, pp. 1–15, Oct. 2020.

[6] X. Ma, Y. Yu, X. Li, Y. Qi, and Z. Zhu, "A survey of weight vector adjustment methods for decomposition-based multiobjective evolutionary algorithms," *IEEE Trans. Evol. Comput.*, vol. 24, no. 4, pp. 634–649, Aug. 2020.

[7] J. Qiao, H. Zhou, C. Yang, and S. Yang, "A decomposition-based multiobjective evolutionary algorithm with angle-based adaptive penalty," *Appl. Soft Comput.*, vol. 74, pp. 190–205, Jan. 2019.

[8] L. Ma, N. Li, Y. Guo, X. Wang, S. Yang, M. Huang, and H. Zhang, "Learning to optimize: Reference vector reinforcement learning adaption to constrained many-objective optimization of industrial copper burdening system," *IEEE Trans. Cybern.*, early access, Jul. 14, 2021, doi: 10.1109/TCYB.2021.3086501.

[9] Z. Shang, Y. Qin, Y. Wang, F. Li, H. Shen, and J. Wang, "The IGD+ indicator and reference vector guided evolutionary algorithm for many-objective optimization problems," in *Proc. Austral. New Zealand Control Conf. (ANZCC)*, Nov. 2021, pp. 161–166.

[10] L. Ma, M. Huang, S. Yang, R. Wang, and X. Wang, "An adaptive localized decision variable analysis approach to large-scale multiobjective and many-objective optimization," *IEEE Trans. Cybern.*, early access, Jan. 21, 2021, doi: 10.1109/TCYB.2020.3041212.

[11] J. Li, G. Chen, M. Li, and H. Chen, "An enhanced-indicator based many-objective evolutionary algorithm with adaptive reference point," *Swarm Evol. Comput.*, vol. 55, Jun. 2020, Art. no. 100669.

[12] Z.-Z. Liu, Y. Wang, and B.-C. Wang, "Indicator-based constrained multi-objective evolutionary algorithms," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 51, no. 9, pp. 5414–5426, Sep. 2021.

[13] E. Zitzler and S. Künzli, "Indicator-based selection in multiobjective search," in *Proc. Int. Conf. Parallel Problem Solving Nature*. Berlin, Germany: Springer, 2004, pp. 832–842.

[14] H. Li, J. Deng, Q. Zhang, and J. Sun, "Adaptive epsilon dominance in decomposition-based multiobjective evolutionary algorithm," *Swarm Evol. Comput.*, vol. 45, pp. 52–67, Mar. 2019.

[15] Z. Liang, T. Luo, K. Hu, X. Ma, and Z. Zhu, "An indicator-based many-objective evolutionary algorithm with boundary protection," *IEEE Trans. Cybern.*, vol. 51, no. 9, pp. 4553–4566, Sep. 2021.

[16] G. Dhiman, M. Soni, H. Pandey, A. Slowik, and H. Kaur, "A novel hybrid hypervolume indicator and reference vector adaptation strategies based evolutionary algorithm for many-objective optimization," *Eng. Comput.*, vol. 37, no. 4, pp. 3017–3035, Feb. 2021.

[17] J. G. Falcon-Cardona, H. Ishibuchi, C. A. Coello Coello, and M. Emmerich, "On the effect of the cooperation of indicator-based multiobjective evolutionary algorithms," *IEEE Trans. Evol. Comput.*, vol. 25, no. 4, pp. 681–695, Aug. 2021.

[18] X. Cai, Z. Mei, Z. Fan, and Q. Zhang, "A constrained decomposition approach with grids for evolutionary multiobjective optimization," *IEEE Trans. Evol. Comput.*, vol. 22, no. 4, pp. 564–577, Aug. 2018.

[19] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE Trans. Evol. Comput.*, vol. 6, no. 2, pp. 182–197, Apr. 2002.

[20] K. Deb and H. Jain, "An evolutionary many-objective optimization algorithm using reference-point-based nondominated sorting approach, part I: Solving problems with box constraints," *IEEE Trans. Evol. Comput.*, vol. 18, no. 4, pp. 577–601, Aug. 2014.

[21] M. V. Patil and A. J. Kulkarni, "Pareto dominance based multiobjective cohort intelligence algorithm," *Inf. Sci.*, vol. 538, pp. 69–118, Oct. 2020.

[22] E. Zitzler, M. Laumanns, and L. Thiele, "SPEA2: Improving the strength Pareto evolutionary algorithm," in *Proc. Evol. Methods Design, Optim., Control Appl. Ind. Problems (EUROGEN)*, May 2001, pp. 95–100.

[23] M. Li, S. Yang, and X. Liu, "Shift-based density estimation for Pareto-based algorithms in many-objective optimization," *IEEE Trans. Evol. Comput.*, vol. 18, no. 3, pp. 348–365, Jun. 2014.

[24] V. Chankong and Y. Y. Haimes, *Multiobjective Decision Making: Theory and Methodology*. Amsterdam, The Netherlands: North Holland, 1983.

[25] L. Hu, J. Wei, and Y. Liu, "A novel nonlinear expanded dominance relation based evolutionary algorithm for many-objective optimization problems," *IEEE Access*, vol. 9, pp. 17335–17349, 2021.

[26] Y. Tian, R. Cheng, X. Zhang, Y. Su, and Y. Jin, "A strengthened dominance relation considering convergence and diversity for evolutionary many-objective optimization," *IEEE Trans. Evol. Comput.*, vol. 23, no. 2, pp. 331–345, Apr. 2019.

[27] Q. Gu, H. Chen, L. Chen, X. Li, and N. N. Xiong, "A many-objective evolutionary algorithm with reference points-based strengthened dominance relation," *Inf. Sci.*, vol. 554, pp. 236–255, Apr. 2021.

[28] J. Shen, P. Wang, and X. Wang, "A controlled strengthened dominance relation for evolutionary many-objective optimization," *IEEE Trans. Cybern.*, vol. 52, no. 5, pp. 3645–3657, May 2022, doi: 10.1109/TCYB.2020.3015998.

[29] H. Bai, J. Zheng, G. Yu, S. Yang, and J. Zou, "A Pareto-based many-objective evolutionary algorithm using space partitioning selection and angle-based truncation," *Inf. Sci.*, vol. 478, pp. 186–207, Apr. 2019.

[30] L. Zhen, M. Li, D. Peng, and X. Yao, "Objective reduction for visualising many-objective solution sets," *Inf. Sci.*, vol. 512, pp. 278–294, Feb. 2020.

[31] R. S. Mendes, V. Lush, E. F. Wanner, F. V. C. Martins, J. F. M. Sarubbi, and K. Deb, "Online clustering reduction based on parametric and nonparametric correlation for a many-objective vehicle routing problem with demand responsive transport," *Expert Syst. Appl.*, vol. 170, May 2021, Art. no. 114467.

[32] H. K. Singh, A. Isaacs, and T. Ray, "A Pareto corner search evolutionary algorithm and dimensionality reduction in many-objective optimization problems," *IEEE Trans. Evol. Comput.*, vol. 15, no. 4, pp. 539–556, Aug. 2011.

[33] S. Yang, M. Li, X. Liu, and J. Zheng, "A grid-based evolutionary algorithm for many-objective optimization," *IEEE Trans. Evol. Comput.*, vol. 17, no. 5, pp. 721–736, Oct. 2013.

[34] Y. Liu, J. Wei, X. Li, and M. Li, "Generational distance indicator-based evolutionary algorithm with an improved niching method for many-objective optimization problems," *IEEE Access*, vol. 7, pp. 63881–63891, 2019.

[35] X. Cai, H. Sun, and Z. Fan, "A diversity indicator based on reference vectors for many-objective optimization," *Inf. Sci.*, vols. 430–431, pp. 467–486, Mar. 2018.

[36] V. Palakonda and R. Mallipeddi, "Pareto dominance-based algorithms with ranking methods for many-objective optimization," *IEEE ACCESS*, vol. 5, pp. 11043–11053, 2017.

[37] J. Yuan, "Dynamic grid-based uniform search for solving constrained multiobjective optimization problems," *Memetic Comput.*, vol. 13, pp. 1–12, Nov. 2021.

[38] D. Corne, J. D. Knowles, and M. Oates, "The Pareto envelope-based selection algorithm for multiobjective optimization," in *Proc. Int. Conf. Parallel Problem Solving Nature*. Berlin, Germany: Springer, 2000, pp. 839–848.

[39] G. G. Yen and H. Lu, "Dynamic multiobjective evolutionary algorithm: Adaptive cell-based rank and density estimation," *IEEE Trans. Evol. Comput.*, vol. 7, no. 3, pp. 253–274, Jun. 2003.

[40] L. Rachmawati and D. Srinivasan, "Dynamic resizing for grid-based archiving in evolutionary multi objective optimization," in *Proc. IEEE Congr. Evol. Comput. (CEC)*, Singapore, Sep. 2007, pp. 3975–3982.

[41] J. Bader and E. Zitzler, "HypE: An algorithm for fast hypervolume-based many-objective optimization," *Evol. Comput.*, vol. 19, no. 1, pp. 45–76, Mar. 2011.
[42] C. Dai, Y. Ji, and J. Li, "An improvement evolutionary algorithm based on grid-based Pareto dominance for many-objective optimization," in *Proc. 14th Int. Conf. Comput. Intell. Secur. (CIS)*, Hangzhou, China, Nov. 2018, pp. 15–19.
[43] J. K. Chong and K. C. Tan, "A novel grid-based differential evolution (DE) algorithm for many-objective optimization," in *Proc. IEEE Congr. Evol. Comput. (CEC)*, Canada, BC, Canada, Jul. 2016, pp. 2776–2783.
[44] M. Javadi, C. Ramirez-Atencia, and S. Mostaghim, "A novel grid-based crowding distance for multimodal multi-objective optimization," in *Proc. IEEE Congr. Evol. Comput. (CEC)*, Glasgow, U.K., Jul. 2020, pp. 1–8.
[45] Q. B. Jin, Y. T. Li, and W. Cai, "A novel grid-based bidirectional local search algorithm for many-objective optimization," in *Proc. IEEE 3rd Adv. Inf. Technol., Electron. Autom. Control Conf. (IAEAC)*, Chongqing, China, Oct. 2018, pp. 274–279.
[46] X. Cai, H. Sun, Q. Zhang, and Y. Huang, "A grid weighted sum Pareto local search for combinatorial multi and many-objective optimization," *IEEE Trans. Cybern.*, vol. 49, no. 9, pp. 3586–3598, Sep. 2019.
[47] S. Rostami and A. Shenfield, "A multi-tier adaptive grid algorithm for the evolutionary multi-objective optimisation of complex problems," *Soft Comput.*, vol. 21, no. 17, pp. 4963–4979, Jun. 2016.
[48] S. Chakraborty, D. Paul, S. Das, and J. Xu, "Entropy weighted power k-means clustering," in *Proc. Int. Conf. Artif. Intell. Statist.*, 2020, pp. 691–701.
[49] L. Bai, J. Liang, and F. Cao, "A multiple k-means clustering ensemble algorithm to find nonlinearly separable clusters," *Inf. Fusion*, vol. 61, pp. 36–47, Sep. 2020.
[50] K. P. Sinaga and M.-S. Yang, "Unsupervised K-means clustering algorithm," *IEEE Access*, vol. 8, pp. 80716–80727, 2020.
[51] E. J. Hughes, "MSOPS-II: A general-purpose many-objective optimiser," in *Proc. IEEE Congr. Evol. Comput.*, Singapore, Sep. 2007, pp. 3944–3951.
[52] S. Jiang and S. Yang, "A strength Pareto evolutionary algorithm based on reference direction for multiobjective and many-objective optimization," *IEEE Trans. Evol. Comput.*, vol. 21, no. 3, pp. 329–346, Jun. 2017.

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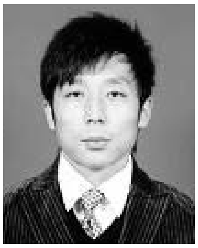

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