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Dynamic Reliability Evaluation of High-Speed Train Gearbox Based on Copula Function

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ABSTRACT As a key component of the high-speed train transmission system, the reliability of the gearbox directly affects the reliability and driving safety of the high-speed train. Taking the key parts in the gearbox: driving gear, driven gear, gearbox housing and bearing as the research object, the dynamic reliability of the gearbox is evaluated. First, combined with classical stress-strength interference theory, the number of stochastic loads is regarded as a Poisson process, and the strength degradation is described by the Gamma process, the Gamma strength degradation parameter of each part is determined from the part material or the part's P-S-N curve, and the reliability model of each key part of the gearbox is obtained. Second, the gearbox is regarded as a series system, and the Gumbel Copula function is introduced to deal with the nonlinear correlation between different parts and between different failure modes of the same part. The nesting relationship of the Copula function is determined, the parameter values of the Gumbel Copula function are determined based on the kernel density estimation and maximum likelihood estimation method, so as to establish the dynamic reliability model of the gearbox system and realize the evaluation of the dynamic reliability of the gearbox. The evaluation results show that the method for evaluating the reliability of high-speed train gearboxes is in line with engineering practice. The evaluation method has practical significance for the reliability check, optimization design and dynamic tracking of the gearbox reliability during the service period of the high-speed train gearbox.

INDEX TERMS High-speed train gearbox, stress-strength interference theory, Gamma process, Gumbel Copula function, Kernel density estimation, maximum likelihood estimation.

I. INTRODUCTION

Gearbox is the key component for high-speed trains to realize power transmission [1]. The main parts include driving gear, driven gear, gear shaft bearing, gear shaft, and gearbox housing [2], as shown in Figure 1. The fault statistics of the actual operation of high-speed trains show that the driving gear, driven gear, bearing, and gearbox housing are parts with high damage frequency and serious damage. During the operation of the train, the gearbox is simultaneously subjected to the excitation of the track, the vibration and shock brought by the vehicle body, and other external excitation loads [3]. There is direct contact and interrelated interfaces between the various parts in the gearbox [4], which makes the failure modes have a certain correlation. The reliability of the parts and the degree of failure correlation between the parts jointly determine the reliability of the gearbox [5], [6]. It is more closer to the engineering practice to measure the reliability of the serial mechanical system by comprehensively considering the dynamic change of the gearbox load, the strength degradation of the parts and the failure correlation between the parts.

Stress-strength interference theory is the basis for the reliability design of mechanical parts [7]. The theory shows that reliability mainly depends on the degree of interference between stress and strength. The stress-strength interference model proposed by Freudenthal [8] was gradually widely used in fatigue reliability design in the 1960s. Traditionally, stress-strength interference theory is used to deal with the reliability of parts and structures under static stress-strength interference [9]. However, during the operation of the mechanical system, the stress and strength of the mechanical parts are random, and the strength of the parts will be degraded due to factors such as corrosion and aging. When

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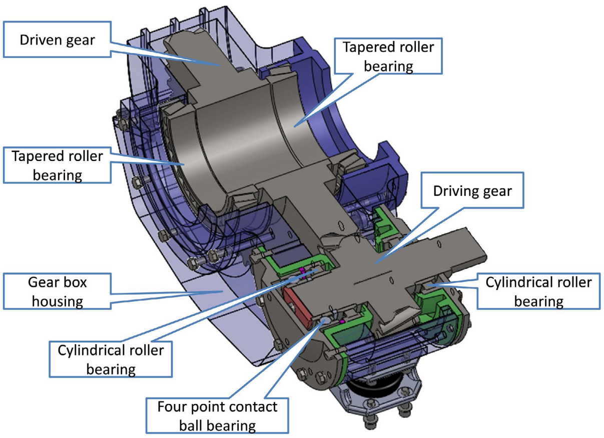

FIGURE 1. **High-speed train gearbox.**

the stress on the parts is greater than their strength, failure will occur.In recent years, the stress-strength interference theory has been widely used in the reliability research of rail vehicles. Song ZhanXun *et al.* [10] established a stress intensity interference model between the transformed S-N curve and the stepped spectrum fitting curve based on Miner's rule, and based on the stress data collected by the actual vehicle, the fatigue damage and life of a railway freight car bogie side frame were analyzed. Li Guangquan [11]determined the equivalent stress based on the stress test data during the operation of the gearbox, established a reliability model of equivalent stress-fatigue strength, and analyzed the fatigue reliability of the gearbox housing under typical working conditions. Xu Chengbin [12] combined the measured dynamic stress data and the stress intensity factor analysis, and considered the material fatigue properties of the EMU axle, and carried out a reliability analysis of the axle based on the stress-strength interference theory. In the research of the above scholars, the stress probability distribution of the research object is obtained by collecting the stress data of the real vehicle, but this collection method is not practical for the bearings, gears and other devices in the gearbox system. In addition, the dependence of stress and strength, and the degradation of strength are not considered in the process of evaluating reliability. Wu Hao *et al.* [13] used Simpack and MATLAB/Simulink to establish the electromechanical model of the gearbox box and traction system of the high-speed train, and analyzed the dynamic stress of the gearbox box, which provided a new idea for establishing the stress distribution in the stress-strength interference model.Li Wei [14] studied the cast aluminum beam under the high-speed train, and studied the influence of the residual strength degradation of the material on the accumulation of damage. The results show that the residual strength degradation has a greater impact on the reliability of components.This shows that it is necessary to study the strength degradation of each part when the reliability analysis of the gearbox system is carried out based on the stress-strength interference theory.

Since Sklar [15] proposed the Copula theory in 1959, Copula theory has been increasingly used in the reliability research of mechanical systems. Research on copulas and their application in statistics has become a new and thriving field. Roger B. Nelsen [16] did a detailed study on the basic properties, correlations, and association measures of Copula.Charpentier *et al.* [17] conducted a comprehensive study of the boundary distribution estimation of Copula. For a long time, copula has gained considerable popularity in many fields of applied mathematics. In recent years, copula theory has been used more and more in reliability research of mechanical parts.The Copula function can connect the joint distribution of multidimensional random variables with marginal distributions, and accurately handle the failure modes of parts or nonlinear correlations between random variables [18], [19]. Zhang Ling *et al.* [20] used the Weibull failure rate distribution function to determine the marginal distribution function of the gear, and analyzed the reliability of the gear system through the Frank Copula function. Xia Yirui *et al.* [21]proposed a reliability evaluation model considering strength degradation, studied the correlation and degree of correlation between different failure modes with the help of Copula function, defined failure thresholds to express the marginal failure probability of correlation between different failure modes, and evaluated the reliability of mechanical structures. Li Zhengwen [22] used the Copula function and combined the stochastic ness and fuzziness of the load to study the dynamic reliability design and calculation of the series and parallel systems. Zhang Jianchun *et al.* [23]used the non-differentiable Copula function to estimate structural reliability, and verified it with a simulation example.

In summary, most of the current research is mainly aimed at the extension of the stress-strength interference theoretical model, combined with the Copula function to evaluate the reliability of a single part or a series and parallel system of the same type of parts. There are few reliability studies applied to complex series and parallel systems. In this paper, a Copula function nested reliability model of multi-type parts series gearbox system is established. Combining the theory of order statistics, the time-aligned Poisson process and the Gamma strength degradation process, the time-varying stress-strength interference model is determined, and the Gamma strength degradation process parameters of the part are determined by the $P-S-N$ curve strength degradation stochastic model of the part or material, which is reliable for the gearbox system. Sexual correlation modeling provides support. In the process of building the model, the structural characteristics of the high-speed train gearbox system, the load characteristics acting on each key part, the strength degradation of each part, and the failure correlation of each key part are fully considered, and its dynamic reliability is evaluated. Make the reliability analysis results closer to reality.

The rest of the paper mainly does the following work. Section II establishes a reliability model considering the stochastic load action process and strength degradation. Section III establishes the Copula reliability model of the series system. Section IV analyzes the dynamic reliability of each key part of the gearbox system. Section V uses the nested Gumbel function to establish the failure correlation model of the gearbox system, and evaluates the dynamic reliability of the gearbox system. Section VI reviews this paper and looks forward to future research.

II. STRESS-STRENGTH INTERFERENCE MODEL CONSIDERING TIME-VARYING LOAD AND STRENGTH DEGRADATION

A. CLASSICAL STRESS-STRENGTH INTERFERENCE MODEL

Assuming that the probability density functions of the stress S and the strength Z of the part are respectively f_s (•) and f_σ (•), the interference of S and Z is shown in figure 2. Failure usually occurs in the interference area and increases with the increase in the area of the interference area. The formula for solving the reliability R is

$$R = P(Z > S) = \int_{-\infty}^{+\infty} f_s(s)[\int_{s}^{+\infty} f_\sigma(z)dz]ds = \int_{-\infty}^{+\infty} f_s(s)(1 - F_\sigma(s))ds \quad (1)$$

where F_σ (•) is the distribution function of component strength.

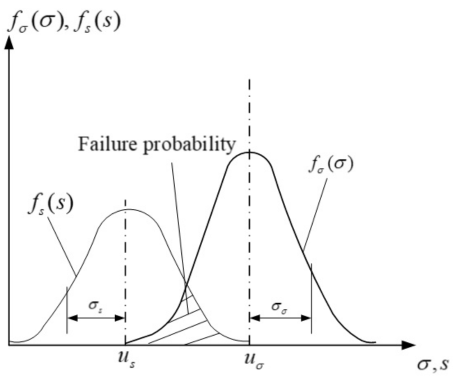

FIGURE 2. Interference between s and σ.

B. STRESS-STRENGTH INTERFERENCE MODEL UNDER MULTIPLE STOCHASTIC LOADS

When the mechanical structure is working, the loads on the parts and components are not constants, but stochastic variables due to the external environment and their own wear, and the components are generally subjected to multiple stochastic loads. If only the classical stress-strength interference model is used to estimate the reliability of components, the estimated result will have a large error compared with the actual reliability.

When the stochastic load acts on the part many times, if the part does not fail under the maximum load, then it is considered that the part does not fail when the stochastic load s acts on the part times. Therefore, when the stochastic load acts n times, the reliability of the component is

$$P(z > s_{\max}) = P(z > s_1, z > s_2, \cdots, z > s_n) \quad (2)$$

where z is the strength of the component,$s_1, s_2, \cdots, s_n$ is a sample drawn from a stochastic load sample,$s_{\max} = \max(s_1, s_2, \cdots, s_n)$. Let $F_s(s)$ and $f_s(s)$ denote the distribution function and probability density function of the stochastic loads respectively, then the distribution function of the maximum load X when the load acts on the part n times is:

$$F_X(x) = [F_s(x)]^n \quad (3)$$

Then the probability density function of stochastic load distribution can be obtained as

$$f_X(x) = n[F_s(x)]^{n-1} f_s(x) \quad (4)$$

When the stochastic load acts on the part n times, combining formula (1) and formula (4), it can be obtained

$$R^{(n)} = \int_{-\infty}^{+\infty} f_X(x) \int_{x}^{+\infty} f_\sigma(z)dzdx = \int_{-\infty}^{+\infty} n[F_s(x)]^{n-1} f_s(x) \int_{x}^{+\infty} f_\sigma(z)dzdx \quad (5)$$

To facilitate understanding, replace x in formula (5) with s, then formula (5) becomes

$$R^{(n)} = \int_{-\infty}^{+\infty} n[F_s(s)]^{n-1} f_s(s) \int_{s}^{+\infty} f_\sigma(z)dzds \quad (6)$$

In the process of reliability analysis of mechanical parts, the cumulative action times of stochastic loads on the parts are consistent with the mathematical model of the Poisson process [24]. The Poisson process can describe events that occur independently at disjoint times and intervals. Therefore, the Poisson process is used in this paper to describe the stochastic load action process. Suppose the total number of load actions is $N(t)$, the probability that a stochastic load acts on a part n times at any time t can then be expressed by a Poisson process with parameter λ as

$$P[N(t) - N(0) = n] = \frac{\lambda t^n}{n!} e^{-\lambda t} \quad (7)$$

Combined with formula (6) and formula (7), the reliability of the part considering the number of dynamic loads can be obtained

$$R(t) = \sum_{n=0}^{\infty} R^{(n)} P[N(t) - N(0) = n] = \sum_{n=0}^{\infty} \frac{\lambda t^n}{n!} e^{-\lambda t} \int_{-\infty}^{\infty} f_\sigma(z) \int_{-\infty}^{\sigma} n[F_s(s)]^{n-1} f_s(s)dsdz \quad (8)$$

Based on the expansion of Taylor's formula of exponential function, (8) can be rewritten as

$$R(t) = \int_{-\infty}^{\infty} f_\sigma(z) e^{-\lambda t} - \lambda t \sum_{n=0}^{\infty} \frac{(\lambda t)^n}{n!} [F_s(z)]^n dz = \int_{-\infty}^{\infty} f_\sigma(z) e^{\{[F_s(z)-1]\lambda t\}} dz \quad (9)$$

C. STRESS-STRENGTH INTERFERENCE MODEL CONSIDERING STRENGTH DEGRADATION

During the operation of mechanical parts or structures, the strength will degrade. Therefore, when establishing the reliability model of the part, the strength degradation of the part needs to be considered. Based on formula (9), the reliability of the part or structure when the strength is degraded can be obtained as

$$R(t) = \int_{-\infty}^{\infty} f_\sigma(z) \mathrm{e}^{\left\{\int_0^t [F_s(z,\tau)-1]\lambda d\tau\right\}} dz \tag{10}$$

$D(t)$ represents the stochastic process of strength degradation, then at any time t, the strength of the part is presented as follows

$$Z(t) = Z - D(t) \tag{11}$$

If stress occurs at time t is event $A(t)$, and then $\overline{A}(t)$ is not appearing. When the stress occurs at any time t, the failure probability function of the structure at that time is

$$P_F(t) = P\{Z - D(t) < S\,|A(t)\} = 1-\mathrm{F}_s(Z - D(t)) \tag{12}$$

The reliability of the part is expressed as

$$R(t) = \int_{-\infty}^{\infty} f_\sigma(z) \mathrm{e}^{\left\{\int_0^t [F_s(Z-D(\tau))-1]\lambda d\tau\right\}} dz \tag{13}$$

The strength degradation of mechanical parts is an incremental monotonic degradation process [25]. The Gamma process is more effective in describing degradation phenomena such as fatigue, cracks, corrosion, and wear. It is suitable to describe the degradation process [26] that monotonously accumulates small increments in the time domain. The probability density function of the Gamma process [27] is shown in (14)

$$\mathrm{Ga}(x\,|v, u) = \frac{u^v}{\Gamma(v)} x^{v-1} \mathrm{e}^{-ux)} I_{(0,\infty)}(x) \tag{14}$$

where $v > 0$ is the shape parameter, $u > 0$ is the scale parameter, $I_A(x)$ is the indicator function,where

$$I_A(x) = \begin{cases} 1 x \in A \\ 0 x \notin A \end{cases}$$

The probability density of the random degradation amount $D(t)$ of the part can be expressed as

$$f_{D(t)} = Ga(x\,|v(t), u) \tag{15}$$

The mean and variance of $D(t)$ are as follows

$$\mathrm{E}(D(t)) = \frac{v(t)}{u}, \mathrm{Var}(D(t)) = \frac{v(t)}{u^2} \tag{16}$$

If the shape parameter $v(t) = at > 0$ is linear, and the size parameter is $u > 0$, the Gamma process has the property of a smooth increase [28]. The strength degradation of mechanical parts will show a steady increase trend with the increase in the number of fatigue loads, and there will be no major fluctuations or jumps. Therefore, a smooth Gamma process can be used to describe the stochastic law of strength degradation. Assume that the stress and strength of the parts respectively obey the following distributions: $Z \sim N(\mu_1, \sigma_1^2)$, $S \sim N(\mu_2, \sigma_2^2)$. Let $Y = Z - S$, the formula (13) can be solved by numerical integration method, and finally the reliability of the part can be obtained as

$$\begin{aligned} R(t) = &\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma_1^2}} \exp\{[-\frac{1}{2}(\frac{z-\mu_1}{\sigma_1})^2]\} \cdot \exp\{\{\text{-}\lambda \\ &\times \cdot \int_0^t \left\{1\text{-}\left\{\int_0^{+\infty} \frac{1}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}}\right.\right. \\ &\times \exp\{\{-\frac{1}{2}[\frac{y-(\mu_1-\mu_2)}{\sqrt{\sigma_1^2+\sigma_2^2}}]^2\}\}\cdot \\ &\left.\left.\times [\int_0^y \frac{u^{v(t)}}{\Gamma(v(t))} x^{v(t)-1} \mathrm{e}^{(-ux)} dx] dy\right\}\right\} d\tau\}\Big\}\mathrm{dz} \end{aligned} \tag{17}$$

Formula (17) can be simplified to

$$\begin{aligned} R(t) = &\exp\{\{\text{-}\lambda \cdot \int_0^t \{1\text{-}\{\int_0^{+\infty} \frac{1}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}} \exp\{\{ \\ &-\frac{1}{2}\cdot[\frac{y-(\mu_1-\mu_2)}{\sqrt{\sigma_1^2+\sigma_2^2}}]^2\}\}\cdot[\int_0^y \frac{u^{v(t)}}{\Gamma(v(t))} \\ &\times x^{v(t)-1} e^{(-ux)} dx] dy\}\} d\tau\}\}\mathrm{dz} \end{aligned} \tag{18}$$

III. RELIABILITY MODEL OF SERIES SYSTEM BASED ON COPULA FUNCTION

A. COPULA FUNCTION THEORY

Copula function is a connection function, which can connect the joint distribution function and its respective marginal distribution functions together, and can accurately deal with the nonlinear correlation between random variables. According to Sklar's[14] theorem, if $F(\bullet, \cdots, \bullet)$ is a multi-dimensional joint distribution function with marginal distribution $F_1(\bullet), F_2(\bullet), F_3(\bullet), \cdots F_N(\bullet)$, there will be a unique Copula function $C(\bullet, \cdots, \bullet)$ that makes equation (19) true

$$F(x_1, x_2, \cdots x_N) = C\left(F_1(x_1), F_2(x_2), \cdots, F_N(x_N)\right) \tag{19}$$

Through the probability density function $c(\bullet, \cdots, \bullet)$ and the marginal distribution $F_1(\bullet), F_2(\bullet), F_3(\bullet), \cdots F_N(\bullet)$ of the Copula function, the probability density function of the N-ary distribution function $F(x_1, x_2, \cdots x_N)$ can be obtained as

$$f(x_1, x_2, \cdots x_N) = c(F_1(x_1), F_2(x_2), \cdots F_N(x_N)) \prod_{n=1}^{N} f_n(x_n) \tag{20}$$

TABLE 1. Three archimedes copula functions.

$Copula$	$Distribution function C(u_1,u_2;\theta)$	$Density function D(u_1,u_2;\theta)$	$Generator function \varphi_\theta(t,\theta)$	$Range of \theta$
$Gumbe$	$\exp\{-[(-\ln u_1)^\theta+(-\ln u_2)^\theta]^{1/\theta}\}$	$\frac{\exp(-S^{1/\theta})(\ln u_1u_2)^{\theta-1}(S^{1/\theta}+\theta+1)}{u_1u_2S^{2-1/\theta}}$	$(-\ln t)^\theta$	$[1,\infty]$
$Clayton$	$(u_1^{-\theta}+u_2^{-\theta}-1)^{-1/\theta}$	$(1+\theta)(u_1u_2)^{-\theta-1}(u_1^{-\theta}+u_2^{-\theta}-1)^{-2-1/\theta}$	$\frac{1}{\theta}(t^{-\theta}-1)$	$(0,\infty)$
$Frank$	$-\frac{1}{\theta}\ln[1+\frac{(e^{-\theta u_1}-1)(e^{-\theta u_2}-1)}{e^{-\theta}-1}]$	$\frac{-\theta(e^{-\theta}-1)e^{-\theta(u_1+u_2)}}{[(e^{-\theta}-1)+(e^{-\theta u_1}-1)(e^{-\theta u_2}-1)]^2}$	$-\ln[\frac{e^{-\theta t}-1}{e^{-\theta}-1}]$	$(-\infty,\infty)\backslash\{0\}$

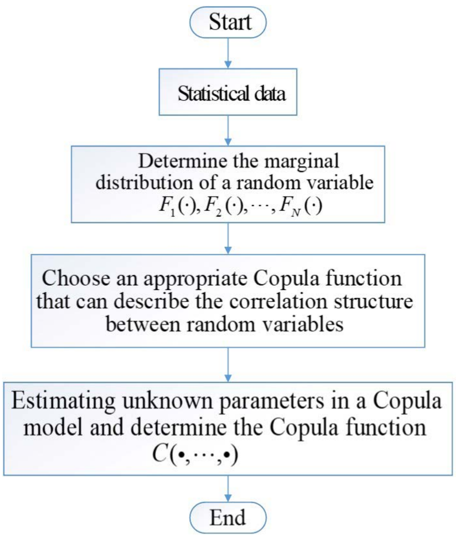

FIGURE 3. The construction process of the copula function model.

The process of building a Copula function model is shown in Figure 3.

There are parametric and nonparametric methods for determining the distribution of random variables. The parametric method assumes that the random variable obeys a certain type of distribution with parameters, and then estimates the parameters in the distribution according to the sample observations, and finally makes a test. The most commonly used nonparametric method is the kernel density estimation method, that is, according to the sample observation data, the kernel density estimation method is used to determine the overall distribution. According to the different generators of the Copula function, the commonly used Copula functions include the normal Copula function, the t-Copula function, and the Archimedes Copula function. Among them, the Archimedes Copula function has the advantages of simple form, symmetry, associativity, etc. that other Copula functions do not have. Among Archimedes Copula functions, Gumbel Copula function, Clayton Copula function and Frank Copula function are commonly used binary Copula functions in reliability research. The description of these three Copula functions is shown in Table 1. The marginal distribution of random variables may contain unknown parameters, and the selected Copula function contains unknown parameters, so parameter estimation is required. Maximum likelihood estimation is widely used because of its better asymptotic properties.Assume that the marginal distribution functions of the continuous random variable $X_1, X_2, \cdots, X_N$ are $F_1(x_1,\theta_1), F_2(x_2,\theta_2), \cdots, F_N(x_N,\theta_N)$ respectively, and the marginal density functions are $f_1(x_1,\theta_1), f_2(x_2,\theta_2), \cdots, f_N(x_N,\theta_N)$ respectively, where $\theta_1,\theta_2,\cdots\theta_N$ is the unknown parameter in the marginal distribution, and the selected Copula distribution function is $C(F_1(x_1,\theta_1), F_2(x_2,\theta_2), \cdots, F_N(x_N,\theta_N);\alpha)$,$\alpha$ is the unknown parameter in the Copula function, and the joint distribution function of $(X_1, X_2, \cdots, X_N)$ is

$$F(x_1,x_2,\cdots x_N;\theta_1,\theta_2,\cdots,\theta_N,\alpha)=C[F_1(x_1;\theta_1), F_2(x_2;\theta_2),\cdots,F_N(x_N;\theta_N);\alpha] \quad (21)$$

The joint density function of $(X_1, X_2, \cdots, X_N)$ is

$$\begin{aligned} f(x_1,x_2,\cdots x_N;\theta_1,\theta_2,\cdots,\theta_N,\alpha) &= c[F_1(x_1;\theta_1),F_2(x_2;\theta_2),\cdots,F_N(x_N;\theta_N);\alpha] \\ &\quad \times f_1(x_1;\theta_1)f_2(x_2;\theta_2)\cdots f_N(x_N;\theta_N) \end{aligned} \quad (22)$$

The likelihood function of the sample is

$$\begin{aligned} &L(\theta_1,\theta_2,\cdots\theta_N,\alpha) \\ &= \prod_{i=1}^{n} f(x_1,x_2,\cdots x_N;\theta_1,\theta_2,\cdots,\theta_N,\alpha)=\prod_{i=1}^{n} c[F_1(x_1;\theta_1), \\ &\quad \times F_2(x_2;\theta_2),\cdots,F_N(x_N;\theta_N);\alpha]f_1(x_1;\theta_1) \\ &\quad \times f_2(x_2;\theta_2)\cdots f_N(x_N;\theta_N) \end{aligned} \quad (23)$$

The log-likelihood function is

$$\begin{aligned} &\ln L(\theta_1,\theta_2,\cdots\theta_N,\alpha) \\ &= \sum_{i=1}^{n} \ln c[F_1(x_1;\theta_1),F_2(x_2;\theta_2),\cdots,F_N(x_N;\theta_N);\alpha] \\ &\quad + \sum_{i=1}^{n} \ln f_1(x_1;\theta_1)+\sum_{i=1}^{n} \ln f_2(x_2;\theta_2)\cdots \\ &\quad + \sum_{i=1}^{n} \ln f_N(x_N;\theta_N) \end{aligned} \quad (24)$$

By solving the maximum point of the log-likelihood function, the maximum likelihood estimation of the unknown parameters in the marginal distribution and the Copula function can be obtained as

$$\hat{\theta}_1,\hat{\theta}_2,\cdots\hat{\theta}_N,\hat{\alpha}=\arg\max \ln L(\theta_1,\theta_2,\cdots\theta_N,\alpha) \quad (25)$$

B. RELIABILITY MODEL OF SERIES MECHANICAL SYSTEM

Suppose the series system is composed of n units, and its joint distribution function is $F_y(y_1, y_2, \cdots y_n)$, $Y_i(t) = \sigma_i - s_i$, Y_i represents the functional function of the ith part at time t, σ_i represents the strength of the part, and s_i represents the stress of the part, the dynamic reliability of the series system is as follows

$$\begin{aligned} R_s(t) &= P(Y_1(t) > 0, Y_2(t) > 0, \cdots, Y_n(t) > 0) \\ &= \Delta^1_{1-R_1(t)}\Delta^1_{1-R_2(t)}\cdots\Delta^1_{1-R_n(t)}C(w_1, w_2, \cdots w_n) \end{aligned} \tag{26}$$

where Δ represents the difference symbol, $\Delta^{w_2}_{w_1}C(w, \gamma_0) = C(w_2, \gamma_0) - C(w_1, \gamma_0)$ represents the one-fold difference, $\Delta^{\gamma_2}_{\gamma_1}\Delta^{w_2}_{w_1}C(w, \gamma) = C(w_2, \gamma_2) - C(w_2, \gamma_1) - C(w_1, \gamma_2) + C(w_1, \gamma_1)$, represents the double integral, and so on, $R_j(t)$ is the dynamic reliability of the part, $w_j = F_j(t) = 1 - R_j(t)$, $j = 1, 2, \cdots, n$.

When calculating the reliability of a specific mechanical system, the multivariate Gauss Copula function [29]and the Archimedes Copula function are usually selected to deal with the correlation problem.

IV. DYNAMIC RELIABILITY OF KEY PARTS

A. DYNAMIC RELIABILITY OF DRIVING AND DRIVEN GEARS

In the process of transmitting power, the driving and driven gears in the gearbox of high-speed trains are simultaneously subjected to external excitations transmitted from other parts and internal excitations during the meshing process of gear pairs. The failure modes of the driving and driven gears are mainly tooth root fracture and tooth surface damage [30]. Based on the failure mode of the gear, the bending stress and contact stress are calculated. Considering the stochastic load, the degradation of tooth root bending strength and tooth surface contact strength, the reliability models of the driving and driven gears are established, and the bending fatigue and contact fatigue dynamic reliability of the driving and driven gears are obtained.

In this paper, the material of the driving and driven gears is 17CrNiMo6. According to GB/T3480-1997, it can be known that the calculated contact stress and the calculated bending stress of the gear are

$$s_H = Z_B Z_H Z_E Z_\varepsilon Z_\beta \sqrt{K_A K_V K_{H\beta} K_{H\alpha} \frac{F_t}{d_1 b} \frac{u+1}{u}} \tag{27}$$

$$s_F = K_A K_V K_{F\beta} K_{F\alpha} Y_F Y_S Y_\beta \frac{F_t}{b_F m_n} \tag{28}$$

where K_A is the use factor, taken K_A=1.35; K_V is the dynamic load factor, according to the gear 6 precision grade and rated speed, K_V=1.2; $K_{H\beta}K_{F\beta}$ are respectively the tooth load distribution coefficient, $K_{H\beta}$=1.14, $K_{F\beta}$=1.104; $K_{H\alpha}K_{F\alpha}$ are respectively the load distribution coefficient between teeth, $K_{H\alpha}$=$K_{F\alpha}$=1; Y_F, Y_s are respectively the tooth profile coefficient and the stress correction coefficient, calculating the coefficient of the driving and driven gear as $Y_{F1} = 2.562$, $Y_{S1} = 1.65$, $Y_{F2} = 2.315$, $Y_{S2} = 1.777$; Y_β is the helix angle coefficient, $Y_\beta = 0.833$; Z_B is the single pair of tooth meshing coefficient, $Z_B = 1$; Z_H is the node area coefficient, $Z_H = 2.158$; Z_E is the elasticity coefficient, look up the table to get $Z_E = 189.8\sqrt{MPa}$; Z_ε is the coincidence coefficient, calculated as $Z_\varepsilon = 0.925$; Z_β is the helix angle coefficient, calculated as $Z_\beta = 0.969$.

The probability distribution of the contact stress on the tooth surface of the driving gear and the driven gear obeys $s_H \sim N(458, 35.51^2)$. The probability distribution of the bending stress at the root of the driving gear obeys $s_{F1} \sim N(95.74, 1.32^2)$, and the probability distribution of the bending stress at the root of the driven gear obeys $s_{F2} \sim N(93.17, 1.29^2)$.

The tooth surface contact fatigue strength and tooth root bending fatigue strength of the driving and driven gears are respectively obeyed: $\sigma_H \sim N(1031.25, 61.88^2)$, $\sigma_F \sim N(575.79, 46.06^2)$.

The strength degradation amount is a stochastic variable. The degradation law of the strength of the parts depends on the properties of the material of the parts, the structure of the parts and the working state of the parts, so the degradation law of the strength of the driving and driven gears will also be different. The P-S-N curve of a material or part can reflect a more comprehensive stress-life relationship, and the P-S-N curve set can reflect three aspects of probability information, as shown in Figure 4. Combining the P-S-N curve of the part material to estimate the characteristic parameters in the Gamma degradation model. The specific process of Gamma parameter estimation is shown in Figure 5. In Figure 5, S is the maximum stress loaded, N is the fatigue life, m and C are two constants, related to the material and the test, t is the time, $S_{p_l}(t_p)$ represents the strength of the $S-t$ curve with the survival rate P_l in $[t_p, t_{p+1}]$, $\hat{u}$ and $\hat{a}$ are the estimated values of the characteristic parameters.Based on the $P-S-N$ curve of the gear material and the Gamma parameter estimation process, the Gamma process parameters of the driving gear and the driven gear can be calculated as shown in Table 2.The contact fatigue intensity of the driving gear obeys the gamma process of the shape parameter $v(t) = at = 0.0136t$, and the scale parameter $u = 0.3184$, the bending fatigue intensity

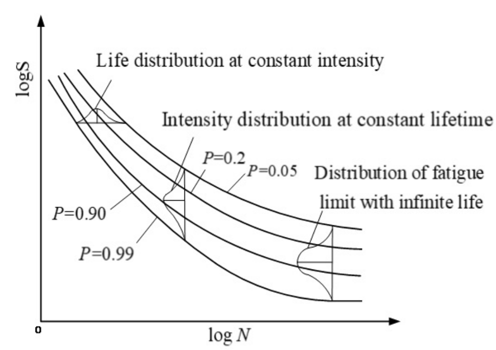

FIGURE 4. P-S-N curve.

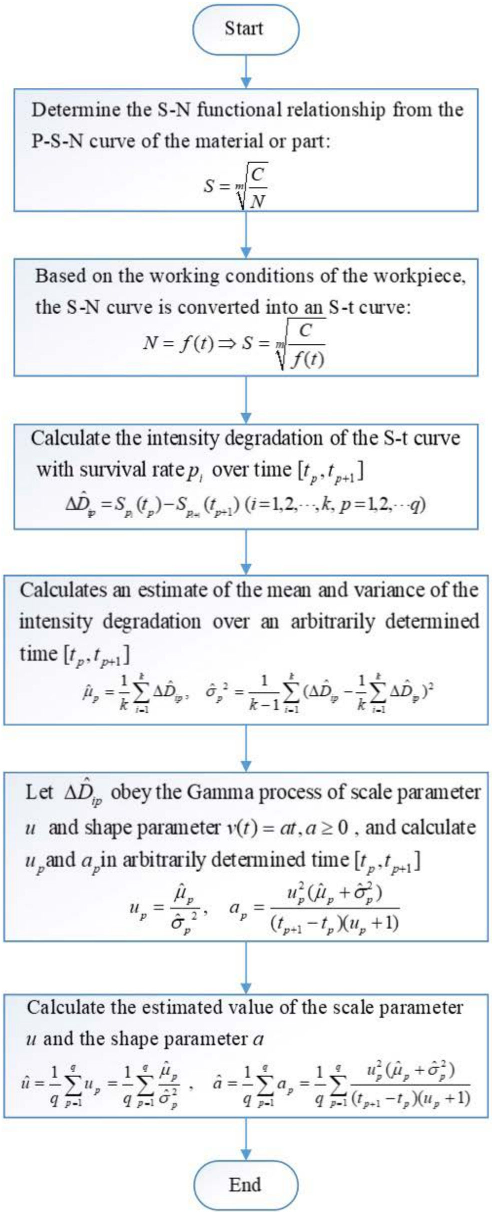

FIGURE 5. The specific process of Gamma parameter estimation.

TABLE 2. Gamma strength degradation parameters of the driving and driven gears.

	Degradation of contact fatigue strength		Degradation of bending fatigue strength	
	$v(t)$	u	$v(t)$	u
Driving gear	0.0136t	0.3184	0.0124t	0.1466
Driven gear	0.0105t	0.2034	0.0104t	0.0799

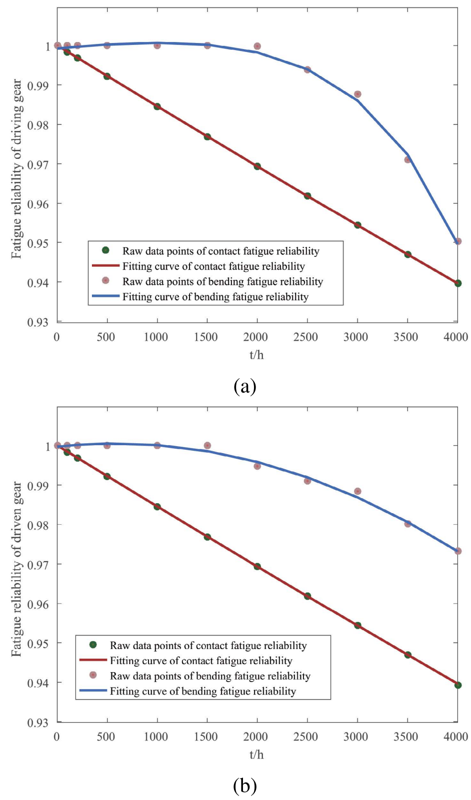

FIGURE 6. (a) Fatigue reliability of driving gear (b) Fatigue reliability of driven gear.

of the driving gear obeys the gamma process of the shape parameter $v(t) = at = 0.0124t$, and the scale parameter $u = 0.1466$. The contact fatigue intensity of the driven gear obeys the gamma process of the shape parameter $v(t) = at = 0.0136t$, and the scale parameter $u = 0.3184$, the bending fatigue intensity of the driving gear obeys the gamma process of the shape parameter $v(t) = at = 0.0104t$, and the scale parameter $u = 0.0799$.

Combining the stress distribution, strength distribution and strength degradation process parameters determined above, the dynamic reliability of the driving and driven gears can be obtained, as shown in Figure 6.

It can be seen from Figure 6 that the contact fatigue reliability of the gear decreases faster than the tooth root bending fatigue reliability, which is consistent with the main failure mode of the gear being contact fatigue failure. In addition, when the operating time is between 2000 and 2500 hours, the contact fatigue reliability of the gear is about 0.97, which is in line with the secondary maintenance standard of this type of high-speed train, and the gear box is maintained for oil change. This shows that the above calculation method is in line with engineering practice.

B. DYNAMIC RELIABILITY OF BEARINGS

The bearing arrangement of the input shaft and output shaft of the gearbox is shown in Figure 7. The cylindrical roller bearings at both ends of the driving gear of the input shaft are used to bear the radial load when the driving shaft rotates, and the four-point contact ball bearing bears the axial load when the motor drives the driving gear. The tapered roller bearings at both ends of the driven gear on the output shaft bear both

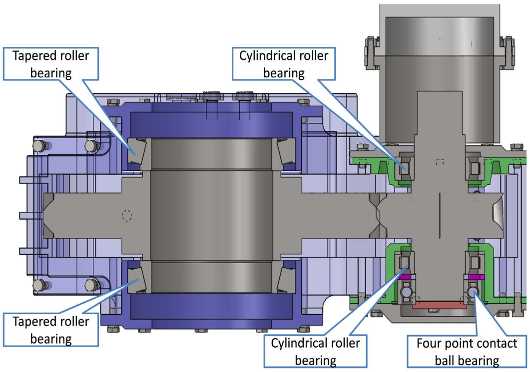

FIGURE 7. Bearing arrangement of the gearbox.

radial and axial loads.The distribution of contact stress and the distribution of strength of rolling bearings are regarded as normal distribution [31].

The contact stress calculation of rolling bearings in gearboxes is calculated by referring to the algorithm in Harris and Kotzalas [32]. After calculation and analysis, it can be obtained: the surface contact stress distribution of the four-point contact ball bearing obeys $s_{B1} \sim N(1066.5, 106.65^2)$, the surface contact stress distribution of the outer cylindrical roller bearing of the driving gear obeys $s_{B2} \sim N(671.29, 67.12^2)$, and the surface contact stress distribution of the inner cylindrical roller bearing of the driving gear obeys $s_{B3} \sim N(575.89, 57.59^2)$, the surface contact stress distribution of the outer tapered roller bearing of the driven gear obeys $s_{B4} \sim N(852.34, 85.23^2)$, and the surface contact stress of the inner tapered roller bearing of the driven gear obeys $s_{B5} \sim N(696.64, 69.66^2)$, the fatigue strength of the bearing material obeys $s_{B5} \sim N(1718, 171.8^2)$. Based on the algorithm in Figure 5 and the P-S-N curve of bearing steel GCr15A, the Gamma strength degradation parameters of each bearing are determined as shown in Table 3.The parameters in the table are the same as the intensity degradation parameters of the gears, and will not be described here.

TABLE 3. Gamma strength degradation parameters of the bearings.

	Degradation of contact fatigue strength of rolling bearings	
	$v(t)$	u
Four-point contact ball bearing	0.0379t	0.8753
Outer cylindrical roller bearing	0.0379t	0.8753
Inner cylindrical roller bearing	0.0379t	0.8753
Outer tapered roller bearing	0.0508t	0.6799
Inner tapered roller bearing	0.0508t	0.6799

Based on the above stress distribution, strength distribution and strength degradation process parameters of each bearing, the dynamic reliability of each bearing can be obtained, as shown in Figure 8.

It can be seen from Figure 8 that among various types of bearings, the reliability of the four-point contact ball bearing decreases faster than that of other bearings, which is related to the complexity of the load-bearing of the four-point ball bearing. In addition, when the operating time is between 2000 and 2500 hours, the reliability of the four-point ball bearing is maintained above 0.96, and the reliability of other types of bearings is above 0.99, which is in line with the secondary maintenance standard. Therefore, the above reliability calculation for the bearing is consistent with the actual engineering.

C. DYNAMIC RELIABILITY OF THE GEARBOX HOUSING

The gearbox housing is subjected to reciprocating vibration caused by various external loads such as wheel rail, drive motor and gear pair meshing, resulting in fatigue damage [33]. The gearbox housing is different from other standard parts due to its complicated shape. The equivalent stress of the weak point of the gearbox housing is calculated through finite element analysis or the stress spectrum of the travel route[10], and the probability distribution of the equivalent stress is regarded as the stress distribution of the gearbox housing. Based on Miner's law and the S-N curve of the gearbox housing material [34], the equivalent stress at the weak point of the gearbox housing is

$$s_{equ} = [\frac{L}{L_1 N}(\sum_{i=1}^{N} n_i s_{ai}^{m})]^{1/m} \tag{29}$$

where L is the total mileage of the high-speed train in its lifetime; L_1 is the actual operating mileage of the high-speed train; n_i is the number of cycles of dynamic stress amplitudes at all levels, m is the S-N curve index, $m = 7$; s_{ai} is the dynamic stress amplitude at all levels, and N represents the total series of load spectrum.

The process of determining the probability distribution of equivalent stress is outlined here. The distribution of equivalent stress calculated under specified operating conditions obeys $s \sim N(24.91, 0.83^2)$.

The fatigue strength of the gearbox housing depends on its own material properties, which can be considered to obey a normal distribution.The material of the gearbox housing is AlSi7Mg0.3, according to the BS EN 1999-1-3-2007 standard, the coefficient of variation of fatigue strength is taken as 0.1, and the fatigue strength of AlSi7Mg0.3 obeys $s\ N\left(29.94, 2.99^2\right)$. According to the requirements of the "EN13749" standard, under the combined action of fatigue loads, the maximum stress on the gearbox housing occurs at the meshing position of the driving and driven gears [35]. Combined with the P-S-N curve of the gearbox housing [36] to determine the Gamma strength degradation parameter of the gearbox case, the scale parameter is $v(t) = 0.0559t$ nd the shape parameter is $u = 66.847$.

Based on the parameters of the stress distribution, strength distribution and strength degradation of the gearbox housing,

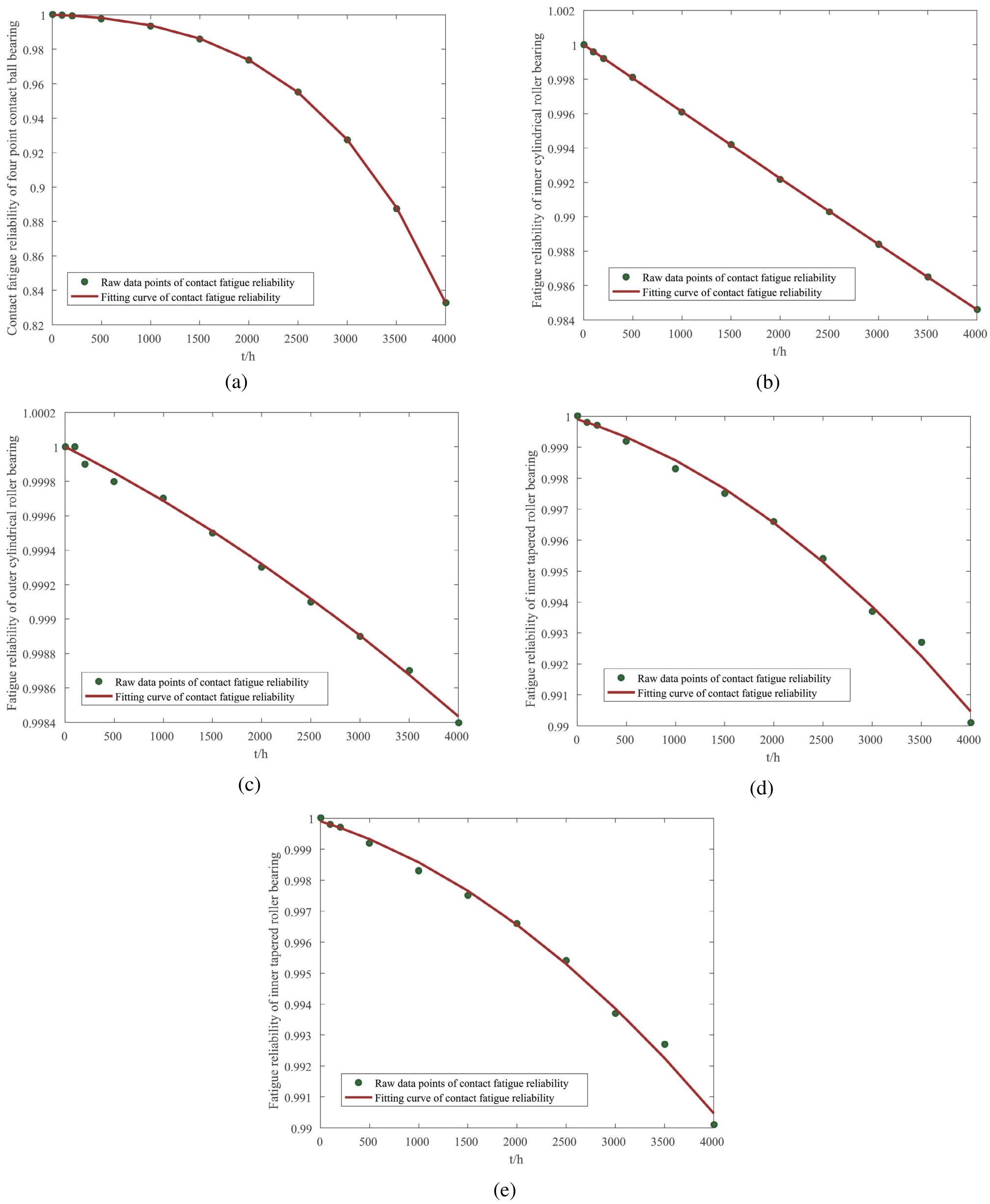

FIGURE 8. **(a) Fatigue reliability of the four-point angular contact ball bearing. (b) Fatigue reliability of the inner cylindrical roller bearing. (c) Fatigue reliability of the outer cylindrical roller bearing. (d) Fatigue reliability of the inner tapered roller bearing. (e) Fatigue reliability of the outer tapered roller bearing.**

the dynamic reliability of the gearbox housing can be obtained as shown in Figure 9.

It can be seen from Figure 9 that under the premise that the casting quality is satisfied and there is no excessive external impact, the reliability decline rate of the gearbox housing increases with the increase of the running time, indicating that the wear state of the driving system parts will have an impact on it. When the running time is between 2000 and 4000 hours, the reliability of the gearbox housing can be maintained above 0.98, which meets the mileage requirements of the third- level and fourth-level maintenance of this type of high-speed train, so the above calculation is in line with the actual engineering.

V. DYNAMIC RELIABILITY ANALYSIS OF GEARBOX BASED ON COPULA FUNCTION

A. ESTABLISH THE COPULA FUNCTION MODEL OF THE GEARBOX

In order to establish the Copula model of the gearbox system, first, the marginal distribution of random variables is determined by the failure functional functions of each key part of the gearbox; then, according to the characteristics of various Copula functions and the characteristics of the research object, select a Copula function that can describe the correlation between random variables; finally, a suitable estimation algorithm is choosen to solve the unknown parameters in the Copula model.

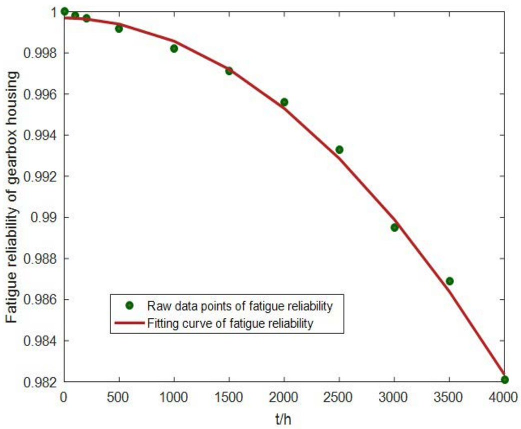

FIGURE 9. Fatigue reliability of the gearbox housing.

If the failure correlation of all the key parts of the gearbox system studied in this article is described by a high-dimensional Copula function, it will be very complicated in parameter estimation and reliability calculation. Therefore, based on the hierarchical correlation of system failures, we consider nesting Copula functions to realize the modeling of high-dimensional Copula functions. When establishing the Copula function model of the gearbox system, the failure correlations on two different levels of the system are considered, that is, the failure correlations between different failure modes of the same part and the failure correlations between different parts of the system. The three Archimedes Copula functions commonly used in mechanical reliability analysis are the Gumbel Copula, Clayton Copula, and Frank Copula [37]. Any N-ary Archimedes Copula function can be equivalent to a binary Archimedes Copula function, and conversely, it can also be constructed and calculated by a binary Archimedes Copula function[25], this property provides the possibility to construct nested Copula function models. When constructing the Copula reliability model of the series system, it is more appropriate to use the Gumbel Copula function to deal with the failure correlation[10], so it is used to analyze the correlation between the key parts and the failure modes. The nesting relationship of the copula function of the gearbox in this paper is shown in Figure 10.

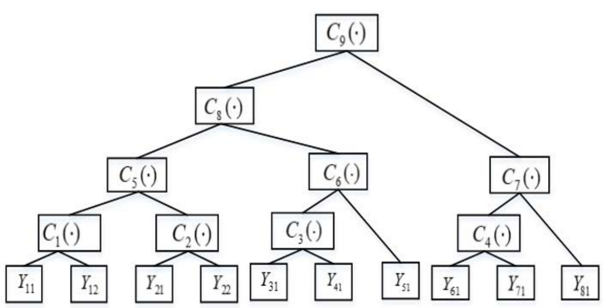

FIGURE 10. The nested Copula function relationship of the gearbox system.

In the figure 10, $Y_{\alpha\beta}(\alpha = 1, 2, \cdots 7, \beta = 1, 2)$ is the performance function of the kth part in the lth failure mode, $\alpha = 1, 2, \cdots 7$ respectively represent the driving gear, driven gear, outer cylindrical roller bearing, inner cylindrical roller bearing, four-point contact ball bearing, outer tapered roller bearing, inner tapered roller bearing, $\beta = 1, 2$ means contact fatigue failure and bending fatigue failure respectively, Y_8 represents the fatigue failure function of the gearbox housing. $C_\gamma(\bullet)(\gamma = 1, 2)$ is the Copula function describing the two failure modes of the driving and driven gears, and $C_\gamma(\bullet)(\gamma = 3, 4, \cdots 9)$ is a Copula function that describes the failure correlation between parts.

The methods of estimating Copula parameters include maximum likelihood estimation, semi-parametric estimation, distribution estimation, etc. [38]. However, the premise of applying these methods is that the distribution law of the sample is known or assumed, and the estimation accuracy is difficult to guarantee. This paper combines the nonparametric kernel density estimation method and the maximum likelihood estimation method to solve the marginal distribution of each random variable and the parameters of the Copula function. Before calculating the parameters of the Copula function by the maximum likelihood estimation method, the failure samples of different parts or different failure modes are determined, so as to determine the marginal distribution function of the failure samples of the parts. Based on the stress and strength distribution functions of each part and each failure mode determined above, the Monte Carlo method is used to conduct random sampling, and the part where the difference between the strength and stress of each part or each failure mode is negative is recorded as the failure sample. The marginal probability distribution functions are determined based on nonparametric kernel density estimates. According to the previous maximum likelihood estimation process, the parameters of the Copula functions at all levels are determined as shown in Table 4.

TABLE 4. Parameter values for each Gumbel Copula function.

Copula function	Parameter values for the Gumbel Copula function
$C_1(\bullet)$	3.0914
$C_2(\bullet)$	3.9372
$C_3(\bullet)$	22.9112
$C_4(\bullet)$	24.6116
$C_5(\bullet)$	72.0174
$C_6(\bullet)$	11.8653
$C_7(\bullet)$	50.8609
$C_8(\bullet)$	16.6826
$C_9(\bullet)$	32.6629

B. DYNAMIC RELIABILITY OF GEARBOX

Based on the dynamic reliability model of the nested Copula function of the gearbox established above, combined with

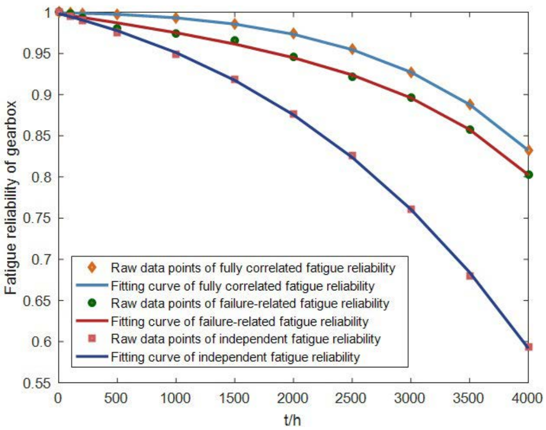

FIGURE 11. Dynamic reliability of gearboxes in three different situations.

the reliability functional functions of each nested subsystem, the dynamic reliability of the gearbox was calculated in MATLAB. For comparison, the reliability of the gearbox system is calculated according to the failure independent hypothesis theory and the failure fully correlated weak link theory, and the calculation results are shown in Figure 11.

According to Figure 11, the dynamic reliability of the gearbox will gradually decrease with the increase of running time, and the dynamic reliability value of the gearbox under failure correlation is between the calculated value of the weak link theory and the independent hypothesis theory. The calculated values are consistent with actual engineering. The dynamic reliability evaluation of high-speed train gearboxes considering random loads, strength degradation and failures can predict and evaluate the reliability of the gearbox in the design stage, so as to optimize and improve the early design. The dynamic reliability assessment of the gearbox can also realize the real-time monitoring of the reliability of the gearbox during the service cycle, so as to ensure the reliability of the gearbox operation and the timeliness of inspection and maintenance.

VI. CONCLUSION

In this paper, we establish a Copula function nested reliability model for a multi-type parts series gearbox system. The model can describe various failure correlations in the gearbox system comprehensively and flexibly, avoiding the problem of 'dimension explosion'. The time-varying stress-strength interference model of each part of G is determined by combining order statistics theory, time-level Poisson process and Gamma strength degradation process. By means of the $P-S-N$ curve strength degradation stochastic model of the part or material, the parameters of the Gamma strength degradation process of the part are determined, which provides support for the reliability correlation modeling of the gearbox system. Through MATLAB, the dynamic reliability of each part and the dynamic reliability of the gearbox system are solved, and compared with the two cases of complete correlation and mutual independence of the parts, the solution results are in line with the actual engineering.The reliability evaluation method in this paper can predict the reliability of the gearbox in the design stage, and can also perform dynamic reliability monitoring during the actual operating life cycle of the gearbox. In this paper, the Gamma process is used to describe the random law of strength degradation of parts. This process conforms to the general characteristics of strength degradation, but whether it conforms to the internal mechanism of strength degradation of each part needs to be further verified based on subsequent engineering practice. The failure correlation model of the gearbox system is constructed by the Copula function. Although it has a strict mathematical basis, the physical nature of the failure correlation of the gearbox system needs further research. On the basis of the previous two studies, the fatigue damage characteristics of key parts in the actual operation process are collected, the $P-S-N$ curve of the parts is more accurately determined, and the multi-parameter composite Copula function is established to accurately calculate the failure-related reliability, so as to achieve accurate evaluation of dynamic reliability of high-speed train gearboxes.

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