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Fuzzy System-Based Position Tracking Iterative Learning Control for Tank Gun Control Systems With Error Constraints

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ABSTRACT In order to get accurate position tracking and effective system constraint together for tank gun control systems, a fuzzy system-based barrier adaptive iterative learning control scheme is proposed. Firstly, the error tracking strategy is applied to solve the initial position problem of tank gun iterative learning control systems. Then, a barrier Lyapunov function is adopted to controller design for the system constraint. In addition, a fuzzy system is used as an approximator to compensate for the nonparametric uncertainties, and difference learning learning approach is used to estimate the optimal parameters of fuzzy systems. It is shown that the system constraints are guaranteed and position tracking error converges to a tunable residual set as the iteration number increases.

INDEX TERMS Tank gun control systems, iterative learning control, fuzzy systems, Barrier Lyapunov function.

I. INTRODUCTION

Iterative learning control (ILC) is well-known for the structural simplicity and model-free feature, and the prominent ability in repeated tracking control or periodic disturbance rejection [1], such that it is suitable for the controller designs of repetitive motion systems and repetitive process systems. By utilizing the repetitive nature in the learning process, ILC algorithms can improve the tracking performance gradually and derive perfect tracking over the iteration domain, which has enticed sustained research interest from control community during the past decades, resulting in a large number of reports on the subject [2]–[9]. To overcome the several difficulties or limitations of traditional ILC, Lypunov-based ILC has been studied in the past two decades, which is also called as adaptive ILC. Adaptive ILC can be regarded as a combination of ILC and adaptive control [10]–[12], which tunes control parameters between successive iterations instead of directly adjusting the control input.

Tanks are different from normal weapons in that they play both offensive roles and defensive roles in modern battles, i.e., they can improve soldiers' surviving ability and enhance efficiency of artillery firepower meanwhile. During fighting, tank need fire shells in complicated battlefield circumstances. Thereby, tank gun control systems have to face some characteristics, including frictional moment, gear backlash and parameter drifts. This makes the underlying control problem challenging and nontrivial. Some reports on the subject, such as PID control [13], variable structure control [14], optimal control [15], adaptive control [16], active disturbance rejection control [17], have been published during the past decades. Thanks to the above-mentioned advantages of adaptive ILC, the design of tank gun adaptive ILC systems has enticed sustained research interest from control community during the recent years.

Currently, in the research of ILC on tank gun control systems, there are some major challenges and interesting topics, three of which will be addressed in this work. The first challenge is about system constraint of tank gun control systems during operations. In a real application, sometimes, the system output, or the tracking error, should remain in a certain compact set for system specification and safety consideration. The violation of such a constraint may cause damage to the system or even hazards to human operators or users. Over the past three decades, several corresponding solutions to system constraints, e.g., maximal output admissible set strategy [18],

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constrained model predictive control [19], reference governor approach [20], convex optimization strategy [21] and barrier Lyapunov function approach [23], have been proposed. In the field of ILC, adaptive ILC with system constraints has been explored during the past decade [24]–[26]. These achievements further promotes the barrier adaptive ILC research and application of tank gun ILC systems [27]. Overall, the related result is very few.

The second issue is about initial position problem of tank gun ILC systems. As for general systems, zero initial error is a fundamental prerequisite for traditional ILC systems. That is, if the prerequisite is broken, traditional ILC algorithms can not work well, and even if the nonzero initial error in traditional ILC systems is very slight, the divergence of the tracking error may still happen [22]. It should be noted that the zero initial-error condition cannot be met because the perfect system resetting in each iteration is not be practically implementable. Hence, it is necessary to develop ILC schemes which can work well under nonzero initial error condition [28]–[30]. Over the past years, several adaptive ILC algorithms for tank gun control systems have been reported in literature, most of which cover the initial position problem of ILC. In [31]–[33], the velocity tracking problem of tank gun control systems are discussed, with alignment condition [31], time-varying boundary layer method [32] and error-tracking strategy [33] being applied to overcome nonzero initial system errors, respectively. The angle position tracking control for tank gun control systems with periodic reference signal is studied in [34], where an adaptive repetitive control strategy is used to compensate for nonparametric uncertainties. To date, few literature has reported ILC algorithms for tank gun control systems with system constraints under zero initial error condition [27]. How to design an error-tracking ILC algorithm for tank gun control systems with system constraints is still unclear.

In tank gun control systems, there exist complicated uncertainties and external disturbances. Parametric uncertainties are common parametric uncertainties in nonlinear systems [22], including tank gun control systems. ILC can deal with unknown iterative-independent functions well through learning method, whether they are time-invariant or time-varying. Specifically, for the unknown time-invariant constants in system model, both differential learning approach and difference learning approach are effective to estimate them; for unknown iteration-independent time-varying parameters, difference learning approach is the proper estimation tool. Compared to parametric uncertainties in tank gun control systems, there still exist unparameterizable uncertainties to be handled [1]. There are usually two strategies to handle nonparametric uncertainties. While the nonparametric uncertainty meets Lipschitz(-like) continuous condition, robust learning approach is a proper solution. On another hand, both fuzzy systems and neural networks may be approximate nonparametric uncertainties in the system models. Up to now, there have been several literature in which fuzzy system-based ILC design was reported [35]–[37]. Up to now, no work discusses the fuzzy system-based approximation to the nonparametric uncertainties in tank gun control systems.

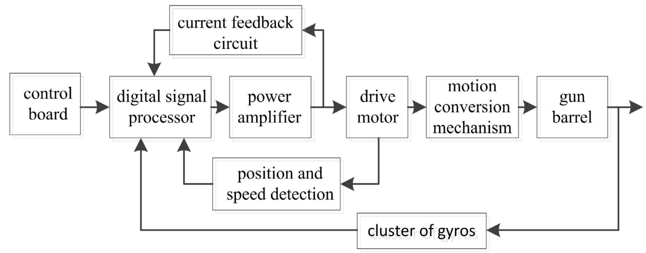

FIGURE 1. The configuration of tank AC all-electric gun control system.

Inspired by the aforementioned studies, this work investigates the adaptive ILC design for tank gun control systems with the constraint requirement on the angle position tracking error and angular velocity tracking error. To address this requirement, we adopt a filtering-error based barrier Lyapunov function to design controller. To overcome the obstacle of nonzero initial error in ILC design, we construct a desired error tracking trajectory for implementing error tracking strategy. We show that under the proposed adaptive ILC scheme, the difference between the tracking error and the desired error trajectory converges along the iteration axis. The main contributions of this work can be summarized as follows:

(i) The error-tracking ILC design stratey is considered for tank gun control systems with error constraints;

(ii) A novel construction method of desired error trajectory for tank gun control systems is proposed;

(iii) To deal with the nonparametric uncertainty in the considered tank gun ILC system, the approximation approach of difference learning fuzzy system is adopted.

This paper is organized as follows. The description of the system mathematical model is introduced in Section II. A barrier error-tracking adaptive iterative leaning controller is designed in the Section III. In addition, the stability of the proposed control method is shown in Section IV. In Section V, numerical simulation results are presented to compare the proposed error-tracking barrier adaptive ILC against the barrier-free adaptive ILC. Finally, Section VI concludes the work.

II. PROBLEM FORMULATION

In this work, the considered all electric gun control system adopts a speed-current dual closed-loop control strategy. The control structure of this control system is shown in Fig. 1. The system is made up of a vertical subsystem and a horizontal subsystem, where the vertical subsystem is actually a AC servo driving system.

The block diagram of this AC servo driving system, a careful reduction of a complex nonlinear simulation model, is shown in Fig. 2. The definition of corresponding variables and parameters in this figure is presented in Table 1.

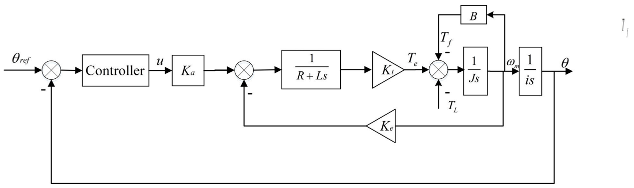

FIGURE 2. Transfer function block diagram of AC servo driving system.

TABLE 1. The definitions of symbols.

Symbol	Definition
θ_{ref}	reference position angle
θ	real position angle
ω_m	angular velocity
R	resistance
L	inductance of the motor armature circuit
u	output voltage of the position loop
K_a	amplifier gain
K_t	motor torque factor
K_e	electric torque coefficient
T_e	motor torque
T_L	load torque disturbance
T_f	friction torque disturbance
J	total moment of inertia to the rotor
B	viscous friction coefficient
i	moderating ratio
s	Laplace operator

Now, three relation formulas may be derived from Fig. 2 as follows:

$$\frac{(K_a u(s) - K_e \omega_m(s))K_t}{R + Ls} = T_e(s), \tag{1}$$

$$\frac{T_e(s) - T_L(s)}{Js + B} = \omega_m(s) \tag{2}$$

and

$$\omega_m(s) = is\theta(s). \tag{3}$$

For the fact that $\frac{L}{R} \ll 1$ holds [38], we may deduce

$$\frac{1}{R + Ls} = \frac{1}{R(1 + Ls/R)} \approx \frac{1}{R}. \tag{4}$$

Substituting (1) into (4), we have

$$\frac{(K_a u(s) - K_e \omega_m(s))K_t}{R} = T_e(s). \tag{5}$$

Then, through algebraic operation, from (2), we can easily get

$$T_e(s) = (Js + B)\omega_m(s) + T_L(s). \tag{6}$$

The two above formulas yields

$$\frac{K_t(K_a u(s) - K_e \omega_m(s))}{R} = T_L(s) + (Js + B)\omega_m(s). \tag{7}$$

By substituting (3) into (7), we have

$$iRJs^2\theta(s) + (K_t K_e + RB)is\theta(s) - K_t K_a u(s) + RT_L(s) = 0. \tag{8}$$

Further, we can get the time domain expression of (8) as

$$\ddot{\theta} = -(\frac{B}{J} + \frac{K_e K_t}{J(R + Ls)})\dot{\theta} + \frac{K_a K_t}{iJ(R + Ls)}u - \frac{T_f + T_L}{iJ}. \tag{9}$$

Define $x_1 = \theta, x_2 = \dot{\theta}$ and $y = x_1$. From (9), we get the dynamics of tank gun control systems in the kth iteration as follows:

$$\begin{cases} \dot{x}_{1,k} = x_{2,k}, \\ \dot{x}_{2,k} = -(\frac{B}{J} + \frac{K_e K_t}{JR})x_{2,k} + \frac{K_a K_t}{iJR}u_k \\ \quad - \frac{T_{f,k} + T_{L,k}}{iJ} \\ y_k = x_{1,k} \end{cases} \tag{10}$$

The control objective of this work is to design an adaptive learning controller u_k to let y_k accurately track y_d as iteration number increases.

III. CONTROL SYSTEM DESIGN

Define $e_{1,k} = x_{1,k} - x_{1,d}, e_{2,k} = x_{2,k} - x_{2,d}, \boldsymbol{e}_k(t) = [e_{1,k}, e_{2,k}]^T$, where $x_{1,d} = y_d, x_{2,d} = \dot{y}_d$. From (10), we can obtain

$$\begin{cases} \dot{e}_{1,k} = e_{2,k}, \\ \dot{e}_{2,k} = -(\frac{B}{J} + \frac{K_e K_t}{JR})x_{2,k} + \frac{K_a K_t}{iJR}u_k - \frac{T_{f,k} + T_{L,k}}{iJ} \\ \quad - \dot{x}_{2,d}. \end{cases}$$

Let $\varepsilon_{1,k} = e_{1,k} - e^*_{1,k}$ and $\varepsilon_{2,k} = e_{2,k} - e^*_{2,k}$, where $e^*_{1,k}$ and $e^*_{2,k}$ are formed as follows:

$$\begin{aligned} e^*_{1,k}(t) &= e_{1,k}(0)h(t) + e_{2,k}(0)h(t)\sin(t), \\ e^*_{2,k}(t) &= e_{2,k}(0)\dot{h}(t) + e_{2,k}(0)\dot{h}(t)\cos(t), \end{aligned} \tag{11}$$

where t_ε is a time point between 0 and T,

$$h(t) = \begin{cases} (1 - \frac{t}{t_\varepsilon})^3, & 0 < t < t_\varepsilon \\ 0, & t_\varepsilon \le t \le T \end{cases} \tag{12}$$

From the above constructions, we can see that (i) $\boldsymbol{e}^*_k(0) = \boldsymbol{e}_k(0)$, (ii) $\boldsymbol{e}^*_k(t) = 0$ for $t \in [t_\varepsilon, T]$, and (iii) $\boldsymbol{e}^*_k(t)$ is continuously differentiable for $0 < t < T$. If $\boldsymbol{e}_k(t)$ can follow $\boldsymbol{e}^*_k(t)$ over $[0, T]$, then the excellent trajectory tracking from $\boldsymbol{x}_k(t)$ to $\boldsymbol{x}_d(t)$ may be achieved during $[t_\varepsilon, T]$. In the next step, we will design an adaptive iterative learning controller to achieve the control task. For the sake of brevity, in this paper, arguments are sometimes omitted when no confusion is likely to arise.

According to the definitions of $\varepsilon_{1,k}$ and $\varepsilon_{2,k}$, we have

$$\begin{cases} \dot{\varepsilon}_{1,k} = \varepsilon_{2,k}, \\ \dot{\varepsilon}_{2,k} = -(\frac{B}{J} + \frac{K_e K_t}{JR})x_{2,k} + \frac{K_a K_t}{iJR}u_k \\ \quad - \frac{T_{f,k} + T_{L,k}}{iJ} - \dot{x}_{2,d} - \dot{e}^*_{2,k}. \end{cases} \tag{13}$$

Let

$$s_{\varepsilon,k} = \alpha\varepsilon_{1,k} + \varepsilon_{2,k} \tag{14}$$

with $\alpha > 0$. Then, combining (13) with (14) yields

$$\dot{s}_{\varepsilon,k} = \alpha\varepsilon_{2,k} - (\frac{B}{J} + \frac{K_eK_t}{JR})x_{2,k} + \frac{K_aK_t}{iJR}u_k - \frac{T_{f,k} + T_{L,k}}{iJ} - \ddot{x}_d - \dot{e}^*_{2,k}. \quad (15)$$

Define a barrier Lyapunov function in the kth iteration as

$$V_k = \frac{s^2_{\varepsilon,k}}{2\varpi(b_s^2 - s^2_{\varepsilon,k})} \quad (16)$$

with $\varpi = \frac{K_aK_t}{iJR}$. The time derivative of V_k is

$$\begin{aligned}\dot{V}_k &= \frac{b_s^2}{(b_s^2 - s^2_{\varepsilon,k})^2}s_{\varepsilon,k}[\varpi^{-1}\alpha\varepsilon_{2,k} - \varpi^{-1}(\frac{B}{J} + \frac{K_eK_t}{JR}) \\ &\quad \times x_{2,k} + u_k - \frac{T_{f,k} + T_{L,k}}{\varpi iJ} - \varpi^{-1}\ddot{x}_d - \varpi^{-1}\dot{e}^*_{2,k}] \\ &= \frac{b_s^2}{(b_s^2 - s^2_{\varepsilon,k})^2}s_{\varepsilon,k}[\boldsymbol{p}^T\boldsymbol{\varphi}_k - \frac{T_{f,k} + T_{L,k}}{\varpi iJ} + u_k], \end{aligned} \quad (17)$$

in which $\boldsymbol{p} = [\varpi^{-1}\alpha, -\varpi^{-1}(\frac{B}{J} + \frac{K_eK_t}{JR}), -\varpi^{-1}\ddot{x}_d, -\varpi^{-1}]$, $\boldsymbol{\varphi}_k = [\varepsilon_{2,k}, x_{2,k}, 1, \dot{e}^*_{2,k}]^T$.

Without loss of generality, we make the following assumption as

$$-\frac{T_{f,k} + T_{L,k}}{\varpi iJ} = f(\boldsymbol{x}_k(t)) + d_k(t) \quad (18)$$

where $f(\boldsymbol{x}_k(t))$ is an unknown real continuous nonlinear function, $d_k(t)$ represents the bounded noncontinuous disturbance. According to (17) and (18), we have

$$\dot{V}_k = \frac{b_s^2}{(b_s^2 - s^2_{\varepsilon,k})^2}s_{\varepsilon,k}[\boldsymbol{p}^T\boldsymbol{\varphi}_k + f(\boldsymbol{x}_k) + d_k + u_k]. \quad (19)$$

To compensate for the unknown nonlinear function $f(\boldsymbol{x}_k(t))$, a fuzzy system $\hat{f}(\boldsymbol{x}_k(t), \boldsymbol{\theta}_k(t))$, which performs as the approximator of $f(\boldsymbol{x}_k(t))$, is described as follows:

$$\begin{aligned}\hat{f}(\boldsymbol{x}_k(t), \boldsymbol{\theta}_k(t)) &= \frac{\sum_{l=1}^{m}\theta_{l,k}(t)\prod_{j=1}^{2}\mu_{f_{j,l}}(x_{j,k}(t))}{\sum_{l=1}^{m}\prod_{j=1}^{2}\mu_{f_{j,l}}(x_{j,k}(t))} \\ &= \sum_{l=1}^{m}\theta_{l,k}(t)z_l(\boldsymbol{x}_k) \end{aligned} \quad (20)$$

where

$$z_l(\boldsymbol{x}_k) = \frac{\prod_{j=1}^{2}\mu_{f_{j,l}}(x_{j,k}(t))}{\sum_{l=1}^{m}\prod_{j=1}^{2}\mu_{f_{j,l}}(x_{j,k}(t))}. \quad (21)$$

Let $\boldsymbol{\theta}_k = [\theta_{1,k}, \theta_{2,k}, \ldots, \theta_{m,k}]^T$ and $\boldsymbol{z}_k = [z_{1,k}, z_{2,k}, \ldots, z_{m,k}]^T$. Then, (20) can be rewritten into a compact form as follows:

$$\hat{f}(\boldsymbol{x}_k(t), \boldsymbol{\theta}_k(t)) = \boldsymbol{\theta}_k^T\boldsymbol{z}_k. \quad (22)$$

In the representation of (20), m is the number of fuzzy rules, $w_{l,k}(t)$ is the consequent parameter, and $\mu_{f_{jl}}(x_{j,k}(t))$ is the fuzzy membership function for the fuzzy system $\hat{f}(\boldsymbol{x}_k(t), \boldsymbol{\theta}_k(t))$. The fuzzy system is expressed as a series of radial basis functions expansion with the basis functions as $z_l(\boldsymbol{x}_k)$. It is well known that the fuzzy system (20) can uniformly approximate real continuous nonlinear function on a compact set $\mathcal{A}_c \subset \mathbb{R}^{2\times 1}$. There exists optimal weight $\boldsymbol{\theta}^*$ such that

$$\sup_{\boldsymbol{x}_k \in \mathcal{A}_c} |f(\boldsymbol{x}_k(t)) - \boldsymbol{\theta}^{*T}\boldsymbol{z}_k| \le \epsilon^* \quad (23)$$

for arbitrary $\epsilon^* > 0$ [39].

According to (23), we have

$$s_{\varepsilon,k}f(\boldsymbol{x}_k(t)) \le s_{\varepsilon,k}\boldsymbol{\theta}^{*T}\boldsymbol{z}_k + |s_{\varepsilon,k}|\epsilon^* \quad (24)$$

On the basis of (19) and (24), we have

$$\dot{V}_k = \frac{b_s^2 s_{\varepsilon,k}(\boldsymbol{p}^T\boldsymbol{\varphi}_k + \boldsymbol{\theta}^{*T}\boldsymbol{z}_k + u_k)}{(b_s^2 - s^2_{\varepsilon,k})^2} + \frac{b_s^2|s_{\varepsilon,k}|\rho}{(b_s^2 - s^2_{\varepsilon,k})^2}, \quad (25)$$

where $\rho \triangleq \epsilon^* + \sup_{t\in[0,T]}(d_k)$. On the basis of (24), we design the iterative learning controller as follows:

$$u_k = -\gamma s_{\varepsilon,k} - \boldsymbol{p}_k^T\boldsymbol{\varphi}_k - \boldsymbol{\theta}_k^T\boldsymbol{z}_k - \rho_k \mathrm{sat}_{-1,1}\left(\frac{\varrho s_{\varepsilon,k}}{\delta + |s_{\varepsilon,k}|}\right), \quad (26)$$

$$\boldsymbol{p}_k = \mathrm{sat}_{\underline{p},\bar{p}}(\boldsymbol{p}_{k-1}) + \frac{b_s^2\mu_1 s_{\varepsilon,k}\boldsymbol{\varphi}_k}{(b_s^2 - s^2_{\varepsilon,k})^2}, \boldsymbol{p}_{-1} = 0, \quad (27)$$

$$\boldsymbol{\theta}_k = \mathrm{sat}_{\underline{\theta},\bar{\theta}}(\boldsymbol{\theta}_{k-1}) + \frac{b_s^2\mu_2 s_{\varepsilon,k}\boldsymbol{z}_k}{(b_s^2 - s^2_{\varepsilon,k})^2}, \boldsymbol{\theta}_{-1} = 0, \quad (28)$$

$$\rho_k = \mathrm{sat}_{0,\bar{\rho}}(\rho_{k-1}) + \frac{b_s^2\mu_3|s_{\varepsilon,k}|}{(b_s^2 - s^2_{\varepsilon,k})^2}, \rho_{-1} = 0, \quad (29)$$

where $\varrho > 1$, $0 < \delta < 1$, $\gamma > 0$, $\mu_1 > 0$, $\mu_2 > 0$, $\mu_3 > 0$, and $\boldsymbol{p}_k$, $\boldsymbol{\theta}_k$ and ρ_k are used to estimate $\boldsymbol{p}$, $\boldsymbol{\theta}$ and ρ, respectively. For the definition of saturation function $\mathrm{sat}_{\cdot,\cdot}(\cdot)$, seen [27].

Remark 1: In (26), to get better control performance, $\mathrm{sat}_{-1,1}\left(\frac{\varrho s_{\varepsilon,k}}{\delta+|s_{\varepsilon,k}|}\right)$ is adopted in the design of robust control term, instead of $\mathrm{sign}(s_{\varepsilon,k})$ and $\mathrm{sat}_{-1,1}\left(\frac{s_{\varepsilon,k}}{\delta}\right)$.

Remark 2: As shown in (27), the optimal weight of adaptive iterative learning fuzzy system in this work is estimated by using difference learning approach; whereas in the existing adaptive fuzzy system control algorithms, the optimal weight is estimated by using differential learning approach.

IV. CONVERGENCE ANALYSIS

Theorem 1: Consider the closed-loop tank gun adaptive learning control system consisting of the plant (10), the fuzzy system-based adaptive iterative learning controller (26) and the difference learning laws (27)-(29). The tracking performance and system stability are guaranteed as follows:

t1) Constraints on the system signals will not be violated as $|s_{\varepsilon,k}(t)| \le \beta b_s$, $|\varepsilon_{1,k}(t)| \le \frac{\beta b_s}{\alpha}$, $|\varepsilon_{2,k}(t)| \le 2\beta b_s$, with the definition of β given in (49);

t2) As the iteration number k increases, the system error converges in the sense that $|s_{\varepsilon,k}(t)| \le \frac{\delta}{\varrho-1}$ for $t \in [0, T]$, which means $|e_{1,k}(t)| \le \frac{\delta}{\alpha(\varrho-1)}$ and $|e_{2,k}(t)| \le \frac{2\delta}{\varrho-1}$ hold for $t \in [t_\varepsilon, T]$ as the iteration number k increases;

t3) All signals in the closed-loop tank gun control system are bounded.

Proof:

t1) Substituting (26) into (25) yields

$$\dot{V}_k \le \frac{b_s^2}{(b_s^2 - s_{\varepsilon,k}^2)^2}(s_{\varepsilon,k}\tilde{\boldsymbol{p}}_k^T \boldsymbol{\varphi}_k + s_{\varepsilon,k}\tilde{\boldsymbol{\theta}}_k^T \boldsymbol{z}_k + |s_{\varepsilon,k}|\rho) - \frac{\gamma b_s^2 s_{\varepsilon,k}^2}{(b_s^2 - s_{\varepsilon,k}^2)^2} - s_{\varepsilon,k}\rho_k \mathrm{sat}_{-1,1}\left(\frac{\varrho s_{\varepsilon,k}}{\delta + |s_{\varepsilon,k}|}\right). \tag{30}$$

where $\tilde{\boldsymbol{p}}_k = \boldsymbol{p} - \boldsymbol{p}_k$ and $\tilde{\boldsymbol{\theta}}_k = \boldsymbol{\theta}^*(t) - \boldsymbol{\theta}_k$. Note that we can see $\rho_k \ge 0$ for any k from (29).

While $s_{\varepsilon,k} > \frac{\delta}{\varrho-1}$, due to $\frac{\delta}{\varrho-1} > 0$, it is obvious that $(\varrho-1)s_{\varepsilon,k} > \delta$ and $\frac{(\varrho-1)s_{\varepsilon,k}+s_{\varepsilon,k}}{\delta+|s_{\varepsilon,k}|} > 1$ hold. According to this and the definition of saturation function, we can see that $\mathrm{sat}_{-1,1}\left(\frac{\varrho s_{\varepsilon,k}}{\delta+|s_{\varepsilon,k}|}\right) = \mathrm{sat}_{-1,1}\left(\frac{(\varrho-1)s_{\varepsilon,k}+s_{\varepsilon,k}}{\delta+|s_{\varepsilon,k}|}\right) = 1$. Therefore,

$$s_{\varepsilon,k}\mathrm{sat}_{-1,1}\left(\frac{\varrho s_{\varepsilon,k}}{\delta + |s_{\varepsilon,k}|}\right) = |s_{\varepsilon,k}|, \forall s_{\varepsilon,k} > \frac{\delta}{\varrho - 1}. \tag{31}$$

While $s_{\varepsilon,k} < -\frac{\delta}{\varrho-1} < 0$, it is obvious that $(\varrho-1)s_{\varepsilon,k} < -\delta$ and $\frac{(\varrho-1)s_{\varepsilon,k}+s_{\varepsilon,k}}{\delta+|s_{\varepsilon,k}|} < -1$ hold. According to this and the definition of saturation function, we get $\mathrm{sat}_{-1,1}\left(\frac{\varrho s_{\varepsilon,k}}{\delta+|s_{\varepsilon,k}|}\right) = \mathrm{sat}_{-1,1}\left(\frac{(\varrho-1)s_{\varepsilon,k}+s_{\varepsilon,k}}{\delta+|s_{\varepsilon,k}|}\right) = -1$, such that

$$s_{\varepsilon,k}\mathrm{sat}_{-1,1}\left(\frac{\varrho s_{\varepsilon,k}}{\delta + |s_{\varepsilon,k}|}\right) = |s_{\varepsilon,k}|, \forall s_{\varepsilon,k} < -\frac{\delta}{\varrho - 1}. \tag{32}$$

From (29), we know $\rho_k \ge 0$ holds for each iteration. Hence, combining (31) with (32), we can derive that

$$s_{\varepsilon,k}\rho_k\mathrm{sat}_{-1,1}\left(\frac{\varrho s_{\varepsilon,k}}{\delta + |s_{\varepsilon,k}|}\right) = |s_{\varepsilon,k}|\rho_k, \forall |s_{\varepsilon,k}| > \frac{\delta}{\varrho - 1}. \tag{33}$$

Combining (30) with (33), while $|s_{\varepsilon,k}| > \frac{\delta}{\varrho-1}$, we deduce that

$$\begin{aligned}\dot{V}_k &\le \frac{b_s^2}{(b_s^2 - s_{\varepsilon,k}^2)^2}(s_{\varepsilon,k}\tilde{\boldsymbol{p}}_k^T \boldsymbol{\varphi}_k + s_{\varepsilon,k}\tilde{\boldsymbol{\theta}}_k^T \boldsymbol{z}_k) + \frac{b_s^2|s_{\varepsilon,k}|(\rho - \rho_k)}{(b_s^2 - s_{\varepsilon,k}^2)^2} \\ &\quad - \frac{\gamma b_s^2 s_{\varepsilon,k}^2}{(b_s^2 - s_{\varepsilon,k}^2)^2} \\ &= \frac{b_s^2}{(b_s^2 - s_{\varepsilon,k}^2)^2}(s_{\varepsilon,k}\tilde{\boldsymbol{p}}_k^T \boldsymbol{\varphi}_k + s_{\varepsilon,k}\tilde{\boldsymbol{\theta}}_k^T \boldsymbol{z}_k + |s_{\varepsilon,k}|\tilde{\rho}_k) \\ &\quad - \frac{\gamma b_s^2 s_{\varepsilon,k}^2}{(b_s^2 - s_{\varepsilon,k}^2)^2}\end{aligned} \tag{34}$$

holds.

Define a candidate barrier Lyapunov functional as follows:

$$L_k = V_k + \frac{1}{2\mu_1}\int_0^t \tilde{\boldsymbol{p}}_k^T\tilde{\boldsymbol{p}}_k d\tau + \frac{1}{2\mu_2}\int_0^t \tilde{\boldsymbol{\theta}}_k^T\tilde{\boldsymbol{\theta}}_k d\tau + \frac{1}{2\mu_3}\int_0^t \tilde{\varrho}_k^2 d\tau. \tag{35}$$

whose time derivative is

$$\begin{aligned}\dot{L}_k &\le \frac{b_s^2(s_{\varepsilon,k}\tilde{\boldsymbol{p}}_k^T \boldsymbol{\varphi}_k + s_{\varepsilon,k}\tilde{\boldsymbol{\theta}}_k^{*T} \boldsymbol{z}_k + |s_{\varepsilon,k}|\tilde{\rho}_k)}{(b_s^2 - s_{\varepsilon,k}^2)^2} + \frac{1}{2\mu_1}\tilde{\boldsymbol{p}}_k^T\tilde{\boldsymbol{p}}_k \\ &\quad + \frac{1}{2\mu_2}\tilde{\boldsymbol{\theta}}_k^T\tilde{\boldsymbol{\theta}}_k + \frac{1}{2\mu_3}\tilde{\rho}_k^2 - \frac{\gamma b_s^2 s_{\varepsilon,k}^2}{(b_s^2 - s_{\varepsilon,k}^2)^2}\end{aligned} \tag{36}$$

According to the learning laws (27)-(29), the three inequalities can be respectively deduced as follows:

$$\begin{aligned}&\frac{b_s^2}{(b_s^2 - s_{\varepsilon,k}^2)^2}s_{\varepsilon,k}\tilde{\boldsymbol{p}}_k^T \boldsymbol{\varphi}_k + \frac{1}{2\mu_1}\tilde{\boldsymbol{p}}_k^T\tilde{\boldsymbol{p}}_k \\ &= \frac{1}{2\mu_1}(\boldsymbol{p} - \boldsymbol{p}_k)^T(2\boldsymbol{p}_k - 2\mathrm{sat}_{\underline{p},\bar{p}}(\boldsymbol{p}_{k-1}) + \boldsymbol{p} - \boldsymbol{p}_k) \\ &= -\frac{1}{2\mu_1}[\boldsymbol{p}_k - \mathrm{sat}_{\underline{p},\bar{p}}(\boldsymbol{p}_{k-1})]^T[\boldsymbol{p}_k - \mathrm{sat}_{\underline{p},\bar{p}}(\boldsymbol{p}_{k-1})] \\ &\quad + \frac{1}{2\mu_1}[\mathrm{sat}_{\underline{p},\bar{p}}(\boldsymbol{p}_{k-1}^T)\mathrm{sat}_{\underline{p},\bar{p}}(\boldsymbol{p}_{k-1}) + \boldsymbol{p}^T\boldsymbol{p} \\ &\quad - 2\boldsymbol{p}^T\mathrm{sat}_{\underline{p},\bar{p}}(\boldsymbol{p}_{k-1})] \\ &\le \frac{1}{2\mu_1}[\mathrm{sat}_{\underline{p},\bar{p}}(\boldsymbol{p}_{k-1}^T)\mathrm{sat}_{\underline{p},\bar{p}}(\boldsymbol{p}_{k-1}) + \boldsymbol{p}^T\boldsymbol{p} \\ &\quad - 2\boldsymbol{p}^T\mathrm{sat}_{\underline{p},\bar{p}}(\boldsymbol{p}_{k-1})],\end{aligned} \tag{37}$$

$$\begin{aligned}&\frac{b_s^2}{(b_s^2 - s_{\varepsilon,k}^2)^2}s_{\varepsilon,k}\tilde{\boldsymbol{\theta}}_k^T \boldsymbol{z}_k + \frac{1}{2\mu_2}\tilde{\boldsymbol{\theta}}_k^T\tilde{\boldsymbol{\theta}}_k \\ &= \frac{1}{2\mu_2}(\boldsymbol{\theta}^* - \boldsymbol{\theta}_k)^T(2\boldsymbol{\theta}_k - 2\mathrm{sat}_{\underline{\theta},\bar{\theta}}(\boldsymbol{\theta}_{k-1}) + \boldsymbol{\theta}^* - \boldsymbol{\theta}_k) \\ &= -\frac{1}{2\mu_2}[\boldsymbol{\theta}_k - \mathrm{sat}_{\underline{\theta},\bar{\theta}}(\boldsymbol{\theta}_{k-1})]^T[\boldsymbol{\theta}_k - \mathrm{sat}_{\underline{\theta},\bar{\theta}}(\boldsymbol{\theta}_{k-1})] \\ &\quad + \frac{1}{2\mu_2}[\mathrm{sat}_{\underline{\theta},\bar{\theta}}(\boldsymbol{\theta}_{k-1}^T)\mathrm{sat}_{\underline{\theta},\bar{\theta}}(\boldsymbol{\theta}_{k-1}) + \boldsymbol{\theta}^{*T}\boldsymbol{\theta}^* \\ &\quad - 2\boldsymbol{\theta}^{*T}\mathrm{sat}_{\underline{\theta},\bar{\theta}}(\boldsymbol{\theta}_{k-1})] \\ &\le \frac{1}{2\mu_2}[\mathrm{sat}_{\underline{\theta},\bar{\theta}}(\boldsymbol{\theta}_{k-1}^T)\mathrm{sat}_{\underline{\theta},\bar{\theta}}(\boldsymbol{\theta}_{k-1}) + \boldsymbol{\theta}^{*T}\boldsymbol{\theta}^* \\ &\quad - 2\boldsymbol{\theta}^{*T}\mathrm{sat}_{\underline{\theta},\bar{\theta}}(\boldsymbol{\theta}_{k-1})]\end{aligned} \tag{38}$$

and

$$\begin{aligned}&\frac{b_s^2}{(b_s^2 - s_{\varepsilon,k}^2)^2}|s_{\varepsilon,k}|\tilde{\rho}_k + \frac{1}{2\mu_3}\tilde{\rho}_k^2 \\ &= \frac{1}{2\mu_3}[-\rho_k^2 + \rho^2 - 2\rho\mathrm{sat}_{\underline{0},\bar{\rho}}(\rho_{k-1}) + 2\rho_k\mathrm{sat}_{\underline{0},\bar{\rho}}(\rho_{k-1})] \\ &= \frac{1}{2\mu_3}[\mathrm{sat}_{\underline{0},\bar{\rho}}(\rho_{k-1})\mathrm{sat}_{\underline{0},\bar{\rho}}(\rho_{k-1}) + \rho^2 - 2\rho\mathrm{sat}_{\underline{0},\bar{\rho}}(\rho_{k-1})] \\ &\quad - \frac{1}{2\mu_3}[\rho_k - \mathrm{sat}_{\underline{0},\bar{\rho}}(\rho_{k-1})]^2 \\ &\le \frac{1}{2\mu_3}[\mathrm{sat}_{\underline{0},\bar{\rho}}(\rho_{k-1})\mathrm{sat}_{\underline{0},\bar{\rho}}(\rho_{k-1}) + \rho^2 - 2\rho\mathrm{sat}_{\underline{0},\bar{\rho}}(\rho_{k-1})].\end{aligned} \tag{39}$$

By the property of saturation function, from (37)-(39), we can see that there exist positive numbers c_1, c_2 and c_3, which satisfy

$$\frac{b_s^2}{(b_s^2 - s_{\varepsilon,k}^2)^2} s_{\varepsilon,k} \tilde{\boldsymbol{p}}_k^T \boldsymbol{\varphi}_k + \frac{1}{2\mu_1} \tilde{\boldsymbol{p}}_k^T \tilde{\boldsymbol{p}}_k \le \frac{c_1}{2\mu_1}, \tag{40}$$

$$\frac{b_s^2}{(b_s^2 - s_{\varepsilon,k}^2)^2} s_{\varepsilon,k} \tilde{\boldsymbol{\theta}}_k^T \boldsymbol{z}_k + \frac{1}{2\mu_2} \tilde{\boldsymbol{\theta}}_k^T \tilde{\boldsymbol{\theta}}_k \le \frac{c_2}{2\mu_2} \tag{41}$$

and

$$\frac{b_s^2}{(b_s^2 - s_{\varepsilon,k}^2)^2} |s_{\varepsilon,k}| \tilde{\rho}_k + \frac{1}{2\mu_3} \tilde{\rho}_k^2 \le \frac{c_3}{2\mu_3}, \tag{42}$$

respectively. Then, substituting (40)- (42) into (36), we derive

$$\dot{L}_k \le -\frac{\gamma b_s^2 s_{\varepsilon,k}^2}{(b_s^2 - s_{\varepsilon,k}^2)^2} + \frac{c_1}{2\mu_1} + \frac{c_2}{2\mu_2} + \frac{c_3}{2\mu_3}. \tag{43}$$

Since $L_k(0) = 0$ holds in each iteration, from (43), we have

$$L_k(t) \le \frac{c_1 T}{2\mu_1} + \frac{c_2 T}{2\mu_2} + \frac{c_3 T}{2\mu_3}, \tag{44}$$

which implies that

$$V_k(t) = \frac{s_k^2(t)}{2\varpi(b_s^2 - s_{\varepsilon,k}^2(t))} \le \frac{c_1 T}{2\mu_1} + \frac{c_2 T}{2\mu_2} + \frac{c_3 T}{2\mu_3}. \tag{45}$$

Through simple algebraic calculation,

$$b_s^2 - s_{\varepsilon,k}^2(t) \ge \frac{s_{\varepsilon,k}^2(t)}{2\varpi(\frac{c_1 T}{2\mu_1} + \frac{c_2 T}{2\mu_2} + \frac{c_3 T}{2\mu_3})} \tag{46}$$

Define $\lambda = \frac{1}{2\varpi(\frac{c_1 T}{2\mu_1} + \frac{c_2 T}{2\mu_2} + \frac{c_3 T}{2\mu_3})}$. From (54), we have

$$b_s^2 \ge (1+\lambda) s_{\varepsilon,k}^2(t), \tag{47}$$

which means

$$|s_{\varepsilon,k}(t)| \le \beta b_s. \tag{48}$$

with

$$\beta = \sqrt{\frac{1}{1+\lambda}} \tag{49}$$

It follows from (48) that

$$|\varepsilon_{1,k}(t)| \le \frac{\beta b_s}{\alpha}, \tag{50}$$

$$|\varepsilon_{2,k}(t)| \le 2\beta b_s \tag{51}$$

hold [40]. This proves t1) of the theorem.

t2) According to the definition of $\varepsilon_{1,k}$ and $\varepsilon_{2,k}$ and the property of $e_{1,k}^*$ and $e_{2,k}^*$, we can see $\varepsilon_{1,k}(0) = 0$ and $\varepsilon_{2,k}(0) = 0$. Then, by the definition of V_k, we can conclude that

$$V_k(0) = 0. \tag{52}$$

Integrating (34) from 0 to t, we deduce that

$$V_k \le V_k(0) + \int_0^t \frac{b_s^2}{(b_s^2 - s_{\varepsilon,k}^2)^2} (s_{\varepsilon,k} \tilde{\boldsymbol{p}}_k^T \boldsymbol{\varphi}_k + s_{\varepsilon,k} \tilde{\boldsymbol{\theta}}_k^T \boldsymbol{z}_k + |s_{\varepsilon,k}| \tilde{\rho}_k) d\tau - \int_0^t \frac{\gamma b_s^2 s_{\varepsilon,k}^2}{(b_s^2 - s_{\varepsilon,k}^2)^2} d\tau. \tag{53}$$

holds if $|s_{\varepsilon,k}| > \frac{\delta}{\varrho - 1}$. Combining (35), (52) with (53) yields

$$\begin{aligned}
&L_k - L_{k-1} \\
&\quad = V_k - V_{k-1} + \frac{1}{2\mu_1} \int_0^t (\tilde{\boldsymbol{p}}_k^T \tilde{\boldsymbol{p}}_k - \tilde{\boldsymbol{p}}_{k-1}^T \tilde{\boldsymbol{p}}_{k-1}) d\tau \\
&\qquad + \frac{1}{2\mu_2} \int_0^t (\tilde{\boldsymbol{\theta}}_k^T \tilde{\boldsymbol{\theta}}_k - \tilde{\boldsymbol{\theta}}_{k-1}^T \tilde{\boldsymbol{\theta}}_{k-1}) d\tau \\
&\qquad + \frac{1}{2\mu_3} \int_0^t (\tilde{\rho}_k^2 - \tilde{\rho}_{k-1}^2) d\tau \\
&\quad \le \int_0^t \frac{b_s^2}{(b_s^2 - s_{\varepsilon,k}^2)^2} (s_{\varepsilon,k} \tilde{\boldsymbol{p}}_k^T \boldsymbol{\varphi}_k + s_{\varepsilon,k} \tilde{\boldsymbol{\theta}}_k^T \boldsymbol{z}_k + |s_{\varepsilon,k}| \tilde{\rho}_k) d\tau \\
&\qquad - \int_0^t \frac{\gamma b_s^2 s_{\varepsilon,k}^2}{(b_s^2 - s_{\varepsilon,k}^2)^2} d\tau - V_{k-1} + \frac{1}{2\mu_1} \int_0^t (\tilde{\boldsymbol{p}}_k^T \tilde{\boldsymbol{p}}_k \\
&\qquad - \tilde{\boldsymbol{p}}_{k-1}^T \tilde{\boldsymbol{p}}_{k-1}) d\tau + \frac{1}{2\mu_2} \int_0^t (\tilde{\boldsymbol{\theta}}_k^T \tilde{\boldsymbol{\theta}}_k - \tilde{\boldsymbol{\theta}}_{k-1}^T \tilde{\boldsymbol{\theta}}_{k-1}) d\tau \\
&\qquad + \frac{1}{2\mu_3} \int_0^t (\tilde{\rho}_k^2 - \tilde{\rho}_{k-1}^2) d\tau.
\end{aligned} \tag{54}$$

Applying the property $(\boldsymbol{p} - \boldsymbol{p}_{k-1})^T (\boldsymbol{p} - \boldsymbol{p}_{k-1}) \ge (\boldsymbol{p} - \mathrm{sat}_{\underline{p},\bar{p}}(\boldsymbol{p}_{k-1}))^T (\boldsymbol{p} - \mathrm{sat}_{\underline{p},\bar{p}}(\boldsymbol{p}_{k-1}))$, from (27), we obtain

$$\begin{aligned}
&\frac{1}{2\mu_1} (\tilde{\boldsymbol{p}}_k^T \tilde{\boldsymbol{p}}_k - \tilde{\boldsymbol{p}}_{k-1}^T \tilde{\boldsymbol{p}}_{k-1}) + \frac{b_s^2}{(b_s^2 - s_{\varepsilon,k}^2)^2} s_{\varepsilon,k} \tilde{\boldsymbol{p}}_k^T \boldsymbol{\varphi}_k \\
&\quad \le \frac{1}{2\mu_1} [(\boldsymbol{p} - \boldsymbol{p}_k)^T (\boldsymbol{p} - \boldsymbol{p}_k) - (\boldsymbol{p} - \mathrm{sat}_{\underline{p},\bar{p}}(\boldsymbol{p}_{k-1}))^T \\
&\qquad \times (\boldsymbol{p} - \mathrm{sat}_{\underline{p},\bar{p}}(\boldsymbol{p}_{k-1}))] + \frac{b_s^2}{(b_s^2 - s_{\varepsilon,k}^2)^2} s_{\varepsilon,k} \tilde{\boldsymbol{p}}_k^T \boldsymbol{\varphi}_k \\
&\quad \le \frac{1}{2\mu_1} (2\boldsymbol{p} - \boldsymbol{p}_k - \mathrm{sat}_{\underline{p},\bar{p}}(\boldsymbol{p}_{k-1}))^T (\mathrm{sat}_{\underline{p},\bar{p}}(\boldsymbol{p}_{k-1}) - \boldsymbol{p}_k) \\
&\qquad + \frac{b_s^2}{(b_s^2 - s_{\varepsilon,k}^2)^2} s_{\varepsilon,k} \tilde{\boldsymbol{p}}_k^T \boldsymbol{\varphi}_k \\
&\quad \le \frac{1}{\mu_1} (\boldsymbol{p} - \boldsymbol{p}_k)^T [\mathrm{sat}_{\underline{p},\bar{p}}(\boldsymbol{p}_{k-1}) - \boldsymbol{p}_k + \frac{\mu_1 b_s^2 s_{\varepsilon,k} \boldsymbol{\varphi}_k}{(b_s^2 - s_{\varepsilon,k}^2)^2}] \\
&\quad = 0.
\end{aligned} \tag{55}$$

Applying the property $(\boldsymbol{\theta}^* - \boldsymbol{\theta}_{k-1})^T (\boldsymbol{\theta}^* - \boldsymbol{\theta}_{k-1}) \ge (\boldsymbol{\theta}^* - \mathrm{sat}_{\underline{\theta},\bar{\theta}}(\boldsymbol{\theta}_{k-1}))^T (\boldsymbol{\theta}^* - \mathrm{sat}_{\underline{\theta},\bar{\theta}}(\boldsymbol{\theta}_{k-1}))$, from (28), we derive

$$\begin{aligned}
&\frac{1}{2\mu_2} (\tilde{\boldsymbol{\theta}}_k^T \tilde{\boldsymbol{\theta}}_k - \tilde{\boldsymbol{\theta}}_{k-1}^T \tilde{\boldsymbol{\theta}}_{k-1}) + \frac{b_s^2}{(b_s^2 - s_{\varepsilon,k}^2)^2} s_{\varepsilon,k} \tilde{\boldsymbol{\theta}}_k^T \boldsymbol{z}_k \\
&\quad \le \frac{1}{2\mu_2} [(\boldsymbol{\theta}^* - \boldsymbol{\theta}_k)^T (\boldsymbol{\theta}^* - \boldsymbol{\theta}_k) - (\boldsymbol{\theta}^* - \mathrm{sat}_{\underline{\theta},\bar{\theta}}(\boldsymbol{\theta}_{k-1}))^T \\
&\qquad \times (\boldsymbol{\theta} - \mathrm{sat}_{\underline{\theta},\bar{\theta}}(\boldsymbol{\theta}_{k-1}))] + \frac{b_s^2}{(b_s^2 - s_{\varepsilon,k}^2)^2} s_{\varepsilon,k} \tilde{\boldsymbol{\theta}}_k^T \boldsymbol{z}_k
\end{aligned}$$

$$
\begin{aligned}
&\le \frac{1}{2\mu_2}(2\boldsymbol{\theta}^* - \boldsymbol{\theta}_k - \mathrm{sat}_{\underline{\theta},\bar{\theta}}(\boldsymbol{\theta}_{k-1}))^T(\mathrm{sat}_{\underline{\theta},\bar{\theta}}(\boldsymbol{\theta}_{k-1}) - \boldsymbol{\theta}_k) \\
&\quad + \frac{b_s^2}{(b_s^2 - s_{\varepsilon,k}^2)^2} s_{\varepsilon,k}\tilde{\boldsymbol{\theta}}_k^T \boldsymbol{z}_k \\
&\le \frac{1}{\mu_2}(\boldsymbol{\theta}^* - \boldsymbol{\theta}_k)^T[\mathrm{sat}_{\underline{\theta},\bar{\theta}}(\boldsymbol{\theta}_{k-1}) - \boldsymbol{\theta}_k + \frac{\mu_2 b_s^2 s_{\varepsilon,k}\boldsymbol{z}_k}{(b_s^2 - s_{\varepsilon,k}^2)^2}] \\
&= 0. \qquad (56)
\end{aligned}
$$

Similarly, applying the property $(\rho - \rho_{k-1})^2 \ge (\rho - \mathrm{sat}_{\underline{\rho},\bar{\rho}}(\rho_{k-1}))^2$, from (29), we have

$$
\begin{aligned}
&\frac{1}{2\mu_3}(\tilde{\rho}_k^2 - \tilde{\rho}_{k-1}^2) + \frac{b_s^2}{(b_s^2 - s_{\varepsilon,k}^2)^2}|s_{\varepsilon,k}|\tilde{\rho}_k \\
&\le \frac{1}{\mu_3}(\rho - \rho_k)\big[\mathrm{sat}_{0,\bar{\rho}}(\rho_{k-1}) - \rho_k + \mu_3 \frac{b_s^2}{(b_s^2 - s_{\varepsilon,k}^2)^2}|s_{\varepsilon,k}|\big] \\
&= 0. \qquad (57)
\end{aligned}
$$

Substituting (55)-(57) into (54) leads to

$$
L_k - L_{k-1} \le -\int_0^t \frac{\gamma b_s^2 s_{\varepsilon,k}^2}{(b_s^2 - s_{\varepsilon,k}^2)^2} d\tau - V_{k-1}, \qquad (58)
$$

which means

$$
L_k(t) \le L_0(t) - \frac{1}{2\varpi}\sum_{j=0}^{k-1} s_{\varepsilon,j}^2(t). \qquad (59)
$$

Note that (59) is a deduction on the premise of $|s_{\varepsilon,k}| > \frac{\delta}{\varrho-1}$. From (44), we can see that $L_0(t)$ is bounded for $t \in [0, T]$. Suppose that $|s_{\varepsilon,k}| > \frac{\delta}{\varrho-1}$ always holds as the iteration number increases, then

$$
L_k(t) < 0 \qquad (60)
$$

would happen when the iteration number k increases to a certain extent. Therefore,

$$
|s_{\varepsilon,k}(t)| \le \frac{\delta}{\varrho - 1} \qquad (61)
$$

holds for $t \in [0, T]$ as the iteration number k increases. It follows from (61) that

$$
|\varepsilon_{1,k}(t)| \le \frac{\delta}{\alpha(\varrho - 1)} \qquad (62)
$$

and

$$
|\varepsilon_{2,k}(t)| \le \frac{2\delta}{\varrho - 1} \qquad (63)
$$

hold for $t \in [0, T]$ as the iteration number k increases [40], which implies

$$
|e_{1,k}(t)| \le \frac{\delta}{\alpha(\varrho - 1)} \qquad (64)
$$

and

$$
|e_{2,k}(t)| \le \frac{2\delta}{\varrho - 1} \qquad (65)
$$

hold for $t \in [t_\varepsilon, T]$ as the iteration number k increases. This proves t2) of the theorem.

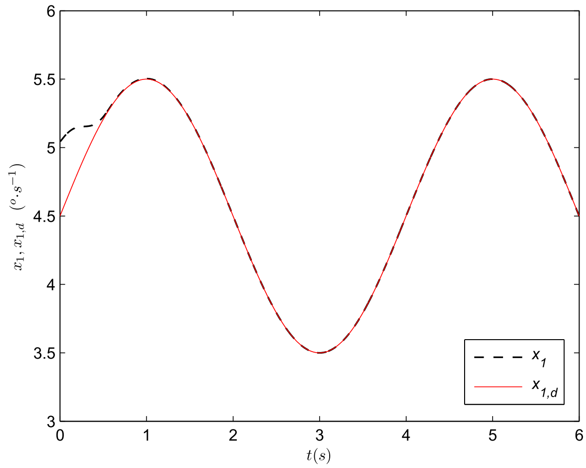

FIGURE 3. Position trajectory x_1 (barrier ILC).

t3) As shown in (48)-(51), $s_{\varepsilon,k}$, $\varepsilon_{1,k}$, $\varepsilon_{2,k}$, are bounded during each iteration. Based on this and the effect of saturation functions, $\boldsymbol{p}_k$, $\boldsymbol{\theta}_k$ and ρ_k are also guaranteed to be bounded. Further, we can verify u_k and all other signals in the closed-loop system are bounded. This proves t3) of the theorem. ■

In this work, the error constraint is achieved by constraining the maximum of $|s_{\varepsilon,k}|$ in a predetermined range, which is useful to improve the robustness and the safety of closed-loop tank gun control systems.

V. NUMERICAL SIMULATION

In this simulation, the values of parameter in the tank gun control systems are given as follows [33]: $R = 0.4\Omega$, $J = 0.0067\mathrm{kg}\cdot\mathrm{m}^2$, $i = 1039$, $L = 2.907\times10^{-3}\mathrm{H}$, $K_t = 0.195N\cdot\mathrm{m/A}$, $K_e = 0.197\, V/(\,\mathrm{rad}\cdot s^{-1})$, $B = 1.43\times10^{-4}$ N·m, $K_a = 2$, $f(\boldsymbol{x}_k) = 5.3+0.5x_1+0.7x_2+x_1x_2$; $d(\boldsymbol{x}_k) = 0.2\mathrm{sign}(x_2)+0.2\sin(0.5t)\mathrm{rand}(t)$. $x_{1,k}(0) = 5 + 0.1\mathrm{rand}(k)$, $x_{2,k}(0) = 0.8 + 0.02\mathrm{rand}(k)$. Here rand(·) represent random numbers between 0 and 1. $\boldsymbol{x}_d = [4.5 + \sin(\frac{\pi}{2}t), \frac{\pi}{2}\cos(\frac{\pi}{2})]^T$.

For fuzzy approximation, the following membership functions are chosen as $\mu_{f_{j,l}}(x_{j,k}(t)) = \exp[-\frac{(x_{j,k}-8+2\times l)^2}{7}]$, $l = 1,\ldots,7, j = 1,2$. The ILC law (26) and adaptive learning laws (27)-(29) are adopted for this simulation. The control parameters and learning gains are chosen as: $m = 7, T = 6, t_\varepsilon = 0.8, \alpha = 2, \gamma = 10, \mu_1 = 1$, $\mu_2 = 4, \mu_3 = 0.05, \varrho = 2, \delta = 0.001, \underline{p} = -50, \bar{p} = 50$, $\underline{\theta} = -50, \bar{\theta} = 50, \bar{\rho} = 20, b_s = 1$.

After 30 iteration cycles, the simulation results are depicted in Figs. 3-10. Figs. 3-4 show the position trajectory and velocity trajectory during the 30th iteration, respectively. From Figs. 3-4, we can see that the $\boldsymbol{x}_k(t)$ can accurately $\boldsymbol{x}_d(t)$ for $t \in [t_\varepsilon, T]$. The profiles of corresponding error are given in Figs. 5 - 6, respectively. The error tracking profiles of tank gun control system are given in Figs. 7 - 8. From Figs. 5-8, we can see that $\boldsymbol{e}_k(t)$ can follow $\boldsymbol{e}_k^*(t)$ for $t \in [0, T]$ as the iteration number increases. The system control input during

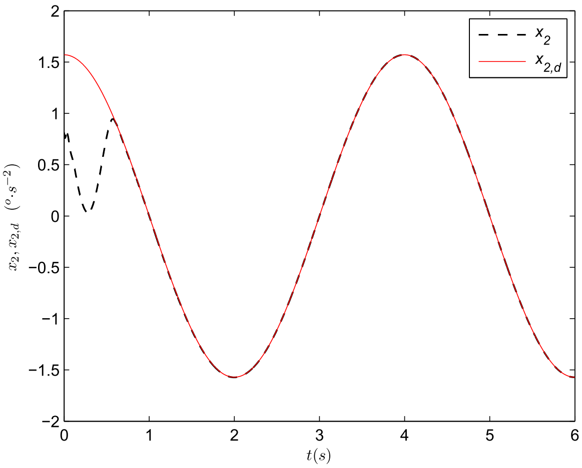

FIGURE 4. Velocity trajectory x_2 (barrier ILC).

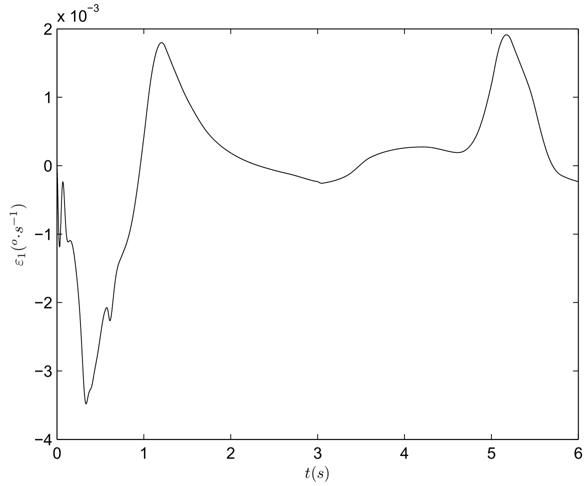

FIGURE 7. Difference between e_1 and e_1^* (barrier ILC).

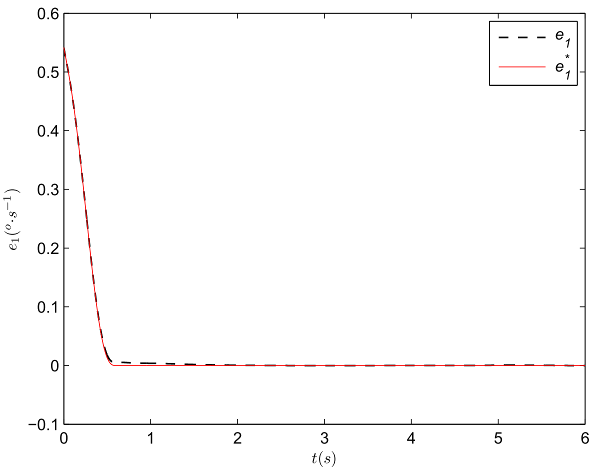

FIGURE 5. Position state error e_1 (barrier ILC).

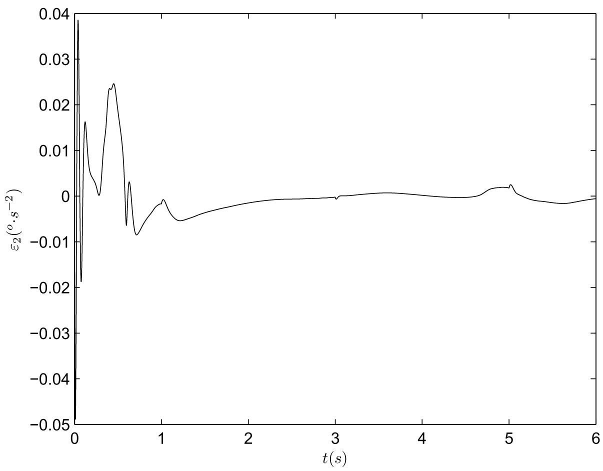

FIGURE 8. Difference between e_2 and e_2^* (barrier ILC).

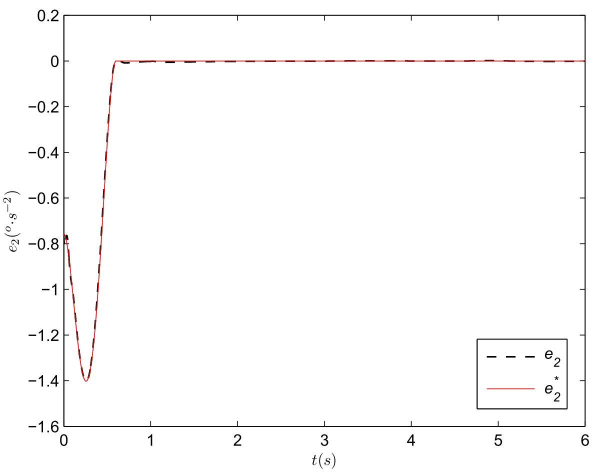

FIGURE 6. Velocity state error e_2 (barrier ILC).

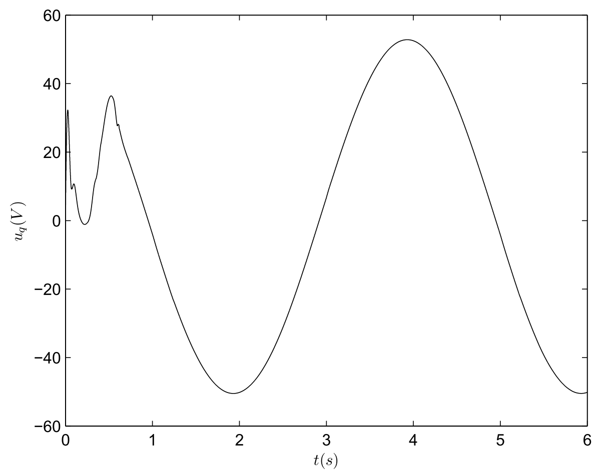

FIGURE 9. Control input(barrier ILC).

30s is shown in Fig. 9. The converge history of $s_{\varepsilon,k}$ is shown in Fig. 10, where $J_k := \max_{t\in[0,T]}(|s_{\varepsilon,k}(t)|)$.

For comparison, the barrier-free ILC algorithm (66)-(68) is adopted for simulation.

$$u_k = -\gamma s_{\varepsilon,k} - \boldsymbol{\vartheta}_k^T \boldsymbol{\phi}(X_k) - \frac{2\eta_k s_{\varepsilon,k}}{\varepsilon + |s_{\varepsilon,k}|}, \tag{66}$$

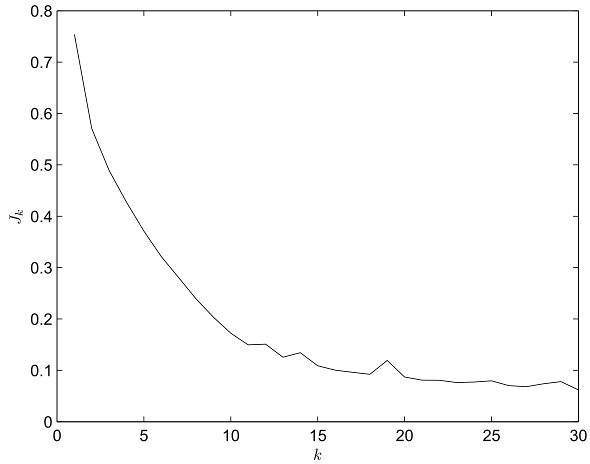

FIGURE 10. Maximum of $|s_{\varepsilon,k}|$ in the iteration domain(barrier ILC).

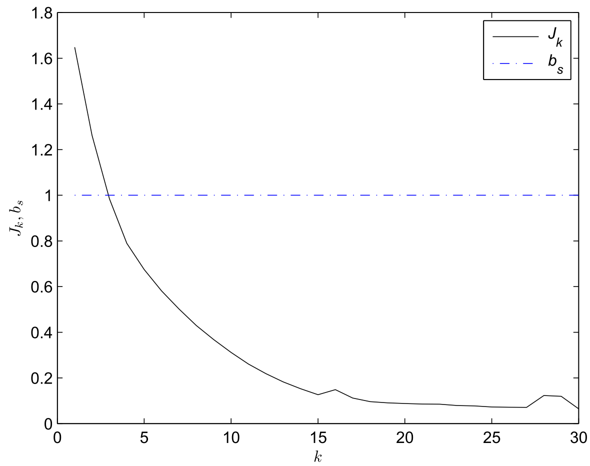

FIGURE 11. Maximum of $|s_{\varepsilon,k}|$ in the iteration domain(barrier-free ILC).

$$\boldsymbol{\vartheta}_k = \mathrm{sat}_{\underline{\vartheta},\bar{\vartheta}}(\boldsymbol{\vartheta}_{k-1}) + \mu_1 s_{\varepsilon,k}\boldsymbol{\phi}(X_k), \boldsymbol{\theta}_{-1} = 0, \quad (67)$$

$$\eta_k = \mathrm{sat}_{0,\bar{\eta}}(\eta_{k-1}) + \mu_3|s_{\varepsilon,k}|, \eta_{-1} = 0, \quad (68)$$

where $\gamma = 10, \mu_1 = 1, \mu_3 = 0.05, \varepsilon = 0.001$, $\underline{\vartheta} = -50, \bar{\vartheta} = 50, \bar{\eta} = 20$, $X_k = [e^*_{1,k}, e^*_{2,k}, e_{1,k}, e_{2,k}, y_d, \dot{y}_d, \ddot{y}_d]^T$, $\boldsymbol{\phi}(X_k) = [\phi_{1,k}, \phi_{2,k}, \ldots, \phi_{m,k}]^T$. For $j = 1, 2, \ldots, 7$, $\phi_{j,k}$ is defined as follows: $\phi_{j,k} = \exp(-\frac{\|\boldsymbol{X}_k - \boldsymbol{c}_j\|}{b_j})$, in which, $b_j = 7$, and $\boldsymbol{c}_j$ evenly spaced on $[-6, 6] \times [-6, 6] \times [-6, 6] \times [-6, 6] \times [-6, 6] \times [-6, 6] \times [-6, 6]$. The maximum of $|s_{\varepsilon,k}(t)|$ during the kth iteration is shown in Fig. 11. Comparing Fig. 10 and Fig. 11, we can see that the barrier property is observed in barrier ILC and no barrier property is observed in barrier-free ILC. The above simulation results have verified the effectiveness of the proposed fuzzy system-based adaptive ILC scheme.

VI. CONCLUSION

In this work, the accurate position control tracking problem is studied for tank gun control systems. A fuzzy system-based adaptive ILC scheme is proposed to get excellent tracking performance. For constraining the system error during each iteration, a barrier Lyapunov function is adopted to controller design. Error tracking strategy is introduced to solve the initial problem of tank gun ILC systems. The fuzzy system is used as an approximator to compensate for the nonparametric uncertainties in tank gun control systems. The optimal parameters of fuzzy systems are estimated by using difference learning learning approach. As the iteration number increases, the position tracking error can converge to a tunable residual set. The theoretical analysis and simulations show that the closed-loop tank gun ILC system has better control performance.

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