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Zebra Optimization Algorithm: A New Bio-Inspired Optimization Algorithm for Solving Optimization Algorithm

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ABSTRACT In this paper, a new bio-inspired metaheuristic algorithm called Zebra Optimization Algorithm (ZOA) is developed; its fundamental inspiration is the behavior of zebras in nature. ZOA simulates the foraging behavior of zebras and their defense strategy against predators' attacks. The ZOA steps are described and then mathematically modeled. ZOA performance in optimization is evaluated on sixty-eight benchmark functions, including unimodal, high-dimensional multimodal, fixed-dimensional multimodal, CEC2015, and CEC2017. The results obtained from ZOA are compared with the performance of nine well-known algorithms. The simulation results show that ZOA can solve optimization problems by creating a suitable balance between exploration and exploitation and has a superior performance compared to nine competitor algorithms. ZOA's ability to solve real-world problems has been tested on four engineering design problems, namely, tension/compression spring, welded beam, speed reducer, and pressure vessel. The optimization results show that ZOA is an effective optimizer in determining the values of the design variables of these problems compared to the nine competitor algorithms.

INDEX TERMS Bio-inspired, exploitation, exploration, engineering design, optimization, zebra.

I. INTRODUCTION

The optimization problem is a problem that has more than one feasible solution, and optimization is the process of achieving the best solution among all the available solutions to this problem. Each optimization problem is determined by its decision variables, constraints, and objective function [1]. Various analytical methods have been proposed to solve optimization problems, including gradient-based methods and numerical calculations. Methods such as gradient-based are limited to solving simple derivative functions, and when the condition of continuity and derivability of functions does not exist, the gradient method is unusable. On the other hand, the accuracy of numerical calculation methods depends on choosing the appropriate initial solution despite their wide application in solving optimization problems. In such methods, improper selection of initial solutions leads to the optimal local solution [2].

In an optimization problem, it is not only important to find a solution, but also the cost of achieving the solution and its efficiency. Global optimization problems in real applied optimization problems have high dimensions and complexity because they involve many multiple decision variables and many complex nonlinear relationships. The complex and non-convex nature of the problems and their unknown search space in real-world applications make analytical methods almost unusable [3]. The weakness of analytical mathematical methods in solving optimization problems has led to the creation of a special type of intelligent search algorithms called meta-heuristic algorithms. Meta-heuristic algorithms are stochastic methods that, inspired by nature and its mechanisms, try to send their initial population to the global optimum and provide appropriate solutions close to the global optimum in a reasonable time [3]. Because these solutions may not be the same as the global optima of optimization problems, the solutions obtained from metaheuristic algorithms are called quasi-optimal [4].

Metaheuristic algorithms based on the two concepts of exploration and exploitation are able to find appropriate solutions to optimization problems. The concept of exploration represents the ability of the algorithm to globally search the search space to scan it to identify the optimal area accurately.

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The concept of exploitation represents the ability of the algorithm to locally search the search space to converge to better solutions [5]. Optimization algorithms must strike a suitable balance between exploration and exploitation to perform well in achieving the appropriate solution [6].

In designing metaheuristic algorithms, simulations of evolution-based processes in nature, physical phenomena, biological sciences, animal behavior, and other living organisms have been used. Hence, in a general category, metaheuristic algorithms fall into three groups: swarm-based, evolutionary-based, and physics-based methods.

Simulation of swarming processes in the behavior of animals, aquatic animals, birds, and other living things has been an inspiration in the development of swarm-based methods. The most famous algorithms in this group are Ant Colony Algorithm (ACO) [7], Particle Swarm Optimization (PSO) [8], and Artificial Bee Colony (ABC) [9]. The natural behavior of the swarm of ants in discovering the shortest path between the nest and the food has been the main inspiration in ACO design. PSO is inspired by the swarm behavior and movement of birds or fish seeking food in nature. ABC has been introduced by imitating the intelligent behavior of a bee colony in search of food. The strategy of living organisms when hunting and trapping prey has been the main idea of various metaheuristic algorithms such as Grey Wolf Optimizer (GWO) [10], Spotted Hyena Optimizer (SHO) [11], Whale Optimization Algorithm (WOA) [12], Chameleon Swarm Algorithm (CSA) [13], and Marine Predator Algorithm (MPA) [2]. The use of foraging and nutritional behavior modeling led to the design of metaheuristic algorithms such as Tunicate Swarm Algorithm (TSA) [14], Raccoon Optimization Algorithm (ROA) [15], and Salp Swarm Algorithm (SSA) [16].

Simulation of biological evolutionary concepts and natural selection theory led to the design of evolution-based algorithms. Genetic Algorithm (GA) [17] and Differential Evolution (DE) [18] can be named the most popular evolution-based methods. In designing GA and DE, random operators including selection, crossover, and mutation have been used according to the concepts of natural selection and the reproductive process. Other evolutionary-base methods can be referred to as Genetic programming (GP), Cultural algorithm (CA) [19], Evolution strategy (ES).

Mathematical modeling of physical laws and phenomena has been effective in the development of physics-based methods. Among the most popular physics-based optimization approaches are Simulated Annealing (SA) [20], Gravitational Search Algorithm (GSA) [21], and Big Bang-Big Crunch Algorithm (BB-BC) [22]. The physical phenomenon of metal melting and their cooling process have been the origin of SA design. Gravitational law and Newtonian laws of motion have been the main idea in GSA design. Big Bang and Big Crunch theories are employed as two concepts of the evolution of the universe in the design of BB-BC. Other physics-based metaheuristics can be referred to Water Cycle Algorithm (WCA) [23], Nuclear Reaction Optimization (NRO) [24], Ray Optimization (RO) algorithm [25], Central Force Optimization (CFO) algorithm [26], Galaxy Based Search Algorithm (GbSA) [27].

TABLE 1. Recently published metaheuristic algorithms.

Algorithm	year
Aquila Optimizer (AO) [28]	2021
Chameleon Swarm Algorithm (CSA) [13]	2021
African Vultures Optimization Algorithm (AVOA) [29]	2021
Artificial Gorilla Troops Optimizer (GTO) [30]	2021
Horse Herd Optimization (HHO) [31]	2021
Quantum-based Avian Navigation Optimizer Algorithm (QANA) [32]	2021
Wild Horse Optimizer (WHO) [33]	2021
Golden Eagle Optimizer (GEO) [34]	2021
Artificial lizard Search Optimization (ALSO) [35]	2021
Artificial Hummingbird Algorithm (AHA) [36]	2022
Reptile Search Algorithm (RSA) [37]	2022
Diversity-maintained Multi-trial Vector Differential Evolution (DMDE) [38]	2022
Ebola Optimization Search Algorithm [39]	2022
Orca Predation Algorithm (OPA) [40]	2022
Honey Badger Algorithm (HBA) [41]	2022
Trees Social Relations Optimization Algorithm (TSR) [42]	2022
Predator–Prey Optimization (PPO) [43]	2022
Starling Murmuration Optimizer (SMO) [44]	2022
Gaze Cues Learning-based Grey Wolf Optimizer (GGWO) [45]	2022
Dwarf Mongoose Optimization Algorithm (DMO) [46]	2022

Numerous metaheuristic algorithms have been published in recent years, some of which are listed in Table 1.

In the study of optimization algorithms, the main research question is that despite the numerous metaheuristic algorithms designed, what is the need to develop newer optimization algorithms? The answer to this question based on the No Free Lunch (NFL) theorem [47] is that the optimal performance of an algorithm in optimizing a set of objective problems and functions does not guarantee the optimal performance of that algorithm in solving other optimization problems. The NFL states that any algorithm can never be declared the best optimizer for all optimization issues. Hence, the NFL encourages researchers to design new metaheuristic algorithms to be able to solve optimization problems more effectively by providing better solutions. The NFL motivated the authors of this paper to develop a new metaheuristic algorithm that is highly efficient in achieving optimal solutions to optimization problems.

Zebras are herbivores whose main diet consists of various grasses and plant materials such as leaves and sprouts. The zebra is a social animal that always lives in a herd to protect

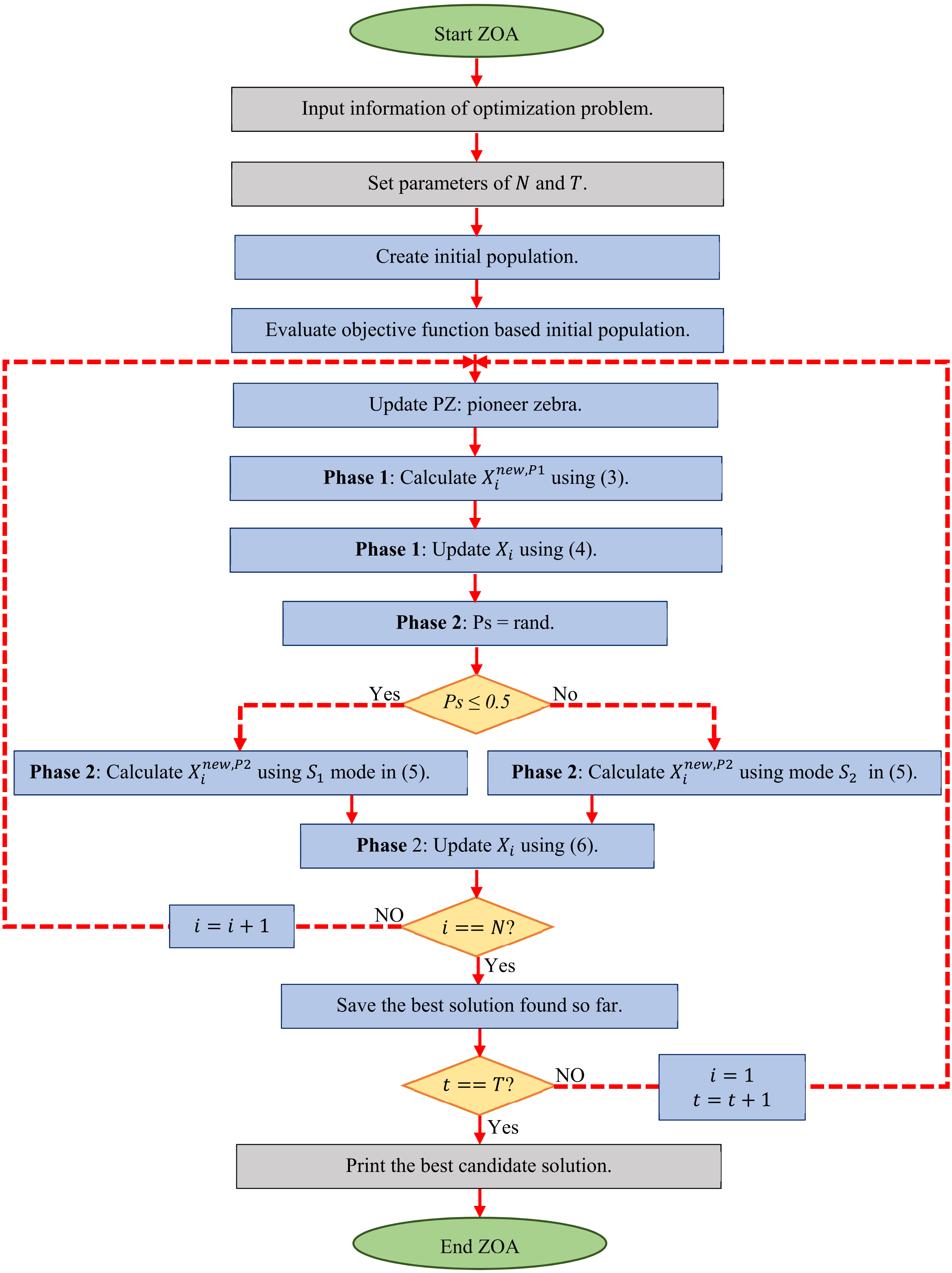

FIGURE 1. Flowchart of ZOA.

TABLE 2. Parameter values for the competitor algorithms.

Algorithm	Parameter	Value
QANA	number of flocks	$k = 10$
	K'	9
	K''	50
MPA		
	Constant number	$P = 0.5$
	Random vector	R is a vector of uniform random numbers from $[0, 1]$.
	Fish aggregating devices (FADs)	FADs = 0.2
	Binary vector	$U = 0$ or 1
TSA		
	P_{min} and P_{max}	1, 4
	c_1, c_2, c_3	random numbers lie in the interval $[0, 1]$.
WOA		
	Convergence parameter (a)	a: Linear reduction from 2 to 0.
	r is a random vector in $[0, 1]$.	
	l is a random number in $[-1, 1]$.	
GWO		
	Convergence parameter (a)	a: Linear reduction from 2 to 0.
TLBO		
	T_F: teaching factor	T_F = round $[(1 + \text{rand})]$
	random number	rand is a random number between $[0, 1]$.
GSA		
	Alpha, G_0, R_{norm}, R_{power}	20, 100, 2, 1
PSO		
	Topology	Fully connected
	Cognitive and social constant	$(c_1, c_2) = (2, 2)$
	Inertia weight	Linear reduction from 0.9 to 0.1
	Velocity limit	10% of dimension range
GA		
	Type	Real coded
	Selection	Roulette wheel (Proportionate)
	Crossover	Whole arithmetic (Probability = 0.8, $\alpha \in [-0.5, 1.5]$)
	Mutation	Gaussian (Probability = 0.05)

itself from predators. Although the animal's first instinctive move is to escape the predator, it sometimes confuses or frightens the predator by gathering together to form a defensive structure.

Based on the best knowledge of the literature, simulation of zebra's social behavior in nature has not been employed in the design of any optimization algorithm. In this paper, a new optimizer based on simulation of foraging behavior and defensive strategy of zebras is developed to address this research gap.

This paper's novelty and scientific contribution are to design a new metaheuristic algorithm called Zebra Optimization Algorithm (ZOA). ZOA's fundamental inspiration is to model the social behavior of herds of zebras in the wild. The ZOA steps are stated, its mathematical modeling is presented. ZOA performance has been tested on sixty-eight benchmark functions of a variety of unimodal, high-dimensional multimodal, fixed-dimensional multimodal, CEC2015, and CEC2017. The optimization results obtained from ZOA are compared with nine well-known algorithms. The claim of this study is that based on the mathematical simulation of zebras' life, a new and powerful metaheuristic algorithm can be designed for optimization applications. The results of optimization and experiments on objective functions, comparison of ZOA performance with several well-known metaheuristic algorithms, and various analyzes confirm that ZOA is highly efficient in optimization applications.

In the following, the paper is organized so that the proposed algorithm is introduced in Section 2. Simulation studies and analysis of the proposed algorithm are presented in Section 3. The efficiency of the proposed algorithm in solving engineering design problems is evaluated in Section 4. Conclusions and several suggestions for future studies are provided in Section 5.

II. ZEBRA OPTIMIZATION ALGORITHM

In this section, the proposed nature-inspired Zebra Optimization Algorithm (ZOA) is introduced and its mathematical modeling is presented.

A. INSPIRATION

Zebras are equine animals and come from eastern and southern Africa. This animal is famous for its black-and-white striped coat on its body. These stripes are usually located vertically on the neck and body, and they are effective in hiding zebras from predators as well as an inhibitory agent against biting flies. Specifications and descriptions of their conditions are as follows: They have a body length of 210–300 cm with a tail long 38–75 cm, 110–160 cm shoulder height, and weigh 175–450 kg [48]. The zebra is a heavy animal whose long and slender legs help the animal run at high speeds if necessary. Like wild equines, zebras have only one toe on each foot, a long neck, and a head that makes it easy to feed on the grass on the ground [49]. Among the social life behavior of zebras in nature, the two types of behavior are the most important: foraging and defense strategy against predators.

In the foraging process, a pioneer zebra opens the way for other zebras to move to the forage. Therefore, other zebras in the herd move in the plains under the guidance of this pioneer zebra [50].

TABLE 3. Optimization results of ZOA and competitor algorithms on unimodal test function.

		ZOA	QANA	TSA	MPA	WOA	GWO	GSA	TLBO	GA	PSO
F_1	Ave	6.61E-124	4.77E-75	7.83E-38	3.32E-21	2.21E-09	1.11E-58	2.06E-17	8.46E-60	13.439108	1.80E-05
	std	2.45E-108	1.41E-74	7.11E-21	4.68E-21	7.51E-25	5.22E-74	1.15E-32	5.02E-76	4.84E-15	6.54E-21
F_2	Ave	3.00E-64	3.84E-40	8.61E-39	1.59E-12	0.554393	1.31E-34	2.41E-08	7.28E-35	2.516591	0.3462165
	std	1.32E-54	1.25E-39	6.01E-41	1.44E-12	1.76E-16	1.94E-50	5.26E-24	6.79E-50	2.27E-15	7.56E-17
F_3	Ave	3.24E-90	9.05E-51	1.17E-21	0.087696	1.79E-08	7.52E-15	283.53406	2.79E-15	1559.9497	598.33438
	std	1.68E-88	3.54E-50	6.80E-21	0.146566	1.05E-23	5.73E-30	1.23E-13	2.69E-31	6.71E-13	7.22E-13
F_4	Ave	1.86E-58	2.56E-31	1.35E-23	2.64E-08	2.94E-05	1.28E-14	3.30E-09	9.56E-15	2.125613	4.022851
	std	3.69E-46	5.17E-31	1.17E-22	9.39E-09	1.23E-20	1.07E-29	2.07E-24	2.15E-30	2.27E-15	2.02E-16
F_5	Ave	25.1737	26.8601	29.294423	46.739735	42.403351	27.263611	36.648554	148.65325	315.08371	51.016387
	std	1.84E-15	3.68E-15	4.83E-03	0.4282285	2.58E-14	0	3.14E-14	1.94E-14	2.13E-13	1.61E-14
F_6	Ave	0	0.6468	7.21E-21	0.40397	1.63E-09	0.6519345	0	0.4501525	14.76825	20.55375
	std	0	0.2726	1.14E-25	0.194271	4.69E-25	6.30E-17	0	4.28E-16	3.23E-15	7.67E-04
F_7	Ave	1.91E-05	2.51E-04	3.78E-04	0.001827	0.0208075	0.000812	0.020909	0.0017255	5.77E-03	0.115101
	std	2.71E-21	5.12E-20	5.17E-05	0.001015	1.57E-18	7.38E-20	2.76E-18	3.94E-19	7.87E-19	4.41E-17
Wins of ZOA in 7 functions			7								
Ties of ZOA in 7 functions			0								
Losses of ZOA in 7 functions			0								

TABLE 4. Optimization results of ZOA and competitor algorithms on high dimensional multimodal test function.

		ZOA	QANA	TSA	MPA	WOA	GWO	GSA	TLBO	GA	PSO
F_8	Ave	-6618.5746	-5281.28	-5826.4439	-3648.0757	-1688.9379	-5973.394	-2891.8085	-7519.7399	-8307.1804	-7012.2856
	std	6.34E-01	563.2137	42.1225	823.49641	727.09444	474.52651	268.31687	521.28208	845.71475	635.00917
F_9	Ave	0	0	5.79E-03	142.22566	4.2641165	8.65E-15	16.511513	10.402228	63.347571	57.91722
	std	0	0	1.48E-03	26.707086	4.43E-15	5.73E-30	3.23E-15	5.64E-15	2.58E-14	6.45E-15
F_{10}	Ave	8.88E-16	4.44E-15	9.95E-14	9.84E-12	0.3342395	1.73E-14	3.62E-09	0.2798355	3.270127	2.186919
	std	7.94E-16	2.16E-14	4.58E-12	6.22E-12	2.02E-12	2.79E-14	3.75E-08	2.60E-14	5.24E-11	8.06E-12
F_{11}	Ave	0	0	1.02E-07	0	0.1206835	0.0037555	3.7935625	0.617323	1.248653	0.046893
	std	0	0	7.57E-07	0	9.13E-17	1.28E-18	2.82E-15	2.02E-16	8.57E-16	3.15E-18
F_{12}	Ave	2.87E-05	0.016112	0.037352	0.0863765	1.767521	0.037758	0.036743	0.0206045	0.047705	0.487809
	std	2.60E-08	0.007672	1.57E-02	0.005278	8.26E-02	4.41E-07	6.30E-07	7.87E-06	4.72E-07	1.89E-06
F_{13}	Ave	0.0199	1.253473	3.0018625	0.4974515	0.350784	0.5849445	0.00203	0.3342395	1.2266275	0.516026
	std	4.87E-16	0.460513	1.59E-12	0.196098	3.30E-12	2.52E-15	4.33E-14	2.14E-14	3.28E-14	5.04E-15
Wins of ZOA in 6 functions				3							
Ties of ZOA in 6 functions				2							
Losses of ZOA in 6 functions				1							

The zebras' first strategy against predators is to escape in a zigzag motion pattern. However, sometimes by gathering, they try to confuse or frighten the predator [48].

Mathematical modeling of these two types of intelligent zebra behavior is the fundamental inspiration for the proposed ZOA design.

B. MATHEMATICAL MODELLING

In this subsection, mathematical simulations of zebra's natural behaviors are presented to model ZOA.

1) INITIALIZATION

ZOA is a population-based optimizer that zebras are members of its population. From a mathematical point of view, each zebra is a candidate solution to the problem and the plain in which the zebras are in the search space for the problem. The position of each zebra in the search space determines the values for the decision variables. Thus, each zebra as a member of the ZOA can be modeled using a vector, while the elements of this vector represent the values of the problem variables. The population of zebras can be mathematically modeled using a matrix. The initial position of the zebras in the search space is randomly assigned. The ZOA population matrix is specified in (1).

$$X = \begin{bmatrix} X_1 \\ \vdots \\ X_i \\ \vdots \\ X_N \end{bmatrix}_{N\times m} = \begin{bmatrix} x_{1,1} & \cdots & x_{1,j} & \cdots & x_{1,m} \\ \vdots & \ddots & \vdots & \iddots & \vdots \\ x_{i,1} & \cdots & x_{i,j} & \cdots & x_{i,m} \\ \vdots & \iddots & \vdots & \ddots & \vdots \\ x_{N,1} & \cdots & x_{N,j} & \cdots & x_{N,m} \end{bmatrix}_{N\times m}, \tag{1}$$

TABLE 5. Optimization results of ZOA and competitor algorithms on fixed dimensional multimodal test function.

		ZOA	QANA	TSA	MPA	WOA	GWO	GSA	TLBO	GA	PSO
F_{14}	Ave	0.9980	0.9980	1.9923	0.9980	0.9980	3.7408	3.5913	2.2721	0.9986	2.1735
	std	9.26E-18	5.62E-17	2.69E-07	4.34E-16	9.58E-16	6.55E-15	8.06E-16	2.02E-16	1.59E-15	8.06E-16
F_{15}	Ave	0.00030	0.00040	0.000406	0.003045	0.0049735	0.0063945	0.002436	0.0033495	5.48E-02	0.0543025
	std	5.64E-16	6.91E-15	9.15E-14	4.16E-15	3.54E-14	1.18E-14	2.95E-14	1.24E-15	7.19E-15	3.94E-14
F_{16}	Ave	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316
	std	1.44E-16	4.68E-16	5.6514E-16	4.4652E-16	9.9301E-16	3.9720E-16	5.9580E-16	1.4398E-15	7.9441E-16	3.4755E-16
F_{17}	Ave	0.3978	0.3978	0.3991	0.3979	0.4047	0.3978	0.3978	0.3978	0.4369	0.7854
	std	6.11E-18	1.51E-16	2.1596E-16	9.1235E-15	2.4825E-14	8.6888E-16	9.9301E-16	7.4476E-16	4.9650E-14	4.9650E-15
F_{18}	Ave	3	3	3	3	3	3.0000	3	3.0009	4.3592	3
	std	2.97E-18	4.75E-16	2.72E-15	2.01E-15	5.84E-15	2.14E-15	7.12E-16	1.63E-15	6.11E-16	3.77E-15
F_{19}	Ave	-3.8627	-3.8627	-3.8066	-3.8627	-3.8627	-3.8621	-3.8627	-3.8609	-3.85434	-3.8627
	std	4.63E-16	5.07E-15	2.69E-15	4.33E-15	3.26E-15	2.53E-15	8.51E-15	7.50E-15	1.01E-13	9.12E-15
F_{20}	Ave	-3.322	-3.31011	-3.287394	-3.287889	-3.209976	-3.219777	-3.009204	-3.169386	-2.795661	-3.229281
	std	6.70E-17	3.66E-15	5.63E-15	1.13E-11	7.86E-16	2.16E-15	2.16E-14	1.77E-15	3.93E-11	2.95E-12
F_{21}	Ave	-10.1532	-10.1532	-5.447079	-10.1532	-7.327584	-9.548748	-5.097114	-9.082854	-4.26096	-5.335209
	std	2.39E-17	1.06E-15	5.41E-13	2.51E-11	2.36E-11	6.49E-15	2.95E-14	8.45E-15	1.57E-12	1.47E-13
F_{22}	Ave	-10.4029	-10.4029	-5.011875	-10.4029	-8.728335	-10.298475	-8.933661	-9.938511	-5.066226	-7.555977
	std	1.05E-16	3.42E-15	8.38E-14	2.79E-11	6.68E-15	1.97E-15	1.63E-12	1.51E-14	6.23E-15	7.51E-15
F_{23}	Ave	-10.5364	-10.5360	-10.257687	-10.431036	-9.900297	-10.028898	-8.815455	-9.197595	-6.496479	-6.103152
	std	7.18E-16	1.36E-15	7.57E-12	3.95E-11	9.04E-15	4.52E-15	7.08E-14	6.13E-15	3.83E-15	2.75E-15
Wins of ZOA in 10 functions			3								
Ties of ZOA in 10 functions			7, But based on the "std" index, the ZOA approach is superior.								
Losses of ZOA in 10 functions			0								

where X is the zebra population, X_i is the ith zebra, $x_{i,j}$ is the value for the jth problem variable proposed by the ith zebra, N is the number of population members (zebras), and m is the number of decision variables.

Each zebra represents a candidate solution to the optimization problem. Therefore, the objective function can be evaluated based on the proposed values of each zebra for the problem variables. The values obtained for the objective function are specified as a vector using (2).

$$F = \begin{bmatrix} F_1 \\ \vdots \\ F_i \\ \vdots \\ F_N \end{bmatrix}_{N \times 1} = \begin{bmatrix} F(X_1) \\ \vdots \\ F(X_i) \\ \vdots \\ F(X_N) \end{bmatrix}_{N \times 1}, \tag{2}$$

where F is the vector of objective function values, and F_i is the objective function value obtained for the ith zebra. Comparing the values obtained for the objective function effectively analyzes the quality of their corresponding candidate solutions and identifies the best candidate solution for the given problem. In minimization problems, the zebra with the least value of objective function is the best candidate solution. In contrast, in maximization problems, the zebra with the highest value of the objective function is the best candidate solution. Since in each iteration, the positions of the zebras and consequently the values of the objective function are updated, the best candidate solution must also be identified in each iteration.

Two natural behaviors of zebras in the wild have been used to update ZOA members. These two types of behavior include

(i) foraging and

(ii) defense strategies against predators.

Therefore, in each iteration, members of the ZOA population are updated in two different phases.

2) PHASE 1: FORAGING BEHAVIOR

In the first phase, population members are updated based on simulations of zebra behavior when searching for forage. The main diet of zebras is mainly grasses and sedges, but if their favorite foods are scarce, they may also eat buds, fruits, bark, roots, and leaves. Depending on the quality and availability of vegetation, zebras may spend 60–80 percent of their time eating [51]. Among the zebras, there is a zebra called the plains zebra, which is a pioneer grazer, by devouring the canopy of upper and less nutritious grass, provides conditions for other species that need shorter and more nutritious grasses below [50]. In ZOA, the best member of the population is considered as the pioneer zebra and leads other population members towards its position in the

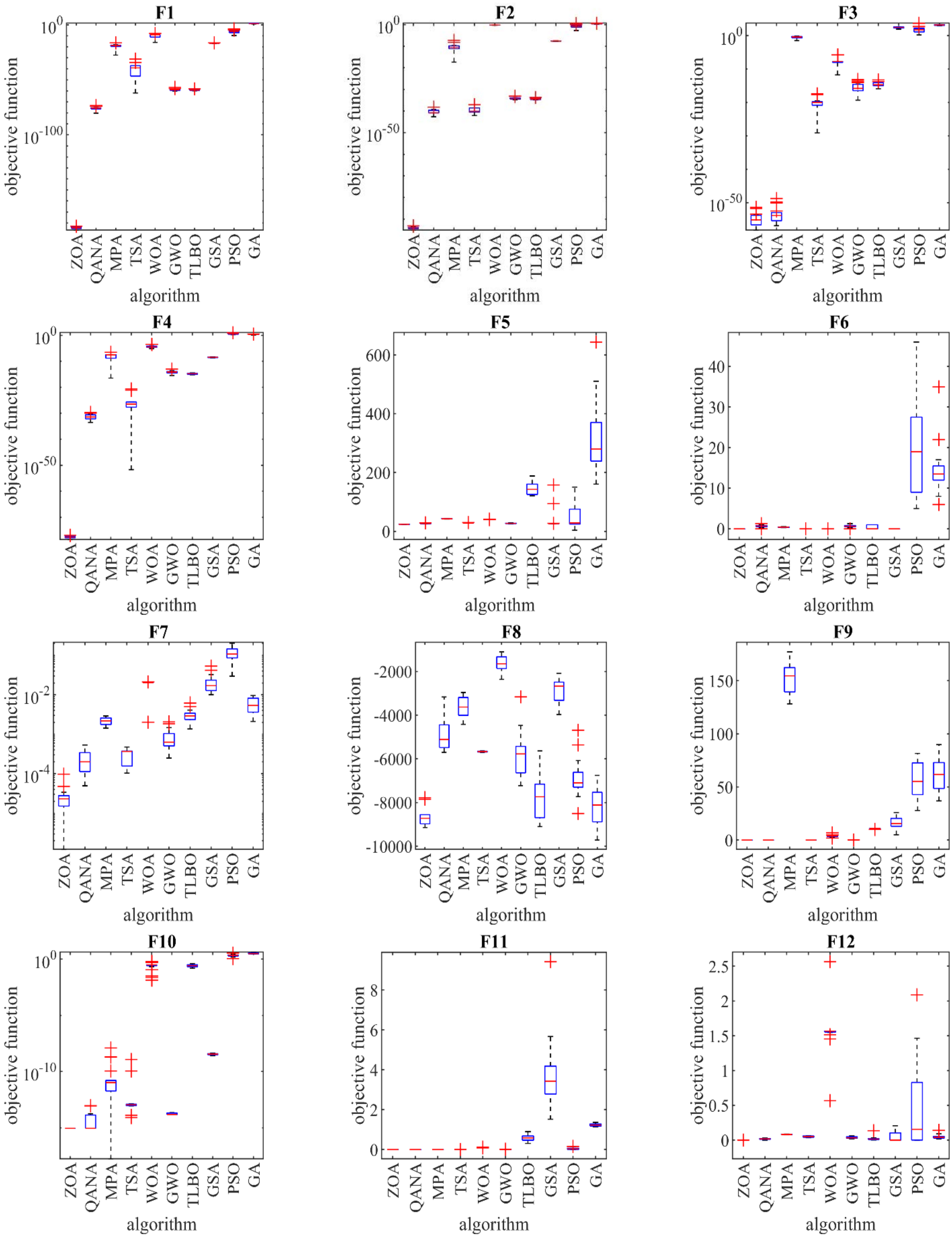

FIGURE 2. Boxplot of performance of ZOA and competitor algorithms in solving test functions.

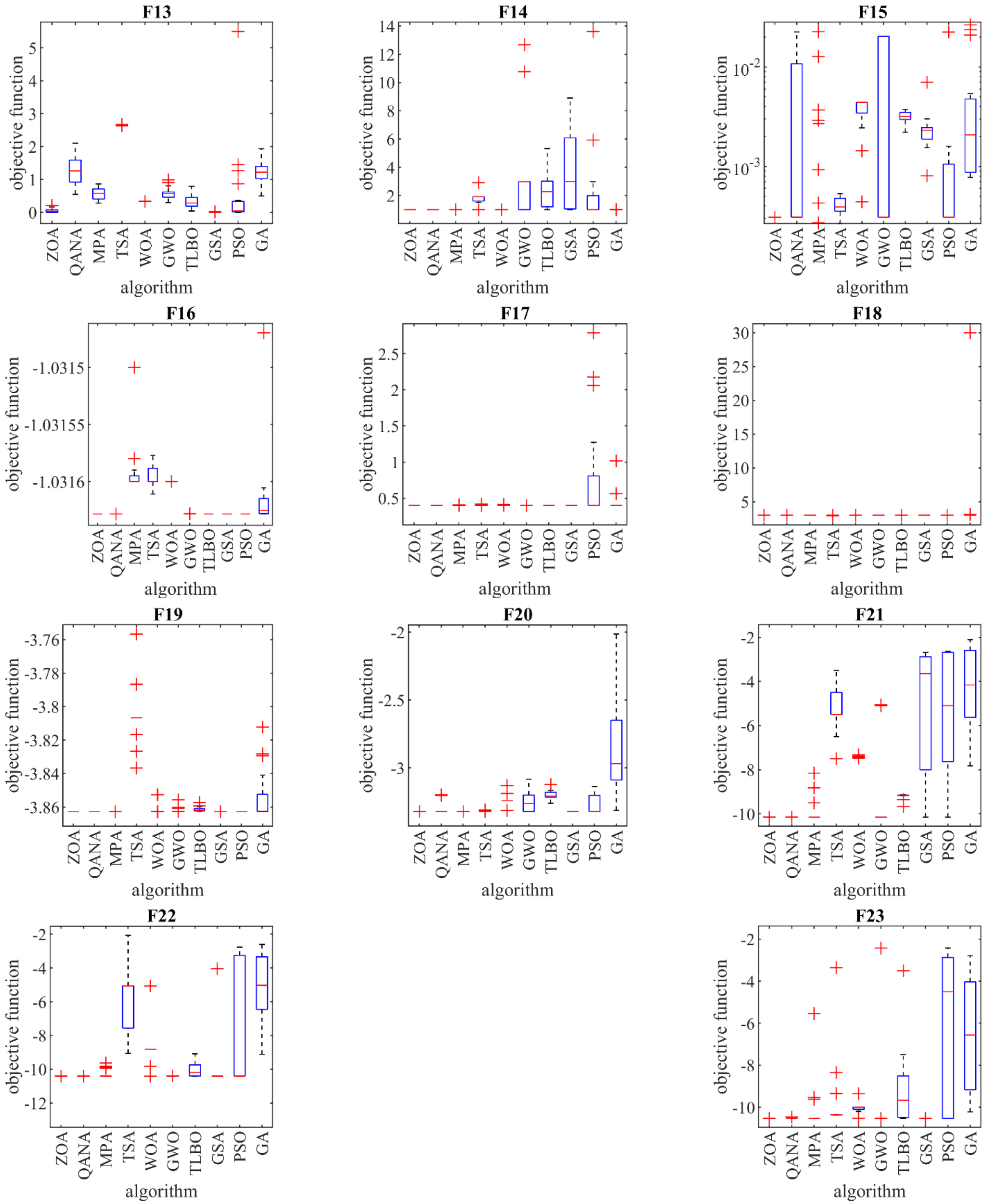

FIGURE 2. *(Continued.)* **Boxplot of performance of ZOA and competitor algorithms in solving test functions.**

search space. Therefore, updating the position of zebras in the foraging phase can be mathematically modeled using (3) and (4).

$$x_{i,j}^{new,P1} = x_{i,j} + r \cdot (PZ_j - I \cdot x_{i,j}) \tag{3}$$

$$X_i = \begin{cases} X_i^{new,P1}, & F_i^{new,P1} < F_i; \\ X_i, & else, \end{cases} \tag{4}$$

where $X_i^{new,P1}$ is the new status of the ith zebra based on first phase, $x_{i,j}^{new,P1}$ is its jth dimension value, $F_i^{new,P1}$ is its

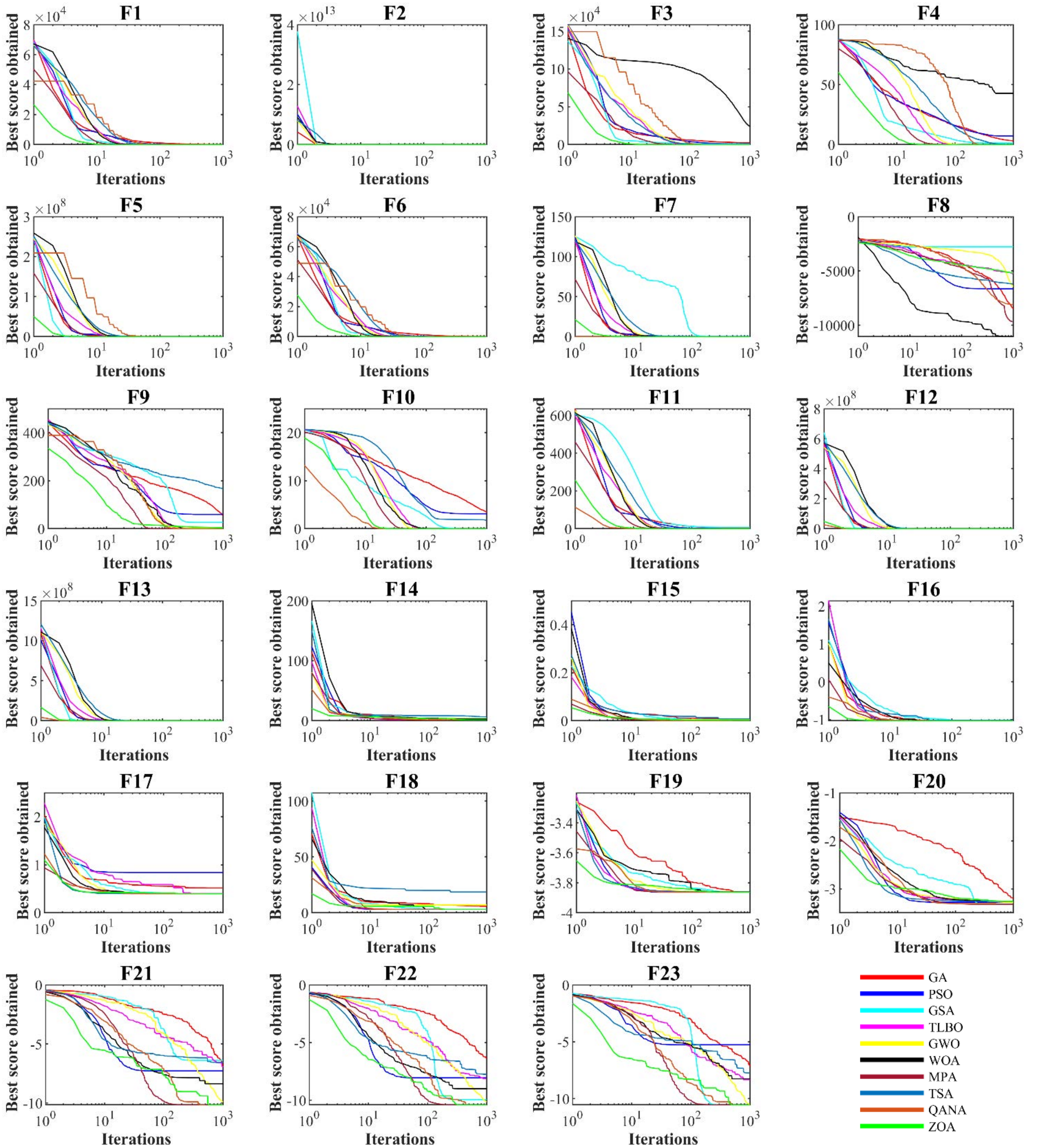

FIGURE 3. Convergence curves of ZOA and competitor algorithms.

objective function value, PZ is the pioneer zebra which is the best member, PZ_j is its jth dimension, r is a random number in interval [0, 1], $I = \text{round}\,(1 + rand)$, where $rand$ is a random number in the interval [0, 1]. Thus, $I \in \{1, 2\}$ and if parameter $I = 2$, then there are much more changes in population movement.

3) PHASE 2: DEFENSE STRATEGIES AGAINST PREDATORS

In the second phase, simulations of the zebra's defense strategy against predator attacks are employed to update the position of population members of ZOA in the search space. The main predators of zebras are lions; however, they are threatened by cheetahs, leopards, wild dogs, brown hyenas,

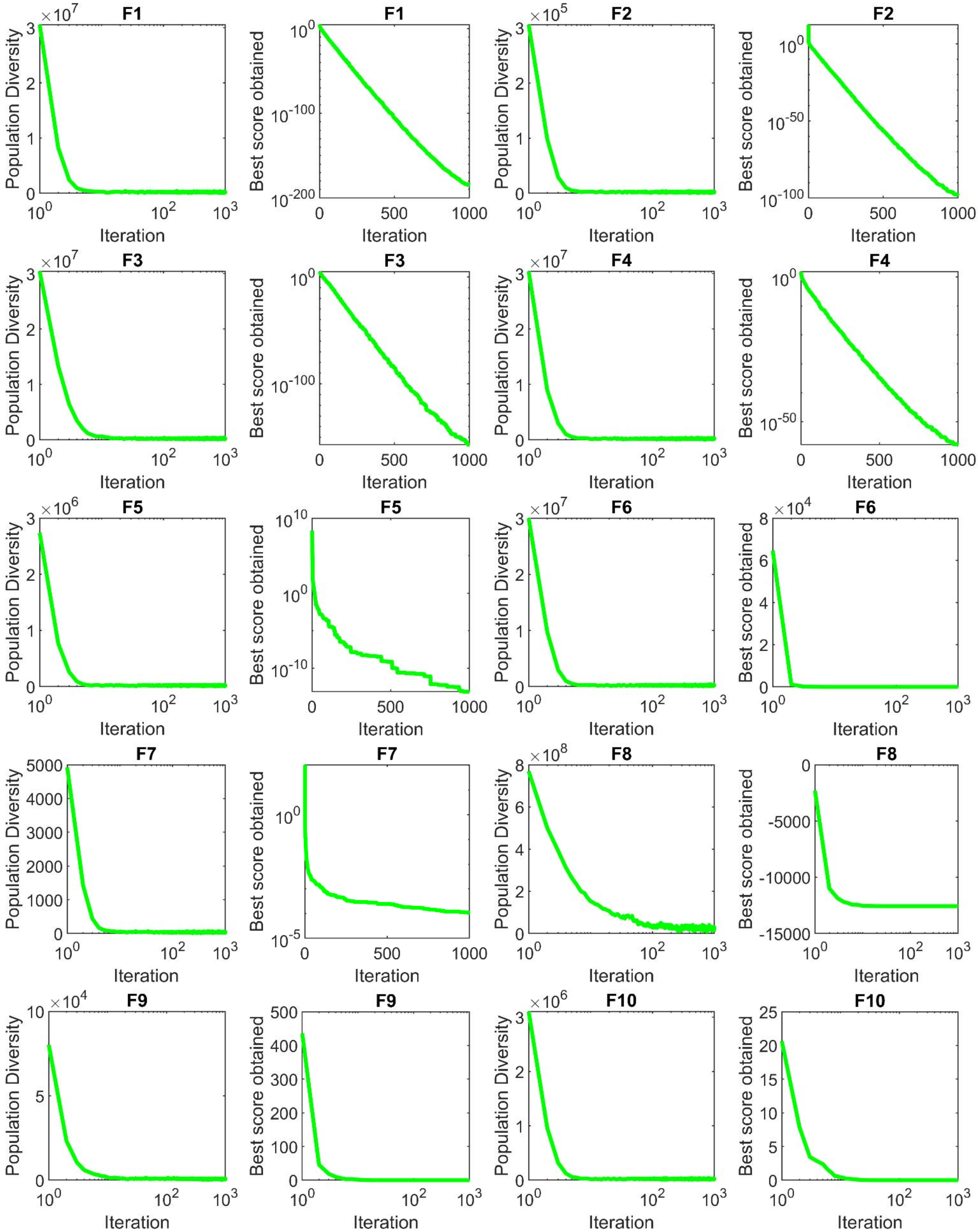

FIGURE 4. The population diversity and convergence curves of the ZOA.

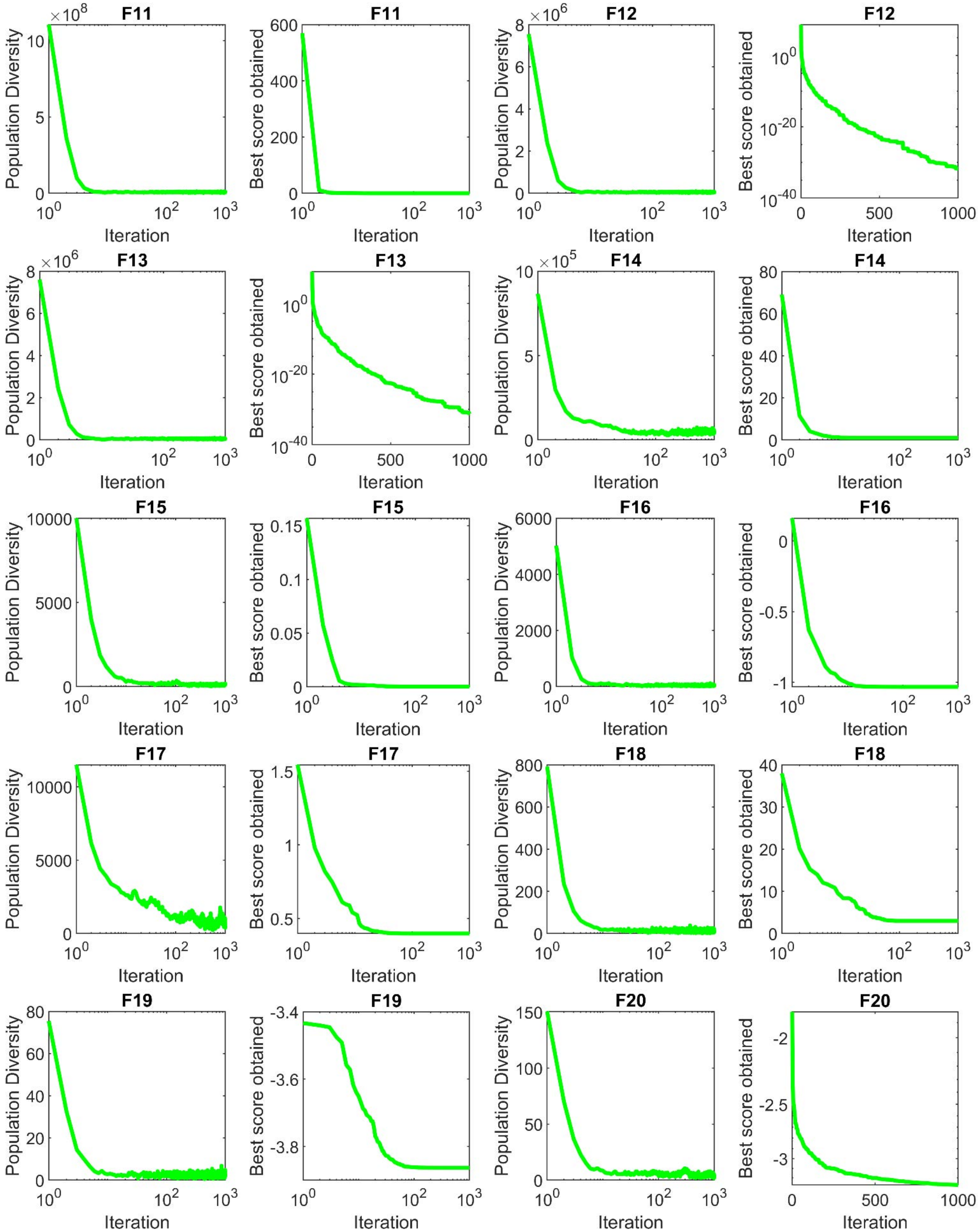

FIGURE 4. ***(Continued.)*** **The population diversity and convergence curves of the ZOA.**

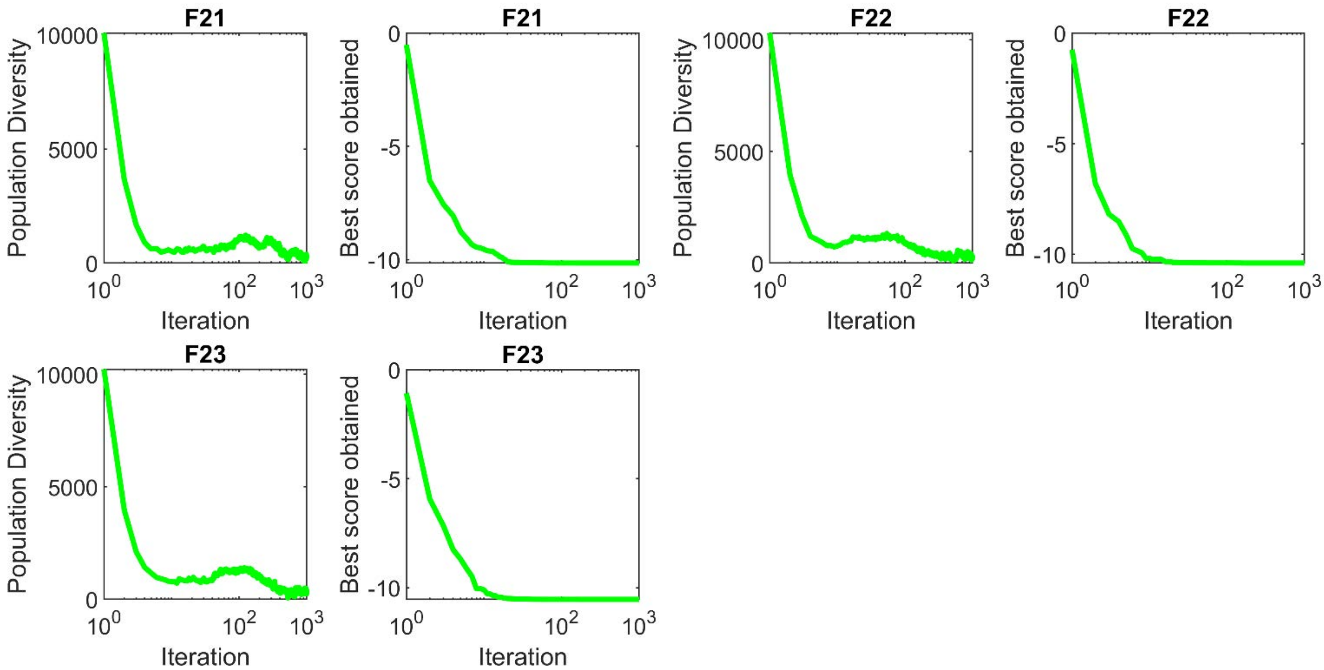

FIGURE 4. ***(Continued.)*** **The population diversity and convergence curves of the ZOA.**

TABLE 6. ***p*-values obtained from Wilcoxon rank sum test.**

Compared Algorithms	Functions type		
	Unimodal	High-Multimodal	Fixed-Multimodal
ZOA vs. QANA	0.015625	0.03261	0.00423601
ZOA vs. MPA	0.015625	0.04625	0.01953125
ZOA vs. TSA	0.015625	0.03125	0.00390625
ZOA vs. WOA	0.015625	0.03125	0.0078125
ZOA vs. GWO	0.015625	0.03125	0.01171875
ZOA vs. TLBO	0.015625	0.03125	0.005859375
ZOA vs. GSA	0.03125	0.03125	0.01953125
ZOA vs. PSO	0.015625	0.03125	0.00390625
ZOA vs. GA	0.015625	0.03125	0.001953125

TABLE 7. **Friedman test results.**

Compared Algorithms	Functions type					
	Unimodal		High-Multimodal		Fixed-Multimodal	
	Avg. Rank	Overall Rank	Avg. Rank	Overall	Avg. Rank	Overall
ZOA	2.2857143	1	1.1666667	1	1.5	1
QANA	2.5714286	3	3.8333333	3	3.6	2
MPA	5.7142857	6	6	7	3.9	3
TSA	2.4285714	2	5	5	4.9	6
WOA	6.5714286	8	6.5	8	4.8	5
GWO	4.4285714	4	3.6666667	2	5.2	8
TLBO	4.8571429	5	4.1666667	4	5	7
GSA	6	7	5.3333333	6	4.2	4
PSO	8.7142857	9	6	7	5.5	9
GA	8.8571429	10	6.5	8	7.4	10

and spotted hyenas [48]. Crocodiles are another predator of zebras when they approach water [52]. Zebras' defense strategy varies depending on the predator. The zebra's defensive strategy against lion attacks is to escape in a zigzag pattern and random sideways turning movements [53]. Zebras are more aggressive against attacks by smaller predators, such as hyenas and dogs, which confuse and frighten the hunter by gathering [48]. In the ZOA design, it is assumed that one of the following two conditions occurs with the same probability:

(i) the lion attacks the zebra, and thus, the zebra chooses an escape strategy;

(ii) other predators attack the zebra, and the zebra will choose the offensive strategy.

In the first strategy, when the zebras are attacked by lions, the zebras escape from the lion's attack in the vicinity of the situation in which they are located. Therefore, mathematically, this strategy can be modeled using the mode S_1 in (5). In the second strategy, when other predators attack one of the zebras, the other zebras in the herd move towards the attacked zebra and try to frighten and confuse the predator by creating a defensive structure. This strategy of zebras is mathematically modeled using the mode S_2 in (5). In updating the position of zebras, the new position is accepted for a zebra if it has a better value for the objective function in that new position. This update condition is modeled using (6).

$$x_{i,j}^{new,P2} = \begin{cases} S_1 : x_{i,j} + R \cdot (2r - 1) \\ \qquad \cdot (1 - \frac{t}{T}) \cdot x_{i,j}, & P_s \leq 0.5; \\ S_2 : x_{i,j} + r \cdot (AZ_j - I \cdot x_{i,j}), & else, \end{cases} \quad (5)$$

$$X_i = \begin{cases} X_i^{new,P2}, & F_i^{new,P2} < F_i; \\ X_i, & else, \end{cases} \quad (6)$$

TABLE 8. Scalability analysis of the ZOA.

Objective Function	Dimension (m)					
	10	30	50	100	500	1000
F_1	2E-101	6.6E-124	4.25E-94	6.01E-95	6.24E-87	2.14E-88
F_2	8.79E-55	3E-64	1.68E-51	7.13E-50	3.65E-43	6.75E-44
F_3	8.47E-87	3.24E-90	1.47E-58	1.55E-45	9.92E-29	9.45E-20
F_4	1.35E-49	1.86E-58	1.43E-45	4.22E-43	3.05E-42	1.67E-41
F_5	7.93447	25.1737	48.23722	98.04907	0.081543	0.136514
F_6	0	0	0	0	0	0
F_7	9.83E-05	1.91E-05	0.000132	0.000136	8.33E-05	0.000121
F_8	-2514.63	-6618.57	-8190.56	-12588.6	5186.986	6860.817
F_9	1.398534	0	0	0	0	0
F_{10}	8.88E-16	8.88E-16	2.31E-15	2.13E-15	1.58E-15	1.09E-15
F_{11}	0.009226	0	0	0	0	0
F_{12}	1.06E-10	2.87E-05	0.002484	0.024505	0.020946	0.039158
F_{13}	8.79E-09	0.019902	1.862478	8.255436	0.133711	0.091979

where $X_i^{new,P2}$ is the new status of the ith zebra based on second phase, $x_{i,j}^{new,P2}$ is its jth dimension value, $F_i^{new,P2}$ is its objective function value, t is the iteration contour, T is the maximum number of iterations, R is the constant number equal to 0.01, P_s is the probability of choosing one of two strategies that are randomly generated in the interval [0, 1], AZ is the status of attacked zebra, and AZ_j is its jth dimension value.

C. REPETITIONS PROCESS, FLOWCHART, AND PSEUDO-CODE OF ZOA

Each ZOA iteration is completed by updating the population members based on the first and second phases. The process of updating the algorithm population continues based on (3) to (6) until the end of the full implementation of the algorithm. The best candidate solution is updated and saved during successive iterations. Once fully implemented, ZOA makes the best candidate solution available as the optimal solution to the given problem. The ZOA steps are presented as flowcharts in Figure 1 and its pseudocode in Algorithm 1.

D. COMPUTATIONAL COMPLEXITY

In this subsection, the computational complexity of ZOA is investigated. ZOA initialization preparation is equal to $O(N \cdot m)$ where N is the number of zebras and m is the number of problem variables. ZOA includes the number of T iterations, so that in each iteration, each population member is updated in two phases and its objective function is evaluated. The computational complexity of this update process is equal to $O(2 \cdot N \cdot m \cdot T)$. Thus, the total computational complexity of ZOA is equal to $O(N \cdot m \cdot (1 + 2 \cdot T))$.

III. SIMULATION STUDIES AND DISCUSSION

In this section, the efficiency of the proposed algorithm in optimizing and providing optimal solutions is evaluated.

Algorithm. 1: Pseudo-Code of Proposed ZOA

Start ZOA.
1. Input: The optimization problem information.
2. Set the number of iterations (T) and the number of zebras' population (N).
3. Initialization of the position of zebras and evaluation of the objective function.
4. For $t = 1$: T
5. Update pioneer zebra (PZ).
6. For $i = 1$: N
7. **Phase 1: Foraging behavior**
8. Calculate new status of the ith zebra using (3).
9. Update the ith zebra using (4).
10. **Phase 2: Defense strategies against predators**
11. If $Ps < 0.5$, $Ps =$ rand
12. **Strategy 1:** against lion (exploitation phase)
13. Calculate new status of the ith zebra using mode S_1 in (5).
14. else
15. **Strategy 2:** against other predator (exploration phase)
16. Calculate new status of the ith zebra using mode S_2 in (5).
17. end if
18. Update the ith zebra using (6).
19. end for $i = 1$: N
20. Save best candidate solution so far.
21. end for $t = 1$: T
22. Output: The best solution obtained by ZOA for given optimization problem.

End ZOA.

Sixty-eight benchmark functions have been employed to test the performance of the proposed algorithm. These functions include unimodal, high-dimensional multimodal, fixed-dimensional multimodal, CEC2015, and CEC2017. The ability of the proposed algorithm is compared with the performance of the nine famous metaheuristics GWO, TLBO, GA, MPA, PSO, QANA, TSA, WOA, and GSA. Table 2 shows the values of the control parameters of these algorithms. The proposed algorithm and each of the

TABLE 9. Sensitivity analysis of the ZOA for the number of population members.

Objective Function	Number of Population Members			
	20	**30**	**50**	**100**
F_1	1.43E-89	6.6E-124	1.4E-175	3.7E-215
F_2	3.42E-46	3E-64	1.49E-89	4.6E-115
F_3	1.93E-63	3.24E-90	4.8E-114	1.2E-134
F_4	4.79E-41	1.86E-58	8.25E-78	5.6E-98
F_5	26.1358	25.1737	25.1036	25.0647
F_6	0	0	0	0
F_7	1.49E-05	1.91E-05	9.54E-06	5.67E-06
F_8	-6259.86	-6618.57	-6667.26	-6788.74
F_9	0	0	0	0
F_{10}	8.88E-16	8.88E-16	8.88E-16	8.88E-16
F_{11}	0	0	0	0
F_{12}	5.77E-05	2.87E-05	2.1E-06	1.63E-06
F_{13}	0.024599	0.019902	0.012295	0.01219
F_{14}	0.998004	0.998	0.998	0.998
F_{15}	0.000307	0.0003	0.0003	0.0003
F_{16}	-1.03163	-1.03163	-1.03163	-1.03163
F_{17}	0.397887	0.3978	0.3978	0.3978
F_{18}	3	3	3	3
F_{19}	-3.86278	-3.86278	-3.86278	-3.86278
F_{20}	-3.322	-3.322	-3.322	-3.322
F_{21}	-10.1532	-10.1532	-10.1532	-10.1532
F_{22}	-10.4029	-10.4029	-10.4029	-10.4029
F_{23}	-10.5364	-10.5364	-10.5364	-10.5364

TABLE 10. Sensitivity analysis of the ZOA for the maximum number of iterations.

Objective Function	Maximum Number of Iterations			
	100	**500**	**800**	**1000**
F_1	1.81E-29	6.6E-99	1.4E-122	6.6E-124
F_2	8.01E-16	7.27E-47	4.7E-59	3E-64
F_3	2.04E-15	6.29E-61	3.87E-74	3.24E-90
F_4	1.09E-14	3.1E-43	5.69E-57	1.86E-58
F_5	28.3168	27.0292	26.2471	25.1737
F_6	0	0	0	0
F_7	0.000512	2.43E-05	2.16E-05	1.91E-05
F_8	-5948.32	-6231.85	-6548.67	-6618.57
F_9	0	0	0	0
F_{10}	4.44E-15	8.88E-16	8.88E-16	8.88E-16
F_{11}	0	0	0	0
F_{12}	0.065762	0.000826	8.48E-05	2.87E-05
F_{13}	1.227694	0.38808	0.03474	0.019902
F_{14}	0.998004	0.998004	0.998	0.998
F_{15}	0.000308	0.000308	0.000308	0.0003
F_{16}	-1.03163	-1.03163	-1.03163	-1.03163
F_{17}	0.3978	0.3978	0.3978	0.3978
F_{18}	3	3	3	3
F_{19}	-3.8627	-3.86278	-3.86278	-3.86278
F_{20}	-3.32179	-3.322	-3.322	-3.322
F_{21}	-10.1532	-10.1532	-10.1532	-10.1532
F_{22}	-10.4029	-10.4029	-10.4029	-10.4029
F_{23}	-10.5364	-10.5364	-10.5364	-10.5364

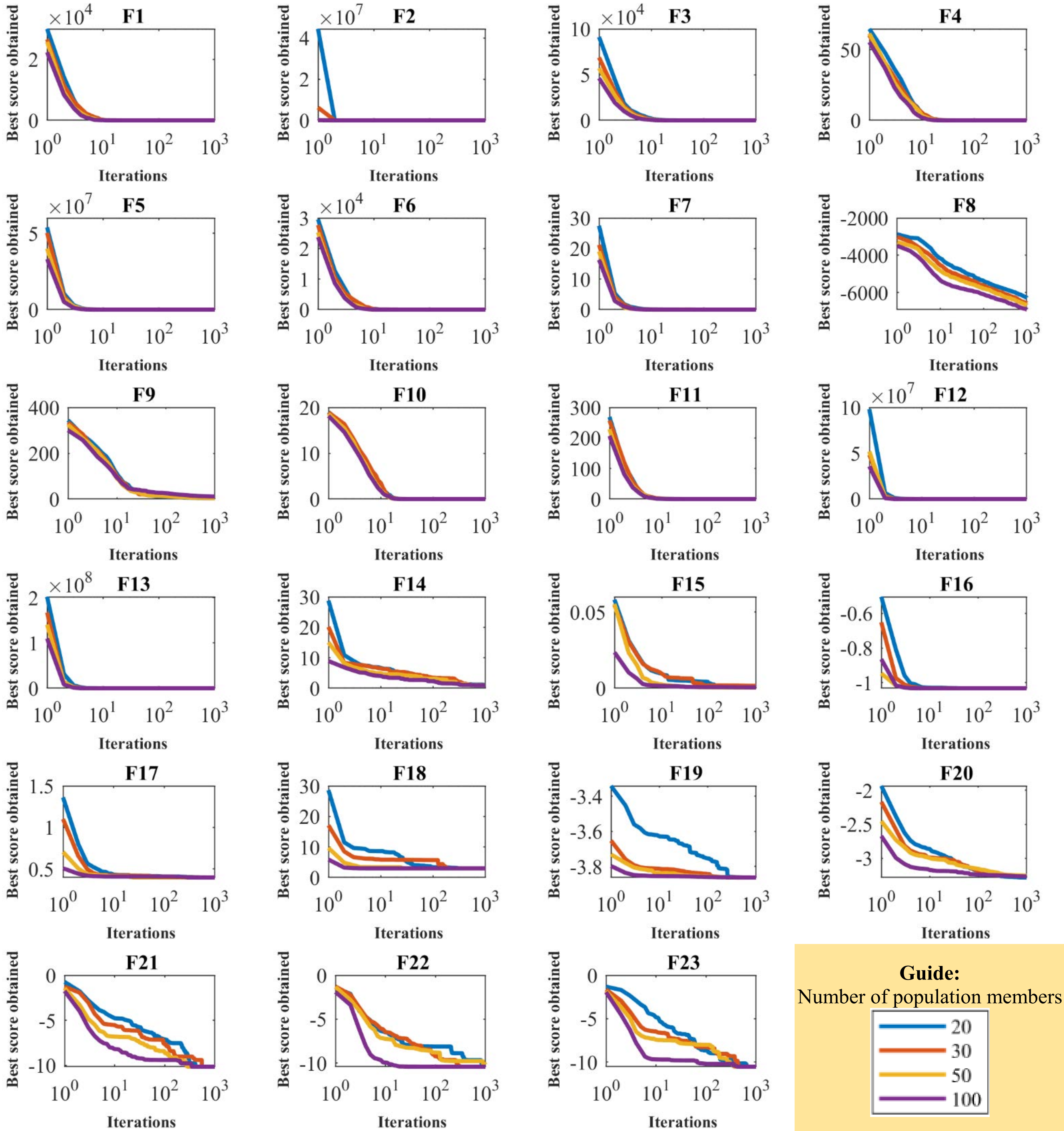

FIGURE 5. Sensitivity analysis of the ZOA for the number of population members.

mentioned algorithms are employed in twenty independent implementations, while each execution contains 1000 repetitions. The experiments are done in the Matlab R2020a version in the environment of Microsoft Windows 10 with 64 bits on the Core i-7 processor with 2.40 GHz and 6 GB memory. The optimization results obtained for the benchmark functions have been reported using two indicators: the average of the optimal solutions obtained (avg) and the standard deviation of these solutions (std).

A. EVALUATION OF UNIMODAL BENCHMARK

The unimodal functions F1 to F7 are a good set to evaluate the exploitability of optimization algorithms because they have only one main solution without having any local solutions.

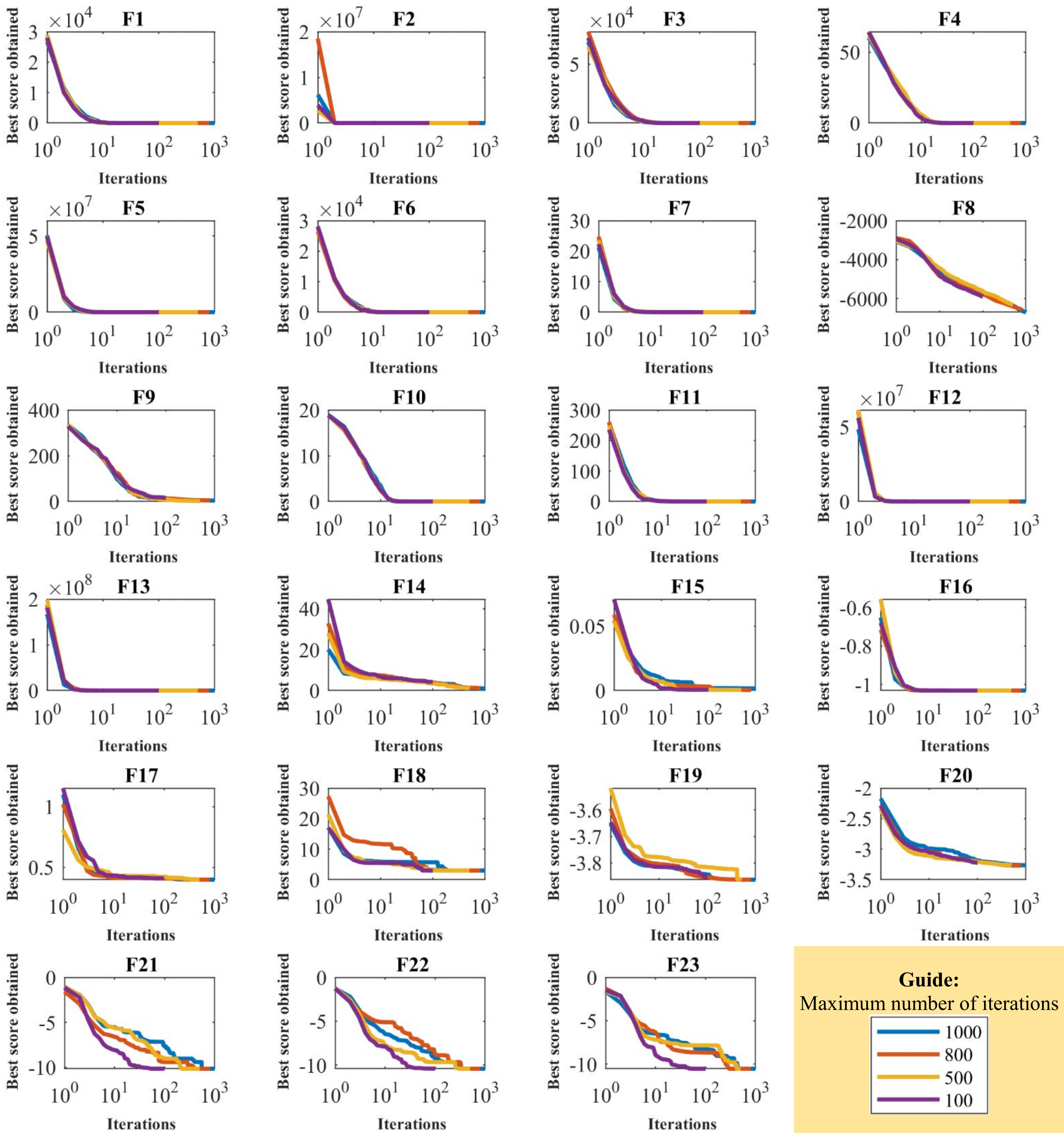

FIGURE 6. Sensitivity analysis of the ZOA for the maximum number of iterations.

Table 3 shows the results of optimizing the functions F1 to F7. The optimization results show that ZOA with high exploitation power has provided the global optimal in F6 solution. ZOA is the first best optimizer compared to the competitor algorithms in optimizing F1, F2, F3, F4, F5, and F7. The simulation results show that ZOA has a superior performance in optimizing the unimodal functions F1 to F7 against nine competitor algorithms.

B. EVALUATION OF HIGH-DIMENSIONAL MULTIMODAL BENCHMARK

The high-dimensional multimodal functions F8 to F13 are a good set to evaluate the exploration power of optimization algorithms because in addition to the main optimal solution, they also have several local solutions in the search space. Table 4 presents the optimization results of functions F8 to F13 using the proposed ZOA and nine competitor algorithms.

TABLE 11. Evaluation results of CEC2015 test functions.

		ZOA	QANA	MPA	TSA	WOA	GWO	TLBO	GSA	GA	PSO
CEC1	Ave	2.13E+05	2.14E+05	2.02E+06	4.37E+05	1.47E+06	6.06E+05	7.65E+06	1.50E+06	1.50E+06	3.20E+07
	std	2.36E+05	2.67E+05	2.08E+06	4.73E+05	2.63E+06	5.02E+05	3.07E+06	1.21E+06	1.21E+06	8.37E+06
CEC2	Ave	1.04E+04	1.05E+04	5.65E+06	9.41E+03	1.97E+04	1.43E+04	7.33E+08	6.70E+06	6.70E+06	4.58E+04
	std	6.92E+03	7.78E+03	6.03E+06	1.08E+04	1.46E+04	1.03E+04	2.33E+08	1.34E+08	1.34E+08	1.09E+04
CEC3	Ave	3.20E+02	3.22E+02	3.20E+02	3.20E+02	3.20E+02	3.20E+02	3.20E+02	3.20E+02	3.20E+02	3.20E+02
	std	5.21E-02	5.74E-02	7.08E-02	8.61E-02	9.14E-02	3.19E-02	7.53E-02	1.16E-03	1.16E-03	1.11E-05
CEC4	Ave	4.10E+02	4.13E+02	4.16E+02	4.08E+02	4.26E+02	4.18E+02	4.42E+02	4.40E+02	4.19E+02	4.39E+02
	std	1.62E+01	1.87E+01	1.03E+01	3.96E+00	1.17E+01	1.03E+01	7.72E+00	5.61E+01	5.61E+01	7.25E+00
CEC5	Ave	8.31E+02	8.33E+02	9.20E+02	8.65E+02	1.33E+03	1.09E+03	1.76E+03	9.81E+02	9.81E+02	1.75E+03
	std	5.12E+01	6.13E+01	1.78E+02	2.16E+02	3.45E+02	2.81E+02	2.30E+02	2.06E+02	2.06E+02	2.79E+02
CEC6	Ave	1.18E+03	1.19E+03	2.26E+04	1.86E+03	7.35E+03	3.82E+03	2.30E+04	4.05E+03	4.05E+03	3.91E+06
	std	2.37E+02	2.74E+02	2.45E+04	1.93E+03	3.82E+03	2.44E+03	2.41E+04	1.05E+04	1.05E+04	2.70E+06
CEC7	Ave	7.02E+02	7.06E+02	7.02E+02	7.02E+02	7.02E+02	7.02E+02	7.06E+02	7.02E+02	7.02E+02	7.08E+02
	std	2.90E-02	3.22E-02	7.07E-01	7.75E-01	1.10E+00	9.40E-01	9.07E-01	5.50E-01	5.50E-01	1.32E+00
CEC8	Ave	1.40E+03	1.40E+03	3.49E+03	3.43E+03	9.93E+03	2.58E+03	6.73E+03	1.47E+03	1.47E+03	6.07E+05
	std	1.12E+03	1.33E+03	2.04E+03	2.77E+03	8.74E+03	1.61E+03	3.36E+03	2.34E+03	2.34E+03	4.81E+05
CEC9	Ave	1.00E+03	1.00E+03	1.00E+03	1.00E+03	1.00E+03	1.00E+03	1.00E+03	1.00E+03	1.00E+03	1.00E+03
	std	4.82E-03	5.35E-03	1.28E-01	7.23E-02	2.20E-01	5.29E-02	9.79E-01	1.51E+01	1.51E+01	5.33E+00
CEC10	Ave	2.06E+03	2.07E+03	4.00E+03	3.27E+03	8.39E+03	2.62E+03	9.91E+03	1.23E+03	1.23E+03	3.42E+05
	std	1.98E-01	2.25E-01	2.82E+03	1.84E+03	1.12E+04	1.78E+03	8.83E+03	2.51E+04	2.51E+04	1.74E+05
CEC11	Ave	1.27E+03	1.28E+03	1.40E+03	1.35E+03	1.37E+03	1.39E+03	1.35E+03	1.35E+03	1.35E+03	1.41E+03
	std	1.60E+00	1.91E+00	5.81E+01	1.12E+02	8.97E+01	5.42E+01	1.11E+02	1.41E+01	1.41E+01	7.73E+01
CEC12	Ave	1.30E+03	1.31E+03	1.30E+03	1.30E+03	1.30E+03	1.30E+03	1.31E+03	1.30E+03	1.30E+03	1.31E+03
	std	8.22E-02	9.47E-02	6.69E-01	6.94E-01	9.14E-01	8.07E-01	1.54E+00	7.50E+00	7.50E+00	2.05E+00
CEC13	Ave	1.30E+03	1.31E+03	1.30E+03	1.30E+03	1.30E+03	1.30E+03	1.30E+03	1.30E+03	1.30E+03	1.35E+03
	std	8.91E-04	1.04E-03	1.92E-03	5.44E-03	1.04E-03	2.43E-03	3.78E-03	6.43E-03	6.43E-03	4.70E+01
CEC14	Ave	6.18E+03	6.21E+03	7.29E+03	7.10E+03	7.60E+03	7.34E+03	7.51E+03	6.22E+03	6.22E+03	9.30E+03
	std	1.23E+02	1.40E+02	2.45E+03	3.12E+03	1.29E+03	2.47E+03	1.52E+03	2.12E+03	2.12E+03	4.04E+02
CEC15	Ave	1.60E+03	1.61E+03	1.61E+03	1.60E+03	1.61E+03	1.60E+03	1.62E+03	1.60E+03	1.60E+03	1.64E+03
	std	1.35E+00	1.59E+00	4.94E+00	2.66E+07	1.13E+01	1.80E+02	3.64E+00	5.69E+01	5.69E+01	1.12E+01
Wins of ZOA in 15 functions			8								
Ties of ZOA in 15 functions			5, But based on the "std" index, the ZOA approach is superior								
Losses of ZOA in 15 functions			2								

ZOA with high exploration power has been able to provide the global optimum for functions F9 and F11 after identifying the optimal area. ZOA is the first best optimizer for functions F10 and F12. In optimization of F8, ZOA is the fourth optimizer after GA, TLBO, and PSO. In optimization of F13, ZOA is the second-best optimizer after GSA. The simulation results show the acceptable exploration power of ZOA in accurately scanning the search space and passing local optimal areas.

FIGURE 7. Schematic view of tension/compression spring problem.

C. EVALUATION OF FIXED- DIMENSIONAL MULTIMODAL BENCHMARK

The fixed-dimensional multimodal functions F14 to F23 challenge the exploration ability of optimization algorithms to find the optimal region in low-dimensional problems. Table 5 releases the implementation results of the proposed ZOA and nine competitor algorithms in solving functions F14 to F23. ZOA is the first best optimization for F15 and F20 functions. In addition, the simulation results show that in solving the functions F14, F16, F17, F18, F19, F21, F22, and F23, although ZOA is similar in avg

TABLE 12. Evaluation results of CEC2017 test functions.

		ZOA	QANA	MPA	TSA	WOA	GWO	TLBO	GSA	GA	PSO
C1	Ave	6.54E+02	2.47E+03	2.38E+05	4.47E+04	2.12E+05	1.57E+05	6.16E+04	3.30E+06	7.75E+05	8.05E+07
	std	7.19E+02	2.43E+03	2.28E+07	4.83E+06	2.18E+07	2.73E+07	5.12E+06	8.47E+07	3.17E+07	3.52E+06
C2	Ave	2.00E+02	2.02E+02	3.23E+04	9.51E+03	5.75E+05	1.07E+03	1.53E+04	4.68E+03	7.43E+07	6.91E+06
	std	5.16E+00	5.07E+02	4.29E+06	1.18E+05	6.13E+07	1.56E+05	1.13E+05	1.19E+04	2.43E+09	3.14E+05
C3	Ave	3.00E+02	3.10E+02	3.30E+02	3.30E+02	3.30E+02	3.30E+02	3.30E+02	3.30E+02	3.30E+02	3.30E+02
	std	1.61E-03	2.64E-02	3.86E-03	8.71E-03	7.18E-03	9.24E-03	3.29E-03	1.21E-02	7.63E-03	7.18E-01
C4	Ave	4.00E+02	4.04E+02	4.21E+02	4.19E+02	4.26E+02	4.36E+02	4.28E+02	4.49E+02	4.52E+02	5.62E+02
	std	1.56E+01	8.70E-01	1.81E+02	3.06E+01	1.13E+02	1.27E+02	1.13E+02	7.35E+02	7.82E+02	4.88E+03
C5	Ave	5.09E+02	5.11E+02	9.23E+02	8.75E+02	9.30E+02	1.43E+04	1.19E+04	1.85E+03	1.86E+03	1.64E+04
	std	9.34E+01	4.04E+00	1.95E+03	2.26E+03	1.88E+03	3.55E+04	2.91E+03	2.89E+03	2.40E+03	3.46E+02
C6	Ave	6.31E+02	6.50E+02	1.39E+04	1.96E+03	2.36E+03	7.45E+03	3.92E+04	3.01E+05	2.40E+04	1.09E+06
	std	6.49E+02	1.65E-04	1.25E+05	1.03E+04	2.55E+05	3.92E+04	2.54E+04	2.80E+07	2.51E+05	2.62E+06
C7	Ave	7.12E+02	7.21E+02	7.12E+02	7.12E+02	7.12E+02	7.12E+02	7.12E+03	7.18E+02	7.16E+02	7.65E+02
	std	4.73E-03	6.31E+00	6.86E-02	7.85E-02	7.17E-02	1.20E+01	9.50E-02	1.42E+01	9.17E-02	3.47E+00
C8	Ave	8.07E+02	8.10E+02	1.96E+03	3.53E+03	3.59E+04	9.03E+03	2.68E+04	6.17E+04	6.83E+03	1.70E+03
	std	7.96E+02	3.21E+00	1.08E+04	2.87E+04	2.14E+04	8.84E+05	1.71E+04	4.91E+08	3.46E+04	3.14E+03
C9	Ave	9.12E+02	9.15E+02	1.10E+03	1.10E+03	1.10E+04	1.10E+03	1.10E+03	1.10E+04	1.10E+03	1.12E+03
	std	1.14E-03	2.50E-02	1.53E-02	7.33E-02	1.38E-02	2.30E-01	5.39E-03	5.43E+01	9.89E-02	0.00E+01
C10	Ave	1.37E+03	1.42E+03	2.10E+03	3.37E+03	4.10E+04	8.49E+04	2.72E+03	3.52E+04	9.01E+04	3.14E+03
	std	1.74E+02	2.88E+02	2.83E+04	1.94E+04	2.92E+04	1.22E+05	1.88E+04	1.84E+06	8.93E+04	3.67E+02
C11	Ave	1.10E+03	1.11E+03	1.48E+03	1.45E+03	1.50E+04	1.47E+03	1.49E+03	1.51E+03	1.45E+03	2.79E+03
	std	1.39E+01	5.52E+00	2.52E+02	1.22E+03	5.91E+02	8.07E+02	5.52E+02	7.83E+02	1.21E+03	3.33E+03
C12	Ave	1.40E+03	1.03E+04	1.40E+03	1.40E+03	1.40E+03	1.40E+03	1.40E+05	1.41E+06	1.41E+03	1.68E+04
	std	1.14E-03	1.08E+04	7.99E-02	6.04E-02	6.79E-02	9.24E-02	8.17E-02	2.15E+01	1.64E+01	5.23E+01
C13	Ave	1.40E+02	8.02E+03	1.40E+02	1.40E+03	1.40E+06	1.40E+04	1.40E+03	1.45E+02	1.40E+04	3.06E+04
	std	2.38E-06	7.39E+03	2.86E-05	5.54E-04	1.02E-05	1.14E-04	2.53E-05	4.80E+02	3.88E-04	2.12E+04
C14	Ave	1.41E+03	1.46E+03	4.35E+04	1.40E+03	7.39E+03	7.70E+04	7.44E+04	9.40E+03	7.61E+03	6.50E+03
	std	6.45E+04	3.58E+01	1.83E+04	3.22E+04	2.55E+04	1.39E+04	2.57E+04	4.14E+03	1.62E+04	1.87E+02
C15	Ave	1.51E+03	1.59E+03	1.70E+03	1.70E+03	1.71E+04	1.71E+06	1.70E+03	1.74E+06	1.72E+06	2.39E+06
	std	3.85E+01	5.28E+01	3.86E+01	2.76E+02	4.04E+01	1.23E+02	1.90E+02	1.22E+01	3.74E+01	2.49E+05
C16	Ave	1.60E+03	1.65E+03	3.28E+05	5.37E+05	3.02E+06	2.47E+05	7.06E+05	4.20E+06	8.65E+05	4.51E+05
	std	3.60E-01	5.60E+01	3.18E+09	5.73E+08	3.08E+09	3.63E+09	6.02E+09	9.37E+09	4.07E+09	1.38E+04
C17	Ave	1.72E+03	1.73E+03	4.13E+04	8.41E+04	6.65E+06	2.97E+04	2.43E+05	5.58E+03	8.33E+06	2.83E+04
	std	3.09E+00	1.99E+01	5.19E+04	2.08E+04	7.03E+05	2.46E+04	2.03E+03	2.09E+03	3.33E+07	8.61E+04
C18	Ave	4.20E+02	1.25E+04	4.20E+02	4.20E+02	4.20E+02	4.20E+02	4.20E+03	4.20E+03	4.20E+02	2.43E+03
	std	3.20E-05	1.25E+04	4.76E-04	9.61E-04	8.08E-04	8.14E-04	4.19E-04	2.11E-07	8.53E-04	2.02E+03
C19	Ave	1.91E+01	1.95E+03	5.11E+03	1.09E+01	5.16E+02	5.26E+02	5.18E+03	5.39E+03	5.42E+02	1.41E+03
	std	2.92E+00	5.18E+01	2.71E+02	4.90E-01	2.03E+02	2.17E+02	2.03E+02	8.25E+02	8.72E+02	2.26E+03
C20	Ave	8.01E+02	2.02E+03	8.13E+03	7.65E+02	8.20E+02	2.33E+03	2.09E+03	2.75E+03	2.76E+04	1.40E+04
	std	1.49E+01	2.45E+01	2.85E+01	3.16E+01	2.78E+00	4.45E+02	3.81E+01	3.79E+01	3.30E+02	7.74E+03
C21	Ave	2.20E+03	2.31E+03	2.29E+03	2.86E+03	3.26E+04	8.35E+04	4.82E+04	4.91E+06	3.30E+04	2.19E+04
	std	4.55E-13	2.31E+01	2.15E+04	2.93E+03	3.45E+05	4.82E+03	3.44E+04	3.70E+07	3.41E+04	3.77E+03
C22	Ave	8.01E+02	2.30E+03	8.02E+02	8.02E+02	8.02E+03	8.02E+02	8.02E+03	8.08E+02	8.06E+03	8.49E+03
	std	2.76E-01	2.02E+01	7.76E-01	8.75E-01	8.07E-01	2.10E+01	8.40E-01	2.32E+00	8.07E-02	1.75E+03
C23	Ave	2.60E+03	2.62E+03	2.86E+03	4.43E+04	4.49E+04	8.93E+04	3.58E+03	7.07E+04	7.73E+03	4.30E+04
	std	1.20E+00	6.08E+00	2.98E+04	3.77E+04	3.04E+04	9.74E+04	2.61E+04	5.81E+06	4.36E+04	6.24E+03
C24	Ave	2.50E+03	2.74E+03	2.00E+04	2.00E+04	2.00E+04	2.00E+04	2.00E+04	2.00E+04	2.00E+04	5.07E+04
	std	0.00E+00	7.59E+00	2.43E-02	7.23E-03	2.28E-02	3.20E-02	6.29E-03	6.33E+01	8.79E-02	4.13E+01
C25	Ave	2.90E+03	2.93E+03	3.00E+04	4.27E+04	5.00E+04	9.39E+04	3.62E+04	4.42E+06	8.91E+04	4.81E+03
	std	1.23E+00	2.18E+01	3.73E+04	2.84E+04	3.82E+04	2.12E+05	2.78E+04	2.74E+06	6.83E+04	2.80E+03
C26	Ave	1.90E+03	2.97E+03	2.38E+04	2.35E+05	2.40E+04	2.37E+04	2.39E+04	2.41E+04	2.35E+04	1.13E+04
	std	6.43E-13	1.82E+02	3.42E+02	2.12E+03	6.81E+02	9.97E+02	6.42E+02	8.73E+02	2.11E+03	5.62E+02
C27	Ave	3.09E+03	3.09E+03	2.30E+04	2.30E+04	2.30E+04	2.30E+04	2.30E+04	2.31E+04	2.31E+04	5.11E+03
	std	1.02E-01	2.46E+00	8.89E-02	7.94E-02	7.69E-02	8.14E-02	9.07E-02	3.05E+00	3.54E+01	1.11E+02
C28	Ave	3.10E+03	3.30E+03	5.30E+04	5.30E+04	5.30E+04	5.30E+04	5.30E+04	5.35E+04	5.30E+04	4.60E+04

TABLE 12. ***(Continued.)*** **Evaluation results of CEC2017 test functions.**

	std	4.55E-13	1.47E+02	3.76E-05	6.44E-04	3.92E-05	2.04E+04	3.43E-05	4.70E+02	4.78E-04	6.84E+00
C29	Ave	3.16E+03	3.17E+03	5.25E+04	8.10E+04	8.29E+03	8.60E+03	8.34E+04	8.30E+04	8.51E+03	5.63E+03
	std	1.27E+01	2.72E+01	2.73E+04	4.12E+04	3.45E+04	2.29E+05	3.47E+05	5.04E+03	2.52E+04	1.32E+01
C30	Ave	4.28E+03	2.97E+05	2.60E+04	2.60E+04	2.61E+04	2.61E+04	2.60E+04	2.64E+04	2.62E+04	6.05E+05
	std	4.58E+02	5.05E+05	4.76E+01	3.66E-04	5.94E+01	2.13E+02	2.80E-03	2.12E+02	4.64E+01	2.97E+04
Wins of ZOA in 30 functions			27								
Ties of ZOA in 30 functions			0								
Losses of ZOA in 30 functions			3								

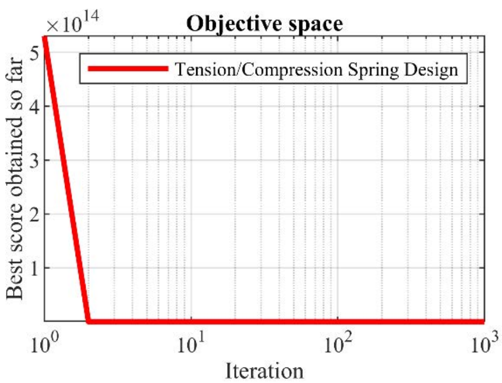

FIGURE 8. **Convergence analysis of the ZOA for the tension/compression spring design optimization problem.**

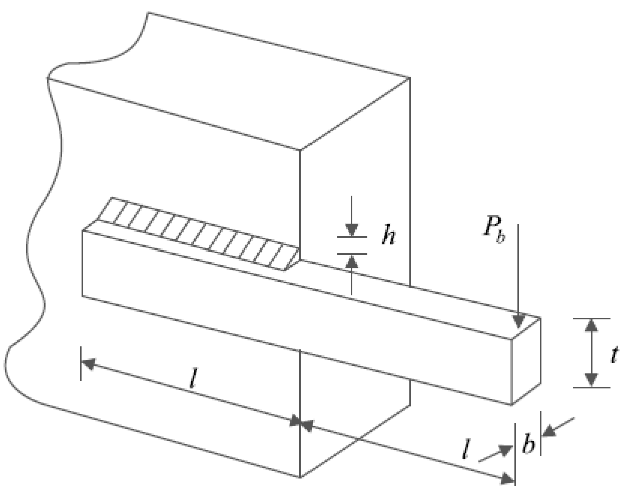

FIGURE 9. **Schematic view of the welded beam design problem.**

criterion to some competitor algorithms, but has better std criterion. Therefore, ZOA is a more effective optimizer in solving these objective functions. The simulation results show that ZOA has a superior performance in solving fixed-dimensional multimodal functions compared to nine competitor algorithms.

The performance of ZOA and nine competitor algorithms in optimizing functions F1 to F23 as boxplot is presented in Figure 2. In addition, Figure 3 plots the convergence curves of ZOA and competing algorithms to achieve a solution.

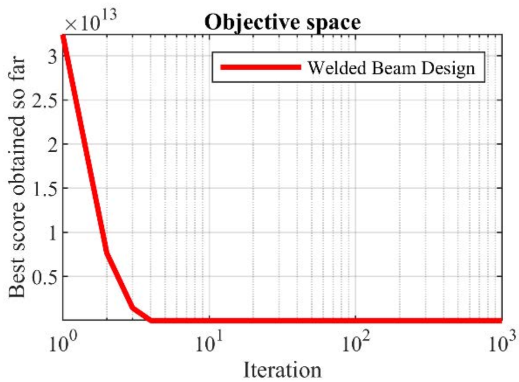

FIGURE 10. **Convergence analysis of the ZOA for the welded beam design optimization problem.**

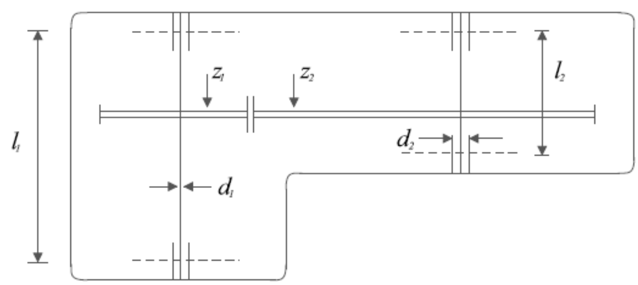

FIGURE 11. **Schematic view of speed reducer design problem.**

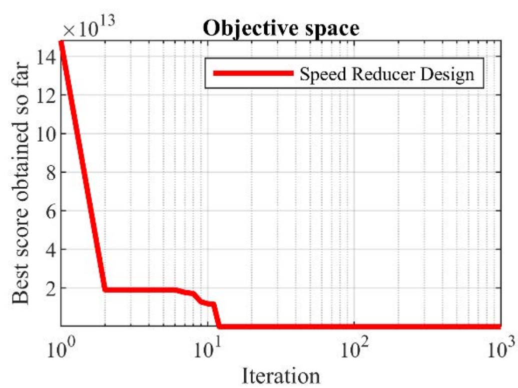

FIGURE 12. **Convergence analysis of the ZOA for the speed reducer design optimization problem.**

D. STATISTICAL ANALYSIS

In this subsection, a statistical analysis is presented to determine whether the superiority of ZOA over nine competitor

TABLE 13. Comparison results for the tension/compression spring design problem.

Algorithm	Optimum Variables			Optimum Cost
	d	*D*	*P*	
ZOA	0.0520983	0.366644	10.7299	**0.012668010**
QANA	0.051926	0.362432	10.961632	0.012701236
TSA	0.051655	0.3471885	12.216455	0.0128007
MPA	0.05068	0.3449564	12.194225	0.0128051
WOA	0.0505	0.3135181	15.15	0.0133245
GWO	0.0505	0.3191156	14.368492	0.0129451
TLBO	0.051288	0.3381268	12.849917	0.0128368
GSA	0.0505	0.3204851	14.370957	0.0130026
PSO	0.050601	0.3132121	14.14	0.0131666
GA	0.050753	0.319515	15.392	0.012904

TABLE 14. Statistical results for the tension/compression spring design problem.

Algorithm	Best	Mean	Worst	SD	Median
ZOA	**0.012668010**	0.012681274	0.012710289	0.000023	0.012681792
QANA	0.012701236	0.012723624	0.012748010	0.000056	0.012716240
TSA	0.0128007	0.012810947	0.012842337	2.73E-05	0.012814166
MPA	0.0128051	0.012824087	0.012847965	4.14E-05	0.012826683
WOA	0.0133245	0.014965353	0.018041132	0.00229472	0.013324506
GWO	0.0129451	0.014609016	0.018018134	0.00163822	0.014161449
TLBO	0.0128368	0.012968033	0.013128432	7.88E-05	0.012973111
GSA	0.0130026	0.01357326	0.014353848	0.00028987	0.013501567
PSO	0.0131666	0.014176617	0.016413937	0.00209373	0.013132389
GA	0.012904	0.013200571	0.015366372	0.00037875	0.013081663

TABLE 15. Comparison results for the welded beam design problem.

Algorithm	Optimum Variables				Optimum Cost
	h	*l*	*t*	*b*	
ZOA	0.205739	3.470261	9.036623	0.205740	**1.7249160**
QANA	0.205730	3.470489	9.036624	0.205730	1.7249556
TSA	0.205666	3.4765834	9.0403169	0.2059139	1.7265238
MPA	0.205781	3.4771407	9.0414825	0.2063321	1.7278585
WOA	0.19751	3.3167185	10.005	0.2014957	1.8213052
GWO	0.205714	3.4738391	9.0454515	0.2058119	1.7263347
TLBO	0.204797	3.5380591	9.0087921	0.21013	1.7600526
GSA	0.147172	5.4934894	10.005	0.2178339	2.1739444
PSO	0.164253	4.0345573	10.005	0.2237588	1.874908
GA	0.20659	3.63769	10.005	0.203351	1.837168

algorithms is statistically significant. Wilcoxon rank sum test [54] is a statistical test that is used to compare two data samples with the aim of detecting significant differences between them. In this test, a p-value is the criterion for determining the superiority of one algorithm over another algorithm. The Wilcoxon rank sum test is implemented on the optimization results of ZOA and nine competitor algorithms and the results are presented in Table 6. What can be deduced from the results of statistical analysis is that ZOA has a significant superiority over the corresponding competitor algorithm in cases where the p-value is less than 5%.

The Friedman test [55] is used to analyze the superiority of ZOA based on ranking its performance in achieving solutions to optimization problems. The results of the Friedman test on the performance of optimization algorithms in handling the functions F1 to F23 are presented in Table 7. What is evident from the simulation results is that ZOA offers superior performance compared to competitor algorithms.

E. POPULATION DIVERSITY ANALYSIS

Population diversity plays an important role in increasing the global search capability of optimization algorithms in order

TABLE 16. Statistical results for the welded beam design problem.

Algorithm	Best	Mean	Worst	SD	Median
ZOA	**1.7249160**	1.725326	1.725402	0.000010	1.725206
QANA	1.7249556	1.725412	1.725561	0.000127	1.752468
TSA	1.7265238	1.72669	1.72693	0.00029	1.72665
MPA	1.7278585	1.72799	1.72843	0.00116	1.72795
WOA	1.8213052	2.23143	3.04976	0.32469	2.24579
GWO	1.7263347	1.73054	1.74252	0.00487	1.72828
TLBO	1.7600526	1.81857	1.87434	0.02756	1.82104
GSA	2.1739444	2.54551	3.00516	0.25599	2.49636
PSO	1.874908	2.1203	2.32129	0.03484	2.0981
GA	1.837168	1.36421	2.03626	0.13955	1.93672

TABLE 17. Comparison results for the speed reducer design problem.

Algorithm	Optimum Variables							Optimum Cost
	b	m	p	l_1	l_2	d_1	d_2	
ZOA	3.50112	0.7	17	7.3423	7.80116	3.35194	5.28818	**2998.5189**
QANA	3.50012	0.7	17	7.90557	7.80004	3.35219	5.2867	3002.2645
TSA	3.519098	0.7	17	7.3	7.8	3.3680264	5.3151837	3013.5435
MPA	3.524223	0.7	17	7.380933	7.815726	3.3746362	5.3132018	3016.2944
WOA	3.517519	0.7	17	8.3	7.8	3.3691741	5.3131486	3020.7918
GWO	3.526045	0.7	17	7.392843	7.816034	3.3748634	5.3132109	3017.9426
TLBO	3.526299	0.7	17	7.3	7.8	3.4783251	5.3156591	3045.7158
GSA	3.618	0.7	17	8.3	7.8	3.3865063	5.3156701	3066.3756
PSO	3.527804	0.7	17	8.35	7.8	3.379012	5.3141616	3082.8988
GA	3.537725	0.7	17	8.37	7.8	3.3838049	5.3151626	3044.147

TABLE 18. Statistical results for the speed reducer design problem.

Algorithm	Best	Mean	Worst	SD	Median
ZOA	**2998.5189**	2999.212	3002.614	1.36421	2999.018
QANA	3002.2645	3004.1640	3005.0318	1.42368	3003.4691
TSA	3013.54	3014.64	3018.91	1.94159	3014.18
MPA	3016.29	3020.87	3023.8	5.86713	3019.54
WOA	3020.79	3120.78	3227.23	80.0363	3120.78
GWO	3017.94	3043.99	3076.26	13.0837	3042.17
TLBO	3045.72	3081.25	3120.3	18.1646	3080.94
GSA	3066.38	3186.19	3380.69	93.0355	3172.54
PSO	3082.9	3202.46	3329.76	17.2042	3214.18
GA	3044.15	3311.81	3637.56	57.3086	3305.1

to prevent optimization algorithms from getting stuck in local optimal solutions. In this subsection, population diversity analysis on the performance of ZOA in the optimization process is presented. In order to analyze the ZOA's population diversity while achieving the solution, the I_c index is employed, which is calculated according to (7) and (8) [56].

$$I_C = \sum\nolimits_{j=1}^{m} \sum\nolimits_{i=1}^{N} \left(x_{i,j} - c_j\right)^2, \tag{7}$$

$$c_j = \frac{1}{N} \sum\nolimits_{i=1}^{N} x_{i,j}, \tag{8}$$

where $c_j, j = 1, 2, \ldots, m$, are the centroids, and I_C is the index of the spreading of population members.

The population diversity analysis in optimizing objective functions F1 to F23 is shown in Figure 4. For each objective function, ZOA's convergence curve and its population diversity are presented. Simulation results and Figure 4 show that ZOA has a high population diversity in the optimizing process of most objective functions.

F. SCALABILITY ANALYSIS

In this subsection, the scalability analysis of ZOA in the optimization process is presented. In this analysis, ZOA is implemented on functions F1 to F13 for different dimensions of 10, 30, 50, 100, 500, and 1000. The simulation results are presented in Table 8. What emerges from the review of the

TABLE 19. Comparison results for the pressure vessel design problem.

Algorithm	Optimum Variables				Optimum Cost
	T_s	T_h	R	L	
ZOA	0.7781084	0.3859585	40.31504	199.9663	**5887.2057**
QANA	0.7781687	0.384649	40.319619	200.000000	5912.6436
TSA	0.83079	0.41641	42.7726	169.43	6051.81
MPA	0.77942	0.38485	40.348	199.75	5892.31
WOA	0.77935	0.38488	40.3411	200.1	5894.33
GWO	0.84614	0.41877	43.8382	156.46	6014.52
TLBO	0.81799	0.41814	41.7703	183.664	6140.44
GSA	1.08634	0.95009	49.3699	169.572	11556.1
PSO	0.75274	0.39974	40.4727	198.102	5893.27
GA	1.100073	0.907032	44.47863	179.7487	6553.298

TABLE 20. Statistical results for the pressure vessel design problem.

Algorithm	Best	Mean	Worst	SD	Median
ZOA	**5887.2057**	5890.1084	5892.2106	1.1621	5889.0846
QANA	5912.6436	5916.5174	5919.0619	1.9814	5914.5891
TSA	6051.81	6055.65	6074.29	2.89445	6053.25
MPA	5892.31	5894.47	5897.57	13.917	5893.6
WOA	5894.33	6534.77	7398.29	534.386	6419.32
GWO	6014.52	6480.54	7254.54	327.171	6400.68
TLBO	6140.44	6329.92	6515.61	126.672	6321.48
GSA	11556.1	23354	33242.9	5793.52	24022
PSO	5893.27	6267.14	7009.25	496.376	6115.75
GA	6553.298	6647.31	8009.44	657.852	7589.8

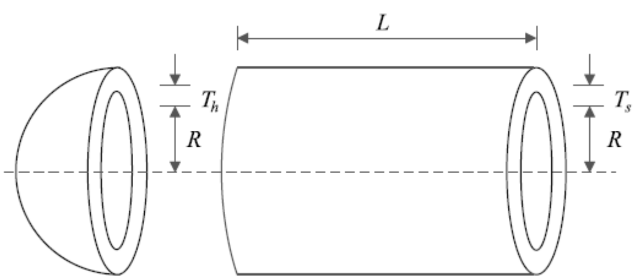

FIGURE 13. Schematic view of pressure vessel design problem.

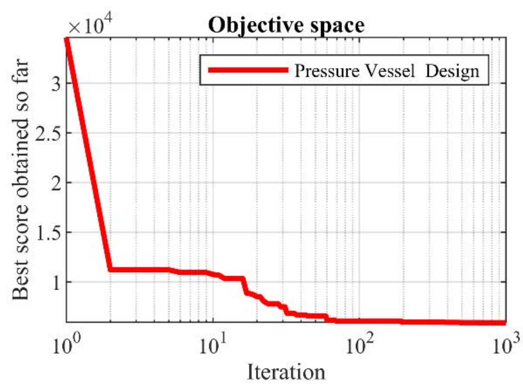

FIGURE 14. Convergence analysis of the ZOA for the pressure vessel design optimization problem.

results is that the ZOA, while increasing the dimensions of the problem, still maintains its efficiency and provides acceptable solutions.

G. SENSITIVITY ANALYSIS

ZOA is able to solve optimization problems in an iteration-based process and based on scanning the search space by its population members. As a result, changes in the number of zebras' population (N) and the maximum number of iterations (T) affect ZOA performance. In this subsection, a sensitivity analysis on ZOA performance with respect to the parameters N and T are presented.

To sensitivity analyze to parameter N, the proposed ZOA has been employed for zebras' populations of sizes 20, 30, 50, and 100 in the optimization of F1 to F23. The results of sensitivity analysis to parameter N are presented in Table 9. ZOA convergence curves under the influence of parameter N changes are shown in Figure 5. What can be deduced from the simulation results is that increasing the value of N improves the algorithm's exploration power in identifying the optimal area and thus ZOA provides better solutions.

In order to sensitivity analyze to parameter T, the proposed ZOA has been employed for the maximum number of iterations of 100, 500, 800, and 1000 in the optimization of F1 to F23. The behavior of the ZOA convergence curves under the sensitivity analysis to the T parameter is shown

TABLE 21. **Unimodal objective functions.**

Objective Function	Range	*Dim*	F_{min}
$F_1(X)=\sum_{i=1}^{m} x_i^2$	$[-100, 100]$	*30*	*0*
$F_2(X)=\sum_{i=1}^{m}\lvert x_i\rvert+\prod_{i=1}^{m}\lvert x_i\rvert$	$[-10, 10]$	30	0
$F_3(X)=\sum_{i=1}^{m}\left(\sum_{j=1}^{i} x_i\right)^2$	$[-100, 100]$	30	0
$F_4(X)=max\{\lvert x_i\rvert\}, \quad 1\le i\le m$	$[-100, 100]$	30	0
$F_5(X)=\sum_{i=1}^{m-1}\left[100\left(x_{i+1}-x_i^2\right)^2+(x_i-1)^2)\right]$	$[-30, 30]$	30	0
$F_6(X)=\sum_{i=1}^{m}([x_i+0.5])^2$	$[-100, 100]$	30	0
$F_7(X) = \sum_{i=1}^{m} ix_i^4+r,$ $where\ r\ is\ a\ random\ real\ number\ in\ the\ range\ 0\ to\ 1$	$[-1.28, 1.28]$	30	0

in Figure 6. The simulation results of this analysis are reported in Table 10. What is evident from the results of ZOA sensitivity analysis to the T parameter is that increasing the value of T gives the algorithm more opportunity to converge towards better solutions based on the exploitation power.

H. EVALUATION OF CEC 2015 BENCHMARK

The ability of ZOA and nine competitor algorithms to optimize CEC1 to CEC15 functions of the CEC2015 benchmark test has been evaluated. The simulation results are presented in Table 11. Analysis and comparison of the results show that ZOA has performed better than the nine competitor algorithms in optimizing CEC1, CEC3, CEC5, CEC6, CEC7, CEC8, CEC9, CEC10, CEC11, CEC12, CEC13, CEC14, and CEC15. TSA has performed better in optimizing CEC2 and CEC4, while ZOA is the second optimizer to solve these functions. Analysis of the CEC2015 optimization results shows the superior performance of ZOA over the nine compared algorithms in most cases.

I. EVALUATION OF CEC 2017 BENCHMARK

The performance of ZOA and nine competitor algorithms have been tested on the optimization of benchmark functions C1 to C30. The optimization results of CEC2017 test functions are presented in Table 12. What can be deduced from the simulation results is that ZOA has provided the optimal solution for C1, C2, C3, C4, C5, C6, C7, C8, C9, C10, C11, C12, C13, C15, C16, C17, C18, C21, C22, C23, C24, C25, C26, C27, C28, C29, and C30. TSA is the first-best optimizer to solve C14, C19, and C20 while ZOA is the second-best optimizer in these functions. Based on the analysis of CEC2017 simulation results, it is clear that ZOA is superiority optimizer over nine competitor algorithms for optimization in most cases.

IV. ZOA APPLICATION FOR ENGINEERING DESIGN PROBLEMS

In this section, the ability of the proposed algorithm to solve real-world problems is tested on four engineering design challenges including tension/compression spring, welded beam, speed reducer, and pressure vessel design problem.

A. TENSION/COMPRESSION SPRING DESING OPTIMIZATION PROBLEM

Tension/compression spring design is a minimization problem, in which the main goal in this design is to reduce the tension/compression spring weight. Figure 7 shows the schematic of the tension/compression spring design problem. The values obtained for the design variables in the tension/compression spring design problem are presented in Table 13. The simulation results show that ZOA provides the optimal solution for this engineering design with the values of the decision variables equal to (0.0520983, 0.366644, 10.7299), and the value of the corresponding objective function is equal to 0.012668010. The statistical results of the performance of the proposed ZOA and nine competitor

TABLE 22. High-dimensional multimodal objective functions.

Objective Function	Range	Dim	F_{min}
$F_8(X)=\sum_{i=1}^{m} -x_i \sin(\sqrt{\lvert x_i\rvert})$	$[-500, 500]$	30	−12569
$F_9(X)=\sum_{i=1}^{m} [\,x_i^2-10\cos(2\pi x_i)+10]$	$[-5.12, 5.12]$	30	0
$F_{10}(X)=-20\exp\left(-0.2\sqrt{\frac{1}{m}\sum_{i=1}^{m} x_i^2}\right)-\exp\left(\frac{1}{m}\sum_{i=1}^{m} cos(2\pi x_i)\right)+20+e$	$[-32, 32]$	30	0
$F_{11}(X)=\frac{1}{4000}\sum_{i=1}^{m} x_i^2-\prod_{i=1}^{m} cos\left(\frac{x_i}{\sqrt{i}}\right)+1$	$[-600, 600]$	30	0
$F_{12}(X)=\frac{\pi}{m}\ \{10\sin(\pi y_1)+\sum_{i=1}^{m}(y_i-1)^2[1+10\sin^2(\pi y_{i+1})]+(y_n-1)^2\}+\sum_{i=1}^{m} u(x_i, 10{,}100{,}4)$, where $y_i = 1+\frac{x_i+1}{4},\ u(x_i,a,i,n)=\begin{cases} k(x_i-a)^n, & x_i>a;\\ 0, & -a\le x_i\le a;\\ k(-x_i-a)^n, & x_i<-a,\end{cases}$	$[-50, 50]$	30	0
$F_{13}(X)=0.1\{\sin^2(3\pi x_1)+\sum_{i=1}^{m}(x_i-1)^2[1+\sin^2(3\pi x_i+1)]+(x_n-1)^2\,[1+\sin^2(2\pi x_m)]\}+\sum_{i=1}^{m} u(x_i, 5{,}100{,}4)$, where $u(x_i,a,i,n)=\begin{cases} k(x_i-a)^n, & x_i>a;\\ 0, & -a\le x_i\le a;\\ k(-x_i-a)^n, & x_i<-a.\end{cases}$	$[-50, 50]$	30	0

algorithms are presented in Table 14. These results show that ZOA with outstanding values in statistical indicators performs better than competitor algorithms. The convergence curve of the proposed ZOA while achieving the solution to the tension/compression spring design problem is shown in Figure 8.

B. WELDED BEAM DESING OPTIMIZATION PROBLEM

This engineering design is a minimization problem whose main challenge is to reduce the fabrication cost of the welded beam. Figure 9 shows the schematic of the welded beam design problem. The simulation results obtained from the ZOA and nine competitor algorithms in solving the welded beam design problem are presented in Table 15. The analysis of the results of this table shows that the proposed ZOA has presented a better performance in optimizing this design with the values of decision variables equal to (0.205739, 3.470261, 9.036623, 0.205740), and the value of the corresponding objective function equal to 1.724916. Statistical results for the welded beam design problem obtained from the mentioned algorithms are presented in Table 16. It can be concluded from this table that the proposed ZOA has provided a more efficient performance in optimizing this problem. The convergence curve behavior of ZOA in optimizing the welded beam design problem is shown in Figure 10.

C. SPEED REDUCER DESING OPTIMIZATION PROBLEM

The design of the speed reducer is one of the minimization problems because the primary goal of this problem is to find its structure so that this speed reducer has the minimum weight. Figure 11 shows the schematic of the speed reducer design problem. The values obtained for decision variables in speed reducer design are reported in Table 17. The proposed ZOA provides the optimal solution with the values of the decision variables equal to (3.50112, 0.7, 17,7.3423, 7.80116, 3.35194, 5.28818), and the value of the corresponding objective function is equal to 2998.5189. The statistical results obtained from the performance of ZOA and nine competitor algorithms on the speed reducer design problem are presented in Table 18. According to this table, the ZOA is superior to competing algorithms with a better position in statistical indicators. The convergence curve of the proposed ZOA in achieving the solution to the speed reducer design problem is shown in Figure 12.

D. PRESSURE VESSEL DESING OPTIMIZATION PROBLEM

This engineering design is a minimization problem whose objective function is to reduce the total cost of material, forming, and welding of a cylindrical vessel. Figure 13 shows the schematic of the pressure vessel design problem. The proposed values for the decision variables in the design of

TABLE 23. **Fixed-dimensional multimodal objective functions.**

Objective Function	Range	Dim	F_{min}
$F_{14}(X)=\left(\frac{1}{500}+\sum_{j=1}^{25}\frac{1}{j+\sum_{i=1}^{2}(x_i-a_{ij})^6}\right)^{-1}$	$[-65.53, 65.53]$	2	0.998
$F_{15}(X)=\sum_{i=1}^{11}\left[a_i-\frac{x_1(b_i^2+b_ix_2)}{b_i^2+b_ix_3+x_4}\right]^2$	$[-5, 5]$	4	0.00030
$F_{16}(X)=4x_1^2-2.1\cdot x_1^4+\frac{1}{3}x_1^6+x_1x_2-4x_2^2+4x_2^4$	$[-5, 5]$	2	−1.0316
$F_{17}(X)=\left(x_2-\frac{5.1}{4\pi^2}x_1^2+\frac{5}{\pi}x_1-6\right)^2+10\left(1-\frac{1}{8\pi}\right)cosx_1+10$	[-5, 10] × [0, 15]	2	0.398
$F_{18}(X)=[1+(x_1+x_2+1)^2(19-14x_1+3x_1^2-14x_2+6x_1x_2+3x_2^2)]$ $[30+(2x_1-3x_2)^2(18-32x_1+12x_1^2+48x_2-36x_1x_2+27x_2^2)]$	$[-5, 5]$	2	3
$F_{19}(X)=-\sum_{i=1}^{4}c_i\exp(-\sum_{j=1}^{3}a_{ij}(x_j-p_{ij})^2)$	$[0, 1]$	3	−3.86
$F_{20}(X)=-\sum_{i=1}^{4}c_i\exp(-\sum_{j=1}^{6}a_{ij}(x_j-p_{ij})^2)$	$[0, 1]$	6	−3.22
$F_{21}(X)=-\sum_{i=1}^{5}[(X-a_i)\cdot(X-a_i)^T+6c_i]^{-1}$	$[0, 10]$	4	−10.1532
$F_{22}(X)=-\sum_{i=1}^{7}[(X-a_i)\cdot(X-a_i)^T+6c_i]^{-1}$	$[0, 10]$	4	−10.4029
$F_{23}(X)=-\sum_{i=1}^{10}[(X-a_i)\cdot(X-a_i)^T+6c_i]^{-1}$	$[0, 10]$	4	−10.5364

the pressure vessel are presented in Table 19. The simulation results show that ZOA provides the optimal solution to this problem by giving the values of the decision variables equal to (0.7781084, 0.3859585, 40.31504, 199.9663), and the value of the corresponding objective function is equal to 5887.2057. The statistical results of the proposed ZOA and nine competitor algorithms are reported in Table 20. This table shows the superiority of ZOA over competitor algorithms in having better statistical indicators for optimizing the pressure vessel design problem. The behavior of the ZOA's convergence curve when solving the problem of design of a pressure vessel is shown in Figure 14.

V. CONCLUSION AND FUTURE WORKS

I In this paper, a new metaheuristic algorithm called Zebra Optimization Algorithm (ZOA), which mimics the natural behaviors of zebras in the wild, was developed. These types of behavior include foraging and defense strategies against predators. The ZOA steps were stated, and then its mathematical modeling was presented. ZOA performance in solving optimization problems was evaluated on sixty-eight benchmark functions, including types of unimodal, high-dimensional multimodal, fixed-dimensional multimodal, CEC2015, and CEC2017. The optimization results showed that ZOA is able to provide optimal solutions for objective functions by creating the appropriate balance between exploitation and exploration. To evaluate the quality of the ZOA, we compared its results with nine other known algorithms, GWO, TLBO, GA, MPA, PSO, TSA, WOA, and GSA. The simulation results show that ZOA performs better in most cases and has better performance against nine competitor algorithms by providing better solutions. ZOA's ability

TABLE 24. IEEE CEC-2015 benchmark test functions.

	Functions	Related basic functions	Dim	*Fmin*
CEC1	Rotated Bent Cigar Function	Bent Cigar Function	30	100
CEC2	Rotated Discus Function	Discus Function	30	200
CEC3	Shifted and Rotated Weierstrass Function	Weierstrass Function	30	300
CEC4	Shifted and Rotated Schwefel's Function	Schwefel's Function	30	400
CEC5	Rotated Katsuura	Katsuura Function	30	500
CEC6	Shifted and Rotated HappyCat Function	HappyCat Function	30	600
CEC7	Shifted and Rotated HGBat Function	HGBat Function	30	700
CEC8	Shifted and Rotated Expanded Griewank's plus Rosenbrock's Function	Griewank's Function Rosenbrock's Function	30	800
CEC9	Shifted and Rotated Expanded Scaffer's F6 Function	Expanded Scaffer's F6 Function	30	900
CEC10	Hybrid Function 1 ($N = 3$)	Schwefel's Function Rastrigin's Function High Conditioned Elliptic Function	30	1000
CEC11	Hybrid Function 2 ($N = 4$)	Griewank's Function Weierstrass Function Rosenbrock's Function Scaffer's F6 Function	30	1100
CEC12	Hybrid Function 3 ($N = 5$)	Katsuura Function HappyCat Function Expanded Griewank's plus Rosenbrock's Function Schwefel's Function Ackley's Function	30	1200
CEC13	Composition Function 1 ($N = 5$)	Rosenbrock's Function High Conditioned Elliptic Function Bent Cigar Function Discus Function High Conditioned Elliptic Function	30	1300
CEC14	Composition Function 2 ($N = 3$)	Schwefel's Function Rastrigin's Function High Conditioned Elliptic Function	30	1400
CEC15	Composition Function 3 ($N = 5$)	HGBat Function Rastrigin's Function Schwefel's Function Weierstrass Function High Conditioned Elliptic Function	30	1500

to optimize real-world problems was studied in four engineering design problems. The optimization results showed the high capability of ZOA to provide optimal solutions in engineering design applications.

The authors make several suggestions for future studies, such as developing binary and multi-objective versions of ZOA. The application of ZOA in solving optimization problems in various sciences and other real-world problems is another future perspective of this study.

APPENDIX A

See Tables 21–25.

APPENDIX B
TENSION/COMPRESSION SPRING DESIGN PROBLEM

$$\textit{Consider } X = [x_1, x_2, x_3] = [d, D, P]$$

$$\textit{Minimize } f(x) = (x_3 + 2)\, x_2 x_1^2.$$

$$\textit{Subject to}: \; g_1(x) = 1 - \frac{x_2^3 x_3}{71785 x_1^4} \le 0,$$

$$g_2(x) = \frac{4x_2^2 - x_1 x_2}{12566(x_2 x_1^3)} + \frac{1}{5108 x_1^2} - 1 \le 0,$$

$$g_3(x) = 1 - \frac{140.45 x_1}{x_2^2 x_3} \le 0,$$

TABLE 25. IEEE CEC-2017 benchmark test functions.

	functions	fmin
C1	Shifted and Rotated Bent Cigar Function	100
C2	Shifted and Rotated Sum of Different Power Function	200
C3	Shifted and Rotated Zakharov Function	300
C4	Shifted and Rotated Rosenbrock's Function	400
C5	Shifted and Rotated Rastrigin's Function	500
C6	Shifted and Rotated Expanded Scaffer's Function	600
C7	Shifted and Rotated Lunacek Bi_Rastrigin Function	700
C8	Shifted and Rotated Non-Continuous Rastrigin's Function	800
C9	Shifted and Rotated Levy Function	900
C10	Shifted and Rotated Schwefel's Function	1000
C11	Hybrid Function 1 ($N = 3$)	1100
C12	Hybrid Function 2 ($N = 3$)	1200
C13	Hybrid Function 3 ($N = 3$)	1300
C14	Hybrid Function 4 ($N = 4$)	1400
C15	Hybrid Function 5 ($N = 4$)	1500
C16	Hybrid Function 6 ($N = 4$)	1600
C17	Hybrid Function 6 ($N = 5$)	1700
C18	Hybrid Function 6 ($N = 5$)	1800
C19	Hybrid Function 6 ($N = 5$)	1900
C20	Hybrid Function 6 ($N = 6$)	2000
C21	Composition Function 1 ($N = 3$)	2100
C22	Composition Function 2 ($N = 3$)	2200
C23	Composition Function 3 ($N = 4$)	2300
C24	Composition Function 4 ($N = 4$)	2400
C25	Composition Function 5 ($N = 5$)	2500
C26	Composition Function 6 ($N = 5$)	2600
C27	Composition Function 7 ($N = 6$)	2700
C28	Composition Function 8 ($N = 6$)	2800
C29	Composition Function 9 ($N = 3$)	2900
C30	Composition Function 10 ($N = 3$)	3000

$$g_4(x) = \frac{x_1 + x_2}{1.5} - 1 \leq 0.$$

With

$$0.05 \leq x_1 \leq 2, 0.25 \leq x_2 \leq 1.3 \text{ and } 2 \leq x_3 \leq 15.$$

APPENDIX C
WELDED BEAM DESIGN PROBLEM

$$\textit{Consider } X = [x_1, x_2, x_3, x_4] = [h, l, t, b].$$

$$\textit{Minimize } f(x) = 1.10471 x_1^2 x_2 + 0.04811 x_3 x_4 (14.0 + x_2).$$

$$\textit{Subject to}: \ g_1(x) = \tau(x) - 13600 \leq 0,$$

$$g_2(x) = \sigma(x) - 30000 \leq 0,$$

$$g_3(x) = x_1 - x_4 \leq 0,$$

$$g_4(x) = 0.10471 x_1^2 + 0.04811 x_3 x_4 (14 + x_2) - 5.0 \leq 0,$$

$$g_5(x) = 0.125 - x_1 \leq 0,$$

$$g_6(x) = \delta(x) - 0.25 \leq 0,$$

$$g_7(x) = 6000 - p_c(x) \leq 0.$$

where

$$\tau(x) = \sqrt{\tau' + (2\tau\tau')\frac{x_2}{2R} + (\tau'')^2},$$

$$\tau' = \frac{6000}{\sqrt{2}\, x_1 x_2},$$

$$\tau'' = \frac{MR}{J},$$

$$M = 6000\left(14 + \frac{x_2}{2}\right),$$

$$R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2},$$

$$J = 2\left\{x_1 x_2 \sqrt{2}\left[\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right]\right\},$$

$$\sigma(x) = \frac{504000}{x_4 x_3^2}$$

$$\delta(x) = \frac{65856000}{\left(30 \cdot 10^6\right) x_4 x_3^3},$$

$$p_c(x) = \frac{4.013\left(30\cdot 10^6\right)\sqrt{\frac{x_3^2x_4^6}{36}}}{196}\left(1-\frac{x_3}{28}\sqrt{\frac{30\cdot 10^6}{4(12\cdot 10^6)}}\right).$$

With

$$0.1 \le x_1, x_4 \le 2 \text{ and } 0.1 \le x_2, x_3 \le 10.$$

APPENDIX D
SPEED REDUCER DESIGN PROBLEM

$$\textit{Consider } X = \left[x_1, x_2, x_3, x_4, x_5, x_6, x_7\right] = \left[b, m, p, l_1, l_2, d_1, d_2\right].$$

$$\textit{Minimize } f(x) = 0.7854x_1x_2^2\left(3.3333x_3^2 + 14.9334x_3 - 43.0934\right) - 1.508x_1\left(x_6^2 + x_7^2\right) + 7.4777\left(x_6^3 + x_7^3\right) + 0.7854(x_4x_6^2 + x_5x_7^2).$$

$$\textit{Subjectto}: \; g_1(x) = \frac{27}{x_1x_2^2x_3} - 1 \le 0,$$

$$g_2(x) = \frac{397.5}{x_1x_2^2x_3} - 1 \le 0,$$

$$g_3(x) = \frac{1.93x_4^3}{x_2x_3x_6^4} - 1 \le 0,$$

$$g_4(x) = \frac{1.93x_5^3}{x_2x_3x_7^4} - 1 \le 0,$$

$$g_5(x) = \frac{1}{110x_6^3}\sqrt{\left(\frac{745x_4}{x_2x_3}\right)^2 + 16.9\times 10^6} - 1 \le 0,$$

$$g_6(x) = \frac{1}{85x_7^3}\sqrt{\left(\frac{745x_5}{x_2x_3}\right)^2 + 157.5\times 10^6} - 1 \le 0,$$

$$g_7(x) = \frac{x_2x_3}{40} - 1 \le 0,$$

$$g_8(x) = \frac{5x_2}{x_1} - 1 \le 0,$$

$$g_9(x) = \frac{x_1}{12x_2} - 1 \le 0,$$

$$g_{10}(x) = \frac{1.5x_6 + 1.9}{x_4} - 1 \le 0,$$

$$g_{11}(x) = \frac{1.1x_7 + 1.9}{x_5} - 1 \le 0.$$

With

$$2.6 \le x_1 \le 3.6,\ 0.7 \le x_2 \le 0.8,\ 17 \le x_3 \le 28,\ 7.3 \le x_4 \le 8.3,\ 7.8 \le x_5 \le 8.3,\ 2.9 \le x_6 \le 3.9,\ \text{and } 5 \le x_7 \le 5.5.$$

APPENDIX E
PRESSURE VESSEL DESIGN PROBLEM

$$\textit{Consider } X = [x_1, x_2, x_3, x_4] = [T_s, T_h, R, L].$$

$$\textit{Minimize } f(x) = 0.6224x_1x_3x_4 + 1.778x_2x_3^2 + 3.1661x_1^2x_4 + 19.84x_1^2x_3.$$

$$\textit{Subject to}: \; g_1(x) = -x_1 + 0.0193x_3 \le 0,$$

$$g_2(x) = -x_2 + 0.00954x_3 \le 0,$$

$$g_3(x) = -\pi x_3^2x_4 - \frac{4}{3}\pi x_3^3 + 1296000 \le 0,$$

$$g_4(x) = x_4 - 240 \le 0.$$

With

$$0 \le x_1, x_2 \le 100 \text{ and } 10 \le x_3, x_4 \le 200.$$

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