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Finite-Time Bound Synchronization of the New Chaotic System With Energy Consumption Estimation

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ABSTRACT As chaotic systems are widely used in many fields, the study of them is becoming more and more in-depth. This paper first presents a new single-equilibrium chaotic system which is only three terms, and some fundamental dynamical feature of the new chaotic system are discussed, such as equilibria, dissipativity, Poincaré diagram, bifurcation graph, etc. Secondly, a new finite-time controllers is designed by using Lyapunov stability theory, and it can be used for bound synchronization of the general chaotic systems. In contrast to the current finite time controller of the chaotic system, the designed controller in this paper does not contain exponential term, it can be simple and eliminate the chattering phenomenon during synchronizationis, which may be easier to implement in practical application. In addition, under the finite time controller, the bound of control energy consumption of the chaotic system is estimated. Finally, the finite-time controllers for the new chaotic system are advanced using the design method of finite-time controller of the general chaotic system, and the result of numerical simulation is given to check its validity by the designed method.

INDEX TERMS Chaos, bifurcation, synchronization.

I. INTRODUCTION

Since the first chaotic attractor has been discovered by Lorenz, Lorenz system [1] has been extensively and deeply studied as a typical chaotic model. The 1970s and 1980s were the period of the development of chaotic theory. During this period, Rössler constructed a simple three-dimensional chaotic system [2]. Buskirk and Jeffries presented a chaotic oscillator by using the exponential function [3]. In 1999, Chen successfully proposed a Chen system similar to the Lorenz system but not topologically equivalent by using chaotic anti-control method [4]. In 2002, Lü *et al.* discovered Lü system and unified chaotic system by utilizing chaotic anti-control method [5]. Later, chaos in engineering was gradually shifted from being a scientific exploration to a potential engineering application [6]–[9]. Therefore, purposeful generation of chaos is not only the need for chaos theory research, but also the key problem of chaos application [6].

In recent decades, many new continuous chaotic systems have been discovered in three-dimensional differential systems by using various chaotic construction methods, such as chaotic systems with six terms [10], chaotic systems without linear terms [11], chaotic systems with only no-equilibrium point [12], chaotic systems with signum nonlinearity [13], chaotic systems with absolute nonlinearity [14], chaotic systems with exponential nonlinearity [15], chaotic systems with hyperbolic tangent nonlinearity [16], chaotic systems with exponential nonlinear term [15], [17]–[20], etc. Generally, how many terms are required for a 3D dynamical system to produce chaotic characteristics? It has always been an interesting topic.

At present, various control methods of chaos system have been discussed, like sliding mode control [21], backstepping control [22], active control [23], passive control [24], adaptive

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control [25], fuzzy control [26], etc. However, finite-time control of complex systems have attracted much attention due to their good robustness and anti-disturbance performance in recent decades [27]–[29]. Stability of finite time refers to the system condition always does not exceed a given threshold in a given initial condition and a known finite time. Generally, the shorter the synchronization time between two chaotic systems, the less the control cost. In order to shorten the control time of chaotic systems, people pay more and more attention to finite-time control technology, so various finite-time control methods are proposed [27], [32]–[35]. In the existing finite-time control methods, the general controller contains a symbolic function, which is a discontinuous functional controller [31], [32]. In order to eliminate this phenomenon, continuous finite-time controller has been proposed successively [32], [33], [35], [37], [38]. However, in the design of finite time controllers for complex systems, they have a common feature, that is, the designed controller contains exponential form. In other words, the controller generally adopted nonlinear feedback controller. Then, whether the linear feedback controller can also realize the finite-time control of complex systems is also a very meaningful problem.

So far, there are few chaotic systems with one equilibrium and three terms discussed in three-dimensional ordinary differential equations. One of the contributions of this paper is to propose a new three terms coupled chaotic system in ordinary differential equations. It has just one equilibrium point, and some basic dynamic characteristics of the system are discussed. In addition, since most of the existing finite-time controllers contain exponential terms, the second contribution of this paper is to design a novel finite-time controller with only linear terms to study its bounded synchronization. Finally, control energy consumption of chaotic system is also estimated.

The structure and main content of the article is divided into five sections: in the second section, a new chaotic system is given, and a few fundamental dynamical feature are discussed. In the third section, the finite-time synchronization condition of chaotic system. In the fourth section, an illustrative example is provided. See the fifth section for conclusions.

II. A SINGLE EQUILIBRIUM NEW CHAOTIC SYSTEM

The following dynamical system is discussed

$$\begin{cases} \dot{x} = \ln(a + h\exp(y - x)), \\ \dot{y} = -\ln(b + \exp(xz)), \\ \dot{z} = \ln(c + \exp(xy - d)), \end{cases} \tag{1}$$

which x, y and z indicate the state variables of the system, and when the parameter $a = 0.1$, $b = 0.1$, $c = 0.1$, $d = 0.9$, $h = 9$, the system (1) is chaotic (See Figs.1-4).

Remark 1: At present, in the three-dimensional dynamic system, most chaotic systems contain four terms or more terms [1], [2], [4]–[7], [10], but few have only three expressions. We have constructed a chaotic system with only three terms in this paper. Of course, whether we can construct a simpler chaotic system with only three terms will be our future research topic.

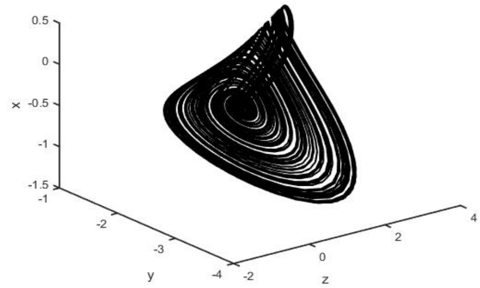

FIGURE 1. The system (1) in x-y-z plane.

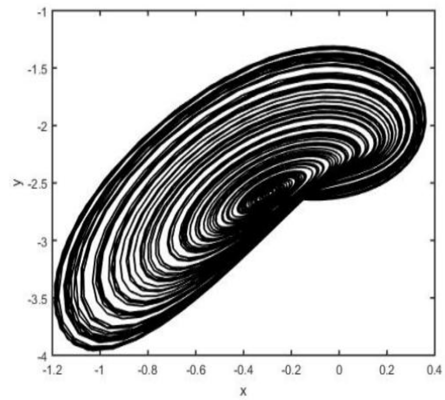

FIGURE 2. The system (1) in x-y plane.

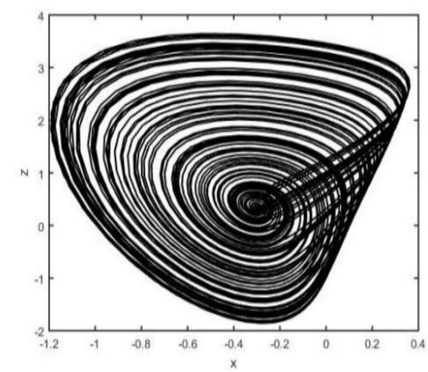

FIGURE 3. The system (1) in x-z plane.

Obviously, when $a = b = c = 0$, the system (1) conversion to the system (2), and the system (2) is not chaotic for the starting values (1, 3, 3) (See Fig.5).

$$\begin{cases} \dot{x} = y - x + \ln h, \\ \dot{y} = -xz, \\ \dot{z} = xy - d. \end{cases} \tag{2}$$

Next, the basic characteristics of the system (1) are analyzed.

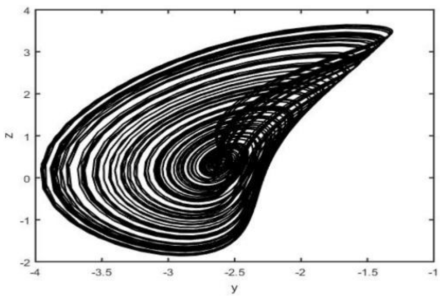

FIGURE 4. The system (1) in y-z plane.

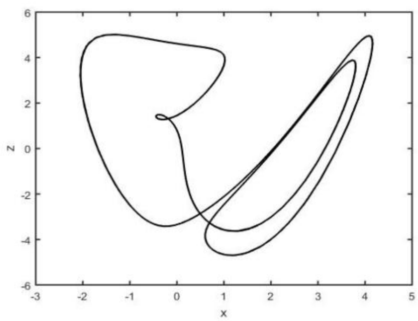

FIGURE 5. The system (2) in x-z plane.

A. EQUILIBRIA

Let

$$\begin{cases} \ln(a + h\exp(y-x)) = 0, \\ -\ln(b + \exp(xz)) = 0, \\ \ln(c + \exp(xy - d)) = 0. \end{cases} \tag{3}$$

When $a, b, c < 1$, the system (1) has an unique equilibrium point

$$E = \left(\frac{d + \ln(1-c)}{1 + \ln(1-a)/h}, 1 + \ln(1-a)/h, \frac{\ln(1-b)(1 + \ln(1-a)/h)}{d + \ln(1-c)} \right).$$

If $a = 0.1$, $b = 0.1$, $c = 0.1$, $d = 0.9$, $h = 9$, then $E = (-0.6100, -1.3026, 0.1727)$.

For $E = (-0.6100, -1.3026, 0.1727)$, Jacobian matrix is obtained by linearization system (1)

$$J = \begin{pmatrix} -\dfrac{h\exp(y-x)}{a+h\exp(y-x)} & \dfrac{h\exp(y-x)}{a+h\exp(y-x)} & 0 \\ -\dfrac{z\exp(xz)}{b+\exp(xz)} & 0 & -\dfrac{x\exp(xz)}{b+\exp(xz)} \\ \dfrac{y\exp(xy-d)}{c+\exp(xy-d)} & \dfrac{x\exp(xy-d)}{c+\exp(xy-d)} & 0 \end{pmatrix}$$

If $|\lambda I - J|_E = 0$, then

$$\begin{aligned} \lambda_1 &= -1.2245, \\ \lambda_2 &= 0.1231 + 0.8601i, \\ \lambda_3 &= 0.1231 - 0.8601i. \end{aligned} \tag{4}$$

Therefore, E is unstable.

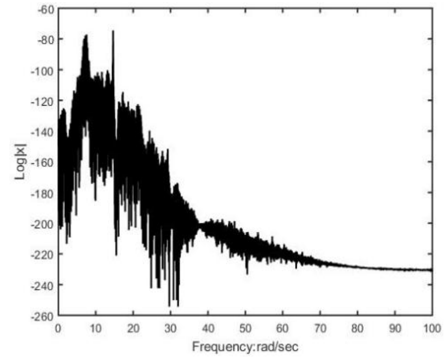

FIGURE 6. Frequency spectrum diagram.

B. DISSIPATIVITY

For the system (1),

$$\nabla \cdot V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = -\frac{h\exp(y-x)}{a + h\exp(y-x)},$$

as $-\frac{9\exp(y-x)}{0.1+9\exp(y-x)} < 0$, so the system (1) is a dissipative system, and an exponential contraction of the system (1) is $\exp\left(-\frac{9\exp(y-x)}{0.1+9\exp(y-x)}\right)$.

C. CHAOTIC BEHAVIOR OF THE SYSTEM (1)

If the initial value is (1, 3, 3), the Lyapunov exponent of the system (1) is $L_1 = 0.1635, L_2 = 0.2245, L_3 = -0.1656$, since the system (1) has two positive Lyapunov exponents. It has chaotic characteristics.

In addition,

$$\begin{aligned} D_L &= j + \frac{1}{|L_{j+1}|} \sum_{i=1}^{j} L_i = 2 + \frac{L_1 + L_2}{|L_3|} \\ &= 2 + \frac{0.3880}{|-0.1656|} = 4.3430, \end{aligned}$$

so the Lyapunov dimension of the system (1) is fractional.

The continuous broadband characteristics of the spectrum of system (1) is shown in Fig.6. For $z = 0$, $x = -0.15$ and $y = -2.5$, the Poincaré diagram of the system (1) are shown in Figs.7 (a)-(c), respectively, and several attractor graphs are shown. When the parameters of the system change in an interval, the bifurcation graph can better induce the complex evolution behavior of the system (1). Fig.8 shows the Lyapunov exponents of the system (1) for $0.5 \le d \le 1.5$. Fig.9 shows the bifurcation evolution of the system (1) for $0.5 \le d \le 1.5$.

III. FINITE-TIME SYNCHRONIZATION OF THE NEW CHAOTIC SYSTEM

Considering the following chaotic system

$$\dot{u} = f(u, t), \tag{5}$$

where $u = (u_1, \cdots, u_n)^T$ is the state variable of the system, $f(u) : R^n \to R^n$ means continuous function.

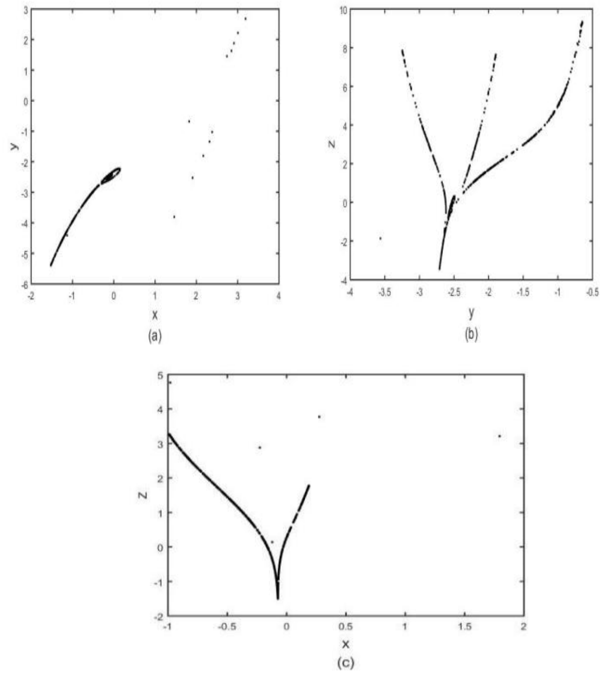

FIGURE 7. Poincaré diagram for (a) $z = 0$, (b) $x = -0.15$, (c) $y = -2.5$.

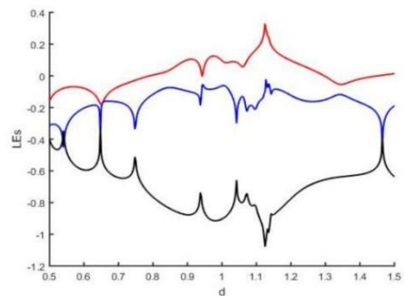

FIGURE 8. Lyapunov exponents diagram of system (1) for $0.5 \le d \le 1.5$.

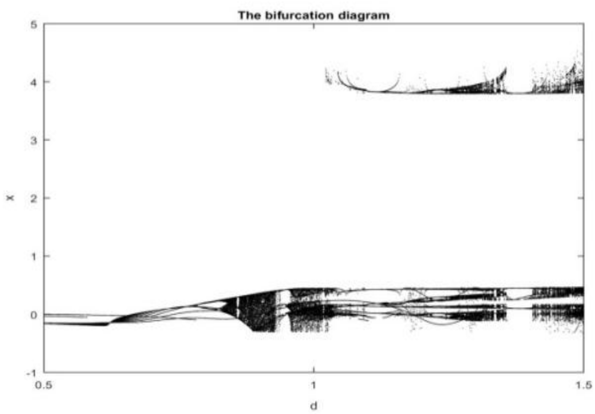

FIGURE 9. Bifurcation diagram of system (1) for $0.5 \le d \le 1.5$.

The controlled chaotic system is

$$\dot{v} = f(v, t) + r(u, v, t), \tag{6}$$

where $v = (v_1, \cdots, v_n)^T$ is the state variable of the system, $r(u, v, t)$ means the controller.

As can be seen from equations (5) and (6), the error system may be described as follows

$$\dot{e} = f(v, t) - f(u, t) + r(u, v, t). \tag{7}$$

Assumptions 1 (A1): Assuming f satisfies

$$|f(v, t) - f(u, t)| \le \rho e, \quad \rho \in R,$$

where $e = (e_1, e_2, \cdots, e_n)^T = (v_1 - u_1, v_2 - u_2, \cdots, v_n - u_n)^T$.

Theorem 1: Under A1, the systems (5) and (6) come true finite-time bound synchronization by the controller designed as follows

$$r(t) = -k_1 e. \tag{8}$$

Then the settling time is

$$T \le \frac{1}{2(k_1 - \lambda - \eta)} \ln\left(1 + \frac{2(k_1 - \lambda - \eta)}{k_2} v(0)\right),$$

and the estimation of energy consumption

$$E \le \frac{k_2 k_1^2}{\lambda} T,$$

where $k_1 > \eta + \lambda, \lambda > 0, k_2 > 0$.

Proof: Let

$$v(t) = \frac{1}{2} e^T(t) e(t),$$

then

$$\begin{aligned} \dot{v} &= e^T(t) \dot{e}(t) = e^T(t) (f(v(t)) - f(u(t)) + r(t)) \\ &\le \eta e^T(t) e(t) - k_1 e^T(t) e(t) \\ &= -2(k_1 - \lambda - \eta) v(t) - k_2 - 2\lambda v(t) + k_2, \end{aligned}$$

so $\dot{v}(t) \le -2(k_1 - \lambda - \eta) v(t) - k_2$ if $v(t) > \frac{k_2}{2\lambda}$. From the results in [37], the decrease of in finite-time drives the trajectories of $v(t)$ the closed-loop system into $v(t) \le \frac{k_2}{2\lambda}$, that is, $|e(t)| \le \sqrt{\frac{k_2}{\lambda}}$, so the settling time T can be estimated as

$$\begin{aligned} T &= \int_0^T dt \le -\int_{v(0)}^0 \frac{dv(t)}{2(k_1 - \lambda - \eta) v(t) + k_2} \\ &= \int_0^{v(0)} \frac{dv}{2(k_1 - \lambda - \eta) v(t) + k_2} \\ &= \frac{1}{2(k_1 - \lambda - \eta)} \ln\left(1 + \frac{2(k_1 - \lambda - \eta)}{k_2} v(0)\right). \end{aligned}$$

In [37] and [38], the authors discuss the energy consumption of complex systems, the energy consumption is defined

$$E = \lim_{t \to T} \int_0^t \|r(s)\|^2 ds,$$

so the estimation of energy consumption is

$$E = \lim_{t \to T} \int_0^t \|-k_1 e\|^2 ds$$

$$
\begin{aligned}
&= \lim_{t\to T}\left(k_1^2\int_0^t \|e\|^2\,ds\right)\\
&\le \lim_{t\to T}\int_0^t\left(k_1^2\int_0^t \frac{k_2}{\lambda}ds\right)\\
&= \frac{k_2k_1^2}{\lambda}T. \qquad (9)
\end{aligned}
$$

The proof is completed.

Remark 2: In the existing finite time control research for complex systems, the controller generally adopts nonlinear feedback controller. For example, in [27], [32]–[35], the controller contains exponential term, $|e(t)|^{\theta}$, this kind of controller is discontinuous controller, while another kind of controller contains exponential term and symbolic function, such as $sign(e(t))\,|e(t)|^{\theta}$ in [30] and [31], this kind of controller is continuous controller. Theorem 1 only uses linear feedback controller to realize finite time control of chaotic system. This kind of controller contains neither exponential term nor symbolic function. Obviously, the controller in Theorem 1 is simpler than the existing controller in [27], [30]–[35].

Remark 3: In [21], the finite-time synchronization of chaotic systems was realized based on the integral dynamic sliding mode control method, which has better response speed. Similar to [21], the controller (8) can also eliminate the chattering phenomenon during synchronization.

Theorem 2: Under A1, the systems (5) and (6) come true finite-time bound synchronization by the controller designed as follows

$$r(t) = -k_1 e + \frac{k_3}{\|e\|^2+\alpha}e, \qquad (10)$$

then the settling time is

$$T \le \frac{1}{2(k_1-\eta)}\ln\left(1-\frac{2(k_1-\eta)}{k_3}v(0)\right),$$

and the estimation of energy consumption

$$E \le \frac{k_3k_1^2T}{k_1-\eta}+\frac{k_3^2T}{\alpha},$$

where $k_1 > \eta, k_3 > 0, \alpha > 0$.

Proof: Let

$$v(t) = \frac{1}{2}e^T(t)\,e(t),$$

then

$$
\begin{aligned}
\dot v &= e^T(t)\,\dot e(t) = e^T(t)\left(f(v(t))-f(u(t))+r(t)\right)\\
&\le \eta e^T(t)\,e(t) - k_1 e^T(t)\,e(t) + \frac{k_3}{\|e\|^2+\alpha}e^T(t)\,e(t)\\
&\le -2(k_1-\eta)\,v(t)+k_3 \qquad (11)
\end{aligned}
$$

so $\dot v(t) < 0$ when $v(t) > \frac{k_3}{2(k_1-\eta)}$. From the results in [36], it can be concluded that the time derivative of $v(t)$ is negative outside a compact residual set Θ, $\Theta = \left\{v(t) \le \frac{k_3}{2(k_1-\eta)}\right\}$.

From Inequality (11)

$$v(t) \le e^{-2(k_1-\eta)t}v(0) + \frac{k_3}{2(k_1-\eta)}\left(1-e^{-2(k_1-\eta)t}\right),$$

when $t \to T$, let $v(t) \to 0$, that is, $e^{-2(k_1-\eta)t}v(0) + \frac{k_3}{2(k_1-\eta)}\left(1-e^{-2(k_1-\eta)t}\right) \le 0$. So, in compact residual set Θ, the settling time T can be estimated as

$$T \le \frac{1}{2(k_1-\eta)}\ln\left(1-\frac{2(k_1-\eta)}{k_3}v(0)\right),$$

the error bound is

$$|e(t)| \le \sqrt{\frac{k_3}{k_1-\eta}},$$

and the estimation of energy consumption is

$$
\begin{aligned}
E &= \lim_{t\to T^*}\int_0^t\left\|-k_1e-\frac{k_3}{\|e\|^2+\alpha}e\right\|^2 ds\\
&\le \lim_{t\to T}\left(\int_0^t \|-k_1e\|^2\,ds + \int_0^t\left\|\frac{k_3}{\|e\|^2+\alpha}e\right\|^2 ds\right)\\
&\le \lim_{t\to T}k_1^2\int_0^t\|e\|^2\,ds + \lim_{t\to T}k_3^2\int_0^t\frac{\|e\|^2+\alpha}{\left\|\,\|e\|^2+\alpha\right\|^2}ds\\
&= \lim_{t\to T}k_1^2\int_0^t\|e\|^2\,ds + \lim_{t\to T}k_3^2\int_0^t\frac{1}{\left\|\,\|e\|^2+\alpha\right\|}ds\\
&\le \lim_{t\to T}k_1^2\int_0^t\frac{k_3}{k_1-\eta}ds + \lim_{t\to T}k_3^2\int_0^t\frac{1}{\alpha}ds\\
&= \frac{k_3k_1^2T}{k_1-\eta}+\frac{k_3^2T}{\alpha}. \qquad (12)
\end{aligned}
$$

The proof is completed.

Remark 4: From the energy consumptions (9) and (12), the control energy consumption required by the system will be different due to different controllers of the systems.

IV. NUMERICAL SIMULATIONS

Let the master and slave systems are as follows

$$
\begin{cases}
\dot u_1 = \ln(a+h\exp(u_2-u_1)),\\
\dot u_2 = -\ln(b+\exp(u_1u_3)),\\
\dot u_3 = \ln(c+\exp(u_1u_2-d)),
\end{cases}
$$

$$
\begin{cases}
\dot v_1 = \ln(a+h\exp(v_2-v_1)) - k_1e_1,\\
\dot v_2 = -\ln(b+\exp(v_1v_3)) - k_1e_2,\\
\dot v_3 = \ln(c+\exp(v_1v_2-d)) - k_1e_3.
\end{cases}
$$

For

$$
f(v)-f(u) = \begin{pmatrix}
\ln\left[(a+h\exp(v_2-v_1))\,/\,(a+h\exp(u_2-u_1))\right]\\
\ln\left[(b+\exp(v_1v_3))\,/\,(b+\exp(u_1u_3))\right]\\
\ln\left[(c+\exp(v_1v_2-d))\,/\,(c+\exp(u_1u_2-d))\right]
\end{pmatrix}.
$$

If

$$\exp(v_2-v_1) > \exp(u_2-u_1) > 0,$$

then

$$1 < \frac{a+\exp(v_2-v_1)}{a+\exp(u_2-u_1)} < \frac{\exp(v_2-v_1)}{\exp(u_2-u_1)} = \exp(e_2-e_1),$$

if

$$\exp(v_2 - v_1) < \exp(u_2 - u_1),$$

then

$$\frac{\exp(v_2 - v_1)}{\exp(u_2 - u_1)} < \frac{a + \exp(v_2 - v_1)}{a + \exp(u_2 - u_1)} < 1.$$

Therefore,

$$\ln\left[\frac{a + \exp(v_2 - v_1)}{a + \exp(u_2 - u_1)}\right] < \ln\left[\max\{1, \exp(e_2 - e_1)\}\right].$$

Similarly,

$$\ln\left[\frac{b+\exp(u_1u_3)}{b+\exp(v_1v_3)}\right] < \ln\left[\max\{1, \exp(u_1u_3 - v_1v_3)\}\right],$$

$$\ln\left[\frac{c+\exp(v_1v_2-d)}{c+\exp(u_1u_2-d)}\right] < \ln\left[\max\{1, \exp(v_1v_2-u_1u_2)\}\right].$$

So,

$$\begin{aligned}
&\|f(v) - f(u)\| \\
&= \left\|\begin{pmatrix} \ln\left[(a + h\exp(v_2 - v_1)) / (a + h\exp(u_2 - u_1))\right] \\ \ln\left[(b + \exp(u_1u_3)) / (b + \exp(v_1v_3))\right] \\ \ln\left[(c + \exp(v_1v_2 - d)) / (c + \exp(u_1u_2 - d))\right] \end{pmatrix}\right\| \\
&< \left\|\begin{pmatrix} -1 & 1 & 0 \\ -v_3 & 0 & -u_1 \\ u_2 & v_1 & 0 \end{pmatrix}\right\| \left\|\begin{matrix} e_1 \\ e_2 \\ e_3 \end{matrix}\right\|.
\end{aligned}$$

If the starting values of the master-slave systems are assumed to be $(2, 1, 3)$ and $(3, 2, 1)$, respectively, and $k_1 = 10$. By estimating the simulation, we have $\|u\|, \|v\| < 5$, so,

$$\left\|\begin{pmatrix} -1 & 1 & 0 \\ -v_3 & 0 & -u_1 \\ u_2 & v_1 & 0 \end{pmatrix}\right\| < 8.6701.$$

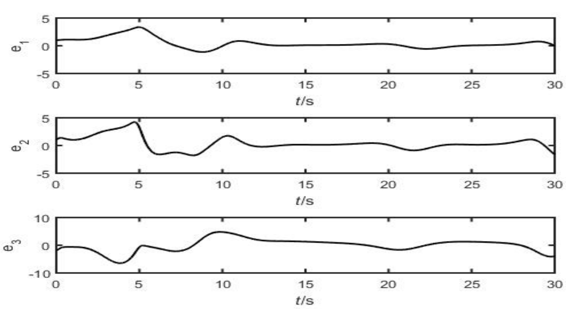

FIGURE 10. Error evolution.

When the system has no controller, two chaotic systems cannot achieve synchronization, as shown in Fig 10. When there is a controller, two chaotic systems realize synchronous evolution, as shown in Fig 11.

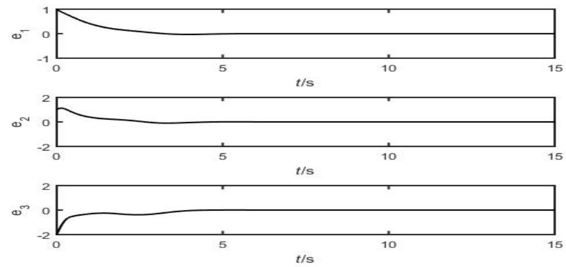

FIGURE 11. Synchronization of the drive and the response system.

V. CONCLUSION

Firstly, the chaotic dynamic system with three terms and one equilibrium was proposed, and its basic dynamic behavior was also studied. Secondly, the finite-time bounded synchronization for the new chaotic system was discussed, and finite-time controller designed did not contain exponential term. Moreover, control energy consumption of the chaotic system is also estimated. Finally, numerical simulation verifies the effectiveness of the designed finite-time controller.

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