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Consensus of Discrete-Time Nonlinear Multiagent Systems Using Sliding Mode Control Based on Optimal Control

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ABSTRACT This article investigates the optimal sliding mode control approach for consensus of nonlinear discrete-time high-order multi-agent systems (MASs). First, the nonlinearity and communication delay in the MAS is solved by designing a distributed discrete-time integral sliding mode control law, together with a proof of reachability of the sliding mode surface, as well as a proof that the chattering of the system is attenuated. In addition, the optimal controller is designed based on the model obtained after the distributed sliding mode control law is applied to the system. The merits of the proposed distributed sliding mode controller with the fusion of optimal control are that it can reduce the chattering of the MASs and their existence of quasi-sliding modes, as well as tolerate the negative impact caused by communication delay among agents. The MASs can achieve consensus quickly with the combined action of the sliding mode controller and the optimal controller. Finally, two examples are given to verify the effectiveness of the control method proposed in this paper.

INDEX TERMS Sliding mode control, optimal control, nonlinear multi-agent system, high-order consensus, communication delay.

I. INTRODUCTION

Individual units in real physical systems have become capable of sufficient computation and execution as a result of the rapid development of distributed computing, sensor network, and communication technology in recent decades, giving rise to the complex system science of MASs [1]. In MASs, mutual coordination among agents not only enhances the overall behavioural performance of the system but also makes it more collaborative and adaptable. As a result, researchers have paid more attention on distributed cooperative control of MAS [2]–[8].

The main goal of consensus of MASs is to make the same state with the access of communicating networks under appropriate distributed control mechanism [9]. Wide variety of consensus control techniques for MAS have been developed by scholars and engineers throughout the world [10]–[18]. For the possible failure of controllers of continuous-time linear MASs, [10] proposed a distributed adaptive fault-tolerant consensus control method [12] explored the distributed optimum cooperation issue and proposed a distributed event-triggered controller for continuous-time heterogeneous linear MASs [15] investigated the consensus problem of continuous-time MASs containing communication time delay. For continuous-time linear MASs, [11] and [16] considered event-triggered control methods to achieve consensus in MAS. The finite-time consensus problem for continuous-time MAS with time-varying topologies was studied in [17] [18] presented a consensus control technique and the convergence rate for a class of linear discrete-time dynamic topological MASs [19] proposed two consensus methods for first- and second-order discrete-time linear heterogeneous integrator MASs, respectively. Since the nonlinearity of the system is more practical [20], [21] proposed a distributed consensus control protocol for high-order nonlinear discrete-time MASs

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containing mismatched disturbances, using inversion techniques that do not require Laplace matrix information and Lippincott restrictions. Due to the limited communication distance and communication channel capacity between actual agents, [22] investigated the leader-follower H-infinite consensus problem for nonlinear discrete-time MASs containing time lag, interference and parameter uncertainty, and designed an output feedback control protocol that enables MASs to achieve leader-follower state consensus.

Due to the extensive usage of computers in modern industrial processes, discrete systems are more versatile in their applications thereby discrete-time sliding mode control (DSMC) has become a more popular way to handle uncertainty inevitably existed in MASs for strengthening the robustness of MASs with the goal of achieving consensus. Patel proposed two DSMC protocols that used the Gao's reaching law and the Power rate reaching law for the consensus of discrete-time MASs with matched disturbance [23]. But this study did not take the negative effect of communication delay into account and the system under consideration is a linear MAS. Communication delay cannot be missed in the research of MASs due to the limited communication distance and communication channel capacity between actual agents. [24] solved the tracking problem of second-order discrete-time nonlinear MAS, where communication delays and packet loss in the system are considered.

It is worth pointing out that the DSMC for the consensus of MASs with uncertainty and communication delay has not been fully investigated up to now. The practical desire of finding a technique that can ensure the consensus of high-order nonlinear MASs with strong robustness to uncertainty and communication delay motivates our research of this paper. Therefore, this paper is devoted to developing a novel slide model control method for the consensus of MASs, such that the systems can have a strong ability to reject the uncertainty and disturbance and tolerate the negative impact caused by communication delay. However, the coupling relationships among agents, the high-order nonlinearity and communication delay among agents pose great challenges for designing the optimal sliding mode controller of MASs with the objective of reaching the consensus via an optimal approach. Moreover, how to reduce the chattering of sliding mode surfaces for MASs is another big challenge. The main contribution in this paper is that a novel distributed sliding mode controller is designed with the full consideration of communication delay and coupling relationships among agents, enabling the MASs to have strong robustness to uncertainty and communication delay and to reduce the chattering of sliding mode surfaces.

The following is the rest of the paper's structure: Section II covers the necessary graph theory and problem formulation. Section III covers the design of sliding mode control and sliding mode reachability analysis, part IV covers the optimal control law and system stability analysis, Section V covers two simulation implementations to demonstrate the method's effectiveness, and Section VI covers the paper's conclusions.

II. PROBLEM STATEMENT

A. GRAPH THEORY [25]

Denote the directed graph by $G = (\vartheta, \varepsilon, A)$, where the graph's collection of N nodes is denoted by $\vartheta = \{v_1, v_2, \cdots, v_N\}$, the node serial number belongs to a fixed set of node sequences $\Xi = \{1, 2, \cdots, N\}$, ε represented the set of edges of the graph, and the non-negative matrix $A = [a_{ij}]$ represents the adjacency matrix of the graph G. The edges of graph G are denoted by e_{ij}, and e_{ij} denotes the flow of information from e_i to e_j. Assuming $a_{ii} = 0, i \in \Xi$, if $e_{ij} \in \varepsilon \Leftrightarrow e_{ij} \in \varepsilon$,then G is an undirected graph. Define the Laplacian matrix $\mathcal{L}$ of the graph G as $\mathcal{L}(G) = L = D - A$, where $D = diag\{\sum_{j=1}^{N} a_{ij}, i = 1, 2, \cdots, N\}$ is the degree matrix of G. $b_i = 1$ denotes the agent a_ij can obtain the state of target agent.

B. CONSENSUS PROBLEM DESCRIPTION

Consider a high-order discrete-time nonlinear MAS containing disturbances composed of N multi-agents

$$x_i(k+1) = Ax_i(k) + B(u_i(k) + f(x_i(k)) + d_i(k)) \tag{1}$$

where $i = 1, 2, \cdots, N$, $x_i(k) \in \mathbb{R}^n$ is the system state vector of agent i, $u_i(k) \in \mathbb{R}^m$ denotes the control vector for agent $i, f(x_i(k))$ is a nonlinear function with the system state variables, $d_i(k) \in \mathbb{R}^m$ is the matched disturbance acting on ith system, A and B are the appropriate dimensional system matrix and input matrix, respectively.

Also consider a leader dynamics as

$$x_0(k+1) = Ax_0(k) + B(f(x_0(k)) + d_0(k)) \tag{2}$$

where $x_0(k) \in \mathbb{R}^n$ denotes the system state vector of leader, $f(x_0(k))$ is a nonlinear function with the system state variables of leader, $d_0(k)$ is the disturbance of leader.

For the purpose of presenting the sliding mode approach for MASs consensus, the following assumption and definition are presented.

Assumption 1. A and B are controllable.

Definition 1: For any agent $i = 1, 2, \cdots, N$, if the tracking error of each order of the follower agents finally converges to zero, namely

$$\lim_{k\to\infty} ||x_i(k) - \bar{x}_0(k)|| = 0, \forall i \tag{3}$$

for any initial state, then the MAS completes consensus tracking.

Eq. (1) can be rewritten in compact form as

$$X(k+1) = \bar{A}X(k) + \bar{B}(U(k) + F(X(k)) + D(k)) \tag{4}$$

where $\bar{A} = I_N \bigotimes A,\ \bar{B} = I_N \bigotimes B$.

To study the consensus problem of leader-follower, let us define the local neighbourhood tracking error:

$$\delta_i(k) = \sum_{j\in N_i} a_{ij}[x_i(k) - x_j(k)] + b_i[x_i(k) - x_0(k)] \tag{5}$$

Further from (5), there are

$$\delta_i(k+1) = \sum_{j\in N_i} a_{ij}[x_i(k+1) - x_j(k+1)] + b_i[x_i(k+1) - x_0(k+1)] \quad (6)$$

To simplify the form of (1), write $f(x_i(k)) + u_i(k)$ as V_i^t, i.e. $V_i^t(k) = f(x_i(k)) + u_i(k)$. Substituting equations (1)-(2) into (6) we have

$$\begin{aligned}
&\delta_i(k+1) \\
&= \sum_{j\in N_i} a_{ij}[Ax_i(k) + B(V_i^t(k) + d_i(k)) \\
&\quad -Ax_j(k) - B(V_j^t(k) + d_j(k))] \\
&\quad +b_i[Ax_i(k) + B(V_i^t(k) + d_i(k)) \\
&\quad -Ax_0(k) - B(f(x_0(k)) + d_0(k))] \\
&= A\sum_{j\in N_i} a_{ij}[x_i(k) - x_j(k)] + b_i[x_i(k) - x_0(k)] \\
&\quad +B\sum_{j\in N_i} a_{ij}[V_i^t(k) - V_j^t(k)] + b_i[V_i^t(k) - f(x_0(k))] \\
&\quad +B\sum_{j\in N_i} a_{ij}(d_i(k) - d_j(k)) + b_i(d_i(k) - d_0(k)) \quad (7)
\end{aligned}$$

Considering the time-delay in the actual MAS communication network, we define

$$V_i^t(k) = \frac{\sum_{j\in N_i} a_{ij}V_j^t(k-\tau) + b_i f(x_0(k)) + \tilde{u}_i(k)}{\sum_{j\in N_i} a_{ij} + b_i} \quad (8)$$

where τ is the time-delay, $\tilde{u}_i(k)$ is a composite control law.

$$\tilde{u}_i(k) = u_{si}(k) + u_{ri}(k) \quad (9)$$

Remark 1: Theoretically, agent i gets information from its neighboring agents j and designs its own controller based on the most recent information obtained. Because we cannot directly utilize $V_j^t(k)$ in updating controller $V_i^t(k)$ due to the time delay caused by calculation time, we use the time-delay signal $V_j^t(k-\tau)$ instead. From the definition (8), $V_i^t(k) = f(x_i(k)) + u_i(k)$ and (9), one can find that if the virtual controllers (9) are designed, then $u_i(k)$ can be easily obtained by (8) and $u_i(k) = V_i^t(k) - f(x_i(k))$.

From (7)-(8), we have the discrete-time MAS neighbourhood error dynamics

$$\delta_i(k+1) = A\delta_i(k) + B\tilde{u}_i(k) + BH_{ij} + B\cdot\Delta_{ij}(k) \quad (10)$$

where $\Delta_{ij}(k) = \sum_{j\in N_i} a_{ij}(d_i(k) - d_j(k)) + b_i(d_i(k) - d_0(k))$, $H_{ij} = \sum_{j\in N_i} a_{ij}[V_j^t(k-\tau) - V_j^t(k)]$

The target of this paper is to design control law (9) that guarantees the consensus via an optimal approach for MASs subject to communication delay. The sequel is going to achieve our target by presenting a novel sliding controller design method.

III. DESIGN OF OPTIMAL SLIDING MODE CONTROLLER

In this section, the discrete-time integral sliding surface and the distributed optimal sliding controller will be presented and proof of the reachability of the discrete sliding surface is given.

A. DESIGN OF INTEGRAL SLIDING MODE CONTROLLER

According to (10), the discrete integral sliding surface is selected as follows

$$s_i(k) = M_i\delta_i(k) - M_i\delta_i(0) + \sigma_i(k) \quad (11)$$

where

$$\sigma_i(k) = \sigma_i(k-1) - M_i(A - BK_i - I)\delta_i(k-1) \quad (12)$$

where $\sigma_i(0) = 0$, K_i is a state feedback matrix that needs to be designed later, $M_i \in \mathbb{R}^{1\times n}$ is the constants matrix to be created, and it satisfies M_iB invertible.

Remark 2: Similar to [26], we set $\sigma_i(0) = 0$ and the system state error to be initially positioned on the switching surface, then the initial value of sliding mode function is zero, i.e. $s_i(0) = 0$. The benefit of this is that discrete integral sliding mode control proposed in this work can remove the sliding mode control convergence process and ensure the robust of the discrete-time MASs.

Further, according to (11), the sliding mode function at $k+1$ time can be expressed as

$$\begin{aligned}
s_i(k+1) &= M_i\delta_i(k+1) - M_i\delta_i(0) + \sigma_i(k) \\
&\quad -M_i(A - BK_i - I)\delta_i(k) \\
&= M_i[A\delta_i(k) + B\tilde{u}_i(k) + BH_{ij} + B\cdot\Delta_{ij}(k)] \\
&\quad -M_i\delta_i(0) + \sigma_i(k) - M_i(A - BK_i - I)\delta_i(k) \\
&= M_i[A\delta_i(k) + Bu_{ri}(k) + Bu_{si}(k) + BH_{ij} \\
&\quad +B\cdot\Delta_{ij}(k)] - M_i\delta_i(0) + \sigma_i(k) - M_iA\delta_i(k) \\
&\quad -M_iBu_{ri}(k) + M_i\delta_i(k) \quad (13)
\end{aligned}$$

The distributed state error feedback control law is chosen as

$$u_{ri}(k) = -K_i\delta_i(k) \quad (14)$$

Then, (13) becomes

$$\begin{aligned}
s_i(k+1) &= M_iB(u_{si}(k) + H_{ij} + \Delta_{ij}(k)) + M_i\delta_i(k) \\
&\quad +\sigma_i(k) - M_i\delta_i(0) \\
&= M_iB(u_{si}(k) + H_{ij} + \Delta_{ij}(k)) + s_i(k) \quad (15)
\end{aligned}$$

The reaching law for choosing agent i is as follows

$$\begin{aligned}
s_i(k+1) &= (1 - q_iT_i)s_i(k) \\
&\quad -(\alpha_i + \beta_i|M_iB|)T_i sgn(s_i(k)) \quad (16)
\end{aligned}$$

Combining (15) and (16), the distributed sliding mode control law for agent i is chosen as

$$\begin{aligned}
u_{si}(k) &= -(M_iB)^{-1}[q_iT_is_i(k) + (\alpha_i + \beta_i|M_iB|) \\
&\quad \cdot T_i sgn(s_i(k))] - H_{ij} - \Delta_{ij}(k) \quad (17)
\end{aligned}$$

where $0 < 1 - q_iT_i < 1$, $\alpha_i > 0$.

According to the error dynamics equation (10), if the sliding mode surface is determined by (17), then the design of the control law formula (17) can ensure the reach conditions of the discrete sliding mode system.

Remark 3: For the actual systems, in order to solve the distributed sliding mode control law (17), it is necessary to first solve H_{ij}. Because the time delay τ will not be very large, H_{ij} will be instead of $H_{ij}(k-\tau) = \sum_{j\in N_i} a_{ij}[V_j^t(k-2\tau) - V_j^t(k-\tau)]$ due to the unavailability of $V_j^t(k)$.

B. REACHABILITY ANALYSIS

Inspired by [27]–[29], the following two theorems are derived to ensure the existence of sliding modes and the arrival conditions of sliding modes and to reduce the jitter of the system

Theorem 1: The trajectory of the system will converge to the sliding mode surface, if $(s_i(k+1) - s_i(k)) \cdot s_i(k) < 0$.

Proof: According to (15)

$$s_i(k+1) - s_i(k) = M_iB(u_{si}(k) + H_{ij} + \Delta_{ij}(k)) \tag{18}$$

Substituting (17) into (18) yields

$$\begin{aligned} s_i(k+1) - s_i(k) &= -q_iT_is_i(k) \\ &\quad -(\alpha_i + \beta_i|M_iB|)T_isgn(s_i(k)) \end{aligned} \tag{19}$$

thus, we have

$$\begin{aligned} s_i(k)(s_i(k+1) - s_i(k)) &= -q_iT_is_i^2(k) - \alpha_iT_i|s_i(k)| \\ &\quad -\beta_iT_i|M_iB||s_i(k)| \\ &< 0 \end{aligned} \tag{20}$$

The discrete sliding mode achieves the condition of proof. □

Theorem 2: The chattering of the system will be decreasing, if $|s_i(k+1)| < |s_i(k)|$.

Proof: From (19), we can get

$$\begin{aligned} s_i(k+1) &= (1-q_iT_i)s_i(k) \\ &\quad -(\alpha_i + \beta_i|M_ib|)T_isgn(s_i(k)) \end{aligned} \tag{21}$$

when [27]

$$s_i(k) > \frac{\alpha_iT_i + \beta_iT_i|M_iB|}{1-q_iT_i} > 0 \tag{22}$$

for some $k \geq 0$,we have

$$\begin{aligned} s_i(k+1) &= (1-q_iT_i)s_i(k) - (\alpha_i + \beta_i|M_iB|)T_i \\ &= (1-q_iT_i)s_i(k) - \alpha_iT_i - \beta_iT_i|M_iB| \\ &> 0 \end{aligned} \tag{23}$$

when

$$s_i(k) < \frac{-\alpha_iT_i - \beta_iT_i|M_iB|}{1-q_iT_i} < 0 \tag{24}$$

for some $k \geq 0$,we have

$$\begin{aligned} s_i(k+1) &= (1-q_iT_i)s_i(k) + (\alpha_i + \beta_i|M_iB|)T_i \\ &= (1-q_iT_i)s_i(k) + \alpha_iT_i + \beta_iT_i|M_iB| \\ &< 0 \end{aligned} \tag{25}$$

From (21)-(25), when

$$s_i(k) > \frac{\alpha_iT_i + \beta_iT_i|M_iB|}{1-q_iT_i}$$

or

$$s_i(k) < \frac{-\alpha_iT_i - \beta_iT_i|M_iB|}{1-q_iT_i}$$

$s_i(k+1)$ and $s_i(k)$ are of the same sign. According to the (20), we can derive $|s_i(k+1)| < |s_i(k)|$, when $s_i(k)$ satisfies certain conditions. Besides, when

$$0 \leq s_i(k) \leq \frac{\alpha_iT_i + \beta_iT_i|M_iB|}{1-q_iT_i} \tag{26}$$

for some $k \geq 0$, one has

$$\begin{aligned} -\alpha_iT_i - \beta_iT_i|M_iB| &\leq s_i(k+1) \\ &\leq (1-q_iT_i)s_i(k) - \alpha_iT_i \\ &\leq \beta_iT_i|M_iB| + \alpha_iT_i \end{aligned} \tag{27}$$

and, when

$$\frac{-\alpha_iT_i - \beta_iT_i|M_iB|}{1-q_iT_i} \leq s_i(k) \leq 0 \tag{28}$$

for some $k \geq 0$, one obtains

$$\begin{aligned} -\alpha_iT_i - \beta_iT_i|M_iB| &\leq s_i(k+1) \\ &\leq \beta_iT_i|M_iB| + \alpha_iT_i \end{aligned} \tag{29}$$

Define $\varepsilon = \beta_iT_i|M_iB| + \alpha_iT_i$, thus it follows from (27) and (29) that $|s_i(k+1)| \leq \varepsilon$ as $|s_i(k)| \leq \varepsilon$ for some $k \geq 0$. □

Remark 4: According to the proof 1 and the proof 2, adopting the control law (17) can not only make the system reach the sliding mode surface but also reduce the chattering of the system, and the system has a quasi-sliding mode bandwidth (QSMB).

Remark 5: From remark 4 and the error dynamics system (10) of agent i, it is known that the communication delay between multi-agents can be well resolved in the neighbourhood of 2ε- switching surface $s_i(k) = 0$ by using the distributed control law (17).

Remark 6: In [23], there are two convergence law reachability demonstrations, but the communication delay is not taken into account, and the control approach utilized is more traditional. [24] addressed communication delay, in which, however, the system is just a second-order system, such the sliding mode technique cannot work for high-order synchronization problem of nonlinear multi-agent systems focused in this paper.

IV. OPTIMAL CONTROL LAW DESIGN AND STABILITY ANALYSIS

In this section, the optimal controller is designed based on the previously designed sliding mode controller and the stability proof of the system is given.

A. DESIGN OF OPTIMAL CONTROLLERS

For ideal sliding mode [27], satisfies $s_i(k+1) = s_i(k) = 0$, when $k \geq k^*$. Thus, (18) can be expressed as

$$s_i(k+1) = M_iB(u_{si}(k) + H_{ij} + \Delta_{ij}(k)) = 0 \tag{30}$$

The equivalent control law is obtained from (30)

$$u_{si}^{(eq)}(k) = -H_{ij} - \Delta_{ij}(k) \tag{31}$$

Substituting (31) into (10), we can derive the following linear error dynamics of DMAS as:

$$\delta_i(k+1) = A\delta_i(k) + Bu_{ri}(k) \tag{32}$$

Remark 7: When the system reaches the sliding surface and remains on the sliding surface, it ideally concludes $s_i(k) = 0$ and $s_i(k+1) = 0$. Therefore, the (30) can be derived from (18).

To design optimal controllers u_{ri}^*, a performance index is minimized for agent i in terms of (32)

$$J_i = \frac{1}{2}\sum_{k=0}^{\infty} \delta_i^T(k)Q_i\delta_i(k) + u_{ri}^T(k)R_iu_{ri}(k) \tag{33}$$

where $Q_i \geq 0$, $R_i > 0$. According to the infinite-time linear discrete system quadratic optimal control theory and dynamic programming theory [30], from (33) one has

$$V_i(k) = \frac{1}{2}[\delta_i{}^T(k)Q_i\delta_i(k) + u_{ri}^T(k)R_iu_{ri}] + V_i(k+1) \tag{34}$$

where $V_i(k) = \frac{1}{2}\sum_{s=k}^{\infty} \delta_i^T(s)Q_i\delta_i(s) + u_{ri}^T(s)R_iu_{ri}(s)$. For linear dynamics (32), the function $V_i(k)$ has quadratic form below

$$V_i(k) = \frac{1}{2}\delta_i{}^T(k)P_i\delta_i(k) \tag{35}$$

where $P_i > 0$. From (34)-(35), it follows

$$\begin{aligned}\delta_i{}^T(k)P_i\delta_i(k) = \delta_i{}^T(k)[Q_i + A^TP_iA]\delta_i(k) \\ +2u_{ri}^TB^TP_iA\delta_i(k) \\ +u_{ri}^T[R_i + B^TP_iB]u_{ri}\end{aligned} \tag{36}$$

The following optimal control law is obtained by taking the partial derivative of u_{ri} on the right-hand side of (36) and making it equal to 0.

$$\begin{aligned}u_{ri}^*(k) &= -K_i\delta_i(k) \\ &= -(R_i + B^TP_iB)^{-1}B^TP_iA\delta_i(k)\end{aligned} \tag{37}$$

Substituting (37) into (32) yields the following discrete-time algebraic Riccati equation

$$P_i = A^TP_iA - A^TP_iB(R_i + B^TP_iB)^{-1}B^TP_iA + Q_i \tag{38}$$

Substituting (37) into (32) yield the optimal closed-loop error system.

$$\delta_i(k+1) = (A - BK_i)\delta_i(k) \tag{39}$$

It is worth noting that the iterative method is used for the solution of P_i. According to the optimal control theory [30], the closed-loop system (39) is asymptotically stable, and the solution as in (37) is used as the optimal trajectory of the system.

Remark 8: As can be seen from (32), when $k \geq k^*$, the DMAS error dynamics equation is a linear system, that is, at this time the use of sliding mode control eliminates the MAS itself exists interference and communication delay.

B. STABILITY ANALYSIS

Substituting (9) into (10), the dynamic equation of the closed-loop system with state error is obtained

$$\begin{aligned}\delta_i(k+1) = (A - BK_i)\delta_i(k) - B((M_iB)^{-1}[q_iT_is_i(k) \\ +(\alpha_i + \beta_i|M_iB|T_isgn(s_i(k))))\end{aligned} \tag{40}$$

taking into (19), we have

$$\begin{aligned}\delta_i(k+1) = (A - BK_i)\delta_i(k) + B((M_iB)^{-1} \\ \cdot(s_i(k+1) - s_i(k)))\end{aligned} \tag{41}$$

Assume (A, B) is controllable, solving (37) and (38) will get the stabilizing controller gain K_i satisfying the eigenvalues of $A - BK_i$ within the unit circle [30]. Thus, the system eventually stabilizes asymptotically.

Remark 9: Based on the foregoing research, this work integrates integral sliding mode control with optimum control to produce optimal integral sliding mode control, which may minimize the impact of external disturbances on the system and decrease chattering. Therefore, the use of (9) enables fast consensus.

V. NUMERICAL SIMULATION

In this section, two examples are used to verify the effectiveness of the proposed method. Example 1 considers the second-order nonlinear multi-agent dynamics. Example 2 will verify the implementation on a multi-agent system consisting of multiple 2-DOF (degrees of freedom) helicopter systems with pitch and yaw angles with the objective of synchronizing velocities with the pilot [23].

According to [24], communication delay always exists, and in order to verify the effectiveness of the proposed control method, the communication delay is chosen to be $\tau = 0.5s$. For both examples, the sample time is $0.01s$, i.e. $T_i = 0.01s$. Consider the directed graph with four agents shown in Fig.1, where symbols are defined as follows:

F 1: Follower 1
F 2: Follower 2
F 3: Follower 3
F 4: Follower 4
L: Leader

A. EXAMPLE 1

Consider a discrete-time MAS as a second-order MAS with a system matrix of

$$A = \begin{bmatrix} 0.898 & 0.056 \\ 0.968 & -0.084 \end{bmatrix}, \quad B = \begin{bmatrix} 0.87 \\ -1.8 \end{bmatrix}$$

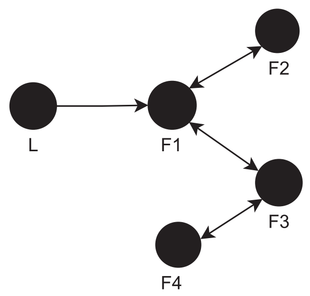

FIGURE 1. Communication topology diagram.

and

$$f(x_i(k)) = \sin(0.01\ x_i(k)), f(x_0(k)) = \sin(0.01\ x_0(k))$$
$$d_i = 0.1\ \sin(0.05\ k), d_0 = 0.1\ \sin(0.05\ k)$$

The initial states of the MAS are chosen randomly as follows:

$$x_0 = \begin{bmatrix} -8.14 \\ 30.33 \end{bmatrix}, \quad x_1 = \begin{bmatrix} -12.23 \\ 8.93 \end{bmatrix}, x_2 = \begin{bmatrix} -14.31 \\ 2.08 \end{bmatrix}$$
$$x_3 = \begin{bmatrix} -4.11 \\ -1.31 \end{bmatrix}, \quad x_4 = \begin{bmatrix} -4.57 \\ 14.28 \end{bmatrix}$$

The parameters will be used are set as

F 1: $M_1 = [0.73\ 0.84]$, $q_1 = 10$, $\alpha_1 = 0.3$, $\beta_1 = 3$, $Q_1 = diag\{50, 50\}$, $R_1 = 10$.

F 2: $M_2 = [0.68\ 0.78]$, $q_2 = 8$, $\alpha_2 = 0.2$, $\beta_2 = 6$, $Q_2 = diag\{33, 33\}$, $R_2 = 15$.

F 3: $M_3 = [0.8\ 0.98]$, $q_3 = 10$, $\alpha_3 = 0.23$, $\beta_3 = 10$, $Q_3 = diag\{45, 45\}$, $R_3 = 19$.

F 4: $M_4 = [0.65\ 0.72]$, $q_4 = 9$, $\alpha_4 = 0.14$, $\beta_4 = 8$, $Q_4 = diag\{64, 64\}$, $R_4 = 21$.

The parameters of the optimal control law $u_{ri}(k)$ are set as

$$K_1 = [0.351\ 0.056]$$
$$K_2 = [0.344\ 0.054]$$
$$K_3 = [0.345\ 0.052]$$
$$K_4 = [0.348\ 0.055]$$

Fig. 2 – Fig. 6 depicts the simulation findings. From Fig. 2 and Fig. 3, it can be seen that the followers follow the leader's trajectory at about 0.6s. Fig. 5 depicts the trajectory of the four agents' sliding mode surface states, with each agent's sliding mode states entering the quasi-sliding mode band after around 0.83s.

B. EXAMPLE 2

Consider the same discrete-time MAS dynamics of the 2-DOF helicopter as in [23]

$$A = \begin{bmatrix} 1 & 0 & 0.0262 & 0 \\ 0 & 1 & 0 & 0.0285 \\ 0 & 0 & 0.7571 & 0 \\ 0 & 0 & 0 & 0.9004 \end{bmatrix},$$

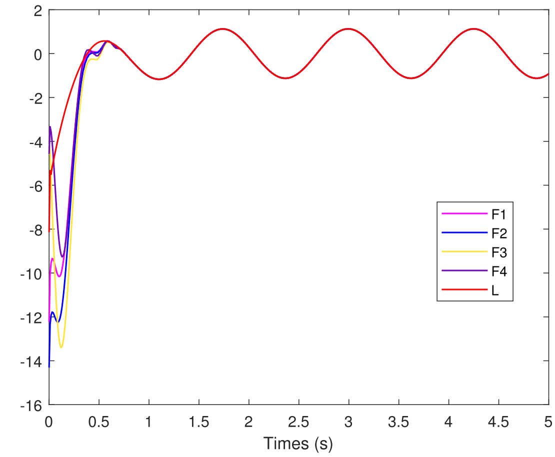

FIGURE 2. The first state tracking trajectory.

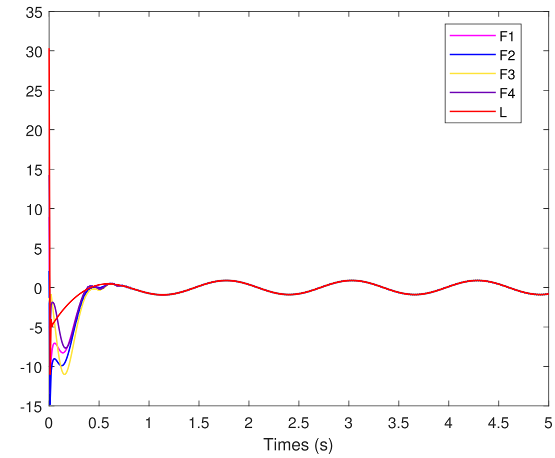

FIGURE 3. The second state tracking trajectory.

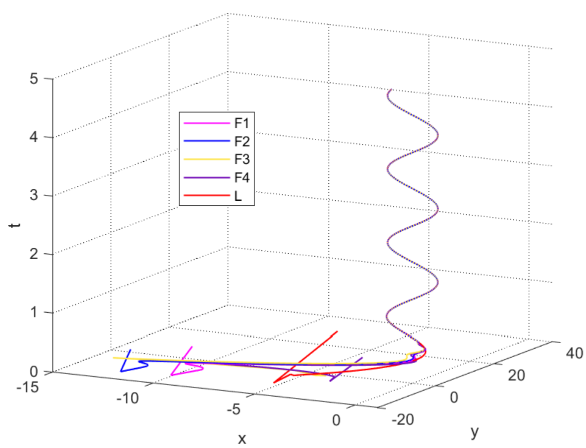

FIGURE 4. The tracking trajectories of the leader and four followers.

$$B = \begin{bmatrix} 0.0010 & 0 \\ 0.0001 & 0.0003 \\ 0.0620 & 0.0021 \\ 0.0069 & 0.0225 \end{bmatrix}$$
$$d_i(k) = 0.002\ \cos(0.86\ k)$$

and the leader has the same dynamics as the follower.

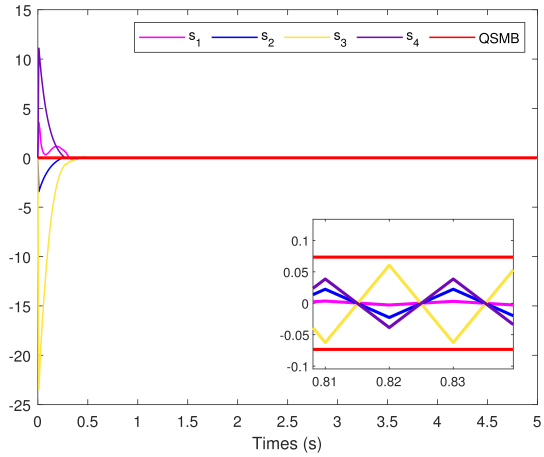

FIGURE 5. Sliding mode variable $s_i(k)$ trajectory.

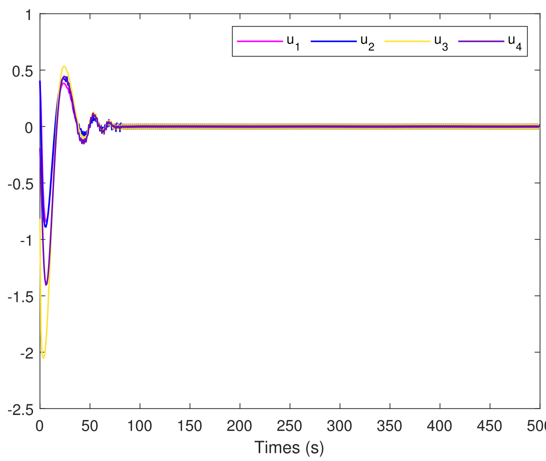

FIGURE 6. Control input signal trajectory $u_i(k)$.

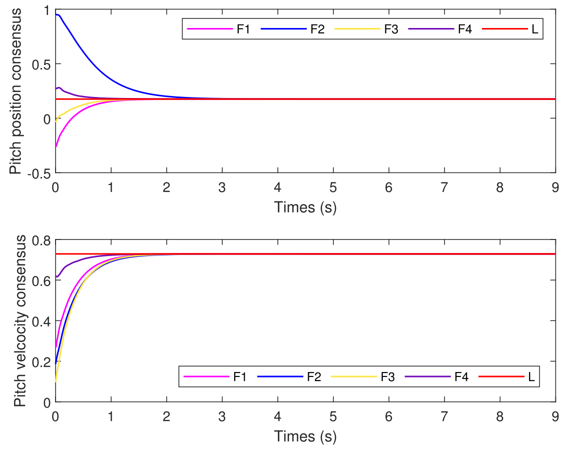

FIGURE 7. Pitch tracking trajectory.

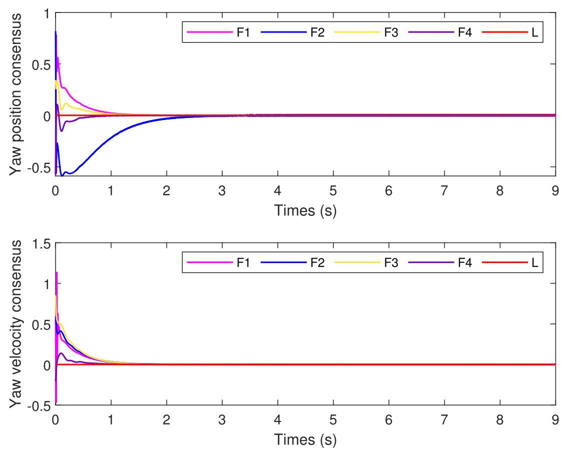

FIGURE 8. Yaw tracking trajectory.

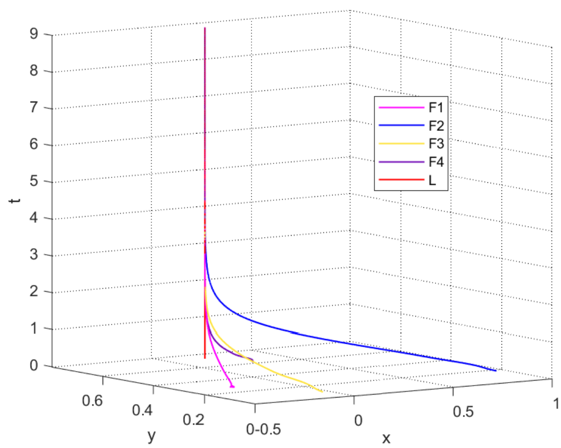

FIGURE 9. The tracking trajectories of the leader and four followers.

The initial states of the MAS are chosen randomly as follows:

$$x_0 = \begin{bmatrix} 0.17 \\ 0.73 \\ 0.01 \\ 0.01 \end{bmatrix}, \quad x_1 = \begin{bmatrix} -0.26 \\ 0.95 \\ 0.03 \\ 0.28 \end{bmatrix}, \quad x_2 = \begin{bmatrix} 0.28 \\ 0.19 \\ 0.09 \\ -0.57 \end{bmatrix}$$

$$x_3 = \begin{bmatrix} -0.58 \\ 0.81 \\ 0.34 \\ -0.57 \end{bmatrix}, \quad x_4 = \begin{bmatrix} 0.55 \\ 0.59 \\ 0.85 \\ 0.21 \end{bmatrix}$$

The parameters are set as

F 1: $q_1 = 10, \ \alpha_1 = 0.1, \ \beta_1 = 1, \ R_1 = 1$

$Q_1 = diag\{764, 764, 764, 764\}$

$$M_1 = \begin{bmatrix} 0.01 & 0.29 & 0.90 & 0.02 \\ 0.63 & 0.96 & 0.39 & 0.57 \end{bmatrix}$$

F 2: $q_2 = 12, \ \alpha_2 = 0.13, \ \beta_2 = 1.5, \ R_2 = 3$

$Q_2 = diag\{815, 815, 815, 815\}$

$$M_2 = \begin{bmatrix} 0.13 & 0.27 & 0.80 & 0.05 \\ 0.48 & 0.98 & 0.99 & 0.38 \end{bmatrix}$$

F 3: $q_3 = 11, \ \alpha_3 = 0.11, \ \beta_3 = 1.4, \ R_3 = 2$

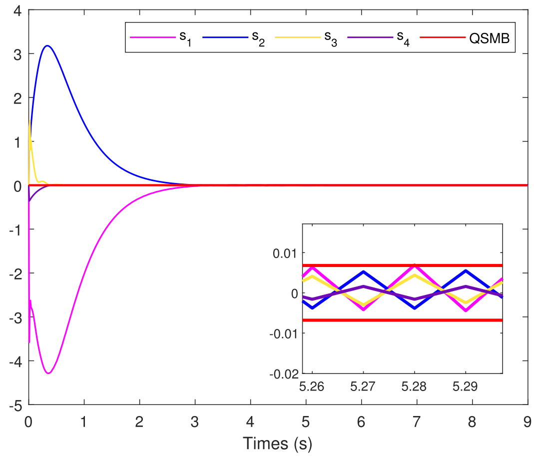

FIGURE 10. Sliding mode variable $s_i(k)$ state trajectory.

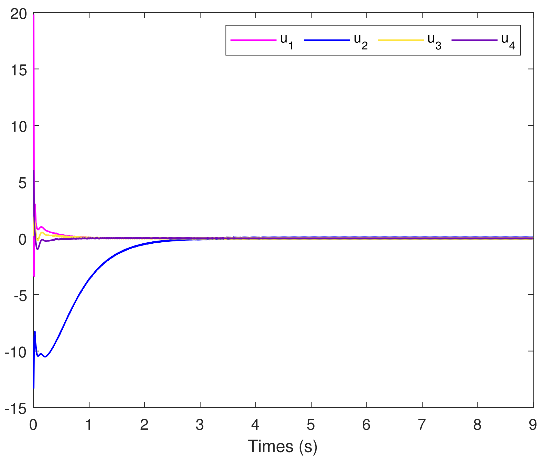

FIGURE 11. Control input signal trajectory $u_i(k)$.

$Q_3 = diag\{800, 800, 800, 800\}$

$$M_3 = \begin{bmatrix} 0.10 & 0.11 & 0.85 & 0.52 \\ 0.97 & 0.93 & 0.50 & 0.51 \end{bmatrix}$$

F 4: $q_4 = 5,\ \alpha_4 = 0.13,\ \beta_4 = 1.2,\ R_4 = 5$

$Q_4 = diag\{811, 811, 811, 811\}$

$$M_4 = \begin{bmatrix} 0.50 & 0.28 & 0.90 & 0.02 \\ 0.53 & 0.10 & 0.89 & 0.67 \end{bmatrix}$$

When the optimal controller gains K_i are

$$K_1 = \begin{bmatrix} 13.105 & 0.220 & 9.748 & 0.281 \\ -2.338 & 21.016 & -1.302 & 18.134 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} 10.511 & 0.536 & 7.328 & 0.550 \\ -1.567 & 13.955 & -0.651 & 11.265 \end{bmatrix}$$

$$K_3 = \begin{bmatrix} 11.525 & 0.453 & 8.266 & 0.478 \\ -1.815 & 16.284 & -0.853 & 13.514 \end{bmatrix}$$

$$K_4 = \begin{bmatrix} 9.090 & 0.595 & 6.0358 & 0.592 \\ -1.283 & 11.238 & -0.439 & 8.678 \end{bmatrix}$$

Fig.7 – Fig.11 show the response curves of the 2-DOF helicopter MASs model containing perturbations under the sliding controllers designed in this paper. By comparing Fig. 7 and Fig.8 with Fig.4 and Fig.5 in [23], this paper takes only 2.5s to complete the consensus because the communication delay $\tau = 0.5s$ between multi-agents is considered and the sliding mode controller is designed based on the optimal approach, while it takes about 4.5s with Gao's reaching law and 4s with Power rate reaching law to achieve consensus in [23], in which the communication delay is ignored. It can be seen from Fig.10 that at around 5.28s, all followers enter into the sliding mode band.

VI. CONCLUSION

This article has addressed the leader-follower consensus problem for the high-order discrete-time nonlinear MASs with communication delay and disturbance. In order to solve the nonlinearities and uncertainty existing in the system, a discrete-time integral sliding mode control method based on optimal control is proposed. The communication delay between the MASs is also well handled when designing the distributed sliding mode control law. Simulation results show the effectiveness of the control method in this paper.

REFERENCES

[1] R. A. Flores-Mendez, "Towards a standardization of multi-agent system framework," *XRDS: Crossroads, ACM Mag. Students*, vol. 5, no. 4, pp. 18–24, Jun. 1999.
[2] R. Olfati-Saber, J. A. Fax, and R. M. Murray, "Consensus and cooperation in networked multi-agent systems," *Proc. IEEE*, vol. 95, no. 1, pp. 215–233, Jan. 2007.
[3] J. Hu, Y. Hong, and L. Gao, "Tracking control for multi-agent consensus with an active leader and variable topology," *Automatica*, vol. 42, no. 7, pp. 1177–1182, Jul. 2007.
[4] R. Olfati-Saber and R. M. Murray, "Distributed cooperative control of multiple vehicle formations using structural potential functions," *IFAC Proc. Volumes*, vol. 35, no. 1, pp. 495–500, 2002.
[5] W. Ren and R. W. Beard, *Distributed Consensus in Multi-Vehicle Cooperative Control*, vol. 27, no. 2. London, U.K.: Springer, 2008, pp. 71–82.
[6] R. M. Murphey, "Recent research in cooperative control of multivehicle systems," *J. Dyn. Syst., Meas., Control*, vol. 129, no. 5, pp. 571–583, 2007.
[7] R. Murphey and P. M. Pardalos, *Cooperative Control and Optimization*, vol. 66. Springer, 2002.
[8] J. Li, Z. Xiao, J. Fan, T. Chai, and F. L. Lewis, "Off-policy Q-learning: Solving nash equilibrium of multi-player games with network-induced delay and unmeasured state," *Automatica*, vol. 136, Feb. 2022, Art. no. 110076.
[9] N. A. Lynch, *Distributed Algorithms*. Amsterdam, The Netherlands: Elsevier, 1996.
[10] S. Chen, D. W. C. Ho, L. Li, and M. Liu, "Fault-tolerant consensus of multi-agent system with distributed adaptive protocol," *IEEE Trans. Cybern.*, vol. 45, no. 10, pp. 2142–2155, Oct. 2015.
[11] W. Hu, L. Liu, and G. Feng, "Consensus of linear multi-agent systems by distributed event-triggered strategy," *IEEE Trans. Cybern.*, vol. 46, no. 1, pp. 148–157, Jan. 2016.
[12] Z. Li, Z. Wu, Z. Li, and Z. Ding, "Distributed optimal coordination for heterogeneous linear multiagent systems with event-triggered mechanisms," *IEEE Trans. Autom. Control*, vol. 65, no. 4, pp. 1763–1770, Apr. 2020.
[13] A. U. Rehman, M. Rehan, N. Iqbal, and C. K. Ahn, "LPV scheme for robust adaptive output feedback consensus of Lipschitz multiagents using Lipschitz nonlinear protocol," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 51, no. 11, pp. 7040–7050, Nov. 2021.
[14] L. Ma, Z. Wang, and H.-K. Lam, "Mean-square H_∞ consensus control for a class of nonlinear time-varying stochastic multi-agent systems: The finite-horizon case," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 47, no. 7, pp. 1050–1060, Jan. 2017.
[15] Z. Wang, J. Xu, X. Song, and H. Zhang, "Consensus conditions for multi-agent systems under delayed information," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 65, no. 11, pp. 1773–1777, Nov. 2018.

[16] W. Zhu, Z.-P. Jiang, and G. Feng, "Event-based consensus of multi-agent systems with general linear models," *Automatica*, vol. 50, no. 2, pp. 552–558, Feb. 2014.
[17] P. Tong, S. Chen, and L. Wang, "Finite-time consensus of multi-agent systems with continuous time-varying interaction topology," *Neurocomputing*, vol. 284, pp. 187–193, Apr. 2018.
[18] J. Qin, C. Yu, and H. Gao, "On discrete-time convergence for general linear multi-agent systems under dynamic topology," *IEEE Trans. Autom. Control*, vol. 59, no. 4, pp. 1054–1059, Apr. 2014.
[19] H. Zhao and S. Fei, "Distributed consensus for discrete-time heterogeneous multi-agent systems," *Int. J. Control*, vol. 91, no. 6, pp. 1376–1384, Jun. 2018.
[20] J. Li, J. Ding, T. Chai, F. L. Lewis, and S. Jagannathan, "Adaptive interleaved reinforcement learning: Robust stability of affine nonlinear systems with unknown uncertainty," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 33, no. 1, pp. 270–280, Jan. 2022.
[21] K. Yang, W. Zou, Z. Xiang, and R. Wang, "Fully distributed consensus for higher-order nonlinear multi-agent systems with unmatched disturbances," *Discrete Continuous Dyn. Syst.-S*, vol. 14, no. 4, p. 1535, 2021.
[22] S. Liang, Z. Liu, and Z. Chen, "Leader-following H_∞ consensus of discrete-time nonlinear multi-agent systems based upon output feedback control," *Trans. Inst. Meas. Control*, vol. 42, no. 7, pp. 1323–1333, 2020.
[23] K. Patel and A. Mehta, "Discrete-time sliding mode protocols for leader-following consensus of discrete multi-agent system with switching graph topology," *Eur. J. Control*, vol. 51, pp. 65–75, Jan. 2020.
[24] J. Zhang, S.-C. Chai, B.-H. Zhang, and G.-P. Liu, "Distributed model-free sliding-mode predictive control of discrete-time second-order nonlinear multiagent systems with delays," *IEEE Trans. Cybern.*, pp. 1–11, 2021.
[25] X. Wang, J. Zhu, and Z. Cheng, "Synchronization reachable topology and synchronization of discrete-time linear multi-agent systems," *IEEE Trans. Autom. Control*, vol. 60, no. 7, pp. 1927–1932, Jul. 2015.
[26] Y.-W. Liao and J. K. Hedrick, "Robust model predictive control with discrete-time integral sliding surface," in *Proc. Amer. Control Conf. (ACC)*, Jul. 2015, pp. 1641–1646.
[27] Y. Niu, D. W. C. Ho, and Z. Wang, "Improved sliding mode control for discrete-time systems via reaching law," *IET Control Theory Appl.*, vol. 4, no. 11, pp. 2245–2251, Nov. 2010.
[28] S. Sarpturk, Y. Istefanopulos, and O. Kaynak, "On the stability of discrete-time sliding mode control systems," *IEEE Trans. Autom. Control*, vol. 32, no. 10, pp. 930–932, Oct. 1987.
[29] H. Ma, J. Wu, and Z. Xiong, "Discrete-time sliding-mode control with improved quasi-sliding-mode domain," *IEEE Trans. Ind. Electron.*, vol. 63, no. 10, pp. 6292–6304, Oct. 2016.
[30] F. L. Lewis, D. Vrabie, and V. L. Syrmos, *Optimal Control*. Hoboken, NJ, USA: Wiley, 2012.

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