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Time-Varying Group Formation With Adaptive Control for Second-Order Multi-Agent Systems

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ABSTRACT In this study, adaptive time-varying group formation control for second-order multi-agent systems with an undirected topology is studied. First, a conventional multi-group adaptive time-varying formation control protocol is presented, and the stability of the system is analyzed based on the Lyapunov method. Considering that traditional methods of dealing with nonlinear dynamics cannot be directly applied to the time-varying formation problem, the parameter estimation method is used to track the unknown internal nonlinear dynamics of an agent. A virtual leader and filter are introduced to solve the problem of variable derivation and repeated derivation of the system state, and the adaptive rate of time-varying formation converges to a stable value. Finally, the feasibility of the results is verified using two simulation results.

INDEX TERMS Time-varying formation, multi-agent systems, adaptive control, nonlinear dynamics, parameter estimation.

I. INTRODUCTION

In recent decades, the cooperative control of multi-agent systems has achieved significant achievements and development [1]–[5]. With the progress in technology and the increase in the complexity and accuracy of tasks, formation control has been widely studied in recent years. Compared with consensus, formation can not only give full play to individual advantages, but also increase the ability of overall cooperation. According to different task objectives, the expected formation vector can be set as a fixed constant vector and a time-varying vector. For these two different situations, the leader-following method [6], [7] has always been an important subject in formation control research.

Many practical tasks require multi-agent systems to form a predetermined geometric structure to achieve, such as the aerial energy supply of aircraft, terrain and resource exploration of robots, cooperative positioning [8] and encapsulation of target objects [9]. Ren [10] conducted a lot of pioneering work on formation control, which showed that leader-follower, virtual structure and behavior-based formation control could all be unified as consensus problems. Han *et al.* [11] discussed multi-group formation control with nonlinear dynamics under the condition of a fixed topology and switching topology. Xie and Wang [12] introduced a decentralized information feedback mechanism in an undirected communication network, thereby enabling a group of mobile robots to achieve and stably maintain a predetermined geometric structure. The works in [11] and [12] dealt with time-invariant formation tracking control, that is, the formation vector is expected to be a fixed constant vector. In this case, it can be perfectly combined with a typical consensus analysis because it does not affect the derivative calculation of the system state variables. The time-varying vector is a function of time. For the second-order system model, the time derivative leads to the influence of the time-varying vector on the system. Therefore, the processing of time-varying formations requires a new analytical thinking and demand environment. Dong *et al.* first proposed the concept of time-varying formation control in [13], and the expected time-varying geometric structure of the entire multi-agent system can be realized using predefined time-varying vectors. The distributed time-varying formation control problem of second-order multi-agent systems is studied based on the leader-following method. The feasibility constraint of forming a stable time-varying formation is given, and a design method for time-varying formation control is proposed by solving an algebraic Riccati equation.

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In [14], the formation problem and protocol design of average time-varying formation control with multiple leaders were discussed by using neighbor relative information, and the sufficient and necessary conditions for formation stability were given. However, [13] and [14] did not take into account the internal nonlinear dynamics and uncertainties or disturbances observed in most mechanical systems [15]–[16]. The purpose of formation is to make the system state and formation vector form a structural relationship with the target state, whereas nonlinear dynamics is a complex function of time and system state. Therefore, formation control cannot deal with nonlinear dynamics like tracking consensus, and other methods need to be developed.

In the formation control problem, a large number of agents are often required to cooperate with each other. When the geometric structure is complex and large, multiple leaders are required to decompose the entire formation structure into several sub-structures, and each leader undertakes its own sub-structure formation. Based on the general linear multi-agent system, Lu *et al.* [17] studied the formation-containment control with multiple subgroups and multiple leaders. Hu *et al.* [18] studied the average consensus problem with coupled subgroups, and the information transfer between different individuals of the system was discontinuous. They designed a new hybrid protocol, which enabled the entire agent group to achieve multiple consistent states. Pu *et al.* [19] studied the time-delay group consensus problem in competitive cooperative networks, calculated the upper limit on time delay and provided adaptive regulatory protocols. However, in [17]–[19], they require that the row sum of the non-diagonal subblocks of the Laplace matrix is zero, that is, the entry equilibrium assumption.

In addition, many physical systems in the process of longtime operation, the effects on the system due to its internal wear, and the change in environmental conditions on the system of the state of the indirect interference, will lead to large fluctuations in the physical system parameters compared to the initial physical parameters, which causes failure or quality control to be unqualified. Therefore, it is necessary to design an adaptive controller for the system model [20]. Yu *et al.* [21] designed a time-varying formation tracking control protocol by introducing a three-step algorithm. The adaptive neural network can not only deal with heterogeneous nonlinear dynamic and time-varying bounded interference among agents, but also demonstrate that the error caused by the interference to the formation structure can be controlled to an arbitrarily small bounded amount. However, only the case of multiple leaders in a single group was considered. Yu and Xia [22] designed an adaptive control protocol by parameterizing the unknown nonlinear dynamics of an individual in a multi-agent system. And they discussed a first-order system containing only the position states, which did not have the cross effect of system states in the error analysis. The problem of fully adaptive time-varying tracking control of high-order nonlinear multi-agent system was studied in [23], and the uncertainty was estimated using a fuzzy logic system.

Motivated by the above research situation, this study investigates the adaptive control of time-varying formation with multiple subgroups based on the existence and absence of unknown internal nonlinear dynamics and disturbances. The main contributions of this paper are as follows: First, there is no conservative assumption for the Laplacian matrix of the communication topology of the multi-agent system, that is, the sum of adjacent weights from each node in one group to all nodes in other subgroups is zero, which increases the degree of freedom of the system. Second, by introducing the formation compensation vector into the control protocol and combining it with the feasibility constraints of formation tracking, the analysis of time-varying formation system is simplified. Third, the unknown internal nonlinear dynamics in the individual are estimated by parameterization, and virtual leader and filter [24] are designed in the protocol to overcome the instability caused by cross variables in the second-order system.

In the second section, related graph theory knowledge and auxiliary lemma are provided, and a preliminary system model is described. Then two adaptive time-varying formation control protocols designed in Section 3 are given, and sufficient conditions for the system to realize formation are given in the theorem. In Section 4, the results of the data simulation are provided, and the conclusion is given at the end.

For simplicity, R and R^n represents real numbers and real N-dimensional vectors, respectively, in the entire article. $I_n \in R^{n\times n}$ is the $n \times n$ identity matrix, and the symmetric matrix $P \in R^{n\times n}$, $P > 0$ is said to be positive definite. Let 0 be a zero matrix or zero vector of the appropriate dimension. $\|P\|$ represents the matrix 2-norm of matrix P. $\|P\|_\infty$ is equal to the largest absolute value of its elements.

II. PRELIMINARIES

A. GRAPH THEORY

Consider $N+M$ agents, of which N agents are followers and M agents are leaders. $G = \{V, E, A\}$ is used to represent a weighted undirected topology of N followers, where $V = \{1, \ldots, N\}$, $E \subset V \times V$, and $A = [\omega_{ij}] \subset R^{N\times N}$ are vertex set, edge set, and non-negative-weighted adjacency matrix, respectively. For each node $i \in 1, 2, \ldots, n$, there is no self-connection, that is, $\omega_{ii} = 0$. $E = [\omega_{ij}, \omega_{ij} > 0]$, for the undirected topology, since communication is mutual, so $\omega_{ij} = \omega_{ji}$, and A is a symmetric matrix. Divide N followers into M subgroups corresponding to the number of leaders, then $V = \{V_1, V_2, \ldots, V_M\}$ and $E = \{E_{11}, E_{12}, \ldots, E_{1M}, \ldots, E_{MM}\}$ are obtained, where V_i is the node-set of the subgroup i, E_{ii} is the intra-group communication of the subgroup i, and $E_{ij} i \neq j$ is the inter-group communication between the subgroup i and the subgroup j. L is the Laplacian matrix of the undirected topology G, defining the degree matrix $D = diag(d_i)$, where $d_i = \sum_{j=1}^{N} \omega_{ij}$, then $L = D - A$. The connection matrix between leaders and followers is represented by $B = diag(b_i)$, and $b_i > 0$ means there is communication between leaders and followers.

B. SYSTEMS DESCRIPTION

For the convenience of discussion, the index set of M subgroups is defined as $V_1 = \{1, \ldots, n_1\}, V_2 = \{n_1+1, \ldots, n_2\}, \ldots, V_M = \left\{\sum_{j=1}^{M-1} n_j + 1, \ldots, N\right\}$, corresponding to M leaders, respectively. To this end, an index set mapping $\delta(i) = j, i \in \sum_{k=0}^{j-1} n_k + 1, \ldots, \sum_{k=0}^{j} n_k, k \in 0, 1, \ldots, M, j \in 1, 2, \ldots M$ is constructed in particular, where $n_0 = 0$.

The dynamic model of follower i is described as

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = u_i(t), \quad i \in 1, 2, \ldots, N, \end{cases} \tag{1}$$

where $x_i(t) \in R^n, v_i(t) \in R^n$ represents the position and velocity of the follower i respectively, $u_i(t) \in R^n$ is the control input.

The dynamic model of leader j is described as

$$\begin{cases} \dot{x}_j^0(t) = v_j^0(t) \\ \dot{v}_j^0(t) = 0, \qquad j \in 1, 2, \ldots, M \end{cases} \tag{2}$$

where $x_j^0(t) \in R^n, v_j^0(t) \in R^n$ represents the position and velocity of the leader j respectively.

Definition 1: For multi-agent systems (1) with multiple leaders (2) to achieve multi-group formation control, if for any bounded initial state, we have

$$\begin{aligned} &x_i(t) - h_{ix}(t) - x_{\delta(i)}^0(t) \to 0, \quad t \to \infty \\ &v_i(t) - h_{iv}(t) - v_{\delta(i)}^0(t) \to 0, \quad t \to \infty \ (i \in V). \end{aligned} \tag{3}$$

Remark 1: The formation vector is set up according to the desired geometry of the system, therefore its existence is completely known. Corresponding to the two system states of position and velocity considered by the second-order system, its formation vector is described as $h_i(t) = \left[h_{ix}^T(t), h_{iv}^T(t)\right]^T$, where $h_{ix}, h_{iv} \in R^n$ is the formation component corresponding to the position and velocity states. Compared with consensus control, formation control considers one more known vector into consideration in its error system.

Lemma 1 ([25]): For a connected undirected graph G, if at least one follower is connected to the leader, that is $B \geq 0$, then $L + B > 0$.

Lemma 2 ([6]): For any vector $x, y \in R^n$ of suitable dimensions and a symmetric positive definite matrix $Z \in R^{n\times n}$ of suitable dimensions, the following inequality holds

$$\pm 2x^T y \leq x^T Z x + y^T Z^{-1} y. \tag{4}$$

Lemma 3 ([26]): For any positive constant P, Q, if $\dot{V}(t) \leq -PV(t) + Q$, the following inequality holds

$$V(t) \leq \left[V(0) - \frac{P}{Q}\right] e^{-Pt} + \frac{P}{Q}. \tag{5}$$

Lemma 4 ([27]): Matrix inequality (Schur's complement lemma)

$$\begin{bmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{bmatrix} > 0,$$

where $Q(x) = Q^T(x), R(x) = R^T(x)$, equivalent to one of the following conditions

(1) $Q(x) > 0, R(x) - S^T(x) Q^{-1}(x) S(x) > 0$;
(2) $R(x) > 0, Q(x) - S^T(x) R^{-1}(x) S(x) > 0$.

III. MAIN RESULTS

A. ADAPTIVE CONTROL

To realize adaptive multi-group time-varying formation control, the following distributed protocol is considered

$$\begin{aligned} u_i(t) &= a_i(t) \left[\sum_{j=1}^{N} \omega_{ij} \left[\left(x_j(t) - h_{jx}(t)\right) - \left(x_i(t) - h_{ix}(t)\right) \right] \right. \\ &\quad + \sum_{j=1}^{N} \omega_{ij} \left[\left(v_j(t) - h_{jv}(t)\right) - \left(v_i(t) - h_{iv}(t)\right) \right] \\ &\quad - b_i \left(x_i(t) - h_{ix}(t) - x_{\delta(i)}^0(t) + v_i(t) - h_{iv}(t) - v_{\delta(i)}^0(t) \right) \\ &\quad \left. + \sum_{j=1}^{N} l_{ij} x_{\delta(i)}^0 + \sum_{j=1}^{N} l_{ij} v_{\delta(i)}^0 \right] + \dot{h}_{iv}(t) \\ \dot{a}_i(t) &= \kappa_i \left[\sum_{j=1}^{N} \omega_{ij} \left[\left(x_i(t) - h_{ix}(t)\right) - \left(x_j(t) - h_{jx}(t)\right) \right] \right. \\ &\quad + \sum_{j=1}^{N} \omega_{ij} \left[\left(v_i(t) - h_{iv}(t)\right) - \left(v_j(t) - h_{jv}(t)\right) \right] \\ &\quad + b_i \left(x_i(t) - h_{ix}(t) - x_{\delta(i)}^0(t) + v_i(t) - h_{iv}(t) - v_{\delta(i)}^0(t) \right) \\ &\quad \left. - \sum_{j=1}^{N} l_{ij} x_{\delta(i)}^0 - \sum_{j=1}^{N} l_{ij} v_{\delta(i)}^0 \right]^T \\ &\quad \times \left[\sum_{j=1}^{N} \omega_{ij} \left[\left(x_i(t) - h_{ix}(t)\right) - \left(x_j(t) - h_{jx}(t)\right) \right] \right. \\ &\quad + \sum_{j=1}^{N} \omega_{ij} \left[\left(v_i(t) - h_{iv}(t)\right) - \left(v_j(t) - h_{jv}(t)\right) \right] \\ &\quad + b_i \left(x_i(t) - h_{ix}(t) - x_{\delta(i)}^0(t) + v_i(t) - h_{iv}(t) - v_{\delta(i)}^0(t) \right) \\ &\quad \left. - \sum_{j=1}^{N} l_{ij} x_{\delta(i)}^0 - \sum_{j=1}^{N} l_{ij} v_{\delta(i)}^0 \right] \end{aligned} \tag{6}$$

where κ_i is a positive constant, and $a_i(t)$ is the designed adaptive parameter.

Assumption 1: The information connection of the followers of the multi-agent system is a connected topology, and for each subgroup $j \in 1, 2, \ldots, M$, there is at least one that can obtain the information of the leader corresponding to this subgroup.

Assumption 2: Vector $h_i(t), \ (i \in 1, 2, \ldots, N)$ satisfies $h_{iv} = \dot{h}_{ix}$, that is, the derivative of the formation vector of the system's position state is equal to the formation vector of its velocity state.

According to Definition 1, the formation error variables are constructed as follows

$$\bar{x}_i(t) = x_i(t) - h_{ix}(t) - x_{\delta(i)}(t)$$
$$\bar{v}_i(t) = v_i(t) - h_{iv}(t) - v_{\delta(i)}(t) \quad i \in V.$$

Remark 2: The error variable is the algebraic sum of the individual state and its corresponding leader state and formation vector, and the state with the mapping subscript is the state of the leader of the subgroup corresponding to the individual. It can be seen that the error vector contains a time-varying formation vector, which affects the differential operation of the error system. Assumption 2 and the introduction of $\dot{h}_{iv}(t)$ in protocol (6) can eliminate this effect, and thus turning it into a routine consensus problem that can uncomplicate the analysis. In addition, the use of $\sum_{j=1}^{N} l_{ij}x^0_{\delta(i)} + \sum_{j=1}^{N} l_{ij}v^0_{\delta(i)}$ in the protocol ensures that the Laplacian matrix of the system does not need to meet the constraint of input balance, and can still deal with the communication connection problem of agents between groups.

Theorem 1: If the multi-agent system (1) with the leaders (2) meets the Assumption (1,2), the designed adaptive control protocol (6) can realize time-varying formation control.

Proof: Let $\bar{x}(t) = \left[\bar{x}_1^T(t), \bar{x}_2^T(t), \ldots, \bar{x}_N^T(t)\right]^T, \bar{v}(t) = \left[\bar{v}_1^T(t), \bar{v}_2^T(t), \ldots, \bar{v}_N^T(t)\right]^T$. From the control protocol (6), we obtain

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = a_i(t)\left[\sum_{j=1}^{N} \omega_{ij}\left[\left(x_j(t) - h_{jx}(t)\right) - \left(x_i(t) - h_{ix}(t)\right)\right]\right. \\ \quad + \sum_{j=1}^{N} \omega_{ij}\left[\left(v_j(t) - h_{jv}(t)\right) - \left(v_i(t) - h_{iv}(t)\right)\right] \\ \quad - b_i\left(x_i(t) - h_{ix}(t) - x^0_{\delta(i)}(t) + v_i(t) - h_{iv}(t) - v^0_{\delta(i)}(t)\right) \\ \quad \left. + \sum_{j=1}^{N} l_{ij}x^0_{\delta(i)} + \sum_{j=1}^{N} l_{ij}v^0_{\delta(i)}\right] + \dot{h}_{iv}(t) \quad i \in 1, 2, \ldots, N, \end{cases}$$

rewrite the above equation as an error system using Assumption 2

$$\begin{aligned} \dot{\bar{x}}_i(t) &= \bar{v}_i(t) \\ \dot{\bar{v}}_i(t) &= a_i(t)\left[\sum_{j=1}^{N} \omega_{ij}\left(\bar{x}_j(t) - \bar{x}_i(t)\right)\right. \\ &\quad \left. + \sum_{j=1}^{N} \omega_{ij}\left(\bar{v}_j(t) - \bar{v}_i(t)\right) - b_i\left(\bar{x}_i(t) + \bar{v}_i(t)\right)\right], \end{aligned} \tag{7}$$

and then write (7) in matrix form

$$\begin{aligned} \dot{\bar{x}}(t) &= \bar{v}(t) \\ \dot{\bar{v}}(t) &= -\left[\left(a(t)(L+B)\right) \otimes I_n\right]\left(\bar{x}(t) + \bar{v}(t)\right), \end{aligned} \tag{8}$$

where $a(t) = diag\left(a_1(t), a_2(t), \ldots, a_N(t)\right)$.

The Lyapunov functional is constructed as follows

$$V(t) = \left(\bar{x}^T, \bar{v}^T\right)\left[\begin{pmatrix} \hat{a}H^2 & \frac{H}{2} \\ \frac{H}{2} & \frac{H}{2} \end{pmatrix} \otimes I_n\right]\begin{pmatrix} \bar{x} \\ \bar{v} \end{pmatrix} + \sum_{i=1}^{N} \frac{\left(a_i(t) - \hat{a}\right)^2}{2\kappa_i}, \tag{9}$$

where $\hat{a}$ is a positive undetermined constant, $H = L + B$.

Taking the time derivative of (9) along the trajectory (8) yields

$$\begin{aligned} \dot{V}(t) &= 2\hat{a}\bar{x}^T(t)\left(H^2 \otimes I_n\right)\bar{v}(t) + \bar{v}^T(t)\left(H \otimes I_n\right)\bar{v}(t) \\ &\quad + \bar{x}^T(t)\left(H \otimes I_n\right)\left[-a(t)\left(H \otimes I_n\right)\left(\bar{x}(t) + \bar{v}(t)\right)\right] \\ &\quad + \bar{v}^T(t)\left(H \otimes I_n\right)\left[-a(t)\left(H \otimes I_n\right)\left(\bar{x}(t) + \bar{v}(t)\right)\right] \\ &\quad + \sum_{i=1}^{N}\left(a_i(t) - \hat{a}\right)\left(\bar{x}^T(t) + \bar{v}^T(t)\right)\left(H^2 \otimes I_n\right) \\ &\quad \times \left(\bar{x}(t) + \bar{v}(t)\right) \end{aligned}$$

notice that

$$\begin{aligned} &\sum_{i=1}^{N}\left(a_i(t) - \hat{a}\right)\left(\bar{x}^T(t) + \bar{v}^T(t)\right)\left(H^2 \otimes I_n\right)\left(\bar{x}(t) + \bar{v}(t)\right) \\ &= \left(\bar{x}^T(t) + \bar{v}^T(t)\right)\left(Ha(t)H \otimes I_n\right)\left(\bar{x}(t) + \bar{v}(t)\right) \\ &\quad - \hat{a}\left(\bar{x}^T(t) + \bar{v}^T(t)\right)\left(H^2 \otimes I_n\right)\left(\bar{x}(t) + \bar{v}(t)\right) \end{aligned} \tag{10}$$

combining (10) can be obtained

$$\begin{aligned} \dot{V}(t) &= -\hat{a}\bar{x}^T(t)\left(H^2 \otimes I_n\right)\bar{x}(t) \\ &\quad - \left(\hat{a} - 1\right)\bar{v}^T(t)\left(H^2 \otimes I_n\right)\bar{v}(t). \end{aligned} \tag{11}$$

From Lemma 1, $H = L + B$ is positive definite, and Lemma 4 shows that there is a certain constant $\gamma > 0$, and when $\hat{a} > \gamma$, $V(t)$ is positive definite. According to (11), when $\hat{a} > 1$, $\dot{V}(t)$ is negative definite. In conclusion, the constructed Lyapunov functional $V(t)$ is monotonically decreases and has a lower bound, thus when $t \to \infty$,we have $\bar{x}(t) \to 0, \bar{v}(t) \to 0$. The proof is completed.

B. ADAPTIVE CONTROL WITH NONLINEAR DYNAMICS

The problem of multi-group time-varying formation control with adaptive ability is discussed for second order multi-agent systems. However, in general, the nonlinear dynamics of an agent cannot be ignored and are usually unknown. This section considers adaptive formation control with external disturbances and internal nonlinear dynamics.

Systems (1) and (2) are modified to the following model

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = f_i\left(x_i, v_i, t\right) + u_i(t) + \Delta_i(t), \quad i \in 1, 2, \ldots, N, \end{cases} \tag{12}$$

where $f_i(x_i, v_i, t), \Delta_i(t) \in R^n$ represents the internal dynamics and external bounded disturbance of the follower i respectively.

$$\begin{cases} \dot{x}_j^0(t) = v_j^0(t) \\ \dot{v}_j^0(t) = f_j^0(t), \quad j \in 1, 2, \ldots, M \end{cases} \tag{13}$$

where $f_j^0(t) \in R^n$ is a function of time t, like $f_i(x_i, v_i, t)$, assuming they are unknown.

To estimate the dynamic of each agent, it is assumed that $f_i(x_i, v_i, t)$ and $f_j^0(t)$ have Lipschitz conditions and are parameterizable, that is, we can select a set of definite basis functions to express them to ensure that they are smooth and continuous. In this subsection, for the focus of unambiguous analysis, and assuming that the system state is one-dimensional, the Kronecker product extends the results to any arbitrary finite dimension. In addition, here let $M = 2$, which divides the system into two subgroups.

Parameterization of the unknown dynamic is described as

$$\begin{aligned} f_i(x_i, v_i, t) &= \varphi_{Fi}^T(x_i, v_i, t)\,\theta_{Fi} \quad i \in 1, 2, \ldots, N \\ f_j^0(t) &= \varphi_j^{0T}(t)\,\theta_j^0 \quad j = 1, 2. \end{aligned} \tag{14}$$

where $\varphi_{Fi}(x_i, v_i, t), \varphi_j^0(t) \in R^m$ is the basis function column vector and $\theta_{Fi}, \theta_j^0 \in R^m$ is the unknown parameter column vector to be estimated.

For the nonlinear dynamics, they are deterministic, but for their neighbors the parameter vectors are unknown and to be estimated. Here $\hat{\theta}_{Fi}$ represents the parameter estimation column vector of θ_{Fi}, and $\hat{\theta}_{\delta(i)i}^0$ represents the parameter estimation column vector of agent i for its leader's internal nonlinear dynamics.

Estimation dynamics can be expressed as

$$\begin{aligned} \hat{f}_{\delta(i)}^0(t) &= \varphi_{\delta(i)}^{0T}(t)\,\hat{\theta}_{\delta(i)i}^0 \\ \hat{f}_i(x_i, v_i, t) &= \varphi_{Fi}^T(x_i, v_i, t)\,\hat{\theta}_{Fi} \quad i \in 1, 2, \ldots, N. \end{aligned} \tag{15}$$

Construct local formation error vector

$$\begin{aligned} e_{xi}(t) &= \sum_{j=1}^{N} \omega_{ij}\left[(x_i(t) - h_{ix}(t)) - \left(x_j(t) - h_{jx}(t)\right)\right] \\ &\quad + b_i\left(x_i(t) - h_{ix}(t) - x_{\delta(i)}^0(t)\right) - \sum_{j=1}^{N} l_{ij} x_{\delta(i)}^0 \\ e_{vi}(t) &= \sum_{j=1}^{N} \omega_{ij}\left[(v_i(t) - h_{iv}(t)) - \left(v_j(t) - h_{jv}(t)\right)\right] \\ &\quad + b_i\left(v_i(t) - h_{iv}(t) - v_{\delta(i)}^0(t)\right) - \sum_{j=1}^{N} l_{ij} v_{\delta(i)}^0 \\ &\quad i \in 1, 2, \ldots, N, \end{aligned} \tag{16}$$

let $e_x(t) = [e_{x1}(t), e_{x2}(t), \ldots, e_{xN}(t)]^T$, $e_v(t) = [e_{v1}(t), e_{v2}(t), \ldots, e_{vN}(t)]^T$, $h_x(t) = [h_{1x}(t), h_{2x}(t), \ldots, h_{Nx}(t)]^T$, $h_v(t) = [h_{1v}(t), h_{2v}(t), \ldots, h_{Nv}(t)]^T$, $x_\delta^0(t) = \left[x_{\delta(1)}^0(t), x_{\delta(2)}^0(t), \ldots, x_{\delta(N)}^0(t)\right]$, $v_\delta^0(t) = \left[v_{\delta(1)}^0(t), v_{\delta(2)}^0(t), \ldots, v_{\delta(N)}^0(t)\right]$.

Rewrite (16) in matrix form as

$$\begin{aligned} e_x(t) &= (L + B)\left[x(t) - h_x(t) - x_\delta^0(t)\right] \\ e_v(t) &= (L + B)\left[v(t) - h_v(t) - v_\delta^0(t)\right] \end{aligned} \tag{17}$$

Assumption 3: We reduce the condition in Assumption 2, and the vector $h_i(t), \; i \in 1, 2, \ldots, N$ satisfies $h_{iv} \to \dot{h}_{ix}, t \to \infty$. This relatively loose restriction can also be interpreted as the formation control having adaptive ability and robustness to small changes in internal parameters of the system and bounded uncertainties in the external environment.

For systems (12) and (13), combined with (14), we rewrite (17) as

$$\begin{aligned} \dot{e}_x(t) &= e_v(t) + (L + B)\left(h_v(t) - \dot{h}_x(t)\right) \\ \dot{e}_v(t) &= (L + B)\Big[\Phi_F(x, v, t)\,\theta_F + u(t) + \Delta(t) \\ &\qquad - \dot{h}_v(t) - \Phi_\delta^0(t)\,\theta_\delta^0\Big], \end{aligned} \tag{18}$$

where $\Phi_F(x, v, t) = diag\left[\varphi_{F1}^T(x_1, v_1, t), \varphi_{F2}^T(x_2, v_2, t), \ldots, \varphi_{FN}^T(x_N, v_N, t)\right]$,
$\Phi_\delta^0(t) = diag\left[\varphi_{\delta(1)}^{0T}(t), \varphi_{\delta(2)}^{0T}(t), \ldots, \varphi_{\delta(N)}^{0T}(t)\right]$,
$\theta_F = \left[\theta_{F1}^T, \theta_{F2}^T, \ldots, \theta_{FN}^T\right]$,
$\theta_\delta^0 = \left[\theta_{\delta(1)1}^{0T}, \theta_{\delta(2)2}^{0T}, \ldots, \theta_{\delta(N)N}^{0T}\right]^T$,
$u(t) = [u_1(t), u_2(t) \ldots, u_N(t)]^T$,
$\Delta(t) = [\Delta_1(t), \Delta_2(t), \ldots, \Delta_N(t)]^T$.

To realize adaptive time-varying formation control for second-order multi-agent systems with unknown neighbor and leader dynamics, the following control protocol is designed

$$\begin{aligned} u_i(t) &= -e_{xi}(t) - d y_{vi}(t) - \varphi_{Fi}^T(x_i, v_i, t)\,\hat{\theta}_{Fi} + \varphi_{\delta(i)}^{0T}(t)\,\hat{\theta}_{\delta(i)i}^0 \\ &\quad + \dot{h}_{iv}(t) - a_i(t)\,sign\,(y_{vi}(t)) \\ y_{vi}(t) &= e_{vi}(t) - \Gamma_{2i}(t) \\ \dot{a}_i(t) &= \tau_0\,|y_{vi}(t)| \\ \dot{\hat{\theta}}_{Fi} &= \alpha\varphi_{Fi}(x_i, v_i, t)\left[\sum_{j=1}^{N} \omega_{ij}\left(y_{vi}(t) - y_{vj}(t)\right) + b_i y_{vi}(t)\right] \\ \dot{\hat{\theta}}_{\delta(i)i}^0 &= -\beta\varphi_{\delta(i)}^0(t)\left[\sum_{j=1}^{N} \omega_{ij}\left(y_{vi}(t) - y_{vj}(t)\right) + b_i y_{vi}(t)\right] \\ \dot{\Gamma}_{2i}(t) &= \left(\Gamma_{1i}(t) - \Gamma_{2i}(t)\right)/k \\ \Gamma_{1i}(t) &= -c e_{xi}(t) \\ \bar{\Gamma}_i(t) &= \Gamma_{2i}(t) - \Gamma_{1i}(t) \quad i \in 1, 2, \ldots, N. \end{aligned} \tag{19}$$

where α, β, c, d, k and τ_0 are positive constants, $a_i(t)$ is an adaptive controller, $e_{xi}(t)$ and $y_{vi}(t)$ are dynamic surfaces, $\Gamma_{1i}(t)$ and $\Gamma_{2i}(t)$ are virtual leading signal and filtering signal, respectively, and $\bar{\Gamma}_i(t)$ is the filtering error.

Remark 3: Compared with the parameterization of the individual unknown internal nonlinear dynamics of first-order system consensus control in [22], the formation control of second-order system discussed in this paper has a more complex environment and more severe challenges. The designs of $\Gamma_{1i}(t)$ and $\Gamma_{2i}(t)$ in (19) can solve the instability derived from cross variables that does not exist in first-order systems.

According to the designed control protocol (19), the system (12) can be written in the following form

$$\begin{cases} \dot{x}(t) = v(t) \\ \dot{v}(t) = \Phi_F(x,v,t)\,\theta_F - e_x(t) - dy_v(t) - \Phi_F(x,v,t)\,\hat{\theta}_F \\ \quad + \Phi_\delta^0(t)\,\theta_\delta^0 + \Delta(t) - a(t)\,sign(y_v(t)), \end{cases} \tag{20}$$

Theorem 2: The multi-agent systems (12) and (13) satisfy Assumption 1 and 3, and the designed control protocol (19) can realize adaptive robust time-varying formation control if the parameters in the system satisfy the requirements

$$c > \lambda_{\max}(L+B), \quad k < \frac{2}{\lambda_{\max}(L+B)+1}. \tag{21}$$

Proof: Define parameter estimation residual vector $\bar{\theta}_{Fi} = \hat{\theta}_{Fi} - \theta_{Fi}, \bar{\theta}^0_{\delta(i)i} = \hat{\theta}^0_{\delta(i)i} - \theta^0_{\delta(i)i}$, let $\bar{\theta}_F = \left[\bar{\theta}^T_{F1}, \bar{\theta}^T_{F2}, \ldots, \bar{\theta}^T_{FN}\right]^T, \bar{\theta}^0_\delta = \left[\theta^{0T}_{\delta(1)1}, \theta^{0T}_{\delta(2)2}, \ldots, \theta^{0T}_{\delta(N)N}\right]^T$.

By combining with (19), the local formation error system (18) can be rewritten as

$$\begin{aligned} \dot{e}_x(t) &= y_v(t) - ce_x(t) + \bar{\Gamma}(t) + (L+B)\left(h_v(t) - \dot{h}_x(t)\right) \\ \dot{e}_v(t) &= (L+B)\left[-\Phi_F(x,v,t)\,\bar{\theta}_F - e_x(t) - dy_v(t)\right. \\ &\quad \left. + \Phi_\delta^0(t)\,\bar{\theta}_\delta^0 + \Delta(t) - a(t)\,sign(y_v(t))\right], \end{aligned} \tag{22}$$

where

$$\begin{aligned} sign(y_v(t)) &= [sign(y_{v1}(t)), sign(y_{v2}(t)), \ldots, \\ &\qquad sign(y_{vN}(t))]^T, \\ a(t) &= diag(a_1(t), a_2(t), \ldots, a_N(t)), \\ \bar{\Gamma}(t) &= \left[\bar{\Gamma}_1(t), \bar{\Gamma}_2(t), \ldots, \bar{\Gamma}_N(t)\right]^T, \\ y_v(t) &= [y_{v1}(t), y_{v2}(t), \ldots, y_{vN}(t)]^T \\ &= e_v(t) - \Gamma_2(t). \end{aligned}$$

Construct the Lyapunov functional as follows

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t) \tag{23}$$

where

$$\begin{aligned} V_1(t) &= \frac{1}{2}e_x^T(t)(L+B)\,e_x(t) \\ V_2(t) &= \frac{1}{2}y_v^T(t)\,y_v(t) \\ &\quad + \frac{1}{2\tau_0} row\left(a_i(t) - \hat{a}\right)(L+B)\,col\left(a_i(t) - \hat{a}\right) \\ V_3(t) &= \frac{1}{2\alpha}\bar{\theta}_F^T\bar{\theta}_F \\ V_4(t) &= \frac{1}{2\beta}\bar{\theta}_\delta^{0T}\bar{\theta}_\delta^0 \\ V_5(t) &= \frac{1}{2}\bar{\Gamma}^T(t)\,\bar{\Gamma}(t). \end{aligned}$$

where $row\left(a_i(t) - \hat{a}\right)$ is a row vector with $a_i(t) - \hat{a}$ as its elements, and $col\left(a_i(t) - \hat{a}\right)$ is a column vector with $a_i(t) - \hat{a}$ as its elements.

Since each term of $V(t)$ is in the form of the system state variable squared, and Lemma 1 states that $(L+B)$ is positively definite, so we can easily conclude that our constructed Lyapunov function is positively definite. They are differentiated separately according to the linear property of differentiation

$$\begin{aligned} \dot{V}_1(t) &= e_x^T(t)(L+B)\left[y_v(t) - ce_x(t) + \bar{\Gamma}(t)\right. \\ &\quad \left. + (L+B)\left(h_v(t) - \dot{h}_x(t)\right)\right] \\ &\le e_x^T(t)(L+B)\,y_v(t) + \frac{\lambda_{\max}(L+B)}{2}\bar{\Gamma}^T(t)\,\bar{\Gamma}(t) \\ &\quad - (c - \lambda_{\max}(L+B))\,e_x^T(t)\,e_x(t) \\ &\quad + \frac{\lambda_{\max}(L+B)}{2}\|L+B\|^2\left\|h_v(t) - \dot{h}_x(t)\right\|^2 \end{aligned} \tag{24}$$

obtained by (19) and (22)

$$\begin{aligned} &\dot{V}_2(t) \\ &= -y_v^T(t)(L+B)\,e_x(t) - dy_v^T(t)(L+B)\,y_v(t) \\ &\quad - y_v^T(t)(L+B)\,\Phi_F(x,v,t)\,\bar{\theta} + y_v^T(t)(L+B)\,\Phi_\delta^0(t)\,\bar{\theta}_\delta^0 \\ &\quad + y_v^T(t)(L+B)\left[\Delta(t) - (L+B)^{-1}\dot{\Gamma}_2(t)\right] \\ &\quad - \left|y_v^T(t)\right|(L+B)\,col(\hat{a}) - \left|y_v^T(t)\right|(L+B)\,col\left(a_i(t) - \hat{a}\right) \\ &\quad + row\left(a_i(t) - \hat{a}\right)(L+B)\,|y_v(t)| \end{aligned} \tag{25}$$

Suppose that $\Delta(t)$ and $\dot{\Gamma}_2(t)$ are bounded. Since $L+B$ is positively bounded, $(L+B)^{-1}$ is positively bounded, $(L+B)^{-1}\dot{\Gamma}_2(t)$ is bounded, there exists a positive constant $\hat{a} = \|\Delta(t)\|_\infty + \lambda_{\max}(L+B)^{-1}\left\|\dot{\Gamma}_2(t)\right\|_\infty$ such that

$$\left\|\Delta(t) - (L+B)^{-1}\dot{\Gamma}_2(t)\right\|_\infty \le \hat{a} \tag{26}$$

it follows that

$$\begin{aligned} &y_v^T(t)(L+B)\left[\Delta(t) - (L+B)^{-1}\dot{\Gamma}_2(t)\right] \\ &\quad - \left|y_v^T(t)\right|(L+B)\,col(\hat{a}) \le 0, \end{aligned}$$

and notice that

$$\begin{aligned} &-\left|y_v^T(t)\right|(L+B)\,col\left(a_i(t) - \hat{a}\right) \\ &\quad + row\left(a_i(t) - \hat{a}\right)(L+B)\,|y_v(t)| = 0, \end{aligned}$$

we can get

$$\begin{aligned} \dot{V}_2(t) &\le -y_v^T(t)(L+B)\,e_x(t) - dy_v^T(t)(L+B)\,y_v(t) \\ &\quad - y_v^T(t)(L+B)\,\Phi_F(x,v,t)\,\bar{\theta} \\ &\quad + y_v^T(t)(L+B)\,\Phi_\delta^0(t)\,\bar{\theta}_\delta^0. \end{aligned} \tag{27}$$

According to (19), the estimated vector is rewritten in matrix form

$$\begin{aligned} \dot{\hat{\theta}}_F &= \alpha\Phi_F^T(x,v,t)(L+B)\,y_v(t) \\ \dot{\hat{\theta}}_\delta^0 &= -\beta\Phi_\delta^{0T}(t)(L+B)\,y_v(t), \end{aligned} \tag{28}$$

taking the time derivative of $V_3(t)$ and $V_4(t)$ along the trajectory (28) yields

$$\begin{aligned}\dot{V}_3(t) &= \bar{\theta}_F^T \Phi_F^T(x, v, t)(L+B) y_v(t)\\ \dot{V}_4(t) &= -\bar{\theta}_\delta^{0T} \Phi_\delta^{0T}(t)(L+B) y_v(t).\end{aligned} \tag{29}$$

And then we take the derivative with respect to $V_5(t)$

$$\begin{aligned}\dot{V}_5(t) &= \bar{\Gamma}^T(t)\left(\dot{\Gamma}_2(t) - \dot{\Gamma}_1(t)\right)\\ &= -\frac{1}{k}\bar{\Gamma}^T(t)\bar{\Gamma}(t) - \bar{\Gamma}^T(t)\dot{\Gamma}_1(t)\\ &\le -\left(\frac{1}{k} - \frac{1}{2}\right)\bar{\Gamma}^T(t)\bar{\Gamma}(t) + \frac{1}{2}\left\|\dot{\Gamma}_1(t)\right\|^2.\end{aligned} \tag{30}$$

Combining (24)-(30) can be obtained

$$\begin{aligned}&\dot{V}(t)\\ &= \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t) + \dot{V}_5(t)\\ &\le e_x^T(t)(L+B) y_v(t) - (c - \lambda_{\max}(L+B)) e_x^T(t) e_x(t)\\ &\quad - d y_v^T(t)(L+B) y_v(t) - y_v^T(t)(L+B) e_x(t)\\ &\quad + \frac{\lambda_{\max}(L+B)}{2}\|L+B\|^2\left\|h_v(t) - \dot{h}_x(t)\right\|^2\\ &\quad + \frac{\lambda_{\max}(L+B)}{2}\bar{\Gamma}^T(t)\bar{\Gamma}(t)\\ &\quad - y_v^T(t)(L+B)\Phi_F(x, v, t)\bar{\theta} + y_v^T(t)(L+B)\Phi_\delta^0(t)\bar{\theta}_\delta^0\\ &\quad + \bar{\theta}_F^T \Phi_F^T(x, v, t)(L+B) y_v(t) - \bar{\theta}_\delta^{0T}\Phi_\delta^{0T}(t)\\ &\quad \times (L+B) y_v(t)\\ &\quad - \left(\frac{1}{k} - \frac{1}{2}\right)\bar{\Gamma}^T(t)\bar{\Gamma}(t) + \frac{1}{2}\left\|\dot{\Gamma}_1(t)\right\|^2\\ &\le -(c - \lambda_{\max}(L+B)) e_x^T(t) e_x(t) - d y_v^T(t)(L+B) y_v(t)\\ &\quad - \left(\frac{1}{k} - \frac{\lambda_{\max}(L+B)+1}{2}\right)\bar{\Gamma}^T(t)\bar{\Gamma}(t)\\ &\quad + \frac{1}{2}\left\|\dot{\Gamma}_1(t)\right\|^2 + \frac{\lambda_{\max}(L+B)}{2}\|L+B\|^2\\ &\quad \times \left\|h_v(t) - \dot{h}_x(t)\right\|^2\end{aligned} \tag{31}$$

let $Q = \frac{\lambda_{\max}(L+B)}{2}\|L+B\|^2\left\|h_v(t) - \dot{h}_x(t)\right\|^2 + \frac{1}{2}\left\|\dot{\Gamma}_1(t)\right\|^2$, $P = \min\left(c - \lambda_{\max}(L+B), d\lambda_{\max}(L+B), \frac{1}{k} - \frac{\lambda_{\max}(L+B)+1}{2}\right)$, we have

$$\dot{V}(t) \le -PV(t) + Q. \tag{32}$$

According to Lemma 3, $V(t)$ is finally uniformly bounded. By our definition of $V(t)$, state $e_x(t), y_v(t), \bar{\theta}_F, \bar{\theta}_\delta^0, \bar{\Gamma}(t)$ are also uniformly bounded. From the construction of $e_x(t), e_v(t)$ and $\Gamma_2(t)$, we get $\lim_{t\to\infty} e_x^T(t) e_x(t) \le \lim_{t\to\infty} 2V(t)/\lambda_{\min}(L+B) \le \varepsilon_1$, $\lim_{t\to\infty}\Gamma_2^T(t)\Gamma_2(t) \le \varepsilon_2$, and $\lim_{t\to\infty} e_v^T(t) e_v(t) = \lim_{t\to\infty}(y_v(t) + \Gamma_2(t))^T (y_v(t) + \Gamma_2(t)) \le \varepsilon_3$, where $\varepsilon_1 + \varepsilon_3 = \|L+B\|\varepsilon$, $\varepsilon_1, \varepsilon_2, \varepsilon_3$ and ε are some suitable positive constants. Let $\Lambda_i(t) = [x_i(t), v_i(t)]^T$, $\Lambda_{\delta(i)}^0(t) = \left[x_{\delta(i)}^0(t), v_{\delta(i)}^0(t)\right]^T$, $\Lambda(t) = [\Lambda_1(t), \Lambda_2(t), \ldots \Lambda_N(t)]^T$, $\left[\Lambda_{\delta(1)}^0(t), \Lambda_{\delta(2)}^0(t), \ldots \Lambda_{\delta(N)}^0(t)\right]^T = \Lambda_\delta^0(t)$, and $h(t) = [h_1(t), h_2(t), \ldots h_N(t)]^T$. One obtains $\left\|(L+B)\left(\Lambda(t) - h(t) - \Lambda_\delta^0(t)\right)\right\| \le \|e_x(t)\| + \|e_v(t)\| \le \varepsilon_1 + \varepsilon_3$, then, $\left\|(\Lambda(t) - h(t) - \Lambda_\delta^0(t))\right\| \le (\varepsilon_1 + \varepsilon_3)/\|(L+B)\| = \varepsilon$, which means that the multi-agent systems (12) and (13) realizes time-varying formation control under the control protocol (19). The proof is completed.

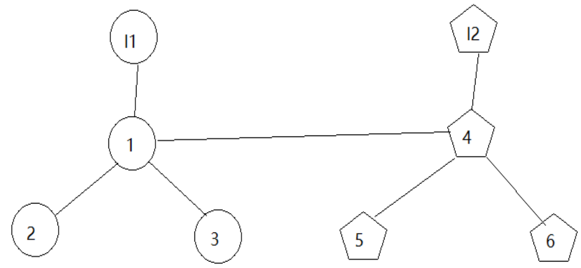

FIGURE 1. Topology structure of agents.

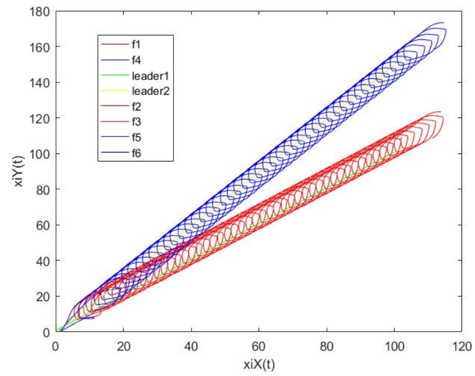

FIGURE 2. States trajectories of followers and leader.

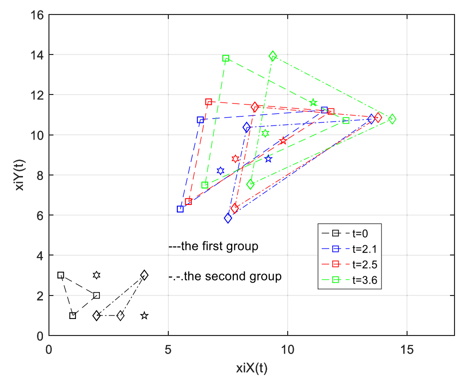

FIGURE 3. Time cross section formation diagram of system trajectory.

IV. NUMERICAL SIMULATIONS

Two simulation results are presented to demonstrate that the proposed control protocol can realize adaptive multi-group tracking control.

First, the communication topology of the multi-agent system is shown in Figure 1. The circular structure is the first group, the pentagonal structure is the second group, and its connection matrix and Laplace matrices are

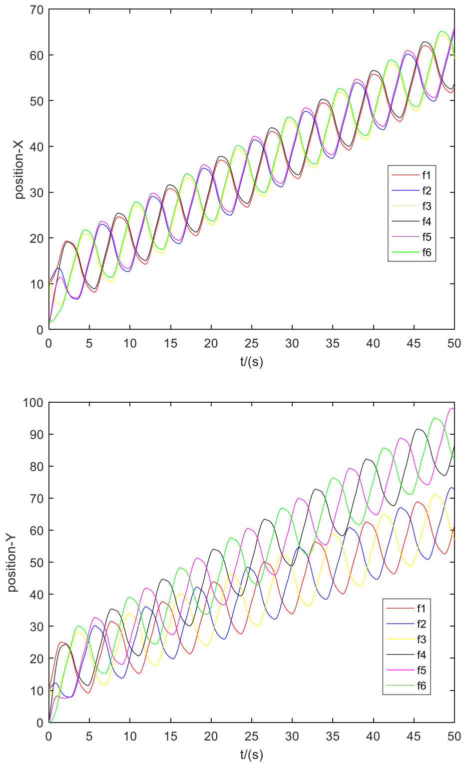

FIGURE 4. Position changes of the x-and y-axes.

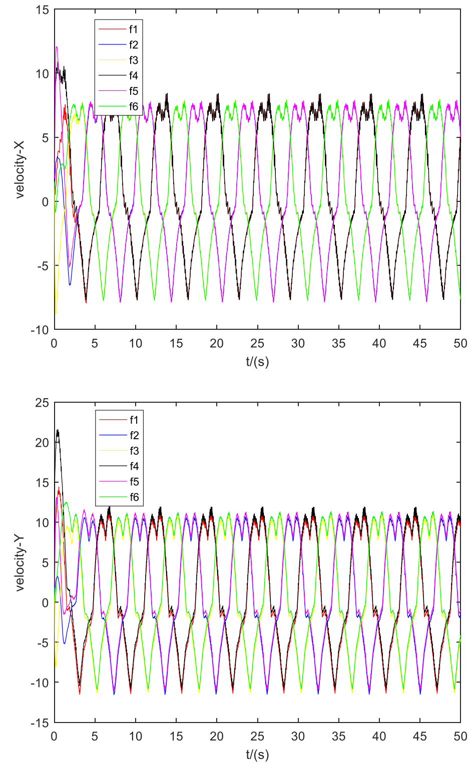

FIGURE 5. Velocity changes of the x-and y-axes.

$B = diag\,(1, 0, 0, 1, 0, 0)$ and

$$L = \begin{bmatrix} 3 & -1 & -1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 3 & -1 & -1 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix}.$$

$\lambda_{\max}(L + B) = 5.4495$, $\lambda_{\min}(L + B) = 0.2679$, let $\kappa_i = 1$. The formation vector of the followers of each subgroup is

$$\begin{bmatrix} r\sin(\omega t + 2(i-1)\pi/3) - r\cos(\omega t + 2(i-1)\pi/3) \\ 2r\sin(\omega t + 2(i-1)\pi/3) \\ r\omega\sin(\omega t + 2(i-1)\pi/3) + r\omega\cos(\omega t + 2(i-1)\pi/3) \\ 2r\omega\cos(\omega t + 2(i-1)\pi/3) \end{bmatrix},$$

and $r = 10, \omega = 1$. It can be seen that its topological relationship and formation vector conform to Assumption 1 and 2. Figure 2 shows the movement track of the agents, and Figure 3 shows its formation at different times. The hexagonal is the leader of the first group, and the pentagon is the leader of the second group. The formation of the followers of both groups gradually becomes regular over time. In combination with Figure 2-3, it can be seen that the followers of the two groups rotate around their respective leaders. Figure 4 and 5 show the position and speed of the follower along different coordinate axes. Figure 6 shows time-varying adaptive control gain.

The second simulation result still follows the topological relationship and formation vector shown in Figure 1. Let

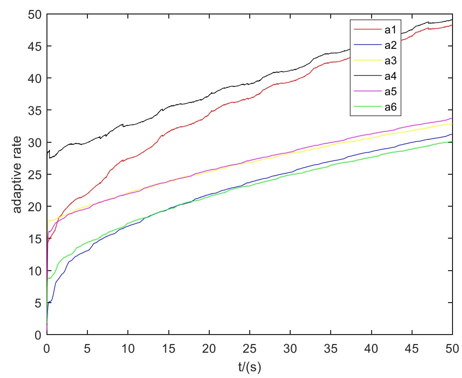

FIGURE 6. Time-varying control gains $a_i(t)$.

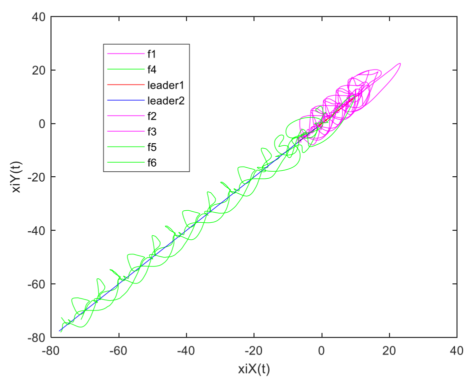

FIGURE 7. States trajectories of followers and leader.

$\alpha = 0.3, \beta = 0.1, c = 5, d = 1, k = 0.1$, the parameter value conforms to the limit in theorem 2. The nonlinear

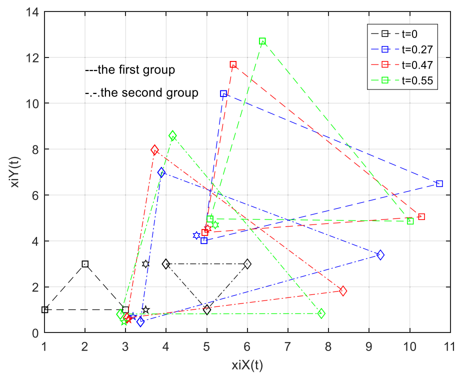

FIGURE 8. Time cross section formation diagram of system trajectory.

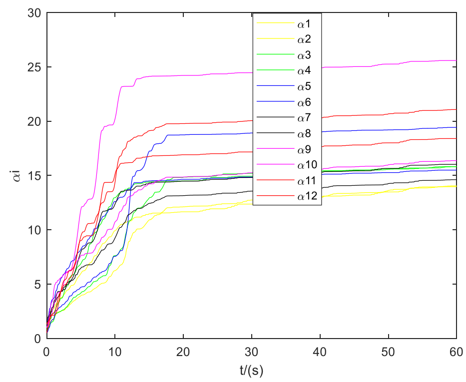

FIGURE 9. Time-varying control gains $\alpha_i\,(t)$.

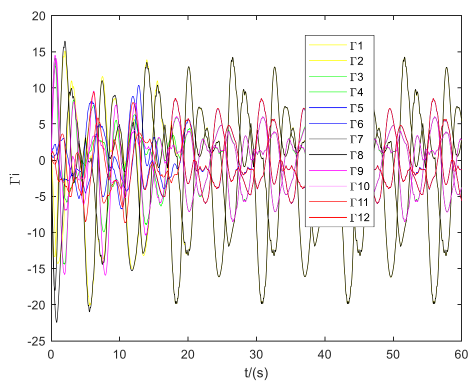

FIGURE 10. Filtering signal.

dynamic parameterization of an individual is described as

$$
\begin{aligned}
& f_i\,(x_i, v_i, t) \\
&\quad = \varphi_{Fi}^{T}\,(x_i, v_i, t)\,\theta_{Fi} \\
&\quad = \begin{bmatrix} x_{i1}\sin(t) & 0 & v_{i1}\cos(t) & 0 \\ 0 & x_{i2}\sin(t) & 0 & v_{i2}\cos(t) \end{bmatrix} \\
&\qquad \times \begin{bmatrix} 1 & -1 & 1 & -1 \end{bmatrix}^{T} \quad i = 1, 2, 3, 4, 5, 6, \\
& f_j^{0}\,(t) \\
&\quad = \varphi_j^{0T}\,(t)\,\theta_j^{0} \\
&\quad = \begin{bmatrix} \sin(0.5t) & 0 & \cos(0.8t) & 0 \\ 0 & \sin(0.5t) & 0 & \cos(0.8t) \end{bmatrix}
\end{aligned}
$$

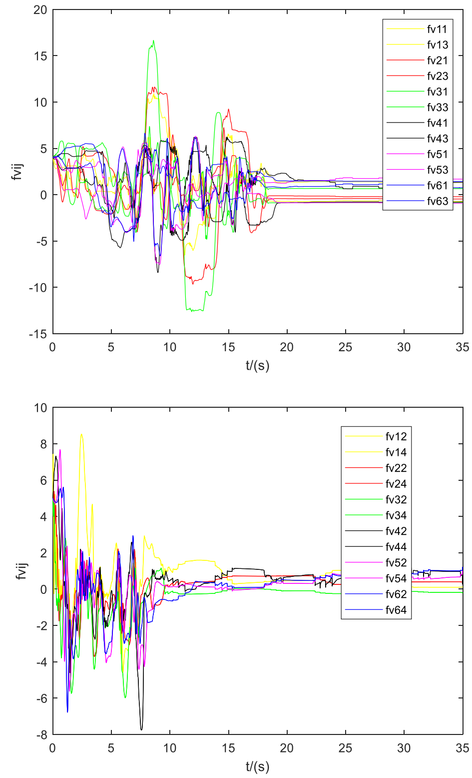

FIGURE 11. Parameter estimation of the follower.

$$
\begin{aligned}
&\qquad \times \begin{bmatrix} 0.5 & 0.8 & 0.5 & 0.8 \end{bmatrix}^{T} \quad j = 1, 2. \\
&\Delta_i = 0.1\sin(t) + 0.2\cos(t).
\end{aligned}
$$

Similarly, Figs 7 and 8 show the trajectory and formation of two groups of followers in periodic movements around the leader. As shown in Fig 9, the time-varying adaptive control gain tends to stabilize after the 15th second. Fig 10 shows the filter signal curve. Fig 11 shows an estimation of the nonlinear dynamics of the follower. In the case of system bounded interference, the dynamic estimation error fluctuates within 0.2.

V. CONCLUSION

In this study, adaptive time-varying formation control for second-order multi-agent systems is studied. First, the time-vary formation conditions are given under ideal environment, and an adaptive control protocol is designed. Subsequently, the influences of the external bounded disturbance and internal nonlinear dynamics on the system model are considered. The virtual leader and filter introduced in the new protocol not only avoid the repeated derivation of the system state, but also solve the unstable influence of the cross variables in the second-order system, and make the adaptive rate converge to a stable value. Finally, simulation results demonstrate the effectiveness of the proposed conclusions. Considering that individuals in multi-agent systems may have master-slave relationships, cooperative and competitive relationships and undertake different objectives, we may study bipartite formation-containment control with multiple groups in future work.

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