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A Novel Approach for Asymmetric Quantum Error Correction With Syndrome Measurement

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ABSTRACT Most of the quantum error correction methods are symmetric. Symmetric methods are implemented by considering the amplitude of bit flip(X) and phase flip(Z) errors as same. With the quantum experiments, it is observed that the amplitude of Z errors are more compared to X errors. Due to which the need of asymmetric error correction has increased. This paved a path for the development of asymmetric error correction methods. In this paper, we discussed the concept of asymmetric quantum error correction (AQEC) and proposed an efficient approach for AQEC with encoding, syndrome measurement and decoding operations with increased fidelity to 85.89% and reduced circuit depth to 48%.

INDEX TERMS Quantum information, phase errors, asymmetric quantum error correction, encoding, syndrome measurement, decoding.

I. INTRODUCTION

The features of quantum mechanics is paving a path towards various approaches for transmitting the information [1]. The main problem observed is noise which is caused by unwanted interaction with the external environment. Therefore the error correction methods plays a crucial role in quantum information process(QIP). Error detection and correction in quantum computation is a difficult task. Because as per the no-cloning theorem [2], copying of quantum information is not possible. If one try to copy the quantum information then it's actual state will be disturbed. The existence of noise leads to the errors. The errors that are possible in QIP are: Bit flip errors(X), Phase flip errors(Z) and Bit & Phase flip errors(Y). The error correction methods are to be implemented in order to handle errors. The quantum information process includes three major steps as in figure 1.

Initially,the quantum information will be encoded using extra qubits and if any error occurs in between it will be detected and corrected using error correction process. After performing error correction, the original information will be decoded by performing decoding operations. Based on this architecture, the QEC methods were introduced in 1995 [3]. Initially the repetition code [4] was implemented for QEC. It is similar to the classical repetition code and is more reliable for bit flip errors but the major drawback is, it cannot detect the error if two qubits are modified. As a solution to this, Peter Shor [5] developed a QEC code for the detection and correction of a single qubit information against one pauli error with nine qubits. The efficiency of QEC methods increases with reduced qubits. For that reason, Calderbank and Shor and Steane [6], [7] developed 7 qubit QEC code using self orthogonal classical linear codes to protect single qubit information against one pauli error. Later it has been reduced to 5 qubits with the development of stabilizer codes [8], [9]. The QEC methods uses the ancilla qubits [10] along with quantum gates to perform the syndrome measurement. The serialized error correction code [11] has been implemented using R, CPHASE gates and these will be applied to ancilla qubits in order to detect the error. To reduce the amplitude and phase damping noise, CWS code [12] is developed. Dissipation control based 3-qubit code has been developed for error correction [13]. Other QEC codes are developed using real time feed back encoding [14], Pauli to binary conversions [15], transversal gates [16]. Along with these, surface codes [17], fault tolerant error correction using surface Gottesman-KitaevPreskill (GKP) code [18], [19] with reduced gate error and additive, non-additive quantum codes [20], entanglement assisted codes [21], [22] were also implemented for error correction. The Holonomic based quantum error correction [23] for nontrivial and matrix valued quantum states has been proposed for universal quantum computation. Quantum machine learning concepts are used for implementing auto encoders for quantum error correction [24]. Decoherence of quantum states also leads to the errors while transmitting

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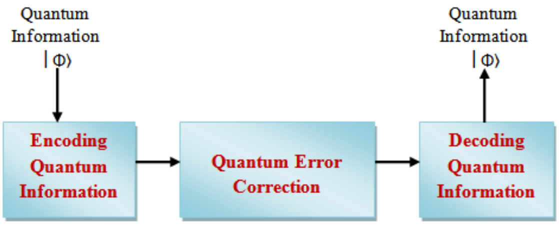

FIGURE 1. **Quantum information process with error correction approach.**

the information. The nature of decoherence and the error correction abilities and automated error correction systems like fault tolerant computation and short error correction codes are discussed [25]–[29]. Some of the error correction methods are implemented with topological codes [30], Continuous variables [31], tanner decoders [32], Bosonic error correction codes [33], [34], network models using holographic tensor products [35], co-variant codes [36], trapped ion based suppression method [37], Noncommutative and Coupling based system [38], 3 dimensional topological toric codes [39], [40] and etc. But the problem with phase flips are still alive with larger probabilities [41]. From the literature it has been observed that the possibility of phase shifts are more while transmitting the information. In order to overcome this problem it is necessary to implement asymmetric quantum error correction methods. Thus, in this paper we propose an efficient asymmetric quantum error correction method for detecting and correcting single arbitrary error.

The remaining paper is outlined as follows. Section II discuss about the preliminaries for quantum computation like structure of a qubit and the quantum Pauli errors. Section III discuss about the proposed methodology on AQEC. The results are analyzed and discussed in section IV. Finally, the paper is concluded in section V.

II. PRELIMINARIES FOR QUANTUM COMPUTATION

A. STRUCTURE OF A QUANTUM BIT

Quantum information is measured in the form of qubits. A qubit is a superposition of multiple quantum states [42]. Quantum states are represented with a state matrix which is the combination of complex state vectors. For example, $\rho, |\phi\rangle$ are the representations of state matrix and a state vector then a new pure quantum state can be represented with the outer product of state vectors as $\rho = |\phi\rangle\langle\phi|$. But after performing the operations on qubits, a pure quantum state will be modified as mixed state due to the interaction with the external environment and it can be represented as

$$\rho = \Sigma_n P_n |\phi_n\rangle\langle\phi_n| \tag{1}$$

where P_n is the probability of quantum state $|\phi_n\rangle$.

B. QUANTUM PAULI ERRORS

If we consider a quantum gate on single qubit, then operation O_p will be performed with unitary group of 2 states which represents the possible Pauli errors for all $4(2 \times 2)$ possible states as follows

$$O_p = E_I\Gamma_I + E_X\Gamma_X + E_Z\Gamma_Z + E_Y\Gamma_Y$$

Here $\Gamma_I, \Gamma_X, \Gamma_Y, \Gamma_Z$ represents the Pauli operators named identity, qubit shift, phase shift, qubit & phase shift and are represented with matrices as given below

$$\Gamma_I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \Gamma_X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$
$$\Gamma_Y = \begin{pmatrix} 0 & -i \\ 1 & 0 \end{pmatrix}, \quad \Gamma_Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

E_I, E_X, E_Y, E_Z are the probabilities with real and complex numbers and which satisfies

$$|E_I|^2 + |E_X|^2 + |E_Y|^2 + |E_Z|^2 = 1$$

For error calculation, only X and Z errors will be considered. By considering these two we can also cover Y errors because Y = XZ [43]. In symmetric error correction methods, it is observed that the possibility of X and Z errors as same but in reality, it is not true. In many experiments, it has been observed that the possibility of Z errors are more compared to the X errors [44]. To solve these kind of problems and to perform error correction, asymmetric error correction methods are introduced. while working with the quantum computer, it is proved that detecting or correcting Z errors are difficult and need complex architectures.

C. FIDELITY MEASUREMENT

In quantum system, the actual and retrieved outputs will not be same. Because currently available quantum systems are in Noisy Intermediate Scale Quantum (NISQ) level. That is the result of a quantum architecture which are executed on a quantum computer will never be 100% due to the noise which is generated while the quantum states are interacted with the environment. Hence fidelity measurement is used to measure the efficiency of an architecture after running it on Quantum computer. The outcome of fidelity measurement represents the closeness between the actual and retrieved outcomes [45]. For example, if the transmitted information is $|\phi\rangle$ and after decoding the information in quantum system, the retrieved information is $|\Phi\rangle$ then the fidelity of this can be calculated using the equation

$$F(|\phi\rangle \& |\Phi\rangle) = \langle\phi|\Phi\rangle \tag{2}$$

Here $F(|\phi\rangle \& |\Phi\rangle)$ is the fidelity of states $|\phi\rangle \& |\Phi\rangle$. The maximum value of fidelity measurement is equal to 1 which is also represented as 100%.

III. METHODOLOGY AND RESULTS

The proposed asymmetric quantum error correction architecture is capable of finding both bit and phase flip errors. But it mainly concentrates on the phase flips because of its higher probability of occurrence while transmitting the information. For this three steps are to be performed: Encoding, Error detection and correction using syndrome measurement and

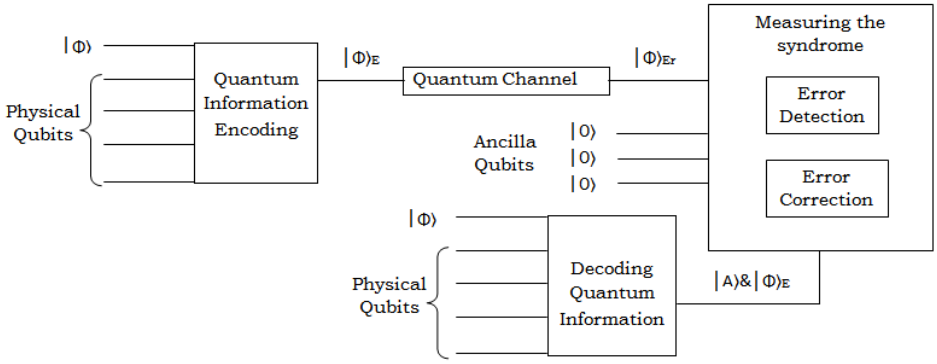

FIGURE 2. **Asymmetric quantum error correction with encoding, syndrome measurement and decoding procedure.**

Decoding. The overall architecture of proposed method is represented in figure 2.

From the figure, we can observe that to perform Quantum error correction, initially an encoding operation is performed. In encoding process, additional redundancy will be added in advance to handle the noise with extra qubits and quantum operations. After encoding the information, it will be transmitted through quantum channel and if any error occurs then that will be measured using syndrome measurement with bounding function and finally after detecting and correcting the error it will be decoded by performing decoding operations. The detailed discussion about each step with Quantum results are as follows

A. ENCODING

To perform QEC, a single logical qubit is encoded into 5 physical qubits. In existing QEC methods, Hadamard(H) and phase shift(Z) gates are used for encoding the information. But in proposed method, single X gate is used to perform the HZH operations. To encode the quantum information, X, S, U and CNOT gates are applied on input data. The detailed algorithm for encoding quantum information is represented in algorithm 1.

Initially a single logical qubit $|\phi\rangle$ is considered for transmitting the information. But to strengthen the logical information from noise,the single logical qubit will be encoded into 5 physical qubits $|\phi\rangle_E$. To do so, S gate with angle $\pi/2$ has been applied on initial qubit and that rotates the quantum state along with the Z-axis with $\pi/2$ angle and then the unitary operation with angle $\pi/4$ has been applied on first qubit q[0] to perform the rotation over X, Y and Z-axis with $\pi/4$ angle to prevent from the phase shifts. The combination of S and U gates leads to the U3 gate with the rotation along with X, Y and Z gates with $3\pi/4$, $\pi/2$ and $\pi/2$ angles respectively. With these rotations the phase of the quantum state will be maintained. The experiment is performed on quantum system. In this the default input will be considered as $|0\rangle$. But the phase shifts cannot be measured for $|0\rangle$ state because the phase shift on state $|0\rangle \rightarrow |0\rangle$ but for state $|1\rangle \rightarrow -|1\rangle$. Thus X gate is used to represent the input as $|1\rangle$. After performing phase and unitary operations on qubit q[0],

Algorithm 1 Procedure for Encoding the Quantum Information

Input: Quantum Information $|\phi\rangle$
Output: Encoded Quantum Information $|\phi\rangle_E$

Initialize the number of qubits n as 5
Initialize Quantum register q and Classical register c with size 5
Apply NOT operation on initial qubit
Apply S and U operations along with X,Y and Z axis
for $i = 0$ to $n - 2$ **do**
    Perform CNOT operation on q[i] & q[i+1]
end for
for $i = 0$ to $n - 1$ **do**
    Store the information from quantum register to Classical register
end for
Measure the information stored in classical register

CNOT operations will be performed on q[0] & q[1], q[1] & q[2], q[2] & q[3], q[3] & q[4] to encode it into 5 qubits. After running the algorithm 1 in quantum computer, the retrieved result for logical input $|1\rangle$ is shown in figure 3 and the resulted quantum states of the proposed method is represented with the density matrix as in figure 4.

Density matrix is used to represent all the possible quantum states of the system and mathematically it is simplified by using the equation 1. For example, the possible outcomes of single qubit system will be either $|0\rangle$ or $|1\rangle$. To represent all possible quantum states, the equation 1 is simplified with 2^1 states as follows

$$\rho = P_1|0\rangle\langle 0| + P_2|1\rangle\langle 1|$$

The vector simplification of above equation results in the density matrix with all possible states with $2^1 \times 2^1$ matrix with probabilities P_1, P_2. Similarly for n qubit system, the density matrix of all possible outcomes will be of size $2^n \times 2^n$. The calculated density matrix of proposed encoding method with 5-qubit system with 32×32 matrix is graphically represented as in figure 4.

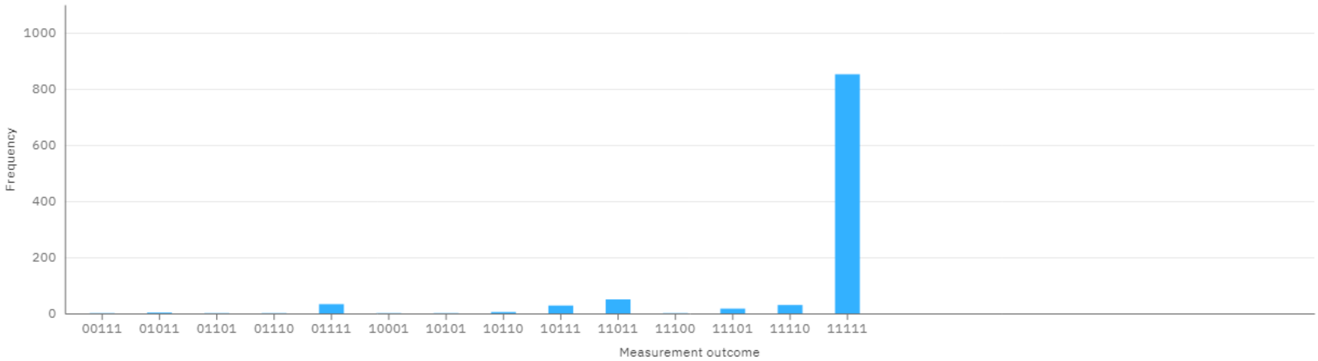

FIGURE 3. **Quantum results for Algorithm 1: Encoding quantum information.**

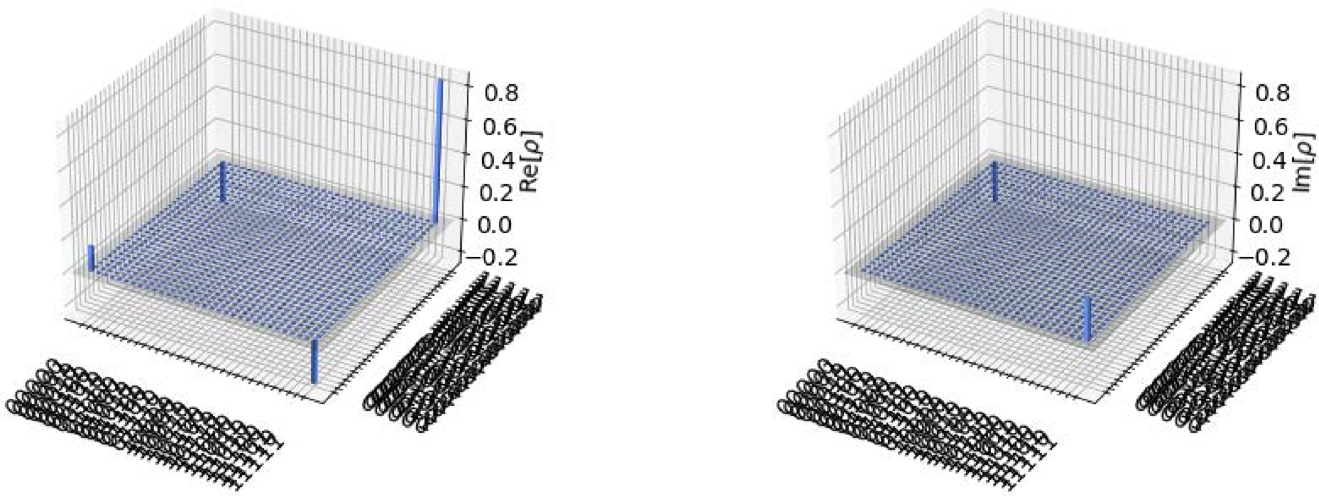

FIGURE 4. **Density matrix of encoding method with all possible states with real and imaginary values Re[ρ], Im[ρ] respectively.**

The error which is produced due to noise is measured by performing Positive Operator Value Measures(POVM's) and is expressed as

$$E_{rr} = (1 - P)|\phi_a\rangle\langle\phi_a| + P|\phi_b\rangle\langle\phi_b| \qquad (3)$$

Here E_{rr} is Error measurement, P is probability of error. The error measurement for the proposed encoding method is

$$E_{rr} = (1 - P)|00000\rangle\langle 00000| + P|11111\rangle\langle 11111|$$

After running it in quantum computer the probability of state $|11111\rangle\langle 11111|$ is 0.852 i.e., P = 0.852 from this we can calculate the error value i.e., $1 - \mathrm{P} = 0.14$. The same result is expressed using the density matrix represented in figure 4.

After transmitting the encoded information, the next step is to perform the error detection and correction.

B. ERROR DETECTION AND CORRECTION

Before performing the error correction it is important to know that due to which error the encoded state has corrupted. This can be done by performing syndrome measurement. In classical error correction, syndrome measurement is used to find which error occurred on which input state based on that we can apply the same error to correct it. Similarly in quantum error correction, the syndrome measurement is used to retrieve the error information like which pauli error(X, Y or Z) has occurred and on which physical qubit. Based on this information, the same pauli operator will be applied on corrupted qubit to revert the error effect.

1) SYNDROME MEASUREMENT AND ERROR CORRECTION

To measure the syndrome, the initial 5-qubits will not be sufficient. We need extra qubits to detect and correct the error. For that 3-ancilla qubits are used $|A\rangle$. Hence total 8-qubit operations will be performed with CNOT operations. Ancilla qubits are extra qubits which are used to perform specific operations in quantum computation. Mainly these are used to convert the complicated gate architectures in to simple ones. In proposed method, the combination of ancilla qubits are used to detect the error on exactly which qubit it has occurred and then the operations are performed to correct it. For this we have considered hamming bound as base and then it has been modified by considering phase flips with highest probability.

2) BOUNDING FUNCTION CALCULATION

To measure the syndrome, hamming bound with [n,k,d] parameters has considered. Where n is the number of physical

Algorithm 2 Step by Step Procedure for Syndrome Measurement

Input: Encoded Quantum Information transmitted through quantum channel $|\phi\rangle_{Er}$, Ancilla qubits $|A\rangle$

Output: Encoded information along with the syndrome measurement $|A\rangle$ & $|\phi\rangle_E$

Initialize number of qubits n as 8
Initialize Quantum register q, Classical register c with size 5
Initialize Ancilla Register a with size 3
$q[0] \leftarrow Z$
$q[0]\&q[5] \leftarrow$ Controlled NOT operation
$q[1]\&q[6] \leftarrow$ Controlled NOT operation
$q[2]\&q[5] \leftarrow$ Controlled NOT operation
$q[2]\&q[6] \leftarrow$ Controlled NOT operation
$q[3]\&q[7] \leftarrow$ Controlled NOT operation
$q[4]\&q[5] \leftarrow$ Controlled NOT operation
$q[4]\&q[7] \leftarrow$ Controlled NOT operation
for $i = 0$ to $n/3$ **do**
    Store the information from quantum register to Ancilla register
end for
Measure the information stored in ancilla register
Initialize A_r with the the combination of ancilla qubits
if $A_r = 0$ **then**
    Append Z gate on initial qubit
end if
for $i = 0$ to n **do**
    Store the information from Quantum register to Classical register
end for
Measure the information stored in classical register

qubits that are used to represent the k number of logical qubits with code distance of d. The bounding function can be expressed as $\sum_{w=0}^{d} \binom{n}{w} d^w$. In proposed method the code distance considered is 3. Hence the final parameters are [5,1,3]. With these parameters the bounding function can be expressed as $\sum_{w=0}^{d} \binom{n}{w} 3^w$. To apply this bounding function, the minimum n value will be 5. After encoding the data, if any syndrome is occurred while transmitting the information can be measured and corrected with the help of the bounding function and CNOT operations. The detailed algorithm for syndrome measurement with error correction is described in algorithm 2.

After running the algorithm 2 in Quantum simulator with phase error at initial qubit q[0], the obtained results are represented in figure 5.

Figure 5 shows that, the first 5 qubits (right to left) represents the encoded information and the remaining 3 qubits represents the ancilla qubits. To check the efficiency of the proposed algorithm, a phase error has been added at the first qubit of the encoded data. After running the algorithm

Algorithm 3 Procedure for Decoding the Quantum Information

Input: Encoded information along with the syndrome measurement $|A\rangle$ & $|\phi\rangle_E$

Output: The original information that has been shared initially $|\phi\rangle$

Initialize the number of qubits n as 5
Initialize Quantum register q, Classical register c with size 5
for $i = n - 1$ to 0 **do**
    Perform CNOT operation on q[i] & q[i-1]
end for
$q[0] \leftarrow$ Unitary operation
$q[0] \leftarrow S$
for $i = 0$ to $n - 1$ **do**
    Store the information from Quantum register to Classical register
end for
Measure the information stored in Classical register

the retrieved result with quantum state $|00111111\rangle$ represents that the error has been occurred at first qubit q[0] with ancilla combination 001. By performing CNOT operations the error will be detected and corrected. The fidelity of obtained result is 84.28%. Due to the noise, the state $|00000000\rangle$ is obtained. The error calculation for syndrome measurement using the equation 3 is 0.15 i.e.15%. After performing the error detection and correction,the next step is to decode the original information.

C. DECODING

To decode the information, the encoding operations are performed in backward direction. After encoding the information if any syndrome arises in between then that will be measured by using syndrome calculation and will be corrected. After that the actual data will be decoded using the decoding operations as explained in algorithm 3.

After running the algorithm 3 in quantum system, the retrieved results are outlined in figure 6.

Initially the input considered is $|00001\rangle$. After performing the encoding operations, it has been converted into $|11111\rangle$. By performing decoding operations we obtained the same state as an output with the efficiency of 883 times.The figure 7 shows the density matrix for the decoding algorithm.

From the density matrix, we can observe that the results are giving 88.3% accuracy and 11.7% is reduced due to the noise while operating with the qubits. The error measurement for decoding using the equation 3 is 0.117. The result of final Quantum Error Correction algorithm by combining encoding, syndrome calculation and decoding algorithms and the comparison with existing works is discussed clearly in following section.

IV. DISCUSSION AND ANALYSIS

The proposed asymmetric error correction method is used to transmit the single logical qubit information using 5-physical

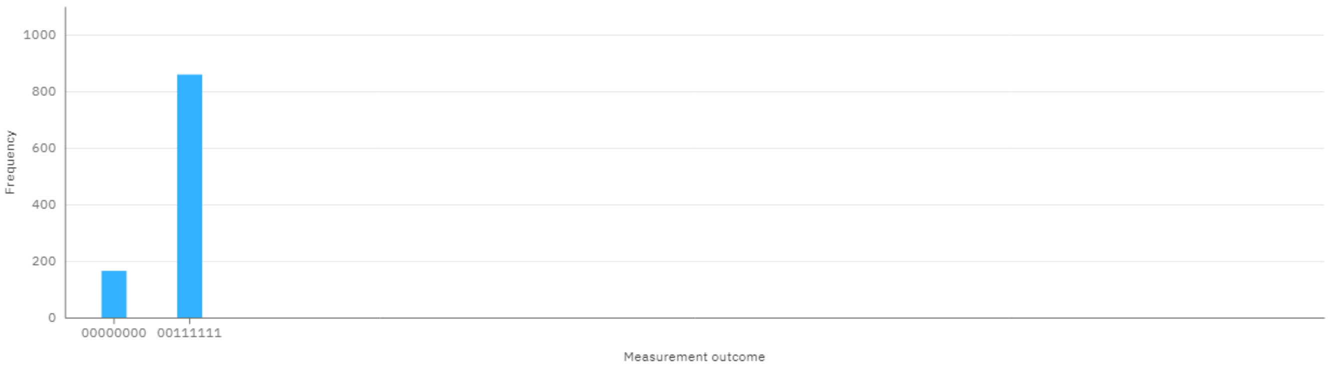

FIGURE 5. **Quantum results for Algorithm 3: Syndrome measurements.**

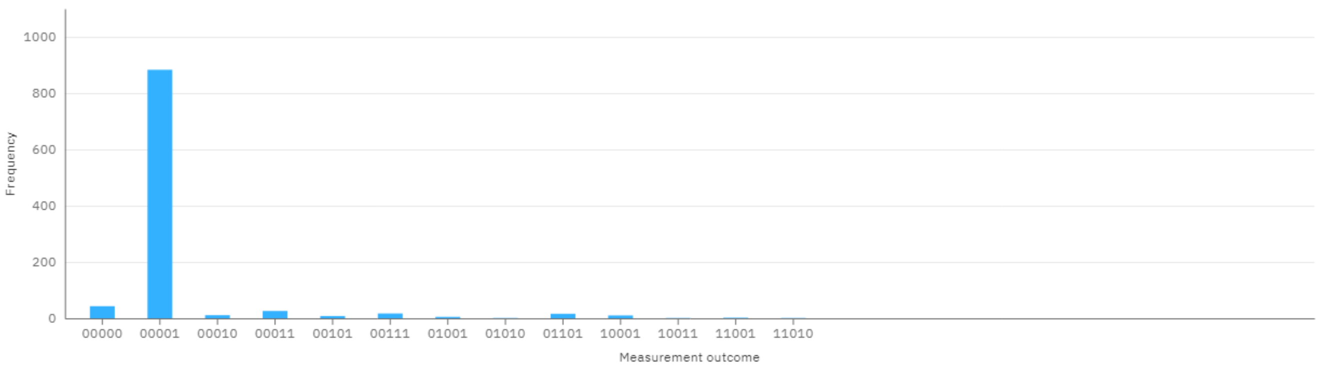

FIGURE 6. **Quantum results for Algorithm 2: Decoding the quantum information.**

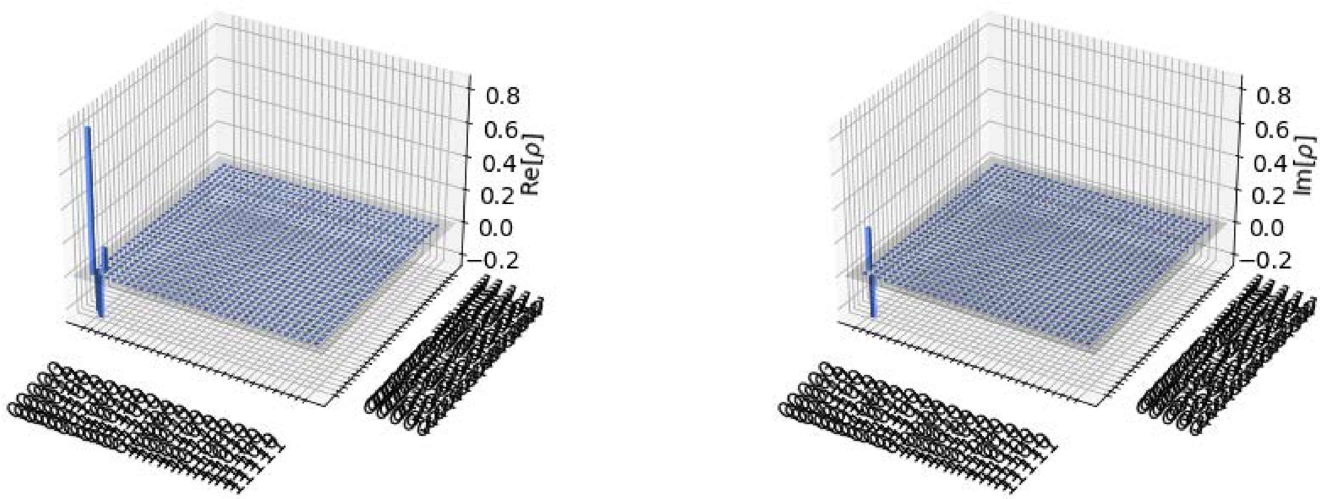

FIGURE 7. **Density matrix of decoding method with all possible states with real and imaginary values Re[ρ], Im[ρ] respectively.**

qubits and 3-ancilla qubits. The overall algorithm of QEC contains the encoding, syndrome measurement and decoding operations. After running all these operations combinedly, the final output is as in figure 8.

For the QEC algorithm, a logical qubit $|00001\rangle$ is given as an input. To transmit this information, the encoding operation has been performed by adding extra physical qubits to strengthen the quantum information. After this operation, the output will be $|11111\rangle$. While transmitting the encoded information if any syndromes are arises then those can be detected and corrected by performing syndrome measurement operations. The resulted state after performing this operation is $|00111110\rangle$, where it detects the syndrome at qubit q[0]. After detecting the error it will be corrected by performing CNOT operations, the results of this is $|00111111\rangle$ where the error at first qubit has been corrected. Finally the data will be decoded by performing the decoding operations. The final output after this stage with fidelity

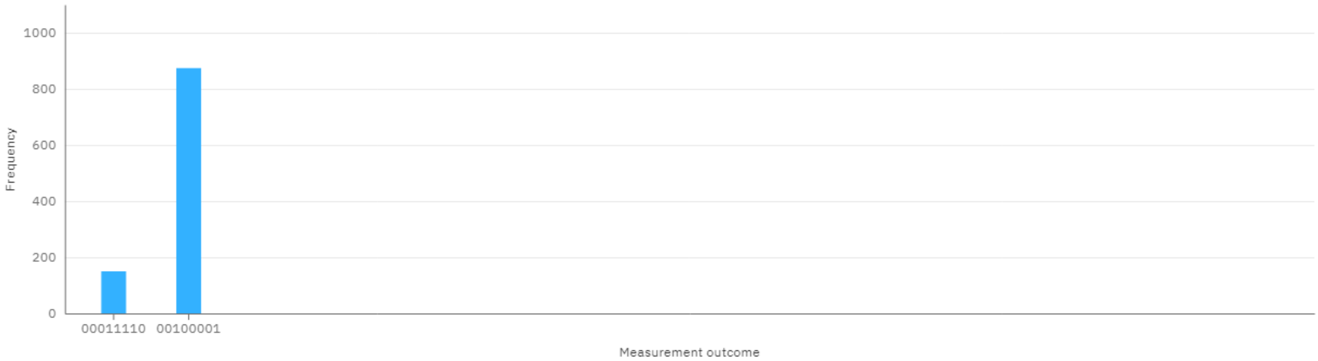

FIGURE 8. Quantum results for the proposed asymmetric quantum error correction method.

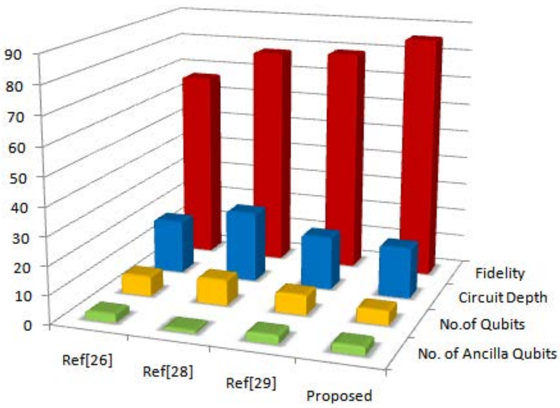

FIGURE 9. Result analysis of proposed AQEC with existing methods.

85.89% is $|00100001\rangle$ which represents that the first 5 qubits as quantum information and the remaining 3 qubits as ancilla qubits. The retrieved results are efficient in terms of fidelity, number of ancilla qubits and circuit depth. Circuit depth is used to represent the number of quantum gates used in the longest path of a quantum operation. The circuit depth and the fidelity of the quantum operations are inversely proportional to each other i.e., if the circuit depth is reduced then the fidelity will increases. The proposed QEC method produces an efficient result compared to the existing methods as represented in figure 9.

From the figure, it is observed that the proposed QEC model improves the fidelity of the expected outcome with less number of ancilla qubits and with reduced circuit depth.

V. CONCLUSION

The proposed asymmetric quantum error correction method detects and corrects both bit and phase flip errors. It works efficiently on phase flip errors when compared with bit flip errors. Because the phase flips are much more probable than qubit flips. To measure these syndromes, bounding function has been used. It not only detects the error but also finds exactly on which qubit the error has occurred. With this it will be easy to correct the error by applying the same error again. The major challenge observed here is the decoherence and noise that will be generated while executing the algorithms in quantum systems. The proposed work performs efficiently to overcome these issues. With the unavailability of universal quantum system, the performance of new methods may vary.

The proposed work has implemented on IBM quantum system using Qiskit tool, it is observed that the results are efficient compared to the existing methods. With the presented AQEC method, the fidelity is increased to 85.89% with less number of ancilla qubits, error rate and circuit depth are reduced to 14% and 48%. This method also helpful for the applications where the secure data transmission is required with less error rate and it also proficient in various applications like Quantum key distribution, Quantum teleportation, Quantum random number generator, etc. In future the advantage of entanglement can be applied along with the asymmetric quantum error correction to get better results.

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