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# Evaluation of Critical Node Groups in Cyber-Physical Power Systems Based on Pinning Control Theory

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**ABSTRACT** Blackouts have severely damaged cyber-physical power systems over the years, resulting in malfunctions that have rapidly spread within the electrical network. Selecting a set of critical nodes for human control can avoid similar situations. We propose a critical node evaluation method based on pinning control theory that uses a minimum nonzero eigenvalue of a modified Laplacian matrix as the evaluation index. Computational complexity can be markedly reduced using matrix analysis theory to sort screening conditions. Multiple nodes are controlled to form critical node groups in the directed weighted cyber-physical power system. We can thus more accurately find an optimal set of controlled nodes compared with other critical node evaluation strategies. Through theoretical and simulated verifications, we conclude that the addition of two nodes, which are used as active and standby dispatching centers of the communication network, is more effective and can result in a cyber-physical power system with better connectivity.


**INDEX TERMS** Cyber-physical power system, critical node group, Laplace matrix, pinning control theory.

## I. INTRODUCTION

Power grid blackouts are caused by both internal and external factors, including overload, control or protection failures, and natural disasters, which cause some virtual nodes in a power communication network to withdraw from an operation, thereby causing cascading failures [1]–[6]. Therefore, focusing on the critical node strength in daily operations is conducive to improving the safety of power grid operation and reducing the probability of large-scale power outages. Liu *et al.* [7] combined node centrality and the degree of cascading failure decoupling and proposed an evaluation method for the importance of coupled network nodes to distinguish the critical nodes in a power communication network; [8]–[11] evaluated the critical nodes in the power communication network based on the comprehensive evaluation method of multi-attribute decision-making; [12]–[14] used the information entropy weight method to calculate the objective weight of the degree centrality and betweenness centrality to determine the importance of each node by the TOPSIS

Algorithm to complete the identification of critical nodes in the power communication network; [15] used the node contraction method to evaluate the importance of nodes in the communication network; [16] developed an index set of communication networks using complex system theory that included the node combination degree and communication efficiency to evaluate node importance; and Zhou *et al.* [17] introduced information indexing to identify and rank critical nodes in a cyber-physical power system. However, in these studies, the importance of each node was only studied for a single node in the system.

However, in some natural systems, some nodes are not important when considered individually but were important when combined with other nodes. For example, Fig. 1 shows a simplified IEEE-14 node connection diagram. Assuming that the greater the influence of a node is, the more critical the node is as a measurement criterion, then node 4 is the node with the greatest influence in the system when considered individually because the number of nodes that node 4 affects or reaches is 5, which is the largest among other nodes. For example, nodes 2, 5, 6 and 9 each affect or reach 4 other nodes. Therefore, node 4 is more important than nodes 2, 5,

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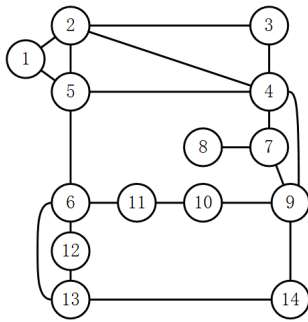


FIGURE 1. IEEE-14 node model spot-line illustration.

6, and 9. However, when considering the node group, when we compare  $\{2, 4\}$  with  $\{5, 6\}$ , we observe the number of nodes that they influence together; when a node with the same effect occurs within a group, it only counts once. The number of nodes that  $\{2, 4\}$  jointly connect to are 1, 3, 5, 7 and 9, and the total number of nodes is 5, while the nodes that  $\{5, 6\}$  jointly influence or reach are 1, 2, 4, 11, 12 and 13, and the number of nodes is 6. Thus, the former group affects fewer nodes than the latter. In this case, the importance of the node group containing node 4 is weaker than that of the node group not including node 4. When nodes are combined, we cannot measure the importance of the node group by the evaluation method of the importance of a single node. The importance evaluation of the node group is different from the importance evaluation of a single node, and the common influence of the nodes in the node group is more important. Therefore, the study of node groups is more meaningful than that of individual nodes.

Although studies have investigated critical node groups in the power grid [18]–[21], they have only considered a single network. Therefore, based on these problems, we investigate the critical node groups in a cyber-physical power system in this study. The primary questions to be answered are as follows:

- 1) How can vital node groups be defined and identified for a double-layer coupled power information network?
- 2) How can a small set of nodes be chosen to maximize the control scale?
- 3) How can the group of nodes that have the most impact on the system and satisfy a certain cost be selected?

The primary contributions of this study are as follows:

- 1) By establishing a directed weighted cyber-physical power system, the minimum non-zero eigenvalue of a modified Laplacian matrix is used as the evaluation index, and a vital node group evaluation strategy based on pinning control theory is proposed;
- 2) The indicators described above are used to filter the essential node groups in the power grid and communication network, and then, they are combined to obtain the critical node groups in the cyber-physical power system.
- 3) Compared with other methods, the proposed algorithm can be used to find critical node groups more

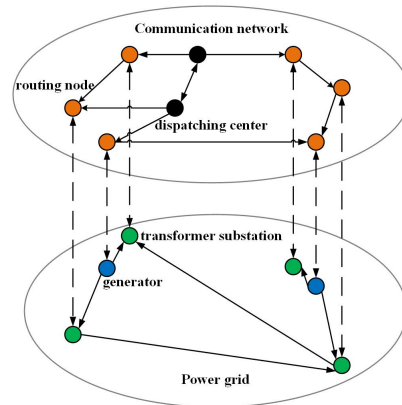


FIGURE 2. System model.

accurately, which reduces the complexity of the screening process.

## II. CRITICAL NODE GROUP EVALUATION ALGORITHM

### A. SYSTEM MODEL AND MATRICES

The system model used in this study is shown in Fig. 2. In the power grid, we simplify the considered equipment as follows: generators and substations are nodes, transmission lines are edges, and reactance values are the edge weights. In the communication network, we simplify dispatch centers as nodes, communication lines as edges, and link use rates as edge weights. The nodes of the power network and the communication network thus constitute a “one-to-one” coupling connection. The communication network uses the nodes with the highest degree as the primary and standby dispatch centers. These nodes do not connect with the power grid nodes directly. The energy and information flow on a coupled line is bidirectional. In addition, the power grid and the communication network each have different topological structures and node states.

Network control is an important research topic. To make a network tend to be controllable, we often choose how to control the nodes, but in many scenarios, it is unrealistic to control all the nodes in the network, particularly in large-scale networks. Therefore, to save costs, we can control the entire network using controllers on some nodes, and using coupling relationships, this strategy can achieve pinning control. We believe that nodes that can play an important role using a small number of controllers must be the most important group of nodes in the network. We thus focus on screening critical node groups in a network based on this concept. Based on pinning control theory [22]–[25], we assume that  $A$  is the power grid;  $B$  is the communication network;  $l_A$  and  $V^A = \{v_1^A, v_2^A, \dots, v_{l_A}^A\}$  are the number of controlled nodes and the set of controlled nodes in the power grid, respectively;  $l_B$  and  $V^B = \{v_1^B, v_2^B, \dots, v_{l_B}^B\}$  are the number of controlled nodes and the set of controlled nodes in the communication network, respectively; and  $S^A$  and  $S^B$  are the target states of the power grid and the communication network, respectively. Applying pinning control theory, the target states  $S^A$  and  $S^B$  of the power grid and the communication network nodes

tend to be consistent; then, the cyber-physical power system can achieve intralayer synchronization through the coupling relationship.

The numbers of nodes in the power grid and communication network are  $N_1$  and  $N_2$ , respectively, and their topology can be represented by adjacency matrices  $A = [a_{ij}]_{N_1 \times N_1}$  and  $B = [b_{ij}]_{N_2 \times N_2}$ . For the weighted directed cyber-physical power system described in this paper, the value of element  $a_{ij}$  in the power grid adjacency matrix  $A$  is:

$$\begin{cases} a_{ij} = \omega_{ij}^A, & \text{when node } i \text{ points to node } j \\ a_{ij} = 0, & \text{when node } i \text{ does not point to node } j \end{cases} \quad (1)$$

$\omega_{ij}^A$  is the weight of the power grid edge  $ij$ , and the adjacency matrix  $A$  is:

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1N_1} \\ \vdots & \ddots & \vdots \\ a_{N_1 1} & \cdots & a_{N_1 N_1} \end{bmatrix}_{N_1 \times N_1} \quad (2)$$

The value of element  $b_{ij}$  in the communication network adjacency matrix  $B$  is:

$$\begin{cases} b_{ij} = \omega_{ij}^B, & \text{when node } i \text{ points to node } j \\ b_{ij} = 0, & \text{when node } i \text{ does not point to node } j \end{cases} \quad (3)$$

where  $\omega_{ij}^B$  is the weight of the communication network edge  $ij$  and the adjacency matrix  $B$  is:

$$B = \begin{bmatrix} b_{11} & \cdots & b_{1N_2} \\ \vdots & \ddots & \vdots \\ b_{N_2 1} & \cdots & b_{N_2 N_2} \end{bmatrix}_{N_2 \times N_2} \quad (4)$$

The Laplacian matrix  $L_A = [l_{ij}^A]_{N_1 \times N_1}$  corresponding to the power grid can be calculated by the adjacency matrix  $A$ , and the value of the element  $l_{ij}^A$  is as follows:

$$\begin{cases} l_{ij}^A = -a_{ij}, & \text{when node } i \text{ points to node } j \\ l_{ii}^A = \sum_{j=1, j \neq i}^{N_1} a_{ij}, & i = j \end{cases} \quad (5)$$

The Laplace matrix  $L_A$  is:

$$L_A = \begin{bmatrix} l_{11}^A & \cdots & l_{1N_1}^A \\ \vdots & \ddots & \vdots \\ l_{N_1 1}^A & \cdots & l_{N_1 N_1}^A \end{bmatrix}_{N_1 \times N_1} = \begin{bmatrix} \sum_{j=2}^{N_1} a_{1j} & \cdots & -a_{1N_1} \\ \vdots & \ddots & \vdots \\ -a_{N_1 1} & \cdots & \sum_{j=1, j \neq N_1}^{N_1} a_{N_1 j} \end{bmatrix}_{N_1 \times N_1} \quad (6)$$

The Laplacian matrix  $L_B = [l_{ij}^B]_{N_2 \times N_2}$  corresponding to the communication network can be calculated by the adjacency matrix  $B$ , and the value of the element  $l_{ij}^B$  is as follows:

$$\begin{cases} l_{ij}^B = -b_{ij}, & \text{when node } i \text{ points to node } j \\ l_{ii}^B = \sum_{j=1, j \neq i}^{N_2} b_{ij}, & i = j \end{cases} \quad (7)$$

The Laplace matrix  $L_B$  is:

$$L_B = \begin{bmatrix} l_{11}^B & \cdots & l_{1N_2}^B \\ \vdots & \ddots & \vdots \\ l_{N_2 1}^B & \cdots & l_{N_2 N_2}^B \end{bmatrix}_{N_2 \times N_2} = \begin{bmatrix} \sum_{j=2}^{N_2} b_{1j} & \cdots & -b_{1N_2} \\ \vdots & \ddots & \vdots \\ -b_{N_2 1} & \cdots & \sum_{j=1, j \neq N_2}^{N_2} b_{N_2 j} \end{bmatrix}_{N_2 \times N_2} \quad (8)$$

For the convenience of subsequent descriptions, we let  $L$  be the Laplace matrix  $L_A$  or  $L_B$  corresponding to the power grid or the communication network, respectively, where  $L$  is a matrix of  $N$  rows and  $N$  columns, and  $N = N_1$  or  $N_2$ . The element in  $L$  is the reactance of the power grid or the link usage rate of the communication network, which characterizes the topological properties of power grids and communication networks:

$$\theta = \begin{bmatrix} w_{p_1} & \cdots & \cdots \\ \vdots & \ddots & \vdots \\ \cdots & \cdots & w_{p_{N-1}} \end{bmatrix} = \text{diag} \{w_{p_1}, \dots, w_{p_{N-1}}\} \quad (9)$$

where  $\theta$  is a diagonal matrix, and  $l$  is the number of controlled nodes in the power grid or communication network. All elements of this matrix are 0 except for along the diagonal. Additionally,  $P = \{p_1, \dots, p_{N-1}\}$  is an uncontrolled set of nodes among  $N$  nodes, and  $V = \{v_1, \dots, v_l\}$  is a controlled set of nodes among  $N$  nodes. In a directed weighted power grid or communication network, the element  $\{w_1, \dots, w_{p_{N-1}}\}$  is the sum of the weights of the edges, where each node in  $P$  points to a node in  $V$ . In the power grid, this expression is the sum of the reactance values of the edges. In the communication network, edge links are treated as the sum of utilization.

## B. SYSTEM RAYLEIGH QUOTIENT AND MINIMUM EIGENVALUE

The definition of the Rayleigh quotient  $R(x)$  for any vector  $x \in C$ , where  $C$  is the set of complex numbers, in the power grid and communication network is shown in Definition 1, where the maximum and minimum values correspond to the eigenvalues of the Laplacian matrix of the power grid and communication network, respectively, as shown in Theorem 1.

*Definition 1:* The Rayleigh quotient  $R_L(x)$  of  $L$  for any vector  $x$  satisfies the relation:  $R_L(x) = (x^H L x) / (x^H x)$  ( $x \neq 0$ ,  $x^H$  is the conjugate transpose of  $x$ ).

The Rayleigh quotient  $R_\theta(x)$  of  $\theta$  for any vector  $x$  satisfies the relation [26]:  $R_\theta(x) = (x^H \theta x) / (x^H x)$  ( $x \neq 0$ ,  $x^H$  is the conjugate transpose of  $x$ ).

In these quotients,  $x$  is a convenient vector for calculation, and its value is unrelated to the power network and communication network.

*Theorem 1:* For the Laplacian matrix  $L$  corresponding to the power grid or communication network, its nonzero eigenvalues satisfy  $\lambda_{\max} = \lambda_N \geq \lambda_{N-1} \geq \dots \geq \lambda_1 = \lambda_{\min}$  [27];

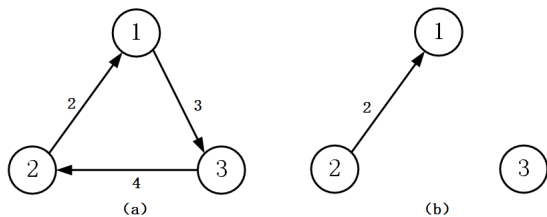


FIGURE 3. Comparison of two directed weighted graphs. (a) The first directed weighted graph. (b) The second directed weighted graph.

then, there are the following relationships:

$$\lambda_1 x^H x \leq x^H L x \leq \lambda_N x^H x \quad (\forall x \in C) \quad (10)$$

$$\lambda_{\max} = \lambda_N = \max_{x \neq 0} R_L(x) = \max_{x^H x = 1} x^H L x \quad (11)$$

$$\lambda_{\min} = \lambda_1 = \min_{x \neq 0} R_L(x) = \min_{x^H x = 1} x^H L x \quad (12)$$

Similarly, for the diagonal matrix  $\theta$ , its nonzero eigenvalue  $\lambda_{\max} = \lambda_N \geq \lambda_{N-1} \geq \dots \geq \lambda_1 = \lambda_{\min}$  also satisfies the above relationship, and its expression is as follows:

$$\lambda_1 x^H x \leq x^H \theta x \leq \lambda_N x^H x \quad (\forall x \in C) \quad (13)$$

$$\lambda_{\max} = \lambda_N = \max_{x \neq 0} R_{\theta}(x) = \max_{x^H x = 1} x^H \theta x \quad (14)$$

$$\lambda_{\min} = \lambda_1 = \min_{x \neq 0} R_{\theta}(x) = \min_{x^H x = 1} x^H \theta x \quad (15)$$

When the matrix  $\theta$  is in this relationship, the minimum nonzero eigenvalue  $\lambda_1$  can be the minimum reactance value in the power grid, and these link the rates used in the communication network.

The minimum nonzero eigenvalue of the Laplace matrix characterizes the connectivity of the power grid and the communication network. For example, in the two weighted directed graphs shown in Fig. 3, the minimum nonzero eigenvalue of Fig. 3(a) is calculated as -0.1256 according to Eq. (12), which is greater than the minimum nonzero eigenvalue of Fig. 3(b), i.e., -0.4142. Fig. 3 also shows that (a) is more connected than (b). Therefore, the node group with the largest minimum nonzero eigenvalue is optimal when comparing the same number of node groups.

**Definition 2:** For the graph  $G = (v, \epsilon)$  of the power grid or communication network,  $G' = (v', \epsilon')$  is a subgraph of  $G$ , where  $v' \in v, \epsilon' \in \epsilon$  are the points and edges of the power grid or communication network, respectively;  $H$  is a subgraph of  $G$  containing the uncontrolled node set  $P$ ;  $L$  is the Laplace matrix of the power grid or communication network; and  $L_{N-l}$  is the remaining matrix after deleting the corresponding rows and columns of the control node set  $V$  in the Laplacian matrix  $L$  from the power grid or communication network. The following relationship exists [28]:

$$L_{N-l} = L(H) + \theta \quad (16)$$

**Theorem 2:** The set of controlled nodes of the power grid or communication network is known to be  $V = \{v_1, \dots, v_l\}$ , the set of uncontrolled nodes is  $P = \{p_1, \dots, p_{N-l}\}$ , and the sum of the weights of the edges, which refer to the edges connecting the node in  $P$  pointing to the nodes in  $V$  in power

grids or communication grids, is  $\{w_1, \dots, w_{p_{N-l}}\}$ . Then, the minimum nonzero eigenvalue  $\lambda_1 (L_{N-l})$  corresponding to  $L_{N-l}$  of the power grid or communication network satisfies the following relationship:

$$\lambda_1 (L_{N-l}) \leq \frac{w_{p_1} + \dots + w_{p_{N-l}}}{N-l} \quad (17)$$

*Proof:* From Definition 2,  $H$  is a subgraph of  $G$  containing the uncontrolled node set  $P$  in the power grid or communication network.

We let  $x_0 = (1/(N-l)^{1/2}) * (1, 1, \dots, 1)^T$  be an  $(N-l) \times 1$  column vector. From Theorems 1 and 2, we deduce the following:

$$\begin{aligned} \lambda_1 (L_{N-l}) &= \min_{x^H x = 1} [x^T L(H)x + x^T \theta x] \\ &\leq x_0^T L(H)x_0 + x_0^T \theta x_0 \\ &= 0 + \left( \frac{1}{\sqrt{N-l}}, \dots, \frac{1}{\sqrt{N-l}} \right) * \begin{pmatrix} \frac{1}{\sqrt{N-l}} \\ \vdots \\ \frac{1}{\sqrt{N-l}} \end{pmatrix} \\ &= \frac{w_{p_1} + \dots + w_{p_{N-l}}}{N-l} \end{aligned} \quad (18)$$

### C. ALGORITHM TO REDUCE COMPUTATIONAL COMPLEXITY

Based on matrix analysis theory, we can begin the screening process to reduce the amount of calculation.

If the vital node group is directly evaluated by Theorem 2, the number of node combinations  $C_N^l$  is large, and selecting the optimal node group from these combinations would be time-consuming, particularly in large networks. Therefore, we use two screening processes to reduce the computational complexity.

#### 1) SINGLE NODE FILTER

The variable  $e$  represents the sum of the weights of some edges inside the controlled node set (i.e., the sum of the weights of the edges of the controlled nodes pointing to other controlled nodes in the power grid or communication network), and  $k_{v_i}^{in}$  represents the in-degree of the controlled node  $v_i$ , which represents the sum of the weights of the edges pointing to this node in the power grid or communication network. Then, the following holds:

$$= \frac{w_{p_1} + \dots + w_{p_{N-l}}}{N-l} \quad (19)$$

From Eqn. (18) and (19), we obtain:

$$\begin{aligned} \lambda_1 (L_{N-l}) &\leq \frac{k_{v_1}^{in} + k_{v_2}^{in} + \dots + k_{v_l}^{in} - e}{N-l} \\ &\leq \frac{k_{v_1}^{in} + k_{v_2}^{in} + \dots + k_{v_l}^{in}}{N-l} \end{aligned} \quad (20)$$

We can find a set of critical node groups through Eqn. (20) because only the total in-degree of nodes can satisfy this formula. This formula is not satisfied when the total in-degree of the formed node group is small, which may occur when



selecting  $l$  nodes for nodes with very small in-degree values in a node group, even if the first  $l - 1$  nodes with the largest in-degree combine with them. For example, we assume that there are four nodes with in-degrees of 1, 3, 4, and 5 in the network. When the number of control nodes is 3 and using the same  $\lambda_1$ , then selecting nodes with in-degrees of 3, 4, and 5 to calculate the right-hand side of the inequality of Eqn. (20) gives the minimum value that satisfies this equation. Even if these nodes combine with nodes in degrees 4 and 5, this nodal relationship in degree one cannot be met. The first screening excludes the nodes whose in-degree  $k^{in}$  does not satisfy Eqn. (21) in the power grid and communication network:

$$k^{in} \geq (N - l) \times \lambda_1^* - \sum_{i=1}^{l-1} k_i^{in} \quad (21)$$

where  $\lambda_1^*$  is the minimum nonzero eigenvalue of the modified Laplacian matrix in the current power grid or communication network. Eqn. (21) is the condition for screening a single node. If the number of nodes to be filtered out is  $a$ , then the number of combinations can be reduced by  $C_N^l - C_{N-a}^l$ , which shows that the first screening effectively reduces the amount of calculation. By screening the nodes through Eqn. (21), if the remaining nodes are combined, there will be two cases. One case occurs when nodes with smaller in-degree connect with the node with the largest in-degree. Then, the number of combinations is  $\sum_{i=1}^q \binom{C_m^i C_{N-a-m}^{l-i}}$ , where  $m$  is the number of nodes with small in-degree, and  $q = \min(l, m)$ . The second case occurs with a combination of nodes with a general in-degree, and the number of combinations is  $C_{N-a-m}^l$ . The total number of combinations is  $\sum_{i=1}^q \binom{C_m^i C_{N-a-m}^{l-i}} + C_{N-a-m}^l$ , which is much smaller than  $C_N^l$  before screening.

## 2) NODE GROUP FILTER

After the first screening, which combines nodes according to the number of pinned nodes  $l$ , the number of node groups will be significant. This situation is not conducive to screening essential node groups. Therefore, a second screening is conducted to reduce the number of node groups. First, we sort the nodes after the first screening according to the node in-degree to select the first  $l$  nodes. According to Eqn. (22), we can obtain  $\lambda^*$  as the basis for subsequent screening. \* in the upper right corner indicates the optimal  $\lambda$ . The corresponding value of the parameter of the node of  $\lambda^*$  is calculated as follows:

$$\lambda^* = \frac{k_{v_1}^{in*} + k_{v_2}^{in*} + \dots + k_{v_l}^{in*} - e^*}{N - l} \quad (22)$$

To determine whether there is a  $\lambda_1(L_{N-l})$  larger than  $\lambda^*$  in the critical node group under the same control node number  $l$ :

$$\lambda_1(L_{N-l}) \geq \lambda^* \quad (23)$$

$$\frac{k_{v_1}^{in} + k_{v_2}^{in} + \dots + k_{v_l}^{in} - e}{N - l} > \lambda^* \quad (24)$$

$$\frac{k_{v_1}^{in} + k_{v_2}^{in} + \dots + k_{v_l}^{in}}{N - l} > \lambda^* \quad (25)$$

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### Algorithm 1 Reduction of Computational Complexity

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**Require:**

Power grid/communication network node;

**Ensure:**

The most critical node group of the power grid/communication network;

- 1: For  $l \leftarrow 1 : N$  do
  - 2: Filter out nodes with low in-degree by Eqn.(21);
  - 3: Calculate  $\lambda^*$  by Eqn.(22);
  - 4: Try to find the node group that satisfies Eqn.(23);
  - 5: Filter by Eqn. (24);
  - 6: If there is a larger  $\lambda_1(L_{N-l})$ , then take it as the latest  $\lambda^*$  and repeat steps 3-5;
  - 7: Find the node group with the largest  $\lambda_1(L_{N-l})$ ;
  - 8: End
  - 9: Remove node groups with the same  $\lambda_1(L_{N-l})$  but larger than  $l$ ;
- 

If there is a larger  $\lambda_1(L_{N-l})$  afterward, then the current  $\lambda_1(L_{N-l})$  is considered to be the latest  $\lambda^*$ , and screening occurs again by the above steps. The final maximum  $\lambda_1(L_{N-l})$  is used as the optimal eigenvalue when the number of control nodes is  $l$ . Assuming that the number of control nodes is different, but the maximum nonzero eigenvalue is the same, then we select a group of critical nodes with a relatively small number of nodes while considering the cost and control difficulty.

The computational complexity will reduce significantly after two screenings, as follows:

If we do not filter, there are  $C_N^l$  node groups to evaluate together, the complexity is  $O(n^l)$ ,  $l$  is the number of pinning nodes, and its value range is an integer greater than or equal to 1. As the number of pinning nodes increases, the complexity will gradually increase. We can combine  $\sum_{i=1}^q \binom{C_m^i C_{N-a-m}^{l-i}} + C_{N-a-m}^l$  node groups after the first step of screening; to evaluate the critical node group, the complexity is  $O(\log n)$ . This result shows that with an increasing number of pinning nodes, the complexity after the first screening decreases below  $O(n^l)$ . In the second screening, if a larger  $\lambda_1(L_{N-l})$  cannot appear, only the node groups after the first screening to complete the evaluation of the critical node groups are also less than the number of node combinations  $C_N^l$  without filtering. The complexity is also less than  $O(n^l)$ . If there is a larger  $\lambda_1(L_{N-l})$  after the second screening, we can reduce the number of node combinations again according to steps 3-5 of Algorithm 1. Then, the complexity will be lower than  $O(\log n)$ .

The two screening steps to reduce the computational complexity into Algorithm 1 are summarized below.

Algorithm 1 finds the node group that maximizes the smallest nonzero eigenvalue of the power grid and communication network with the least computation. This node group can allow the network to maintain the best connectivity when the number of controlled nodes is  $l$ . Therefore, the most critical node group is considered under this number

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**Algorithm 2** Finding the Critical Node Groups of a Cyber-Physical Power System According to the Number of Pinned Nodes
 

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**Require:**

Directed weighted adjacency matrix for the power grid/communication network;

**Ensure:**

Optimal pinning node group  $V = \{v_1, \dots, v_l\}$  of the cyber-physical power system;

- 1: for  $l \leftarrow 1 : N$  do
  - 2: Select the first  $l$  nodes according to the in-degree ranking of the power grid/communication network, and calculate  $\lambda^*$ ;
  - 3: Screen the power grid/communication network through Algorithm 1;
  - 4: Combine critical node groups of power grids and communication grids;
  - 5: End
  - 6: Remove node groups with the same  $\lambda_1 (L_{N-l})$  but larger than 1;
- 

of controlled nodes. According to the network size, control, and computational difficulty requirements investigated in this study, we only observe node groups with five or fewer nodes in the single-layer network and then combine the critical node groups of the two-layer network. The most critical node group in the power network can affect the power network with the largest range and can affect some nodes in the communication network through the coupling relationship between the power network and the communication network. However, it is not necessarily the communication network with the largest range. In the same way, the most critical node group in the communication network can affect the communication network with the largest range. The coupling relationship can also affect some power network nodes, but it is not necessarily the power network with the largest range. Combining the node groups can maximize the influence of the cyber-physical power system under the number of pinning nodes and achieve a better result. Similarly, we compare the size of the smallest nonzero eigenvalue after the combination. If the minimum nonzero eigenvalues of multiple groups of nodes are the same, we select a smaller number of nodes as the current optimal node group.

#### D. CRITICAL NODE GROUP ALGORITHM FOR THE CYBER-PHYSICAL POWER SYSTEM

We will now introduce the general steps to judge the critical node groups of the cyber-physical power system, which are described in Algorithm 1.

Because the focus of this study is the selection strategy of pinning nodes and the optimization of the pinning control strategy, the network can still maintain good connectivity when controlling fewer nodes. The smallest nonzero eigenvalue of the Laplacian matrix characterizes the network's connectivity, indicating that the larger the eigenvalue is, the better the connectivity. Therefore, an eigenvalue can be used

to evaluate the node selection strategy. Next, some matrix analysis theories are used to facilitate the proposed derivation of the control node group to reduce the amount of calculation.

### III. EXPERIMENTAL ANALYSIS

In this section, the IEEE-30 node system and the IEEE-118 node system are used to create the power network and are coupled with two corresponding scale-free communication networks for simulation. The power network also determines whether the cost of two more nodes paid by the communication network is worthwhile. The simulation analysis shows the effectiveness of the proposed algorithm in reducing the number of required calculations. Compared with other critical node group evaluation methods, when selecting the same number of node sets, the algorithm in this paper can make the network have better connectivity. In addition, the optimal pinning node group will have different results under different coupling methods.

#### A. DISCUSSION OF CRITICAL NODE GROUPS

First, the evaluation of the essential node groups is observed in the small-scale cyber-physical power system, where  $\lambda_A$ ,  $\lambda_B$  and  $\lambda$  are the minimum non-zero eigenvalues of the power grid, the communication network, and the cyber-physical power system, respectively; and the same is true for the latter table.

In the power grid layer with one control node, the most critical node group is  $\{30\}$ , and the corresponding  $\lambda_A = -0.0655$ . With two control nodes,  $\{10, 30\}$  is selected as the critical node group, and the corresponding  $\lambda_A = -0.0653$ . Using the proposed algorithm and taking these values as the training index and screening groups, respectively, the node group  $\{27, 29\}$  yields the smallest nonzero eigenvalue to be the largest value, which is  $\lambda_A = -0.0643$ . Therefore, this node group is optimal when the number of control nodes is two. With three pinning control nodes,  $\{10, 27, 30\}$  is selected as the node group for the first training, and the minimum nonzero eigenvalue is  $\lambda_A = -0.0641$ . Next, the proposed algorithm is used, and there are no node combinations with larger eigenvalues. Therefore,  $\{10, 27, 30\}$  is the critical node group when the number of pinning control nodes is three. With four pinning control nodes, the optimal node group derived by the proposed method is  $\{10, 24, 27, 30\}$ , and the minimum nonzero eigenvalue is  $\lambda_A = -0.0559$ . With five pinning control nodes, the optimal node group is  $\{10, 24, 27, 29, 30\}$ , which has the same minimum nonzero eigenvalue as the node group with fewer nodes. This result indicates that the connectivity effect brought by the two groups of nodes is the same. Thus, with four or five control nodes,  $\{10, 24, 27, 30\}$  is the best group. These conditions are shown in Table 1.

Based on the same concept, the first communication network layer has two more nodes than the power network. In the communication network, these additional nodes can act as active and standby dispatch centers, which are responsible for the safe operation and economic dispatch of the entire

**TABLE 1. Critical node groups in the power grid (IEEE-30).**

l	node-set	$\lambda_A$
1	{30}	-0.0655
2	{27, 29}	-0.0643
3	{30, 10, 27}	-0.0641
4	{30, 10, 27, 24}	-0.0559
5	{30, 10, 27, 24, 29}	-0.0559(not)

**TABLE 2. Critical node groups in the communication network (scale-free network with 32 nodes).**

l	node-set	$\lambda_B$
1	{3}	-0.0094
2	{3, 5}	-0.0082
3	{3, 5, 15}	0.0012
4	{3, 5, 15, 2}	0.0012(not)
5	{3, 5, 15, 2, 4}	0.0012(not)

**TABLE 3. Critical node groups in the communication network (scale-free network with 32 nodes).**

l	node-set	$\lambda_B$
1	{3}	-0.2952
2	{3, 4}	-0.2948
3	{3, 4, 7}	-0.0604
4	{3, 4, 7, 6}	-0.0604(not)
5	{3, 4, 7, 6, 18}	-0.0604(not)

power grid according to the prescribed power generation plan and monitoring principles. These nodes, which are not coupled directly with the power grid to prevent the occurrence of faults and affect functionality, can still typically operate. With one control node, the node group is {3}, and the minimum nonzero eigenvalue is  $\lambda_B = -0.0094$ . With two control nodes, the critical node group is {3, 5}, and the minimum nonzero eigenvalue is  $\lambda_B = -0.0082$ . With three control nodes, the optimal node group is {3, 5, 15}, and the minimum nonzero eigenvalue is  $\lambda_B = 0.0012$ . With four control nodes, the minimum nonzero eigenvalue is always maintained at 0.0012. Thus, with three, four or five control nodes, the node group when the number of control nodes is three is selected (i.e., {3, 5, 15}). These conditions are shown in Table 2 below. Similarly, the second communication network (i.e., the number of nodes in the communication network) is the same as that of the power grid and includes active and standby dispatch centers, which are coupled “one-to-one” with the power grid. These results are shown in Table 3.

From these results for the power grid layer, the critical node group does not add nodes to the original node group as the number of nodes increases and changes. Due to its scale-free characteristics, the degree difference between nodes is very large for the communication network layer. In this case, the nodes that contribute the most to the network connectivity are usually those with large degrees; thus, the node increment increases. However, we must still choose the critical node group because the number of control nodes is different, and the minimum nonzero eigenvalue is the same. A greedy algorithm cannot be used to evaluate the critical node group directly. Previous results show that the larger the minimum nonzero eigenvalue is, the better the connectivity of the network. In comparing the two communication networks, the

**TABLE 4. Critical node groups in the cyber-physical power system (IEEE-30 and 32 scale-free “one-to-one” connection).**

l	node-set	$\lambda$
2	{30}, {3}	-0.0568
4	{27, 29}, {3, 5}	-0.0556
6	{30, 10, 27}, {3, 5, 15}	-0.0555
7	{30, 10, 27, 24}, {3, 5, 15}	-0.0480

**TABLE 5. Critical node groups in the cyber-physical power system (IEEE-30 and 30 scale-free “one-to-one” connection).**

l	node-set	$\lambda$
2	{30}, {3}	-0.2681
4	{27, 29}, {3, 4}	-0.2677
6	{30, 10, 27}, {3, 4, 7}	-0.0509
7	{30, 10, 27, 24}, {3, 4, 7}	-0.0509(not)

connectivity of the second type of communication network is not as good as that of the first. Therefore, in communication network modeling, the cost of two more nodes is worthwhile. This conclusion must be verified in a cyber-physical power system.

We now evaluate the critical node groups in the cyber-physical power system. First, the power grid is coupled with the first communication network. When each network layer has a pinning node, the minimum nonzero eigenvalue is  $\lambda = -0.0568$ . With two pinning nodes, this value is  $\lambda = -0.0556$ ; when each network layer has three pinning nodes, the minimum nonzero eigenvalue is  $\lambda = -0.0555$ . With four nodes each, the communication network can only have three nodes to form a critical node group when combined because the optimal number of nodes in the communication network after the comparison in Table 2 is 3, and the minimum nonzero eigenvalue is  $\lambda = -0.0480$ . With 10 control nodes, the power grid can only have four nodes, and the communication network can only have three nodes. The combined effect is the same as that in the previous case, which shows that as the number of control nodes increases, the minimum nonzero eigenvalue also increases. However, because it is impractical to control many nodes when selecting a node group, it is necessary to choose a node group according to the requirements. This situation is described in Table 4. Similarly, the coupling results with the second communication network are shown in Table 5.

Comparing Tables 4 and 5 shows that the connectivity of the second type of cyber-physical power system is worse than that of the first. These results again indicate that the price paid by two nodes yields better network connectivity while avoiding failures that affect functionality. In the same way, the evaluation of essential node groups of large-scale cyber-physical power systems uses the same method. Although no descriptions are provided in this study, similar results are observed, and the evaluation of critical node groups in the cyber-physical power system is shown in Tables 6-10.

The two communication networks are also in the same situation in the large-scale cyber-physical power system, which shows that the price paid by the two additional communication network nodes is still beneficial. Therefore, we continue to use the first communication network for

**TABLE 6.** Critical node groups in the power grid (IEEE-118).

l	node-set	$\lambda_A$
1	{106}	-0.0527
2	{70, 106}	-0.0520
3	{24, 66, 106}	-0.0449
4	{17, 24, 66, 106}	-0.0447
5	{24, 62, 86, 106, 113}	-0.0403

**TABLE 7.** Critical node groups in the communication network (scale-free network of 120 nodes).

l	node-set	$\lambda_B$
1	{120}	-0.0052
2	{5, 3}	-0.0035
3	{5, 3, 2}	-0.0035(not)
4	{5, 3, 2, 4}	-0.0034
5	{5, 3, 2, 4, 23}	-0.0029

**TABLE 8.** Critical node groups in the communication network (scale-free network of 118 nodes).

l	node-set	$\lambda_B$
1	{2}	-0.7642
2	{2, 7}	-0.4793
3	{2, 7, 1}	-0.4793(not)
4	{2, 7, 1, 22}	-0.3782
5	{2, 7, 1, 22, 3}	-0.3782(not)

**TABLE 9.** Critical node groups in the cyber-physical power system (IEEE-118 and 120 scale-free “one-to-one” connection).

l	node-set	$\lambda$
2	{106}, {120}	-0.0460
4	{70, 106}, {5, 3}	-0.0439
5	{24, 66, 106}, {5, 3, 2}	-0.0412
8	{17, 24, 66, 106}, {5, 3, 2, 4}	-0.0412(not)
10	{24, 62, 86, 106, 113}, {5, 3, 2, 4, 23}	-0.0347

connection purposes in future research. The number of nodes in a single-layer network is different, but the minimum nonzero eigenvalue is the same, which may also occur in the cyber-physical power system. As shown in Table 9, when the total number of control nodes is 5 and 8, the minimum nonzero eigenvalues are the same; thus, we investigate the case where the total number of control nodes is 5.

In the cyber-physical power system, the difference in the importance of the node group composed of all nodes of the power grid and the communication network is shown in Tables 11 and 12.

These two tables show that the minimum nonzero eigenvalue corresponding to the node group composed of all power grid nodes is larger than that composed of the communication network in a small-scale or large-scale cyber-physical power system. This result shows that the node group consisting of all power grid nodes is more important than the communication network node group.

We are taking a small-scale cyber-physical power system as an example if we are looking for critical node groups in the cyber-physical power system from an overall perspective. The results show in Table 13.

Comparing Table 13 and Table 4, When the number of pinning nodes is 2 and 4, the minimum non-zero eigenvalues obtained by this method are smaller than that of the way in this paper, which indicates that the connectivity of the

**TABLE 10.** Critical node groups in the cyber-physical power system (IEEE-118 and 118 scale-free “one-to-one” connection).

	node-set	$\lambda$
2	{106}, {2}	0.7605
4	{70, 106}, {2, 7}	0.4670
5	{24, 66, 106}, {2, 7}	0.4670(not)
8	{17, 24, 66, 106}, {2, 7, 1, 22}	0.3686
9	{24, 62, 86, 106, 113}, {2, 7, 1, 22}	0.3686(not)

**TABLE 11.** Comparison of the importance of node groups composed of all nodes in power grid and communication network (IEEE-30 and 32 scale-free “one-to-one” connection).

network	$\lambda$
power grid	-0.0086
communication network	-0.1952

**TABLE 12.** Comparison of the importance of node groups composed of all nodes in power grid and communication network (IEEE-118 and 120 scale-free “one-to-one” connection).

network	$\lambda$
power grid	-0.0026
communication network	-0.0683

**TABLE 13.** Find critical node groups from an overall perspective (IEEE-30 and 32 scale-free “one-to-one” connection).

l	node-set	$\lambda$
2	{27, 29}	-0.0643
4	{30, 10, 27, 24}	-0.0559
6	{30, 10, 27, 24, 29, 26}	-0.0426
8	{30, 10, 27, 24, 29, 26, 15, 25}	-0.0423
10	{30, 10, 27, 24, 29, 26, 15, 25, 20, 17}	-0.0330

network is not as good as the method in this paper. In the case of many pinning nodes, this method can make the network connectivity more dominant. However, we can find that the nodes in the critical node group of this method are all from the power grid under the same number of pinning nodes. If the power grid is severely damaged, we cannot know which nodes in the communication network could be controlled to improve the connectivity. Therefore, the method in this paper is more effective.

**B. DISCUSSION OF THE PROPOSED ALGORITHM TO REDUCE THE NUMBER OF CALCULATIONS**

Algorithm 1 primarily reduces the number of calculations by performing two screenings in Steps 2 and 5. Step 2 of Algorithm 1 can reduce the number of nodes and simplify the calculation only for a single node’s in-degree. However, Step 5 of Algorithm 1 can reduce the number of node combinations and increase the accuracy, and the two screening processes can reduce the amount of calculation. Tables 14-17 show the results of reducing the number of calculations.

As shown in Tables 14-15, the number of nodes in the power grid is either 30 or 118 before passing through Step 2 of Algorithm 1. The number of node groups decreases after filtering, e.g., in a 120-node scale-free communication network, when the number of control nodes is small (i.e.,  $l = 2$ ). Although the reduced number of nodes is only two, the number of subsequent node combinations will be reduced by  $C_{120}^2 - C_{118}^2 = 237$ , which contributes to a reduction in the amount of computation. As shown in Tables 16-17



TABLE 14. Results after the first screening of the power grid (IEEE-30/IEEE-118).

l	l=1	l=2	l=3	l=4	l=5
number of nodes(30/118)	29/117	28/116	27/115	26/114	25/113

TABLE 15. Results after the first screening of the communication network (32-node/120-node).

l	l=1	l=2	l=3	l=4	l=5
number of nodes(32/120)	31/119	30/118	29/117	28/116	27/115

TABLE 16. Results after the second screening of the power grid (IEEE-30/IEEE-118).

l	l=1	l=2	l=3	l=4	l=5
number of nodes(30/118)	29/117	378/ 6670	2925/ $2.47 \times 10^5$	14950/ $6.61 \times 10^6$	$10/1.4 \times 10^8$

TABLE 17. Results after the second screening of the communication network (32-node/120-node).

l	l=1	l=2	l=3	l=4	l=5
number of nodes(32/120)	31/119	435/6903	665/ $2.60 \times 10^5$	277/ $7.01 \times 10^6$	$1/1.53 \times 10^8$

TABLE 18. Critical node groups in the cyber-physical power system (IEEE-30 and 32 scale-free “multiple-to-multiple” connection).

l	node-set	$\lambda$
2	{30},{3}	-0.0449
4	{27, 29},{3, 5}	-0.0436
6	{30, 10, 27},{3, 5, 15}	-0.0433
9	{30, 10, 27, 24},{3, 5, 15}	-0.0355

for the IEEE-30 power grid and the 32-node communication network, the number of node groups after screening gradually decreases with an increasing number of pinned nodes. This result likely occurs because the fundamental node group with the largest in-degree may be critical in itself with this number of restrained nodes. Therefore, if its  $\lambda$  is larger, there will be fewer remaining node groups that meet the second filter condition. After passing through Step 5 of Algorithm 1, the number of node combinations decreases even further (e.g., in the IEEE-30 power grid). With five pinning control nodes, the number of node combinations is reduced by  $C_{30}^5 - 10 = 142506$  groups after the second screening, which markedly reduces the number of calculations. These results also show that the proposed algorithm effectively reduces the number of calculations and screens many node combinations with low values.

C. COMPARISON WITH OTHER CRITICAL NODE EVALUATION METHODS

In this section, we compare the proposed method with the other three methods:

- 1) Betweenness centrality (BC): This strategy uses the betweenness of a node as an indicator and considers that the more times a node acts as an intermediary node, the more critical it is;
- 2) Degree centrality (DC): This strategy uses the degree of the node as an indicator and considers that the greater the degree of the node, the more critical it is; and
- 3) Information entropy strategy (S): This strategy considers that the greater the probability of a node being a neighbor node by other nodes, the more critical it is;

Figs. 4 and 5 visually compare the usage of the four strategies on the cyber-physical power system to demonstrate the

superior performance of the proposed algorithm when evaluating critical node groups. These figures show that the minimum nonzero eigenvalue obtained by the proposed method is the largest with different numbers of pinning control nodes, whether a small-scale or a large-scale cyber-physical power system is used. Also, the network’s connectivity is optimal, reflecting the proposed algorithm’s superiority. The other methods mentioned in this article can find in single-node importance evaluation articles that perform well in the single-node evaluation and have their basis. Nevertheless, the figure shows that when using these excellent single node importance detection algorithms to assess critical node groups, they cannot make the network have excellent connectivity. Because these methods do not consider the common influence of nodes, their influence nodes may have influenced nodes multiple times. They do not delete the number of affected nodes from the statistics, which creates the illusion that the node group has a lot of influence. If it affects many nodes repeatedly, there will jointly affect fewer nodes, and the affected range will be smaller, which cannot optimize connectivity. All single-node evaluation strategies may face this problem in critical node group evaluation. Even if a method performs optimally in a single-node evaluation strategy, it will face this problem. The effect of the critical node group evaluation may be the same as this paper, but it will not exceed. In addition, these figures show that within a certain number of control nodes, and an increasing number of pinned nodes, the minimum nonzero eigenvalue remains unchanged or increases accordingly. This result shows that the more control nodes there are, the more connected the network. However, it is unrealistic to control a large number of control nodes. Therefore, the number of nodes cannot be deduced when selecting a pinning node, which increases the cost and control difficulty.

D. EVALUATION OF CRITICAL NODE GROUPS UNDER DIFFERENT COUPLING METHODS

The impact of different coupling methods on evaluating critical node groups in the cyber-physical power system is now

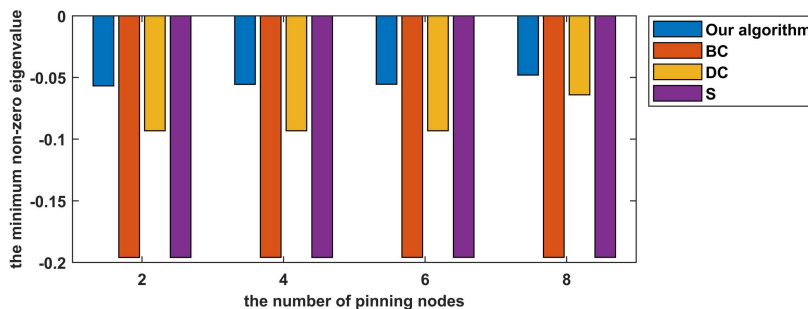


FIGURE 4. Comparison chart of the minimum nonzero eigenvalue trend based on the number of pinned nodes from 2 to 8 using different strategies on the “30-32 scale-free” cyber-physical power system.

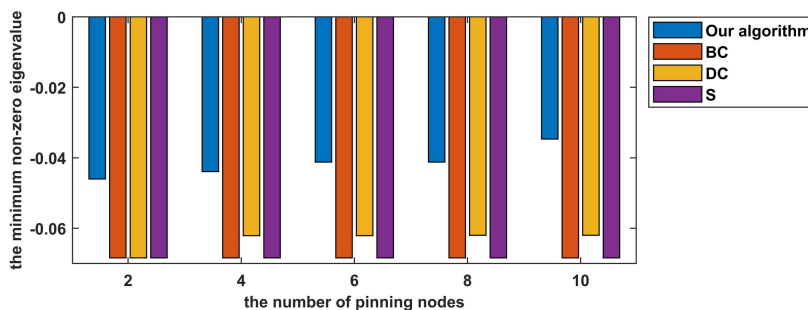


FIGURE 5. Comparison chart of the minimum nonzero eigenvalue trend based on the number of pinned nodes from 2 to 10 using different strategies on the “118-120 scale-free” cyber-physical power system.

described. First, we list the cases when the coupling mode is “multiple-to-multiple”, as shown in Tables 18 and 19.

These tables show that compared with the “one-to-one” coupling connection of critical node groups, the “multiple-to-multiple” coupling method is more effective in both large- and small-scale cyber-physical power systems. This result indicates that using the “multiple-to-multiple” coupling connection can improve network connectivity.

**E. VARIATION TREND OF MINIMUM NON-ZERO EIGENVALUE WITH THE NUMBER OF PINNED NODES**

For an adjacency matrix  $T$  of a power network or communication network, its characteristic equation is  $|T - \lambda E| = 0$ , where  $\lambda$  is all eigenvalues of  $T$  and  $E$  is the identity matrix of the same dimension as  $T$ . The minimum nonzero eigenvalue  $\lambda_1$  of the modified Laplacian matrix  $L_{N-l}$  of the power network or communication network can be expressed as:

$$\lambda_1 = \min \{ \lambda | |L_{N-l} - \lambda E| = 0, \lambda \neq 0 \} \quad (26)$$

Considering  $L = D_{in} - T$ , where  $D_{in}$  is the diagonal matrix composed of the in-degrees of all nodes of the power grid or communication network and  $L_{N-l} = L(H) + \theta$  of definition 2 into Eqn. (26), the following can be deduced:

$$\begin{aligned} \lambda_1 &= \min \{ \lambda | |L(H) + \theta - \lambda E| = 0, \lambda \neq 0 \} \\ &= \min \{ \lambda | |D'_{in} - T' + \theta - \lambda E| = 0, \lambda \neq 0 \} \end{aligned} \quad (27)$$

where  $D'_{in}$  and  $T'$  are the parts that remain after deleting the corresponding rows and columns of the  $D_{in}$  and  $T$  node groups (i.e., deleting the rows and columns corresponding

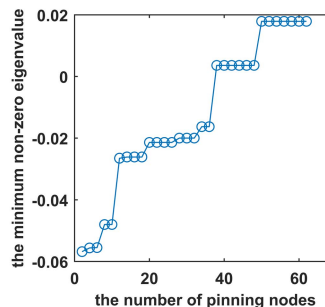


FIGURE 6. Process of changing the minimum nonzero eigenvalue with the change in the number of pinned nodes (IEEE-30 and 32 scale-free “multiple-to-multiple”).

to the controlled nodes of the power grid or communication network).

This section considers a small-scale cyber-physical power system with “multiple-to-multiple” coupling connections. We use Algorithm 1 to determine the critical node groups of the power grid and communication network. Then, these groups are combined and introduced into Eqn. (27) to calculate the minimum nonzero eigenvalue. Fig. 6 shows how the minimum nonzero eigenvalue changes with the number of pinned nodes.

Fig. 6 shows that as the number of pinned nodes increases, the minimum nonzero eigenvalue gradually increases or remains unchanged. The more pinned nodes there are, the less efficient the change in the minimum nonzero eigenvalue. The more nodes there are, the better the network’s connectivity; the connectivity may also eventually stabilize. The minimum

**TABLE 19.** Critical node groups in the cyber-physical power system (IEEE-118 and 120 scale-free “multiple-to-multiple” connection).

$l$	node-set	$\lambda$
2	{106},{120}	-0.0442
4	{70,106},{5,3}	-0.0421
5	{24,66,106},{5,3}	-0.0361
8	{17,24,66,106},{5,3,2,4}	-0.0361(not accepted)
10	{24,62,86,106,113},{5,3,2,4,23}	-0.0324

nonzero eigenvalue remains unchanged because a single power grid and a single communication network have the same minimum nonzero eigenvalues for a given number of pinned nodes. This result differs from other numbers of pinned nodes with the same network connectivity. For this given number of pinned nodes, selecting a group with the least number of pinned nodes under the same nonzero eigenvalue reduces the control cost and difficulty. When the minimum nonzero eigenvalue of the cyber-physical power system is -0.0261, three groups of nodes use the same group of nodes as the optimal pinning control node group because the number of restraining nodes in the power grid and the communication network is 7, 8, and 9, which all have the same nonzero eigenvalues. This result means that the same group of nodes will be used as the optimal number of nodes. When the minimum nonzero eigenvalue is -0.0214, there will be four groups using the same node group. When the minimum nonzero eigenvalue is -0.02, there will be two groups using the same optimal node group. When the minimum nonzero eigenvalue is 0.0036, there will be six groups using the same node group. When the minimum nonzero eigenvalue is 0.0045, there will be seven groups using the same node group. Each of these nonzero eigenvalues is similar to the previous group; therefore, their changes can be ignored.

#### IV. CONCLUSION

We propose an algorithm based on pinning control theory to evaluate critical node groups in a directed weighted cyber-physical power system. This algorithm uses the smallest nonzero eigenvalue of the modified Laplace matrix as the evaluation index with the help of matrix analysis theory. After many simulations, the conclusions are as follows:

- 1) Computational complexity is markedly reduced through the two screening processes in the algorithm proposed in this paper. The critical node groups in the directed weighted cyber-physical power system can be more accurately determined compared with other essential node evaluation strategies.
- 2) The active and standby dispatch center nodes do not directly couple with the power grid of the two communication networks. Although there is a cost when adding two nodes as the active and standby dispatch centers, better results are achieved. The node group composed of all nodes of the power grid is more important than the node group consisting of all the nodes of the communication network large- and small-scale cyber-physical power systems.

- 3) As the number of control nodes increases, the connectivity of the cyber-physical power system improves until it tends to stabilize. Compared with the “one-to-one” coupling connection method, the “multiple-to-multiple” coupling connection method can result in better connectivity of the cyber-physical power system under a node group with the same number of nodes.

In future work, we plan to use more methods to reduce computational complexity in a larger-scale directed weighted cyber-physical power system.

#### REFERENCES

- [1] R. Zhou, M. Peng, and X. Gao, “Vulnerability assessment of power cyber-physical system considering nodes load capacity,” in *Proc. 6th Int. Conf. Intell. Comput. Signal Process. (ICSP)*, Apr. 2021, pp. 1438–1441.
- [2] Z. Zhang, W. An, and F. Shao, “Cascading failures on reliability in cyber-physical system,” *IEEE Trans. Rel.*, vol. 65, no. 4, pp. 1745–1754, Dec. 2016.
- [3] Z. Huang, C. Wang, M. Stojmenovic, and A. Nayak, “Characterization of cascading failures in interdependent cyber-physical systems,” *IEEE Trans. Comput.*, vol. 64, no. 8, pp. 2158–2168, Aug. 2015.
- [4] Y. Zhang and O. Yagan, “Robustness of interdependent cyber-physical systems against cascading failures,” *IEEE Trans. Autom. Control*, vol. 65, no. 2, pp. 711–726, Feb. 2020.
- [5] Y. Chen, Y. Li, W. Li, X. Wu, Y. Cai, Y. Cao, and C. Rehtanz, “Cascading failure analysis of cyber physical power system with multiple interdependency and control threshold,” *IEEE Access*, vol. 6, pp. 39353–39362, 2018.
- [6] P. Sun and Y. Dong, “On vulnerability analysis of nodes against cross-domain cascading failures propagation in active distribution network cyber-physical system,” in *Proc. 7th Int. Conf. Dependable Syst. Their Appl. (DSA)*, Nov. 2020, pp. 355–363.
- [7] F. Liu, G. Xie, and Z. Zhao, “Identification of important nodes in power-information coupled network,” in *Proc. Chin. Autom. Congr. (CAC)*, Nov. 2020, pp. 6329–6334.
- [8] H. Liu, S. Luo, J. Gan, and Y. Wu, “Research on node importance of power communication network based on multi-attribute analysis,” in *Proc. IEEE 4th Inf. Technol., Netw., Electron. Autom. Control Conf. (ITNEC)*, Jun. 2020, pp. 2683–2687.
- [9] Z. Min, W. Muqing, Q. Lilin, A. Quanbiao, and L. Sixu, “Evaluation of cross-layer network vulnerability of power communication network based on multidimensional and multi-layer node importance analysis,” *IEEE Access*, early access, Sep. 3, 2021, doi: 10.1109/ACCESS.2021.3109902.
- [10] F. Liu, B. Xiao, H. Li, and J. Xue, “Complex network node centrality measurement based on multiple attributes,” in *Proc. 10th Int. Conf. Model., Identificat. Control (ICMIC)*, Jul. 2018, pp. 1–5.
- [11] S. Wang and J. Zhao, “Multi-attribute integrated measurement of node importance in complex networks,” *Chaos, Interdiscipl. J. Nonlinear Sci.*, vol. 25, no. 11, Nov. 2015, Art. no. 113105.
- [12] M. Zhou, L. Rui, X. Qiu, Z. Xia, and B. Li, “Evaluation of the node importance in power grid communication network and analysis of node risk,” in *Proc. IEEE/IFIP Netw. Oper. Manage. Symp. (NOMS)*, Apr. 2018, pp. 1–5.
- [13] F. Luo, Y. Qiu, C. Zheng, and Y. Liu, “Critical node identification of power wireless private communication network based on complex network,” in *Proc. IEEE 3rd Adv. Inf. Manage., Communicates, Electron. Autom. Control Conf. (IMCEC)*, Oct. 2019, pp. 1605–1609.
- [14] L. M. Pecora and T. L. Carroll, “Master stability functions for synchronized coupled systems,” *Phys. Rev. Lett.*, vol. 80, p. 2109, Mar. 1998.

- [15] J. Wu and Y.-J. Tan, "Finding the most vital node by node contraction in communication networks," in *Proc. Int. Conf. Commun., Circuits Syst.*, 2005, p. 1286.
- [16] S. Guo, S. Miao, H. Yin, Z. Wang, and J. Han, "Construction of coupling model and evaluation of node importance for electric sensor communication network," in *Proc. IEEE 1st China Int. Youth Conf. Electr. Eng. (CIYCEE)*, Nov. 2020, pp. 1–6.
- [17] R. Zhou, M. Peng, and X. Gao, "Vulnerability assessment of power cyber-physical system considering nodes load capacity," in *Proc. 6th Int. Conf. Intell. Comput. Signal Process. (ICSP)*, Apr. 2021, pp. 1438–1441.
- [18] W. Liu, W. Gu, W. Sheng, X. Meng, S. Xue, and M. Chen, "Pinning-based distributed cooperative control for autonomous microgrids under uncertain communication topologies," *IEEE Trans. Power Syst.*, vol. 31, no. 2, pp. 1320–1329, Mar. 2016.
- [19] S. Manaffam, M. Talebi, A. K. Jain, and A. Behal, "Intelligent pinning based cooperative secondary control of distributed generators for microgrid in islanding operation mode," *IEEE Trans. Power Syst.*, vol. 33, no. 2, pp. 1364–1373, Mar. 2018.
- [20] J. Liu, X. Lu, C. Chen, and B. Chen, "Leader selection in robust pinning-based distributed control for islanded microgrids," in *Proc. IEEE Energy Convers. Congr. Exposit. (ECCE)*, Sep. 2019, pp. 5411–5416.
- [21] H. Zhu, K. Liu, and J. Lu, "Cooperative pinning synchronization of a class of undirected complex networks," in *Proc. 34th Chin. Control Conf. (CCC)*, Jul. 2015, pp. 6861–6865.
- [22] Y.-W. Wang and C. Wen, "A survey on pinning control of complex dynamical networks," in *Proc. 10th Int. Conf. Control, Autom., Robot. Vis.*, Dec. 2008, pp. 64–67.
- [23] C. J. Vega, E. N. Sanchez, and O. J. Suarez, "Discrete-time neural sliding-mode pinning control for synchronization of complex networks," in *Proc. 16th Int. Conf. Electr. Eng., Comput. Sci. Autom. Control (CCE)*, Sep. 2019, pp. 1–5.
- [24] T. Yu, H. Wang, J. Cao, and Y. Yang, "On impulsive synchronization control for coupled inertial neural networks with pinning control," *Neural Process. Lett.*, vol. 51, no. 3, pp. 2195–2210, Jun. 2020.
- [25] B. Li, J. Lu, Y. Liu, and Z.-G. Wu, "The outputs robustness of Boolean control networks via pinning control," *IEEE Trans. Control Netw. Syst.*, vol. 7, no. 1, pp. 201–209, Mar. 2020.
- [26] Q. T. Zhang, "A simple approach to probability of error for equal gain combiners over Rayleigh channels," *IEEE Trans. Veh. Technol.*, vol. 48, no. 4, pp. 1151–1154, Jul. 1999.
- [27] T. Blaszczyk, "Application of the Rayleigh-Ritz method to solve a class of fractional variational problem," in *Proc. 20th Int. Conf. Methods Models Autom. Robot. (MMAR)*, Aug. 2015, pp. 83–86.
- [28] R. Diestel, *Graph Theory* (Graduate Texts in Mathematics), 2nd ed. New York, NY, USA: Springer, 2000.



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