Received March 15, 2022, accepted April 18, 2022, date of publication April 25, 2022, date of current version May 6, 2022.
Digital Object Identifier 10.1109/ACCESS.2022.3170033

Efficient User Subset Selection for Multiuser Space-Time Line Code Systems

SANGCHOON KIM
Department of Electronics, Dong-A University, Busan 49315, South Korea
e-mail: sckim@dau.ac.kr

This work was supported by the research funds from the Dong-A University.

ABSTRACT This paper considers the problem of user subset (US) selection for minimizing the bit error rate (BER) of multi-user space-time line code (MU-STLC) multiple-input multiple-output systems with fairness-aware per-user power allocation. The optimal selection criterion suitable for MU-STLC transmissions based on zero forcing (ZF) precoding is given and two efficient algorithms are then proposed. First, an incremental search approach is presented for US selection in the MU-STLC systems. The proposed suboptimal solution to BER minimization starts an empty US and adds users one by one, where the low-complexity recursive computation of the block matrix inverse is further performed. Second, by avoiding recurring matrix computations in each incremental procedure of the first algorithm, a more efficient algorithm is developed. It is observed through simulation results that the proposed incremental-based algorithms achieve most US selection gains with very low complexity. In addition, it is demonstrated that when there are N_T transmit antennas, U users (each user has 2 receive antennas), and K selected users, the achievable upper diversity order of the ZF precoding-based MU-STLC systems with optimal US selection is given as $2(N_T - K + 1)(U - K + 1)$. The analytical diversity order is well-matched with simulation results.

INDEX TERMS User selection, multiple input multiple output (MIMO), precoding, multiuser diversity, space-time line code.

I. INTRODUCTION

Recently, a space-time line code (STLC) in [1] has been designed as a novel transmission approach. In a STLC scheme achieving a full spatial diversity gain, two information symbols are encoded by channel gains coming from multiple receive antennas and are sent consecutively in time [1]–[3]. It requires the perfect knowledge of the full channel state information (CSI) at the transmitter and utilizes a simple STLC combining structure without CSI at the receiver. Furthermore, a multi-user (MU) STLC system, which can simultaneously deliver multiple STLC streams to multiple users, has been proposed for multiple-input multiple-output (MIMO) downlink transmissions [4], [5]. By employing a zero-forcing (ZF) precoder, the MU-STLC system with fairness-aware per-user (FAPU) power allocation has been shown to be capable of offering near optimal performance in terms of the sum achievable rate.

In wireless communication systems, the number of downlink multiple users (U) is often larger than the number of transmit antennas (N_T) and/or the number of users that can be served at the same time. Then, the base station has to perform user subset (US) selection based on the CSI of all the available users in an MU MIMO communication system where ZF-based precoding is employed in downlink transmissions [6]–[15]. The optimal US selection can be performed by using an exhaustive search, but due to its impractical computational complexity, the development of efficient suboptimal selection algorithms has drawn great attention. Several suboptimal US selection algorithms fall into two categories such as incremental search-based algorithms and decremental-based algorithms. In [7], suboptimal greedy US selection schemes based on ZF dirty-paper (DP) precoding and simple ZF beamforming without DP coding have been considered for wireless broadcast channels. Since the linear ZF precoding has many advantages over DP coding

The associate editor coordinating the review of this manuscript and approving it for publication was Jie Tang.

in practical systems, much of the existing US selection algorithms are based on ZF precoding. In [8] and [9], a semi-orthogonal US (SUS) selection algorithm has been developed under a ZF beamforming strategy. Low-complexity SUS incrementally selects a new user with the largest effective channel norm that is nearly orthogonal to the channels of the other selected users. In [11], a decremental US selection algorithm based on ZF beamforming has been proposed when the number of users is less than the number of transmit antennas. In [12], a novel greedy US selection procedure with swap has been presented.

Although various US selection schemes have been presented for MU MIMO transmissions, they are unsuitable for the MU-STLC MIMO systems owing to different transmission methods. To the best of our knowledge, no study has been previously made for US selection in the MU-STLC systems. Moreover, previous works on US selection for different MIMO systems have mainly focused on the problem of achieving the maximum sum rate. Although the sum achievable rate could be employed as an US selection criterion in terms of information-theoretic point of view, it would be more practical to use a minimum error probability for a given modulation scheme and a MIMO detector [16], [17]. It should be noted that one key problem in US selection is to optimally design a proper selection criterion.

In this paper, an US selection criterion suitable to the ZF-based MU-STLC MIMO systems with FAPU power allocation is first presented to minimize the bit error rate (BER) performance. It has been shown in [4] that the FAPU power allocation in the MU-STLC systems is able to offer fairness in terms of allocated power as well as low-complexity power allocation. Thus this work assumes a simple FAPU power allocation. Efficient incremental search-based suboptimal algorithms are proposed to greatly reduce the computational complexity of the optimal US selection algorithm. Furthermore, the original SUS selection algorithm of [8] with low-complexity is not suitable for the MU-STLC transmission systems. It should be adapted for use by MU- STLC systems. Therefore, we present a modified SUS algorithm with lower complexity for MU-STLC, which is simulated as a benchmark for comparison with the proposed incremental-based US selection algorithms. By exploring an analytical bound on pairwise error probability (PEP), the achievable diversity order is derived. We demonstrate that the ZF-based MU-STLC MIMO systems with FAPU power allocation can achieve an upper diversity order of $2(N_T - K + 1)(U - K + 1)$, where K users are selected, which simultaneously achieves both receive/transmit and MU diversity. Simulation results verify that the achievable diversity order matches well with the analytical one. The analytical and numerical results show that the proposed incremental US selection strategy can efficiently decrease the computational complexity to yield an US.

The main contributions of this study are summarized as follows:

- The efficient US selection algorithm based on the incremental search approach is developed for the MU-STLC systems. Another proposed incremental US selection can reduce efficiently the computational complexity by exploiting the orthogonal STLC encoding structure as well as the recursive block matrix inversion operations. To our knowledge, the proposed efficient US selection algorithms are the first efforts in the MU-STLC systems to provide a low computational complexity.
- The computational complexity of the proposed US selection algorithms is analyzed and compared to the optimal and conventional SUS selection schemes. The complexity comparison proves the efficiency of the proposed algorithms.
- The overall diversity order achieved in the ZF-based MU-STLC system with optimal US selection is analytically provided and verified by simulation results.

The remainder of this paper is organized as follows. In Section II, a system model for the MU-STLC transmission with US selection, based on the ZF precoder, is briefly presented. In Section III, three efficient US selection algorithms for MU-STLC systems are presented together with the computational complexity analysis. The achievable diversity order is analyzed in Section IV. The simulation results are presented in Section V. Finally, some conclusions are drawn in Section VI.

Notation: Throughout this paper, the boldface lower-case and upper-case letters represent the vectors and matrices, respectively. We use the superscripts $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^H$ to denote the complex conjugate, the transpose, and the Hermitian transpose operations, respectively. $tr(\cdot)$ and $(\cdot)^{-1}$ represent the trace operation and inverse operation, respectively. $E[\cdot]$, $\|\cdot\|$, and $\|\cdot\|_F$ denote the statistical mean, the Euclidean norm, and the Frobenius norm, respectively. $\mathbf{I}_{n\times n}$, $\mathbf{0}_{n\times m}$, $|\cdot|$, and $Q(\cdot)$ are the $n \times n$ identity matrix, the $n \times m$ zero matrix, the absolute value, and the Q function, respectively. In addition, $\mathbf{X}(k : k+1, :)$ denotes the k-th and $(k+1)$-th two row vectors of matrix $\mathbf{X}$. $\mathbf{X}([1 : (2k-2)\ (2k+1) : end], :)$ represents the remaining submatrix obtained by deleting the $(2k-1)$-th and $2k$-th two row vectors in matrix $\mathbf{X}$.

II. SYSTEM MODEL OF MU-STLC WITH US SELECTION

We consider a downlink MU-STLC time division duplex (TDD) system, which has N_T transmit antennas and U users. Each user has two receive antennas for STLC [1]–[6] as shown in Fig. 1. In this work, it is assumed that $K(\leq U)$ users are selected from the U users. Let $x_{k,t}$ be the t-th transmitted symbol of the k-th user, with $E[x_{k,t}x_{k,t}^*] = \sigma_x^2$. Then the MU-STLC signal matrix is defined as

$$\mathbf{U}_S = [\mathbf{u}_1 \quad \mathbf{u}_2] \triangleq \mathbf{W}_S\mathbf{X} \in C^{N_T\times 2} \tag{1}$$

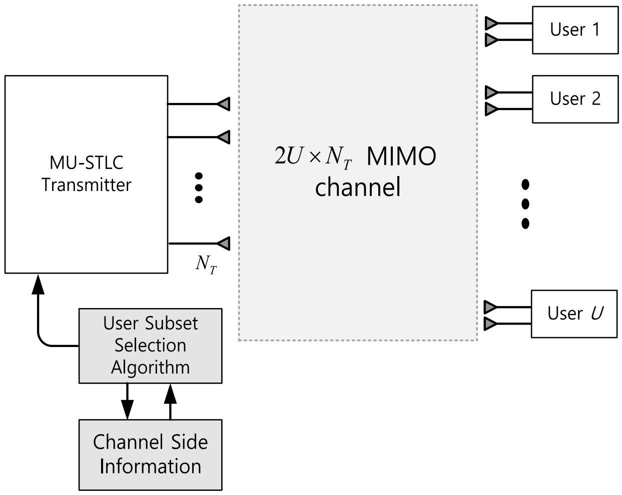

FIGURE 1. Block diagram of an MU-STLC TDD system with user subset selection.

where $\mathbf{u}_t = [u_{1,t}\ u_{2,t} \cdots u_{N_T,t}]^T \in C^{N_T \times 1}, t = 1, 2$, with $u_{b,t}, b = 1, 2, \cdots, N_T$, denoting the (b, t)-th element of the matrix $\mathbf{W}_S\mathbf{X}$ and

$$\mathbf{X} = [\mathbf{X}_1 \quad \mathbf{X}_2 \quad \cdots \quad \mathbf{X}_K]^T \in C^{2K \times 2} \tag{2}$$

$$\mathbf{X}_k = \begin{bmatrix} x_{k,1} & x_{k,2} \\ -x_{k,2}^* & x_{k,1}^* \end{bmatrix} \in C^{2 \times 2} \tag{3}$$

and $\mathbf{W}_S \in C^{N_T \times 2K}$ is the MU-STLC precoding matrix for all users such that $\|\mathbf{W}_S\|_F^2 = 1$.

The received signals with US selection are represented as

$$[\mathbf{r}_1 \quad \mathbf{r}_2] = \mathbf{H}_S\mathbf{U}_S + \mathbf{Z} \in C^{2K \times 2} \tag{4}$$

where $\mathbf{r}_t = \left[\mathbf{r}_{1,t}^T\ \mathbf{r}_{2,t}^T\ \cdots\ \mathbf{r}_{K,t}^T\right]^T \in C^{2K \times 1}$. Here $\mathbf{r}_{k,t} = \left[r_{k,t}^1\ r_{k,t}^2\right]^T \in C^{2 \times 1}$ is the received signal vector where $r_{k,t}^a$ is the received signal at the a-th receive antenna of the k-th user at time t. $\mathbf{H}_S \in C^{2K \times N_T}$ denotes the channel submatrix obtained by selecting K users from the full channel matrix $\mathbf{H} = [\mathbf{H}_1^T\ \mathbf{H}_2^T\ \cdots\ \mathbf{H}_U^T]^T \in C^{2U \times N_T}$, which is assumed to be perfectly known at the transmitter. In TDD mode, channel reciprocity between uplink and downlink channels can be exploited to estimate the CSI by using the pilot/training signals from all the available users [4], [18]. Here $\mathbf{H}_u \in C^{2 \times N_T}$, $u = 1, 2, \cdots, U$, is a channel matrix between all transmit antennas and each user, which is static for $t = 1$ and $t = 2$, and whose elements are independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian random variables with zero mean and unit variance. $\mathbf{Z} = [\mathbf{Z}_1\ \mathbf{Z}_2\ \cdots\ \mathbf{Z}_K]^T \in C^{2K \times 2}$ with $\mathbf{Z}_k = [\mathbf{z}_1\ \mathbf{z}_2] \in C^{2 \times 2}$ and $\mathbf{z}_t = [z_{1,t}\ z_{2,t}]^T \in C^{2 \times 1}$, $t = 1, 2$, is an i.i.d. additive white Gaussian noise (AWGN) matrix whose elements are the zero-mean circular complex white Gaussian noise component of a variance of σ_z^2.

For the MU-STLC decoding, the received signal matrix of (4) is re-expressed in a linear form as [5]

$$\mathbf{r} \triangleq \left[\mathbf{r}_1^T \quad \mathbf{r}_2^H\right]^T = \begin{bmatrix} \mathbf{H}_S\mathbf{W}_S \\ (\mathbf{H}_S\mathbf{W}_S)^* \mathbf{Q}_{2K} \end{bmatrix} \mathbf{x} + \mathbf{z} \in C^{4K \times 1} \tag{5}$$

where

$$\mathbf{x} \triangleq [x_{1,1} \quad x_{1,2} \quad \cdots \quad x_{k,1} \quad x_{k,2} \quad \cdots \quad x_{K,1} \quad x_{1k,2}]^T \tag{6}$$

$$\mathbf{Q}_{2K} = blkdiag\{\mathbf{Q}_2, \cdots, \mathbf{Q}_2\} \in R^{2K \times 2K} \tag{7}$$

$$\mathbf{Q}_2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \tag{8}$$

and $\mathbf{z} \in C^{4K \times 1}$ is the AWGN vector with $E[\mathbf{z}\,\mathbf{z}^H] = \sigma_z^2\mathbf{I}_{4K}$.

By the simple STLC combining procedure at the receiver, which is described in [5], user k conducts STLC combining as $\mathbf{r}_{k,1} + \mathbf{Q}_2^T\mathbf{r}_{k,2}^*$. Therefore the MU combined-STLC received signal vector can be given as

$$\mathbf{y} = \left[\mathbf{I}_{2K \times 2K} \quad \mathbf{Q}_{2K}^T\right]\mathbf{r} = \tilde{\mathbf{H}}_S\tilde{\mathbf{W}}_S\mathbf{x} + \mathbf{z}' \in C^{2K \times 1} \tag{9}$$

where the expression (5) is used and

$$\tilde{\mathbf{H}}_S \triangleq \left[\mathbf{H}_S \quad \mathbf{Q}_{2K}^T\mathbf{H}_S^*\right] \in C^{2K \times 2N_T} \tag{10}$$

$$\tilde{\mathbf{W}}_S \triangleq \left[\mathbf{W}_S^T \quad \mathbf{Q}_{2K}^T\mathbf{W}_S^H\right]^T \in C^{2N_T \times 2K} \tag{11}$$

and the combined AWGN vector $\mathbf{z}' \in C^{2K \times 1}$ follows the distribution $\mathcal{CN}(\mathbf{0}_{2K \times 2K}, 2\sigma_z^2\mathbf{I}_{2K \times 2K})$. Here the MU-STLC ZF precoding matrix $\tilde{\mathbf{W}}_S$ can be given by

$$\tilde{\mathbf{W}}_S = \tilde{\beta}_S\tilde{\mathbf{H}}_S^H(\tilde{\mathbf{H}}_S\tilde{\mathbf{H}}_S^H)^{-1} \tag{12}$$

where the power normalization factor related with the selected US is given as

$$\tilde{\beta}_S = \sqrt{\frac{1}{tr\left[(\tilde{\mathbf{H}}_S\tilde{\mathbf{H}}_S^H)^{-1}\right]}} \tag{13}$$

III. US SELECTION ALGORITHMS

In this section, we present optimal and two suboptimal incremental US selection algorithms for the ZF-precoded MU-STLC systems. It can be easily shown from (9) that maximizing the received signal-to-noise ratio (SNR) of the ZF-precoded MU-STLC systems is equivalent to maximizing the term $\tilde{\beta}_S^2$ of (13). It is pointed out that the US selection design objective of max $\tilde{\beta}_S^2$ is identical to minimizing the error probability from the PEP expression (55), which will be presented in section IV. Then the optimal US selection algorithm for the ZF-precoded MU-STLC system can be described as

$$S_{opt} = \underset{S \in \{S_n, n=1,2,\cdots,C_K^U\}}{\arg\min} \; tr\left[(\tilde{\mathbf{H}}_S\tilde{\mathbf{H}}_S^H)^{-1}\right] \tag{14}$$

where S_n is the n-th enumeration of the set of all available USs. Here C_K^U is the total number of combinations of selecting K users out of U available users. Obviously, the exhaustive search algorithm to solve (14) requires C_K^U matrix inverse operations, whose computational complexity is huge, especially when the number of all the possible USs is large. To evaluate its computational complexity, we take account of the number of real multiplications (RMs) and the number of real summations (RSs). Here a complex multiplication corresponds to 4 RMs and 2 RSs whereas a complex summation employs 2 RSs. By performing the similar computational complexity analysis used in [6], [19]–[23], they are given as

$$N_{optimal}^{RM} = C_K^U(16N_TK^2 + 8N_TK + 16K^3 + 24K^2) \quad (15)$$

$$N_{optimal}^{RS} = C_K^U(16N_TK^2 + 8N_TK + 16K^3 + 24K^2 - 2K) \quad (16)$$

It should be pointed out that US selection algorithms are closely coupled with an employed transmission method. The conventional low-complexity US selection algorithms are based on the channel $\mathbf{H}_S$(or $\mathbf{H}$), not $\tilde{\mathbf{H}}_S$(or $\tilde{\mathbf{H}}$) and thus unsuitable for US selection in the MU-STLC systems. Because of the difference between two channel matrices of $\mathbf{H}_S \in C^{2K\times N_T}$ and $\tilde{\mathbf{H}}_S \in C^{2K\times 2N_T}$, they are inappropriate for direct use in the MU-STLC systems. The succeeding efficient US selection algorithm (Algorithm 5 in subsection III.D) designed for the MU-STLC systems by effectively utilizing the orthogonal STLC encoding structure of the definition (10) in this work is distinct from the conventional US selection algorithms.

Algorithm 1 Original SUS Selection

Inputs: $\mathbf{H}$, U, K
1: $\mathbf{\Psi} = \mathbf{0}_{2U\times N_T}$
2: for $i = 1 : K$
3:   for $j = 1 : (U - i + 1)$
4:     $\mathbf{T} = \mathbf{0}_{N_T\times N_T}$
5:     for $p = 1 : (i - 1)$
6:       $\mathbf{\Theta} = \mathbf{\Psi}(2p - 1 : 2p, :)$
7:       $\mathbf{T} = \mathbf{T} + \mathbf{\Theta}^H(\mathbf{\Theta}\mathbf{\Theta}^H)^{-1}\mathbf{\Theta}$
8:     end
9:     $\mathbf{F}(2j - 1 : 2j, :) = \mathbf{H}(2j - 1 : 2j, :)(\mathbf{I}_{N_T\times N_T} - \mathbf{T})$
10:     $\alpha_j = tr(\mathbf{F}(2j - 1 : 2j, :)\mathbf{F}^H(2j - 1 : 2j, :))$
11:   end
12:   $v(i) = \arg\max\{\alpha_1, \alpha_2, \cdots, \alpha_{U-i+1}\}$
13:   $\mathbf{H}_S(2i - 1 : 2i, :) = \mathbf{H}(2v(i) - 1 : 2v(i), :)$
14:   $\mathbf{\Psi}(2i - 1 : 2i, :) = \mathbf{F}(2v(i) - 1 : 2v(i), :)$
15:   $\mathbf{H} = \mathbf{H}([1 : 2v(i) - 2 2v(i) + 1 : \text{end}], :)$
16: end
Output: $\mathbf{H}_S$

A. ORIGINAL SUS SELECTION ALGORITHM

The semi-orthogonal US selection method with the full channel matrix $\mathbf{H} \in C^{2U\times N_T}$ originally proposed in [8]

Algorithm 2 SUS Selection for MU-STLC

Inputs: $\mathbf{H}$, $\mathbf{Q}_{2U}$, U, K
$\tilde{\mathbf{H}} = \left[\mathbf{H}\ \mathbf{Q}_{2U}^T\mathbf{H}^*\right]$
$\tilde{\mathbf{\Psi}} = \mathbf{0}_{2U\times 2N_T}$
for $i = 1 : K$
  for $j = 1 : (U - i + 1)$
    $\tilde{\mathbf{T}} = \mathbf{0}_{2N_T\times 2N_T}$
    for $p = 1 : (i - 1)$
      $\tilde{\mathbf{\Theta}} = \tilde{\mathbf{\Psi}}(2p - 1 : 2p, :)$
      $\tilde{\mathbf{T}} = \tilde{\mathbf{T}} + \tilde{\mathbf{\Theta}}^H(\tilde{\mathbf{\Theta}}\tilde{\mathbf{\Theta}}^H)^{-1}\tilde{\mathbf{\Theta}}$
    end
    $\tilde{\mathbf{F}}(2j - 1 : 2j, :) = \tilde{\mathbf{H}}(2j - 1 : 2j, :)(\mathbf{I}_{2N_T\times 2N_T} - \tilde{\mathbf{T}})$
    $\alpha_j = tr(\tilde{\mathbf{F}}(2j - 1 : 2j, :)\tilde{\mathbf{F}}^H(2j - 1 : 2j, :))$
  end
  $v(i) = \arg\max\{\alpha_1, \alpha_2, \cdots, \alpha_{U-i+1}\}$
  $\mathbf{H}_S(2i - 1 : 2i, :) = \mathbf{H}(2v(i) - 1 : 2v(i), :)$
  $\tilde{\mathbf{\Psi}}(2i - 1 : 2i, :) = \tilde{\mathbf{F}}(2v(i) - 1 : 2v(i), :)$
  $\tilde{\mathbf{H}} = \tilde{\mathbf{H}}([1 : 2v(i) - 2\ 2v(i) + 1 : \text{end}], :)$
end
Output: $\mathbf{H}_S$

can be described as Algorithm 1, termed as SUS. It can incrementally find K best users with large channel gains that have also a good level of orthogonality. In the i-th iteration, the components of channel vectors, $\{\mathbf{H}(2j - 1, :),\ \mathbf{H}(2j, :)\}$, of the j-th user orthogonal to the subspace spanned by $\{\mathbf{\Psi}(1, :), \mathbf{\Psi}(2, :), \cdots, \mathbf{\Psi}(2i - 1, :), \mathbf{\Psi}(2i, :)\}$ can be obtained by [8]

$$\mathbf{F}(2j - 1 : 2j, :) = \mathbf{H}(2j - 1 : 2j, :)\left(\mathbf{I}_{N_T\times N_T} - \sum_{p=1}^{i-1}\mathbf{T}_p\right) \quad (17)$$

where

$$\mathbf{T}_p = \frac{\mathbf{\Psi}^H(2p - 1, :)\mathbf{\Psi}(2p - 1, :)}{\|\mathbf{\Psi}(2p - 1, :)\|^2} + \frac{\mathbf{\Psi}^H(2p, :)\mathbf{\Psi}(2p, :)}{\|\mathbf{\Psi}(2p, :)\|^2} \quad (18)$$

and $\mathbf{\Psi}(2p - 1 : 2p, :) = \mathbf{H}(2v(p) - 1 : 2v(p), :)(\mathbf{I}_{N_T\times N_T} - \mathbf{T})$ has been computed at the $(i - 1)$-th iteration as presented in lines 9, 12, and 14 of Algorithm 1. Note that each user in the MU-STLC system has 2 receive antennas. That's why two channel vectors are selected for each user at any given time. It is noted that Algorithm 1 does not force semi-orthogonality among users for simple use and comparison. It means that we do not adopt the step of measuring the orthogonality of $\mathbf{H}(2j - 1 : 2j, :)$ and $\mathbf{\Psi}(2p - 1 : 2p, :)$ and comparing it to a predetermined threshold, which has been employed in [8]. It is shown in Section V that the modified low-complexity SUS selection algorithm (Algorithm 3) without this step has negligible difference in BER performance compared to the proposed incremental-based algorithms.

Algorithm 3 LC-SUS Selection

Inputs: $\mathbf{H}$, $\mathbf{Q}_{2U}$, U, K
$\tilde{\mathbf{H}} = \left[\mathbf{H}\ \mathbf{Q}_{2U}^T\mathbf{H}^*\right]$
$\tilde{\mathbf{\Psi}} = \mathbf{0}_{2U\times 2N_T}$
$\tilde{\mathbf{T}} = \mathbf{0}_{2N_T\times 2N_T}$
for $i = 1 : K$
  if $i > 1$
    $\tilde{\mathbf{\Theta}} = \tilde{\mathbf{\Psi}}(2i-3 : 2i-2, :)$
    $\tilde{\mathbf{T}} = \tilde{\mathbf{T}} + \tilde{\mathbf{\Theta}}^H(\tilde{\mathbf{\Theta}}\tilde{\mathbf{\Theta}}^H)^{-1}\tilde{\mathbf{\Theta}}$
  end
  $\tilde{\mathbf{V}} = \mathbf{I}_{2N_T\times 2N_T} - \tilde{\mathbf{T}}$
  for $j = 1 : (U-i+1)$
    $\tilde{\mathbf{F}}(2j-1 : 2j, :) = \tilde{\mathbf{H}}(2j-1 : 2j, :)\tilde{\mathbf{V}}$
    $\alpha_j = tr(\tilde{\mathbf{F}}(2j-1 : 2j, :)\tilde{\mathbf{F}}^H(2j-1 : 2j, :))$
  end
  $v(i) = \arg\max\{\alpha_1, \alpha_2, \cdots, \alpha_{U-i+1}\}$
  $\mathbf{H}_S(2i-1 : 2i, :) = \mathbf{H}(2v(i)-1 : 2v(i), :)$
  $\tilde{\mathbf{\Psi}}(2i-1 : 2i, :) = \tilde{\mathbf{F}}(2v(i)-1 : 2v(i), :)$
  $\tilde{\mathbf{H}} = \tilde{\mathbf{H}}([1 : 2v(i)-2\ 2v(i)+1 : \text{end}], :)$
end
Output: $\mathbf{H}_S$

B. MODIFIED SUS SELECTION ALGORITHM WITH LOW-COMPLEXITY

The original SUS selection approach in Algorithm 1 is based on the full channel matrix $\mathbf{H} \in C^{2U\times N_T}$. It will be shown through simulations in Section V that its performance is very poor. Therefore, it should be modified using the channel $\tilde{\mathbf{H}} \in C^{2U\times 2N_T}$ to apply to the MU-STLC systems for a fair comparison. Then the SUS selection algorithm for MU-STLC signals can be presented as Algorithm 2.

The computational complexity of the SUS selection algorithm in terms of the RMs and RSs, respectively, can be obtained as

$$N_{SUS}^{RM} = \sum_{i=1}^{K}\sum_{j=1}^{U-i+1}((i-1)(32N_T^2+56N_T+40)+24N_T) \tag{19}$$

$$N_{SUS}^{RS} = \sum_{i=1}^{K}\sum_{j=1}^{U-i+1}\Big((i-1)(32N_T^2+48N_T+18) + 32N_T^2+18N_T-5\Big) \tag{20}$$

To reduce the computational complexity of Algorithm 2, a step to make up the subspace $\{\tilde{\mathbf{\Psi}}(1, :), \tilde{\mathbf{\Psi}}(2, :), \cdots, \tilde{\mathbf{\Psi}}(2i-1, :), \tilde{\mathbf{\Psi}}(2i, :)\}$ can be relocated outside from inside of 'a for-loop with index j' as shown in Algorithm 3, which is a low-complexity SUS (called LC-SUS) selection algorithm for the MU-STLC. Then

$$N_{LC-SUS}^{RM} = (K-1)(32N_T^2+56N_T+40) + \sum_{i=1}^{K}(U-i+1)(32N_T^2+24N_T) \tag{21}$$

Algorithm 4 Inc-US Selection

Inputs: $\mathbf{H}$, $\mathbf{Q}_{2U}$, $\mathbf{Q}_2$, U, K
$v(1) = \underset{u\in\{1,2,\cdots,U\}}{\arg\min}\ tr(\mathbf{H}_u\mathbf{H}_u^H)$
$\mathbf{H}_S(1 : 2, :) = \mathbf{H}_{v(1)}$
$\tilde{\mathbf{H}}_S(1 : 2, :) = \left[\mathbf{H}_S(1 : 2, :)\ \mathbf{Q}_2^T\mathbf{H}_S^*(1 : 2, :)\right]$
$\tilde{\mathbf{V}}_{(1)} = (\tilde{\mathbf{H}}_S(1 : 2, :)\tilde{\mathbf{H}}_S^H(1 : 2, :))^{-1}$
$\mathbf{H}_R = \mathbf{H}([1 : 2v(1)-2\ 2v(1)+1 : \text{end}], :)$
for $i = 2 : K$
  $\mathbf{H}_{S,temp} = \mathbf{Q}_{2K}^T(1 : 2i-2, 1 : 2i-2)\mathbf{H}_S^*(1 : 2i-2, :)$
  $\tilde{\mathbf{H}}_S = \left[\mathbf{H}_S(1 : 2i-2, :)\ \mathbf{H}_{S,temp}\right]$
  for $j = 1 : (U-i+1)$
    $\tilde{\mathbf{H}}_R = \left[\mathbf{H}_R(2j-1 : 2j, :)\ \mathbf{Q}_2^T\mathbf{H}_R^*(2j-1 : 2j, :)\right]$
    $\tilde{\mathbf{\Lambda}}_{ij} = \tilde{\mathbf{V}}_{(i-1)}\tilde{\mathbf{H}}_S\tilde{\mathbf{H}}_R^H$
    $\tilde{\mathbf{\Xi}}_{ij} = (\tilde{\mathbf{H}}_R\tilde{\mathbf{H}}_R^H - \tilde{\mathbf{H}}_R\tilde{\mathbf{H}}_S^H\tilde{\mathbf{\Lambda}}_{ij})^{-1}$
    $\tilde{\mathbf{\Omega}}_i = \left[\tilde{\mathbf{V}}_{(i-1)} + \tilde{\mathbf{\Lambda}}_{ij}\tilde{\mathbf{\Xi}}_{ij}\tilde{\mathbf{\Lambda}}_{ij}^H - \tilde{\mathbf{\Xi}}_{ij}\tilde{\mathbf{\Lambda}}_{ij}^H; -\tilde{\mathbf{\Lambda}}_{ij}\tilde{\mathbf{\Xi}}_{ij}\tilde{\mathbf{\Xi}}_{ij}\right]$
    $\alpha_j = tr(\tilde{\mathbf{\Omega}}_i)$
  end
  $v(i) = \arg\min\{\alpha_1, \alpha_2, \cdots, \alpha_{U-i+1}\}$
  $\tilde{\mathbf{V}}_{(i)} = \tilde{\mathbf{\Omega}}_{(v(i))}$
  $\mathbf{H}_S(2i-1 : 2i, :) = \mathbf{H}_R(2v(i)-1 : 2v(i), :)$
  $\mathbf{H}_R = \mathbf{H}_R([1 : 2v(i)-22v(i)+1 : \text{end}], :)$
end
Output: $\mathbf{H}_S$

$$N_{LC-SUS}^{RS} = (K-1)(32N_T^2+48N_T+18) + 2KN_T + \sum_{i=1}^{K}(U-i+1)(32N_T^2+16N_T-5) \tag{22}$$

C. INCREMENTAL US SELECTION ALGORITHM

For the US selection with reduced complexity, another incremental user selection strategy is introduced. Note that this work focuses on the incremental-based selection algorithm because its complexity is substantially smaller than the decremental method when the number of all the available users is large and simultaneously much larger than K. An US is constructed by adding users one by one in the incremental manner. Assuming that $(i-1)$ users are selected, the i-th user is selected according to the following criterion.

$$v(i) = \underset{j\in\{1,2,\cdots,U-i+1\}}{\arg\min}\ tr\left[(\tilde{\mathbf{P}}\tilde{\mathbf{P}}^H)^{-1}\right] \tag{23}$$

where $\tilde{\mathbf{P}} = [\tilde{\mathbf{H}}_{S,(i-1)}; \tilde{\mathbf{H}}_{R,j}] \in C^{2i\times 2N_T}$. $\tilde{\mathbf{H}}_{S,(i-1)} \in C^{2(i-1)\times 2N_T}$ corresponds to the channel submatrix selected during $(i-1)$ iterations and $\tilde{\mathbf{H}}_R$ is the channel submatrix remained after removing the row vectors associated with the users selected up to the $(i-1)$-th iteration. Thus, $\tilde{\mathbf{H}}_{R,j}$ is the j-th block matrix of $\tilde{\mathbf{H}}_R$ formed after $(i-1)$ iterations.

Algorithm 5 R-inc-US Selection

Inputs: $\mathbf{H}$, $\mathbf{Q}_{2U}$, $\mathbf{Q}_2$, U, K

$v(1) = \arg_{u\in\{1,2,\cdots,U\}}\max\ tr(\mathbf{H}_u\mathbf{H}_u^H)$

$\mathbf{H}_S(1:2,:) = \mathbf{H}_{v(1)}$

$\tilde{\mathbf{H}}_S(1:2,:) = \left[\mathbf{H}_S(1:2,:)\ \mathbf{Q}_2^T\mathbf{H}_S^*(1:2,:)\right]$

$\tilde{\mathbf{V}}_{(1)} = ((\tilde{\mathbf{H}}_S(1,:)\tilde{\mathbf{H}}_S^H(1,:))\mathbf{I}_{2\times 2})^{-1}$

$\mathbf{H}_R = \mathbf{H}([1:2v(1)-2\ 2v(1)+1:\text{end}],:)$

for $i = 2:K$

  $\mathbf{H}_{S,temp} = \mathbf{Q}_{2K}^T(1:2i-2, 1:2i-2)\mathbf{H}_S^*(1:2i-2,:)$

  $\tilde{\mathbf{H}}_S = \left[\mathbf{H}_S(1:2i-2,:)\ \mathbf{H}_{S,temp}\right]$

  for $j = 1:(U-i+1)$

    $\tilde{\mathbf{H}}_R = \left[\mathbf{H}_R(2j-1:2j,:)\ \mathbf{Q}_2^T\mathbf{H}_R^*(2j-1:2j,:)\right]$

    $\tilde{\mathbf{H}}_{SR,temp} = \tilde{\mathbf{H}}_S(1:2:(2(i-1)-1),:)\tilde{\mathbf{H}}_R^H$

    $\tilde{\mathbf{H}}_{SR} = []$

    for $p = 1:(i-1)$

      $\tilde{\mathbf{H}}_{SR}^{(1)} = \left[\tilde{\mathbf{H}}_{SR,temp}(p,1)\tilde{\mathbf{H}}_{SR,temp}(p,2)\right]$

      $\tilde{\mathbf{H}}_{SR}^{(2)} = \left[-\tilde{\mathbf{H}}_{SR,temp}^H(p,2)\tilde{\mathbf{H}}_{SR,temp}^H(p,1)\right]$

      $\tilde{\mathbf{H}}_{SR} = \left[\tilde{\mathbf{H}}_{SR};\ \tilde{\mathbf{H}}_{SR}^{(1)};\ \tilde{\mathbf{H}}_{SR}^{(2)}\right]$

    end

    $\tilde{\mathbf{\Lambda}}_{ij}(:,1) = \tilde{\mathbf{V}}_{(i-1)}\tilde{\mathbf{H}}_{SR}(:,1)$

    $\tilde{\mathbf{\Theta}}_{ij} = (\tilde{\mathbf{H}}_{SR}^H(:,1)\tilde{\mathbf{\Lambda}}_{ij}(:,1))\mathbf{I}_{2\times 2}$

    $\tilde{\mathbf{H}}_{RR} = (\tilde{\mathbf{H}}_R(1,:)\tilde{\mathbf{H}}_R^H(1,:))\mathbf{I}_{2\times 2}$

    $\tilde{\mathbf{\Xi}}_{ij} = (\tilde{\mathbf{H}}_{RR} - \tilde{\mathbf{\Theta}}_{ij})^{-1}$

    $\alpha_j = tr(\tilde{\mathbf{\Xi}}_{ij})$

  end

  $v(i) = \arg\min\{\alpha_1, \alpha_2, \cdots, \alpha_{U-i+1}\}$

  $\tilde{\mathbf{\Psi}}_{iv(i)} = []$, $\tilde{\mathbf{Z}}_{iv(i)} = []$

  for $q = 1:(i-1)$

    $\tilde{\mathbf{\Lambda}}_{iv(i)}(2q-1,:) = \left[\tilde{\mathbf{\Lambda}}_{iv(i)}(2q-1,1) - \tilde{\mathbf{\Lambda}}_{iv(i)}^H(2q,1)\right]$

    $\tilde{\mathbf{\Lambda}}_{iv(i)}(2q,:) = \left[\tilde{\mathbf{\Lambda}}_{iv(i)}(2q,1)\tilde{\mathbf{\Lambda}}_{iv(i)}^H(2q-1,1)\right]$

    $\tilde{\mathbf{V}}_{v(i)}^{(1)}(1,q) = \tilde{\mathbf{\Xi}}_{iv(i)}(1,:)\tilde{\mathbf{\Lambda}}_{iv(i)}^H(2q-1,:)$

    $\tilde{\mathbf{V}}_{v(i)}^{(2)}(1,q) = \tilde{\mathbf{\Xi}}_{iv(i)}(1,:)\tilde{\mathbf{\Lambda}}_{iv(i)}^H(2q,:)$

    $\tilde{\mathbf{V}}_{temp} = \left[\tilde{\mathbf{V}}_{v(i)}^{(1)}(1,q)\tilde{\mathbf{V}}_{v(i)}^{(2)}(1,q);\ -\tilde{\mathbf{V}}_{v(i)}^{(2)^H}(1,q)\tilde{\mathbf{V}}_{v(i)}^{(1)^H}(1,q)\right]$

    $\tilde{\mathbf{\Psi}}_{iv(i)} = \left[\tilde{\mathbf{\Psi}}_{iv(i)}\tilde{\mathbf{V}}_{temp}\right]$

    $\tilde{\mathbf{Z}}_{iv(i)} = \left[\tilde{\mathbf{Z}}_{iv(i)}\ \tilde{\mathbf{\Lambda}}_{iv(i)}\tilde{\mathbf{V}}_{temp}\right]$

  end

  $\tilde{\mathbf{V}}_{(i)} = \left[\tilde{\mathbf{V}}_{(i-1)} + \tilde{\mathbf{Z}}_{iv(i)}\ \ -\tilde{\mathbf{\Psi}}_{iv(i)};\ -\tilde{\mathbf{\Psi}}_{iv(i)}^H\ \ \tilde{\mathbf{\Xi}}_{iv(i)}\right]$

  $\mathbf{H}_S(2i-1:2i,:) = \mathbf{H}_R(2v(i)-1:2v(i),:)$

  $\mathbf{H}_R = \mathbf{H}_R([1:2v(i)-2\,2v(i)+1:\text{end}],:)$

end

Output: $\mathbf{H}_S$

Furthermore, (23) can be re-expressed as

$$v(i) = \mathop{\arg\min}_{j\in\{1,2,\cdots,U-i+1\}} tr\left(\begin{bmatrix} \tilde{\mathbf{\Pi}}_{(i-1)} & \tilde{\mathbf{B}}_{ij} \\ \tilde{\mathbf{B}}_{ij}^H & \tilde{\mathbf{D}}_{jj} \end{bmatrix}^{-1}\right) \tag{24}$$

where

$$\tilde{\mathbf{\Pi}}_{(i-1)} = \tilde{\mathbf{H}}_{S,(i-1)}\tilde{\mathbf{H}}_{S,(i-1}^H \tag{25}$$

$$\tilde{\mathbf{H}}_{S,(i-1)} = \Big[\mathbf{H}_S(1:2i-2,:)\quad \mathbf{Q}_{2K}^T(1:2i-2,1:2i-2) \times \mathbf{H}_S^*(1:2i-2,:)\Big] \tag{26}$$

$$\tilde{\mathbf{B}}_{ij} = \tilde{\mathbf{H}}_{S,(i-1)}\tilde{\mathbf{H}}_{R,j}^H \tag{27}$$

$$\tilde{\mathbf{H}}_{R,j} = \left[\mathbf{H}_R(2j-1:2j,:)\ \mathbf{Q}_2^T\mathbf{H}_R^*(2j-1:2j,:)\right] \tag{28}$$

$$\tilde{\mathbf{D}}_{jj} = \tilde{\mathbf{H}}_{R,j}\tilde{\mathbf{H}}_{R,j}^H \tag{29}$$

To further reduce the computational complexity, we employ the block matrix inverse [12] and then have the following result.

$$\begin{bmatrix} \tilde{\mathbf{\Pi}}_{(i-1)} & \tilde{\mathbf{B}}_{ij} \\ \tilde{\mathbf{B}}_{ij}^H & \tilde{\mathbf{D}}_{jj} \end{bmatrix}^{-1} = \begin{bmatrix} \tilde{\mathbf{A}}_{ij} & -\tilde{\mathbf{\Pi}}_{(i-1)}^{-1}\tilde{\mathbf{B}}_{ij}\tilde{\mathbf{S}}_{ij}^{-1} \\ -\tilde{\mathbf{S}}_{ij}^{-1}\tilde{\mathbf{B}}_{ij}^H\tilde{\mathbf{\Pi}}_{(i-1)}^{-1} & \tilde{\mathbf{S}}_{ij}^{-1} \end{bmatrix} \tag{30}$$

where we assume that $\tilde{\mathbf{\Pi}}_{(i-1)}$ and $\tilde{\mathbf{S}}_{ij}$ are both nonsingular and

$$\tilde{\mathbf{A}}_{ij} = \tilde{\mathbf{\Pi}}_{(i-1)}^{-1} + \tilde{\mathbf{\Pi}}_{(i-1)}^{-1}\tilde{\mathbf{B}}_{ij}\tilde{\mathbf{S}}_{ij}^{-1}\tilde{\mathbf{B}}_{ij}^H\tilde{\mathbf{\Pi}}_{(i-1)}^{-1} \tag{31}$$

$$\tilde{\mathbf{S}}_{ij} = \tilde{\mathbf{D}}_{jj} - \tilde{\mathbf{B}}_{ij}^H\tilde{\mathbf{\Pi}}_{(i-1)}^{-1}\tilde{\mathbf{B}}_{ij} \tag{32}$$

Here $\tilde{\mathbf{\Pi}}_{(i-1)}^{-1}$ needed for the i-th user selection has been updated without matrix inversion at the $(i-1)$-th iteration. The proposed incremental US selection algorithm with low-complexity is summarized in Algorithm 4 and called as inc-US, where the first user is determined by maximizing the trace of the Gram matrix $\mathbf{H}_u\mathbf{H}_u^H$, $u = 1, 2, \cdots, U$ other than minimizing $(\mathbf{H}_u\mathbf{H}_u^H)^{-1}$ for the complexity reduction. Note in Algorithm 4 that $\tilde{\mathbf{\Pi}}_{(i-1)}^{-1}$ is replaced with another matrix symbol notation of $\tilde{\mathbf{V}}_{(i-1)}$. The RM and RS complexity of the inc-US, respectively, can be evaluated as

$$N_{inc\text{-}US}^{RM} = 12N_TU + 24N_T + 40 + \sum_{i=2}^{K}(U-i+1)\,(64\,i^2 + (32N_T-64)i - 8N_T + 40) \tag{33}$$

$$N_{inc\text{-}US}^{RS} = (12N_T-5)U + 24N_T + 18 + \sum_{i=2}^{K}(U-i+1)\,(56i^2 + (32N_T-70)i - 8N_T + 33) \tag{34}$$

D. INCREMENTAL US SELECTION ALGORITHM WITH REDUCED COMPLEXITY

In the incremental US selection algorithm proposed in subsection 3.C, the block matrix computation of (30) is involved with several matrix-by-matrix multiplications at each incremental step (*for double for-loops in Algorithm 4*). By exploiting the definition of (10) used for the STLC encoding, we can reduce the computational complexity for the matrix-by-matrix multiplications.

First of all, we can easily show from (10) that

$$\tilde{\mathbf{H}}_S(1:2,:)\tilde{\mathbf{H}}_S^H(1:2,:) = \left(\sum_{a=1}^{2}\sum_{b=1}^{N_T}\left|h_{S,ab}\right|^2\right)\mathbf{I}_{2\times 2} \tag{35}$$

where $h_{S,ab}$ is the (a,b)-th element of the channel matrix $\mathbf{H}_S(1:2,:) \in C^{2\times N_T}$. Thus the computation of $\tilde{\mathbf{H}}_S(1:2,:)\tilde{\mathbf{H}}_S^H(1:2,:)$ can be reduced by calculating only $\left|h_{S,ab}\right|^2$. The RM and RS computations of $\tilde{\mathbf{H}}_S(1:2,:)\tilde{\mathbf{H}}_S^H(1:2,:)$ by the matrix-by-matrix multiplication are given as $24N_T$ and $24N_T - 6$, respectively, whereas (35) requires $8N_T$ and $8N_T - 2$, respectively, in terms of the RM and RS.

Next, let $\mathbf{H}_S(2i-3:2i-2,:) = [\mathbf{h}_{S,1}^T \ \mathbf{h}_{S,2}^T]^T \in C^{2\times N_T}$ where $\mathbf{h}_{S,1}$ and $\mathbf{h}_{S,2}$ are the first and second row vectors of $\mathbf{H}_S(2i-3:2i-2,:)$. Further, let $\mathbf{H}_{R,j} = [\mathbf{h}_{R,1}^T \ \mathbf{h}_{R,2}^T]^T \in C^{2\times N_T}$ where $\mathbf{h}_{R,1}$ and $\mathbf{h}_{R,2}$ are the first and second row vectors of $\mathbf{H}_{R,j}$. Then we have

$$\begin{aligned}&\tilde{\mathbf{H}}_S(2i-3:2i-2,:)\\&= \left[\mathbf{H}_S(2i-3:2i-2,:) \ \ \mathbf{Q}_2^T\mathbf{H}_S^*(2i-3:2i-2,:)\right]\\&= \left[\tilde{\mathbf{h}}_{S,1}^T \quad \tilde{\mathbf{h}}_{S,2}^T\right]^T\end{aligned} \tag{36}$$

$$\tilde{\mathbf{H}}_{R,j} = \left[\mathbf{H}_{R,j} \ \ \mathbf{Q}_2^T\mathbf{H}_{R,j}^*\right] = \left[\tilde{\mathbf{h}}_{R,1}^T \quad \tilde{\mathbf{h}}_{R,2}^T\right]^T \tag{37}$$

where

$$\tilde{\mathbf{h}}_{S,1} = \left[\mathbf{h}_{S,1} \quad \mathbf{h}_{S,2}^*\right] \tag{38}$$

$$\tilde{\mathbf{h}}_{S,2} = \left[\mathbf{h}_{S,2} \quad -\mathbf{h}_{S,1}^*\right] \tag{39}$$

$$\tilde{\mathbf{h}}_{R,1} = \left[\mathbf{h}_{R,1} \quad \mathbf{h}_{R,2}^*\right] \tag{40}$$

$$\tilde{\mathbf{h}}_{R,2} = \left[\mathbf{h}_{R,2} \quad -\mathbf{h}_{R,1}^*\right] \tag{41}$$

Then it can be easily shown that the $(i-1)$-th block submatrix of (27) is expressed as

$$\tilde{\mathbf{H}}_S(2i-3,2i-2:)\tilde{\mathbf{H}}_{R,j}^H = \begin{bmatrix}\mathbf{h}_{SR,1} & \mathbf{h}_{SR,2}\\ -\mathbf{h}_{SR,2}^* & \mathbf{h}_{SR,1}^*\end{bmatrix} \tag{42}$$

where

$$\mathbf{h}_{SR,1} = \mathbf{h}_{S,1}\mathbf{h}_{R,1}^H + \mathbf{h}_{S,2}^*\mathbf{h}_{R,2}^T \tag{43}$$

$$\mathbf{h}_{SR,2} = \mathbf{h}_{S,1}^*\mathbf{h}_{R,2}^H - \mathbf{h}_{S,2}^*\mathbf{h}_{R,1}^T \tag{44}$$

Thus the computation of $\tilde{\mathbf{H}}_S(2i-3,2i-2:)\tilde{\mathbf{H}}_{R,j}^H$ can be reduced by obtaining only $\mathbf{h}_{SR,1}$ and $\mathbf{h}_{SR,2}$ instead of performing a matrix-by-matrix multiplication. Then, to reduce the computational complexity of $\tilde{\mathbf{B}}_{ij}^H\tilde{\boldsymbol{\Pi}}_{(i-1)}^{-1}\tilde{\mathbf{B}}_{ij}$ in (32), it can be represented as

$$\begin{aligned}&\tilde{\mathbf{H}}_{R,j}\tilde{\mathbf{H}}_S^H(2i-3,2i-2:)\tilde{\boldsymbol{\Pi}}_{(i-1)}^{-1}\tilde{\mathbf{H}}_S(2i-3,2i-2:)\tilde{\mathbf{H}}_{R,j}^H\\&= \begin{bmatrix}\mathbf{h}_{SR,1} & \mathbf{h}_{SR,2}\\ -\mathbf{h}_{SR,2}^* & \mathbf{h}_{SR,1}^*\end{bmatrix}^H \begin{bmatrix}\kappa & 0\\ 0 & \kappa\end{bmatrix}\begin{bmatrix}\mathbf{h}_{SR,1} & \mathbf{h}_{SR,2}\\ -\mathbf{h}_{SR,2}^* & \mathbf{h}_{SR,1}^*\end{bmatrix}\\&= \kappa\left(\mathbf{h}_{SR,1}\mathbf{h}_{SR,1}^H + \mathbf{h}_{SR,2}\mathbf{h}_{SR,2}^H\right)\mathbf{I}_{2\times 2}\end{aligned} \tag{45}$$

where $\tilde{\boldsymbol{\Pi}}_{(i-1)}^{-1}$ is a diagonal matrix with the same diagonal element κ. Therefore, the matrix-by-matrix multiplications of $\tilde{\mathbf{B}}_{ij}^H\tilde{\boldsymbol{\Pi}}_{(i-1)}^{-1}\tilde{\mathbf{B}}_{ij}$ in (32) can be obtained by the expression of (45). Thus the RM and RS computations of lines 11 and 12 in Algorithm 4 by the matrix-by-matrix multiplication are calculated as $32i^2 + (32N_T - 32)i - 8N_T + 40$ and $32i^2 + (32N_T - 48)i - 8N_T + 34$, respectively, whereas utilizing (45) the RM and RS complexities of lines 11, 18, 19, 20, and 21 in Algorithm 5 are obtained as $16i^2 + (16N_T - 24)i - 8N_T + 48$ and $16i^2 + (16N_T - 32)i - 8N_T + 37$, respectively.

In a similar way to the expression (35), we get

$$\tilde{\mathbf{D}}_{jj} = \tilde{\mathbf{H}}_{R,j}\tilde{\mathbf{H}}_{R,j}^H = \left(\sum_{a=1}^{2}\sum_{b=1}^{N_T}\left|h_{R,ab}\right|^2\right)\mathbf{I}_{2\times 2} = \xi_R\mathbf{I}_{2\times 2} \tag{46}$$

where $h_{R,ab}$ is the (a,b)-th element of the channel matrix $\mathbf{H}_R(2j-1:2j,:) \in C^{2\times N_T}$ and ξ_R is the squared sum of absolute values of all elements of $\mathbf{H}_{R,j}$.

The proposed incremental US selection algorithm using the above-mentioned complexity reduction methods is described in Algorithm 5 (named as R-inc-US), where $\tilde{\mathbf{V}}_{(i-1)}$ takes replace of $\tilde{\boldsymbol{\Pi}}_{(i-1)}^{-1}$. In the algorithm, the optimization criterion of (24) is modified for further reduction of the complexity. The US selection criterion of (24) and (30) can be rewritten as

$$v(i) = \underset{j\in\{1,2,\cdots,U-i+1\}}{\arg\min}\ tr\left(\tilde{\mathbf{A}}_{ij}\right) + tr\left(\tilde{\mathbf{S}}_{ij}^{-1}\right) \tag{47}$$

where

$$tr\left(\tilde{\mathbf{A}}_{ij}\right) = tr\left(\tilde{\boldsymbol{\Pi}}_{(i-1)}^{-1}\right) + tr\left(\tilde{\boldsymbol{\Pi}}_{(i-1)}^{-1}\tilde{\mathbf{B}}_{ij}\tilde{\mathbf{S}}_{ij}^{-1}\tilde{\mathbf{B}}_{ij}^H\tilde{\boldsymbol{\Pi}}_{(i-1)}^{-1}\right) \tag{48}$$

The second term of (48) can be rewritten as

$$\begin{aligned}&tr\left(\tilde{\boldsymbol{\Pi}}_{(i-1)}^{-1}\tilde{\mathbf{B}}_{ij}\tilde{\mathbf{S}}_{ij}^{-1}\tilde{\mathbf{B}}_{ij}^H\tilde{\boldsymbol{\Pi}}_{(i-1)}^{-1}\right)\\&= tr\left(\xi_R^2\tilde{\boldsymbol{\Pi}}_{(i-1)}^{-1}\tilde{\mathbf{H}}_{S,(i-1)}(\mathbf{I}_{2\times 2} - \tilde{\mathbf{H}}_{S,(i-1)}^H\tilde{\boldsymbol{\Pi}}_{(i-1)}^{-1}\tilde{\mathbf{H}}_{S,(i-1)})^{-1}\right.\\&\quad\left.\times\ \tilde{\mathbf{H}}_{S,(i-1)}^H\tilde{\boldsymbol{\Pi}}_{(i-1)}^{-1}\right)\end{aligned} \tag{49}$$

By exploiting *Lemma 2* in [24], it can be verified that (49) is positive. It is also obvious that $tr\left(\tilde{\boldsymbol{\Pi}}_{(i-1)}^{-1}\right) > 0$. Then the following inequality is satisfied.

$$tr\left(\tilde{\mathbf{A}}_{ij}\right) + tr\left(\tilde{\mathbf{S}}_{ij}^{-1}\right) > tr\left(\tilde{\mathbf{S}}_{ij}^{-1}\right) \tag{50}$$

Then the optimization problem can be formulated as

$$v(i) = \underset{j\in\{1,2,\cdots,U-i+1\}}{\arg\min}\ tr\left(\tilde{\mathbf{S}}_{ij}^{-1}\right) \tag{51}$$

Finally, it should be noted that to compute the updated $\tilde{\boldsymbol{\Pi}}_{(i)}^{-1}$ such as the right side of (30), the same complexity-reduction approach as before is employed after finding the user index $v(i) = \arg\min\{\alpha_1, \alpha_2, \cdots, \alpha_{U-i+1}\}$ as in line 24 of Algorithm 5. In the updating step of $\tilde{\boldsymbol{\Pi}}_{(i)}^{-1}$, the recurring matrix computations can be avoided and the final step to obtain the matrix $\tilde{\boldsymbol{\Omega}}_{(v(i))}$ given in line 17 of Algorithm 4 can be described in lines 25∼35 of Algorithm 5.

Then, the RM and RS complexities of the R-inc-US, respectively, are expressed as

$$N^{RM}_{R\text{-}inc\text{-}US} = 12N_T U + 8N_T + 1 + \sum_{i=2}^{K}\Big\{(U-i+1)(16\,i^2 + (16N_T - 24)i - 8N_T + 48) + 32i^2 - 48i + 16\Big\} \tag{52}$$

$$N^{RS}_{R\text{-}inc\text{-}US} = (12N_T - 5)U + 8N_T - 1 + \sum_{i=2}^{K}\Big\{(U-i+1)(16\,i^2 + (16N_T - 32)i - 8N_T + 38) + 32i^2 - 52i + 20\Big\} \tag{53}$$

IV. ACHIEVABLE DIVERSITY ORDER

Using the combined symbols in (9) and (12), the maximum likelihood (ML) detection can be performed at the receiver side by minimizing the following metric:

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}\in\Delta^K} \left\|\mathbf{y} - \tilde{\beta}_{S_n}\mathbf{x}\right\|^2 \tag{54}$$

where Δ is the set of user message symbols from the M-ary signal constellation. To derive the achievable diversity order of the MU-STLC system with optimal US selection, we consider the PEP of detecting $\check{\mathbf{x}}$ when $\mathbf{x}$ is transmitted with the assumption that the optimal US for a given channel realization $\tilde{\mathbf{H}}_S$ is denoted by $S_{\hat{n}}$, which is given as

$$PEP(\mathbf{x}\to\check{\mathbf{x}}) = Q\left(\sqrt{\frac{\left\|\tilde{\beta}_{S_{\hat{n}}}(\mathbf{x}-\check{\mathbf{x}})\right\|_F^2}{4\sigma_z^2}}\right) \le \frac{1}{2}\exp\left(-\frac{\left\|\tilde{\beta}_{S_{\hat{n}}}(\mathbf{x}-\check{\mathbf{x}})\right\|_F^2}{8\sigma_z^2}\right) \tag{55}$$

Here, to examine the distribution of $1/[(\tilde{\mathbf{H}}_{S_{\hat{n}}}\tilde{\mathbf{H}}^H_{S_{\hat{n}}})^{-1}]_{q,q}$, where $[\cdot]_{q,q}$ denotes the (q,q) entry of matrix $[\cdot]$, $\tilde{\beta}^2_{S_{\hat{n}}}$ can be re-expressed as

$$\tilde{\beta}^2_{S_{\hat{n}}} = \frac{1}{\sum_{t=1}^{2K}\left[(\tilde{\mathbf{H}}_{S_{\hat{n}}}\tilde{\mathbf{H}}^H_{S_{\hat{n}}})^{-1}\right]_{q,q}} = \frac{1}{2KE\left\{\left[(\tilde{\mathbf{H}}_{S_{\hat{n}}}\tilde{\mathbf{H}}^H_{S_{\hat{n}}})^{-1}\right]_{q,q}\right\}} = \frac{1}{2K}E\left\{\tilde{\beta}^2_{S_{\hat{n}},q}\right\} \tag{56}$$

where

$$\tilde{\beta}^2_{S_{\hat{n}},q} = \frac{1}{\left[(\tilde{\mathbf{H}}_{S_{\hat{n}}}\tilde{\mathbf{H}}^H_{S_{\hat{n}}})^{-1}\right]_{q,q}} = \tilde{\mathbf{h}}_q\tilde{\mathbf{\Gamma}}_{S_{\hat{n}}}\tilde{\mathbf{h}}_q^H = \tilde{\mathbf{h}}_q\tilde{\mathbf{U}}^H_{S_{\hat{n}}}\tilde{\mathbf{\Lambda}}_{S_{\hat{n}}}\tilde{\mathbf{U}}_{S_{\hat{n}}}\tilde{\mathbf{h}}_q^H = \sum_{m=1}^{2N_T}\lambda_q^{(m)}\zeta^2_{q,m} \tag{57}$$

and $\tilde{\mathbf{h}}_q \in C^{1\times 2N_T}$ is the t-th row vector of $\tilde{\mathbf{H}}_{S_{\hat{n}}}$ and $\tilde{\mathbf{\Gamma}}_{S_{\hat{n}}}$ is an $2N_T \times 2N_T$ non-negative Hermitian matrix formed from $\tilde{\mathbf{h}}_1,\cdots,\tilde{\mathbf{h}}_{q-1},\tilde{\mathbf{h}}_{q+1},\cdots,\tilde{\mathbf{h}}_{2K}$ and thus is independent of $\tilde{\mathbf{h}}_q$. It is shown that $E\{\tilde{\mathbf{h}}_q^H\tilde{\mathbf{h}}_q\} = diag[1,1,\cdots,1]$ and $\tilde{\mathbf{U}}_{S_{\hat{n}}}\tilde{\mathbf{h}}_q^H = \boldsymbol{\zeta}_q$ where $\boldsymbol{\zeta}_q = [\zeta_{q,1}\ \zeta_{q,2}\cdots\ \zeta_{q,2N_T}]^T$, with a unitary matrix $\tilde{\mathbf{U}}_{S_{\hat{n}}}$. In [25], $\zeta_{q,m}$, $m = 1,2,\cdots,2N_T$, is approximately modeled as the zero-mean Gaussian random variable with unit-variance. Also, we have $\tilde{\mathbf{\Lambda}}_{S_{\hat{n}}} = diag[\lambda_q^{(1)}\ \lambda_q^{(2)}\ \lambda_q^{(3)}\ \cdots\ \lambda_q^{(2N_T-1)}\ \lambda_q^{(2N_T)}]$ where $\lambda_q^{(m)}$ is the eigenvalue of the matrix $\tilde{\mathbf{\Gamma}}_{S_{\hat{n}}}$ and $\lambda_q^{(2b-1)} = \lambda_q^{(2b)}$, $b = 1,2,\cdots,N_T$. Thus, it can be verified as in [25] that the eigenvalues of $\tilde{\mathbf{\Gamma}}_{S_{\hat{n}}}$ with $(2N_T - 2(K-1))$ are equal to 1 and the others with $2(K-1)$ eigenvalues are 0. Hence, the $\tilde{\beta}^2_{S_{\hat{n}},q}$ of expression (57) can be rewritten as

$$\tilde{\beta}^2_{S_{\hat{n}},q} = \sum_{m=1}^{2N_T-2(K-1)}\zeta^2_{q,m} \tag{58}$$

Therefore, the variate $\tilde{\beta}^2_{S_{\hat{n}},q}$ is a central-chi-square distributed random variable with $2(N_T - K + 1)$ degrees of freedom. It is also noted as in [26] that $\tilde{\beta}^2_{S_{\hat{n}}}$ can be approximated by a Gamma distribution. Given an optimally selected US consisting of K users, the MU-STLC system with N_T transmit antennas and 2 receive antennas per user can achieve the diversity order of $2(N_T - K + 1)$.

Next, to obtain an additional diversity order of the MU-STLC system achieved from the optimal US selection scheme selecting K users among U users, the same analysis used in [27] can be straightforwardly applied. Using the Craig's result, the conditional PEP between signal vectors $\mathbf{x}$ and $\check{\mathbf{x}}$ is given as

$$PEP\left(\mathbf{x}\to\check{\mathbf{x}}\middle|\,\tilde{\mathbf{H}}_{S_n}\right) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\exp\left(-\frac{\left\|\tilde{\beta}_{S_n}(\mathbf{x}-\check{\mathbf{x}})\right\|_F^2}{8\sigma_z^2\sin^2\theta}\right)d\theta \tag{59}$$

As shown in [27], the optimal US selection for ML detection with respect to the additional diversity order is to determine K users in the $\hat{n}$-th US, which satisfies

$$\hat{n} = \arg\max_{n\in\{1,2,\cdots,C_K^U\}}\min_{\mathbf{x},\check{\mathbf{x}}\neq\mathbf{x}}\left\|\tilde{\beta}_{S_n}(\mathbf{x}-\check{\mathbf{x}})\right\|_F^2 \tag{60}$$

Thus the PEP between $\mathbf{x}$ and $\check{\mathbf{x}}$ for the MU-STLC system with optimal US selection can be expressed as

$$PEP\left(\mathbf{x}\to\check{\mathbf{x}}\right) = E_{\tilde{\mathbf{H}}}\left\{\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\exp\left(-\frac{\left\|\tilde{\beta}_{S_{\hat{n}}}(\mathbf{x}-\check{\mathbf{x}})\right\|_F^2}{8\sigma_z^2\sin^2\theta}\right)d\theta\right\} \tag{61}$$

Then, the diversity order of the optimal US selection can be obtained by [27]

$$g_{US} = \min_{\mathbf{x},\check{\mathbf{x}}\neq\mathbf{x}}\left[\lim_{\sigma_z^2\to 0}\frac{\log PEP\left(\mathbf{x}\to\check{\mathbf{x}}\right)}{\log\sigma_z^2}\right] \tag{62}$$

Therefore, by employing the same analysis in [27], it can be easily shown from the lower bound on the PEP between $\mathbf{x}$ and

$\check{\mathbf{x}}$ that the upper bound on g_{US} is given as $U-K+1$. Thus it is concluded that the MU-STLC system with US selection can support an overall achievable upper diversity order of $G = 2(N_T-K+1)(U-K+1)$, where the product gain of $2(N_T-K+1)$ is achieved by STLC full receive antenna diversity gain of 2 and transmit antenna diversity gain of (N_T-K+1) while another of $(U-K+1)$ is offered by US diversity selection.

V. SIMULATION RESULTS

This section presents simulation results to evaluate the BER performance of the proposed efficient US selection algorithms such as inc-US and R-inc-US in static Raleigh flat-fading channels. The ZF-based MU-STLC system with N_T transmit antennas, U available users, and K selected users is represented by (N_T, U, K). Recall that each user has 2 receive antennas. The QPSK modulation is adopted. The SNR is defined by σ_x^2/σ_z^2. The optimal algorithm and LC-SUS are simulated here as reference and benchmark, respectively. In a case of $U = K$, no US selection is performed. Moreover, the additional BER reference lines are plotted using a form of c/SNR^G, where c is an appropriately selected positive constant and G is the diversity order.

Fig. 2 shows the BER performance of the proposed US selection algorithms for $(2, U, 2)$ MU-STLC systems with respect to SNR in dB. Here three different scenarios of 3, 4, and 6 users are considered for US selection. Note that the (2, 2, 2) scenario representing no US selection is simulated as other reference. It can be observed that as the total number of all available users increases, the performance gets improved. The (2, 2, 2), (2, 3, 2), (2, 4, 2), and (2, 6, 2) systems have the diversity orders of $G = 2$, 4, 6, and 10, respectively. To draw the BER reference black solid lines, the constants associated with $G = 2$, 4, 6, and 10, are selected as $c = 8.9$, 950, 1.8×10^5, and 1.9×10^{10}, respectively. Given that the analytical diversity orders are in good agreement with the simulation results, we can verify the accuracy of our diversity order analysis. It is shown that the proposed inc-US selection algorithm offers a large BER performance gain compared with no US selection case, which has only 2 receive diversity gain. For example, the proposed US selection algorithm for (2, 6, 2) yields about 8.3 dB gain at a BER of 10^{-3}. It is obtained by the product of a diversity order of 5 (MU diversity) and that of 2 (receive diversity). Although the LC-SUS, inc-US, and R-inc-US selection algorithms display worse performance than optimal one, their BER performance gap is minor.

Fig. 3 considers the (2, 4, 2), (3, 4, 2), and (4, 4, 2) systems, where the number of transmit antennas is different. The achievable diversity orders of (3, 4, 2) and (4, 4, 2) are given as $G = 12$ and 18, respectively, which are calculated by MU diversity and receive diversity in addition to transmit diversity. Here the constants $c = 2.3 \times 10^{11}$ and 3.8×10^{16}, respectively, are used for $G = 12$ and 18. For a comparison purpose, simulation results of the original SUS selection scheme (Algorithm 1) are also presented. It is shown that the

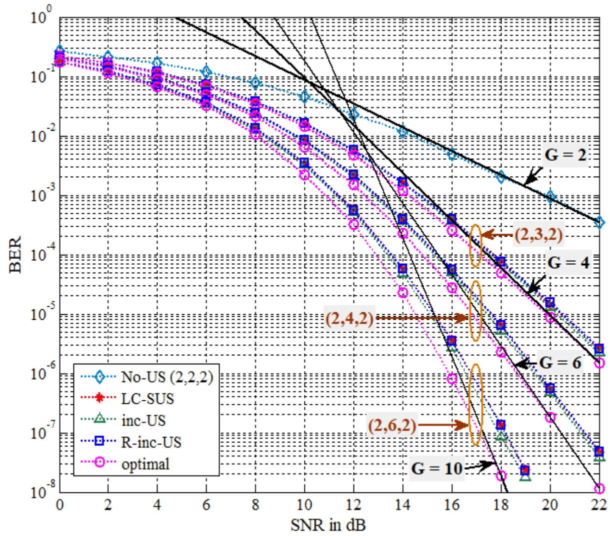

FIGURE 2. BER comparison of the proposed US and optimal selection algorithms for $(2, U, 2)$ MU-STLC systems.

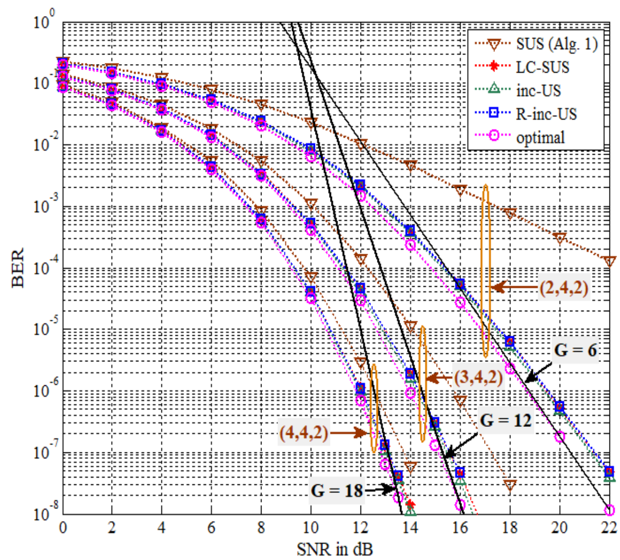

FIGURE 3. BER comparison of the proposed US and optimal selection algorithms for $(N_T, 4, 2)$ MU-STLC systems.

proposed algorithms outperform the original SUS selection algorithm, which cannot achieve the full diversity gain of the MU-STLC transmission systems. Compared with the optimal algorithm, the performance loss of the proposed algorithms is small. Note that, as expected from the diversity order analysis, the BER performance increases as the number of transmit antennas grows.

Fig. 4 compares four different MU-STLC systems with $N_T = U$. That is, (2, 2, 2), (3, 3, 2), (4, 4, 2), and (5, 5, 2) systems are considered. It is noticed that except for (2, 2, 2), the other three systems can achieve three kinds of diversities. Here (3, 3, 2) and (5, 5, 2) can achieve the diversity orders of $G = 8$ and 32, respectively, whose BER reference lines

TABLE 1. Computational complexity of RMs plus RSs.

System Parameters (N_T, U, K)	Optimal	Conventional SUS (Algorithm 2)	LC-SUS (Algorithm 3)	inc-US (Algorithm 4)	R-inc-US (Algorithm 5)
(2,3,2)	2,061	2,719	2,185	1,301	629
(2,4,2)	4,122	3,911	2,847	1,853	860
(2,6,2)	10,305	6,295	4,171	2,957	1,322
(3,4,2)	5,082	7,717	5,795	2,333	1,116
(4,4,2)	6,042	12,803	9,767	2,813	1,372
(5,5,2)	11,670	24,957	18,353	4,253	2,075
(8,12,8)	17,613,585	1,278,292	334,978	256,142	102,612
(10,12,8)	19,767,825	1,933,028	514,706	287,278	117,556
(12,12,8)	21,922,065	2,722,932	732,834	318,414	132,500
(14,12,8)	24,076,305	3,648,004	989,362	349,550	147,444

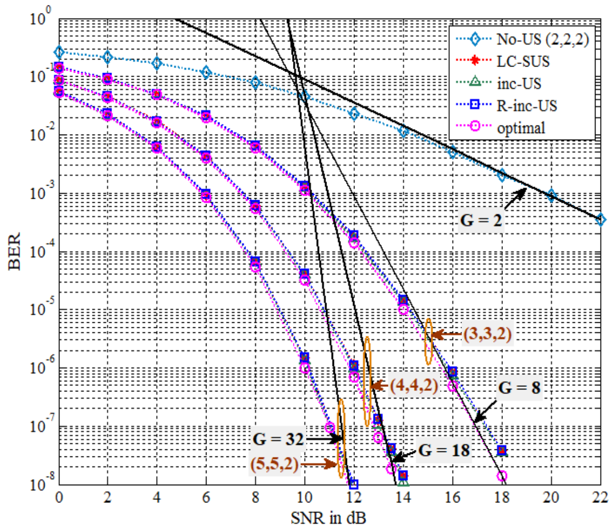

FIGURE 4. BER comparison of the proposed US and optimal selection algorithms for $(N_T, U, 2)$ MU-STLC systems with $N_T = U$.

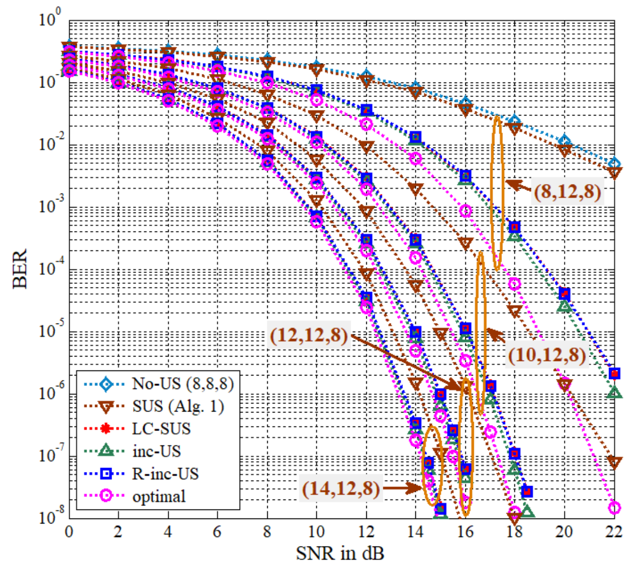

FIGURE 5. BER comparison of the proposed US and optimal selection algorithms for $(N_T, 12, 8)$ MU-STLC systems.

are plotted with $c = 3.5 \times 10^6$ and 6.0×10^{29}. It is clear that as the numbers of available users and transmit antennas simultaneously increase, the diversity gain is greatly improved. Especially in (5, 5, 2), the proposed efficient US selection algorithms offers BER results very close to that of optimal one.

The complexities of the proposed algorithms under the scenarios considered in Figs. 2, 3, and 4 are described in terms of RMs plus RSs in Table 1. We observe that the proposed inc-US and R-inc-US selection algorithms yield a complexity reduction compared to the LC-SUS and optimal one. Especially, the complexity of the R-inc-US selection algorithm is lowest for all the given systems and less than half that of the inc-US. Moreover, as the system parameter values increase, the rates of increase in complexity of the inc-US and R-inc-US are substantially smaller than those in the LC-SUS and optimal one.

In Fig. 5, the BER results of the proposed US selection algorithm are given for (8, 12, 8), (10, 12, 8), (12, 12, 8), and (14, 12, 8) systems with higher numbers of transmit antennas. It is also found that the proposed algorithms make a substantial performance gain over a (8, 8, 8) scenario. The SNR gains for (8, 12, 8), (10, 12, 8), (12, 12, 8), and (14, 12, 8) are about 6 dB, 10 dB, 12 dB, and 13.2 dB, respectively, at BER $= 10^{-2}$. Further, the performance of the proposed algorithms under (14, 12, 8) is close to that of the optimal one. The complexity comparison in the scenarios presented in Fig. 5 is also made in Table 1. Additionally, it is observed that the original SUS selection scheme (Algorithm 1) yields significantly worse BER performance than other algorithms especially for $N_T < U$. Compared to the system parameters used in Figs. 2, 3, and 4, the system parameter in Fig. 5 has larger values. In the scenarios with large numbers of transmit antennas and users, the optimal algorithm has a tremendous complexity. On the other hand, the proposed inc-US and R-inc-US algorithms have much smaller complexity than the LC-SUS. Furthermore, it is found

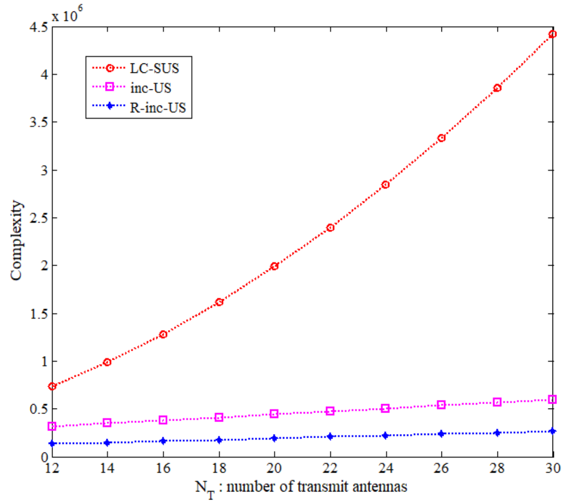

FIGURE 6. **Complexity comparison of the proposed LC-SUS, inc-US, and R-inc-US selection algorithms for (N_T, 12, 8) MU-STLC systems.**

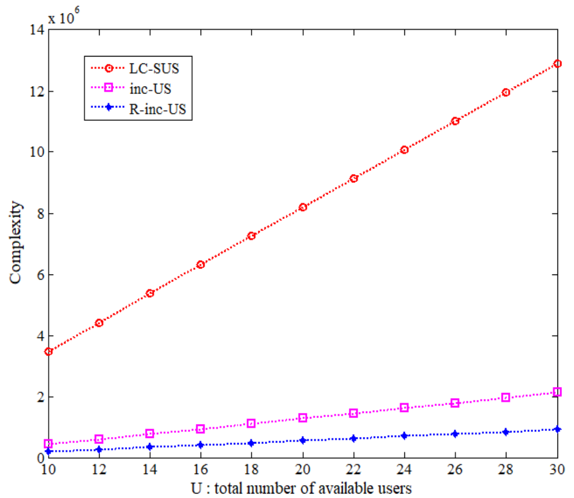

FIGURE 7. **Complexity comparison of the proposed LC-SUS, inc-US, and R-inc-US selection algorithms for (30, U, 8) MU-STLC systems.**

that the proposed R-inc-US algorithm can offer more than 2 times lower complexity than the proposed inc-US algorithm. Hence the complexity reduction of the proposed R-inc-US is remarkable for large numbers of transmit antennas and users.

Next, we investigate the complexity of the proposed algorithms in terms of the number of transmit antennas for $U = 12$ and $K = 8$ in Fig. 6. It is obvious as expected that the complexity of the proposed algorithms grows as the number of transmit antennas increases. The complexity slope of the LC-SUS is much steeper than the R-inc-US and inc-US. The R-inc-US has the lowest complexity. Note that since the optimal algorithm has a huge complexity, its complexity is not included in the plot. Fig. 7 compares the complexity of the proposed algorithms as a function of the number of selected users for $N_T = 30$ and $K = 8$. It is found that the complexity of the inc-US and R-inc-US is lower than that of LC-SUS and the R-inc-US has still the smallest complexity.

VI. CONCLUSION

This paper examines the MU-STLC system based on ZF precoding with US selection and it is shown that it can achieve a large performance gain compared to the system without US selection. The conventional SUS selection algorithm with low-complexity has been modified to be acceptable to MU-STLC transmission systems and thus used as a benchmark for comparison. A more efficient incremental US selection algorithm has been proposed by adopting an incremental strategy in company with the recursive computation of the block matrix inverse. The proposed inc-US algorithm is capable of achieving near-optimal performance with very low complexity. In addition, the inc-US selection scheme has been modified to have more reduced complexity. It has been achieved by exploiting the fact that the matrix-by-matrix computation involved with MU-STLC transmission has recurring operations at each incremental step, which is due to the orthogonal STLC encoding structure. Moreover, we have analyzed an achievable upper diversity order for the MU-STLC system with optimal US selection, which can offer receive and transmit antennas diversity as well as MU diversity. Simulation results have verified the analytical diversity orders.

REFERENCES

[1] J. Joung, "Space–time line code," *IEEE Access*, vol. 6, pp. 1023–1041, 2018.
[2] S.-C. Lim and J. Joung, "Transmit antenna selection for space–time line code systems," *IEEE Trans. Commun.*, vol. 69, no. 2, pp. 786–798, Feb. 2021.
[3] J. Joung and B. C. Jung, "Machine learning based blind decoding for space–time line code (STLC) systems," *IEEE Trans. Veh. Technol.*, vol. 68, no. 5, pp. 5154–5158, May 2019.
[4] J. Joung and E.-R. Jeong, "Multiuser space–time line code with optimal and suboptimal power allocation methods," *IEEE Access*, vol. 6, pp. 51766–51775, 2018.
[5] J. Joung and J. Choi, "Multiuser space–time line code with transmit antenna selection," *IEEE Access*, vol. 8, pp. 71930–71939, 2020.
[6] S. Kim, "Efficient transmit antenna subset selection for multiuser space–time line code systems," *Sensors*, vol. 21, no. 8, p. 2690, Apr. 2021.
[7] G. Dimić and N. D. Sidiropoulos, "On downlink beamforming with greedy user selection: Performance analysis and a simple new algorithm," *IEEE Trans. Signal Process.*, vol. 53, no. 10, pp. 3857–3868, Oct. 2005.
[8] T. Yoo and A. Goldsmith, "On the optimality of multiantenna broadcast scheduling using zero-forcing beamforming," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 3, pp. 528–541, Mar. 2006.
[9] T. Yoo and A. Goldsmith, "Sum-rate optimal multi-antenna downlink beamforming strategy based on clique search," in *Proc. GLOBECOM IEEE Global Telecommun. Conf.*, Nov. 2005, pp. 1510–1514.
[10] G. Lee and Y. Sung, "A new approach to user scheduling in massive multi-user MIMO broadcast channels," *IEEE Trans. Commun.*, vol. 66, no. 4, pp. 1481–1495, Apr. 2018.
[11] S. Huang, H. Yin, H. Li, and V. C. M. Leung, "Decremental user selection for large-scale multi-user MIMO downlink with zero-forcing beamforming," *IEEE Wireless Commun. Lett.*, vol. 1, no. 5, pp. 480–483, Oct. 2012.
[12] S. Huang, H. Yin, J. Wu, and V. C. M. Leung, "User selection for multiuser MIMO downlink with zero-forcing beamforming," *IEEE Trans. Veh. Technol.*, vol. 62, no. 7, pp. 3084–3097, Sep. 2013.
[13] A. Bayesteh and A. K. Khandani, "On the user selection for MIMO broadcast," *IEEE Inf. Theory*, vol. 54, no. 3, pp. 1086–1107, Mar. 2008.

[14] Z. Chen, J. Li, and J. Huang, "Downlink multi-user scheduling with zero-forcing precoding in cognitive hetnets: From performance tradeoff perspective," *IEEE Access*, vol. 6, pp. 50131–50141, 2018.

[15] G. Gupta and A. K. Chaturvedi, "Conditional entropy based user selection for multiuser MIMO systems," *IEEE Commun. Lett.*, vol. 17, no. 8, pp. 1628–1631, Aug. 2013.

[16] J. Choi and F. Adachi, "User selection criteria for multiuser systems with optimal and suboptimal LR based detectors," *IEEE Trans. Signal Process.*, vol. 58, no. 10, pp. 5463–5468, Oct. 2010.

[17] L. Bai, C. Chen, J. Choi, and C. Ling, "Greedy user selection using a lattice reduction updating method for multiuser MIMO systems," *IEEE Trans. Veh. Technol.*, vol. 60, no. 1, pp. 136–147, Jan. 2011.

[18] J. Jose, A. Ashikhmin, T. L. Marzetta, and S. Vishwanath, "Pilot contamination and precoding in multi-cell TDD systems," *IEEE Trans. Wireless Commun.*, vol. 10, no. 8, pp. 2640–2651, Aug. 2011.

[19] R. Hunger, "Floating point operations in matrix-vector calculus (version 1.3)," Technische Universitat Munchen, Munchen, Germany, Tech. Rep., 2007.

[20] S. Kim, "Efficient transmit antenna selection for receive spatial modulation-based massive MIMO," *IEEE Access*, vol. 8, pp. 152034–152044, 2020.

[21] S. Kim, "Decoupled transmit and receive antenna selection for precoding-aided spatial modulation," *IEEE Access*, vol. 9, pp. 57829–57840, 2021.

[22] J. Zheng, "Fast receive antenna subset selection for pre-coding aided spatial modulation," *IEEE Commun. Lett.*, vol. 4, no. 3, pp. 317–320, Jun. 2015.

[23] P. Wen, X. He, Y. Xiao, P. Yang, R. Shi, and K. Deng, "Efficient receive antenna selection for pre-coding aided spatial modulation," *IEEE Commun. Lett.*, vol. 22, no. 2, pp. 416–419, Feb. 2018.

[24] P.-H. Lin and S.-H. Tsai, "Performance analysis and algorithm designs for transmit antenna selection in linearly precoded multiuser MIMO systems," *IEEE Trans. Veh. Technol.*, vol. 61, no. 4, pp. 1698–1708, May 2012.

[25] J. H. Winters, J. Salz, and R. D. Gitlin, "The impact of antenna diversity on the capacity of wireless communication systems," *IEEE Trans. Commun.*, vol. 42, no. 234, pp. 1740–1751, Feb. 1994.

[26] J. Luo, S. Wang, F. Wang, and W. Zhang, "Generalized precoding-aided spatial modulation via receive antenna transition," *IEEE Wireless Commun. Lett.*, vol. 8, no. 3, pp. 733–736, Jun. 2019.

[27] X. Jin and D. Cho, "Diversity analysis on transmit antenna selection for spatial multiplexing systems with ML detection," *IEEE Trans. Veh. Technol.*, vol. 62, no. 9, pp. 4653–4658, Nov. 2013.

SANGCHOON KIM was born in Jeju, South Korea, in 1965. He received the B.S. degree from Yonsei University, Seoul, South Korea, in 1991, and the M.E. and Ph.D. degrees from the University of Florida, Gainesville, FL, USA, in 1995 and 1999, respectively, all in electrical and computer engineering. From 2000 to 2005, he was a Senior Research Engineer at LG Corporate Institute of Technology, Seoul, South Korea, and the Chief Research Engineer at the LG Electronics, Anyang, South Korea, working on a range of research projects in the field of wireless/mobile communications. Since 2005, he has been with Dong-A University, Busan, South Korea, where he is currently a Professor with the Department of Electronics Engineering. His research interests include wireless/mobile communications, signal processing, and antenna design.