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An Improved Ordered Visibility Graph Aggregation Operator for MADM

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ABSTRACT For multi-attribute decision-making (MADM), how to aggregate data and determine attribute weight is still an open issue. Ordered visibility graph aggregation (OVGA) operator can objectively and effectively determine the weight of each attribute value in the network and solve the problem of data fusion. OVGA not only considers the attribute values of nodes in the network, but also synthesizes the influence of the distance between nodes on the weight distribution. However, when there are multiple identical attribute values in the network, the weights assigned by this method are unreasonable. This paper proposes an improved OVGA operator method based on OVGA, which redefines the distance between visual nodes. When there are multiple identical attribute values in the network, the distance formula is redefined in the form of a piecewise function, so that equivalent nodes are given the same weight. The improved method proposed in this paper not only considers the correlation between the visible nodes, but also fully considers the rationality of the weight distribution of the equivalent node support after the fusion of the entire network data. Meanwhile, through several practical application examples which including an application in produced water management, Dongping reservoir tourism resources and the academic ranking of world universities to illustrate the effectiveness and practicability of this method for MADM in complex networks.

INDEX TERMS Visibility graph, aggregation operator, the ordered weighted average operator, multi-attribute decision making.

I. INTRODUCTION

As a representative problem of group decision-making [1]–[4], multi-attribute decision-making (MADM) is mainly to solve the scheduling and optimization problem of finite schemes with multiple attributes. MADM is an important part of modern decision science. Many methods about MADM were applied in lots of field such as risk assessment [5] and single-valued neutrosophic set [6]. MADM is mainly composed of two parts: One is through a certain way, the decision information is gathered and the scheme is sorted and optimized. Another is how to obtain decision information. In this paper, we focus on the problem of decision information. The problem of decision information generally includes two aspects: attribute value and attribute weight. The attribute value can obtain by observed or measured, which usually has three forms: real number, interval number, and language. The attribute weight is usually given by experts, which has subjective and random. Therefore, how to determine the weight of attribute reasonably is very important in the process of MADM. The ordered weighted average (OWA) operator is a useful method to determine the weight of attribute, which was first proposed by Yager [7], and has been widely used in decision-making fields such as risk analysis [8], environmental assessment [9] and so on [10]–[12]. The determination of association weight is a key problem in the aggregation theory of OWA operators. To make the decision in uncertain environment, many OWA operators are introduced [13]–[16].

The OVGA [17] algorithm is proposed on the basis of OWA [7] and the visibility graph (VG). The view of the visibility graph, which based on complex networks, is first published by Lacasa et al [18]. The visibility graph is a new algorithm for covering time series into a complex network [19]. The algorithm considers the mapping of the time series to a complex network, and the reference value of the time series is represented by a vertical bar. If two vertical bars can be seen from each other, then they are linked. In complex network graph, some inherent characteristics of time series

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are retained. For OVGA method, a set of parameter values of an attribute is innovative considered as time series. Then, inspired by Newton's law of universal gravitation, a support function is given to measure the relationship among values of the visibility graph. The value of nodes in the visibility graph and the distance between any two nodes are considered. When two nodes are connected, they are considered to support each other. Meanwhile, the support degree of visible nodes is defined as the linearity between two nodes. The sum of the support degree of visible nodes is determined as the relative weight of nodes. In OVGA operators, the weight of nodes is proportional to the sum of the support of other nodes. On this basis, a weighted network is constructed to determine the weight of each value, and OVGA is applied to production water management [20], which shows that OVGA can deal with MADM. In OVGA method, the complex networks and the visibility graph methods are combined [21]–[24]. Complex networks describe a wide range of systems. The complex network has been widely used to mimic the complex system. For example, dynamics [13], medical science [25], [26], human behavior [27], geography social time series analysis [19], [28], and so on [29]–[32].

However, the method of using OVGA to establish a viewable data fusion weighting cannot solve the multi-attribute decision-making problem of the existence of equivalent nodes in complex networks. Intuitively, if the parameter values of multiple nodes are equal, their support and weight should also be equal. In the OVGA method, when the parameter values of multiple nodes are the same, they get different support and different weights. In other words, the same attribute gets different weights. This is inconsistent with the actual logic. The reason for this result is that these attributes with the same value are placed at different coordinate positions in the visible view. This makes the distance between these equivalent nodes and other nodes different. According to the constraint condition that the nodes in the visible view are visible, the OVGA method causes the disconnection between the nodes with the same attribute value and other visible nodes to become invisible, which leads to the equivalent nodes with the same attribute to obtain different supports and weights. This paper proposes an improved method based on OVGA. The core is the definition of the position and distance of the equivalent node. Aiming at the equivalent nodes with the same attributes appearing in the real network, this paper uses a piecewise function to redefine the position and distance of the node after visualization. Place all the equivalent nodes at positions equidistant from other nodes in the visualized two-dimensional coordinates, while the equivalent nodes in the complex network are located at a position with a distance of ''1'' between each pair in spatial geometric coordinates. Therefore, from the intuitive point of view of planar two-dimensional coordinates, the equivalent node is at the same position relative to other nodes and has the same distance from other nodes. Therefore, the visible nodes connected to the equivalent node are all the same. Equivalent nodes are also visible in pairs and support each other. This is one great contribution of this article. More importantly, when there is no equivalent node with the same attribute in the network, the distance formula defined in this article is consistent with that in the OVGA method, and when there are many equivalent nodes with the same attribute in the actual network, The new method proposed in this paper can accurately and effectively carry out data fusion weighting, and then realize the decision-making of multi-attribute realistic networks.

Another contribution of this paper is to solve the problem of data accuracy in the OVGA algorithm program. In a real network, when performing binary machine operations on batches of data, accuracy errors will occur, causing nodes to perform visibility operations, and invisible nodes are calculated as connected and weighted. For errors in floating-point operations, the improved method takes into account the ''decimal'' program module. Several practical application examples are used to simulate experiments and analysis to show that the method in this paper correctly establishes an ordered weighted view, which reasonably and effectively solves the problem of big data aggregation under uncertainty. It provides a general solution for multi-attribute decision-making in complex networks.

The rest of the paper is organized as follows, some simple basic concepts, which including visibility graph and aggregation operator, are introduced in Section II. In Section III, a new method and an example are proposed to verify the feasibility. In Section IV, a few practical examples illustrate the accuracy, practicability and universal validity of this method. Finally, Some conclusions are summarized in Section V.

II. PRELIMINARIES

In this section, some simple basic concepts, which including visibility graph and aggregation operator, are introduced.

A. THE VISIBILITY GRAPH

The visibility graph is a method of converting the time series of nodes into straight bars [18], [33], [34]. For given a time series $\mathrm{Y} = \{\mathrm{a}_1,\mathrm{a}_2, \cdots \mathrm{a}_i, \cdots \mathrm{a}_n\}$, where a_i is the value of time i and the value of a_i is represented by the height of the vertical in the diagram. The order value a_i and the order i constitute the coordinates (i,a_i). For the visibility graph method, the following sequential visibility criteria can be established: any two data (i,a_i) and (j,a_j) will have visibility, then node i and j will become the two connection nodes of the association graph. The connection of a visible graph of two nodes conforms to linear programming. If there is the other node k, which is between nodes i and j. And then, node i and node j of the graph are connected when these nodes satisfy with:

$$\mathrm{a}_k < \mathrm{a}_j + (\mathrm{a}_i - \mathrm{a}_j)\frac{k-j}{i-j} \tag{1}$$

If the two vertical bars are linked in the picture, they are also linked to each other in the associated graph. In order to transform a time series of size n, we need to check whether all $\frac{(n-1)}{2}$ pairs of nodes can see each other. In order to illustrate

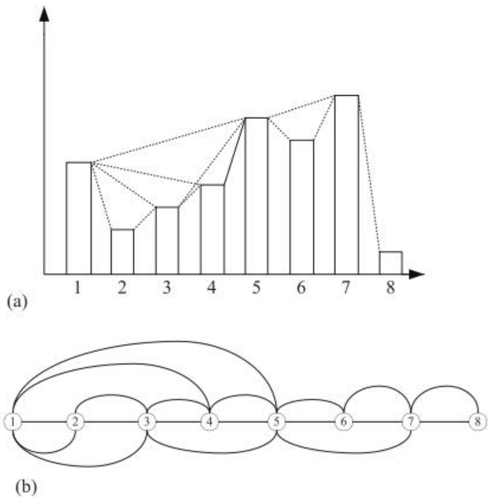


FIGURE 1. The visibility graph with 8 nodes.

this method, an example, which is a time series with 8 nodes, is given and shown in Fig. 1.

From Fig. 1(a), the height of the vertical represents the magnitude of the value of its node. The horizontal coordinate represents the distance between two nodes. According to (1), Fig. 1(a) is converted to Fig. 1(b). From Fig. 1, the visibility graph generation has three properties:

- The network is undirected.
- Each node is connected to at least the adjacent nodes.
- Even if the axis of the proportion of the coordinates of transverse or longitudinal axis dimension changes a certain proportion, or to an affine transformation of coordinate axes, after this method transforms the network remains consistent with the initial visibility.

B. THE ORDERED WEIGHTED AVERAGE OPERATOR

The ordered weighted average (OWA) operator is one of the famous aggregation operators and has been widely used in many fields [15], [21], [34]. OWA operators provide a unified framework for decision-making in an uncertain environment. In this paper, some basics method about OWA operators are introduced as follows.

Yager proposes two measures related to OWA operators [7], "orness meassure" and "dispersion measure".

$$\mathrm{orness(W)} = \frac{1}{n-1}\sum_{i=1}^{n}(n-i)w_i \tag{2}$$

Among them, "orness meassure" is used to measure the degree of the operation of "or" or "and" (it can also be regarded as the optimism of decision makers).

The dispersion measurement related to the weighting function w is defined as,

$$disp(W) = -\sum_{i=1}^{n} w_i \ln w_i \tag{3}$$

Dispersion measure is used to measure the extent to which each data is utilized in the resultant set value.

On the basis of "orness meassure" and "dispersion measure", a maximum entropy programming model is proposed in [14].

$$\begin{aligned} &Maximize(W) = -\sum_{i=1}^{n} w_i \ln w_i \\ &s.t.\ \mathrm{roughness(W)} = \alpha = \frac{1}{n-1}\sum_{i=1}^{n}(n-i)w_i, \quad 0 \le \alpha \le 1 \end{aligned} \tag{4}$$

Equation (4) can be solved analytically and transformed Yager's OWA equation by using the Lagrange multipliers method [35].

Let $\mathrm{Y} = \{\mathrm{a}_1, \mathrm{a}_2, \cdots \mathrm{a}_n\}$ be a set of ordered data, the OWA operator of dimension n is mapping $\mathrm{F}:\mathrm{I}^{\mathrm{n}} \to I, I \in R$

$$F(a_1, a_2, \ldots a_n) = \sum_{i=1}^{n} w_i a_i \tag{5}$$

where a_i is the ith largest element and w_i is the relative weight of a_i, which satisfies $\sum_{i=1}^{n} w_i = 1$ and $0 < w_i < 1$.

III. THE PROPOSED METHOD AND NUMERICAL EXAMPLE

A. CLASSIC ORDINAL VISIBILITY GRAPH AVERAGING (OVGA) OPERATOR

We introduced briefly an ordered visibility graph average aggregation operator in this section. Suppose a_i represent the height of node i vertical line in the visibility graph, and d_{ij} is the distance between nodes i and j. The support degree for nodes i and j, denoted as $Sup(a_i, a_j)$, which has the formula as follows,

$$Sup(a_i, a_j) = \frac{a_i a_j}{d_{ij}^2} \tag{6}$$

For given any two nodes in complex network, and node i can send some information to node j. And that, they are like information carriers and sending messages between their common neighbors. If node i and node j are connected, the information between them can be received from each other. It means that information can be shared by using their contacting. In the visibility graph, each vertical bar has its support degree. The value of the support degree of the vertical bar is bigger, the more important it is.

According to formula (5), there are n reference values here, so we have n corresponding vertical bars here. Let $Y = (a_1, a_2, \ldots a_n)$ be an ordered set of data. The ordered weighted average operator is a mapping $F(a_1, a_2, \ldots a_n) = \sum_{i=1}^{n} w_i a_i$, where a_i is the ith largest element and w_i is the relative weight of a_i, which satisfies $\sum_{i=1}^{n} w_i = 1$ and $0 < w_i < 1$.

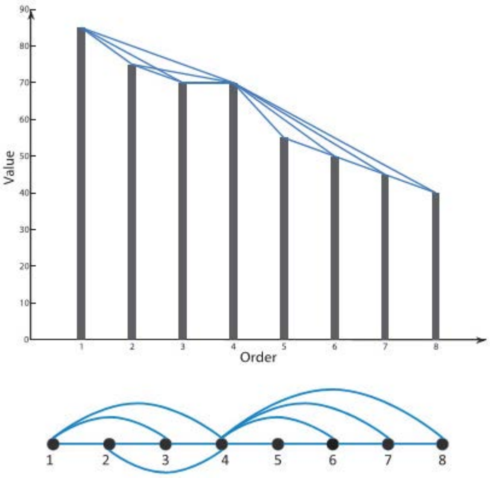


FIGURE 2. The visibility graph of example 1.

The w_i is given as follows,

$$w_i = \frac{K_i}{N} \tag{7}$$

where K_i is the sum of support degree for node i from all other nodes and N is the sum of support degree of all nodes. Their expressions are defined as follows, respectively:

$$K_i = \sum_{\substack{j=1 \\ j \neq i}}^{n} Sup(a_i, a_j) \tag{8}$$

and

$$N = \sum_{i=1}^{n} K_i \tag{9}$$

B. THE SHORTCOMING OF THE OVGA

The following simple example shows that the algorithm proposed in this paper is superior to the classic OVGA method.

Example 1 [17]: there is a set of ordered values $Y = \{85, 75, 70, 70, 55, 50, 45, 40\}$, which are plotted by using vertical bars, and draw the corresponding visibility diagram according to the visible constraints between nodes, and shown in Fig. 2.

In this example, node 3 and node 4 are equivalent (which parameter values are 70). While from Fig. 2, for node 3, there are three vertical bars to support it, which are 85, 75 and 70. For node 4, it is supported by seven nodes except itself. By using the OVGA method, the support degree and weight of nodes are obtained and shown in Table 1. And then, according to formula (7), (8) and (9). The values of K_i and w_i of nodes are given and shown in Table 2.

From Table 1 and Table 2, the support degree of other nodes obtained by equivalent nodes 3 and 4 is different, and the weight obtained is also different, which is unreasonable. In addition, from the visual graph in Fig. 2 that node 4 is supported by seven subsequent nodes, while node 3 is not.

TABLE 1. Support degree of nodes of example 1.

	1	2	3	4	5	6	7	8
1	0	6370	1487.5	670.5	0	0	0	0
2	6370	0	5250	1312.5	0	0	0	0
3	1487.5	5250	0	4900	0	0	0	0
4	670.5	1312.5	4900	0	3850	875	350	175
5	0	0	0	3850	0	2750	0	0
6	0	0	0	875	2750	0	2250	0
7	0	0	0	350	0	2250	0	1800
8	0	0	0	175	0	0	1800	0

TABLE 2. Support and weight of each node.

i	k_i	w_i
1	8528	0.1331
2	12932.5	0.2018
3	11637.5	0.1816
4	12133	0.1893
5	6600	0.1030
6	5875	0.0917
7	4400	0.0687
8	1975	0.0308
Total	64081	1

In OVGA operators, the weight of a node is directly proportional to the sum of support degrees from others. However, for nodes with the same parameter values, the support and weight obtained from other nodes should be the same, which is the shortcoming of OVGA. Just as in elections, the more people support him, the more likely they are to be elected. For candidates with the same strength, they will have equal support from the masses and equally likely to be elected. Meanwhile, there may be some errors for floating-point in process of connecting nodes. Two nodes may be connected since the error of calculating about floating-point while it violated the rule of visibility graph method. The problem about floating-point usually occurs when the value of attributes is non-integer, for example 1, it has not this error. Therefore, the shortcoming about floating-point will be described in Section IV.

C. THE PROPOSED METHOD

For OVGA, how to define the support degree of nodes is still an important problem. In a visibility graph, whether one node can see other nodes is related to the parameter values and arrangement order positions of these nodes. Therefore, it is very important to rank a group of random parameters. In an orderly viewable view, a node can be connected to at least two adjacent nodes. When two nodes are connected, their support degree needs to be considered to determine the weight. However, when there are multiple equivalent parameter values in a group of data, the influence of the sorting method on the node support and weight cannot be ignored. According to formula (6), when the parameter values of multiple nodes are equal, the support degree and weight of that should also be equal.

For the shortcoming of the OVGA method, the key is to place the equivalent attributes. That is, although these

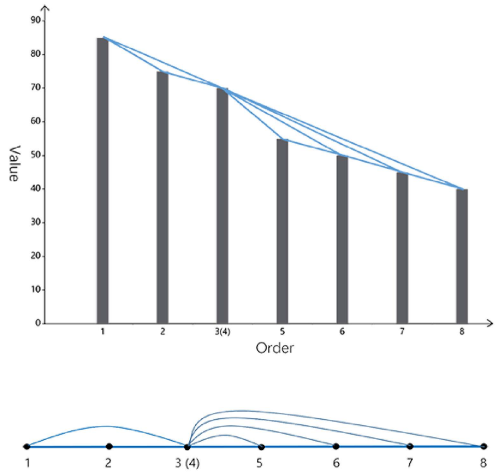


FIGURE 3. The visibility graph.

attributes are placed at different positions, the support and weight of that should be equal since they are the same values of attributes. Therefore, a new distance is defined for equivalent attributes in this paper. For nodes with the same parameter values, because these nodes are placed the same seat, the distance among these nodes is defined as "1" for the equivalent distance. The program module about "decimal" is considered in the improved method to avoid the error of arithmetic about floating-point. And then, an improved OVGA method is proposed and introduced as follows.

Let $Y = (a_1, a_2, \ldots a_n)$ represents a set of parameter values arranged in descending order, where a_j is the jth largest element of the set. The support degree between node i and j is denoted as $\sup'(a_i, a_j)$ and defined as follows,

$$sup'(a_i, a_j) = \frac{y_i y_j}{D_{ij}^2} \tag{10}$$

where D_{ij} is the distance between node i and j and given as follows,

$$D_{ij} = \begin{cases} 1, & y_i = y_j \\ d_j - d_i, & y_i \neq y_j \end{cases} \tag{11}$$

where y_i and y_j are the values of a_i and a_j, respectively. d_i is the site of node i. That is, two equivalent nodes are in the same position relative to other nodes. In (11), D_{ij} is d_{ij} when all value of $y_i (i = 1, 2, \ldots n)$ are different.

In order to described our method, the example 1 to illustrate the feasibility and superiority of the method proposed. The relevant views are shown in Fig. 3.

For nodes 1 and 2, the values of that are 85 and 75, respectively. Then the support degree between them is obtained as follows,

$$sup'(1, 2) = \frac{y_1 y_2}{D_{12}^2} = \frac{85 \times 75}{(2-1)^2} = 6375$$

TABLE 3. support of each node.

	1	2	3	4	5	6	7	8
1	0	6375	1487.5	1487.5	0	0	0	0
2	6375	0	5250	5250	0	0	0	0
3	1487.5	5250	0	4900	3850	875	350	175
4	1487.5	5250	4900	0	3850	875	350	175
5	0	0	3850	3850	0	2750	0	0
6	0	0	875	875	2750	0	2250	0
7	0	0	350	350	0	2250	0	1800
8	0	0	175	175	0	0	1800	0

TABLE 4. Support and weight of each node.

i	k_i	w_i
1	9350	0.1114
2	16875	0.2011
3	16887.5	0.2012
4	16887.5	0.2012
5	10450	0.1245
6	6750	0.0804
7	4750	0.0566
8	1975	0.0235
Total	83925	1

For the equivalent nodes of the first node and the third position, the values of that are 85 and 70, respectively. Then the support degree between them is given as follows,

$$sup'(1, 3) = sup'(1, 4) = \frac{85 \times 70}{(3-1)^2} = 1487.5$$

For equivalent node 3 and 4. According to formula (10) and (11), the distance between them is 1, and then the support degree between node 3 and node 4 is given as follows,

$$sup'(3, 4) = \frac{70 \times 70}{1^2} = 4900$$

The support degree of all nodes are given and shown in Table 3.

We construct the visual icon into a weighted network and get the weight from the support relationship between nodes. In (7), $w_i = \frac{K_i}{N}$, where K_i is the sum of the support for node i from all other nodes and N is the sum of the support of all nodes. The sum weight of each node is given and shown in Table 4.

According to formula (7), the weight of a node is directly proportional to the sum of its support degree. In other words, if one node can get more support degree from other nodes, it will have a greater impact on the final aggregation results. Therefore, it is reasonable for this node to get more weight in the aggregation process. In network, the more similar and close the two influencing factors are, the more support they get from each other. From Table 4, the support degree and weight obtained by equivalent nodes 3 and 4 are the highest and equal, which is logical. From Fig. 3 and Table 3, the more links, the more support degree the nodes get. The support degree between nodes is a measure of the compactness between nodes. Comparing Table 2 and Table 4, the support degree and weight of each node calculated by the two algorithms are different.

TABLE 5. 14 BAT technologies.

14 technologies	abbreviation	best available technologies(BATs)
resource depletion	RD	floatation-A_1
global warming	GW	sparging-A_2
air pollution	AP	coalescence-A_3
critical water mass	CT	hydrocyclones-A_4
solid waste mass	SM	PECT-F or mares Tail-A_5
dissolved oil	DO	centrifuges-A_6
benzene,toluene,ethyl benzene and xylene	BTEX	MPPE-A_7
naphthalene,phenanthrene,dibenzothiophene	NPD	adsorption-A_8
poly-cyclic aromatic hydrocarbons	PAH	C-tour-A_9
heavy metal	HM	membranes-A_{10}
naturally occurring radioactive material	NORM	steam stripping-A_{11}
ease of operation	EO	biological- A_{12}
efficiency	EF	Produced water re-injection- A_{13}
status of technology	ST	down hole separation-A_{14}
control measures	CM	
working capital	WC	
operation ang maintenance	OM	
capital cost	CC	

IV. APPLICATION

A. AN APPLICATION IN PRODUCED WATER MANAGEMENT

In order to compare OVGA and our method, we take the water management as the first example [20]. Modern environmental protection is becoming more and more important. The design and selection of green cleaning processes and products involve the processing of a large number of data related to the environment, economy and technology [36]–[38]. Therefore, it is necessary to use a comprehensive technology to guide aggregation under uncertain conditions to deal with these factors. In order to obtain a comprehensive and feasible technology to deal with the relationship between these uncertain factors [10]. The influencing factors are calculated by OVGA, and the corresponding conclusions are drawn. In this process, 14 best available technologies (BATs) are selected and shown in Table 5, each of which includes 18 indicators. The composed decision matrix is shown in Table 6.

For A_1, the 18 separated parameters are arranged in descending order. According to formula (1), a visual chart is drew and shown in Fig. 4(a). To compare our method and OVGA, the visual chart about A_1 of the OVGA method is given and shown in Fig. 4(b). Nodes with equal parameter values should get equal weights such as nodes 6 and 7, nodes 8, 9 and 10, nodes 12 and 13 for A_1. From Fig. 4(a), the equivalent nodes are in the same position relative to the other nodes, which are the same as the visible nodes, and obtain the same support and weight. However, the weight of equivalent nodes in OVGA is different. From Fig. 4(b), nodes 3, 4, 5 and 7 are connected with node 6, while nodes 4, 5, 6, 8, 9, 10, 17 and 18 are connected with node 7. The ranking position of equivalent nodes in the visual graph affects the visibility of other nodes, resulting in different support and weight of equivalent nodes. The reason is about floating-point error. In fact, according to formula (1), node 3 and node 5, node 10 and node 12 are invisible in Fig. 4(b). While because of the computer programming operation, the decimal conversion binary will produce floating-point error. Therefore, there is a wrong judgment of nodes 3 and 5, nodes 10 and 12 are visible in Fig. 4(b). Finally, according to formula (6), (7) and (10), the support degree of each node of A_1 of our method and OVGA are shown in the first and second line in Table 7, respectively. The corresponding weights of each node are calculated and shown in Table 8.

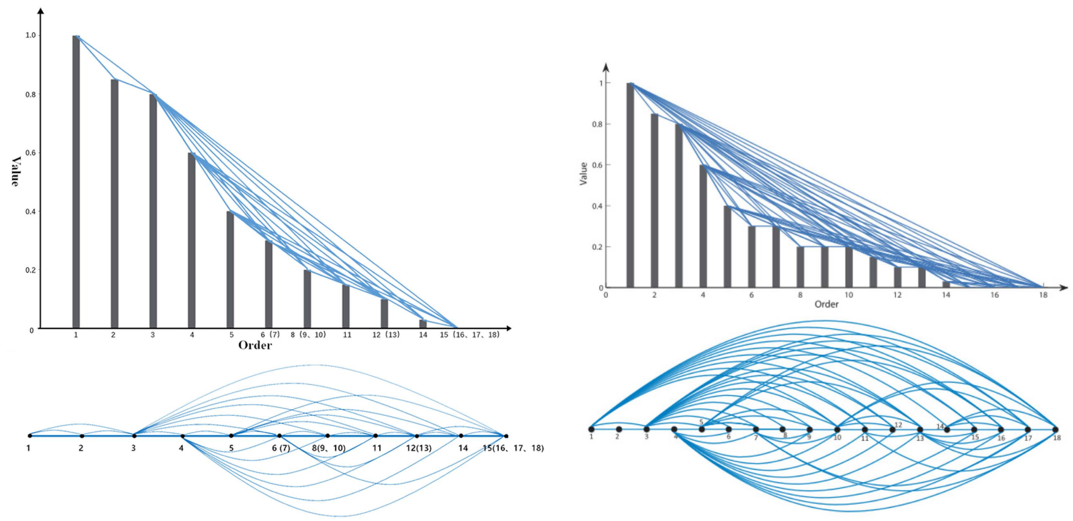
From Table 8, the sum of the parameter weights of each treatment technology is "1", which verifies the accuracy of the method proposed in this paper. The weight value is 0 because the parameter value is 0, which is not supported by other nodes. The weight of each node is determined by the support of other nodes. The method proposed in this paper takes into account the influence of its own parameters and node distance. The special case of equivalence is considered in the form of a piecewise function, which avoids the defects of [17] and makes the calculation results more accurate and reasonable.

Finally, according to formula (5), calculate the final aggregation value for all indicators, and these results are compared with those obtained by the method in [17], [35] and shown inTable 9.

From Table 9, A_{11} is the best choice for OVGA operators. For the OWA operator, we choose two extremum results of 0.1 and 0.9 as reference. From the maximum entropy result of the OWA operator, the result will be different if the value of a is different, but the value of a has no objective basis to choose, and the result is not reasonable if it is affected by subjective factors. Compared with the method of OVGA, the summary results of different process selection in Table 9 are quite different, and the final sorting selection is more convincing. In terms of ranking, A_{11} ranks first, while A_3 and A_{10} rank the same. However, the ranking of the remaining 11 technologies are different because the case of parameter equivalence is considered in our method. In our method, for

TABLE 6. Decision matrix.

	RD	GW	AP	CT	SM	DO	BTEX	NPD	PAH	HM	NORM	EO	EF	ST	CM	WC	OM	CC
A_1	0.03	0.10	0.15	0.30	0.20	0.60	0.00	0.00	0.20	0.00	0.00	0.80	0.85	1.00	0.40	0.10	0.20	0.30
A_2	0.35	0.15	0.20	0.20	0.20	0.60	0.00	0.00	0.30	0.00	0.00	0.80	0.90	1.00	0.50	0.10	0.25	0.35
A_3	0.40	0.20	0.20	0.30	0.30	0.70	0.00	0.00	0.40	0.00	0.00	0.70	0.90	0.80	0.40	0.15	0.15	0.30
A_4	0.55	0.15	0.30	0.35	0.25	0.90	0.00	0.00	0.50	0.00	0.00	0.90	0.95	1.00	0.80	0.20	0.20	0.25
A_5	0.60	0.20	0.30	0.35	0.30	0.95	0.00	0.00	0.70	0.00	0.00	0.80	95	0.75	0.60	0.25	0.20	0.30
A_6	0.45	0.20	0.20	0.15	0.20	0.95	0.00	0.00	0.70	0.00	0.00	0.80	0.95	1.00	0.70	0.20	0.20	0.35
A_7	0.70	0.40	0.50	0.40	0.30	0.90	0.90	0.70	0.90	0.30	0.30	0.60	0.90	0.60	0.30	0.15	0.35	040
A_8	0.60	0.35	0.40	0.35	0.50	0.95	0.95	0.70	0.80	0.10	0.10	0.60	0.90	0.60	0.30	0.15	0.40	0.50
A_9	0.80	0.60	0.55	0.50	0.30	0.95	0.30	0.80	0.90	0.50	0.60	0.70	0.90	0.50	0.40	0.20	0.35	0.50
A_{10}	0.70	0.50	0.30	0.40	0.80	0.95	0.90	0.80	0.90	0.60	0.70	0.60	0.90	0.50	0.30	0.15	0.35	0.50
A_{11}	1.00	1.00	1.00	1.00	0.30	0.60	0.95	0.00	0.00	0.00	0.00	0.70	0.90	0.50	0.20	0.30	0.40	0.60
A_{12}	0.30	0.35	0.30	0.20	1.00	0.90	0.95	0.60	0.70	0.40	0.10	0.90	0.90	0.60	0.10	0.10	0.20	0.50
A_{13}	0.90	0.60	0.45	0.30	0.15	0.90	0.90	0.80	0.80	0.0.50	0.50	0.90	0.95	0.90	0.80	0.25	0.20	0.60
A_{14}	0.85	0.60	0.30	0.35	0.15	0.90	0.95	0.80	0.90	0.0.60	0.50	0.90	0.90	0.70	0.90	0.20	0.60	0.70



(a) Viewable for this article

(b) Viewable for Ref. [17]

FIGURE 4. Viewable comparison of A_1.

TABLE 7. Comparison with A1 results in [17].

	W_1	W_2	W_3	W_4	W_5	W_6	W_7	W_8	W_9	W_{10}	W_{11}	W_{12}	W_{13}	W_{14}	W_{15}	W_{16}	W_{17}	W_{18}
this paper	0.1427	0.2080	0.1975	0.1170	0.0669	0.0652	0.0652	0.0344	0.0344	0.0344	0.0217	0.0057	0.0057	0.0010	0	0	0	0
Ref. [17]	0.1675	0.2428	0.2379	0.1264	0.0760	0.0447	0.0376	0.0169	0.0173	0.0153	0.0086	0.0052	0.0035	0.0005	0	0	0	0

TABLE 8. The obtained weights.

	W_1	W_2	W_3	W_4	W_5	W_6	W_7	W_8	W_9	W_{10}	W_{11}	W_{12}	W_{13}	W_{14}	W_{15}	W_{16}	W_{17}	W_{18}
A_1	0.1427	0.2080	0.1975	0.1170	0.0669	0.0652	0.0652	0.0344	0.0344	0.0344	0.0217	0.0057	0.0057	0.0010	0	0	0	0
A_2	0.1194	0.2147	0.1772	0.1068	0.1090	0.0532	0.0532	0.0446	0.0339	0.0232	0.0232	0.0232	0.0155	0.0028	0	0	0	0
A_3	0.0522	0.1335	0.1517	0.1517	0.0909	0.0909	0.0909	0.0571	0.0571	0.0571	0.0226	0.0226	0.0109	0.0109	0	0	0	0
A_4	0.0715	0.2002	0.1796	0.1796	0.1522	0.0539	0.0475	0.0211	0.0231	0.0195	0.0195	0.0136	0.0136	0.0052	0	0	0	0
A_5	0.1376	0.1376	0.1524	0.1065	0.1088	0.0832	0.0832	0.0528	0.0324	0.0324	0.0324	0.0258	0.0075	0.0075	0	0	0	0
A_6	0.1357	0.1995	0.1995	0.1485	0.0966	0.0411	0.0329	0.0410	0.0185	0.0185	0.0185	0.0185	0.0185	0.0124	0	0	0	0
A_7	0.11161	0.1161	0.1161	0.1161	0.1099	0.1099	0.0595	0.0595	0.0426	0.0211	0.0211	0.0211	0.0285	0.0143	0.0143	0.0143	0.0143	0.0051
A_8	0.0820	0.0820	0.1145	0.0611	0.0868	0.0839	0.0839	0.0839	0.0772	0.0772	0.0392	0.0392	0.0327	0.0327	0.0167	0.0035	0.0015	0.0015
A_9	0.0607	0.1143	0.1143	0.0993	0.0993	0.0785	0.0394	0.0394	0.0715	0.0506	0.0506	0.0506	0.0506	0.0333	0.0191	0.0120	0.0120	0.0043
A_{10}	0.0679	0.1041	0.1041	0.1041	0.1044	0.1044	0.0656	0.0656	0.0567	0.0567	0.0364	0.0364	0.0364	0.0196	0.0161	0.0095	0.0095	0.0024
A_{11}	0.1400	0.1400	0.1400	0.1400	0.1650	0.0657	0.0521	0.0430	0.0430	0.0301	0.0164	0.0099	0.0099	0.0044	0	0	0	0
A_{12}	0.0395	0.1463	0.1446	0.1446	0.1446	0.1149	0.0637	0.0637	0.0427	0.0169	0.0205	0.0160	0.0160	0.0094	0.0094	0.0025	0.0025	0.0025
A_{13}	0.0679	0.0994	0.0994	0.0994	0.0994	0.0994	0.0968	0.0968	0.0968	0.0392	0.0392	0.0218	0.0218	0.0140	0.0033	0.0028	0.0016	0.0008
A_{14}	0.0912	0.1037	0.1037	0.1037	0.1037	0.1037	0.0961	0.0385	0.0493	0.0493	0.0398	0.0398	0.0398	0.0238	0.0060	0.0045	0.0019	0.0014

A_1, nodes 6 and 7 are equivalent, and their support and weight are equal. The support degree is affected by distance, and also changes the weight distribution of each node, therefore the final aggregation result is different.

B. DONGPING RESERVOIR TOURISM RESOURCES

Take Dongping Reservoir as the second example [39] to study how to develop tourism resources of Dongping Reservoir to maximize its comprehensive benefits. According to the laws

TABLE 9. Aggregated results and comparison.

Alternatives	Results of maximal entropy OWA operator (Ref. [36])				Results of OVGA operators (Ref. [17])		Results of the improved OVGA operators	
	$a = 0.1$	$Order$	$a = 0.9$	$Order$	$Results$	$Order$	$Results$	$Order$
A_1	0.0161	14	0.7893	13	0.7071	10	0.6388	13
A_2	0.0261	13	0.8122	12	0.6909	13	0.6486	12
A_3	0.0303	11	0.7447	14	0.6305	14	0.5389	14
A_4	0.0269	12	0.8357	11	0.7991	2	0.7903	4
A_5	0.0390	9	0.8483	10	0.7229	8	0.6963	9
A_6	0.0315	10	0.8843	6	0.7892	4	0.7713	6
A_7	0.2557	4	0.8579	9	0.7503	12	0.7179	8
A_8	0.1722	6	0.8756	7	0.7082	9	0.6571	11
A_9	0.2952	1	0.8750	8	0.7062	11	0.6932	10
A_{10}	0.2788	2	0.8927	5	0.7441	7	0.7467	7
A_{11}	0.0509	8	0.9672	1	0.8581	1	0.8927	1
A_{12}	0.1415	7	0.9175	2	0.7845	6	0.7752	5
A_{13}	0.2499	5	0.9082	4	0.7869	5	0.8215	3
A_{14}	0.2712	3	0.9095	3	0.7969	3	0.8224	2

TABLE 10. The development plan sets of Dongping reservoir tourism resources.

Plan Number	Plan Name	Development Formation	Tourism Resource Endowments
X_1	Sightseeing tour	Natural sightseeing/Historic spots and interest places sightseeing	Liangshan Hill,Tortoise Hill, Phoenix Hill, Water Margin Relics, Huangtu pillar, solution cavity, Dongping reservior, Lashan Hill, Kunshan Hill, Sili Hill, Chenggongaʼs Hall, Stone Tablet Village, Old City of Dongping, Huangshi Cliff, Bai Buddha Hill, Dai Dam, Xiangyu Tomb, Confucian Temple, Baoxiang Temple Tower, Nanwangfenshui Dragon King Temple, stone inscriptions, Cuiping Hill, Hongyuan Pool
X_2	Update sightseeing tour	Micro-landscape/ Archaistic village/ Theme park	Feasible to be built (to be proved)
X_3	Cultural tour	Historical tour/custom tour/ art appreciation tour/nostalgic tour/ historical sites tour/religious tour	The Beijing-Hangzhou Grand Canal (Dai Dam, Nanwangfenshui Dragon King Temple,etc), handicrafts, stone inscription, Water Margin Relics, Chenggongaʼs Hall, Stone Tablet Village, Xiangyu Tomb, Confucian Temple, Baoxiang Temple Tower,Bai Buddha Hill, Old City of Dongping, Hongyuan Pool
X_4	Business tour	Conference tour/ FSE Tourism	Conference tour/FSE Tourism are feasible to be built (to be proved)
X_5	Vacation	Rural Tourism/resort/ Recreational belts around metropolis tour/ water conservancy projects/ Reservoir tour/Camping tour	Feasible (to be proved);the represent water conservancy facilities are Dongping Dam, Xie Hill Tunneling, Flood Basin, The Beijing-Hangzhou Grand Canal(Dai Dam, Nanwangfenshui Dragon King Temple,etc)
X_6	Fitness tour	Sport tour/Medical care tour	Feasible (to be proved)
X_7	Affair tour	Education tour/ Industrial tourism/ Scientific Expedition and geological tourism	Feasible (to be proved)
X_8	Luxurious tour	Leisure tour/gourmet tour	special local products like Carp of Yellow River and ma duck egg, handicrafts like arras and wooden fish stone
X_9	Rising tour	Ecotourism/national park/forest park /photography tour/communitytour/ excursion	Feasible (to be proved); Resettlement community is distinctive

of market demand and resource endowments, a set of alternatives is established, and its development time series is determined. The positioning of Dongping Reservoir's tourism resources is related to the feasibility of Dongping Reservoir's tourism development. It is of practical significance to explore this issue.

Taking into account the tourism resource endowment of Dongping Reservoir, the tourism resources will be developed into the following 9 alternative tourism products. We define the plan sets as $X = \{x_1, x_2 \cdots x_9\}$. The specific plans are shown in Table 10. Then use 8 functional attributes as evaluation indicators, evaluate and rank 9 alternatives, and select the best one. Set 8 functional attributes as $U = \{u_1, u_2, \cdots, u_8\}$, representing the value orientation of the functional attributes(u_1 — Sightseeing and Recreation Value, u_2 — Historical, Cultural, Scientific and Art Value, u_3 — Rare Degree, u_4 — Scale Abundance and its distribution, u_5 — Integrity, u_6 — Popularity and Influence, u_7 — Availability and Application, u_8 — Environment Conservation or Environment Security), and the weight distribution $w_j, j = 1, 2 \cdots 8$ is unknown. According to the survey results of the research team and the local government, 8 functional attributes are evaluated, and 9 alternatives (from 0 to 100 points) are graded to obtain the decision matrix as shown in the table 11.

According to the improved method in this article, firstly arrange the attribute values of the 8 evaluation indicators of the 9 alternatives in descending order to construct the visual view. Then, according to the constructed view and formula (10) and formula (7), calculate the weight distribution

TABLE 11. Decision matrix.

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
x_1	95	90	80	85	70	60	85	75
x_2	50	50	55	70	75	60	95	65
x_3	90	95	90	85	80	75	90	80
x_4	70	60	60	75	70	60	80	65
x_5	85	75	75	70	80	50	80	60
x_6	60	60	50	80	85	50	70	85
x_7	70	90	80	70	70	65	80	85
x_8	60	55	70	85	80	75	95	60
x_9	70	60	75	70	80	65	90	95

TABLE 12. The obtained weights.

	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8
x_1	0.06	0.11	0.26	0.26	0.10	0.09	0.08	0.04
x_2	0.10	0.12	0.11	0.10	0.09	0.08	0.19	0.19
x_3	0.03	0.18	0.18	0.18	0.06	0.16	0.16	0.03
x_4	0.03	0.06	0.14	0.14	0.06	0.19	0.19	0.19
x_5	0.05	0.18	0.18	0.21	0.21	0.08	0.06	0.03
x_6	0.11	0.11	0.09	0.08	0.18	0.18	0.11	0.11
x_7	0.03	0.05	0.15	0.15	0.19	0.19	0.19	0.03
x_8	0.08	0.11	0.11	0.10	0.09	0.23	0.23	0.03
x_9	0.05	0.13	0.10	0.09	0.25	0.25	0.07	0.04

TABLE 13. Aggregated results and comparison.

Alternatives	Results in Ref. [40]		Results of the improved OVGA operators	
	Results	*Order*	*Results*	*Order*
x_1	83.5125	2	82.3368	2
x_2	56.1770	9	62.8149	9
x_3	89.8240	1	86.2081	1
x_4	61.7583	7	64.6898	8
x_5	73.1170	4	75.0770	3
x_6	60.7995	8	66.0359	7
x_7	78.3620	3	74.3561	5
x_8	64.489	6	70.0875	6
x_9	71.2275	5	74.7183	4

$(w_j, j = 1, 2 \cdots 8)$ of the 8 functional attributes, as shown in Table 12. Finally, calculate the aggregate value of each functional attribute index according to formula (5), and these results are compared with those obtained by the method in [39] and shown in Table 13.

It can be seen from Table 13 that the aggregation results obtained by the method in this paper are not much different from the results in [39]. Both ranking results are ranked first in x_3, which is the best travel product. x_1 ranks second and x_2 ranks last. It shows that the method proposed in this paper is effective and universal in dealing with this kind of multi-attribute decision-making problems.

C. THE ACADEMIC RANKING OF WORLD UNIVERSITIES

Take the academic ranking of world universities as the last example [40], select 50 universities as a set of alternatives $(x_i, i = 1, 2 \cdots 50)$, and use the following 6 attribute values as decision-making indicators$(v_j, j = 1, 2 \cdots 6)$. Convert these attribute values into a decision matrix as shown in Table 14.

v_1: Quality of Education (Alumni: Alumni of an institution winning Nobel Prizes and Fields Medals).

TABLE 14. Decision matrix.

	u_1	u_2	u_3	u_4	u_5	u_6
x_1	100	100	100	100	100	79.2
x_2	42.9	89.6	80.1	73.6	73.1	55.8
x_3	65.1	79.4	64.9	68.7	68.4	59
x_4	78.3	96.6	51.3	56.7	67.8	58.5
x_5	69.4	80.7	55.3	71.7	61.7	69.7
x_6	53.3	98	51.3	47.2	42.9	74.4
x_7	49.7	54.9	56.2	55	74.5	46.1
x_8	51	66.7	39.7	57.3	43.6	100
x_9	63.5	65.9	41	53.3	68.9	33.3
x_{10}	59.8	86.3	34	42.7	50.2	44.5
x_{11}	47.6	50.4	44.7	58.4	62.6	37.1
x_{12}	29.5	47.1	58	44.5	71.4	33.4
x_{13}	42	49.8	41	47	60.5	40.9
x_{14}	19.2	35.5	49.2	57.8	63.5	37
x_{15}	21.2	31.6	49.2	52.1	72.6	31
x_{16}	37.7	33.6	38.4	47	71.9	31.1
x_{17}	28.1	36.2	41	41.6	73.9	32.4
x_{18}	31.6	33.8	42.3	39.4	67.7	37.8
x_{19}	29.5	35.5	35.5	50.2	55.6	46.1
x_{20}	36.3	25.3	30.8	47.5	70	29.7
x_{21}	0	39.9	37	52.1	59.3	33.5
x_{22}	14.5	35.8	43.5	32.9	64	39.9
x_{23}	34.4	0	51.3	41.6	76.6	25.8
x_{24}	34.4	24.9	51.3	42	51.7	37.2
x_{25}	15.4	19.2	57.1	38.9	62.1	25.9
x_{26}	15.4	22.1	54.3	35.6	56.9	32.8
x_{27}	19.9	17.2	32.4	38.2	80.1	30.3
x_{28}	32.8	34.8	30.8	35	62.7	24.3
x_{29}	28.1	31.9	32.4	39.5	57.3	22
x_{30}	21.8	18.8	32.4	36.2	65.2	41.9
x_{31}	29.9	36.2	30.8	33.1	55.1	29.1
x_{32}	31.6	37.2	27.1	31.5	58.4	23.8
x_{33}	29.5	16.3	39.7	32.5	64.8	24.1
x_{34}	15.4	18.8	42.3	32.7	64.5	27.2
x_{35}	18.5	32.6	37	26.4	58.4	29
x_{36}	8.9	23.7	39.7	32.6	60.8	33.8
x_{37}	17	59.8	27.1	41.8	19.3	40
x_{38}	12.6	34.1	30.8	36.8	46.2	35.1
x_{39}	33.6	27.4	20.5	29.7	61.9	25.3
x_{40}	17	13.3	35.5	24.8	67.9	32.2
x_{41}	20.5	24.9	32.4	31.3	52.1	26.8
x_{42}	14.5	39.1	32.4	27.3	37.7	38.2
x_{43}	18.5	34.5	30.8	37.6	34.9	27.7
x_{44}	25.6	26.6	22.9	25.1	52.6	40.2
x_{45}	16.2	16.3	29	37	56.3	26.6
x_{46}	30.3	54.3	10.3	17.6	47.9	27.7
x_{47}	19.9	25.3	22.9	30.6	51.8	34.9
x_{48}	34.8	21.6	29	23.3	49.7	34.6
x_{49}	0	31.7	35.5	23.4	53.9	26.2
x_{50}	21.2	21	34	19.6	55.3	27.9

v_2: Quality of Faculty 1 (Award: Staff of an institution winning Nobel Prizes and Fields Medals).

v_3: Quality of Faculty 2 (HiCi: Highly Cited researchers in 21 broad subject categories).

v_4: Papers published in Nature and Science(N&S).

v_5: Papers indexed in Science Citation Index-expanded and Social Science Citation Index (PUB).

v_6: Per capita academic performance of an institution (PCP).

In this MADM problem, let $(w_j, j = 1, 2 \cdots 6)$ be the weight of each attribute index, and it is unknown. First build a visible view according to the method proposed in this paper, calculate the support degree of connected nodes with formula (10), and then calculate the index of the weight of each attribute $(w_j, j = 1, 2 \cdots 6)$ according to formula (7).

TABLE 15. Compare with the ranking result of [40].

Alternatives	Results of the improved OVGA operators		The ranking range of Ref. [41]	
	Results	*Order*	$R^{WA}_{w\in S}$	$R^{OWA}_{w\in S}$
x_1	99.3345	1	[1,1]	[1,1]
x_2	73.1716	2	[2,4]	[2,2]
x_3	67.7751	5	[3,5]	[4,5]
x_4	71.2972	3	[2,5]	[4,5]
x_5	68.9149	4	[2,5]	[3,3]
x_6	66.7257	6	[6,9]	[6,6]
x_7	56.6145	10	[7,9]	[8,8]
x_8	64.8383	7	[6,11]	[7,7]
x_9	58.2480	8	[6,8]	[9,9]
x_{10}	57.1200	9	[8,11]	[10,10]
x_{11}	51.3433	12	[8,11]	[11,11]
x_{12}	51.5867	11	[12,15]	[12,12]
x_{13}	47.2609	16	[12,14]	[13,13]
x_{14}	48.8552	15	[12,15]	[14,14]
x_{15}	49.3575	14	[14,16]	[17,22]
x_{16}	46.1270	18	[13,16]	[15,15]
x_{17}	45.2237	20	[17,20]	[16,20]
x_{18}	43.7857	23	[20,21]	[16,21]
x_{19}	41.3422	28	[17,19]	[16,22]
x_{20}	44.8258	21	[17,20]	[23,25]
x_{21}	45.9178	19	[19,21]	[16,21]
x_{22}	43.0975	25	[25,31]	[22,23]
x_{23}	51.2920	13	[22,33]	[16,24]
x_{24}	42.7626	26	[22,29]	[16,23]
x_{25}	46.3579	17	[27,35]	[26,34]
x_{26}	43.2991	24	[31,37]	[24,25]
x_{27}	44.6443	22	[22,26]	[26,34]
x_{28}	38.7873	35	[22,25]	[26,29]
x_{29}	37.9781	36	[22,26]	[27,31]
x_{30}	41.6201	27	[28,35]	[26,29]
x_{31}	36.5652	39	[27,31]	[29,35]
x_{32}	37.2728	37	[25,31]	[30,34]
x_{33}	40.0866	31	[33,39]	[32,35]
x_{34}	41.0332	29	[35,41]	[36,40]
x_{35}	37.1059	38	[38,49]	[36,38]
x_{36}	39.7372	32	[36,43]	[29,34]
x_{37}	40.1744	30	[23,40]	[38,43]
x_{38}	35.6413	43	[32,38]	[27,35]
x_{39}	36.3465	40	[29,36]	[38,42]
x_{40}	39.7156	33	[44,49]	[41,48]
x_{41}	33.4569	49	[41,44]	[45,48]
x_{42}	34.4661	45	[45,49]	[36,42]
x_{43}	32.2023	50	[37,45]	[40,48]
x_{44}	34.9326	44	[45,47]	[40,44]
x_{45}	35.8027	42	[37,42]	[47,49]
x_{46}	39.3039	34	[30,44]	[36,42]
x_{47}	33.7959	48	[42,44]	[46,49]
x_{48}	34.3248	46	[46,49]	[42,44]
x_{49}	36.3068	41	[47,50]	[45,49]
x_{50}	33.8804	47	[49,50]	[50,50]

Therefore, the MADM problem is transformed into the Yager's OWA operator aggregation problem [7] through the view method. Finally, the aggregation result is calculated by formula (5) and compared with the result in [40], as shown in Table 15.

It can be clearly seen from Table 15 that the ranking results of the 50 universities obtained by the method in this paper are within the ranking range of [40], and the results are more accurate. It shows that the method in this paper is reasonable, effective and accurate in the process of aggregate decision-making.

Analyzing the numerical examples in this part, it can be found that the improved OVGA operator proposed by the combination of the view algorithm and the OWA operator for MADM is reasonable, effective and universal. For multiple equivalent attribute values in practical applications, such as multiple equivalent attribute indicators in the decision matrix (Table 6, Table 11 and Table 14), the method in this paper avoids the limitations of previous methods, and the processing results are accurate, effective and it is in line with the logic of reality.

V. CONCLUSION

MADM is a kind of multi-objective decision-making, that is, the optimal or ranked decision is selected according to certain decision criteria. In MADM, due to the complexity and uncertainty of objective things and the ambiguity of human thinking, people are often unable to give accurate values of the attribute weights of the scheme. Therefore, the study of MADM problems in complex networks has important theoretical significance and practical background. This paper proposes an improved OVGA method, which uses a piecewise function to redefine the position and distance formula of the network nodes in the visible view after sorting. When there is no equivalent node with the same attribute in the network, the distance formula defined in this article is consistent with the distance formula in the OVGA method, but when there are multiple equal target attribute values in a complex network, the method in this article can be accurate and effective Data fusion and weighting are carried out to realize the decision-making on the multi-attribute reality network, and solve the problems that the OVGA method cannot handle. Take several practical applications as examples, the improved method of this article considers the "decimal" program module, correctly establishes an ordered weighted view, and reasonably and effectively solves the problem of big data aggregation under uncertainty. It provides a general solution for MADM in complex networks.

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