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Design Parameters for Massive Communication Systems Under Energy-Efficient Polynomial Precoder

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ABSTRACT Massive multiple-input multiple output (MIMO) systems have been introduced as a resolution for next generation cellular systems. The complexity of computing the precoding in massive MIMO is increased. So, studying a scalable precoding in massive MIMO system is a challenge task. In this paper, we propose a scalable precoder based polynomial for multiuser massive MIMO system, where base station (BS) is equipped with antennas that simultaneously communicate user equipments (UEs). This precoder applies matrix polynomial instead of matrix inversion. An energy efficiency (EE) optimization problem is formulated. This paper also studies optimal design parameters, which are the optimal transmit power, active UEs and number of antennas at BS. Mathematical formula for the EE-maximizing parameter estimations was mathematically analyzed with different orders of polynomial precoder. The impact of increasing polynomial orders is studied on the system performance. Comparison between proposed precoding technique and conventional techniques (i.e., zero forcing (ZF) precoder, minimum mean square error (MMSE) precoder and linear precoder) is provided. Results have shown that maximal EE and area throughput are achieved by deploying polynomial precoder in multiuser massive MIMO system. It can achieve better performance compared with conventional techniques. Utilization of polynomial precoder enhances the performance and provides high EE values.

INDEX TERMS Multiuser, massive MIMO, polynomial precoder, optimization, energy efficiency.


I. INTRODUCTION

Recently, there has been interest in massive MIMO systems due to its ability to maximize the spectral efficiency, energy efficiency, reliability and robustness with low complexity transmit precoding and multiuser detection [1]–[2]. Multiuser massive MIMO is a communication scheme where a BS with a large antennas number communicates with user equipments (UEs). The use of large scale antenna arrays can improve the energy efficiency (EE) for wireless systems due to the improvement in array gain and spatial resolution [3]–[6]. Low cost hardware at both the BS and UE side is achieved by using

massive MIMO. At BS, the expensive and power ineffective hardware is substituted by low cost power units that can work consecutively in massive system [7].

Precoding is a preprocessing method that employs channel state information (CSI) at BS to match the transmission to the instantaneous channel conditions. Linear precoding may be considered a straightforward and efficient method that can reduce the MIMO system complexity [8]. It is considered as optimum in specific situations that using partial CSI [9]. There are several linear precoders like zero forcing (ZF), MMSE, matched filter (MF) and regularized ZF (RZF).

The ZF and MMSE precoding involve channel inversion, with the channel pseudo inverse [8]. The MF precoding is interference limited at high signal to noise ratio (SNR) but

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its performance is preferable than ZF at low SNRs. The RZF precoder uses a regularization parameter in the channel inversion. Peel *et al.* [10] presented a vector perturbation scheme to decrease the RZF transmit power. By doing this, RZF can operate near channel capacity. It is so difficult to implement because large matrices fast inversions in every coherence period is required [8], [10].

The complication of some linear precoding schemes is still uncompromising when antennas number M and UEs number K are large. The arithmetic operations number for these precoding schemes is proportional to K^2M . A prominent exclusion is the MF, whose complexity increases as MK . But, the method needs roughly more antennas at BS to proceed as well as RZF [8], [11]. So, it is more difficult to apply massive MIMO system because it cripples the throughput.

In order to achieve the gain in massive MIMO, a two-stage precoding scheme with limited RF chains is studied [12]. This precoder is divided into a phase only radio frequency (RF) precoder and baseband precoder. The RF precoder is applied for the spatial correlation matrices to overcome inter-cluster interference. In addition, the baseband precoder is applied for channel state information. This precoder can decrease the overhead of the channel state information signaling.

The authors in [13] explain phased zero-forcing precoder for massive multiuser MIMO systems. In this scheme, only phase control at the RF domain is applied, and then the channel based baseband zero-forcing precoding is performed. The system is demonstrated under Rayleigh fading and sparsely scattered millimeter wave channels. This scheme can give desirable performance.

A distributed MIMO system is illustrated in [14], where many transmitters can cooperatively share a common receiver. In order to improve the system performance, a linear Hermitian precoder is studied, where each source has the channel state information of its own path and the other paths have a slow fading. Linear Hermitian precoding can convert the equivalent channel into a Hermitian matrix form.

ZF precoder was investigated in multiuser schemes, where ZF can decouple the multiuser channel into autonomous signal user channels [15]. In addition, ZF precoder includes channel inversion via channel pseudo-inverse or other generalized inverses [16]. Matched filter (MF) precoder is studied for communication system [17]. It is interference limited at high values of signal to noise ratio but at low SNR, it can outperform the ZF precoder [16].

ZF precoder has low performance especially when the channel is ill-conditioned. Vector perturbation approach-based precoding has been explained in [18]. In order to decrease the energy penalty provided by ZF precoder, this scheme is inspired via Tomlinson–Harashima precoding, where a complex vector is added to each data vector. The receiver with a modulo function is applied to calculate the transmitted symbols under noise. A technique based on a linear minimum mean squared error was studied [19]. Also, a method based on optimizing mutual information is illustrated in [20].

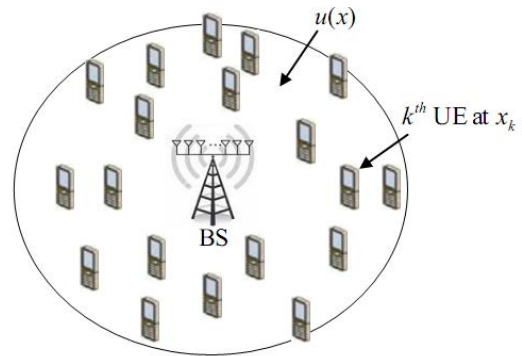


FIGURE 1. Multiuser massive MIMO system in a single cell scenario with random UE distribution $u(x)$.

The problem of ZF and MMSE precoders' massive MIMO system is high complexity and low EE due to matrix inversion for large matrices in massive MIMO [8], [21]. To control this issue, we propose a scalable precoder based polynomial for multiuser massive MIMO system. It employs a matrix polynomial instead of matrix inversion. The main idea depends on approximating the matrix inverse using matrix polynomial with J -terms.

The energy efficient and low complexity precoding scheme in multiuser massive MIMO system represents the primary focus of this paper. In this paper, we propose linear precoding method based on matrix polynomial for the single cell multiuser massive MIMO system. This paper also studies the effect of increasing polynomial orders on the EE and system performance. The EE optimization problem for multiuser massive MIMO system that based on polynomial precoder is mathematically illustrated and derived for different polynomial orders.

The paper is structures as follows. In Section II, we describe the multiuser massive MIMO system. Section III, gives a brief review of the uplink rate and average uplink RF power for ZF detector. The downlink rate and average downlink RF power for proposed polynomial precoder are mathematically presented and derived in Section IV. The model for power consumption is illustrated in Section V. In Section VI, the EE optimization problem for multiuser massive MIMO system based polynomial precoder is mathematically solved and derived. Simulation parameters and results are given in Section VII. Finally, the concluding remarks are given in Section VIII.

II. MULTIUSER MASSIVE MIMO SYSTEM MODEL

Assume a multiuser massive MIMO system with BS of M -antennas simultaneously serves K antenna UEs in the same time-frequency resource [22]. It is operating at bandwidth B Hz in a single cell. Assume the distribution of UEs $u(x)$ in a circular cell is random as shown in Fig. 1. The communication between M antennas at BS with K UEs is chosen in a round-robin scheme from UEs set. The UEs locations are handled as random variables taken from a UE

distribution $u(x)$. We assume the position of BS in the center of a cell [23]–[24].

Let $x_k \in \mathbb{R}^2$ denotes the k th UE physical location and the function $p(x_k)$ represents large-scale fading because of shadowing and path loss at UE location x_k . The spacing among antennas at BS is chosen in a way channel components are uncorrelated among antennas. The channel vector $\mathbf{h}_k = [h_{k,1}, h_{k,2}, \dots, h_{k,M}]^T \in \mathbb{C}^{M \times 1}$ has entries $\{h_{k,n}\}$. A rayleigh-fading distribution is assumed with $\mathbf{h}_k \sim CN(0_M, p(x_k) \mathbf{I}_M)$.

The UE-channels are assumed fixed in time frequency coherence blocks of $U = B_C T_C$, where B_C (in Hz) denotes coherence-bandwidth and T_C (in second) denotes coherence-time. The coherence block U channel uses consists of $U \zeta^{\text{UL}}$ channel uses for uplink transmission and $U \zeta^{\text{DL}}$ channel uses for downlink transmission. The uplink and downlink transmission ratios are represented by ζ^{UL} and ζ^{DL} , respectively, with $\zeta^{\text{DL}} + \zeta^{\text{UL}} = 1$. The pilot signaling occupies $\tau^{\text{UL}} K$, and $\tau^{\text{DL}} K$ channel-uses in the uplink and downlink, with $\tau^{\text{DL}}, \tau^{\text{UL}} \geq 1$ to enable orthogonal pilot sequences among UEs [24], [26].

We consider that the BS and UEs are perfectly synchronized and time division duplex (TDD) protocol is considered [23], [25]. The pilots of uplink make the BS to calculate UE channels. TDD protocol requires the same number of M and K in the uplink and downlink.

III. UPLINK RATE AND AVERAGE UPLINK RF POWER FOR ZF DETECTOR

In the uplink, the BS is assumed to acquire perfect CSI from the uplink pilots. We use ZF algorithm for data detection. The detector function removes the interferers' signals, which is done by inverting the channel response [7], [9]. Denoting by $\mathbf{Y} = [y_1, y_2, \dots, y_K] \in \mathbb{C}^{M \times K}$ the ZF matrix with the column y_k being assigned to the k th UE is [7], [9]:

$$\mathbf{Y} = \mathbf{H} \left(\mathbf{H}^H \mathbf{H} \right)^{-1} \quad (1)$$

where $(\cdot)^H$ denotes the Hermitian transpose operation of channel matrix and $\mathbf{H} = [h_1, h_2, \dots, h_K]$ contains all the user channels. The achievable uplink rate of the k th UE under ZF detector and perfect CSI is given by [23], [26]:

$$R_k^{\text{UL}} = \zeta^{\text{UL}} \left(1 - \frac{\tau^{\text{UL}} K}{U \zeta^{\text{UL}}} \right) \bar{R}_k^{\text{UL}} \quad \text{bit/second} \quad (2)$$

and the uplink gross rate of the transmission from the k th UE is [23], [25]:

$$\bar{R}_k^{\text{UL}} = B \log \left(1 + \frac{p_k^{\text{UL}} |\mathbf{h}_k^H y_k|^2}{\sum_{\ell=1, \ell \neq k}^K p_\ell^{\text{UL}} |\mathbf{h}_\ell^H y_k|^2 + \sigma^2 \|y_k\|^2} \right) \quad (3)$$

where $\mathbf{P}^{\text{UL}} = [p_1^{\text{UL}}, p_2^{\text{UL}}, \dots, p_K^{\text{UL}}]^T$ is the uplink power allocation vector and is given by [9], [11]:

$$\mathbf{P}^{\text{UL}} = \sigma^2 \left(\mathbf{D}^{\text{UL}} \right)^{-1} \mathbf{1}_K \quad \text{Joule/channel use} \quad (4)$$

Here, $\mathbf{1}_K$ is the all one column vector of size K , σ^2 is the noise variance and (k, ℓ) th element of a diagonal matrix $\mathbf{D}^{\text{UL}} \in \mathbb{C}^{K \times K}$ is [11], [30]:

$$\left[\mathbf{D}^{\text{UL}} \right]_{k,\ell} = \begin{cases} \frac{1}{\gamma_k} \left| \mathbf{h}_k^H y_k \right|^2 & \text{for } k = \ell \\ - \left| \mathbf{h}_k^H y_\ell \right|^2 & \text{for } k \neq \ell \end{cases} \quad (5)$$

where γ_k denotes the SINR at the k th UE and computed as $(2^{R/B} - 1)$. The average uplink RF power is given by [24]:

$$\mathbf{P}_{\text{RF}}^{\text{UL}} = \frac{B \zeta^{\text{UL}}}{\eta^{\text{UL}}} \mathbb{E} \left\{ \mathbf{1}_K^T \mathbf{P}^{\text{UL}} \right\} \text{ watt} \quad (6)$$

where $0 < \eta^{\text{UL}} \leq 1$ is the efficiency of power amplifier at UEs.

Lemma 1: If ZF detector is applied with $M \geq K + 1$, then the gross rate is given by [25]:

$$\bar{R} = B \log (1 + \alpha (M - K)) \quad (7)$$

where α denotes the design parameter. The uplink RF power is given by [25]:

$$\mathbf{P}_{\text{RF,ZF}}^{\text{UL}} = S_0 \alpha K \quad (8)$$

where the coefficient S_0 is given in TABLE 4 in Appendix A.

IV. DOWNLINK RATE AND AVERAGE DOWNLINK RF POWER FOR PROPOSED POLYNOMIAL PRECODER

In the downlink, linear processing is used for data precoding. We denote by $\mathbf{Z} = [z_1, z_2, \dots, z_K] \in \mathbb{C}^{M \times K}$ the precoding matrix. We propose precoder based matrix polynomial, which is given by [27]–[29]:

$$\mathbf{Z} = \sum_{\ell=0}^J \omega_\ell \left(\mathbf{H} \mathbf{H}^H \right)^\ell \mathbf{H} \quad (9)$$

where $\omega = [\omega_0, \omega_1, \dots, \omega_J]^T$ denote the real valued coefficients of the precoder matrix polynomial. The achievable downlink rate with linear processing is given by [23], [26]:

$$R_k^{\text{DL}} = \zeta^{\text{DL}} \left(1 - \frac{\tau^{\text{DL}} K}{U \zeta^{\text{DL}}} \right) \bar{R}_k^{\text{DL}} \quad (10)$$

and the downlink gross rate of the transmission is [23], [25]:

$$\bar{R}_k^{\text{DL}} = B \log \left(1 + \frac{p_k^{\text{DL}} |\mathbf{h}_k^H z_k|^2}{\sum_{\ell=1, \ell \neq k}^K p_\ell^{\text{DL}} |\mathbf{h}_\ell^H z_k|^2 + \sigma^2 \|z_k\|^2} \right) \quad (11)$$

where $\mathbf{P}^{\text{DL}} = [p_1^{\text{DL}}, p_2^{\text{DL}}, \dots, p_K^{\text{DL}}]^T$ is the downlink power allocation vector and is given by [11], [30]:

$$\mathbf{P}^{\text{DL}} = \sigma^2 \left(\mathbf{D}^{\text{DL}} \right)^{-1} \mathbf{1}_K \quad (12)$$

and

$$\left[\mathbf{D}^{\text{DL}} \right]_{k,\ell} = \begin{cases} \frac{1}{\gamma_k} \left| \mathbf{h}_k^H z_k \right|^2 & \text{for } k = \ell \\ - \left| \mathbf{h}_k^H z_\ell \right|^2 & \text{for } k \neq \ell \end{cases} \quad (13)$$

The average downlink RF power is given by [24]:

$$\mathbf{P}_{\text{RF}}^{\text{DL}} = \frac{B\xi^{\text{DL}}}{\eta^{\text{DL}}} \mathbb{E} \left\{ \mathbf{1}_K^T \mathbf{P}^{\text{DL}} \right\} \text{ watt} \quad (14)$$

where $0 < \eta^{\text{DL}} \leq 1$ is the efficiency of power amplifier at BS.

Lemma 2: If first order polynomial precoder ($J = 1$) is used with $M \geq K + 1$, a diagonal matrix $\mathbf{D}^{\text{DL}} \in \mathbb{C}^{K \times K}$ is:

$$\begin{aligned} \left[\mathbf{D}^{\text{DL}} \right]_k^{1^{\text{st}} \text{ order}} &= \frac{1}{\left(2^{\frac{\bar{R}(J=1)}{B}} - 1 \right)} \left| \mathbf{h}_k^H z_k \right|^2 \\ &= \frac{1}{\left(2^{\frac{\bar{R}(J=1)}{B}} - 1 \right)} \left| \mathbf{h}_k^H \sum_{\ell=0}^1 \omega_\ell \left(\mathbf{h}_k \mathbf{h}_k^H \right)^\ell \mathbf{h}_k \right|^2 \end{aligned} \quad (15)$$

and the power allocation is:

$$\begin{aligned} \mathbf{p}^{\text{DL}, 1^{\text{st}} \text{ order}} &= \sigma^2 \left(2^{\frac{\bar{R}(J=1)}{B}} - 1 \right) \\ &\times \left| \sum_{\ell=0}^1 \omega_\ell \left(\mathbf{h}_k \mathbf{h}_k^H \right)^\ell \mathbf{h}_k \mathbf{h}_k^H \right|^{-2} \mathbf{1}_K \end{aligned} \quad (16)$$

$$\begin{aligned} \mathbf{p}^{\text{DL}, 1^{\text{st}} \text{ order}} &= \sigma^2 \left(2^{\frac{\bar{R}(J=1)}{B}} - 1 \right) \\ &\times \left| \sum_{\ell=0}^1 \omega_\ell \left(\mathbf{h}_k \mathbf{h}_k^H \right)^{\ell+1} \right|^{-2} \mathbf{1}_K \end{aligned} \quad (17)$$

Then, the RF power is given by:

$$\mathbf{P}_{\text{RF}}^{\text{DL}, 1^{\text{st}} \text{ order}} = S_1 \frac{\alpha (M + 1)}{\alpha (K - 1) + K} \left| S_2 + \left(\frac{K + 1}{K} \right) S_3 \right|^{-2} \quad (18)$$

where the coefficients S_1, S_2 and S_3 are listed in TABLE 4.

Proof: This result is proved in Appendix B.

Lemma 3: If second order polynomial precoder ($J = 2$) is used with $M \geq K + 1$, a diagonal matrix $\mathbf{D}^{\text{DL}} \in \mathbb{C}^{K \times K}$ is:

$$\begin{aligned} \left[\mathbf{D}^{\text{DL}} \right]_k^{2^{\text{nd}} \text{ order}} &= \frac{1}{\left(2^{\frac{\bar{R}(J=2)}{B}} - 1 \right)} \left| \mathbf{h}_k^H \sum_{\ell=0}^2 \omega_\ell \left(\mathbf{h}_k \mathbf{h}_k^H \right)^\ell \mathbf{h}_k \right|^2 \end{aligned} \quad (19)$$

and the power allocation is:

$$\begin{aligned} \mathbf{p}^{\text{DL}, 2^{\text{nd}} \text{ order}} &= \sigma^2 \left(2^{\frac{\bar{R}(J=2)}{B}} - 1 \right) \left| \sum_{\ell=0}^2 \omega_\ell \left(\mathbf{h}_k \mathbf{h}_k^H \right)^{\ell+1} \right|^{-2} \mathbf{1}_K \end{aligned} \quad (20)$$

Then, the RF power is given by:

$$\begin{aligned} \mathbf{P}_{\text{RF}}^{\text{DL}, 2^{\text{nd}} \text{ order}} &= S_1 \frac{\alpha (M^3 + 2M^2 + 2M + 1)}{\alpha (K - 1) (M^2 + M + 1) + K(1 + M)} \end{aligned}$$

$$\times \left| S_2 + \left(\frac{K + 1}{K} \right) S_3 + \left(\frac{K^2 + 3K + 2}{K^2} \right) S_4 \right|^{-2} \quad (21)$$

where the coefficients S_1, S_2, S_3 and S_4 are listed in TABLE 4.

Proof: This result is proved in Appendix C.

Similarly, the RF powers for third, fourth and fifth orders of proposed polynomial precoder, respectively are given by:

$$\begin{aligned} \mathbf{P}_{\text{RF}}^{\text{DL}, 3^{\text{rd}} \text{ order}} &= S_1 \left(2^{\frac{\bar{R}(J=3)}{B}} - 1 \right) \\ &\times \left| S_2 + \left(\frac{K + 1}{K} \right) S_3 + \left(\frac{K^2 + 3K + 2}{K^2} \right) S_4 \right. \\ &\left. + \left(\frac{K^3 + 6K^2 + 11K + 6}{K^3} \right) S_5 \right|^{-2} \end{aligned} \quad (22)$$

and

$$\begin{aligned} \mathbf{P}_{\text{RF}}^{\text{DL}, 4^{\text{th}} \text{ order}} &= S_1 \left(2^{\frac{\bar{R}(J=4)}{B}} - 1 \right) \\ &\times \left| S_2 + \left(\frac{K + 1}{K} \right) S_3 + \left(\frac{K^2 + 3K + 2}{K^2} \right) S_4 \right. \\ &+ \left(\frac{K^3 + 6K^2 + 11K + 6}{K^3} \right) S_5 \\ &\left. + \left(\frac{K^3 + 6K^2 + 11K + 6}{K^3} \right) S_6 \right|^{-2} \end{aligned} \quad (23)$$

and

$$\begin{aligned} \mathbf{P}_{\text{RF}}^{\text{DL}, 5^{\text{th}} \text{ order}} &= S_1 \left(2^{\frac{\bar{R}(J=5)}{B}} - 1 \right) \\ &\times \left| S_2 + \left(\frac{K + 1}{K} \right) S_3 + \left(\frac{K^2 + 3K + 2}{K^2} \right) S_4 \right. \\ &+ \left(\frac{K^3 + 6K^2 + 11K + 6}{K^3} \right) S_5 \\ &+ \left(\frac{K^3 + 6K^2 + 11K + 6}{K^3} \right) S_6 \\ &\left. + \left(\frac{K^4 + 6K^3 + 13K^2 + 12K + 4}{K^4} \right) S_7 \right|^{-2} \end{aligned} \quad (24)$$

V. POWER CONSUMPTION MODEL

We use a circuit power consumption scheme for multiuser massive MIMO system, which is designed as a function of antennas number at BS(M), UEs(K) number, and design parameter (α). The total power is given by [31]–[34]:

$$\mathbf{P}_T = P_{\text{FIX}} + P_{\text{TC}} + P_{\text{CE}} + P_{\text{C/D}} + P_{\text{BH}} + P_{\text{LP}} \quad (25)$$

where P_{FIX} is the fixed power consumption [31], P_{TC} is the power consumption for transmitters/receivers chains, P_{CE} is the power of the channel estimation process, $P_{\text{C/D}}$ accounts for channel coding and decoding units and P_{BH} accounts for load-dependent backhaul that is proportional to the average

sum rate. The power consumption for linear processing at the BS is [25], [28]–[29], [35]:

$$P_{LP} = \frac{2BMK}{L_{BS}} \left(1 - \frac{(\tau^{DL} + \tau^{UL})K}{U} \right) + \frac{B}{UL_{BS}} \times \left[\frac{J}{2} (K^2 + K) (2M - 1) + MK (2K - 1) \right] \quad (26)$$

In a closed form, the total power is:

$$P_T = \sum_{i=0}^2 C_i K^i + M \sum_{i=0}^2 D_i K^i + AK \left(1 - \frac{(\tau^{DL} + \tau^{UL})K}{U} \right) \bar{R} \quad (27)$$

where the coefficient A , C_i and D_i are listed in TABLE 5 in Appendix D.

Proof: This result is proved in Appendix E.

VI. ENERGY EFFICIENCY OPTIMIZATION WITH POLYNOMIAL PRECODER

A. PROBLEM STATEMENT

Definition 1: The EE denotes the ratio among the mean sum rate and the mean total power consumption for uplink and downlink, which is given by [36]–[38]:

$$EE = \frac{\sum_{k=1}^K (\mathbb{E} \{R_k^{DL}\} + \mathbb{E} \{R_k^{UL}\})}{P_{RF}^{DL} + P_{RF}^{UL} + P_T} \quad (28)$$

Plugging Eq. (2) and (10) into Eq. (28), the EE becomes:

$$EE = \frac{K \left(1 - \frac{(\tau^{DL} + \tau^{UL})K}{U} \right) \bar{R}}{P_{RF}^{DL} + P_{RF}^{UL} + P_T} \quad (29)$$

Problem 1: The optimum EE for multiuser massive MIMO system based polynomial precoder with $M \geq K + 1$, is achieved by solving the optimization problem:

$$\max_{M \in \mathbb{Z}_+, K \in \mathbb{Z}_+, \alpha \geq 0} EE^{1st\ order} = \frac{K \left(1 - \frac{(\tau^{DL} + \tau^{UL})K}{U} \right) \bar{R}}{P_{RF}^{DL} + P_{RF}^{UL} + P_T} \quad (30)$$

B. EE OPTIMIZATION WITH FIRST ORDER POLYNOMIAL PRECODER

In this subsection, we will solve the EE optimization in Problem 1 for first order polynomial precoder. Then, the Problem 1 becomes:

$$\begin{aligned} \max_{M \in \mathbb{Z}_+, K \in \mathbb{Z}_+, \alpha \geq 0} EE^{1st\ order} \\ = \frac{K \left(1 - \frac{(\tau^{DL} + \tau^{UL})K}{U} \right) \bar{R} (J = 1)}{P_{RF}^{DL, 1st\ order} + P_{RF, ZF}^{UL} + P_T^{1st\ order}} \end{aligned} \quad (31)$$

After arranging the terms, the above equation becomes (32), as shown at the bottom of the next page, where the coefficients A , C_i and D_i are given in TABLE 5 with $J = 1$. We will estimate M , K and α values for maximizing the EE. Also, We will deduce the mathematical equation for the optimum EE to solve the optimization problem by using a sequential algorithm

1) OPTIMAL NUMBER OF USERS (K)

We assume that the number of antennas per UE and sum SINR equal to $M/K = \bar{\beta}$ and $\alpha K = \bar{\alpha}$, respectively. We search the K value that maximizes the EE in Eq. (32). Let $R_0 = (\tau^{DL} + \tau^{UL})B/U$, then the optimization problem becomes (33), as shown at the bottom of the next page. We approximate the logarithmic term using two point Gauss-Legendre Log (2P-GLLOG) [43]:

$$\log_2(1+x) = \frac{\ln(1+x)}{\ln(2)} = \frac{6x+3x^2}{(6+6x+x^2)\ln(2)} \quad (34)$$

After that, the optimization problem becomes:

$$\max_{K \in \mathbb{Z}_+} f(K)^{1st\ order} = \frac{f_1(K)^{1st\ order}}{f_2(K)^{1st\ order}} \quad (35)$$

where

$$\begin{aligned} f_1(K)^{1st\ order} = R_1 K^8 + R_2 K^7 + R_3 K^6 + R_4 K^5 \\ + R_5 K^4 + R_6 K^3 + R_7 K^2 + R_8 K + R_9 \end{aligned} \quad (36)$$

and

$$\begin{aligned} f_2(K)^{1st\ order} \\ = R_{10} K^{13} + R_{11} K^{12} + R_{12} K^{11} + R_{13} K^{10} \\ + R_{14} K^9 + R_{15} K^8 + R_{16} K^7 + R_{17} K^6 + R_{18} K^5 \\ + R_{19} K^4 + R_{20} K^3 + R_{21} K^2 + R_{22} K \end{aligned} \quad (37)$$

where the coefficients $\sum_{i=1}^{22} R_i$ are given in TABLE 6 in Appendix F.

Theorem 1: The function $f(K)^{1st\ order}$ is quasi concave for $K \in R$ if the level sets $S_\kappa = \{K : f(K)^{1st\ order} \geq \kappa\}$ are convex for any $\kappa \in \mathbb{R}$ [39, Section 3.4]. This implies that the global maximizer of $f(K)^{1st\ order}$ for K satisfies the stationary condition $\frac{d}{dK} f(K)^{1st\ order} = 0$, which gives the polynomial:

$$\begin{aligned} f_2(K)^{1st\ order} \times \frac{d}{dK} f_1(K)^{1st\ order} - f_1(K)^{1st\ order} \\ \times \frac{d}{dK} f_2(K)^{1st\ order} = 0 \end{aligned} \quad (38)$$

Let $K_\ell^{(o), 1st\ order}$ the real roots of the above polynomial, then the optimal number of UEs that give optimum EE is:

$$K^{1st\ order} = \max_\ell \left[K_\ell^{(o), 1st\ order} \right] \quad (39)$$

where $\lfloor \cdot \rfloor$ is either the closest smaller or larger integer to $K_\ell^{(o), 1st\ order}$.

2) OPTIMAL NUMBER OF ANTENNAS AT BS (M)

We will compute the optimal number of antenna $M^{*,1^{st} order} \geq K + 1$ that maximizes the $\mathbf{EE}^{1^{st} order}$. The optimization problem is (40), as shown at the bottom of the page. We rewrite the denominator as:

$$\begin{aligned} & \left(S_0 \alpha K + \frac{S_1 \alpha}{\alpha(K-1)+K} \left| S_2 + \left(\frac{K+1}{K} \right) S_3 \right|^{-2} + \sum_{i=0}^2 C_i K^i \right) \\ & + M \left(\frac{S_1 \alpha}{\alpha(K-1)+K} \left| S_2 + \left(\frac{K+1}{K} \right) S_3 \right|^{-2} + \sum_{i=0}^2 D_i K^i \right) \\ & + AKB \left(1 - \frac{(\tau^{DL} + \tau^{UL})K}{U} \right) \log \left(1 + \frac{\alpha(M+1)}{\alpha(K-1)+K} \right) \end{aligned} \quad (41)$$

Definition 2: The Lambert function $W(x)$ is defined by the equation $x = W(x)e^{W(x)}$ for any $x \in \mathbb{C}$ [40].

Lemma 4: For the optimization problem

$$\underset{z > -\frac{g}{b}}{\text{maximize}} \frac{g \log(a + bz)}{c + dz + h \log(a + bz)} \quad (42)$$

with constant coefficients $a \in \mathbb{R}$, $c, h \geq 0$, and $b, d, g > 0$. The unique solution is given by [40]:

$$z^* = \frac{e^{W\left(\frac{bc}{de} - \frac{a}{e}\right) + 1} - a}{b} \quad (43)$$

Now, we will apply this solution to the above problem in Eq. (40). Let $a_1 = 1 + \frac{\alpha}{\alpha(K-1)+K}$, $b_1 = \frac{\alpha}{\alpha(K-1)+K}$, $c_1 = \left(S_0 \alpha K + \frac{S_1 \alpha}{\alpha(K-1)+K} \left| S_2 + \left(\frac{K+1}{K} \right) S_3 \right|^{-2} + \sum_{i=0}^2 C_i K^i \right)$, $d_1 =$

$$\begin{aligned} & \left(\frac{S_1 \alpha}{\alpha(K-1)+K} \left| S_2 + \left(\frac{K+1}{K} \right) S_3 \right|^{-2} + \sum_{i=0}^2 D_i K^i \right), \\ g_1 & = KB \left(1 - \frac{(\tau^{DL} + \tau^{UL})K}{U} \right), \quad \text{and} \end{aligned}$$

$h_1 = AKB \left(1 - \frac{(\tau^{DL} + \tau^{UL})K}{U} \right)$. Then, the optimal number of BS antennas is:

$$M^{*,1^{st} order} = \left\lceil \frac{e^{W\left(\frac{b_1 c_1}{d_1 e} - \frac{a_1}{e}\right) + 1} - a_1}{b_1} \right\rceil \quad (44)$$

3) OPTIMAL DESIGN PARAMETER (α)

The optimization problem for the design parameter α is (45), as shown at the bottom of the next page. Let $\ell_1 = KB \left(1 - \frac{(\tau^{DL} + \tau^{UL})K}{U} \right)$, $\ell_2 = \left| S_2 + \left(\frac{K+1}{K} \right) S_3 \right|^{-2}$ and $\ell_3 = \left(\sum_{i=0}^2 C_i K^i + M \sum_{i=0}^2 D_i K^i \right)$. We approximate the logarithmic term using two point Gauss-Legendre Log (2P-GLLOG) [43]. Then, the optimization problem becomes:

$$\begin{aligned} & \underset{\alpha \geq 0}{\text{maximize}} \varphi(\alpha)^{1^{st} order} \\ & = \frac{U_1 \alpha^3 + U_2 \alpha^2 + U_3 \alpha}{U_4 \alpha^4 + U_5 \alpha^3 + U_6 \alpha^2 + U_7 \alpha + U_8} \end{aligned} \quad (46)$$

where the coefficients $\sum_{i=1}^8 U_i$ are given in TABLE 7 in Appendix G. The global maximizer of $\varphi(\alpha)^{1^{st} order}$ for α satisfies the stationary condition $\frac{d}{d\alpha} \varphi(\alpha)^{1^{st} order} = 0$, which gives the polynomial:

$$(U_1 U_4) \alpha^6 + (2U_2 U_4) \alpha^5 + (U_2 U_5 + 3U_3 U_4 - U_1 U_6) \alpha^4$$

$$\begin{aligned} & \underset{M \in \mathbb{Z}_+, K \in \mathbb{Z}_+, \alpha \geq 0}{\text{maximize}} \mathbf{EE}^{1^{st} order} \\ & = \frac{K \left(1 - \frac{(\tau^{DL} + \tau^{UL})K}{U} \right) \bar{R}(J=1)}{S_1 \frac{\alpha(M+1)}{\alpha(K-1)+K} \left| S_2 + \left(\frac{K+1}{K} \right) S_3 \right|^{-2} + S_0 \alpha K + \sum_{i=0}^2 C_i K^i + M \sum_{i=0}^2 D_i K^i + AK \left(1 - \frac{(\tau^{DL} + \tau^{UL})K}{U} \right) \bar{R}(J=1)} \end{aligned} \quad (32)$$

$$\begin{aligned} & \underset{M \in \mathbb{Z}_+, K \in \mathbb{Z}_+, \alpha \geq 0}{\text{maximize}} f(K)^{1^{st} order} \\ & = \frac{K(B - R_0 K) \log_2 \left(1 + \frac{\bar{\alpha} \bar{\beta} + \bar{\alpha}/K}{\bar{\alpha} - \bar{\alpha}/K + K} \right)}{S_1 \frac{\bar{\alpha} \bar{\beta} + \bar{\alpha}/K}{\bar{\alpha} - \bar{\alpha}/K + K} \left| S_2 + \left(\frac{K+1}{K} \right) S_3 \right|^{-2} + S_0 \bar{\alpha} + \sum_{i=0}^2 C_i K^i + \bar{\beta} \sum_{i=0}^2 D_i K^{i+1} + AK(B - R_0 K) \log_2 \left(1 + \frac{\bar{\alpha} \bar{\beta} + \bar{\alpha}/K}{\bar{\alpha} - \bar{\alpha}/K + K} \right)} \end{aligned} \quad (33)$$

$$\begin{aligned} & \underset{K \in \mathbb{Z}_+}{\text{maximize}} \phi(M)^{1^{st} order} \\ & = \frac{KB \left(1 - \frac{(\tau^{DL} + \tau^{UL})K}{U} \right) \log \left(1 + \frac{\alpha(M+1)}{\alpha(K-1)+K} \right)}{S_1 \frac{\alpha(M+1)}{\alpha(K-1)+K} \left| S_2 + \left(\frac{K+1}{K} \right) S_3 \right|^{-2} + S_0 \alpha K + \sum_{i=0}^2 C_i K^i + M \sum_{i=0}^2 D_i K^i + AKB \left(1 - \frac{(\tau^{DL} + \tau^{UL})K}{U} \right) \log \left(1 + \frac{\alpha(M+1)}{\alpha(K-1)+K} \right)} \end{aligned} \quad (40)$$

$$+ 2(U_3U_5 - U_1U_7)\alpha^3 + (U_3U_6 - 3U_1U_8 - U_2U_7)\alpha^2 - (2U_2U_8)\alpha - (U_3U_8) = 0 \quad (47)$$

Let $\alpha_\ell^{(o),1^{st} order}$ the real roots of the above polynomial, then the optimal α that give optimum EE is:

$$\alpha^{*,1^{st} order} = \max_\ell \left[\alpha_\ell^{(o),1^{st} order} \right] \quad (48)$$

C. EE OPTIMIZATION WITH SECOND ORDER POLYNOMIAL PRECODER

The EE optimization problem based second order polynomial precoder is given by:

$$\begin{aligned} & \text{maximize } \mathbf{EE}^{2^{nd} order} \\ & M \in \mathbb{Z}_+, K \in \mathbb{Z}_+, \alpha \geq 0 \\ & = \frac{K \left(1 - \frac{(\tau^{DL} + \tau^{UL})K}{U} \right) \bar{R}(J=2)}{\mathbf{P}_{RF}^{DL,2^{nd} order} + \mathbf{P}_{RF,ZF}^{UL} + \mathbf{P}_T^{2^{nd} order}} \end{aligned} \quad (49)$$

We replace values of $\bar{R}(J=2)$ as in Eq. (50), as shown at the bottom of the next page. The coefficients A , C_i and D_i are calculated and given in TABLE 5 with $J=2$ in Appendix H.

1) OPTIMAL NUMBER OF USERS (K)

We are looking for the K value to maximize the EE in Eq. (50). We assume $M/K = \bar{\beta}$ and $\alpha K = \bar{\alpha}$, then the optimization problem becomes (see Eq. (51), as shown at the bottom of the next page), where $R_0 = (\tau^{DL} + \tau^{UL})B/U$ and $\bar{c}_{J=2} = \log_2 \left(1 + \frac{\bar{\alpha}(\bar{\beta}^3 K^3 + \bar{\beta} K + 1)}{\bar{\beta}(\bar{\alpha}\bar{\beta} + 1)K^3 + (\bar{\alpha}\bar{\beta} - \bar{\alpha}\bar{\beta}^2 + 1 + 1)K^2 + \bar{\alpha}(1 - \bar{\beta})K - \bar{\alpha}} \right)$.

The logarithmic term in $\bar{c}_{J=2}$ is approximated using 2P-GLLOG [34] and is given by:

$$\begin{aligned} \bar{c}_{J=2} & = \frac{o_1 K^6 + o_2 K^5 + o_3 K^4 + o_4 K^3 + o_5 K^2 + o_6 K + o_7}{o_8 K^6 + o_9 K^5 + o_{10} K^4 + o_{11} K^3 + o_{12} K^2 + o_{13} K + o_{14}} \end{aligned} \quad (52)$$

where the coefficients $\sum_{i=1}^{14} o_i$ are listed in TABLE 8. After arranging, the problem becomes:

$$\text{maximize}_{M \in \mathbb{Z}_+} f(K)^{2^{nd} order} = \frac{\sum_{i=0}^{15} g_i K^i}{\sum_{j=0}^{16} G_j K^j} \quad (53)$$

where the coefficients $\sum_{i=0}^{15} g_i$ and $\sum_{j=0}^{16} G_j$ can easily be deduced. The global maximizer of $f(K)^{2^{nd} order}$ for $K \in \mathbb{R}$

satisfies the stationary condition $\frac{d}{dK} f(K)^{2^{nd} order} = 0$, which gives the polynomial:

$$\sum_{j=0}^{16} G_j K^j \times \sum_{i=1}^{15} i g_i K^{i-1} - \sum_{i=0}^{15} g_i K^i \times \sum_{j=1}^{16} j G_j K^{j-1} = 0 \quad (54)$$

The optimal number of UEs that give optimum EE is:

$$K^{2^{nd} order} = \max_\ell \left[K_\ell^{(o),2^{nd} order} \right] \quad (55)$$

where $K_\ell^{(o),2^{nd} order}$ is the real roots of the above polynomial in Eq. (54). When the polynomial orders increase, the number of roots is increased. These roots give more EE values. So, we will choose the root that give maximum EE for optimization problem.

2) OPTIMAL NUMBER OF ANTENNAS AT BS (M)

We will find $M^{*,2^{nd} order}$ that maximizes the $\mathbf{EE}^{2^{nd} order}$. The optimization problem is (56), as shown at the bottom of the next page, where $\bar{c}_{J=2} = \log_2 \left(1 + \frac{\alpha(M^3 + M + 1)}{\alpha(K-1)M^2 + (\alpha K - \alpha + K)M + (\alpha K - \alpha + K)} \right)$.

Let the constants:

$$\begin{aligned} w_1 & = KB \left(1 - \frac{(\tau^{DL} + \tau^{UL})K}{U} \right) \\ w_2 & = \left| S_2 + \left(\frac{K+1}{K} \right) S_3 + \left(\frac{K^2 + 3K + 2}{K^2} \right) S_4 \right|^{-2}, \\ w_3 & = S_0 \alpha K + \sum_{i=0}^2 C_i K^i \\ w_4 & = \sum_{i=0}^2 D_i K^i. \end{aligned}$$

The problem becomes:

$$\begin{aligned} & \text{maximize}_{M \in \mathbb{Z}_+} \phi(M)^{2^{nd} order} \\ & = \frac{w_1 \bar{c}_{J=2}}{\frac{w_2 S_1 \alpha (M^3 + M + 1)}{\alpha(K-1)(M^2 + M + 1) + K(M+1)} + w_3 + w_4 M + A v_1 \bar{c}_{J=2}} \end{aligned} \quad (57)$$

After arranging and approximation the logarithmic term, the problem becomes:

$$\text{maximize}_{M \in \mathbb{Z}_+} \phi(M)^{2^{nd} order} = \frac{\sum_{i=0}^8 v_i M^i}{\sum_{j=0}^9 V_j M^j} \quad (58)$$

$$\begin{aligned} & \text{maximize}_{\alpha \geq 0} \varphi(\alpha)^{1^{st} order} \\ & = \frac{KB \left(1 - \frac{(\tau^{DL} + \tau^{UL})K}{U} \right) \log \left(1 + \frac{\alpha(M+1)}{\alpha(K-1)+K} \right)}{\left(\sum_{i=0}^2 C_i K^i + M \sum_{i=0}^2 D_i K^i \right) + \left(\frac{S_1 \alpha (M+1)}{\alpha(K-1)+K} \left| S_2 + \left(\frac{K+1}{K} \right) S_3 \right|^{-2} + S_0 K \alpha \right) + AKB \left(1 - \frac{(\tau^{DL} + \tau^{UL})K}{U} \right) \log \left(1 + \frac{\alpha(M+1)}{\alpha(K-1)+K} \right)} \end{aligned} \quad (45)$$

where the coefficients $\sum_{i=0}^8 v_i$ are listed in TABLE 9 in Appendix I and $\sum_{j=0}^9 V_j$ can be deduced. The global maximizer of $\phi(M)^{2^{nd} order}$ satisfies the stationary condition $\frac{d}{dM} \phi(M)^{2^{nd} order} = 0$, which gives the polynomial:

$$\sum_{j=0}^9 V_j M^j \times \sum_{i=1}^8 i v_i M^{i-1} - \sum_{i=0}^8 v_i M^i \times \sum_{j=1}^9 j V_j M^{j-1} = 0 \quad (59)$$

The optimum number of antennas at BS that give maximum EE is:

$$M^{*, 2^{nd} order} = \max_{\ell} \left[M_{\ell}^{(o), 2^{nd} order} \right] \quad (60)$$

3) OPTIMAL DESIGN PARAMETER (α)

We will find $\alpha^{*, 2^{nd} order}$ that maximizes the **EE**^{2nd order}.

Let the constants:

$$\begin{aligned} w_1 &= KB \left(1 - \frac{(\tau^{DL} + \tau^{UL})K}{U} \right); \\ w_2 &= \left| S_2 + \left(\frac{K+1}{K} \right) S_3 + \left(\frac{K^2 + 3K + 2}{K^2} \right) S_4 \right|^{-2}; \\ w_5 &= \sum_{i=0}^2 C_i K^i + M \sum_{i=0}^2 D_i K^i \end{aligned}$$

The optimization problem is:

$$\begin{aligned} &\text{maximize } \varphi(\alpha)^{2^{nd} order} \\ &\alpha \geq 0 \\ &= \frac{w_1 \bar{c}_{J=2}}{\frac{w_2 S_1 \alpha (M^3 + M + 1)}{\alpha(K-1)(M^2 + M + 1) + K(M+1)} + S_0 \alpha K + w_5 + A w_1 \bar{c}_{J=2}} \end{aligned} \quad (61)$$

where $\bar{c}_{J=2} = \log_2 \left(1 + \frac{\alpha(M^3 + M + 1)}{\alpha(K-1)M^2 + (\alpha K - \alpha + K)M + (\alpha K - \alpha + K)} \right)$. After arranging and approximation the logarithmic term, the problem becomes:

$$\text{maximize } \varphi(\alpha)^{2^{nd} order} = \frac{Q_1 \alpha^2 + Q_2 \alpha}{Q_3 \alpha^3 + Q_4 \alpha^2 + Q_5 \alpha + Q_6} \quad (62)$$

where the coefficients $\sum_{i=1}^6 Q_i$ are listed in TABLE 10 in Appendix J. The global maximizer of $\varphi(\alpha)^{2^{nd} order}$ satisfies the stationary condition $\frac{d}{d\alpha} \varphi(\alpha)^{2^{nd} order} = 0$, which gives the polynomial:

$$Q_1 Q_3 \alpha^4 + 2Q_2 Q_3 \alpha^3 + (Q_2 Q_4 - Q_1 Q_5) \alpha^2 - 2Q_1 Q_6 \alpha - Q_2 Q_6 = 0 \quad (63)$$

Let $\alpha_{\ell}^{(o), 2^{nd} order}$ the real roots of the above polynomial, then the optimal α that give optimum EE is:

$$\alpha^{*, 2^{nd} order} = \max_{\ell} \left[\alpha_{\ell}^{(o), 2^{nd} order} \right] \quad (64)$$

$$\bar{R}(J=2) = \frac{BK \left(1 - \frac{(\tau^{DL} + \tau^{UL})K}{U} \right) \log_2 \left(1 + \frac{\alpha(M^3 + M + 1)}{\alpha(K-1)(M^2 + M + 1) + K(M+1)} \right)}{\left[\frac{S_1 \alpha (M^3 + M + 1)}{\alpha(K-1)(M^2 + M + 1) + K(M+1)} \left| S_2 + \left(\frac{K+1}{K} \right) S_3 + \left(\frac{K^2 + 3K + 2}{K^2} \right) S_4 \right|^{-2} + S_0 \alpha K + \sum_{i=0}^2 C_i K^i + M \sum_{i=0}^2 D_i K^i \right.} \quad (50)$$

$$\left. + ABK \left(1 - \frac{(\tau^{DL} + \tau^{UL})K}{U} \right) \log_2 \left(1 + \frac{\alpha(M^3 + M + 1)}{\alpha(K-1)(M^2 + M + 1) + K(M+1)} \right) \right]$$

$$\begin{aligned} \max_{M \in \mathbb{Z}_+} f(K)^{2^{nd}} &= \frac{K(B - R_0 K) \bar{c}_{J=2}}{\left[\frac{S_1 \bar{\alpha} (\bar{\beta}^3 K^3 + \bar{\beta} K + 1)}{\bar{\beta} (\bar{\alpha} \bar{\beta} + 1) K^3 + (\bar{\alpha} \bar{\beta} - \bar{\alpha} \bar{\beta}^2 + 1) K^2 + \bar{\alpha} (1 - \bar{\beta}) K - \bar{\alpha}} \left| S_2 + \left(\frac{K+1}{K} \right) S_3 + \left(\frac{K^2 + 3K + 2}{K^2} \right) S_4 \right|^{-2} \right.} \quad (51) \\ &\left. + S_0 \bar{\alpha} + \sum_{i=0}^2 C_i K^i + \bar{\beta} \sum_{i=0}^2 D_i K^{i+1} + AK(B - R_0 K) \bar{c}_{J=2} \right] \end{aligned}$$

$$\begin{aligned} \max_{M \in \mathbb{Z}_+} \phi(M)^{2^{nd} order} &= \frac{KB \left(1 - \frac{(\tau^{DL} + \tau^{UL})K}{U} \right) \bar{c}_{J=2}}{\left[\frac{S_1 \alpha (M^3 + M + 1)}{\alpha(K-1)(M^2 + M + 1) + K(M+1)} \left| S_2 + \left(\frac{K+1}{K} \right) S_3 + \left(\frac{K^2 + 3K + 2}{K^2} \right) S_4 \right|^{-2} + S_0 \alpha K \right.} \quad (56) \\ &\left. + \sum_{i=0}^2 C_i K^i + M \sum_{i=0}^2 D_i K^i + ABK \left(1 - \frac{(\tau^{DL} + \tau^{UL})K}{U} \right) \bar{c}_{J=2} \right] \end{aligned}$$

TABLE 1. Algorithm for optimization problem EE for first order polynomial precoder.

<p>1) Initialization Generate a feasible value for (M, K, α).</p> <p>2) Set $n = 0$;</p> <p>3) Repeat Update the value of K from Eq. (39).</p> $K^{1^{st}order} = \max_{\ell} [K_{\ell}^{(o),1^{st}order}]$ <p>4) Replace the value of M with optimal value from Eq. (44).</p> $M^{*,1^{st}order} = \left\lfloor \frac{e^{W\left(\frac{b_1 c_1}{d_1 e} - \frac{a_1}{e}\right) + 1} - a_1}{b_1} \right\rfloor$ <p>5) Optimize the parameter α according to Eq. (48).</p> $\alpha^{*,1^{st}order} = \max_{\ell} [\alpha_{\ell}^{(o),1^{st}order}]$ <p>6) $n = n + 1$;</p> <p>7) Until The optimal values of M and K are unchanged and constant.</p>
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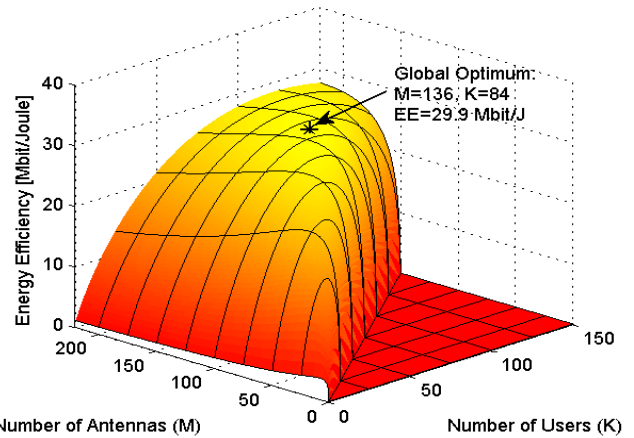


FIGURE 3. Energy efficiency with second order polynomial precoder ($J = 2$) in single-cell scenario with perfect CSI.

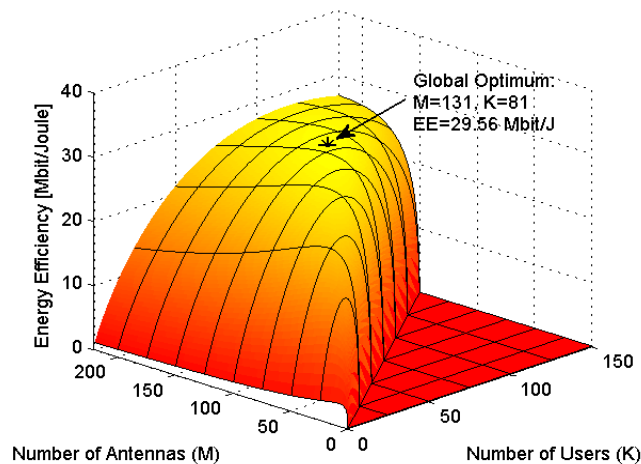


FIGURE 2. Energy efficiency with first order polynomial precoder ($J = 1$) in single-cell scenario with perfect CSI.

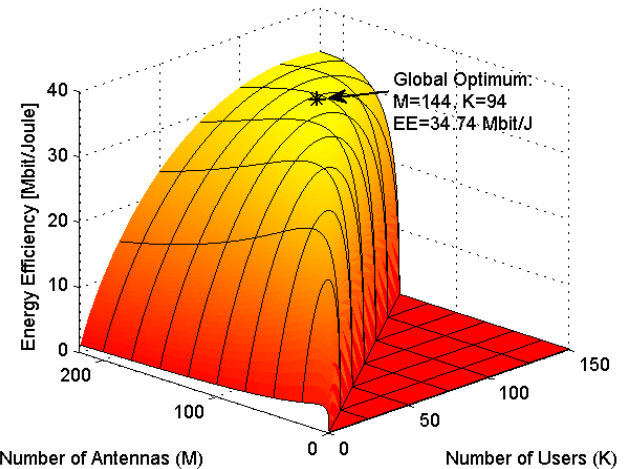


FIGURE 4. Energy efficiency with third order polynomial precoder ($J = 3$) in single-cell scenario with perfect CSI.

D. OPTIMIZATION ALGORITHM

The optimization problem for each M, K and α is obtained separately when the two other parameters are given. The global optimum for M and K is obtained by searching over all possible combinations of the pair and estimating the parameter α that proportional to transmit power and received SINR for each pair (M, K) . We will increase M and K and stop if the EE maximization begins to lower. The optimization for system parameters M, K and α is made sequentially according to alternating optimization algorithm. TABLE 1 shows the algorithm steps of optimization problem EE for first order polynomial precoder. These steps are also made for higher order polynomials $J = 2, 3, 4$ and 5 as illustrated in results.

VII. SIMULATION PARAMETERS AND RESULTS

The simulation parameters of multiuser massive MIMO system are listed in TABLE 2. We simulate the system for polynomial precoder with different orders. We compare and

illustrate the performance of massive MIMO based polynomial precoding with ZF and MMSE precoders. Fig. 2 shows the values of EE for first order polynomial precoder ($J = 1$) in single-cell scenario with perfect CSI. The figure illustrates that there is a global EE-optimum 29.56 Mbit/J at $M = 131$ and $K = 81$.

Fig. 3 shows the corresponding set of achievable EE values under second order polynomial precoder ($J = 2$). The global optimum is found at $M = 136, K = 84$ and $EE = 29.9$ Mbit/J. The optimum value of EE increases from 29.56 to 29.9 Mbit/J, when polynomial order increases from ($J = 1$) to ($J = 2$), respectively. When the polynomial order increases and equals three, there is another global optimum point at $M = 144, K = 94$ and $EE = 34.74$ Mbit/J as shown in Fig. 4.

The proposed polynomial precoder with third order gives global optimum point at $EE = 34.74$ Mbit/J which is high compared to $EE = 30.7$ Mbit/J and $EE = 30.3$ Mbit/J for ZF and MMSE precoders [25], respectively as shown in TABLE 3.

TABLE 2. Simulation parameters [24].

Parameter	Value	Parameter	Value
Cell radius: d_{max}	250 m	P_{FIX}	18 W
Minimum distance: d_{min}	35 m	P_{SYN}	2 W
Transmission bandwidth: B	20 MHz	P_{BS}	1 W
Channel coherence bandwidth: B_C	180 KHz	P_{UE}	0.1 W
Channel coherence time: T_C	10 ms	P_{COD}	0.1 W/(Gbit/s)
Coherence block (channel uses): U	1800	P_{DEC}	0.8 W/(Gbit/s)
Total noise power: $B\sigma^2$	-96 dBm	P_{BT}	0.25 W/(Gbit/s)
Relative pilot lengths: τ^{UL}, τ^{DL}	1	ζ^{DL}, ζ^{UL}	0.6, 0.4
Computational efficiency at BSs: L_{BS}	12.8 Gflops/W	η^{UL}, η^{DL}	0.3, 0.39
Computational efficiency at UEs: L_{UE}	5 Gflops/W	Polynomial orders (J)	1, 2, 3, 4, 5

TABLE 3. EE, M and K for different precoding schemes at optimum point.

	Number of Antennas (M)	Number of Users (K)	EE (Mbit/Joule)
ZF [15]	165	104	30.7
MMSE [15]	145	95	30.3
Polynomial Precoder ($J=1$)	131	81	29.56
Polynomial Precoder ($J=2$)	136	84	29.9
Polynomial Precoder ($J=3$)	144	94	34.74
Polynomial Precoder ($J=4$)	152	99	35.84
Polynomial Precoder ($J=5$)	164	110	37.28

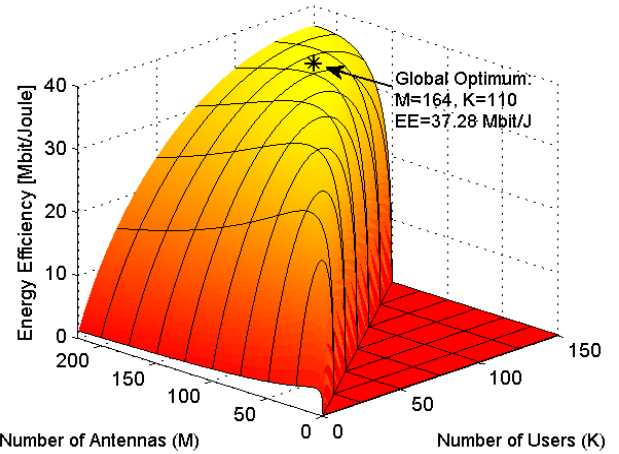


FIGURE 6. Energy efficiency with fifth order polynomial precoder ($J = 5$) in single-cell scenario with perfect CSI.

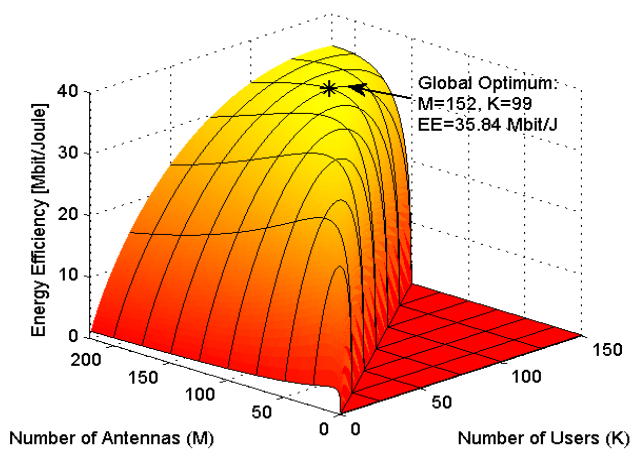


FIGURE 5. Energy efficiency with fourth order polynomial precoder ($J = 4$) in single-cell scenario with perfect CSI.

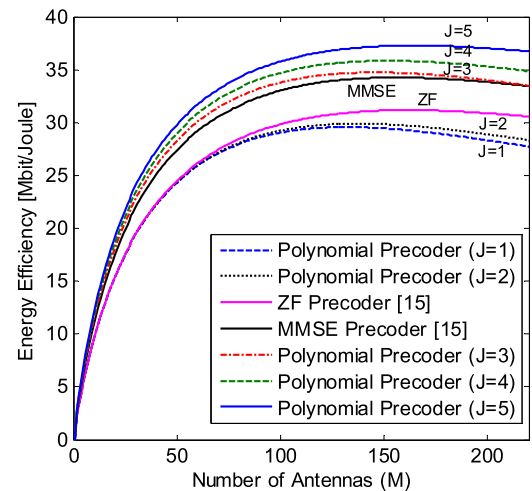


FIGURE 7. EE versus antennas number at BS with various precoder schemes in single-cell scenario with perfect CSI.

We observe that massive MIMO based higher order polynomial precoding achieves higher EE. Fig. 5 and Fig. 6 show energy efficiency with fourth ($J = 4$) and fifth ($J = 5$) orders polynomial precoder, respectively. The global optimum point

for fourth order is $M = 152, K = 99$ and $EE = 35.84$ Mbit/J. We compare different global optimum points at different polynomial orders with ZF and MMSE precoders as illustrated in TABLE 3.

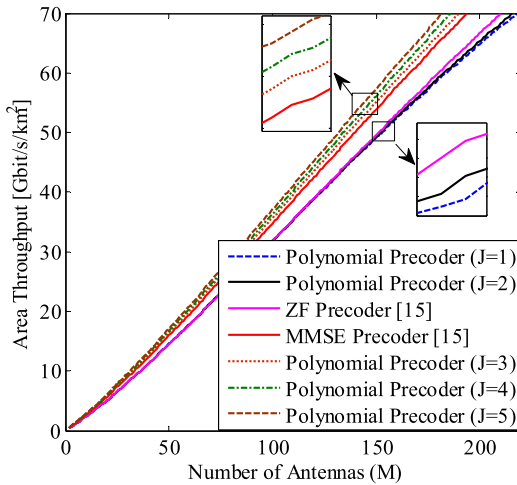


FIGURE 8. Area throughput versus number antennas at BS in the single-cell scenario.

The global optimum point for fifth order is $M = 164$, $K = 110$ and $EE = 37.28$ Mbit/J which is better compared to lower order values. The EE optimum for massive MIMO based higher order polynomial precoder is much higher than system based ZF and MMSE. Fig. 7 illustrates the maximum EE as a function of the BS antennas number. The multiuser massive MIMO system based ZF gives better performance than first and second order for polynomial precoder. The EE is improved when we increase polynomial order.

Fig. 8 depicts the area throughput that maximizes the EE for various M . There was an improvement in optimal EE for higher order polynomial processing as compared to ZF and lower order values for polynomial precoder.

It is extravagant to implement a considerable antennas number at BS and after that co-process them using high complexity precoders like ZF and MMSE scheme that is limiting both the area throughput and energy efficiency. The polynomial precoder has J degrees of freedom that can be optimized. It is desirable to choose the polynomial precoder order J that achieves a maximum EE with respect to precoders based matrix inversion. Polynomial precoder with higher order values can achieve both unprecedented area throughput and great EE. From the results, the multiuser massive MIMO system based polynomial precoder scheme gives better performance than system based classical ZF and MMSE precoders.

Fig. 9 compares the energy efficiency for different precoding techniques. We can observe that the proposed polynomial precoder with $(J = 4)$ with more users can improve the energy efficiency of multiuser massive MIMO system. This figure illustrates that the energy efficiency with polynomial precoder $(J = 4)$ gives the highest energy efficiency although the number of users K increase. MMSE performs the higher energy efficiency than other techniques. The proposed polynomial precoder based massive MIMO system is a good for obtaining the maximal energy efficiency in the future cellular networks.

Fig. 10 compares total transmit power versus number of antennas for different precoding techniques. The proposed

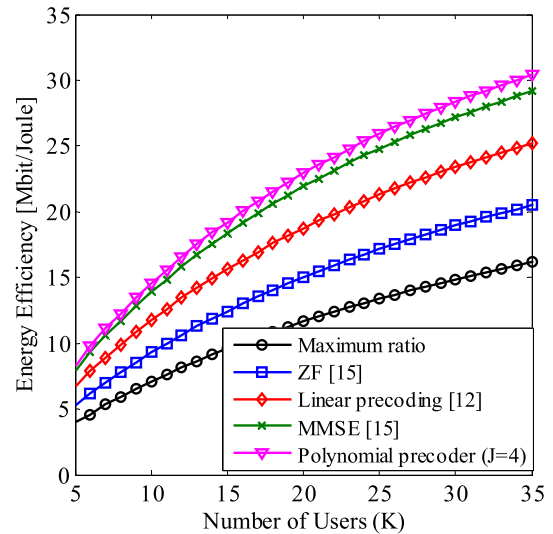


FIGURE 9. Energy efficiency versus users number.

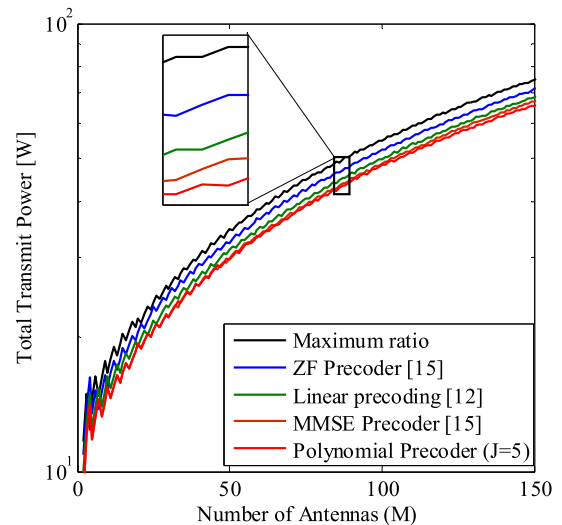


FIGURE 10. Total transmit power versus antennas number at BS in single-cell scenario.

precoder employs low amount of transmission power compared with the other precoders. The transmit power for system based polynomial precoding is low. This illustrates that massive MIMO can be designed using low-power consumer grade transceiver equipment at the BSs.

The transmit power per BS antenna reduces with M as depicted in Fig. 11. The transmit power with polynomial precoder is smaller than ZF, MMSE and linear precoding. The values of the transmit powers are smaller than for classical macro BSs system (which can work at 6.35 W per antenna [44]) and shows that the energy efficiency optimal solution with polynomial precoder can be employed with low power users.

The complexity of the polynomial precoder is calculated and compared with the conventional precoders. We use the number of floating point operations (FLOPs) to determine the computational complexity, where each multiplication or

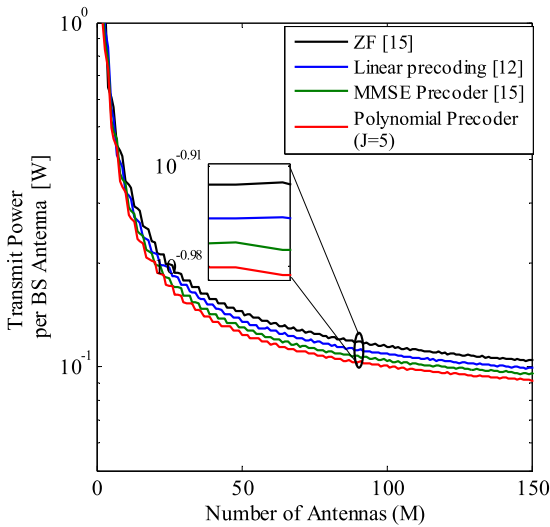


FIGURE 11. Transmit power per BS antenna versus number of antennas.

addition is expressed as one FLOP. Our calculation is based on the FLOPs number at the BS for producing τ vectors of precoded data. The complexity of the polynomial precoder is determined for generation of τ vectors via Horner's rule [45]. The precoded vectors can be expressed as $\mathbf{Z} = \frac{\mathbf{H}^H}{\sqrt{M}} \left(\omega_0 + \omega_1 \mathbf{H}\mathbf{H}^H \left(\mathbf{I} + \frac{\omega_2}{\omega_1} \mathbf{H}\mathbf{H}^H \left(\mathbf{I} + \frac{\omega_3}{\omega_2} \mathbf{H}\mathbf{H}^H \dots \right) \right) \right)$. First, we multiply \mathbf{H}^H with a scaled version of the channel matrix \mathbf{H} . After applying calculations J times, the resulted vector is multiplied with \mathbf{H}^H / \sqrt{M} . We require $(2K - 1)M + (2M - 1)K$ FLOPs for each multiplication with the channel matrix [46]. Therefore, the total complexity is given by $\tau((J + 1)(2K - 1)M + J(2M - 1)K)$.

Another method for calculation is to determine matrix polynomial $\mathbf{Z} = \mathbf{H}^H / \sqrt{M} \sum_{\ell=0}^J \omega_\ell (\mathbf{H}\mathbf{H}^H)^\ell$ and then operate τ vector matrix multiplications to obtain the vectors of transmitted data. Therefore, the complexity is $(2K - 1)M\tau + 0.5L(K^2 + K)(2M - 1) + MK(2K - 1)$ FLOPs.

In case of MMSE precoder, the matrix of precoding is obtained once in every τ symbol intervals. Also, the multiplication is operated τ times. The $0.5(K^2 + K)(2M - 1)$ is required to obtain the covariance matrix $\mathbf{H}\mathbf{H}^H$ from \mathbf{H} . For matrix inversion with size $K \times K$, it operates $(K^3 + K^2 + K)$ [46]. Then, the multiplication of \mathbf{H} with the inverse matrix operates $KM(2K - 1)$. After that, $(2K - 1)M\tau$ is calculated for matrix multiplications and to give τ precoded data vectors. Finally, the overall complexity is $KM(2K - 1) + 0.5(K^2 + K)(2M - 1) + (K^3 + K^2 + K) + (2K - 1)M\tau$.

There are any divisions in the complexity calculations of the polynomial precoder, which becomes attractive in its applications. In addition, the operations are sequential process, where floating point operations are required because of stability issues. It can be applied via processors with parallel fixed point, which can achieve lower implementation complexity.

Finally, the results proved that the proposed multiuser massive MIMO system with polynomial precoder can give

TABLE 4. The coefficients for polynomial orders.

Coefficients $\{S_i\}$	
S_0	$S_0 = \frac{B\zeta^{UL}\sigma^2(d_{max}^{\kappa+2} - d_{min}^{\kappa+2})}{\pi\eta^{UL}\bar{d}(\kappa + 2)(d_{max}^2 - d_{min}^2)}$
S_1	$S_1 = \frac{B\zeta^{DL}\sigma^2\bar{d}}{\pi\eta^{DL}(d_{max}^2 - d_{min}^2)}$
S_2	$S_2 = \left \frac{\omega_0(d_{max}^{2-\kappa} - d_{min}^{2-\kappa})}{(2 - \kappa)} \right $
S_3	$S_3 = \left \frac{\omega_1\bar{d}(d_{max}^{2-2\kappa} - d_{min}^{2-2\kappa})}{(2 - 2\kappa)} \right $
S_4	$S_4 = \left \frac{\omega_2\bar{d}^2(d_{max}^{2-3\kappa} - d_{min}^{2-3\kappa})}{(2 - 3\kappa)} \right $
S_5	$S_5 = \left \frac{\omega_3\bar{d}^3(d_{max}^{2-4\kappa} - d_{min}^{2-4\kappa})}{(2 - 4\kappa)} \right $
S_6	$S_6 = \left \frac{\omega_4\bar{d}^4(d_{max}^{2-\kappa} - d_{min}^{2-\kappa})(d_{max}^{2-4\kappa} - d_{min}^{2-4\kappa})}{\pi(2 - \kappa)(2 - 4\kappa)(d_{max}^2 - d_{min}^2)} \right $
S_7	$S_7 = \left \frac{\omega_5\bar{d}^5(d_{max}^{2-3\kappa} - d_{min}^{2-3\kappa})^2}{\pi(2 - 3\kappa)^2(d_{max}^2 - d_{min}^2)} \right $

maximum energy efficiency in the future cellular networks. The utilization of polynomial precoder helps in improving the system performance and achieves high values of energy efficiency.

VIII. CONCLUSION

In this paper, multiuser massive MIMO system based polynomial precoder was proposed and its performance compared with ZF and MMSE precoders. The optimization problem for average RF power and EE was mathematically derived and analyzed. This paper study the EE optimization problem that illustrate how to choose the active UEs number K , BS antennas number M , and transmit power to maximize the EE in multiuser massive MIMO systems. Mathematical formulas for the EE-maximizing parameter values are expressed under polynomial precoder with different orders in the case of perfect CSI. The complexity of polynomial precoder is low because there is no precoding matrix inversion to compute as found in ZF and MMSE. Also, the value of polynomial order J can be chosen to the available hardware. It is competitive in terms of both number of UEs, number of antennas at BS and implementation complexity. Results have shown that the utilization of polynomial precoder in massive MIMO system enhances the performance and provides high EE values.

APPENDIX A

See Table 4.

APPENDIX B PROOF OF LEMMA 1

The RF power for first order polynomial precoder is:

$$\mathbf{P}_{\text{RF}}^{\text{DL}, 1^{\text{st}} \text{ order}} = \frac{B\zeta^{\text{DL}}}{\eta^{\text{DL}}} \sigma^2 \left(2^{\frac{\bar{R}(J=1)}{B}} - 1 \right) \times \mathbb{E} \left\{ \mathbf{1}_K^T \left| \sum_{\ell=0}^1 \omega_{\ell} \left(\mathbf{h}_k \mathbf{h}_k^H \right)^{\ell+1} \right|^{-2} \mathbf{1}_K \right\} \quad (65)$$

Then,

$$= \frac{B\zeta^{\text{DL}} \sigma^2 (2^{\bar{R}(J=1)/B} - 1)}{\eta^{\text{DL}}} \left| \sum_{\ell=0}^1 \omega_{\ell} \cdot \mathbb{E} \left\{ \text{tr} \left(\left(\mathbf{h}_k \mathbf{h}_k^H \right)^{\ell+1} \right) \right\} \right|^{-2} \quad (66)$$

For fixed user locations, we note that $\mathbf{H}\mathbf{H}^H \in \mathbb{C}^{K \times K}$ has a complex distribution $\mathbf{\Lambda} = \text{diag} (p(x_1), p(x_2), \dots, p(x_k))$ [41]. Then, an eigen-decomposition is applied to the channel covariance matrix:

$$\mathbf{H}\mathbf{H}^H = \mathbf{T}\mathbf{\Lambda}\mathbf{T}^H \quad (67)$$

where $\mathbf{\Lambda}$ and \mathbf{T} denote the channel covariance matrix eigenvalue and eigenvector.

$$\begin{aligned} \mathbb{E}_{\{\mathbf{h}_k, x_k\}} \left\{ \text{tr} \left(\left(\mathbf{h}_k \mathbf{h}_k^H \right)^{\ell+1} \right) \right\} &= \mathbb{E}_{\{x_k\}} \left\{ \text{tr} \left(\left(\mathbf{T}\mathbf{\Lambda}\mathbf{T}^H \right)^{\ell+1} \right) \right\} \\ &= \mathbb{E}_{\{x_k\}} \left\{ \text{tr} \left(\mathbf{T}\mathbf{\Lambda}^{\ell+1}\mathbf{T}^H \right) \right\} \end{aligned} \quad (68)$$

Then,

$$\begin{aligned} \mathbf{P}_{\text{RF}}^{\text{DL}, 1^{\text{st}} \text{ order}} &= \frac{B\zeta^{\text{DL}}}{\eta^{\text{DL}}} \sigma^2 (2^{\bar{R}(J=1)/B} - 1) \left| \omega_0 \cdot \mathbb{E}_{\{x_k\}} \left\{ \text{tr}(\mathbf{\Lambda}) \right\} \right. \\ &\quad \left. + \omega_1 \cdot \mathbb{E}_{\{x_k\}} \left\{ \text{tr}(\mathbf{\Lambda}^2) \right\} \right|^{-2} \end{aligned} \quad (69)$$

Let the distribution function of UEs in a single circular cell is [24]:

$$u(x) = \begin{cases} \frac{\|x\|}{\pi (d_{\max}^2 - d_{\min}^2)} & d_{\min} \leq \|x\| \leq d_{\max} \\ 0 & \text{otherwise.} \end{cases} \quad (70)$$

and large scale fading due to path loss is [24]:

$$p(x) = \bar{d} / \|x\|^{\kappa} \quad \text{for } \|x\| \geq d_{\min} \quad (71)$$

By using [41, Eq. (15) and Eq. (16)], the first and second-order moment is:

$$\mathbb{E}_{\{x_k\}} \left\{ \text{tr}(\mathbf{\Lambda}) \right\} = \mathbb{E}_{\{x_k\}} \left\{ p(x_k) \right\} = \int_{d_{\min}}^{d_{\max}} p(x) u(x) dx \quad (72)$$

Then,

$$\begin{aligned} &= \frac{\bar{d}}{\pi (d_{\max}^2 - d_{\min}^2)} \int_{d_{\min}}^{d_{\max}} \frac{\|x\|}{\|x\|^{\kappa}} dx \\ &= \frac{\bar{d} (d_{\max}^{2-\kappa} - d_{\min}^{2-\kappa})}{\pi (2 - \kappa) (d_{\max}^2 - d_{\min}^2)} \end{aligned} \quad (73)$$

TABLE 5. The coefficients for power consumption model based polynomial precoder.

Coefficients $\{A \text{ and } C_i\}$	
A	$A = P_{\text{COD}} + P_{\text{DEC}} + P_{\text{BT}}$
C_0	$C_0 = P_{\text{FIX}} + P_{\text{SYN}}$
C_1	$C_1 = P_{\text{UE}} - \frac{JB}{2UL_{\text{BS}}}$
C_2	$C_2 = \frac{B}{U} \left(\frac{2\tau^{\text{UL}}}{L_{\text{BS}}} + \frac{4\tau^{\text{DL}}}{L_{\text{UE}}} - \frac{J}{2L_{\text{BS}}} \right)$
Coefficients $\{D_i\}$	
D_0	$D_0 = P_{\text{BS}}$
D_1	$D_1 = \frac{B}{L_{\text{BS}}} \left(2 + \frac{1}{U} (J - 1) \right)$
D_2	$D_2 = \frac{B}{UL_{\text{BS}}} (-2(\tau^{\text{DL}} + \tau^{\text{UL}}) + J + 2)$

After arranging the terms:

$$\mathbb{E}_{\{x_k\}} \left\{ \text{tr}(\mathbf{\Lambda}^2) \right\} = \mathbb{E}_{\{x_k\}} \left\{ (p(x))^2 \right\} + \frac{1}{K} E_{\{x_k\}} \left\{ (p(x))^2 \right\} \quad (74)$$

Then,

$$\begin{aligned} &= \frac{K+1}{K} \int_{d_{\min}}^{d_{\max}} (p(x))^2 u(x) dx \\ &= \left(\frac{K+1}{K} \right) \frac{\bar{d}^2 (d_{\max}^{2-2\kappa} - d_{\min}^{2-2\kappa})}{\pi (2 - 2\kappa) (d_{\max}^2 - d_{\min}^2)} \end{aligned} \quad (75)$$

After that,

$$\begin{aligned} \mathbf{P}_{\text{RF}}^{\text{DL}, 1^{\text{st}} \text{ order}} &= \frac{B\zeta^{\text{DL}} \sigma^2 (2^{\bar{R}(J=1)/B} - 1)}{\eta^{\text{DL}}} \left| \frac{\omega_0 \bar{d} (d_{\max}^{2-\kappa} - d_{\min}^{2-\kappa})}{\pi (2 - \kappa) (d_{\max}^2 - d_{\min}^2)} \right. \\ &\quad \left. + \left(\frac{K+1}{K} \right) \frac{\omega_1 \bar{d}^2 (d_{\max}^{2-2\kappa} - d_{\min}^{2-2\kappa})}{\pi (2 - 2\kappa) (d_{\max}^2 - d_{\min}^2)} \right|^{-2} \end{aligned} \quad (76)$$

and the achievable rate of first order polynomial precoder $\bar{R}(J=1)$, is computed according to [21], [42]:

$$\bar{R}_{J=1}^{\text{DL}} = B \log \left(1 + \frac{\alpha (M+1)}{\alpha (K-1) + K} \right) \quad (77)$$

Hence, the RF power is given in Eq. (18).

APPENDIX C PROOF OF LEMMA 2

The RF power for second order polynomial precoder is:

$$\mathbf{P}_{\text{RF}}^{\text{DL}, 2^{\text{nd}} \text{ order}}$$

TABLE 6. The coefficients $\{R_i\}$.

$$\begin{aligned}
 R_1 &= -R_0 r_1 r_6 \\
 R_2 &= r_6 (Br_1 - R_0 r_5) - R_0 r_1 r_7 \\
 R_3 &= r_6 (Br_5 - R_0 r_2) + r_7 (Br_1 - R_0 r_5) - 6R_0 r_1 \\
 R_4 &= r_6 (Br_2 - R_0 r_3) + r_7 (Br_5 - R_0 r_2) + 6(Br_1 - R_0 r_5) + 3R_0 r_1 \\
 R_5 &= r_6 (Br_3 - R_0 r_4) + r_7 (Br_2 - R_0 r_3) + 6(Br_5 - R_0 r_2) - 3(Br_1 - R_0 r_5) \\
 R_6 &= Br_4 r_6 + r_7 (Br_3 - R_0 r_4) + 6(Br_2 - R_0 r_3) - 3(Br_5 - R_0 r_2) \\
 R_7 &= Br_4 r_7 + 6(Br_3 - R_0 r_4) - 3(Br_2 - R_0 r_3) \\
 R_8 &= 6Br_4 - 3(Br_3 - R_0 r_4) \\
 R_9 &= -3Br_4 \\
 R_{10} &= (\ln 2 r_1 r_6 D_2 / \bar{\alpha}) \\
 R_{11} &= r_{12} (r_5 \bar{\beta} D_2 + r_1 r_9) + \ln 2 r_1 r_6 D_2 (\bar{\beta} + 2) \\
 R_{12} &= r_{12} (r_2 \bar{\beta} D_2 + r_5 r_9 + r_1 r_{10}) + \bar{\alpha} r_{12} (r_5 \bar{\beta} D_2 + r_1 r_9) (\bar{\beta} + 2) + r_1 r_8 \bar{\beta} D_2 - AR_0 r_1 r_6 \\
 R_{13} &= r_{12} r_{13} + 6 \ln 2 (r_2 \bar{\beta} D_2 + r_5 r_9 (\bar{\beta} + 2) / \bar{\alpha} + r_1 r_{10}) + r_8 (r_5 \bar{\beta} D_2 + r_1 r_9) + 2 \ln 2 r_1 \bar{\beta} D_2 (-2\bar{\beta} - 3) \\
 &\quad + r_6 (ABr_1 - AR_0 r_5) - AR_0 r_1 r_7 \\
 R_{14} &= r_{12} r_{14} + \bar{\alpha} r_{12} r_{13} (\bar{\beta} + 2) + r_8 (r_2 \bar{\beta} D_2 + r_5 r_9 + r_1 r_{10}) + 2 \ln 2 (-2\bar{\beta} - 3) (r_5 \bar{\beta} D_2 + r_1 r_9) + \ln 2 r_1 \bar{\beta} D_2 \\
 &\quad + r_6 (ABr_5 - AR_0 r_2) + r_7 (ABr_1 - AR_0 r_5) - 6AR_0 r_1 \\
 R_{15} &= r_{12} (r_4 r_9 + r_3 r_{10} + r_2 r_{11}) + \bar{\alpha} r_{12} r_{14} (\bar{\beta} + 2) + r_8 r_{13} + (Br_2 - R_0 r_3) (r_2 \bar{\beta} D_2 + r_5 r_9 + r_1 r_{10}) + \ln 2 (r_5 \bar{\beta} D_2 + r_1 r_9) \\
 &\quad + r_6 (ABr_2 - AR_0 r_3) + r_7 (ABr_5 - AR_0 r_2) + 6(ABr_1 - AR_0 r_5) + 3AR_0 r_1 \\
 R_{16} &= r_{12} (r_4 r_{10} + r_3 r_{11}) + \bar{\alpha} r_{12} (r_4 r_9 + r_3 r_{10} + r_2 r_{11}) (\bar{\beta} + 2) + r_8 r_{14} - 2 \ln 2 (2\bar{\beta} + 3) r_{13} + \ln 2 (r_2 \bar{\beta} D_2 + r_5 r_9 + r_1 r_{10}) \\
 &\quad + r_6 (ABr_3 - AR_0 r_4) + r_7 (ABr_2 - AR_0 r_3) + 6(ABr_5 - AR_0 r_2) - 3(ABr_1 - AR_0 r_5) \\
 R_{17} &= r_{12} (\bar{\alpha} \bar{\beta} S_1 + r_4 r_{11}) + \bar{\alpha} r_{12} (r_4 r_{10} + r_3 r_{11}) (\bar{\beta} + 2) + r_8 (r_4 r_9 + r_3 r_{10} + r_2 r_{11}) + 2 \ln 2 (-2\bar{\beta} - 3) r_{14} + \ln 2 r_{13} \\
 &\quad + ABr_4 r_6 + r_7 (ABr_3 - AR_0 r_4) + 6(ABr_2 - AR_0 r_3) - 3(ABr_5 - AR_0 r_2) \\
 R_{18} &= (6 \ln 2 S_1 / \bar{\alpha}) + \bar{\alpha} r_{12} (\bar{\alpha} \bar{\beta} S_1 + r_4 r_{11}) (\bar{\beta} + 2) + r_8 (r_4 r_{10} + r_3 r_{11}) - 2 \ln 2 (2\bar{\beta} + 3) (r_4 r_9 + r_3 r_{10} + r_2 r_{11}) + \ln 2 r_{14} \\
 &\quad + ABr_4 r_7 + 6(ABr_3 - AR_0 r_4) - 3(ABr_2 - AR_0 r_3) \\
 R_{19} &= \bar{\alpha} r_{12} \bar{\alpha} S_1 (\bar{\beta} + 2) + r_8 (\bar{\alpha} \bar{\beta} S_1 + r_4 r_{11}) - 2 \ln 2 (2\bar{\beta} + 3) (r_4 r_{10} + r_3 r_{11}) + \ln 2 (r_4 r_9 + r_3 r_{10} + r_2 r_{11}) \\
 &\quad + 6ABr_4 - 3(ABr_3 - AR_0 r_4) \\
 R_{20} &= r_8 \bar{\alpha} S_1 - 2 \ln 2 (2\bar{\beta} + 3) (\bar{\alpha} \bar{\beta} S_1 + r_4 r_{11}) + \ln 2 (r_4 r_{10} + r_3 r_{11}) - 3ABr_4 \\
 R_{21} &= -2 \ln 2 (2\bar{\beta} + 3) \bar{\alpha} S_1 + \ln 2 (\bar{\alpha} \bar{\beta} S_1 + r_4 r_{11}) \\
 R_{22} &= \ln 2 \bar{\alpha} S_1
 \end{aligned}$$

where

$$r_1 = S_2^2 + 2S_2 S_3 + S_3^2 r_2 = (\bar{\alpha} + 1) S_3^2 + \bar{\alpha} S_2^2 r_3 = -\bar{\alpha} (S_3^2 + 2)$$

$$r_4 = -\bar{\alpha} S_3^2 r_5 = (\bar{\alpha} + 2) S_3^2 + \bar{\alpha} S_2^2 + 2(\bar{\alpha} + 1) S_2 S_3 r_6 = 6\bar{\beta} / \bar{\alpha}$$

$$r_7 = 3(2\bar{\beta} + 2/\bar{\alpha} + \bar{\beta}^2) r_8 = \ln 2 (\bar{\beta}^2 + 6(\bar{\beta} - 1/\bar{\alpha} + 1)) r_9 = (C_2 + \bar{\beta} D_1)$$

$$r_{10} = (C_1 + \bar{\beta} D_0) r_{11} = (C_0 + \bar{\alpha} S_0) r_{12} = 6 \ln 2 / \bar{\alpha}^2$$

$$r_{13} = (r_3 \bar{\beta} D_2 + r_2 r_9 + r_5 r_{10} + r_1 r_{11}) r_{14} = (r_4 \bar{\beta} D_2 + r_3 r_9 + r_2 r_{10} + r_5 r_{11})$$

$$= \frac{B\zeta^{\text{DL}}}{\eta^{\text{DL}}} \sigma^2 (2^{\bar{R}(J=2)/B} - 1) \left| \sum_{\ell=0}^2 \omega_{\ell} \cdot \mathbb{E} \left\{ \text{tr} \left(\left(\mathbf{h}_k \mathbf{h}_k^H \right)^{\ell+1} \right) \right\} \right|^{-2} \quad (78)$$

Then,

$$\begin{aligned} \mathbf{P}_{\text{RF}}^{\text{DL}, 2^{\text{nd}} \text{ order}} &= \frac{B\zeta^{\text{DL}}}{\eta^{\text{DL}}} \sigma^2 (2^{\bar{R}(J=2)/B} - 1) \left| \omega_0 \cdot \mathbb{E}_{\{x_k\}} \left\{ \text{tr}(\mathbf{\Lambda}) \right\} \right. \\ &\quad \left. + \omega_1 \cdot \mathbb{E}_{\{x_k\}} \left\{ \text{tr}(\mathbf{\Lambda}^2) \right\} + \omega_2 \cdot \mathbb{E}_{\{x_k\}} \left\{ \text{tr}(\mathbf{\Lambda}^3) \right\} \right|^{-2} \quad (79) \end{aligned}$$

After that,

$$\begin{aligned} \mathbf{P}_{\text{RF}}^{\text{DL}, 2^{\text{nd}} \text{ order}} &= \frac{B\zeta^{\text{DL}} \sigma^2 \left(2^{\frac{\bar{R}(J=2)}{B}} - 1 \right)}{\eta^{\text{DL}}} \\ &\quad \times \left| \frac{\omega_0 \bar{d} \left(d_{\max}^{2-\kappa} - d_{\min}^{2-\kappa} \right)}{\pi (2-\kappa) \left(d_{\max}^2 - d_{\min}^2 \right)} \right. \\ &\quad \left. + \left(\frac{K+1}{K} \right) \frac{\omega_1 \bar{d}^2 \left(d_{\max}^{2-2\kappa} - d_{\min}^{2-2\kappa} \right)}{\pi (2-2\kappa) \left(d_{\max}^2 - d_{\min}^2 \right)} \right. \\ &\quad \left. + \left(\frac{K^2+3K+2}{K^2} \right) \frac{\omega_2 \bar{d}^3 \left(d_{\max}^{2-3\kappa} - d_{\min}^{2-3\kappa} \right)}{\pi (2-3\kappa) \left(d_{\max}^2 - d_{\min}^2 \right)} \right|^{-2} \quad (80) \end{aligned}$$

Arranging the terms, then

$$\begin{aligned} \mathbf{P}_{\text{RF}}^{\text{DL}, 2^{\text{nd}} \text{ order}} &= \frac{B\zeta^{\text{DL}} \sigma^2 \bar{d} \left(2^{\frac{\bar{R}(J=2)}{B}} - 1 \right)}{\pi \eta^{\text{DL}} \left(d_{\max}^2 - d_{\min}^2 \right)} \\ &\quad \times \left| \frac{\omega_0 \left(d_{\max}^{2-\kappa} - d_{\min}^{2-\kappa} \right)}{(2-\kappa)} + \left(\frac{K+1}{K} \right) \frac{\omega_1 \bar{d} \left(d_{\max}^{2-2\kappa} - d_{\min}^{2-2\kappa} \right)}{(2-2\kappa)} \right. \\ &\quad \left. + \left(\frac{K^2+3K+2}{K^2} \right) \frac{\omega_2 \bar{d}^2 \left(d_{\max}^{2-3\kappa} - d_{\min}^{2-3\kappa} \right)}{(2-3\kappa)} \right|^{-2} \quad (81) \end{aligned}$$

and the achievable rate of second order polynomial precoder $\bar{R}(J=2)$, is computed according to [21], [42]:

$$\begin{aligned} \bar{R}_{J=2}^{\text{DL}} &= B \log \left(1 + \frac{\alpha (M^3 + 2M^2 + 2M + 1)}{\alpha (K-1) (M^2 + M + 1) + K(1+M)} \right) \quad (82) \end{aligned}$$

Hence, the RF power is given in Eq. (21).

APPENDIX D

See Table 5.

TABLE 7. The coefficients $\{U_i\}$.

$$\begin{aligned} U_1 &= l_1 u_1 (K-1) \\ U_2 &= 6K l_1 (M+1)(K-1) + K l_1 u_1 \\ U_3 &= 6l_1 K^2 (M+1) \\ U_4 &= \ln 2 S_0 u_2 K (K-1) \\ U_5 &= (K-1) (\ln 2 l_3 u_2 + A l_1 u_1 + 6 \ln 2 S_0 u_3 K^2) \\ &\quad + \ln 2 S_0 u_2 K^2 + \ln 2 S_1 l_2 u_2 (M+1) \\ U_6 &= 6(K-1) (\ln 2 l_3 u_3 K + A l_1 K (M+1) + \ln 2 S_0 K^3) \\ &\quad + K (\ln 2 l_3 u_2 + A l_1 u_1 + 6 \ln 2 S_0 u_3 K^2) \\ &\quad + 6 \ln 2 S_1 l_2 u_3 K (M+1) \\ U_7 &= K (6 \ln 2 l_3 u_3 K + 6 A l_1 K (M+1) + 6 \ln 2 S_0 K^3) \\ &\quad + 6 \ln 2 l_3 (K-1) K^2 + 6 \ln 2 S_1 l_2 (M+1) K^2 \\ U_8 &= 6 \ln 2 l_3 K^3 \end{aligned}$$

where

$$\begin{aligned} u_1 &= 3M^2 + 6K(M+1) - 3 \\ u_2 &= 6K(M+K-1) + M(M-4) + 1 \\ u_3 &= 2K + M - 1 \end{aligned}$$

TABLE 8. The coefficients $\{\Sigma_{i=1}^{14} o_i\}$.

$$\begin{aligned} o_1 &= 3\bar{\alpha}\bar{\beta}^4 (2\bar{\alpha}\bar{\beta} + \bar{\alpha}\bar{\beta}^2 + 2) \\ o_2 &= 6\bar{\alpha}\bar{\beta}^3 (\bar{\alpha}\bar{\beta} - \bar{\alpha}\bar{\beta}^2 + 1) \\ o_3 &= 6\bar{\alpha}^2 \bar{\beta}^3 (1 - \bar{\beta}) \\ o_4 &= \bar{\alpha}\bar{\beta} (-7\bar{\alpha}\bar{\beta}^2 + 2\bar{\alpha}\bar{\beta} + 2) \\ o_5 &= \bar{\alpha} (7\bar{\alpha}\bar{\beta} - 4\bar{\alpha}\bar{\beta}^2 + 1) \\ o_6 &= \bar{\alpha}^2 (1 - 2\bar{\beta}) \\ o_7 &= 2\bar{\alpha}^2 \\ o_8 &= 6\bar{\beta}^2 (\bar{\alpha}\bar{\beta} + 1) (\bar{\alpha}\bar{\beta}^2 + \bar{\alpha}\bar{\beta} + 1) + \bar{\alpha}^2 \bar{\beta}^6 \\ o_9 &= 6\bar{\alpha}\bar{\beta}^3 (\bar{\alpha}\bar{\beta} - \bar{\alpha}\bar{\beta}^2 + 1) \\ o_{10} &= 6\bar{\alpha}^2 \bar{\beta}^2 (1 - 2\bar{\beta}^2 + 2\bar{\beta} + 1/\bar{\alpha}) + 6 \\ o_{11} &= 6\bar{\alpha}\bar{\beta} (-2\bar{\alpha}\bar{\beta}^2 + \bar{\alpha}\bar{\beta} + 1) \\ o_{12} &= 2\bar{\alpha}^2 (3 - 3\bar{\beta} - 2\bar{\beta}^2) \\ o_{13} &= -6\bar{\alpha}^2 \\ o_{14} &= \bar{\alpha}^2 \end{aligned}$$

APPENDIX E

The values of consumption powers are [5]–[6], [31]–[34]:

$$P_{TC} = MP_{BS} + P_{SYN} + KP_{UE} \quad (83)$$

$$P_{CE} = \frac{2B\tau^{\text{UL}} MK^2}{UL_{BS}} + \frac{4B\tau^{\text{DL}} K^2}{UL_{UE}} \quad (84)$$

$$P_{C/D} = K (P_{COD} + P_{DEC}) \left(1 - \frac{(\tau^{\text{DL}} + \tau^{\text{UL}}) K}{U} \right) \bar{R} \quad (85)$$

TABLE 9. The coefficients $\{\sum_{i=0}^8 v_i\}$.

$$\begin{aligned}
 v_0 &= \alpha^3(K^3 + K - 2) + \alpha^2K(2K + 1) + \alpha K^2 \\
 v_1 &= w_1[\alpha^3(8K^2 + 3K + 5) + \alpha K(8K + 3\alpha)] \\
 v_2 &= w_1[3\alpha^3(5K^2 - 8K + 3) + \alpha K(28\alpha K + 13K - 25\alpha)] \\
 v_3 &= w_1[\alpha^3(26K^2 - 52K - 1) + \alpha K(38\alpha K + 12K - 23\alpha)] \\
 v_4 &= w_1[\alpha^3(25K^2 - 47K + 22) \\
 &\quad + \alpha K(36\alpha K + 12K - 36\alpha)] \\
 v_5 &= w_1[24\alpha^3(K^2 - 2K + 1) + 6\alpha K(4\alpha K + K - 4\alpha)] \\
 v_6 &= w_1[3\alpha^3(4K^2 - 7K + 3) + 3\alpha^2K(4K - 3)] \\
 v_7 &= w_1[3\alpha^3(2K^2 - 3K + 1) + 3\alpha^2K] \\
 v_8 &= 3w_1\alpha^3(K - 1)
 \end{aligned}$$

TABLE 10. The coefficients $\{\sum_{i=1}^6 Q_i\}$.

$$\begin{aligned}
 Q_1 &= w_1q_2q_3q_4 \\
 Q_2 &= w_1q_2q_3q_5 \\
 Q_3 &= S_0Kq_2q_3q_6 \\
 Q_4 &= S_1w_2q_1q_6 + q_2q_3(S_0Kq_7 + w_5q_6 + Aw_1q_4) \\
 Q_5 &= S_1w_2q_1q_7 + q_2q_3(6S_0Kq_3 + w_5q_7 + Aw_1q_5) \\
 Q_6 &= 6q_3^2(S_1w_2q_1 + w_5q_2q_3)
 \end{aligned}$$

where

$$\begin{aligned}
 q_1 &= M^3 + M + 1, q_2 = (M^2 + M + 1)(K - 1) \\
 q_3 &= K(M + 1), q_4 = 3q_1(2q_2 + q_1) \\
 q_5 &= 6q_1q_3q_6 = 3 \ln 2(2q_2^2 + 2q_1q_2 + q_1^2) \\
 q_7 &= 6q_3 \ln 2(2q_2 + q_1)
 \end{aligned}$$

$$P_{BH} = K(P_{BT}) \left(1 - \frac{(\tau^{DL} + \tau^{UL})K}{U} \right) \bar{R} \quad (86)$$

Plugging these values into Eq. (25) yields:

$$\begin{aligned}
 P_T &= MP_{BS} + P_{SYN} + P_{FIX} \\
 &+ K \left[P_{UE} - \frac{JB}{2UL_{BS}} + MB \left(\frac{2}{L_{BS}} + \frac{J}{UL_{BS}} - \frac{1}{UL_{BS}} \right) \right] \\
 &+ K^2 \left[\frac{2B\tau^{UL}}{UL_{BS}} + \frac{4B\tau^{DL}}{UL_{UE}} - \frac{JB}{2UL_{BS}} \right. \\
 &\quad \left. + \frac{MB}{UL_{BS}} (J + 2 - 2(\tau^{DL} + \tau^{UL})) \right] \\
 &+ K(P_{COD} + P_{DEC} + P_{BT}) \\
 &\times \left(1 - \frac{(\tau^{DL} + \tau^{UL})K}{U} \right) \bar{R} \quad (87)
 \end{aligned}$$

Rearrange the terms of above equation, and then the total power is given in Eq. (27).

APPENDIX F

See Table 6.

APPENDIX G

See Table 7.

APPENDIX H

See Table 8.

APPENDIX I

See Table 9.

APPENDIX J

See Table 10.

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