Received April 19, 2022, accepted May 7, 2022, date of publication May 11, 2022, date of current version May 17, 2022.

Digital Object Identifier 10.1109/ACCESS.2022.3174345

Robust Cooperative Beamforming and Its Feasibility Analysis in Multiuser Multirelay Networks

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This work was supported by the National Key Research and Development Program under Grant 2021YFB2900300.

ABSTRACT In this paper, we study the optimal cooperative beamforming problem in the amplify-and-forward (AF) multi-user and multi-relay networks, which face the challenges of rapid node number variations and per-node power limits. To achieve extra diversity gain, direct-link (source-to-destination) and distributed relay-link signals are jointly exploited. The optimal cooperative beamforming problem is formulated as the maximum relay transmit power minimization problem, subject to per-relay transmit power and the minimum destination signal-to-noise ratio (SNR) constraints. Since the problem is non-convex, we introduce a phase-regulation (PR) method to transform the non-convex problem into a tractable second-order cone programming (SOCP) problem. It is demonstrated that the proposed method can provide much more robustness against node number variations in terms of worst-case convergence rates than the Lagrange dual and the successive convex approximation (SCA) methods. Furthermore, the closed-form expressions of two necessary feasibility conditions are derived, by which the infeasible channels can be identified and excluded. Consequently, computational costs are reduced. The equivalence of the proposed Necessary Condition I (NC1) and the signal-to-interference ratio (SIR) condition is proved theoretically and numerically. The proposed Necessary Condition II (NC2) has a lower upper-bound than the SIR condition, thus reducing more computational costs. This method is applicable to both direct-link and non-direct-link scenarios.

INDEX TERMS Cooperative beamforming, multiuser, multirelay, direct link, robust, feasibility.

I. INTRODUCTION

With the proliferation of wireless sensor networks (WSN), machine-to-machine (M2M) communications and Internet of Things (IoT), reliable low-rate wireless communications among self-sustained nodes, which are equipped with low-complexity hardware and limited batteries, are highly demanded. Cooperative relaying is regarded as a promising candidate to enhance reliability, coverage and capacity of those networks. Due to the broadcast nature of wireless channels, nodes can share their power, bandwidth and antennas with their neighbors to obtain cooperative diversity [1]–[3].

A. RELATED WORKS AND MOTIVATION

Most previous works on cooperative beamforming ignore *direct-link* signals, due to obstacles or severe attenuation. In [4], the optimal beamforming problem of an AF multi-user multi-channel relaying system is studied. Because the original problem is non-convex, the authors transform its Lagrange dual problem into a semidefinite programming (SDP) counterpart. Nevertheless, the optimal beamforming vectors need to be calculated from the optimal dual variables. Although [5] considers a multi-user multi-relay network and the optimal problem is solved with an alternating optimization method, the cooperative beamforming problem is not investigated. In [6], a joint power-allocation and optimal relay beamforming algorithm is introduced to maximize the weighted sum-rate for non-orthogonal multiple-access (NOMA) AF relay systems. The non-convex optimal beamforming problem is solved via the successive convex approximation (SCA) method. In [7], an artificial noise aided beamforming scheme for a full-duplex AF relay network in the presence of multiple untrusted energy harvesting receivers is presented. The optimization problem is also solved via the

The associate editor coordinating the review of this manuscript and approving it for publication was Stefano Scanzio.

SCA method, which is employed in [8] as well. In [9], a joint relay selection and power allocation method is developed to maximize the instantaneous secrecy rate for millimeter wave (mmWave) relaying systems. Directional beamforming is utilized to compensate the pathloss attenuation. The secure communications between the source and destination are assisted by an untrusted relay in the presence of passive eavesdroppers. Besides the above works, [6], [7], [10] and [11] also assume that there are no direct links between the sources and destinations. Although direct-links are taken into account in [12] and [13], multi-relay cooperative beamforming is not discussed.

Very recently, AF relaying techniques are usually investigated in Rayleigh fading channels. In [14], a cooperative jamming scheme is proposed for three-hop AF relaying networks where the data can be confidentially communicated from one source to one destination through multiple untrusted relays. All the source-to-relay (S-R), relay-to-relay (R-R) and relay-to-destination (R-D) links are modeled as Rayleigh fading channels. Reference [15] studies the block error rate (BLER) of short-packet communications (SPC) in an AF relaying scenario in the presence of channel estimation errors and hardware imperfections. Both Nakagami-m and Rayleigh fading channels are investigated in the S-R and R-D links. The S-R and R-D links are also modeled as Rayleigh fading channels in [16]. Additionally, direct links are also not considered in the above works.

Furthermore, although optimal beamforming problems are discussed in many works, feasibility conditions are rarely studied [17]–[21]. In [22], the distributed beamforming problem in multi-user and multi-relay networks with direct links is studied. However, feasibility conditions are not investigated either.

B. CONTRIBUTIONS

In this paper, we propose a computationally efficient cooperative beamforming approach with robustness against rapid variations of node number.

- In contrast to the works mentioned above, direct-link and relay-link signals are jointly exploited to formulate the cooperative beamforming problem. Thus, the proposed method is applicable for both direct-link and non-direct-link scenarios. Moreover, the S-R and R-D links are modeled as Rician fading channels and the source-to-destination (S-D) links are modeled as Rayleigh fading channels to evaluate the proposed method.
- We introduce a phase-regulation (PR) method to transform the non-convex optimal beamforming problem into a SOCP problem, which can be easily solved with interior-point methods. The closed-form expression of the worst-case convergence rate of the propose method is derived. It is demonstrated that the proposed method can provide much more robustness against user-pair/relay number variations than the Lagrange dual and SCA methods.

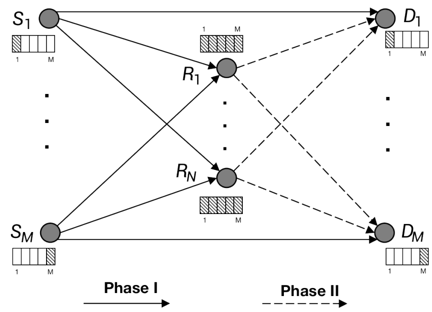

FIGURE 1. Transmissions of multiple S-D pairs via multiple orthogonal subchannels in AF multi-relay networks with direct-links.

- The closed-form expressions of two necessary feasibility conditions, NC1 and NC2, are derived to reduce computational costs. The *equivalence* of the proposed NC1 condition and the SIR condition in [4] is proved theoretically and numerically. The proposed NC2 condition has a lower upperbound than the NC1 and SIR conditions, thus reducing more computational burdens.

C. NOTATIONS

1) NOTATIONS

I denotes Identity Matrix. $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$ denote the complex conjugate, the transpose and the Hermitian operators respectively. $E[\cdot]$ denotes the expectation. $\mathbf{e}_i$ denotes a $\mathbf{0}$ vector except a one as the ith element. diag($\mathbf{a}$) denotes a diagonal matrix with the elements of vector $\mathbf{a}$ as diagonal elements. $[\cdot]_{i,j}$ denotes the (ith, jth) element of a matrix. $\odot$ denotes Hadamard product. $|\cdot|$ and $||\cdot||$ denote the absolute value and Euclidean norm respectively. $\lambda_{min}(\cdot)$ and $\lambda_{max}(\cdot)$ denote the minimal and maximal eigenvalue of a matrix.

II. SYSTEM MODEL

We consider a two-hop multi-user and multi-relay AF cooperative network where self-sustained nodes need to transmit reliable low-rate data to each other. This scenario presents in WSN, IoT, industrial IoT (IIoT) and M2M networks. Owing to the limits of battery and hardware complexity of nodes, high-complexity communication algorithms are not affordable. Because spectrum resources are relatively sufficient in those scenarios, the channel can be divided into multiple orthogonal subchannels to improve communication reliability by avoiding mutual interferences. The AF protocol is utilized to extend coverage and/or improve communication reliability.

As shown in Fig. 1, this network consists of M user-pairs $\{(S_m, D_m)|1 \leq m \leq M\}$, N relays $\{R_n|1 \leq n \leq N\}$ and M orthogonal subchannels $\{L_m|1 \leq m \leq M\}$. Each user-pair is assigned to one orthogonal subchannel. The problem that the user-pair number exceeds the available subchannel number, can be addressed by round-robin or random access subchannel allocation methods.

Data transmissions are divided into two phases. In the first phase, each source S_m broadcasts data to the paired

TABLE 1. Symbol list.

Symbol	Description
S_m	the mth source
D_m	the mth destination
L_m	the mth subchannel
R_n	the nth relay
$h_{sd,m}$	the subchannel coefficient of $S_m - D_m$ link, $CN(0, \lambda_{sd}^2)$
$h_{sr,m,n}$	the Rician subchannel coefficient of $S_m - R_n$ link
$h_{rd,m,n}$	the Rician subchannel coefficient of $R_n - D_m$ link
$n_{sd,m}$	the AWGN noise at D_m in Phase I, $CN(0, \sigma_{sd}^2)$
$n_{sr,m,n}$	the AWGN noise at R_n on L_m , $CN(0, \sigma_{sr}^2)$
$n_{rd,m}$	the AWGN noise at D_m on L_m in Phase II, $CN(0, \sigma_{rd}^2)$
P_S	the common transmit power of the sources
P_R	the common transmit power constraint of the relays
$w_{m,n}$	the beam-weight of R_n on L_m
$\mathbf{W}$	the beam-weight matrix with the (nth,mth) element $w_{m,n}$
x_m	the information-bearing signal of S_m, $CN(0, P_S)$
γ_0	the common minimum destination SNR requirement
γ_m	the receive SNR of D_m
$p_{m,n}$	the transmit power constraint of R_n on L_m
α	the factor of $p_{m,n}$, $p_{m,n} = \alpha P_R$ with $0 \le \alpha \le 1$

destination D_m and the N relays through assigned subchannel L_m. Thus, the channel is composed of M orthogonal broadcast subchannels (BC). In the second phase, the N relays amplify the M received signals respectively and forward them to the paired destinations on their own subchannels respectively. Thus, the channel is composed of M orthogonal multiple access subchannels (MAC). Specifically, in the BC phase, the subchannel L_m consists of one source to destination link $S_m - D_m$ and N source to relay links $S_m - R_n$, $1 \le n \le N$. In the MAC phase, the subchannel L_m only consists of N relay to destination links $R_n - D_m$, $1 \le n \le N$.

A. ASSUMPTIONS

It is assumed that each node is equipped with a single antenna and operates in a half-duplex mode. The second hop channel state information (CSI) is available to all relays. Thereby, the distributed optimal beamforming can be employed. All destinations have the same SNR requirement γ_0. Each relay is constrained with the same transmit power budget P_R. The channel coefficients are assumed to be quasi-static. Assuming that line-of-sight (LoS) links exist for $S - R$ and $R - D$ links. Then, the $S - R$ and $R - D$ links are modeled as Rician fading channels, and $S - D$ links are modeled as Rayleigh fading channels. Noises are i.i.d.

In the BC phase, the received signal at D_m through $S_m - D_m$ link can be expressed as

$$y_{sd,m} = h_{sd,m}x_m + n_{sd,m} \tag{1}$$

where x_m is the broadcast signal of S_m on L_m with transmit power P_S. Assuming that all sources have the same transmit power, i.e., $E[|x_1|^2] = \cdots = E[|x_M|^2] = P_S$. $h_{sd,m}$ is the subchannel coefficient of direct link $S_m - D_m$. $n_{sd,m}$ is a zero-mean complex additive white Gaussian noise (AWGN).

Meanwhile, the received signal at relay R_n through relay link $S_m - R_n$ can be expressed as

$$y_{sr,m,n} = h_{sr,m,n}x_m + n_{sr,m,n} \tag{2}$$

where $h_{sr,m,n}$ is the Rician subchannel coefficient of link $S_m - R_n$. $n_{sr,m,n}$ is a zero-mean complex AWGN noise.

Subsequently, the N received signals of the N relays can be jointly written in a vector form as $\mathbf{y}_{sr,m} = [y_{sr,m,1}, \cdots, y_{sr,m,N}]^T$

$$\mathbf{y}_{sr,m} = \mathbf{h}_{sr,m}x_m + \mathbf{n}_{sr,m} \tag{3}$$

where $\mathbf{h}_{sr,m} = [h_{sr,m,1}, \cdots, h_{sr,m,N}]^T$ is the Rician subchannel coefficient vector. $\mathbf{n}_{sr,m} = [n_{sr,m,1}, \cdots, n_{sr,m,N}]^T$ is the AWGN noise vector with $\mathbf{n}_{sr,m} \sim \mathcal{CN}(\mathbf{0}, \sigma_{sr}^2\mathbf{I}_N)$.

In the MAC phase, the N received signals of the N relays are amplified respectively with the beamforming weights $\{w_{m,n}|1 \le n \le N\}$ and forwarded via subchannel L_m. Then the received signal at D_m in the MAC phase is

$$\begin{aligned} y_{rd,m} &= \sum_{n=1}^{N} h_{rd,m,n} \cdot w_{m,n} \cdot y_{sr,m,n} + n_{rd,m} \\ &= \mathbf{h}_{rd,m}^T \left(\mathbf{w}_m \odot \mathbf{y}_{sr,m}\right) + n_{rd,m} \\ &= \mathbf{h}_{rd,m}^T\mathbf{W}_m\mathbf{h}_{sr,m}x_m + \mathbf{h}_{rd,m}^T\mathbf{W}_m\mathbf{n}_{sr,m} + n_{rd,m} \end{aligned} \tag{4}$$

where $\mathbf{h}_{rd,m} = [h_{rd,m,1}, \cdots, h_{rd,m,N}]^T$ denotes the Rician subchannel coefficient vector of $R_{1-N} - D_m$ links. $\mathbf{w}_m = [w_{m,1}, \cdots, w_{m,N}]^T$ is the distributed relay beamforming vector and $\mathbf{W}_m = diag(\mathbf{w}_m)$. $n_{rd,m}$ is a AWGN noise.

The two signals, received in the BC and MAC phases respectively, at D_m can be jointly written in a vector form

$$\begin{aligned} &\underbrace{\begin{bmatrix} y_{sd,m} \\ y_{rd,m} \end{bmatrix}}_{\mathbf{y}_m} \\ &= \underbrace{\begin{bmatrix} h_{sd,m} \\ \mathbf{h}_{rd,m}^T\mathbf{W}_m\mathbf{h}_{sr,m} \end{bmatrix}}_{\mathbf{h}} x_m + \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ \mathbf{h}_{rd,m}^T\mathbf{W}_m & 0 & 1 \end{bmatrix}}_{\mathbf{B}} \underbrace{\begin{bmatrix} \mathbf{n}_{sr,m} \\ n_{sd,m} \\ n_{rd,m} \end{bmatrix}}_{\mathbf{n}} \end{aligned} \tag{5}$$

Accordingly, the SNR γ_m of D_m can be written as

$$\begin{aligned} \gamma_m &= \frac{E\left\|\mathbf{h}x_m\right\|^2}{E\left\|\mathbf{Bn}\right\|^2} \\ &= \frac{P_S\left|h_{sd,m}\right|^2 + P_S\left|\mathbf{h}_{rd,m}^T\mathbf{W}_m\mathbf{h}_{sr,m}\right|^2}{\sigma_{sr}^2\left\|\mathbf{h}_{rd,m}^T\mathbf{W}_m\right\|^2 + \sigma_{rd}^2 + \sigma_{sd}^2} \\ &= \frac{P_S\left|h_{sd,m}\right|^2 + P_S\left|\mathbf{h}_m^T\mathbf{w}_m\right|^2}{\sigma_{sr}^2\mathbf{w}_m^H\mathbf{\Lambda}_{\gamma,m}\mathbf{w}_m + \sigma_{rd}^2 + \sigma_{sd}^2} \ge \gamma_0 \end{aligned} \tag{6}$$

where $\mathbf{h}_m = \mathbf{h}_{sr,m} \odot \mathbf{h}_{rd,m}$, $\mathbf{\Lambda}_{\gamma,m} = diag(\mathbf{h}_{rd,m} \odot \mathbf{h}_{rd,m}^*)$ is a $N \times N$ dimensional real-valued diagonal matrix.

The transmit power of relay R_n can be written as

$$\begin{aligned} P_n &= \sum_{m=1}^{M} E\left[\left|w_{m,n}y_{sr,m,n}\right|^2\right] \\ &= \sum_{m=1}^{M}\left(P_S\left|w_{m,n}h_{sr,m,n}\right|^2 + \sigma_{sr}^2\left|w_{m,n}\right|^2\right) \\ &= \left[\mathbf{W}\mathbf{\Lambda}_{P,n}\mathbf{W}^H\right]_{n,n} \le P_R \end{aligned} \tag{7}$$

where $\mathbf{W} = [\mathbf{w}_1, \cdots, \mathbf{w}_M]$ is the beamforming matrix. $\mathbf{H}_{sr} = \{h_{sr,m,n}\}_{N\times M} = [\mathbf{h}_{sr,1}, \cdots, \mathbf{h}_{sr,M}]$ is the channel coefficient matrix from the M sources to the N relays in the BC phase. $\mathbf{\Lambda}_{P,n} = P_S diag(\mathbf{H}_{sr}^T\mathbf{e}_n \odot \mathbf{H}_{sr}^H\mathbf{e}_n) + \sigma_{sr}^2\mathbf{I}_M$ is a $M \times M$ dimensional real-valued diagonal matrix.

III. OPTIMAL BEAMFORMING

In order to save relay transmit power and to meet the minimum SNR requirements of the destinations in AF multi-user and multi-relay networks, we formulate the distributed cooperative beamforming problem as the maximum relay transmit power minimization problem subject to relay transmit power and destination SNR constraints.

The original problem can be formulated as

$$\mathcal{P}0: \begin{cases} \min\limits_{\mathbf{W},p} & p \\ s.t. & \gamma_m \geq \gamma_0 \qquad 0 \leq m \leq M \\ & P_n \leq p \leq P_R \quad 0 \leq n \leq N \end{cases} \tag{8}$$

where γ_m and P_n are defined in (6) and (7) respectively.

A. SOCP PROBLEM FORMULATION

When the direct-link signal $y_{sd,m}$ alone can satisfy the SNR requirement of D_m, there is no need for the relays to forward signals to D_m in the MAC phase. The mathematical expression can be written as $|h_{sd,m}|^2\gamma_S \geq \gamma_0$, $\gamma_S = P_S/\sigma_{sd}^2$. We name the above inequality as *direct-link green* (DLG) and name $|h_{sd,m}|^2\gamma_S < \gamma_0$ as *direct-link red* (DLR). Since $h_{sd,m}$ obeys complex Gaussian distribution, $|h_{sd,m}|^2$ obeys exponential distribution. The probabilities of the DLG and DLR can be expressed respectively as follows

$$\begin{aligned} P_r\{DLG\} &= e^{-\frac{\gamma_0}{\lambda_{sd}^2\gamma_S}} \\ {}_r\{DLR\} &= 1 - P_r\{DLG\} = 1 - e^{-\frac{\gamma_0}{\lambda_{sd}^2\gamma_S}} \end{aligned} \tag{9}$$

Because the MAC phase transmissions are omitted in the DLG cases, we particularly discuss the optimal cooperative beamforming problem in the DLR cases. By substituting the DLR inequality into (6), we have

$$\begin{aligned} \gamma_m &= \frac{P_S\left|\mathbf{h}_m^T\mathbf{w}_m\right|^2}{\sigma_{sr}^2\mathbf{w}_m^H\mathbf{\Lambda}_{\gamma,m}\mathbf{w}_m + \sigma_m^2} \\ &= \frac{P_S\mathbf{w}_m^H\mathbf{h}_m^*\mathbf{h}_m^T\mathbf{w}_m}{\sigma_{sr}^2\mathbf{w}_m^H\mathbf{\Lambda}_{\gamma,m}\mathbf{w}_m + \sigma_m^2} \geq \gamma_0 \quad 0 \leq m \leq M \end{aligned} \tag{10}$$

where $\sigma_m^2 = \sigma_{rd}^2 + \sigma_{sd}^2 - \frac{P_S|h_{sd,m}|^2}{\gamma_0} > 0$. The non-direct-link scenarios are the special cases where $\sigma_m^2 = \sigma_{rd}^2$.

Since the function on the left-hand side of the inequality in (10) is non-convex, $\mathcal{P}_0$ is a non-convex problem. Therefore, we propose the *phase-regulation (PR)* method to transform the non-convex SNR constraints into second-order cone constraints [23]. The details are shown as follows.

Suppose that matrix $\hat{\mathbf{W}} \in \mathbb{C}^{N\times M}$ is a feasible point of $\mathcal{P}_0$. And we construct a phase regulating matrix $\Phi \in \mathbb{C}^{N\times M}$ with the (nth,mth) element $[\Phi]_{n,m} = e^{j\phi_{n,m}}$, where $0 \leq \phi_{n,m} \leq 2\pi$ is an arbitrary phase for any $1 \leq n \leq N$ and $1 \leq m \leq M$. Then we compose the matrix $\mathbf{W}$ as

$$\mathbf{W} = \Phi \odot \hat{\mathbf{W}} = \begin{bmatrix} e^{j\phi_{1,1}}\hat{w}_{1,1} & \cdots & e^{j\phi_{1,M}}\hat{w}_{1,M} \\ \vdots & \ddots & \vdots \\ e^{j\phi_{N,1}}\hat{w}_{N,1} & \cdots & e^{j\phi_{N,M}}\hat{w}_{N,M} \end{bmatrix} \tag{11}$$

Substituting $\mathbf{W}$ and $\hat{\mathbf{W}}$ into both (7) and (10), we have the following equations

$$\begin{aligned} &\sum_{m=1}^{M}\left(P_S\left|\hat{w}_{m,n}h_{sr,m,n}\right|^2 + \sigma_{sr}^2\left|\hat{w}_{m,n}\right|^2\right) \\ &= \sum_{m=1}^{M}\left(P_S\left|e^{j\phi_{n,m}}\hat{w}_{m,n}h_{sr,m,n}\right|^2 + \sigma_{sr}^2\left|e^{j\phi_{n,m}}\hat{w}_{m,n}\right|^2\right) \\ &= \sum_{m=1}^{M}\left(P_S\left|w_{m,n}h_{sr,m,n}\right|^2 + \sigma_{sr}^2\left|w_{m,n}\right|^2\right) \end{aligned} \tag{12}$$

and

$$\begin{aligned} &\frac{P_S\left|\mathbf{h}_m^T\hat{\mathbf{w}}_m\right|^2}{\sigma_{sr}^2\hat{\mathbf{w}}_m^H\mathbf{\Lambda}_{\gamma,m}\hat{\mathbf{w}}_m + \sigma_m^2} \\ &= \frac{P_S\left|\mathbf{h}_m^T\Phi_m \odot \hat{\mathbf{w}}_m\right|^2}{\sigma_{sr}^2\left(\Phi_m \odot \hat{\mathbf{w}}_m\right)^H\mathbf{\Lambda}_{\gamma,m}\left(\Phi_m \odot \hat{\mathbf{w}}_m\right) + \sigma_m^2} \\ &= \frac{P_S\left|\mathbf{h}_m^T\mathbf{w}_m\right|^2}{\sigma_{sr}^2\mathbf{w}_m^H\mathbf{\Lambda}_{\gamma,m}\mathbf{w}_m + \sigma_m^2} \end{aligned} \tag{13}$$

where Φ_m, $\hat{\mathbf{w}}_m$ and $\mathbf{w}_m$ are the mth column of Φ, $\hat{\mathbf{W}}$ and $\mathbf{W}$ respectively.

It means that we can regulate Φ to guarantee $\mathbf{h}_m^T\mathbf{w}_m \geq 0$ for any $1 \leq m \leq M$ while both (7) and (10) are satisfied. Thus, the non-convex SNR constraints in (10) can be recast as second-order cone constraints as follows

$$\sqrt{\frac{P_S}{\gamma_0}}\mathbf{h}_m^T\mathbf{w}_m \geq \left\| \begin{matrix} \sigma_{sr}\mathbf{\Lambda}_{\gamma,m}^{\frac{1}{2}}\mathbf{w}_m \\ \sigma_m \end{matrix} \right\| \quad 0 \leq m \leq M \tag{14}$$

The second-order cone constraints can be written in generalized inequality form [24] as

$$\begin{bmatrix} \sqrt{\frac{P_S}{\gamma_0}}\mathbf{h}_m^T\mathbf{w}_m \\ \sigma_{sr}\mathbf{\Lambda}_{\gamma,m}^{\frac{1}{2}}\mathbf{w}_m \\ \sigma_m \end{bmatrix} \succeq_K 0 \quad 0 \leq m \leq M \tag{15}$$

where generalized inequality $\succeq_K$: $\begin{bmatrix} z \\ \mathbf{z} \end{bmatrix} \succeq_K 0 \Leftrightarrow ||\mathbf{z}|| \leq z$.

Then we can recast the original problem $\mathcal{P}0$ as a second-order cone programming (SOCP) problem and express it as a conic form problem as follows [23], [24]

$$\mathcal{P}1:\begin{cases}\min\limits_{\mathbf{W},p} & p\\ s.t. & \begin{bmatrix}\sqrt{\frac{P_S}{\gamma_0}}\mathbf{h}_m^T\mathbf{W}\mathbf{e}_m\\ \sigma_{sr}\mathbf{\Lambda}_{\gamma,m}^{\frac{1}{2}}\mathbf{W}\mathbf{e}_m\\ \sigma_m\end{bmatrix}\succeq_K 0 \quad 0\le m\le M\\ & \begin{bmatrix}\sqrt{p}\\ \mathbf{\Lambda}_{P,n}^{\frac{1}{2}}\mathbf{W}^T\mathbf{e}_n\end{bmatrix}\succeq_K 0 \quad 0\le n\le N\\ & 0\le p\le P_R\end{cases} \tag{16}$$

Thus, the SOCP beamforming problem is formulated.

B. COMPLEXITY ANALYSIS

Hereafter, we analyze the complexity of the proposed SOCP problem in (16). It can be seen that the optimal problem of (16) has M destination SNR constraints and N relay transmit power constraints. Therefore, the per-iteration computational complexity of the proposed problem is $\mathcal{O}(MN)$. This SOCP problem can be efficiently solved via interior-point method software packages such as SeDuMi. The computational complexity of the proposed SOCP problem is lower than that of the SDP problem in [4].

C. WORST-CASE CONVERGENCE ANALYSIS

In order to evaluate the robustness against node number variations of the proposed method, we use the convergence theory in [24] to analyze the worst-case convergence rate of the formulated SOCP problem.

According to [24], the expression of the worst-case convergence rate is given by

$$\left\lceil\frac{log\left(\bar{\theta}/t^{(0)}\epsilon\right)}{log\mu}\right\rceil\cdot\left(\frac{\bar{\theta}\left(\mu-1-log\mu\right)}{\gamma}+c\right) \tag{17}$$

where $\bar{\theta}$ is the sum degree of the optimal problem. ϵ is the duality gap, i.e., $p-p^o\le\epsilon$. p^o is the optimal value. p decreases by at least γ at each inner iteration. t sets the approximation accuracy between the logarithmic barrier function and indicator function. $t^{(0)}$ is the initial value of t. μ is the outer iteration factor for $t^{(i+1)}=\mu t^{(i)}$. c is the constant indicating the tolerance of the Newton Method.

The sum degree of the proposed SOCP problem can be derived as follows

$$\bar{\theta}_{SOCP}=2(M+N)$$

To demonstrate the superiority of the proposed method, the worst-case convergence rate of the proposed method is compared with the counterparts of the Lagrange dual method and the SCA method. The approach to transform the original problem into a SDP problem can refer to [4]. We ignore the computations of the optimal beamforming vector of the Lagrange dual SDP problem, so that the worst-case convergence rates of the SOCP problem and the SDP problem can be compared straightly. Even though the complexity of the Lagrange dual method is underestimated, the proposed SOCP method has lower worst-case convergence rate than the Lagrange dual and the SCA methods. The details are given in the section of Numerical Results.

IV. FEASIBILITY ANALYSIS

An optimal problem is infeasible if there is no point can satisfy all the constraints. Accordingly, the formulated beamforming problem is infeasible, if the destination SNR and relay transmit power constraints can not be met simultaneously. In this condition, we investigate the necessary feasibility conditions to identify and exclude those infeasible channel coefficients. Thus, computational burdens can be reduced.

A. NECESSARY CONDITION I

When there is no transmit power constraint $p_{m,n}$ on the mth subchannel of R_n, a necessary feasibility condition of the beamforming problem is derived.

Lemma 1: For a matrix $\mathbf{A}$, a nonsingular positive definite matrix $\mathbf{B}$ and a real constant β, the eigenvalue $\lambda\left(\mathbf{A}-\beta\mathbf{B}\right)\le 0$ iff $\lambda(\mathbf{B}^{-1}\mathbf{A})\le\beta$.

Proof: According to the properties of the generalized Rayleigh quotient, when $\lambda(\mathbf{B}^{-1}\mathbf{A})\le\beta$, we have $\lambda(\mathbf{B}^{-1}\mathbf{A})=\lambda[\mathbf{A},\mathbf{B}]\le\beta$. $\lambda[\mathbf{A},\mathbf{B}]$ is the generalized eigenvalue of the pencil $[\mathbf{A},\mathbf{B}]$ and $\lambda(\mathbf{B}^{-1}\mathbf{A})$ is the eigenvalue of the matrix $\mathbf{B}^{-1}\mathbf{A}$.

Consequently, the generalized Rayleigh quotient can be written as

$$r\left(\mathbf{x}\right)=\frac{\mathbf{x}^H\mathbf{A}\mathbf{x}}{\mathbf{x}^H\mathbf{B}\mathbf{x}}\le\beta\quad\forall\mathbf{x} \tag{18}$$

Since $\mathbf{B}$ is nonsingular positive definite, we have

$$\mathbf{x}^H\left(\mathbf{A}-\beta\mathbf{B}\right)\mathbf{x}\le 0\quad\forall\mathbf{x} \tag{19}$$

which means $\lambda(\mathbf{B}^{-1}\mathbf{A})\le\beta\Rightarrow\lambda\left(\mathbf{A}-\beta\mathbf{B}\right)\le 0$.

Reversely, the proof holds. ■

Proposition 1: That $\mathbf{G}_m=\gamma_0\sigma_{sr}^2\mathbf{\Lambda}_{\gamma,m}-P_s\mathbf{h}_m^*\mathbf{h}_m^T$ is *not* positive definite, i.e., the minimum eigenvalue of $\mathbf{G_m}$ is no larger than zero, is a necessary feasibility condition of the cooperative beamforming problem (8).

Proof: Because $\sigma_{sr}^2\mathbf{w}_m^H\mathbf{\Lambda}_{\gamma,m}\mathbf{w}_m+\sigma_m^2>0$, the inequality in (10) can be reformulated as

$$\mathbf{w}_m^H\left(\gamma_0\sigma_{sr}^2\mathbf{\Lambda}_{\gamma,m}-P_s\mathbf{h}_m^*\mathbf{h}_m^T\right)\mathbf{w}_m+\gamma_0\sigma_m^2\le 0 \tag{20}$$

It can be seen that when $\mathbf{G}_m$ is positive definite, the inequality (20) never holds. Conversely, (20) holds only if $\mathbf{G}_m$ is not positive definite. Hence, the first necessary condition is given. Equivalently,

$$\lambda_{min}\left(\mathbf{G}_m\right)\le 0 \tag{21}$$

It means that the minimum eigenvalue is no larger than zero is also a necessary feasibility condition of the beamforming problem.

According to *Lemma 1*, it can be proved that

$$\lambda_{min}\left(\mathbf{G}_m\right)\le 0\Leftrightarrow\frac{P_s}{\sigma_{sr}^2}\mathbf{h}_m^T\mathbf{\Lambda}_{\gamma,m}^{-1}\mathbf{h}_m^*\ge\gamma_0 \tag{22}$$

The right inequality above is the SIR necessary condition in [4].

Furthermore, according to *Weyl's Theorem*, we have

$$\gamma_0 \sigma_{sr}^2 \min_n \left\{ \mathbf{\Lambda}_{\gamma,m} \right\} - P_s \|\mathbf{h}_m\|_2^2 \leq 0 \tag{23}$$

where $\min_n \left\{ \mathbf{\Lambda}_{\gamma,m} \right\}$ is the minimum diagonal element of the diagonal matrix $\mathbf{\Lambda}_{\gamma,m}$. Therefore, (23) is a simplified necessary condition of (8) with low computational complexity. ■

B. NECESSARY CONDITION II

When there is a transmit power constraint $p_{m,n}$ on the mth subchannel of R_n, Necessary Condition I can be moved on as Necessary Condition II.

Proposition 2: $\lambda_{min}(\mathbf{G}_m) < -\frac{\gamma_0 \sigma_m^2}{\mathbf{w}_m^H \mathbf{w}_m}$ is a necessary feasibility condition of the beamforming problem.

Proof: Since $\left| w_{m,n} h_{sr,m,n} \right|^2 = \left| h_{sr,m,n} \right|^2 \left| w_{m,n} \right|^2$, we have

$$\begin{aligned} P_S \left| w_{m,n} h_{sr,m,n} \right|^2 &+ \sigma_{sr}^2 \left| w_{m,n} \right|^2 \\ &= \left(P_S \left| h_{sr,m,n} \right|^2 + \sigma_{sr}^2 \right) \left| w_{m,n} \right|^2 \leq p_{m,n} \end{aligned} \tag{24}$$

where $p_{m,n}$ can be expressed as $p_{m,n} = \alpha P_R$ with $0 \leq \alpha \leq 1$.

Then we have

$$\left| w_{m,n} \right|^2 \leq \frac{p_{m,n}}{P_S \left| h_{sr,m,n} \right|^2 + \sigma_{sr}^2} = q_{m,n} \tag{25}$$

Additionally, (20) can be reformulated as

$$\frac{\mathbf{w}_m^H \mathbf{G}_m \mathbf{w}_m}{\mathbf{w}_m^H \mathbf{w}_m} \leq -\frac{\gamma_0 \sigma_m^2}{\mathbf{w}_m^H \mathbf{w}_m} \tag{26}$$

According to *Rayleigh-Ritz theorem*, we have

$$\lambda_{min}(\mathbf{G}_m) \leq \frac{\mathbf{w}_m^H \mathbf{G}_m \mathbf{w}_m}{\mathbf{w}_m^H \mathbf{w}_m} \leq \lambda_{max}(\mathbf{G}_m) \tag{27}$$

Since $\mathbf{w}_m^H \mathbf{w}_m = \sum_{n=1}^N \left| w_{m,n} \right|^2 \leq \sum_{n=1}^N q_{m,n}$,

$$\lambda_{min}(\mathbf{G}_m) \leq -\frac{\gamma_0 \sigma_m^2}{\sum_{n=1}^N q_{m,n}} = \eta_m \tag{28}$$

is a necessary feasibility condition of the beamforming problem. The reason is that when $\lambda_{min}(\mathbf{G}_m) > \eta_m$, the inequality (20) never holds. Conversely, (20) holds only if $\lambda_{min}(\mathbf{G}_m) \leq \eta_m$. ■

V. NUMERICAL RESULTS

In this section, the proposed method is evaluated via Monte Carlo simulations. The common simulation parameters are set to: $\lambda_{sd}^2 = 1, \sigma_{sd}^2 = \sigma_{sr}^2 = \sigma_{rd}^2 = 1, K_c = [2, 5, 10]$ (Rician factor, the variance of scattered component is set to one).

A. COMPARISON OF NECESSARY CONDITIONS

We define feasible rate as: the number of feasible cases/1000, and use this metric to verify the performances of the necessary feasibility conditions. Fig. 2 and Fig. 3 manifest that the the feasible rates of NC1 and NC2 increase with the increase of the relay number N and Rician factor K_c, while they decrease with the increase of destination SNR requirement γ_0. The reasons are: i) the more relays are used, the more power is available to satisfy the receive SNR requirements of the destinations; ii) the higher γ_0 is, the more difficult $\gamma_m \leq \gamma_0$ can be satisfied; iii) high K_c indicates strong relay channels. Consequently, the feasible rates increase. Nevertheless, the effect of K_c shrinks when K_c is large.

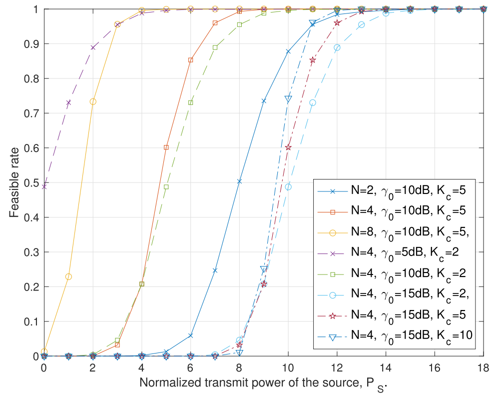

FIGURE 2. The effects of N, γ_0, K_c and P_S on the feasible rate of the Necessary Condition I.

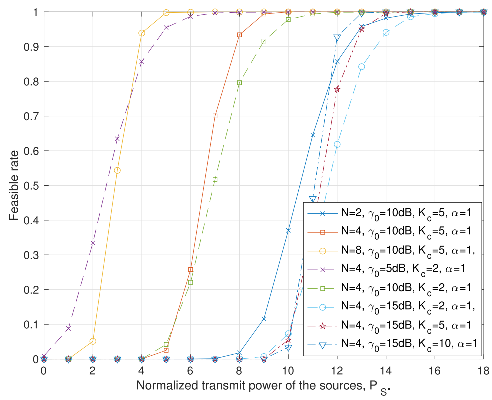

FIGURE 3. The effects of N, γ_0, K_c and P_S on the feasible rate of the Necessary Condition II.

When the transmit power of the nth relay R_n in the mth subchannel is constrained by $p_{m,n}$, i.e.,

$$P_S \left| w_{m,n} h_{sr,m,n} \right|^2 + \sigma_{sr}^2 \left| w_{m,n} \right|^2 \leq p_{m,n} = \alpha P_S$$

where $P_R = P_S$ is assumed for simplicity. The feasible rates of NC2 are displayed in Fig. 4. It can be seen that the feasible rates increase with the increase of α. The reason is that large $p_{m,n}$ relaxes the transmit power constraints, which can be proved by (28).

In Fig. 5, the feasible rates of the proposed NC1, NC2 conditions and the SIR condition in [4] are compared. It can be seen that the NC1 and SIR conditions have the same feasible rates for different parameters, which proves the correctness of *Proposition 1*. Because the NC2 condition has a lower upper-bound than the NC1 and SIR conditions, the

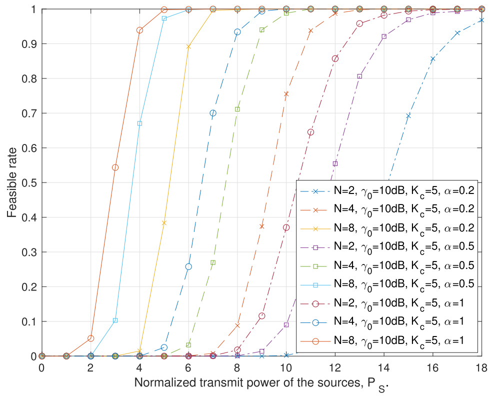

FIGURE 4. The effects of per-relay per-subchannel transmit power constraint, $p_{m,n}$, on the feasible rate of the Necessary Condition II.

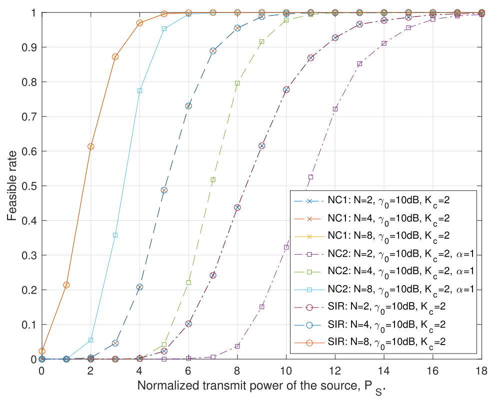

FIGURE 5. The feasible rate comparison among the Necessary Condition I, II and the SIR Necessary Condition.

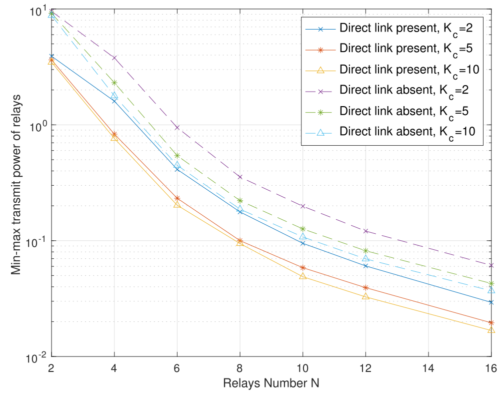

FIGURE 6. The min-max relay transmit power versus relay number N.

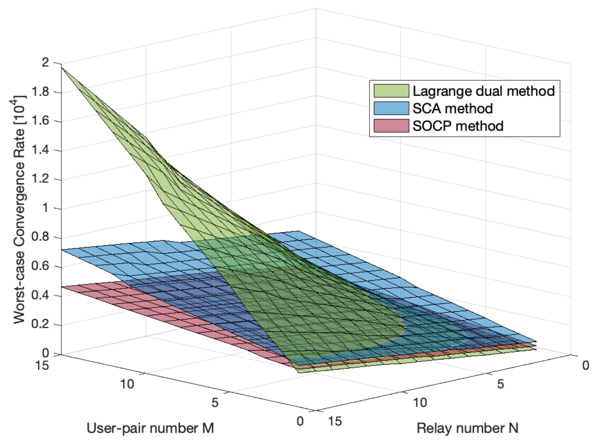

FIGURE 7. The worst-case convergence rate versus user-pair number *M* and relay number *N*.

feasible rate of NC2 is lower than those of the NC1 and SIR conditions. In other words, the NC2 condition can exclude more infeasible cases and reduce more computational costs than the NC1 and SIR conditions.

B. MINMAX RELAY POWER

Fig. 6 depicts the min-max relay transmit power P_R^o versus the relay number N. The normalized source and relay transmit power is set to $P_S = P_R = 10dB$. The minimum receive SNR requirement of destinations is set to $\gamma_0 = 5dB$. The user-pair number is set to $M = 2$. One thousand realizations are used for each curve. For infeasible realizations, P_R^o is set to P_R. Fig. 6 reveals that: i) The min-max relay transmit power P_R^o decreases with the increase of relay number N, because the more relays are used, the lower the average per-relay transmit power is consumed when the minimum destination SNR is fixed. ii) P_R^o decreases quickly at first and harvests marginal gain with the further increase of K_c. In other words, increasing relay number is the main solution to reduce P_R^o and increase capacity. iii) It demonstrates that the proposed method is applicable to both direct-link and non-direct-link scenarios and the relay transmit power can be effectively saved by exploiting direct-link signals.

C. COMPARISON OF ROBUSTNESS

The worst-case convergence rates of the Lagrange dual, the SCA and the proposed methods are compared. The parameters are set to $\mu = 2$, $\epsilon = 10^{-5}$, $\gamma = 0.1$, $c = 6$. Fig. 7 confirms that the proposed method provides much more robustness against node number variations than the Lagrange dual and the SCA methods. Specifically, although the worst-case iteration numbers of all the three methods increase with the increases of both N and M, the increasing trend of the proposed method is much slower than the counterparts of the other two methods. The reason is that, according to (17), the worst-case convergence rate is greatly affected by the sum degree of the formulated optimal problem. The sum degrees of the optimal problems formulated by the three methods are derive as

$$\begin{aligned}\bar{\theta}_{SDP} &= M \cdot N \\ \bar{\theta}_{SCA} &= 4M + 2N \\ \bar{\theta}_{SOCP} &= 2(M+N)\end{aligned} \tag{29}$$

Substituting the sum degrees into (17), we obtain the simulation results. The computational cost of our proposed method is less sensitive to node number variations than those of the other two methods. Therefore, the proposed method can robustly tackle the rapid increase of node number at the cost of slight increase of computational burden.

VI. CONCLUSION

In this paper, we proposed a method to minimize the maximum per-relay transmit power for AF multi-user and multi-relay networks. By jointly exploiting the direct-link and relay-link signals, extra cooperative diversity was obtained. The proposed method exhibits better performance in dealing with rapid user access than the Lagrange dual and the SCA methods. Besides, the optimal beamforming vector can be calculated directly. Necessary feasibility conditions were discussed to prevent futile beamforming in cases of bad channel conditions. Thus, computational costs can be saved.

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