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An Extended MULTIMOORA Based on Trapezoidal Fuzzy Neutrosophic Sets and Objective Weighting Method in Group Decision-Making

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ABSTRACT Multi-objective optimization based on ratio analysis plus full multiplicative form (MULTIMOORA) is an efficient decision-making method for solving multi-criteria group decision-making (MCGDM) processes. It uses three strategies to examine different alternatives and to determine their evaluation values. These strategies include ratio system approach (RSA), reference point approach (RPA), and full multiplicative form (FMF). However, this method presents some challenges in the examination model, such as the ability to aggregate and determine the final result according to these strategies without considering value differences, the complexity of calculating the aggregation and multi-time comparisons, and the probability of distinguishing circular reasoning rules in dominance theory. In addition, determining an appropriate instrument for handling uncertainty, inconsistency, and incompleteness of information and a suitable weight for each criterion and decision-maker for reducing human interventions are also considered and will be a complex MCGDM process. To overcome these weaknesses, we propose an extended MULTIMOORA based on trapezoidal fuzzy neutrosophic numbers (TraFNNs) for MCGDM. We integrated it with ordinal priority approach (OPA) method to decide the initial weights for decision-makers and criteria without human subjective assessment. In addition, we used correlation coefficient and standard deviation (CCSD) technique to statistically compute the relationship between these strategies in resolving unique weights to obtain realistic results and eliminate the above issues. Finally, sensitivity and comparative analyses demonstrate the capability and effectivity of the extended method.

INDEX TERMS Correlation coefficient and standard deviation method, multi-criteria group decision-making, MULTIMOORA method, trapezoidal fuzzy neutrosophic number.

I. INTRODUCTION

The process of multiple-criteria group decision-making (MCGDM) is one of the most crucial and appropriate steps to overcome decision-making problems by emphasizing objective assessment by several policymakers and decision-makers. However, providing precise judgment is a challenging task for decision-makers. In addition, uncertainty in decision making plays the most predominant role and part of every decision-making process when evaluating these factors. To overcome this, Smarandache [1] first raised the idea of the neutrosophic set (NS), which allows policymakers or decision-makers (DMs) to assign three discrete membership values to alternatives over any criteria with values expressing truth-membership, indeterminacy-membership, and falsity-membership. Therefore, many researchers have routinely developed different approaches to address the problems of MCGDM. In this regard, Wang *et al.* [2] simplified this idea

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TABLE 1. List of abbreviations and acronyms.

Abbreviations	Acronyms
MULTIMOORA	Multi-objective optimization based on ratio analysis plus full multiplicative form
MCGDM	Multi-Criteria Group Decision-Making
RSA	Ratio System Approach
RPA	Reference Point Approach
FMF	Full Multiplicative Form
TraFNNs	Trapezoidal Fuzzy Neutrosophic Numbers
OPA	Ordinal Priority Approach
CCSD	Correlation Coefficient and Standard Deviation
NS	Neutrosophic Set
DMs	Decision-Makers
SVNS	Single-Valued Neutrosophic Set
TraFNS	Trapezoidal Fuzzy Neutrosophic Set
TraFN	Trapezoidal Fuzzy Number
SVTraNNs	Single-Valued Trapezoidal Neutrosophic Numbers
WASPAS	Weighted Aggregated Sum Product Assessment
MABAC	Multi-Attribute Border approximation Area Comparison
CODAS	Combinative Distance-based Assessment
EDAS	Evaluation based on Distance from Average Solution
ARAS	Additive Ratio Assessment
SWARA	Stepwise Weight Assessment Ratio Analysis
MOORA	Multi-Objective Optimization based on Ratio Analysis
TraFNNWA	Trapezoidal Fuzzy Neutrosophic Number Weighted Averaging
TraFNNWG	Trapezoidal Fuzzy Neutrosophic Number Weighted Geometric
SAW	Simple Additive Weighting
CSA	Cash Social Assistance

by developing single-valued neutrosophic set (SVNS) model and using it in decision making in engineering and science.

Although SVNS is an appropriate instrument for overcoming uncertainty, inconsistency, and incompleteness, it still represents an object with one value for each degree of membership. In reality, expressing an object using more than one value is crucial, and is expected to represent an appropriate assessment of the evaluated object. To consider this, Ye [3] introduced the trapezoidal fuzzy neutrosophic set (TraFNS) by combining the concept of the trapezoidal fuzzy number (TraFN) with the theory of NS and later proposed the trapezoidal fuzzy neutrosophic number (TraFNN).

A list of acronyms and abbreviations involved throughout this manuscript is provided in Table 1.

Inspired by the capabilities of TraNS, many researchers have employed this approach to solve the MCGDM problem. For instance, Wu *et al.* [4] constructed single-valued trapezoidal neutrosophic numbers (SVTraNNs) to handle the MCGDM issue. They also developed a new operator for SVTraNNs to avoid distortion and loss of information while the evaluating aggregation information. Deli [5] designed other SVTraNNs and introduced their characteristics to group decision making. He also designed novel aggregation operators such as ordered weighted geometric, ordered weighted arithmetic, hybrid geometric, and hybrid arithmetic for SVTraNNs. Chakraborty *et al.* [6] analyzed some types of linear and nonlinear TraFNNs in handling an essential role for the concepts of incompleteness and vagueness. Jana *et al.* [7] developed an interval trapezoidal neutrosophic set integrated with the TraFN and interval NS. Liang *et al.* [8] developed a model that integrates SVTraNNs with a weighted Bonferroni mean to address the MCGDM problem. Deli and Öztürk [9] defined a defuzzification method for obtaining crisp values for SVTraNNs. Aal *et al.* [10] utilized SVTraNNs to propose an evaluation model for information systems.

In addition, the MCGDM process can generally be stated as one that aims to determine the best choice from a defined set of options over a finite set of criteria based on DMs involvement in assessment. Multi-criteria involvement in this process could conflict with each other and bring this process into incredible complexity. In the last two decades, many researchers have developed decision-making methods using different logic procedures. Most aim to handle the complexity of conflicting criteria in the decision-making processes. Each method has its own advantages and disadvantages. Some of these methods are the weighted aggregated sum product assessment (WASPAS) [11], ordinal priority approach (OPA) [12], multi-attribute border approximation area comparison (MABAC) [13], combinative distance-based assessment (CODAS) [14], evaluation based on distance from average solution (EDAS) [15], additive ratio assessment (ARAS) [16], stepwise weight assessment ratio analysis (SWARA) [17], and multi-objective optimization based on ratio analysis (MOORA) [18]. These methods have been successfully applied to overcome decision-making concerns in various scientific, engineering, and industrial fields.

MOORA is a well-known method in decision science, in which its methodology can evaluate objects based on the ratio system approach (RSA) and the reference points approach (RPA) [19]. Compared with other traditional decision-making methods, it has some crucial features, such as simplicity of the algorithm, minimum mathematical computations, good result stability, and fast solving runtime [20].

Along with the growth of decision-making issues, MOORA was redeveloped by Brauers and Zavadskas to increase its robustness. They included a full multiplicative form (FMF) to evaluate multi objects optimally, and then introduced their improved method with the multi-objective optimization based on ratio analysis plus full multiplicative form (MULTIMOORA) method as more effective than the traditional MOORA method. Therefore, to evaluate objects, MULTIMOORA utilizes the strategies of RSA, RPA, and FMF, and aggregates them using dominance theory.

In recent years, researchers have applied the MULTIMOORA method to overcome the complex MCGDM problems. For example, it has been applied to space debris evaluation [21], old building revitalization evaluation [22], electric charging station selection [23], local green-area construction evaluation [24], medical apparatus preference [25], hybrid-vehicle machine selection [26], minimum-carbon tourism strategy judgment [27], and computer technology preference [28].

As presented in the above implementations of MULTIMOORA in previous studies, the method can be considered satisfactory for selecting, examining, and assessing alternatives with no human subjectivity orientations in numerous different phenomena. However, several studies have been conducted to improve the accuracy and efficiency of this method to tackle issues in different environments. Some recent developments in the MULTIMOORA method are described below.

Yang *et al.* [29] evaluated the civil airline safety status using MULTIMOORA based on index fuzzy segmentation. Wu *et al.* [30] utilized the improved Borda rule to enhance MULTIMOORA using probabilistic linguistic information. Stanujkić *et al.* [31] created a novel extension of MULTIMOORA for single-valued bipolar fuzzy circumstances. Xiao *et al.* [32] utilized prospect theory to enlarge MULTIMOORA for an MCGDM problem under multi-valued neutrosophic information. Camgöz-Akdağ *et al.* [33] successfully developed MULTIMOORA based on Einstein interval-valued fuzzy numbers to address decision-making problems. Gou *et al.* [34] also extended MULTIMOORA by combining it with a double-hierarchy hesitant fuzzy method for solving a decision-making task. In addition, Tian *et al.* [35] utilized independent NS entries to effectively upgrade MULTIMOORA under neutrosophic linguistic circumstances. Hafezalkotob *et al.* [36] utilized interval fuzzy numbers to extend MULTIMOORA and applied them to unravel a material-evaluation task. Tian *et al.* [37] proposed MULTIMOORA, based on a picture fuzzy set for various investment project selections. They also developed Schweizer–Sklar aggregation operators. Baidya *et al.* [38] utilized bipolar complex fuzzy sets to integrate MULTIMOORA with CRITIC for logistic provider selection in China.

A. RESEARCH GAP

Although the MULTIMOORA method continues to be developed from various perspectives as described above, most of the above improvement methods still assume a condition where the evaluation results obtained from the three strategies are considered of the same importance while ignoring the distortion of values in each of those strategies. Consequently, dominance theory will potentially have aggregation complexity and must execute multiple comparisons to obtain the final result. In addition, there is the possibility of existing circular reasoning, which makes it difficult to determine an appropriate final result. This is a challenge, and it is necessary to add an extra strategy to handle it to increase the effectiveness and accuracy of the MULTIMOORA method.

In this study, we used the correlation coefficient and standard deviation (CCSD) method to present a weighting method that can enhance the effectiveness and accuracy of the MULTIMOORA method. The CCSD method is a recent factual weighting method that integrates the correlation coefficient and standard deviation concepts to statistically determine the appropriate weight of each criterion.

B. MOTIVATIONS AND CONTRIBUTIONS

The main objective of this study is to extend the original MULTIMOORA method using TraFNN information to overcome the MCGDM problem. Some conflicting criteria, various assessments of DMs, and the determination of a robust decision-making method may impact final decision making. Therefore, in this study, we aimed to develop a proper mathematical model for selecting the best option from a finite set of alternatives using the extended MULTIMOORA method. In addition, we want to overcome some complex situations when we apply TraFNNs to the MCGDM methodology. To realize this, we execute this model in a case study of social aid distribution to select appropriately impoverished families affected by the COVID-19 outbreak in Indonesia.

Meanwhile, our contributions to this study are to provide the MCGDM methodology, which integrates three methods: two methods for the proper weight estimation technique, and another for placing alternatives in the precise order. These are the OPA, CCSD, and extended MULTIMOORA methods, respectively. We used the OPA method to obtain the initial weights of the DMs and criteria. The OPA can reduce human intervention in determining numerical weights. Subsequently, we estimate the objective weights statistically using the CCSD method and use them in the extended MULTIMOORA, especially in fusing the results from the three strategies of MULTIMOORA, for increased accuracy when compared with other MCGDM models. Finally, we assessed and evaluated the recipient candidates for social aid and selected impoverished families with the assistance of the MCGDM methodology using TraFNNs.

The leftover of this paper is organized as follows. Section 2 addresses the main concepts of the NS, SVNS, TraFNS, and TraFNNs. Section 2 also discusses the theory of the original MULTIMOORA and CCSD methods. Later on, Section 3 describes the details of the extended MULTIMOORA method, the weaknesses and challenges found in its original form, and the procedures for handling them. Section 4 explains the extended MULTIMOORA in detail through an illustrative example to demonstrate its capability to select properly impoverished families in a social aid distribution case study. Section 5 provides a sensitivity analysis to show the effect of different weights for the criteria and DMs on final decision making. In addition, in Section 4, we compare the results obtained from the extended MULTIMOORA with those of several other decision-making methods. Finally, conclusions and future studies are presented in Section 5.

II. PRELIMINARIES

In this section, we discuss some basic theories related to this study, such as the neutrosophic set, trapezoidal fuzzy neutrosophic number, MULTIMOORA, dominance, and CCSD methods.

A. THE THEORY OF NEUTROSOPHIC SET

Smarandache [1] introduced the theory of a neutrosophic set (NS) as depicted in Definition 1.

Definition 1 [1]: Let $\mathcal{N}$ be a set in a universe of discourse $\mathbb{D}$, generally identified by t, is defined as an NS if

$$\mathcal{N} = \{[t, \mathcal{T}_{\mathcal{N}}(t), \mathcal{I}_{\mathcal{N}}(t), \mathcal{F}_{\mathcal{N}}(t)] : t \in \mathbb{D}\}$$

where $\mathcal{T}_{\mathcal{N}}, \mathcal{I}_{\mathcal{N}}, \mathcal{F}_{\mathcal{N}} : \mathbb{D} \to]0^-, 1^+[$ is termed the membership functions of truth, indeterminacy, and falsity of $t \in \mathbb{D}$ in the set $\mathcal{N}$. Additionally, $\mathcal{T}_{\mathcal{N}}(t), \mathcal{I}_{\mathcal{N}}(t), \mathcal{F}_{\mathcal{N}}(t)$ portrays the degrees of truth membership, indeterminacy membership, and falsity membership of $t \in \mathbb{D}$, and must satisfy the condition below.

$$0^- \leq \mathcal{T}_{\mathcal{N}}(t) + \mathcal{I}_{\mathcal{N}}(t) + \mathcal{F}_{\mathcal{N}}(t) \leq 3^+, \quad \forall t \in \mathbb{D}$$

Because the above definition is difficult to apply in science and engineering, Wang *et al.* [2] simplified this and proposed the theory of an SVNS, which is also a subclass of NS.

Definition 2 [2]: Let $\tilde{\mathcal{N}}$ be a set in a universe of discourse $\mathbb{D}$, generally identified by t, is defined as an SVNS if

$$\tilde{\mathcal{N}} = \left\{\left[t, \tilde{\mathcal{T}}_{\tilde{\mathcal{N}}}(t), \tilde{\mathcal{I}}_{\tilde{\mathcal{N}}}(t), \tilde{\mathcal{F}}_{\tilde{\mathcal{N}}}(t)\right] : t \in \mathbb{D}\right\}$$

where $\tilde{\mathcal{T}}_{\tilde{\mathcal{N}}}, \tilde{\mathcal{I}}_{\tilde{\mathcal{N}}}, \tilde{\mathcal{F}}_{\tilde{\mathcal{N}}} : \mathbb{D} \to [0, 1]$ is termed the membership functions of truth, indeterminacy, and falsity of $t \in \mathbb{D}$ in the set $\tilde{\mathcal{N}}$. Additionally, $\tilde{\mathcal{T}}_{\tilde{\mathcal{N}}}(t), \tilde{\mathcal{I}}_{\tilde{\mathcal{N}}}(t), \tilde{\mathcal{F}}_{\tilde{\mathcal{N}}}(t)$ portrays the degrees of truth membership, indeterminacy membership, and falsity membership of $t \in \mathbb{D}$, and must satisfy the condition below.

$$0 \leq \tilde{\mathcal{T}}_{\tilde{\mathcal{N}}}(t) + \tilde{\mathcal{I}}_{\tilde{\mathcal{N}}}(t) + \tilde{\mathcal{F}}_{\tilde{\mathcal{N}}}(t) \leq 3, \quad \forall t \in \mathbb{D}$$

1) TRAPEZOIDAL FUZZY NEUTROSOPHIC SET

Ye proposed the concept of TraFNS, which is a combination of a TrFN and SVNS.

Definition 3 [3]: Let $\tilde{\mathcal{X}}$ be a set in a universe of discourse $\mathbb{D}$, generally identified by t, is defined as a TraFNS if

$$\tilde{\mathcal{X}} = \left\{\left[t, \mathcal{T}_{\tilde{\mathcal{X}}}(t), \mathcal{I}_{\tilde{\mathcal{X}}}(t), \mathcal{F}_{\tilde{\mathcal{X}}}(t)\right] : t \in \mathbb{D}\right\}$$

where $\tilde{\mathcal{T}}_{\tilde{\mathcal{N}}}, \tilde{\mathcal{I}}_{\tilde{\mathcal{N}}}, \tilde{\mathcal{F}}_{\tilde{\mathcal{N}}} : \mathbb{D} \to [0, 1]$ is termed the membership function of truth, indeterminacy, and falsity of $t \in \mathbb{D}$ in the set $\tilde{\mathcal{X}}$,

$$\begin{aligned}
\mathcal{T}_{\tilde{\mathcal{X}}}(t) &= \left(\tau^a_{\tilde{\mathcal{X}}}(t), \tau^b_{\tilde{\mathcal{X}}}(t), \tau^c_{\tilde{\mathcal{X}}}(t), \tau^d_{\tilde{\mathcal{X}}}(t)\right) \\
\mathcal{I}_{\tilde{\mathcal{X}}}(t) &= \left(i^a_{\tilde{\mathcal{X}}}(t), i^b_{\tilde{\mathcal{X}}}(t), i^c_{\tilde{\mathcal{X}}}(t), i^d_{\tilde{\mathcal{X}}}(t)\right) \\
\mathcal{F}_{\tilde{\mathcal{X}}}(t) &= \left(f^a_{\tilde{\mathcal{X}}}(t), f^b_{i\tilde{\mathcal{X}}}(t), f^c_{\tilde{\mathcal{X}}}(t), f^d_{\tilde{\mathcal{X}}}(t)\right)
\end{aligned}$$

are TrFNs and depict the trapezoidal degrees of truth membership, indeterminacy membership, and falsity membership of $t \in \mathbb{D}$ into $\tilde{\mathcal{X}}$ and must satisfy the condition below.

$$0 \leq \tau^d_{\tilde{\mathcal{X}}}(t) + i^d_{\tilde{\mathcal{X}}}(t) + f^d_{\tilde{\mathcal{X}}}(t) \leq 3$$

For amenity, those TrFNs are notated by

$$\begin{aligned}
\mathcal{T}_{\tilde{\mathcal{X}}}(t) &= \left(\tau^a_{\tilde{\mathcal{X}}}, \tau^b_{\tilde{\mathcal{X}}}, \tau^c_{\tilde{\mathcal{X}}}, \tau^d_{\tilde{\mathcal{X}}}\right) \\
\mathcal{I}_{\tilde{\mathcal{X}}}(t) &= \left(i^a_{\tilde{\mathcal{X}}}, i^b_{\tilde{\mathcal{X}}}, i^c_{\tilde{\mathcal{X}}}, i^d_{\tilde{\mathcal{X}}}\right) \\
\mathcal{F}_{\tilde{\mathcal{X}}}(t) &= \left(f^a_{\tilde{\mathcal{X}}}, f^b_{i\tilde{\mathcal{X}}}, f^c_{\tilde{\mathcal{X}}}, f^d_{\tilde{\mathcal{X}}}\right)
\end{aligned}$$

Therefore, a trapezoidal fuzzy neutrosophic number (TraFNN) can be expressed by

$$\tilde{n}_1 = \left[\left(\tau^a_1, \tau^b_1, \tau^c_1, \tau^d_1\right), \left(i^a_1, i^b_1, i^c_1, i^d_1\right), \left(f^a_1, f^b_1, f^c_1, f^d_1\right)\right]$$

Definition 4 [3]: Let

$$\begin{aligned}
\tilde{n}_1 &= \left[\left(\tau^a_1, \tau^b_1, \tau^c_1, \tau^d_1\right), \left(i^a_1, i^b_1, i^c_1, i^d_1\right), \left(f^a_1, f^b_1, f^c_1, f^d_1\right)\right], \\
\tilde{n}_2 &= \left[\left(\tau^a_2, \tau^b_2, \tau^c_2, \tau^d_2\right), \left(i^a_2, i^b_2, i^c_2, i^d_2\right), \left(f^a_2, f^b_2, f^c_2, f^d_2\right)\right]
\end{aligned}$$

be two TraFNNs in the universe of discourse $\mathbb{D}$. The mathematical rules for the two TraFNNs are as follows.

a) Addition

$$\tilde{n}_1 \oplus \tilde{n}_2 = \begin{bmatrix} (\tau^a_1 + \tau^a_2 - \tau^a_1\tau^a_2, \tau^b_1 + \tau^b_2 - \tau^b_1\tau^b_2, \\ \tau^c_1 + \tau^c_2 - \tau^c_1\tau^c_2, \tau^d_1 + \tau^d_2 - \tau^d_1\tau^d_2), \\ (i^a_1 i^a_2, i^b_1 i^b_2, i^c_1 i^c_2, i^d_1 i^d_2), \\ (f^a_1 f^a_2, f^b_1 f^b_2, f^c_1 f^c_2, f^d_1 f^d_2) \end{bmatrix}$$

b) Multiplication

$$\tilde{n}_1 \otimes \tilde{n}_2 = \begin{bmatrix} (\tau^a_1\tau^a_2, \tau^b_1\tau^b_2, \tau^c_1\tau^c_2, \tau^d_1\tau^d_2), \\ (i^a_1 + i^a_2 - i^a_1 i^a_2, i^b_1 + i^b_2 - i^b_1 i^b_2, \\ i^c_1 + i^c_2 - i^c_1 i^c_2, i^d_1 + i^d_2 - i^d_1 i^d_2), \\ (f^a_1 + f^a_2 - f^a_1 f^a_2, f^b_1 + f^b_2 - f^b_1 f^b_2, \\ f^c_1 + f^c_2 - f^c_1 f^c_2, f^d_1 + f^d_2 - f^d_1 f^d_2) \end{bmatrix}$$

c) The scalar multiplication

$$\zeta\tilde{n}_1 = \begin{bmatrix} \left(1 - \left(1 - \tau^a_1\right)^\zeta, 1 - \left(1 - \tau^b_1\right)^\zeta, \right. \\ \left. 1 - \left(1 - \tau^c_1\right)^\zeta, 1 - \left(1 - \tau^d_1\right)^\zeta\right), \\ \left(i^{a^\zeta}_1, i^{b^\zeta}_1, i^{c^\zeta}_1, i^{d^\zeta}_1\right), \\ \left(f^{a^\zeta}_1, f^{b^\zeta}_1, f^{c^\zeta}_1, f^{d^\zeta}_1\right) \end{bmatrix}, \quad \zeta > 0$$

d) The power of $\tilde{n}_1$

$$\tilde{n}^\zeta_1 = \begin{bmatrix} \left(\tau^{a^\zeta}_1, \tau^{b^\zeta}_1, \tau^{c^\zeta}_1, \tau^{d^\zeta}_1\right), \\ \left(1 - \left(1 - i^a_1\right)^\zeta, 1 - \left(1 - i^b_1\right)^\zeta, \right. \\ \left. 1 - \left(1 - i^c_1\right)^\zeta, 1 - \left(1 - i^d_1\right)^\zeta\right), \\ \left(1 - \left(1 - f^a_1\right)^\zeta, 1 - \left(1 - f^b_1\right)^\zeta, \right. \\ \left. 1 - \left(1 - f^c_1\right)^\zeta, 1 - \left(1 - f^d_1\right)^\zeta\right) \end{bmatrix}, \quad \zeta \geq 0.$$

Definition 5 [3]: Let

$$\tilde{n}_1 = \left[\left(\tau^a_1, \tau^b_1, \tau^c_1, \tau^d_1\right), \left(i^a_1, i^b_1, i^c_1, i^d_1\right), \left(f^a_1, f^b_1, f^c_1, f^d_1\right)\right]$$

be a TraFNN over a universe of discourse $\mathbb{D}$. Then,

a) The score function of $\tilde{n}_1$ can be represented by

$$\mathcal{S}(\tilde{n}_1) = \frac{1}{3}\left(2 + \frac{\tau_1^a + \tau_1^b + \tau_1^c + \tau_1^d}{4} - \frac{i_1^a + i_1^b + i_1^c + i_1^d}{4} - \frac{f_1^a + f_1^b + f_1^c + f_1^d}{4}\right), \quad \mathcal{S}(\tilde{n}_1) \in [0, 1] \tag{1}$$

(b) The accuracy function of $\tilde{n}_1$ can be represented by

$$\mathcal{A}(\tilde{n}_1) = \frac{\tau_1^a + \tau_1^b + \tau_1^c + \tau_1^d}{4} - \frac{f_1^a + f_1^b + f_1^c + f_1^d}{4}, \quad \mathcal{A}(\tilde{n}_1) \in [-1, 1] \tag{2}$$

Definition 6 [3]: Let $\tilde{n}_1$ and $\tilde{n}_2$ be two TraFNNs in the universe of discourse $\mathbb{D}$. They can then be compared by satisfying the following conditions.

a) If $\mathcal{S}(\tilde{n}_1) > \mathcal{S}(\tilde{n}_2)$ then $\tilde{n}_1 > \tilde{n}_2$;
b) If $\mathcal{S}(\tilde{n}_1) = \mathcal{S}(\tilde{n}_2)$ and
   (i). If $\mathcal{A}(\tilde{n}_1) = \mathcal{A}(\tilde{n}_2)$ then $\tilde{n}_1 = \tilde{n}_2$;
   (ii). If $\mathcal{A}(\tilde{n}_1) > \mathcal{A}(\tilde{n}_2)$ then $\tilde{n}_1 > \tilde{n}_2$.

2) AGGREGATION OPERATORS FOR TraFNNs

In this section, we recall two operators to aggregate TraFNN information and some basic properties.

Definition 7 [3]: Let $\tilde{n}_j\,(j = 1, 2, \ldots, r)$ be a set of TraFNNs and $\theta_j = (\theta_1, \theta_2, \ldots, \theta_r)^T$ be the weight vector of $\tilde{n}_j\,(j = 1, 2, \ldots, r)$ with $\theta_j \in [0, 1]$ and $\sum_{j=1}^{r} \theta_j = 1$, respectively.

1). a trapezoidal fuzzy neutrosophic number weighted averaging (TraFNNWA) operator is a mapping $TraFNNWA_\theta : \tilde{n}^r \rightarrow \tilde{n}$ such that

$$TraFNNWA_\theta = \bigoplus_{j=1}^{r} (\theta_j \tilde{n}_j) = \theta_1 \tilde{n}_1 \oplus \theta_2 \tilde{n}_2 \oplus \ldots \oplus \theta_r \tilde{n}_r \tag{3}$$

2). a trapezoidal fuzzy neutrosophic number weighted geometric (TraFNNWG) operator is a mapping $TraFNNWG_\theta : \tilde{n}^r \rightarrow \tilde{n}$ such that

$$TraFNNWG_\theta = \bigotimes_{j=1}^{r} \left(\theta_j^{\tilde{n}_j}\right) = \theta_1^{\tilde{n}_1} \otimes \theta_2^{\tilde{n}_2} \otimes \ldots \otimes \theta_r^{\tilde{n}_r} \tag{4}$$

Theorem 1: Let $\tilde{n}_j\,(j = 1, 2, \ldots, r)$ be a set of TraFNNs and $\theta_j = (\theta_1, \theta_2, \ldots, \theta_r)^T$ be the weight matrices of $\tilde{n}_j\,(j = 1, 2, \ldots, r)$ with $\theta_j \in [0, 1]$ and $\sum_{j=1}^{r} \theta_j = 1$, respectively.

1). The value of TraFNN aggregation using the TraFNNWA operator should also be a TraFNN and is expressed as

$$TraFNNWA_\theta(\tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_r) = \left[\begin{array}{l} \left(1 - \prod_{j=1}^{r}\left(1 - \tau_j^a\right)^{\theta_j}, 1 - \prod_{j=1}^{r}\left(1 - \tau_j^b\right)^{\theta_j}, \right. \\ \left. 1 - \prod_{j=1}^{r}\left(1 - \tau_j^c\right)^{\theta_j}, 1 - \prod_{j=1}^{r}\left(1 - \tau_j^d\right)^{\theta_j}\right), \\ \left(\prod_{j=1}^{r}\left(i_j^a\right)^{\theta_j}, \prod_{j=1}^{r}\left(i_j^b\right)^{\theta_j}, \prod_{j=1}^{r}\left(i_j^c\right)^{\theta_j}, \prod_{j=1}^{r}\left(i_j^d\right)^{\theta_j}\right), \\ \left(\prod_{j=1}^{r}\left(f_j^a\right)^{\theta_j}, \prod_{j=1}^{r}\left(f_j^b\right)^{\theta_j}, \prod_{j=1}^{r}\left(f_j^c\right)^{\theta_j}, \prod_{j=1}^{r}\left(f_j^d\right)^{\theta_j}\right) \end{array}\right] \tag{5}$$

2). The value of TraFNN aggregation using the TraFNNWG operator should also be a TraFNN and is expressed as

$$TraFNNWG_\theta(\tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_r) = \left[\begin{array}{l} \left(\prod_{j=1}^{r}\left(\tau_j^a\right)^{\theta_j}, \prod_{j=1}^{r}\left(\tau_j^b\right)^{\theta_j}, \prod_{j=1}^{r}\left(\tau_j^c\right)^{\theta_j}, \prod_{j=1}^{r}\left(\tau_j^d\right)^{\theta_j}\right), \\ \left(1 - \prod_{j=1}^{r}\left(1 - i_j^a\right)^{\theta_j}, 1 - \prod_{j=1}^{r}\left(1 - i_j^b\right)^{\theta_j}, \right. \\ \left. 1 - \prod_{j=1}^{r}\left(1 - i_j^c\right)^{\theta_j}, 1 - \prod_{j=1}^{r}\left(1 - i_j^d\right)^{\theta_j}\right), \\ \left(1 - \prod_{j=1}^{r}\left(1 - f_j^a\right)^{\theta_j}, 1 - \prod_{j=1}^{r}\left(1 - f_j^b\right)^{\theta_j}, \right. \\ \left. 1 - \prod_{j=1}^{r}\left(1 - f_j^c\right)^{\theta_j}, 1 - \prod_{j=1}^{r}\left(1 - f_j^d\right)^{\theta_j}\right) \end{array}\right] \tag{6}$$

Proof: Theorem 1 was proven in detail by Ye using mathematical induction and by investigating properties such as idempotency, boundedness, and monotonicity. A complete proof of this can be found in [3].

3) THE MAXIMUM DISTANCE BETWEEN TWO TraFNNs

Definition 8 [39]: Let $t_1 = \left(t_1^a, t_1^b, t_1^c, t_1^d\right)$ be a TrFN in a universe of discourse $\mathbb{D}$ then, the centroid point of t_1 is defined by

$$COG(t_1) = \left(\alpha_{t_1}, \beta_{t_1}\right)$$

where

$$\alpha_{t_1} = \frac{1}{3}\left[t_1^a + t_1^b + t_1^c + t_1^d - \frac{t_1^d t_1^c - t_1^a t_1^b}{\left(t_1^d + t_1^c\right) - \left(t_1^a + t_1^b\right)}\right],$$

$$\beta_{t_1} = \frac{1}{3}\left[1 + \frac{t_1^c - t_1^b}{\left(t_1^d + t_1^c\right) - \left(t_1^a + t_1^b\right)}\right],$$

$L_{t_1} = t_1^b - t_1^a$ is the left spread of t_1, and $R_{t_1} = t_1^d - t_1^c$ is the right spread of t_1.

Definition 9 [40]: Let $x = \left(t_x^a, t_x^b, t_x^c, t_x^d\right)$ and $y = \left(t_y^a, t_y^b, t_y^c, t_y^d\right)$ be two TrFNs. In addition, (α_x, β_x) and (α_y, β_y) are the centroid points, and (L_x, R_x) and (L_y, R_y) are the left/right spreads of x and y, respectively. The distance between these two TrFNs [40] is expressed as

$$\mathfrak{D}_{TrFN}(x, y) = \max\left\{\left|\alpha_x - \alpha_y\right|, \left|\beta_x - \beta_y\right|, \left|L_x - L_y\right|, \left|R_x - R_y\right|\right\}$$

Definition 10: Let $\tilde{n}_1 = [\mathcal{T}_1, \mathcal{I}_1, \mathcal{F}_1]$ and $\tilde{n}_2 = [\mathcal{T}_2, \mathcal{I}_2, \mathcal{F}_2]$ be two TraFNNs in a universe of discourse $\mathbb{D}$, then, the maximum distance between them is computed by:

$$\mathfrak{D}_{\max}(\tilde{n}_1, \tilde{n}_2) = \begin{cases} \mathfrak{D}_{TrFN}(\mathcal{T}_1, \mathcal{T}_2), & \tilde{n}_1, \tilde{n}_2 \in \Omega_{\max} \\ \mathfrak{D}_{TrFN}(\mathcal{F}_1, \mathcal{F}_2), & \tilde{n}_1, \tilde{n}_2 \in \Omega_{\min} \end{cases} \tag{7}$$

where $\Omega_{\max}$ is the set of benefit criteria and $\Omega_{\min}$ is the set of cost criteria.

B. THE ORIGINAL MULTIMOORA METHOD

MULTIMOORA is an extension of the MOORA developed by Brauers and Zavadskas by adding an FMF strategy to increase its performance. Therefore, the three strategies of MULTIMOORA are the RSA, RPA, and FMF [41]. Dominance theory must be applied to determine the final result because of the different inevitable results among the three strategies.

Initially, this method constructs a decision matrix $M = \left(m_{ij}\right)_{mxn}$, which consists of m alternatives and n criteria. Owing to the differentiation of criteria measurement units, the matrix must be transformed into a normalized decision matrix $\tilde{M} = \left(\tilde{m}_{ij}\right)_{mxn}$. Furthermore, to evaluate alternatives based on the three strategies of this method separately, the first step of each approach strategy is always taken from the normalized decision matrix.

1) RATIO SYSTEM APPROACH (RSA)

The RSA strategy considers the values of the overall importance of the alternatives (y_i) ordered in descending order to determine the result. The value of y_i can be obtained using Equation (8).

$$y_i = y_i^+ - y_i^- \tag{8}$$

where y_i^+ is the overall importance value of alternative i for the benefit criteria and y_i^- is the overall importance value of alternative i for the cost criteria. The values of y_i^+ and y_i^- can be obtained using Eqs. (9) and (10), respectively.

$$y_i^+ = \sum\nolimits_{j\in\Omega_{\max}} w_j \tilde{m}_{ij} \tag{9}$$

$$y_i^- = \sum\nolimits_{j\in\Omega_{\min}} w_j \tilde{m}_{ij} \tag{10}$$

where $\Omega_{\max}$ is the set of benefit criteria, $\Omega_{\min}$ is the set of cost criteria, w_j is the weight of criterion j, and $\tilde{m}_{ij}$ is the normalized evaluation value of alternative i over criterion j.

The value of $\tilde{m}_{ij}$ can be obtained using Eq. (11).

$$\tilde{m}_{ij} = \frac{m_{ij}}{\sqrt{\sum_{i=1}^{n} m_{ij}^2}}, \quad i = 1, 2, .., m; j = 1, 2, \ldots, n \tag{11}$$

where m_{ij} is the entry of the matrix M, or the evaluation value of the alternative $i$over the criterion j.

2) REFERENCE POINT APPROACH (RPA)

The RPA strategy considers the maximum distance of the alternatives to the reference point ($D_i^{\max}$) in ascending order to determine the result. The value of $D_i^{\max}$ can be calculated using Eq. (12).

$$D_i^{\max} = \max_j \left(w_j.\left|\tilde{m}_j^* - \tilde{m}_{ij}\right|\right) \tag{12}$$

where $\tilde{m}_j^*$ is the coordinate of reference point j. The values of $\tilde{m}_j^*$ can be obtained using Equation (13).

$$\tilde{m}_j^* = \begin{cases} \max\limits_i \left(m_{ij}\right), & j \in \Omega_{\max} \\ \min\limits_i \left(m_{ij}\right), & j \in \Omega_{\min} \end{cases} \tag{13}$$

3) FULL MULTIPLICATION FORM (FMF)

The FMF strategy considers the overall utility of the alternatives (u_i) in descending order to determine the result. The value of u_i can be obtained using Equation (14).

$$u_i = \frac{b_i}{c_i} \tag{14}$$

where b_i is the product of the weighted evaluation rankings for the benefit criteria and c_i is the product of the weighted evaluation rankings for the cost criteria. The values of b_i and c_i can be calculated using Eq. (15) and (16), respectively:

$$b_i = \prod_{j\in\Omega_{\max}} w_j \tilde{m}_{ij} \tag{15}$$

$$c_i = \prod_{j\in\Omega_{\min}} w_j \tilde{m}_{ij} \tag{16}$$

4) THE THEORY OF DOMINANCE

Brauers and Zavandskas [41] introduced the theory of dominance to optimize a multi-alternatives problem for the three approaches of the MULTIMOORA method. According to these authors, there are seven rules for this theory, as described below.

1) Absolute dominance: If an alternative is stated as absolute dominance over all alternatives, denoted by *(1-1-1)*, this alternative is the first position in the alternative ranking ordered by the three approaches.
2) General dominance: An alternative is defined as general dominance when prioritized using two approaches of the three approaches. Accordingly, if $x\mathbb{P}y$ means x is preferred to y and assuming $v\mathbb{P}w\mathbb{P}x\mathbb{P}y$, the general dominance shows the following conditions.
   - *(z-w-w)* is generally dominant *(y-x-x)*
   - *(v-x-v)* is generally dominant *(w-x-w)*

– *(v-v-y)* is generally dominant *(w-w-x)*

3) Transitiveness: If alternative x dominates y and y dominates z, then x is also the dominant z.
4) Overall dominance: If the three approaches prefer alternative x over the other alternative. For example, *(x-x-x)* represents overall dominance over *(y-y-y)*.
5) Absolute equability: The condition under which the three approaches return the same result for the two alternatives.
6) Partial equability: Two alternatives are stated as partial equability when one of the approaches prioritizes the two alternatives at the same level, but one of those alternatives dominates the other. For example, *(y-z-v)* and *(x-z-y)* indicate partial equability.
7) Circular reasoning: Despite the above situations defined as rules, there is still the possibility of having a circular reasoning condition. For example, the results of the three approaches (RSA-RPA-FMF) for alternatives X, Y, and Z were (12-21-15), (15-17-16), and (16-20-13), respectively. Accordingly, alternative X generally dominates alternative Y, alternative Y generally dominates alternative Z, and alternative Z generally dominates alternative X. Under these conditions, this is called circular reasoning. Thus, the dominance theory gives them the same ranking.

C. THE CCSD METHOD

Wang and Luo [42] introduced the CCSD method, which is an integrated method between the standard deviation (SD) and correlation of each criterion. It has several benefits compared with other weighting methods. According to [42], this method can perform a normalization process with no specific approach and can comprehensively determine the weights. In addition, it can provide convincing weight to criteria through a transparency mechanism. Therefore, these will be reasons why we use it to overcome the constraints of the MULTIMOORA method. The following steps describe the analysis and determination of criteria weights using this integrated method.

1). Construct a decision matrix $A = \left(a_{ij}\right)_{mxn}$ where a_{ij} is the evaluation value of alternative R_i under criterion C_j.

2). Matrix A is normalized to matrix N, the elements of which can be calculated using (17) and (18) for the benefit ($\Omega_{\max}$) and cost criteria ($\Omega_{\min}$), respectively.

$$n_{ij} = \frac{a_{ij} - a_j^-}{a_j^+ - a_j^-}, \quad i = 1, \ldots, n;\ j \in \Omega_{\max} \tag{17}$$

$$n_{ij} = \frac{a_j^+ - a_{ij}}{a_j^+ - a_j^-}, \quad i = 1, \ldots, n;\ j \in \Omega_{\min} \tag{18}$$

where $a_j^+ = \max_{1 \le i \le n}\left\{a_{ij}\right\}$ and $a_j^- = \min_{1 \le i \le n}\left\{a_{ij}\right\}$. Equation (19) includes the values of n_{ij} as the entries of the matrix N.

$$N = \left(n_{ij}\right)_{mxn} = \begin{array}{c} \\ R_1 \\ R_2 \\ \vdots \\ R_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} n_{11} & n_{12} & \cdots & n_{1n} \\ n_{21} & n_{22} & \cdots & n_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ n_{m1} & n_{m2} & \cdots & n_{mn} \end{bmatrix} \end{array} \tag{19}$$

The vector $W = [w_1, w_2, \ldots, w_n]$ is the weight matrix of the criteria that satisfies the conditions in $W \geq 0$ and (20).

$$\sum\nolimits_{j=1}^{n} w_j = 1 \tag{20}$$

3). Calculate the overall judgment of the decision-making alternatives, according to the simple additive weighting (SAW) [43] method using (21).

$$o_i = \sum\nolimits_{j=1}^{n} n_{ij} w_j, \quad i = 1, 2, \ldots, m \tag{21}$$

This overall judgment value is linearly proportional to the criteria weights. So, an alternative obtaining the highest overall judgment value will be the best-selected alternative. We then attempt to erase criterion C_j from the set of criteria and consider its consequence on decision-making. After dropping out the criterion C_j, the overall judgment value of each alternative can be re-calculated by using (22).

$$o_{ij} = \sum\nolimits_{k=1, k \neq j}^{n} n_{ik} w_k, \quad i = 1, 2, \ldots, n \tag{22}$$

The correlation coefficient between the value of C_j and the aforementioned overall judgment value can be calculated using Eq. (23).

$$F_j = \frac{\sum_{i=1}^{m}\left(n_{ij} - \bar{n}_j\right)\left(o_{ij} - \bar{o}_j\right)}{\sqrt{\sum_{i=1}^{m}\left(n_{ij} - \bar{n}_j\right)^2 . \sum_{i=1}^{m}\left(o_{ij} - \bar{o}_j\right)^2}}, \quad j = 1, \ldots, n \tag{23}$$

where the values of $\bar{n}_j$ and $\bar{o}_j$ are expressed as (24) and (25), respectively.

$$\bar{n}_j = \frac{1}{m}\sum\nolimits_{i=1}^{m} n_{ij}, \quad j = 1, 2, \ldots, n \tag{24}$$

$$\bar{o}_j = \frac{1}{m}\sum\nolimits_{i=1}^{m} o_{ij} = \sum\nolimits_{k=1, k \neq j}^{n} \bar{n}_k w_k, \quad j = 1, 2, \ldots, n \tag{25}$$

According to the F_j value, there were three situations. In the first situation, if the F_j is large enough to be close to 1, the criterion C_j and the overall judgment value with the admittance of C_j will acquire nearly identical numerical distributions and rankings. In this situation, the deletion of C_j does not significantly affect the decision-making procedure. Therefore, the weight of C_j is low. In the second situation, if F_j is sufficiently small to be close to 0, the criterion C_j and the overall judgment value with the admittance of C_j will acquire different numerical distributions and rankings. In this situation, the deletion of C_j significantly affects the decision-making procedure. Therefore, the weight of the C_j

TABLE 2. The linguistic values for TraFNN information.

Linguistic Terms (L_i)	TraFNN Values	$S(L_i)$
Absolutely-Low (AL)	[(0.10,0.13,0.14,0.15), (0.85,0.87,0.90,0.95), (0.05,0.07,0.10,0.15)]	0.382
Very-Low (VL)	[(0.15,0.17,0.19,0.20), (0.75,0.80,0.83,0.85), (0.15,0.17,0.19,0.20)]	0.398
Low (L)	[(0.20,0.22,0.24,0.25), (0.67,0.69,0.73,0.75), (0.20,0.21,0.24,0.25)]	0.431
Fairly-Low (FL)	[(0.25,0.27,0.29,0.30), (0.55,0.57,0.63,0.65), (0.25,0.27,0.29,0.30)]	0.467
Medium (M)	[(0.50,0.53,0.54,0.55), (0.45,0.47,0.52,0.55), (0.30,0.35,0.37,0.40)]	0.559
Fairly-High (FH)	[(0.55,0.57,0.63,0.65), (0.25,0.27,0.29,0.30), (0.25,0.27,0.29,0.30)]	0.682
High (H)	[(0.65,0.70,0.73,0.75), (0.20,0.21,0.23,0.25), (0.20,0.21,0.23,0.25)]	0.754
Very-High (VH)	[(0.75,0.80,0.83,0.85), (0.15,0.17,0.19,0.20), (0.15,0.16,0.19,0.20)]	0.818
Absolutely-High (AH)	[(0.85,0.89,0.93,0.95), (0.10,0.11,0.14,0.15), (0.05,0.10,0.13,0.15)]	0.891

should be higher. The third situation occurs when a criterion has the same impact on all alternatives; it can be temporarily excluded from the criteria set with no impact on the decision. In other words, any criterion with a high SD value should be assigned a greater weight than the other criteria with low SDs. Based on the above analysis, the weight of each criterion can be defined as (26).

$$w_j = \frac{\sigma_j\sqrt{1-F_j}}{\sum_{k=1}^{n}\sigma_k\sqrt{1-F_k}}, \quad j = 1, 2, \ldots, n \tag{26}$$

where σ_j is the standard deviation of C_j, which can be obtained using Eq. (27).

$$\sigma_j = \sqrt{\frac{1}{m}\sum_{i=1}^{m}\left(n_{ij}-\bar{n}_j\right)^2}, \quad j = 1, 2, \ldots, n \tag{27}$$

To reduce the difference between the highest and lowest weights, it is necessary to compute the root-squared value of $1 - F_j$. However, the difference between them is higher. Equation (26) is a nonlinear system that obtains n equations and n weight variables. Therefore, to obtain a solution to (26), it is necessary to convert this nonlinear system into a nonlinear optimization model, as defined in (28).

$$\begin{aligned} &\textit{Minimize } J = \sum_{j=1}^{n}\left(w_j - \frac{\sigma_j\sqrt{1-F_j}}{\sum_{k=1}^{n}\sigma_k\sqrt{1-F_k}}\right)^2 \\ &\textit{Subject to } \sum_{j=1}^{n} w_j = 1 \quad w_j \geq 0,\ j = 1, 2, \ldots, n \end{aligned} \tag{28}$$

We can utilize MS. Excel solver or MATLAB to solve this model.

III. THE PROPOSED MULTIMOORA BASED ON TraFNN FOR MCGDM PROBLEM

In this section, we propose an MCGDM methodology that incorporates TraFNN and MULTIMOORA methods. Subsequently, we call this the TraFNN-MULTIMOORA method. Assume that there is a management team of r policymakers or DMs $D = \{D_1, D_2, \ldots, D_r\}$ with the DM's weight matrix of DMs $\mathcal{E} = \{\mathcal{E}_1, \mathcal{E}_2, \ldots, \mathcal{E}_r\}$ that satisfies $\mathcal{E}_i \in [0, 1]$ and $\sum_{i=1}^{r}\varepsilon_i = 1$. They are responsible for examining m alternatives $R = \{R_1, R_2, \ldots, R_m\}$ over n criteria $C = \{C_1, C_2, \ldots, C_n\}$, where the weight matrix of the criteria is $\mathcal{W} = \{\mathcal{W}_1, \mathcal{W}_2, \ldots, \mathcal{W}_n\}$, that satisfies $\mathcal{W}_j \in [0, 1]$ and $\sum_{j=1}^{n}\mathcal{W}_j = 1$.

The TraFNN-MULTIMOORA method comprises eight main steps, as illustrated in Fig. 1. For convenience, the following steps describe the complete calculation procedure of the TraFNN-MULTIMOORA method for MCGDM.

A. STEP 1: DEFINE THE LINGUISTIC TERMS AND VALUES UNDER TraFNN INFORMATION

DMs examine alternatives by providing linguistic values for the different criteria. They can freely determine a set of linguistic values, denoted as $L = \{L_1, L_2, \ldots, L_p\}$, where each linguistic value has a score function denoted as $\mathcal{S}(L_i)|i = 1, 2, \ldots, p$, and p is the number of linguistic values. However, given linguistic values must satisfy the $\mathcal{S}(L_1) < \mathcal{S}(L_2) < \ldots < \mathcal{S}(L_p)$ condition. Assume that L_1 is Absolutely-Low, L_2 is Very-Low, L_3 is Low, L_4 is Fairly-Low, L_5 is Medium, L_6 is Fairly-High, L_7 is High, L_8 is Very-High, and L_9 is Absolutely-High, then the linguistic terms, the linguistic values, and their TraFNN score functions can be seen in Table 2.

B. STEP 2: CONSTRUCT TraFNN DECISION-MAKING MATRICES

In this decision-making process, there are k decision makers, where each DM has her/his own personal judgment on the alternatives under different criteria. The DMs evaluate all alternatives by delivering their own evaluation values based on the identified linguistic terms, as shown in Table 2. The process of assigning an evaluation value constructs a decision-making matrix that can be expressed as (29):

$$\mathcal{T}_k = \begin{array}{c} \\ R_1 \\ R_2 \\ \vdots \\ R_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} t_{11}^k & t_{12}^k & \cdots & t_{1n}^k \\ t_{21}^k & t_{22}^k & \cdots & t_{2n}^k \\ \vdots & \vdots & \ddots & \vdots \\ t_{m1}^k & t_{m2}^k & \cdots & t_{mn}^k \end{bmatrix} \end{array} \tag{29}$$

where t_{ij}^k is a TraFNN depicted as (30).

$$\begin{aligned} t_{ij}^k &= [\mathcal{T}_{ij}^k, \mathcal{I}_{ij}^k, \mathcal{F}_{ij}^k] \\ &= \Big[\left(\tau_{ij}^{a(k)}, \tau_{ij}^{b(k)}, \tau_{ij}^{c(k)}, \tau_{ij}^{d(k)}\right), \left(\iota_{ij}^{a(k)}, \iota_{ij}^{b(k)}, \iota_{ij}^{c(k)}, \iota_{ij}^{d(k)}\right), \\ &\quad \left(f_{ij}^{a(k)}, f_{ij}^{b(k)}, f_{ij}^{c(k)}, f_{ij}^{d(k)}\right)\Big] \end{aligned} \tag{30}$$

In other words, t_{ij}^k is the evaluation value in the TraFNN format given by decision maker D_k for alternative R_i

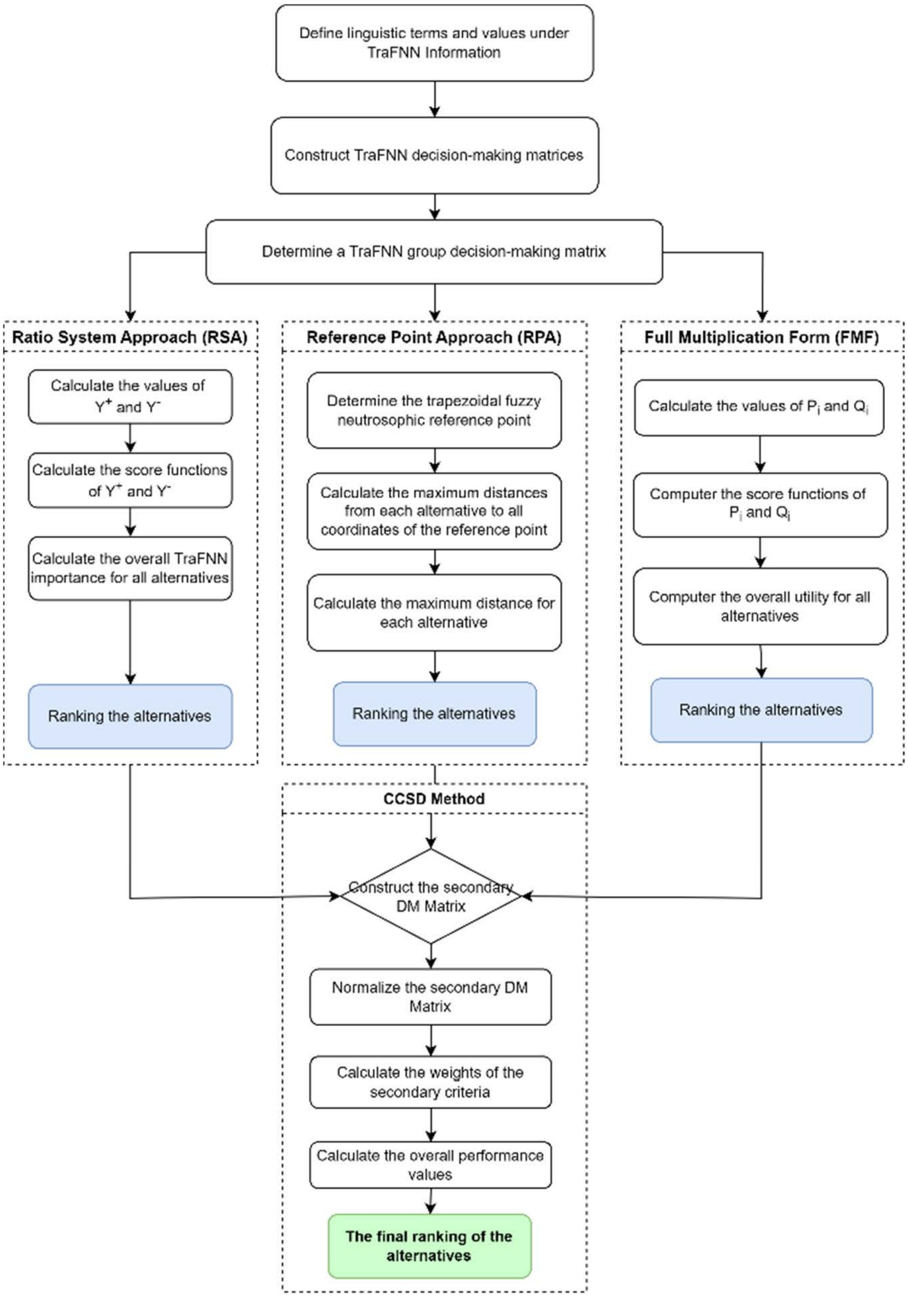

FIGURE 1. The flowchart of the proposed TraFNN-MULTIMOORA algorithm.

corresponding to criterion C_j. For instance, $t_{21}^k = FH$ means that the decision-maker k assesses alternative R_2 for criterion C_1 with a Fairly-High rating.

C. STEP 3: BUILD A TraFNN GROUP DECISION-MAKING MATRIX

The decision result involves several DMs who contribute to their judgments. It is important that differences in the background knowledge, experience, judgment, skills, perceptions, and professionalism of decision-makers reduce subjectivity in assessing alternatives. Therefore, to obtain a representative decision, it is necessary to aggregate all their evaluation information. In this case, we adopt (5) to aggregate all information constructed in the individual TraFNN decision-making matrices into a TraFNN group decision-making matrix using the TraFNNWA operator. Equation (31) shows the results of the construction of the TraFNN group decision-making matrix.

$$\mathcal{G} = \begin{array}{c} \\ R_1 \\ R_2 \\ \vdots \\ R_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1n} \\ g_{21} & g_{22} & \cdots & g_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ g_{m1} & g_{m2} & \cdots & g_{mn} \end{bmatrix} \end{array} \tag{31}$$

We can determine each element of the TraFNN group matrix $\mathcal{G}$ by using (32).

$$g_{ij}=\begin{bmatrix}\left(\mathbf{T}_{ij}^{a},\mathbf{T}_{ij}^{b},\mathbf{T}_{ij}^{c},\mathbf{T}_{ij}^{d}\right),\\ \left(\mathbf{I}_{ij}^{a},\mathbf{I}_{ij}^{b},\mathbf{I}_{ij}^{c},\mathbf{I}_{ij}^{d}\right),\\ \left(\Phi_{ij}^{a},\Phi_{ij}^{b},\Phi_{ij}^{c},\Phi_{ij}^{d}\right)\end{bmatrix} = \begin{bmatrix}\left(1-\prod_{k=1}^{r}\left(1-\tau_{ij}^{a(k)}\right)^{\mathcal{E}_k},1-\prod_{k=1}^{r}\left(1-\tau_{ij}^{b(k)}\right)^{\mathcal{E}_k},\right.\\ \left.1-\prod_{k=1}^{r}\left(1-\tau_{ij}^{c(k)}\right)^{\mathcal{E}_k},1-\prod_{k=1}^{r}\left(1-\tau_{ij}^{d(k)}\right)^{\mathcal{E}_k}\right),\\ \left(\prod_{k=1}^{r}\left(i_{ij}^{a(k)}\right)^{\mathcal{E}_k},\prod_{k=1}^{r}\left(i_{ij}^{b(k)}\right)^{\mathcal{E}_k},\right.\\ \left.\prod_{k=1}^{r}\left(i_{ij}^{c(k)}\right)^{\mathcal{E}_k},\prod_{k=1}^{r}\left(i_{ij}^{d(k)}\right)^{\mathcal{E}_k}\right),\\ \left(\prod_{k=1}^{r}\left(f_{ij}^{a(k)}\right)^{\mathcal{E}_k},\prod_{k=1}^{r}\left(f_{ij}^{b(k)}\right)^{\mathcal{E}_k},\right.\\ \left.\prod_{k=1}^{r}\left(f_{ij}^{c(k)}\right)^{\mathcal{E}_k},\prod_{k=1}^{r}\left(f_{ij}^{d(k)}\right)^{\mathcal{E}_k}\right)\end{bmatrix} \tag{32}$$

D. STEP 4: DETERMINE THE RESULT BASED-ON THE RSA STRATEGY

The first strategy used by the TraFNN-MULTIMOORA method to rank alternatives is RSA. RSA refers to the multiplication of the alternative evaluation value over a criterion in the aggregated matrix with its corresponding criterion weight. Because this case study uses TraFNN to express information, the distance value between the score functions of the overall weighted TraFNN value for benefit criteria and cost criteria will determine a decision. The alternative ranking process based on this strategy for the TraFNN-MULTIMOORA method begins with the TraFNN group decision-making matrix $\mathcal{G}$. The following steps demonstrate how the RSA calculation procedure can rank alternatives.

1) CALCULATE THE VALUES OF $\tilde{\mathcal{Y}}_i^{+}$ AND $\tilde{\mathcal{Y}}_i^{-}$

The overall positive importance value for each alternative refers to the sum of the weighted decision-making values of the benefit criteria ($\Omega_{\max}$). By contrast, the overall negative importance value is the sum of the weighted decision-making values from the cost criteria ($\Omega_{\min}$). Therefore, we can use (33) to calculate the overall positive importance values of alternative, denoted as $\tilde{\mathcal{Y}}_i^{+}$.

$$\tilde{\mathcal{Y}}_i^{+}=\begin{bmatrix}\left(\tilde{\mathbf{T}}_{i}^{a},\tilde{\mathbf{T}}_{i}^{b},\tilde{\mathbf{T}}_{i}^{c},\tilde{\mathbf{T}}_{i}^{d}\right),\\ \left(\tilde{\mathbf{I}}_{i}^{a},\tilde{\mathbf{I}}_{i}^{b},\tilde{\mathbf{I}}_{i}^{c},\tilde{\mathbf{I}}_{i}^{d}\right),\\ \left(\tilde{\Phi}_{i}^{a},\tilde{\Phi}_{i}^{b},\tilde{\Phi}_{i}^{c},\tilde{\Phi}_{i}^{d}\right)\end{bmatrix} = \begin{bmatrix}\left(1-\prod_{j\in\Omega_{\max}}\left(1-\mathbf{T}_{ij}^{a}\right)^{\mathcal{W}_j},1-\prod_{j\in\Omega_{\max}}\left(1-\mathbf{T}_{ij}^{b}\right)^{\mathcal{W}_j},\right.\\ \left.1-\prod_{j\in\Omega_{\max}}\left(1-\mathbf{T}_{ij}^{c}\right)^{\mathcal{W}_j},1-\prod_{j\in\Omega_{\max}}\left(1-\mathbf{T}_{ij}^{d}\right)^{\mathcal{W}_j}\right),\\ \left(\prod_{j\in\Omega_{\max}}\left(\mathbf{I}_{ij}^{a}\right)^{\mathcal{W}_j},\prod_{j\in\Omega_{\max}}\left(\mathbf{I}_{ij}^{b}\right)^{\mathcal{W}_j},\right.\\ \left.\prod_{j\in\Omega_{\max}}\left(\mathbf{I}_{ij}^{c}\right)^{\mathcal{W}_j},\prod_{j\in\Omega_{\max}}\left(\mathbf{I}_{ij}^{d}\right)^{\mathcal{W}_j}\right),\\ \left(\prod_{j\in\Omega_{\max}}\left(\Phi_{ij}^{a}\right)^{\mathcal{W}_j},\prod_{j\in\Omega_{\max}}\left(\Phi_{ij}^{b}\right)^{\mathcal{W}_j},\right.\\ \left.\prod_{j\in\Omega_{\max}}\left(\Phi_{ij}^{c}\right)^{\mathcal{W}_j},\prod_{j\in\Omega_{\max}}\left(\Phi_{ij}^{d}\right)^{\mathcal{W}_j}\right)\end{bmatrix} \tag{33}$$

Next, we perform a calculation operation for the overall negative importance value of alternative, denoted as $\tilde{\mathcal{Y}}_i^{-}$, using (34).

$$\tilde{\mathcal{Y}}_i^{-}=\begin{bmatrix}\left(\underset{\sim}{\mathbf{T}}_{i}^{a},\underset{\sim}{\mathbf{T}}_{i}^{b},\underset{\sim}{\mathbf{T}}_{i}^{c},\underset{\sim}{\mathbf{T}}_{i}^{d}\right),\\ \left(\underset{\sim}{\mathbf{I}}_{i}^{a},\underset{\sim}{\mathbf{I}}_{i}^{b},\underset{\sim}{\mathbf{I}}_{i}^{c},\underset{\sim}{\mathbf{I}}_{i}^{d}\right),\\ \left(\underset{\sim}{\Phi}_{i}^{a},\underset{\sim}{\Phi}_{i}^{b},\underset{\sim}{\Phi}_{i}^{c},\underset{\sim}{\Phi}_{i}^{d}\right)\end{bmatrix} = \begin{bmatrix}\left(1-\prod_{j\in\Omega_{\min}}\left(1-\mathbf{T}_{ij}^{a}\right)^{\mathcal{W}_j},1-\prod_{j\in\Omega_{\min}}\left(1-\mathbf{T}_{ij}^{b}\right)^{\mathcal{W}_j},\right.\\ \left.1-\prod_{j\in\Omega_{\min}}\left(1-\mathbf{T}_{ij}^{c}\right)^{\mathcal{W}_j},1-\prod_{j\in\Omega_{\min}}\left(1-\mathbf{T}_{ij}^{d}\right)^{\mathcal{W}_j}\right),\\ \left(\prod_{j\in\Omega_{\min}}\left(\mathbf{I}_{ij}^{a}\right)^{\mathcal{W}_j},\prod_{j\in\Omega_{\min}}\left(\mathbf{I}_{ij}^{b}\right)^{\mathcal{W}_j},\right.\\ \left.\prod_{j\in\Omega_{\min}}\left(\mathbf{I}_{ij}^{c}\right)^{\mathcal{W}_j},\prod_{j\in\Omega_{\min}}\left(\mathbf{I}_{ij}^{d}\right)^{\mathcal{W}_j}\right),\\ \left(\prod_{j\in\Omega_{\min}}\left(\Phi_{ij}^{a}\right)^{\mathcal{W}_j},\prod_{j\in\Omega_{\min}}\left(\Phi_{ij}^{b}\right)^{\mathcal{W}_j},\right.\\ \left.\prod_{j\in\Omega_{\min}}\left(\Phi_{ij}^{c}\right)^{\mathcal{W}_j},\prod_{j\in\Omega_{\min}}\left(\Phi_{ij}^{d}\right)^{\mathcal{W}_j}\right)\end{bmatrix} \tag{34}$$

2) DECIDE THE SCORE FUNCTIONS OF $\tilde{\mathcal{Y}}_i^{+}$ AND $\tilde{\mathcal{Y}}_i^{-}$

The score function is a mathematical calculation that defines a crisp value from a fuzzy or neutrosophic number. This method can not only rank fuzzy or neutrosophic numbers, but also measure the distance between two fuzzy or neutrosophic numbers. At this stage, we applied (1) to compute the score functions of $\tilde{\mathcal{Y}}_i^{+}$ and $\tilde{\mathcal{Y}}_i^{-}$, denoted as Ψ_i^{+} and Ψ_i^{-}, respectively. Equations (35) and (36) show how the two score functions were determined.

$$\Psi_i^+ = \mathcal{S}\left(\tilde{\mathcal{Y}}_i^+\right) \tag{35}$$

$$\Psi_i^- = \mathcal{S}\left(\tilde{\mathcal{Y}}_i^-\right) \tag{36}$$

3) CALCULATE THE OVERALL TRAPEZOIDAL FUZZY NEUTROSOPHIC IMPORTANCE FOR ALL ALTERNATIVES

We can determine the importance value of the overall TraFNN for an alternative by referring to a difference value between a sum of the weighted decision-making values of the benefit criteria and a sum of the weighted decision-making values of the cost criteria. Equation (37) calculates the overall TraFNN importance value for the alternative i, denoted as Ψ_i.

$$\Psi_i = \Psi_i^+ - \Psi_i^- \tag{37}$$

4) RANK THE ALTERNATIVES

In this step, the RSA ranks all the alternatives in descending order based on the values of Ψ_i.

E. STEP 5: DETERMINE THE RESULT BASED-ON THE RPA STRATEGY

The second strategy used by the TraFNN-MULTIMOORA method to rank alternatives was RPA. The RPA begins with the group matrix $\mathcal{G}$, where each entry in the matrix $\mathcal{G}$ is defined by (32). The following steps demonstrate how the RPA calculation procedure can rank alternatives.

1) DETERMINE THE TRAPEZOIDAL FUZZY NEUTROSOPHIC REFERENCE POINT

First, we defined the coordinates of the TraFNN reference point for all criteria, denoted as $\mathfrak{g}^* = \left\{\tilde{\mathfrak{g}}_1^*, \tilde{\mathfrak{g}}_2^*, \ldots, \tilde{\mathfrak{g}}_n^*\right\}$ in where $\tilde{\mathfrak{g}}_j^*$ represents TraFNN, $j = 1, 2, \ldots, n$, and $M = \{1, 2, \ldots, m\}$. Each coordinate can be calculated using Eq. (38).

$$\tilde{\mathfrak{g}}_j^* = \begin{cases} \left[\mathcal{T}_j^+, \mathcal{I}_j^-, \mathcal{F}_j^-\right] \\ \left[\mathcal{T}_j^-, \mathcal{I}_j^+, \mathcal{F}_j^+\right] \end{cases}
= \begin{cases} \left[\begin{array}{l} \left(\max\limits_{i\in M} \mathbf{T}_{ij}^a, \max\limits_{i\in M} \mathbf{T}_{ij}^b, \max\limits_{i\in M} \mathbf{T}_{ij}^c, \max\limits_{i\in M} \mathbf{T}_{ij}^d\right), \\ \left(\min\limits_{i\in M} \mathbf{I}_{ij}^a, \min\limits_{i\in M} \mathbf{I}_{ij}^b, \min\limits_{i\in M} \mathbf{I}_{ij}^c, \min\limits_{i\in M} \mathbf{I}_{ij}^d\right), \\ \left(\min\limits_{i\in M} \mathbf{\Phi}_{ij}^a, \min\limits_{i\in M} \mathbf{\Phi}_{ij}^b, \min\limits_{i\in M} \mathbf{\Phi}_{ij}^c, \min\limits_{i\in M} \mathbf{\Phi}_{ij}^d\right) \end{array}\right], \\ j \in \mathbf{\Omega}_{\max} \\ \left[\begin{array}{l} \left(\min\limits_{i\in M} \mathbf{T}_{ij}^a, \min\limits_{i\in M} \mathbf{T}_{ij}^b, \min\limits_{i\in M} \mathbf{T}_{ij}^c, \min\limits_{i\in M} \mathbf{T}_{ij}^d\right), \\ \left(\max\limits_{i\in M} \mathbf{I}_{ij}^a, \max\limits_{i\in M} \mathbf{I}_{ij}^b, \max\limits_{i\in M} \mathbf{I}_{ij}^c, \max\limits_{i\in M} \mathbf{I}_{ij}^d\right), \\ \left(\max\limits_{i\in M} \mathbf{\Phi}_{ij}^a, \max\limits_{i\in M} \mathbf{\Phi}_{ij}^b, \max\limits_{i\in M} \mathbf{\Phi}_{ij}^c, \max\limits_{i\in M} \mathbf{\Phi}_{ij}^d\right) \end{array}\right], \\ j \in \mathbf{\Omega}_{\min} \end{cases} \tag{38}$$

2) CALCULATE THE MAXIMUM DISTANCE FROM EACH ALTERNATIVE TO ALL COORDINATES OF THE REFERENCE POINT

After calculating all TraFNN reference points, we measured the maximum distance for each alternative by multiplying the distance between each element of the group matrix $\mathcal{G}$ corresponding to the TraFNN reference point and the corresponding criterion weight $\mathcal{W}_j$. The maximum distance can be calculated using Eq. (39).

$$\mathfrak{D}_{ij}^{\max} = \mathfrak{D}_{\max}\left(g_{11}, \tilde{g}_j^*\right)\mathcal{W}_j \tag{39}$$

where $\mathfrak{D}_{ij}^{\max}$ is the maximum distance of the i-th alternative corresponding to the j-th criterion to all coordinates, and $\mathcal{W}_j$ is the weight of the j-th criterion. The value of $\mathfrak{D}_{\max}\left(g_{11}, \tilde{g}_j^*\right)$ can be obtained precisely using (7).

3) CALCULATE THE MAXIMUM DISTANCE OF EACH ALTERNATIVE

The RPA determines the maximum distance by selecting the maximum value between the maximum value from each alternative and all reference points representing all criteria. This maximum distance is a representative value from decision-makers as an alternative. Equation (40) shows the mathematical notation used to select the maximum distance based on RPA.

$$\mathfrak{D}_i^{\max} = \max_{j\in N} \mathfrak{D}_{ij}^{\max}, \quad N = \{1, 2, \ldots, n\} \tag{40}$$

where $\mathfrak{D}_i^{\max}$ is the maximum distance of the i-th alternative, and n is the number of criteria used in the decision-making process.

4) RANK THE ALTERNATIVES

At this stage, the RPA ranks all alternatives in ascending order based on the values of $\mathfrak{D}_i^{\max}$.

F. STEP 6: DETERMINE THE RESULT BASED-ON THE FMF STRATEGY

The third strategy used by the TraFNN-MULTIMOORA method to rank alternatives is FMF. This strategy analyzes the impact of the weighted performance of the benefit and cost criteria on overall alternative utility. The following steps demonstrate how the FMF calculation procedure can rank alternatives.

1) CALCULATE THE VALUES OF $\tilde{\mathcal{P}}_i$ AND $\tilde{\mathcal{Q}}_i$

Once again, we begin with the group matrix $\mathcal{G}$ and apply the TraFNNWG operator, as presented in (6), to determine the values of the weighted performance product for the benefit criteria ($\Omega_{\max}$) denoted as $\tilde{\mathcal{P}}_i$ and the weighted performance product for the cost criteria ($\Omega_{\min}$) denoted as $\tilde{\mathcal{Q}}_i$ using (41) and (42), respectively.

$$\tilde{\mathcal{P}}_i = \begin{bmatrix} \left(\tilde{\mathcal{T}}_i^a, \tilde{\mathcal{T}}_i^b, \tilde{\mathcal{T}}_i^c, \tilde{\mathcal{T}}_i^d\right), \\ \left(\tilde{\mathcal{I}}_i^a, \tilde{\mathcal{I}}_i^b, \tilde{\mathcal{I}}_i^c, \tilde{\mathcal{I}}_i^d\right), \\ \left(\tilde{\mathcal{F}}_i^a, \tilde{\mathcal{F}}_i^b, \tilde{\mathcal{F}}_i^c, \tilde{\mathcal{F}}_i^d\right) \end{bmatrix} = \begin{bmatrix} \left(\prod_{j\in\Omega_{\max}} \left(\mathbf{T}_{ij}^a\right)^{\mathcal{W}_j}, \prod_{j\in\Omega_{\max}} \left(\mathbf{T}_{ij}^b\right)^{\mathcal{W}_j}, \right. \\ \left. \prod_{j\in\Omega_{\max}} \left(\mathbf{T}_{ij}^c\right)^{\mathcal{W}_j}, \prod_{j\in\Omega_{\max}} \left(\mathbf{T}_{ij}^d\right)^{\mathcal{W}_j}\right), \\ \left(1-\prod_{j\in\Omega_{\max}} \left(\mathbf{1}-\mathbf{I}_{ij}^a\right)^{\mathcal{W}_j}, 1-\prod_{j\in\Omega_{\max}} \left(\mathbf{1}-\mathbf{I}_{ij}^b\right)^{\mathcal{W}_j}, \right. \\ \left. 1-\prod_{j\in\Omega_{\max}} \left(\mathbf{1}-\mathbf{I}_{ij}^c\right)^{\mathcal{W}_j}, 1-\prod_{j\in\Omega_{\max}} \left(\mathbf{1}-\mathbf{I}_{ij}^d\right)^{\mathcal{W}_j}\right), \\ \left(1-\prod_{j\in\Omega_{\max}} \left(\mathbf{1}-\Phi_{ij}^a\right)^{\mathcal{W}_j}, 1-\prod_{j\in\Omega_{\max}} \left(\mathbf{1}-\Phi_{ij}^b\right)^{\mathcal{W}_j}, \right. \\ \left. 1-\prod_{j\in\Omega_{\max}} \left(\mathbf{1}-\Phi_{ij}^c\right)^{\mathcal{W}_j}, 1-\prod_{j\in\Omega_{\max}} \left(\mathbf{1}-\Phi_{ij}^d\right)^{\mathcal{W}_j}\right) \end{bmatrix} \tag{41}$$

$$\tilde{\mathcal{Q}}_i = \begin{bmatrix} \left(\underset{\sim}{\mathcal{T}}_i^a, \underset{\sim}{\mathcal{T}}_i^b, \underset{\sim}{\mathcal{T}}_i^c, \underset{\sim}{\mathcal{T}}_i^d\right), \\ \left(\underset{\sim}{\mathcal{I}}_i^a, \underset{\sim}{\mathcal{I}}_i^b, \underset{\sim}{\mathcal{I}}_i^c, \underset{\sim}{\mathcal{I}}_i^d\right), \\ \left(\underset{\sim}{\mathcal{F}}_i^a, \underset{\sim}{\mathcal{F}}_i^b, \underset{\sim}{\mathcal{F}}_i^c, \underset{\sim}{\mathcal{F}}_i^d\right) \end{bmatrix} = \begin{bmatrix} \left(\prod_{j\in\Omega_{\min}} \left(\mathbf{T}_{ij}^a\right)^{\mathcal{W}_j}, \prod_{j\in\Omega_{\min}} \left(\mathbf{T}_{ij}^b\right)^{\mathcal{W}_j}, \right. \\ \left. \prod_{j\in\Omega_{\min}} \left(\mathbf{T}_{ij}^c\right)^{\mathcal{W}_j}, \prod_{j\in\Omega_{\min}} \left(\mathbf{T}_{ij}^d\right)^{\mathcal{W}_j}\right), \\ \left(1-\prod_{j\in\Omega_{\min}} \left(\mathbf{1}-\mathbf{I}_{ij}^a\right)^{\mathcal{W}_j}, 1-\prod_{j\in\Omega_{\min}} \left(\mathbf{1}-\mathbf{I}_{ij}^b\right)^{\mathcal{W}_j}, \right. \\ \left. 1-\prod_{j\in\Omega_{\min}} \left(\mathbf{1}-\mathbf{I}_{ij}^c\right)^{\mathcal{W}_j}, 1-\prod_{j\in\Omega_{\min}} \left(\mathbf{1}-\mathbf{I}_{ij}^d\right)^{\mathcal{W}_j}\right), \\ \left(1-\prod_{j\in\Omega_{\min}} \left(\mathbf{1}-\Phi_{ij}^a\right)^{\mathcal{W}_j}, 1-\prod_{j\in\Omega_{\min}} \left(\mathbf{1}-\Phi_{ij}^b\right)^{\mathcal{W}_j}, \right. \\ \left. 1-\prod_{j\in\Omega_{\min}} \left(\mathbf{1}-\Phi_{ij}^c\right)^{\mathcal{W}_j}, 1-\prod_{j\in\Omega_{\min}} \left(\mathbf{1}-\Phi_{ij}^d\right)^{\mathcal{W}_j}\right) \end{bmatrix} \tag{42}$$

2) COMPUTE THE SCORE FUNCTIONS OF $\tilde{\mathcal{P}}_i$ AND $\tilde{\mathcal{Q}}_i$

As described in Section 4.2, this stage applies (1) to compute the score functions of $\tilde{\mathcal{P}}_i$ and $\tilde{\mathcal{Q}}_i$, denoted as p_i and q_i, respectively. Equations (43) and (44) show how to determine the values of both the parameters.

$$p_i = \mathcal{S}(\mathcal{P}_1) \tag{43}$$

$$q_i = \mathcal{S}(\mathcal{Q}_1) \tag{44}$$

3) COMPUTE THE OVERALL UTILITY FOR ALL ALTERNATIVES

The overall utility of the alternatives refers to the quotient of the weighted performance product of the benefit and cost criteria. However, in the case of MCGDM problems where the decision-making process does not have any cost criteria, the value of $\mathfrak{q}_i$ is equal to one. Equation (45) computes the overall utility value for alternative i, denoted by $\mathfrak{u}_i$.

$$\mathfrak{u}_i = \begin{cases} \mathfrak{p}_i/\mathfrak{q}_i, & \exists C_j \in \Omega_{\min} \\ \mathfrak{p}_i, & \forall C_j \notin \Omega_{\min} \end{cases} \tag{45}$$

where C_j is a criterion j and $\Omega_{\min}$ is a set of the cost criteria.

4) RANK THE ALTERNATIVES

At this stage, the FMF ranks all alternatives in descending order based on the values of $\mathfrak{u}_i$.

G. STEP 7: DETERMINE THE RESULT BASED ON THE THEORY OF DOMINANCE

Furthermore, we want to observe the results from dominance theory, summarizing the results from the three strategies above. However, it considers two assumptions: (a) the three strategies have an equal priority level for calculating the final result, and (b) the rank scores of all alternatives in each strategy are considered as the basis values for aggregation to yield the final result [41].

However, these assumptions are not realistic in several specific situations. In addition, to produce the final aggregation, they did not delineate the actual distinction of the criteria to produce the final aggregation. Thus, this is a limitation of the TraFNN-MULTIMOORA method. This was improved by adding additional steps to investigate the results using the CCSD method.

H. STEP 8: DETERMINE THE RESULT BASED ON CCSD METHOD

After estimating the weights of the RPA, RSA, and FMF ($\varpi_j(j = 1, 2, 3)$) using (28), we now determine the result based on the CCSD approach and determine it as the final result. The following sections show how the CCSD calculation procedure can rank alternatives.

1) CONSTRUCT THE SECONDARY DECISION MATRIX

First, we constructed the secondary matrix $\mathfrak{S} = \left[s_{ij}\right]_{mx3}$, in which its entries are the values of the performance evaluation of the RPA, RSA, and FMF strategies. Owing to the different logic of these strategies, we then decided that the RPA is a cost criterion, whereas the RSA and FMF are benefit criteria. Therefore, the secondary matrix can be expressed as (46):

$$\mathfrak{S} = \left(s_{ij}\right)_{mx3} = \begin{matrix} \\ R_1 \\ R_2 \\ \vdots \\ R_m \end{matrix} \begin{matrix} RPA & RSA & FMF \\ \left[\begin{matrix} s_{11} \\ s_{21} \\ \vdots \\ s_{m1} \end{matrix}\right. & \begin{matrix} s_{12} \\ s_{22} \\ \vdots \\ s_{m2} \end{matrix} & \left.\begin{matrix} s_{13} \\ s_{23} \\ \vdots \\ s_{m3} \end{matrix}\right] \end{matrix} \tag{46}$$

where $s_{i1} = \mathfrak{D}_i^{\max}$, $s_{i2} = \Psi_i$, $s_{i3} = u_i$, $i = 1, 2, \ldots, m$, and m is the number of alternatives.

2) NORMALIZE THE SECONDARY DECISION MATRIX

We then transformed the secondary decision-making matrix into the normalized secondary decision-making matrix $\tilde{\mathfrak{S}}$, in which each entry can be calculated using (47) for the RSA and FMF, and (48) for the RPA.

$$\tilde{s}_{ij} = \frac{s_{ij} - s_j^-}{s_j^+ - s_j^-}, \quad i = 1, \ldots, n; \ j \in \Omega_{\max} \tag{47}$$

$$\tilde{s}_{ij} = \frac{s_j^+ - s_{ij}}{s_j^+ - s_j^-}, \quad i = 1, \ldots, n; \ j \in \Omega_{\min} \tag{48}$$

where $s_j^+ = \max_{1 \le i \le n} \left\{s_{ij}\right\}$ and $s_j^- = \min_{1 \le i \le n} \left\{s_{ij}\right\}$. Equation (49) includes the values of s_{ij}as entries in the normalized decision matrix $\tilde{\mathfrak{S}}$.

$$\tilde{\mathfrak{S}} = \left(\tilde{s}_{ij}\right)_{mx3} = \begin{matrix} \\ R_1 \\ R_2 \\ \vdots \\ R_m \end{matrix} \begin{matrix} RPA & RSA & FMF \\ \left[\begin{matrix} \tilde{s}_{11} \\ \tilde{s}_{21} \\ \vdots \\ \tilde{s}_{m1} \end{matrix}\right. & \begin{matrix} \tilde{s}_{12} \\ \tilde{s}_{22} \\ \vdots \\ \tilde{s}_{m2} \end{matrix} & \left.\begin{matrix} \tilde{s}_{13} \\ \tilde{s}_{23} \\ \vdots \\ \tilde{n}_{m3} \end{matrix}\right] \end{matrix} \tag{49}$$

3) RANK THE ALTERNATIVE

We calculated the final values of the overall alternative assessment using (50) and, based on these values, ranked them in descending order to obtain the selected value. Thus, the final result was based on the TraFNN-MULTIOORA method.

$$f_i = \sum_{j=1}^{3} \tilde{s}_{ij} \varpi_j, \quad i = 1, 2, \ldots, m \tag{50}$$

IV. APPLICATION IN SOCIAL AID DISTRIBUTION PROBLEM

In this section, we demonstrate an illustrative example of a case study in distributing social aid to poor and marginal families to show the capability of an application and the merits of the TraFNN-MULTIMOORA method.

A. CASE STUDY

The Indonesian government believes that the COVID-19 pandemic is still impacting people and has not subsided until 2021. Through the Ministry of Social Affairs, the government will continue to distribute special social aid, namely, the cash social assistance (CSA) program, to reduce the economic burden on communities affected by the pandemic.

Nevertheless, in the distribution process of the CSA program in several places, it is suspected that there have been practices of the misuse of social assistance and CSA. Unexpected practices, such as corruption, collusion, and nepotism, tend to occur when distributing such assistance. According to some researchers, this is because the implementation and distribution process of this social assistance is still held in a conventional way, where an officer in the office has full authority to select prospective assistance recipients based on their perceptions and assessments. This implies susceptibility to subjectivity and judgments and is considered unfair.

TABLE 3. Relevant criteria for selecting CSA recipients.

Abbreviation	Criteria	Type
C_1	the size of the family	Benefit
C_2	monthly household expense	Benefit
C_3	amount of electricity bill	Benefit
C_4	monthly household income	Cost

TABLE 4. Priority order for DMs.

1st Priority	2nd Priority	3rd Priority	4th Priority
D_2 and D_4	D_1	D_3	-

Another issue in our decision-making case was choosing the appropriate data to serve as a requirement for proposing social assistance. After conducting a preliminary study at the Ministry of Social Affairs and the Central Statistics Agency by interviewing policy-makers of the office and constructing the regulations document of the Minister of Social Affairs, we obtained information related to criteria considered impoverished families. These criteria are the size of the family (C_1), monthly household expenses (C_2), the amount of electricity bill (C_3), and monthly household income (C_4). Table 3 provides information on the relevant criteria for decision-making.

Owing to the crucial issue and effort to avoid arbitrariness from certain parties and to help impoverished families during this pandemic, the case study in our research selects impoverished families who earn cash social assistance. Data collection, validated by an evaluator at the office, included five families. Furthermore, their data were checked and assessed by four officers of the office appointed as decision-making teams. This team has sufficient knowledge of poverty and the social, economic, and sociological aspects of the community.

B. SOLVING THE ILLUSTRATIVE EXAMPLE

One of the most important stages of this research is determining the weights of the criteria and decision makers. This section analyzes the acceptable weight using an attempted example to solve the MCGDM problem.

1) WEIGHTING THE DECISION-MAKERS AND CRITERIA BY OPA METHOD

In this MCGDM research, we utilized the OPA method to determine the weights for the DMs and criteria. Ataei *et al.* first developed the OPA method and applied it to evaluate the criteria weights [12]. The method starts by collecting the priority order of DMs and criteria, as listed in Tables 4 and 5,

TABLE 5. Priority order of criteria based on each DM.

Decision-makers	1st Priority	2nd Priority	3rd Priority	4th Priority
D_1	C_4	C_2	C_1 and C_3	-
D_2	C_2 and C_4	C_1 and C_3	-	-
D_3	C_2	C_4 and C_3	C_1	-
D_4	-	C_4 and C_3	C_1 and C_2	-

TABLE 6. Weights of criteria and DMs.

Decision-makers/ Criteria	Notation	Weight	Rank
D_1	ε_1	0.165957	3
D_2	ε_2	0.459574	1
D_3	ε_3	0.119150	4
D_4	ε_4	0.255319	2
C_1	W_1	0.170213	4
C_2	W_2	0.293617	2
C_3	W_3	0.204255	3
C_4	W_4	0.331915	1

respectively. The weights were then set simultaneously. The OPA method provides some benefits compared to other methods. It does not need to make a pairwise object comparison matrix, provide numerical numbers, construct a normalization matrix, or aggregate the opinions of the DMs. In addition, the OPA opens up the possibility for DMs to assess only the alternatives and the criteria for which they have sufficient knowledge [12]. Therefore, it is one of the applicable methods for deciding the weight of DMs and criteria in the MCGDM.

Because the main goal of this research was to extend the MULTIMOORA method to TraFNN, we did not address the process of determining the criteria weights using OPA. A detailed explanation of how to do this for MCGDM can be found in [12]. After executing the OPA method steps, the weights assigned to each decision-maker and criterion are presented in Table 6.

2) CONSTRUCTING THE DECISION-MAKING MATRIX

After receiving the weights of the DMs and criteria from the OPA method, the DMs were asked to assess all alternatives under the identified criteria using the linguistic term values in Table 2. According to the conversion grade in Table 2, we transformed all the assessed linguistic variables of the decision-making matrices into TraFNNs and obtained the results shown in Tables 7 and 8. Furthermore, these matrices were aggregated using the TraFNNWA operator to establish a group-decision-making matrix, as presented in Table 9.

As described above, according to the procedures of the extended MULTIMOORA method, all the alternatives must be ranked separately using three different strategies (RSA, RPA, and FMF). At the end of the stage, we apply dominance theory to determine the final result from the aggregation of the three ranking lists. The following sections describe the process, computational steps, and evaluation results of each strategy.

3) THE RESULT BASED ON RSA

After constructing the group decision-making matrix, we began to calculate the values of $\tilde{\mathcal{Y}}_i^+$ and $\tilde{\mathcal{Y}}_i^+$ using (33) and (34) and stored them into two different matrices, as presented in Table 10. From the entries in these two matrices, we then performed the de-neutrosophic process for the TraFNN values of $\tilde{\mathcal{Y}}_i^+$ and $\tilde{\mathcal{Y}}_i^+$ using (35) and (36), as shown in Table 9.

The third RSA step is a syntax to compute the difference between the score functions of $\tilde{\mathcal{Y}}_i^+$ and$\tilde{\mathcal{Y}}_i^+$ for each alternative, which can be obtained using (37). Subsequently, based on these values, the RSA offered an evaluation result

TABLE 7. The decision-making matrices of four DMs.

Decision-makers	Alternatives	C_1	C_2	C_3	C_4
D_1	R_1	AL	VL	AL	VL
	R_2	VL	FH	VL	AH
	R_3	AH	VL	AH	AH
	R_4	L	VL	AL	FH
	R_5	VL	H	VH	M
D_2	R_1	M	M	AH	AL
	R_2	L	AH	VH	AL
	R_3	H	FL	AH	M
	R_4	FH	FH	VL	FL
	R_5	FL	L	AL	FH
D_3	R_1	H	AH	AL	M
	R_2	FL	AL	AL	FH
	R_3	FL	L	VH	VH
	R_4	AH	M	AH	FH
	R_5	H	AH	FH	FL
D_4	R_1	FH	L	VL	H
	R_2	VL	H	VL	FL
	R_3	M	VL	AH	FL
	R_4	L	FL	AH	L
	R_5	L	H	FL	H

TABLE 8. The TraFNN decision matrices of four DMs.

Decision-makers	Alternatives	C_1	C_2	C_3	C_4
D_1	R_1	[(0.1,0.13,0.14,0.15), (0.85,0.87,0.9,0.95), (0.05,0.07,0.1,0.15)]	[(0.15,0.17,0.19,0.2), (0.75,0.8,0.83,0.85), (0.15,0.17,0.19,0.2)]	[(0.1,0.13,0.14,0.15), (0.85,0.87,0.9,0.95), (0.05,0.07,0.1,0.15)]	[(0.15,0.17,0.19,0.2), (0.75,0.8,0.83,0.85), (0.15,0.17,0.19,0.2)]
	R_2	[(0.15,0.17,0.19,0.2), (0.75,0.8,0.83,0.85), (0.15,0.17,0.19,0.2)]	[(0.55,0.57,0.63,0.65), (0.25,0.27,0.29,0.3), (0.25,0.27,0.29,0.3)]	[(0.15,0.17,0.19,0.2), (0.75,0.8,0.83,0.85), (0.15,0.17,0.19,0.2)]	[(0.85,0.89,0.93,0.95), (0.1,0.11,0.14,0.15), (0.05,0.1,0.13,0.15)]
	R_3	[(0.85,0.89,0.93,0.95), (0.1,0.11,0.14,0.15), (0.05,0.1,0.13,0.15)]	[(0.15,0.17,0.19,0.2), (0.75,0.8,0.83,0.85), (0.15,0.17,0.19,0.2)]	[(0.85,0.89,0.93,0.95), (0.1,0.11,0.14,0.15), (0.05,0.1,0.13,0.15)]	[(0.85,0.89,0.93,0.95), (0.1,0.11,0.14,0.15), (0.05,0.1,0.13,0.15)]
	R_4	[(0.2,0.22,0.24,0.25), (0.67,0.69,0.73,0.75), (0.2,0.21,0.24,0.25)]	[(0.15,0.17,0.19,0.2), (0.75,0.8,0.83,0.85), (0.15,0.17,0.19,0.2)]	[(0.1,0.13,0.14,0.15), (0.85,0.87,0.9,0.95), (0.05,0.07,0.1,0.15)]	[(0.55,0.57,0.63,0.65), (0.25,0.27,0.29,0.3), (0.25,0.27,0.29,0.3)]
	R_5	[(0.15,0.17,0.19,0.2), (0.75,0.8,0.83,0.85), (0.15,0.17,0.19,0.2)]	[(0.65,0.7,0.73,0.75), (0.2,0.21,0.23,0.25), (0.2,0.21,0.23,0.25)]	[(0.75,0.8,0.83,0.85), (0.15,0.17,0.19,0.2), (0.15,0.16,0.19,0.2)]	[(0.5,0.53,0.54,0.55), (0.45,0.47,0.52,0.55), (0.3,0.35,0.37,0.4)]
D_2	R_1	[(0.5,0.53,0.54,0.55), (0.45,0.47,0.52,0.55), (0.3,0.35,0.37,0.4)]	[(0.5,0.53,0.54,0.55), (0.45,0.47,0.52,0.55), (0.3,0.35,0.37,0.4)]	[(0.85,0.89,0.93,0.95), (0.1,0.11,0.14,0.15), (0.05,0.1,0.13,0.15)]	[(0.1,0.13,0.14,0.15), (0.85,0.87,0.9,0.95), (0.05,0.07,0.1,0.15)]
	R_2	[(0.2,0.22,0.24,0.25), (0.67,0.69,0.73,0.75), (0.2,0.21,0.24,0.25)]	[(0.85,0.89,0.93,0.95), (0.1,0.11,0.14,0.15), (0.05,0.1,0.13,0.15)]	[(0.75,0.8,0.83,0.85), (0.15,0.17,0.19,0.2), (0.15,0.16,0.19,0.2)]	[(0.1,0.13,0.14,0.15), (0.85,0.87,0.9,0.95), (0.05,0.07,0.1,0.15)]
	R_3	[(0.65,0.7,0.73,0.75), (0.2,0.21,0.23,0.25), (0.2,0.21,0.23,0.25)]	[(0.25,0.27,0.29,0.3), (0.55,0.57,0.63,0.65), (0.25,0.27,0.29,0.3)]	[(0.85,0.89,0.93,0.95), (0.1,0.11,0.14,0.15), (0.05,0.1,0.13,0.15)]	[(0.5,0.53,0.54,0.55), (0.45,0.47,0.52,0.55), (0.3,0.35,0.37,0.4)]
	R_4	[(0.55,0.57,0.63,0.65), (0.25,0.27,0.29,0.3), (0.25,0.27,0.29,0.3)]	[(0.55,0.57,0.63,0.65), (0.25,0.27,0.29,0.3), (0.25,0.27,0.29,0.3)]	[(0.15,0.17,0.19,0.2), (0.75,0.8,0.83,0.85), (0.15,0.17,0.19,0.2)]	[(0.25,0.27,0.29,0.3), (0.55,0.57,0.63,0.65), (0.25,0.27,0.29,0.3)]
	R_5	[(0.25,0.27,0.29,0.3), (0.55,0.57,0.63,0.65), (0.25,0.27,0.29,0.3)]	[(0.2,0.22,0.24,0.25), (0.67,0.69,0.73,0.75), (0.2,0.21,0.24,0.25)]	[(0.1,0.13,0.14,0.15), (0.85,0.87,0.9,0.95), (0.05,0.07,0.1,0.15)]	[(0.55,0.57,0.63,0.65), (0.25,0.27,0.29,0.3), (0.25,0.27,0.29,0.3)]
D_3	R_1	[(0.65,0.7,0.73,0.75), (0.2,0.21,0.23,0.25), (0.2,0.21,0.23,0.25)]	[(0.85,0.89,0.93,0.95), (0.1,0.11,0.14,0.15), (0.05,0.1,0.13,0.15)]	[(0.1,0.13,0.14,0.15), (0.85,0.87,0.9,0.95), (0.05,0.07,0.1,0.15)]	[(0.5,0.53,0.54,0.55), (0.45,0.47,0.52,0.55), (0.3,0.35,0.37,0.4)]
	R_2	[(0.25,0.27,0.29,0.3), (0.55,0.57,0.63,0.65), (0.25,0.27,0.29,0.3)]	[(0.1,0.13,0.14,0.15), (0.85,0.87,0.9,0.95), (0.05,0.07,0.1,0.15)]	[(0.1,0.13,0.14,0.15), (0.85,0.87,0.9,0.95), (0.05,0.07,0.1,0.15)]	[(0.55,0.57,0.63,0.65), (0.25,0.27,0.29,0.3), (0.25,0.27,0.29,0.3)]
	R_3	[(0.25,0.27,0.29,0.3), (0.55,0.57,0.63,0.65), (0.25,0.27,0.29,0.3)]	[(0.2,0.22,0.24,0.25), (0.67,0.69,0.73,0.75), (0.2,0.21,0.24,0.25)]	[(0.75,0.8,0.83,0.85), (0.15,0.17,0.19,0.2), (0.15,0.16,0.19,0.2)]	[(0.75,0.8,0.83,0.85), (0.15,0.17,0.19,0.2), (0.15,0.16,0.19,0.2)]
	R_4	[(0.85,0.89,0.93,0.95), (0.1,0.11,0.14,0.15), (0.05,0.1,0.13,0.15)]	[(0.5,0.53,0.54,0.55), (0.45,0.47,0.52,0.55), (0.3,0.35,0.37,0.4)]	[(0.85,0.89,0.93,0.95), (0.1,0.11,0.14,0.15), (0.05,0.1,0.13,0.15)]	[(0.55,0.57,0.63,0.65), (0.25,0.27,0.29,0.3), (0.25,0.27,0.29,0.3)]
	R_5	[(0.65,0.7,0.73,0.75), (0.2,0.21,0.23,0.25), (0.2,0.21,0.23,0.25)]	[(0.85,0.89,0.93,0.95), (0.1,0.11,0.14,0.15), (0.05,0.1,0.13,0.15)]	[(0.55,0.57,0.63,0.65), (0.25,0.27,0.29,0.3), (0.25,0.27,0.29,0.3)]	[(0.25,0.27,0.29,0.3), (0.55,0.57,0.63,0.65), (0.25,0.27,0.29,0.3)]
D_4	R_1	[(0.55,0.57,0.63,0.65), (0.25,0.27,0.29,0.3), (0.25,0.27,0.29,0.3)]	[(0.2,0.22,0.24,0.25), (0.67,0.69,0.73,0.75), (0.2,0.21,0.24,0.25)]	[(0.15,0.17,0.19,0.2), (0.75,0.8,0.83,0.85), (0.15,0.17,0.19,0.2)]	[(0.65,0.7,0.73,0.75), (0.2,0.21,0.23,0.25), (0.2,0.21,0.23,0.25)]
	R_2	[(0.15,0.17,0.19,0.2), (0.75,0.8,0.83,0.85), (0.15,0.17,0.19,0.2)]	[(0.65,0.7,0.73,0.75), (0.2,0.21,0.23,0.25), (0.2,0.21,0.23,0.25)]	[(0.15,0.17,0.19,0.2), (0.75,0.8,0.83,0.85), (0.15,0.17,0.19,0.2)]	[(0.25,0.27,0.29,0.3), (0.55,0.57,0.63,0.65), (0.25,0.27,0.29,0.3)]
	R_3	[(0.5,0.53,0.54,0.55), (0.45,0.47,0.52,0.55), (0.3,0.35,0.37,0.4)]	[(0.15,0.17,0.19,0.2), (0.75,0.8,0.83,0.85), (0.15,0.17,0.19,0.2)]	[(0.85,0.89,0.93,0.95), (0.1,0.11,0.14,0.15), (0.05,0.1,0.13,0.15)]	[(0.25,0.27,0.29,0.3), (0.55,0.57,0.63,0.65), (0.25,0.27,0.29,0.3)]
	R_4	[(0.2,0.22,0.24,0.25), (0.67,0.69,0.73,0.75), (0.2,0.21,0.24,0.25)]	[(0.25,0.27,0.29,0.3), (0.55,0.57,0.63,0.65), (0.25,0.27,0.29,0.3)]	[(0.85,0.89,0.93,0.95), (0.1,0.11,0.14,0.15), (0.05,0.1,0.13,0.15)]	[(0.2,0.22,0.24,0.25), (0.67,0.69,0.73,0.75), (0.2,0.21,0.24,0.25)]
	R_5	[(0.2,0.22,0.24,0.25), (0.67,0.69,0.73,0.75), (0.2,0.21,0.24,0.25)]	[(0.65,0.7,0.73,0.75), (0.2,0.21,0.23,0.25), (0.2,0.21,0.23,0.25)]	[(0.25,0.27,0.29,0.3), (0.55,0.57,0.63,0.65), (0.25,0.27,0.29,0.3)]	[(0.65,0.7,0.73,0.75), (0.2,0.21,0.23,0.25), (0.2,0.21,0.23,0.25)]

consisting of the alternatives' rankings in descending order, as given in Table 10. The results of this strategy were $R_2 > R_4 > R_3 > R_1 > R_5$.

4) THE RESULT BASED ON RPA

Similar to RSA, we started to take the entries of the group matrix to calculate n TraFNN reference points using (38). All

TABLE 9. The TraFNN group decision matrix.

Aggregation Operator	Alternatives	C_1	C_2	C_3	C_4
TraFNNWA	R_1	[(0.48,0.51,0.54,0.56), (0.39,0.41,0.44,0.46), (0.20,0.23,0.26,0.29)]	[(0.46,0.50,0.54,0.56), (0.45,0.47,0.52,0.54), (0.19,0.23,0.26,0.28)]	[(0.61,0.66,0.73,0.77), (0.30,0.32,0.37,0.39), (0.06,0.10,0.13,0.16)]	[(0.34,0.38,0.41,0.42), (0.53,0.55,0.58,0.62), (0.10,0.13,0.16,0.20)]
	R_2	[(0.18,0.20,0.22,0.23), (0.68,0.71,0.75,0.77), (0.18,0.19,0.22,0.23)]	[(0.72,0.77,0.82,0.85), (0.17,0.19,0.22,0.23), (0.09,0.13,0.16,0.19)]	[(0.51,0.56,0.60,0.62), (0.36,0.39,0.42,0.44), (0.13,0.14,0.17,0.19)]	[(0.41,0.45,0.51,0.54), (0.46,0.48,0.52,0.55), (0.09,0.12,0.15,0.19)]
	R_3	[(0.63,0.68,0.72,0.74), (0.24,0.26,0.29,0.31), (0.18,0.21,0.24,0.26)]	[(0.20,0.22,0.24,0.25), (0.64,0.67,0.72,0.74), (0.19,0.21,0.23,0.24)]	[(0.84,0.88,0.92,0.94), (0.10,0.11,0.14,0.15), (0.05,0.10,0.13,0.15)]	[(0.58,0.62,0.66,0.69), (0.32,0.34,0.38,0.41), (0.19,0.24,0.26,0.29)]
	R_4	[(0.49,0.53,0.58,0.61), (0.33,0.36,0.39,0.40), (0.18,0.21,0.24,0.25)]	[(0.42,0.44,0.48,0.50), (0.39,0.41,0.45,0.46), (0.23,0.25,0.27,0.29)]	[(0.55,0.60,0.67,0.71), (0.36,0.38,0.43,0.45), (0.08,0.12,0.14,0.17)]	[(0.34,0.36,0.40,0.41), (0.46,0.48,0.52,0.54), (0.23,0.25,0.27,0.28)]
	R_5	[(0.28,0.31,0.34,0.35), (0.53,0.56,0.60,0.62), (0.21,0.22,0.25,0.26)]	[(0.53,0.58,0.63,0.65), (0.32,0.33,0.36,0.38), (0.16,0.19,0.21,0.23)]	[(0.36,0.40,0.43,0.45), (0.49,0.51,0.55,0.58), (0.10,0.13,0.16,0.20)]	[(0.54,0.57,0.61,0.63), (0.28,0.30,0.33,0.34), (0.24,0.26,0.28,0.30)]

TABLE 10. Determining the alternatives ranking based on RSA strategy.

Alternatives	$\tilde{Y}_i$	$\tilde{Y}_i$	$Y_i^+ = S(\tilde{Y}_i)$	$Y_i^- = S(\tilde{Y}_i)$	$Y_i^+ - Y_i^-$	Rank
R_1	[(0.387,0.426,0.468,0.497), (0.531,0.550,0.589,0.609), (0.270,0.321,0.356,0.386)]	[(0.131,0.150,0.161,0.169), (0.811,0.822,0.837,0.853), (0.474,0.508,0.545,0.587)]	0.513662196	0.264391787	0.2493	4
R_2	[(0.428,0.474,0.524,0.556), (0.460,0.482,0.516,0.532), (0.246,0.286,0.320,0.343)]	[(0.161,0.183,0.211,0.230), (0.773,0.784,0.808,0.821), (0.451,0.498,0.539,0.580)]	0.566164211	0.294001729	0.272	1
R_3	[(0.458,0.506,0.560,0.595), (0.436,0.455,0.497,0.513), (0.258,0.310,0.342,0.361)]	[(0.251,0.278,0.305,0.324), (0.687,0.701,0.731,0.743), (0.582,0.624,0.647,0.663)]	0.578659251	0.314769115	0.2639	3
R_4	[(0.357,0.388,0.438,0.465), (0.513,0.535,0.568,0.583), (0.295,0.335,0.365,0.384)]	[(0.129,0.138,0.155,0.163), (0.773,0.785,0.807,0.815), (0.619,0.633,0.652,0.660)]	0.505634306	0.23653609	0.2691	2
R_5	[(0.313,0.349,0.380,0.401), (0.558,0.575,0.607,0.627), (0.290,0.317,0.350,0.376)]	[(0.229,0.247,0.273,0.285), (0.660,0.673,0.692,0.703), (0.625,0.643,0.658,0.670)]	0.478549602	0.308951902	0.1696	5

TABLE 11. TraFNN reference points.

	C_1	C_2	C_3	C_4
$\tilde{\wp}_j^*$	[(0.635,0.683,0.722,0.748), (0.247,0.260,0.294,0.314), (0.180,0.197,0.222,0.232)]	[(0.723,0.772,0.824,0.854), (0.179,0.192,0.223,0.238), (0.093,0.136,0.166,0.191)]	[(0.840,0.881,0.922,0.943), (0.104,0.115,0.145,0.155), (0.056,0.103,0.132,0.155)]	[(0.340,0.361,0.400,0.415), (0.533,0.554,0.587,0.621), (0.243,0.264,0.284,0.300)]

TABLE 12. Determining the alternatives ranking based on RPA strategy.

Alternatives	$D_{ij}^{\max}$				$D_i^{\max}$	Rank
	C_1	C_2	C_3	C_4		
R_1	0.028746854	0.080317147	0.041045104	0.041	0.080	1
R_2	0.082345464	0	0.065505696	0.044	0.082	2
R_3	0	0.165094256	0	0.008	0.165	5
R_4	0.023537538	0.096002353	0.053136254	0.006	0.096	3
R_5	0.063146436	0.055923418	0.098975483	0.000	0.099	4

of these reference points can be stored in a vector, as presented in Table 11.

Next, the maximum distance between each group matrix element (Table 9) and the corresponding TraFNN reference point must be measured. The result is a matrix provided in Table 12. According to the logic of the extended MULTIMOORA for the RPA strategy, the method must multiply the entries of this matrix by the corresponding criteria weight (Table 6) obtained from the OPA method and put each calculation value into a matrix presented in Table 12. Finally,

TABLE 13. Determining the alternatives ranking based on FMF strategy.

Alternatives	$P_i^{\%}$	$Q_i^{\%}$	$\bar{p}_i = S(P_1)$	$\bar{q}_i = S(Q_1)$	u_i	Rank
R_1	[(0.639,0.673,0.707,0.727), (0.285,0.303,0.339,0.358), (0.109,0.136,0.156,0.175)]	[(0.703,0.730,0.745,0.755), (0.223,0.235,0.254,0.275), (0.036,0.045,0.056,0.071)]	0.74018728	0.811306586	0.9123	2
R_2	[(0.595,0.630,0.661,0.678), (0.293,0.317,0.348,0.365), (0.087,0.107,0.127,0.140)]	[(0.745,0.771,0.800,0.817), (0.185,0.196,0.220,0.234), (0.031,0.042,0.054,0.069)]	0.731504244	0.841719504	0.8691	3
R_3	[(0.559,0.588,0.614,0.628), (0.310,0.332,0.372,0.390), (0.104,0.127,0.145,0.156)]	[(0.835,0.856,0.873,0.885), (0.121,0.130,0.151,0.160), (0.069,0.088,0.099,0.107)]	0.704342126	0.876856748	0.8033	4
R_4	[(0.610,0.639,0.683,0.704), (0.265,0.284,0.313,0.327), (0.123,0.143,0.161,0.172)]	[(0.699,0.713,0.737,0.747), (0.185,0.196,0.218,0.227), (0.085,0.092,0.101,0.105)]	0.737136805	0.806922794	0.9135	1
R_5	[(0.547,0.583,0.613,0.631), (0.319,0.336,0.368,0.388), (0.111,0.126,0.146,0.162)]	[(0.816,0.832,0.852,0.860), (0.105,0.113,0.124,0.132), (0.088,0.096,0.105,0.111)]	0.701370196	0.873705881	0.8028	5

TABLE 14. Results base on the three strategies.

Strategies	Ranking
RSA	$R_2 > R_4 > R_3 > R_1 > R_5$
RPA	$R_1 > R_2 > R_4 > R_5 > R_3$
FMF	$R_4 > R_1 > R_2 > R_3 > R_5$
Dominance Theory	$R_4 = R_2 = R_1 > R_3 > R_5$

based on these values, the RPA presented an evaluation result with the ranking of alternatives ranking in ascending order, as presented in Table 12. The results of this strategy were $R_1 > R_2 > R_4 > R_5 > R_3$.

5) THE RESULT BASED ON FMF

For the evaluation results using the FMF strategy, the overall utility of the alternatives must be computed using (45). The results are presented in Table 13. The results of this strategy were $R_4 > R_1 > R_2 > R_3 > R_5$.

6) THE RESULT BASED ON THE DOMINANCE THEORY

After acquiring the results from the three different strategies, dominant theory plays a role in deciding the final alternative ranking. In this study, the result of the alternative ranking is shown in Table 14, which is $R_4 = R_2 = R_1 > R_3 > R_5$. Thus, the best and worst alternatives were R_4 and R_2.

7) THE FINAL RESULT BASED ON CCSD METHOD

After receiving the resulting values from the three different strategies (RPA, RSA, and FMF), based on the procedure of the extended MULTIMOORA method, it is necessary to construct the secondary decision-making (SDM) matrix and insert those values sequentially as the entries of this matrix, as listed in Table 15.

In this stage, we now consider these three strategies as criteria in which RPA is a cost criterion and RSA and FMF are benefit criteria. Next, we can use (28) to estimate the weight of each strategy. The values are listed in Table 16.

TABLE 15. The secondary decision-making matrix.

Alternatives	RPA	RSA	FMF
R_1	0.080	0.2493	0.9123
R_2	0.082	0.272	0.8691
R_3	0.165	0.2639	0.8033
R_4	0.096	0.2691	0.9135
R_5	0.099	0.1696	0.8028

TABLE 16. The weight of RPA, RSA, and FMF strategies.

	RPA	RSA	FMF
Weight	0.41902	0.44085	0.14014

TABLE 17. The normalized secondary decision-making matrix.

Alternatives	RPA	RSA	FMF
R_1	1	0.77680376	0.989381684
R_2	0.976074709	1	0.598632869
R_3	0	0.919345161	0.004556718
R_4	0.814983006	0.970123603	1
R_5	0.779913045	0	0

Because each criterion has different measurement units, we normalized the entries of the SDM matrix using (47) and (48), as shown in Table 17. We then multiplied them by the corresponding weights to obtain the weighted SDM matrix, as presented in Table 18.

Finally, we used (50) to calculate the overall performance values for all alternatives and ranked them according to their overall values (see Table 19). The final results of the alternative ranking were $R_2 > R_4 > R_1 > R_3 > R_5$, as presented in Table 20.

V. SENSITIVITY ANALYSIS AND COMPARATIVE ANALYSIS

To demonstrate the validity and robustness of the TraFNN-MULTIMOORA method, we conducted the sensitivity analysis and compared the results between

TABLE 18. The weighted normalized secondary decision-making matrix.

Alternatives	RPA	RSA	FMF
R_1	0.419015061	0.342452202	0.138649153
R_2	0.408990004	0.440847765	0.083890718
R_3	0	0.40529126	0.000638566
R_4	0.341490154	0.427676823	0.140137174
R_5	0.326795312	0	0

TABLE 19. The overall performance value and the final result.

Alternatives	Overall Performance Value	Rank
R_1	0.900116415	3
R_2	0.933728487	1
R_3	0.405929825	4
R_4	0.90930415	2
R_5	0.326795312	5

TABLE 20. Results comparison.

Alternatives	RPA	RSA	FMF	Dominance Theory	Our Extended Method
R_1	1	4	2	1	3
R_2	2	1	3	1	1
R_3	5	3	4	4	4
R_4	3	2	1	1	2
R_5	4	5	5	5	5

the TraFNN-MULTIMOORA method and other existing methods.

A. SENSITIVITY ANALYSIS

A sensitivity analysis is demonstrated in this section by providing three different cases. We define three case studies, in which the first case is the obtained result of this work, and the second and the third cases are the other solutions, which are obtained from the calculation results using different criteria weights. In this study, the selection of CSA recipients was determined based on four DMs and four criteria. Subsequently, we employed the OPA method under group decision-making to find the initial criteria weights of those criteria. We then utilized the TraFNN-MULTIMOORA method for each case study to deliver three alternative rankings. At the end of each stage, we used the criteria weights calculated by the CCSD method to obtain the final result.

In the second case study, we utilized the OPA method to deliver appropriate weights based on the second attempt to prioritize the criteria performed by the DMs. Therefore, we received the criteria weights $C_1 = 0.150$, $C_2 = 0.273$, $C_3 = 0.204$, and $C_4 = 0.373$ and considered the weights of DMs as $\varepsilon_1 = 0.165$, $\varepsilon_2 = 0.449$, $\varepsilon_3 = 0.098$, and $\varepsilon_4 = 0.288$, respectively. In this case, the final ranking order was $R_2 > R_4 > R_1 > R_3 > R_5$.

Similar to the previous case, we obtained the criteria weights are $C_1 = 0.16$, $C_2 = 0.283$, $C_3 = 0.194$, and $C_4 = 0.363$ in the third case study and considered the weights

TABLE 21. The comparison results of sensitivity analysis.

Alternatives	1st Case	Rank	2nd Case	Rank	3rd Case	Rank
R_1	0.9001	3	0.7994	3	0.8472	3
R_2	0.9337	1	0.9508	1	0.9282	1
R_3	0.4059	4	0.4599	4	0.3939	4
R_4	0.9093	2	0.9029	2	0.9149	2
R_5	0.3266	5	0.2678	5	0.3503	5

of DMs are $\varepsilon_1 = 0.175$, $\varepsilon_2 = 0.439$, $\varepsilon_3 = 0.109$, and $\varepsilon_4 = 0.277$, respectively. By substituting these weights, the final ranking order was $R_2 > R_4 > R_1 > R_3 > R_5$.

As shown in Table 21, although the alternatives' performance values for these cases change slightly, the results in terms of alternative ranking orders do not switch. Therefore, these three cases demonstrate that our extended method can preserve the sturdiness of the MULTIMOORA method, and is feasible for application to the MCGDM problem.

B. COMPARATIVE ANALYSIS

Many prestigious scholars conducting research on intelligent systems, artificial intelligence, and soft computing have developed novel decision-making concepts to overcome MCGDM problems in an incomplete, uncertain, and vague environment. In this study, we performed a comparative analysis between our extended TraFNN-MULTIMOORA method and other decision-making methods to inspect each characteristic of these methods in terms of application domains, criteria types, calculation operators used, and basic ideas. The characteristics of these methods are listed in Table 22.

As shown in Table 22, we can first state that the six different methods evaluated the MCGDM problem in the same application domain. However, these methods have different logics for examining alternatives, criteria types involved in the decision-making process, and aggregation operators used in their algorithms. Second, the improved MABAC, MCGDM-TNNWAA, and TraFN-TOPSIS methods only utilize the TraFNNWA operator to aggregate decision-making matrices constructed by DMs into a decision-making group matrix. In contrast, an ideal solution and MCGDM-TNNWG methods use the TraFNNWG operator to perform alternative evaluation process. After performing the de-neutrosophic process, these five methods determine their score functions for all alternatives, rank them, and choose the best one.

However, unlike previous methods, the extended MULTIMOORA method successfully explored three aggregation operators (TraFNNWA, TraFNNWG, and Boundedness operator) to investigate the alternatives and generate solutions based on three different strategies (RSA, RPA, and FMF). It provides three outcome options that allow decision-makers to consider and view solutions from various perspectives. Finally, by applying the rules of the theory of dominance and considering the same significance for these three strategies, the extended MULTIMOORA method yields the final result.

TABLE 22. Comparative analysis between TraFNN-MULTIMOORA method and other methods.

Methods	Application Domains	Criteria Types	Used Calculation Operators	Basic Ideas
Improved MABAC [44]	Trapezoidal Fuzzy Neutrosophic Sets	Benefit	TraFNNWA	TraFNN & MABAC
An Ideal Solution [45]	Trapezoidal Fuzzy Neutrosophic Sets	Benefit & Cost	TraFNNWG	TraFNN & Ideal Solution
MCGDM-TNNWAA [46]	Trapezoidal Neutrosophic Sets	Benefit & Cost	TraFNNWA	TraFNN & MCGDM
MCGDM-TNNWGA [46]	Trapezoidal Neutrosophic Sets	Benefit & Cost	TraFNNWG	TraFNN & MCGDM
TraFNN-TOPSIS [46]	Trapezoidal Neutrosophic Sets	Benefit & Cost	TraFNNWA	TraFNN & TOPSIS
Our proposed method	Trapezoidal Fuzzy Neutrosophic Sets	Benefit & Cost	TraFNNWA, Boundedness, The dominance theory, & TraFNNWG	TraFNN, OPA, CCSD, SAW & MULTIMOORA

TABLE 23. Ranking comparisons between TraFNN-MULTIMOORA method and other methods.

Alternatives	TraFNN-MABAC [44]	An Ideal Solution [45]	MCGDM-TNNWAA [46]	MCGDM-TNNWGA [46]	TraFNN-TOPSIS [46]	TraFNN-MULTIMOORA
R_1	3	1	4	5	1	3
R_2	1	3	2	4	3	1
R_3	2	5	1	1	4	4
R_4	4	2	3	3	2	2
R_5	5	4	5	2	5	5

Another important aspect of our method is to add the procedures of the CCSD method to overcome the limitations of the original MULTIMOORA method as described in [47]. The merit of this method is that it considers the condition under which all three strategies (RPA, RSP, and FMF) are assumed to be equally important. Although the diffusion of the scores is distinct for each strategy, the difference in scores for each strategy has a dissimilar significance. Therefore, it statistically generates adaptable weights for these strategies using the concept of correlation coefficients and standard deviation, which can improve the efficiency and accuracy of the TraFNN-MULTIMOORA method. It also provides a suitable tool for determining the criteria weights according to the opinions of different DMs using the OPA method. It precisely reduces the subjectivity of humans in determining the appropriate criteria weights.

As presented in Table 23, we can first obtain that the results based on MCGDM-TNNWAA method were similar to the RSA strategy, as demonstrated in Fig. 2. Second, the results obtained from the ideal solution and TraFNN-TOPSIS methods were identical to those of the RPA strategy, as shown in Fig. 3. Third, the results based on the TraFNN-MULTIMOORA method are slightly different from those of the TraFNN-MABAC method, as shown in Fig. 4.

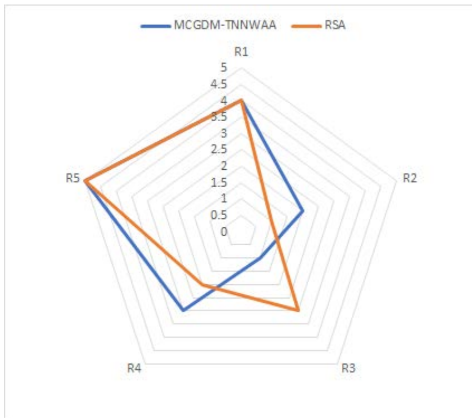

FIGURE 2. Result comparisons between MCGDM-TNNWAA method and the RSA strategy.

C. DISCUSSION

As shown in Table 20, according to the dominance theory, we obtained at least three rules. The first is absolute dominance. If an alternative preferred the three approaches (RPA-RSA-FMF) for alternative R_5*(4-5-5)*, then alternative R_5 is absolute dominance and dominance theory assigned it the fifth alternative in the alternative ranking order. The second rule is transitiveness. If alternative R_1*(1-4-2)* dominates R_2*(2-1-3)* and R_2*(2-1-3)* dominates R_3*(5-3-4)*, then R_1*(1-4-2)* dominates R_3*(5-3-4)*. Another transitive rule is if alternative R_2*(2-1-3)* dominates R_4*(3-2-1)* and R_4*(3-2-1)* dominates R_3*(5-3-4)*, then R_2*(2-1-3)* dominates R_3*(5-3-4)*. Because R_1 dominates R_3 and R_2 dominates R_3, then alternative R_3 is placed in the fourth alternative in the alternative ranking order. The third rule is circular reasoning. As we can see, alternative R_4*(3-2-1)* dominates R_1*(1-4-2)*, alternative R_1*(1-4-2)* dominates R_2*(2-1-3)*, and alternative R_2*(2-1-3)* generally dominates R_4*(3-2-1)*. Thus, dominance theory ranked them with the same ranking as the first alternative. Therefore, the results based on dominance theory for this study were $R_4 = R_1 = R_2 > R_3 > R_5$

In addition, as presented in Table 15, the performance values of FMF were very large compared those of the other two methods. This was unfair if we assigned the same

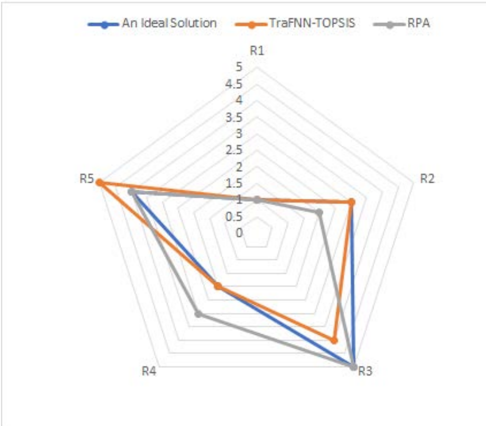

FIGURE 3. Result comparisons between the ideal solution, TraFNN-TOPSIS methods and the RPA strategy.

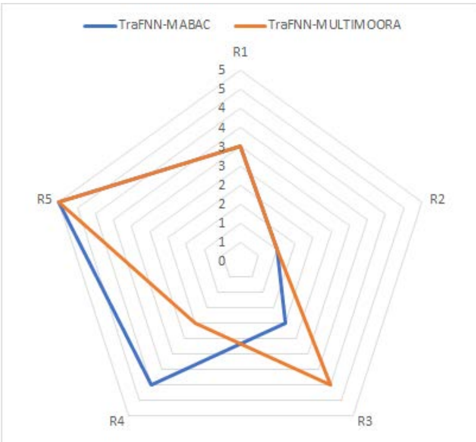

FIGURE 4. Result comparisons between TraFNN-MULTIMOORA and TraFNN-MABAC methods.

priority level to these three strategies. Therefore, it was necessary to determine appropriate weights using the CCSD method, as presented in Table 16. Based on the SAW method, we obtained the following final results: $R_2 > R_4 > R_1 > R_3 > R_5$, as provided in Table 19. Here, we no longer need to use dominance theory to determine the final result and successfully overcome the aforementioned challenges of the MULTIMOORA. Therefore, we can say that, in general, the TraFNN-MULTIMOORA method is an effective tool for overcoming this MCGDM case and is more reasonable, capable, and robust in determining the results of the decision-making process.

VI. CONCLUSION

Generally, there are two ways to find solutions to issues involving human intervention. The first step is to determine the decisions made by individual assessments. This can be achieved when decision-makers have sufficient experience, pieces of knowledge, professionalism, education, skills, and so on. However, studies related to the problem of selecting individuals or groups from several people involved in the selection process have become a serious complex issue. Suitable decisions and policies made and implemented by decision-makers must be representative of and suited to the expectations of all parties. In this process, humans, agencies, or organizations should make decisions according to various criteria and circumstances to solve several issues around them. The outcome of the decision is very complex, as some criteria or attributes often have conflicting priorities, which can be used in the development of decision-making systems for various conditions. Therefore, multi-criteria analysis is becoming increasingly important because it is an instrument to help decision-makers provide objective alternative solutions to support decisions and policymakers.

The second is to determine an effective decision-making method for addressing MCGDM issues. MULTIMOORA is one of the most well-known decision methods that improves the MOORA method by adding a new evaluation approach (FMF) to examine alternatives. It enhances the robustness of MULTIMOORA and is more effective in various decision-making cases without subjectivity. In this study, we improved the accuracy and proficiency of MULTIMOORA under TraFNN circumstances using the CCSD method. Technically, after obtaining the decision results based on the three approaches (RSA, RPA, and FMF), we constructed a secondary decision matrix, in which its entries are the three evaluation result values. Subsequently, utilizing the merit of the CCSD method, which is the ability to analyze the relationship between the relative movement of criteria statistically, we calculated the appropriate weight for each approach and multiplied them by corresponding to the normalized secondary decision matrix. Finally, we computed the overall performance evaluation values of the alternatives using the SAW method and conveyed the final ranking of alternatives. Based on an illustrative example, sensitivity analysis, and comparison analysis, the extended MULTIMOORA has demonstrated its capability, feasibility, and accuracy in overcoming the MCGDM problem, and is a fully robust method for ranking alternatives in the case of CSA recipients' selection in Indonesia.

Studies related to expressing information in uncertain, vague and incomplete environments have been increasingly conducted. Most of these studies emphasize the development of novel approaches to transforming linguistic variables into arbitrary numbers, proposing a new method to measure the distance between linguistic values, extending information concepts, and so on. Considering these opportunities, this study has the potential to be developed for future studies. For instance, we can integrate the MULTMOORA method with the concepts of bipolar trapezoidal neutrosophic sets [48], t-spherical fuzzy sets [49], triangular interval type-2 intuitionistic fuzzy sets [50], and single-valued neutrosophic hesitant fuzzy sets [51] to solve MCGDM problems. Robust averaging geometric aggregation operators [52] can also be utilized to aggregate the evaluation values in MULTIMOORA.

Many researchers have studied the issues of consistency and consensus for linguistic MCGDM problems.

However, these issues are main crucial. They developed consensus-reaching models that emphasize adjusting the judgment relationship of the DM and ignoring individual consistency. However, we did not consider these issues in this study. Therefore, we strongly recommend that this research be conducted by developing the concepts of personalized individual semantics (PISs), dynamic feedback mechanisms, and consensus levels of internal and external subgroups in linguistic MCGDM to control and increase consistency and consensus.

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