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A Polynomial Decentralized Controller Design for a Large-Scale Nonlinear System: SOS Approach

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ABSTRACT This paper proposes a novel approach for designing a decentralized controller to stabilize the large-scale nonlinear system. Unlike the previous studies, the polynomial system framework is employed to model the nonlinear large-scale system to decrease not only the modeling error but also the complexity and computational load significantly. Especially, in case the large-scale nonlinear systems consist of non-polynomial forms (such as sine, cosine, and so on), synthesizing the controller for this system becomes much more challenging. By putting non-polynomial terms inside the matrices, a new approach for designing a decentralized polynomial controller is presented to eliminate the impacts of nonlinear terms and stabilize the system. Based on Lyapunov methodology, the conditions for synthesizing a decentralized polynomial controller expressed under the framework of Sum-of-Square (SOS) are derived in the main theorems. Finally, the effectiveness and superiority of the proposed method are demonstrated in two illustrative examples.

INDEX TERMS Large-scale nonlinear system, polynomial system, polynomial decentralized controller design, SOS.

I. INTRODUCTION

In recent years, it is witnessing the rapid development of modern technology, therefore, the physical systems in practice increasingly become large and complex. A complicated system with a large size is called a ''large-scale system'' [1], [2]. In general, a large-scale system is a combination of a set of interconnection subsystems that interact together. There exist a wide range of applications of the large-scale system in reality such as biological systems, energy systems, automated highway systems, and so on. Due to the popularity in the reality of the large-scale system, for the past decades, there were a lot of studies paying attention to solving the problems of these systems [3]–[16]. For example, an approach to design a decentralized controller for a large-scale nonlinear system with uncertainties was proposed in [4], in which an observer was synthesized to estimate the unmeasurable states. In paper [5], an approach for controller design based on the backstepping control strategy was proposed. However, the limitations of the method in paper [5] are that the backstepping controller in [5] was merely applied for a large-scale nonlinear system with a strict feedback form, otherwise, this method was failed. Additionally, the interconnected terms in this work must satisfy the bounded constraint (see Assumption 3 [5]). In paper [6], the distributed controller was synthesized for the discrete-time networked large-scale system where the communication between the sensor and controller has an issue. Moreover, an observer-based on the sliding-mode technique was synthesized for large-scale nonlinear systems in [7], where the interconnection terms were known, and the uncertainties are bounded. The predictive control combined with an observer was designed for the discrete-time networked large-scale linear system in [8]. The subsystems of large-scale systems in [8] are connected to others via the interconnection parts with the time-delay communication. In paper [13], the large-scale nonlinear system was modeled by decomposing this system into coupled lower-order subsystems in which the computational load was reduced. Besides, the methods to design the adaptive robust fault-tolerant control and observer-based adaptive fault-tolerant control for the large-scale nonlinear system were investigated in papers [14] and [15], respectively to make the error tracking of the system approach to a small region of zero. An event-triggered fault-tolerant adaptive controller was studied for the strict-feedback discrete-time multi-agent systems [16].

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It is noted that directly designing a nonlinear controller for a large-scale nonlinear system is complicated and much more challenging. To overcome this difficulty, the large-scale nonlinear systems are linearized to the large-scale linear system [8], [17], [18]. For example, in paper [18], a decentralized controller was designed for the large-scale power system in which this system was linearized to the large-scale linear systems. Besides, to deal with the complexity of the nonlinear system, an approach called ''model reduction'' method has been introduced to represent the nonlinear system [19]–[21]. However, the drawbacks of these modeling methods are that the obtained large-scale linear systems are approximate to the original large-scale nonlinear system and there exist modeling errors between the obtained system and the original system. These modeling errors will impact the performance of the system.

Nowadays, Takagi-Sugeno (T-S) fuzzy model [22] has increasingly become one of the most popular methods for modeling nonlinear systems. Based on the nonlinearity sector and linearization in [23] and [24], the nonlinear system is transformed into a T-S fuzzy system which composes of a set of linear subsystems and IF-THEN fuzzy rules. Furthermore, there has been increasing interest in applying the T-S fuzzy model to model a large-scale nonlinear system. Numerous studies [25]–[37] concentrating on controller design for large-scale T-S fuzzy systems have been developed in the past few years. For example, a decentralized controller was studied for a large-scale T-S fuzzy system [26] where the interconnection parts had to fulfill both the matching conditions and bounded constraints. Additionally, in [29], an approach based on the technique to synthesize the decentralized controller was investigated for a large-scale T-S fuzzy system with uncertainties and disturbances. In [30], a fuzzy filter was designed for a large-scale T-S fuzzy system in which the premise variables of the system and filter were different. The robust observer has been designed for the large-scale T-S fuzzy system to estimate the unknown states as well as eliminate the impact of the uncertainties. However, there exist limitations for modeling the large-scale nonlinear system under the framework of the large-scale T-S fuzzy system are that if the number of nonlinear terms of the system and/or the number of subsystems increase then the numbers of fuzzy rules and the number of conditions that need to satisfy for the observer/controller synthesis will considerably grow up. This issue considered as ''rule-explosion'' problem [45, p.273]. That leads to the computational load burden, and the complexity of the previous methods will significantly increase. The controller synthesis for the T-S large-scale nonlinear system with the existence of fault and Denial of Service (DoS) was investigated in [36] and [37], respectively.

Besides, recently, a new approach for modeling the nonlinear system called ''polynomial system'' was proposed in [38] and [39]. The polynomial system can represent the original nonlinear system under the framework of the linear system; however, the system matrices are expressed in terms of the polynomial form instead of the constant matrices in the linear system. In other words, a polynomial system is an extended form of a linear system. The advantage of the polynomial system is that the nonlinear terms which need to linearize are put inside the system matrices, therefore, it is unnecessary to linearize the nonlinear system. Nowadays, with the support of SOS tools [40], the polynomial system increasingly plays an important role in resolving the problems of nonlinear systems. Several papers studied the polynomial system [41]–[43] in past few years. For instance, the controllers were synthesized for a discrete-time polynomial system with disturbance and the continuous polynomial with uncertainties in [44] and [45], respectively. Unfortunately, relied on our best knowledge, there is not any previous paper employing the polynomial system for modeling large-scale nonlinear systems.

With the aforementioned discussions, it is seen that the large-scale nonlinear system consists of a large number of nonlinear terms and subsystems. Therefore, if we apply the methods in [8], [17], [18] to linearize the nonlinear terms then the modeling errors become large. Additionally, when the T-S fuzzy model is employed to model the large-scale nonlinear system [25]–[37], the number of fuzzy rules will exponentially increase and the controller procedure becomes much more complicated and the computational load also significantly increase. Due to these reasons, we are inspired to propose a method to synthesize a decentralized polynomial controller for the large-scale polynomial system. The contributions of this paper are emphasized in the following threefold:

1) Firstly, the polynomial framework is employed to represent the nonlinear large-scale system that has not been considered in any previous paper. This approach will let us directly apply the powerful methodologies of the linear system to synthesize the controller for the nonlinear large-scale system instead of using the complicated nonlinear methods.
2) Unlike previous studies [8], [17], [18], the nonlinear terms of the nonlinear large-scale system in this work do not need to linearize. It leads to reduce the modeling error significantly. In addition, the proposed method in this paper also assists to decrease the complexity and computational load with respect to the methods in [25]–[37] when applying the T-S fuzzy framework to model the large-scale nonlinear system.
3) Especially, when the nonlinear large-scale system consists of the non-polynomial terms sin(x), cos(x), tang(x), square root, and so on, it is not able to apply the polynomial system to present the nonlinear large-scale system because we cannot put the non-polynomial terms sin(x), cos(x), tang(x), square roots and so on inside the system matrices. These non-polynomial terms will make the controller design processing much more challenging. To overcome this difficulty, in this work, the non-polynomial terms will be grouped in the time-varying matrices and then a new approach is

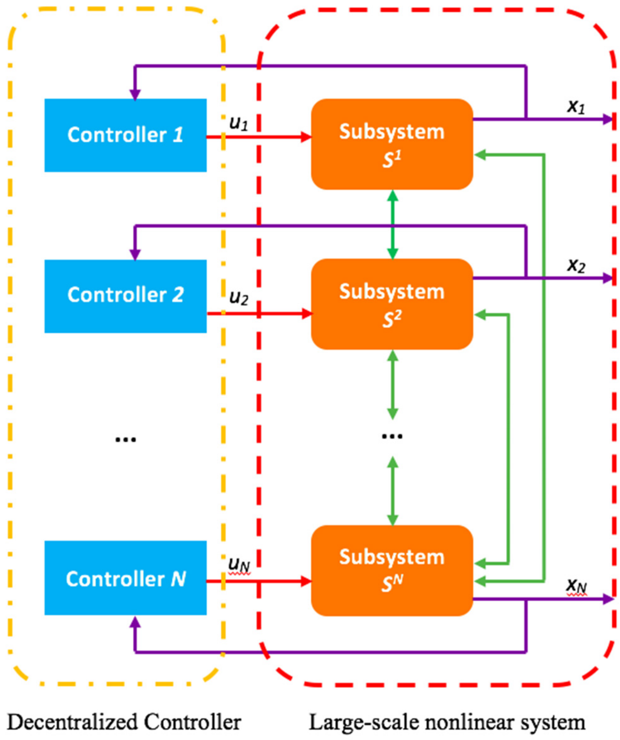

FIGURE 1. Structure of the large-scale nonlinear system.

proposed to synthesize a controller for eliminating the effects of these terms and stabilizing the system.

The remainder of this paper is organized as follows. The system model is described in Section 1 and the considered problems are pointed out in this section as well. The proposed method to design a polynomial controller for a large-scale nonlinear system with and without non-polynomial terms is presented in Section 3 and Section 4, respectively. The illustrative examples are shown in Section 5. Finally, the conclusions are presented in Section 6.

Notations: In this paper, Θ^{-1} and Θ^T stand for inverse and transpose matrix Θ, respectively. $\Theta > 0$ ($\Theta < 0$) infers that the matrix Θ is the positive (negative) definite matrix. I indicates the identity matrix. The symbol $\|\bullet\|$ stands for Euclidean Norm and $\Re^{n_l \times m_l}$ denotes a set of $n_l \times m_l$ matrices.

II. SYSTEM DESCRIPTION AND PROBLEM STATEMENT

A. SYSTEM DESCRIPTION

Considered a class of nonlinear systems expressed under the framework of the large-scale nonlinear system with N interconnection subsystems illustrated in Fig. 1 as follows.

$$\begin{cases} \dot{x}_1 = f_1(x_1)x_1 + g_1(x_1)u_1 + h_1(x) \\ \vdots \\ \dot{x}_l = f_l(x_l)x_l + g_l(x_l)u_l + h_l(x) \\ \vdots \\ \dot{x}_N = f_N(x_N)x_N + g_N(x_N)u_N + h_N(x) \end{cases}$$

The general framework of the l^{th} subsystem is presented in the following equation:

$$\dot{x}_l = f_l(x_l)x_l + g_l(x_l)u_l + h_l(x) \quad l = 1, 2, \ldots, N. \quad (1)$$

where $x_l \in \Re^{n_l}$ and $u_l(t) \in \Re^{m_l}$ are the vectors of state variables and inputs of the l^{th} subsystem. $x = [x_1, x_2, \ldots, x_N]^T$ is the state variable vector which consists of all state variables x_l. $f_l(x_l)$ and $g_l(x_l)$ are the system and input nonlinear functions, respectively. $h_l(x)$ is the nonlinear function of interconnection parts. There are two scenarios for the nonlinear large-scale system (1).

Scenario 1: The functions $f_l(x)$, $g_l(x)$, and $h_l(x)$ merely contain the polynomial terms in x_l, then the system (1) is modeled under the framework of the polynomial large-scale system as follows,

$$\dot{x}_l = A^l(x_l)x_l + B^l(x_l)u_l(t) + \sum_{m=1, m\neq l}^{N} M^{lm}(x_l)x_m \qquad l, m = 1, 2, \ldots, N. \quad (2)$$

in which x_m is the states of m^{th} subsystem, $l \neq m$, which impacts on the l^{th} subsystem. $A^l(x_l) \in \Re^{n_l \times n_l}$ and $B^l(x_l) \in \Re^{n_l \times m_l}$ are the polynomial system and input matrices, respectively. $\sum_{m=1, m\neq l}^{N} M^{lm}(x_l)x_m$ is the interconnection terms that express the interaction of l^{th} subsystem with other subsystems.

Scenario 2: In practice, however, in many large-scale nonlinear systems, the nonlinear function $f_l(x)$ and $g_l(x)$ are not only dependent on the polynomial form of x_l but also include the non-polynomial form of x_l such as $\sin(x_l)$, $\cos(x_l)$, and so on. Hence, in that case, it is impossible to represent the system (1) under the format of the polynomial large-scale system (2). In this case, system (1) is modeled as follows

$$\dot{x}_l = [A^l(x_l) + \tilde{A}^l(x_l)]x_l + [B^l(x_l) + \tilde{B}^l(x_l)]u_l(t) + \sum_{m=1, m\neq l}^{N} M^{lm}(x_l)x_m, \quad l = 1, 2, \ldots, N. \quad (3)$$

where $\tilde{A}^l(x_l)$ and $\tilde{B}^l(x_l)$ consist of the non-polynomial terms such as $\sin(x_l)$, $\cos(x_l)$, and so on.

B. PROBLEM STATEMENT

Suppose that the system (1) is unstable, the objective of this paper is to synthesize a controller to stabilize the system (1). To design a controller for a large-scale nonlinear system (1), many papers directly synthesize controllers for the nonlinear framework [3]–[15]. However, the methods in these works were complicated and the controller merely applied for a specific nonlinear form such as the method in [5] is only applied for the strict feedback nonlinear system. Some methods apply the linearization approach [8], [17], [18] to convert to a large-scale linear system. But this approach will cause modeling errors. Furthermore, in several previous studies, system (1) is modeled under a large-scale T-S fuzzy system [24]–[37]. However, when the number of subsystems and nonlinear terms increases, the number of fuzzy rules will exponentially grow up and it will cause the "rule explosion" problem [45, p.273]. To overcome these challenges, in this study, system (1) is represented under the framework of the polynomial large-scale system (2) and (3) in Scenarios 1 and 2, respectively. Therefore, in this paper, we will propose two methods to synthesize the controller for stabilizing the system (2) and (3) instead of the original large-scale nonlinear system (1) in the next sections.

To design the controller, the following lemmas and propositions are needed for proof of the controller procedure in the consequent sections.

Lemma 1 ([29]): With an arbitrary vector $T_i \in \Re^n$, the following Tchebychev's inequality satisfy

$$\left[\sum_{i=1}^{N} T_i\right]^T \left[\sum_{i=1}^{N} T_i\right] \leq N \times \sum_{i=1}^{N} (T_i)^T T_i$$

Lemma 2 (S- Procedure Lemma [44]): Taken into consideration two arbitrary quadratic forms $\Pi(x) \in \Re^n$ and $\Delta(x) \in \Re^n$, then $\Pi(x) < 0$ for all $x \in \Re^n - \{0\}$ with $\Delta(x) \leq 0$ if and only if there exists $\lambda \geq 0$ such that

$$\Pi(x) - \lambda\Delta(x) < 0$$

Proposition 1 ([39]): Taking into consideration the function $h(x(t))$. The function $h(x(t))$ is called Sum-Of-Square (SOS) if it can be expressed in the form $h(x(t)) = \sum_{i=1}^{n} [c_i(x(t))]^2$ in which $c_i(x(t))$ is the polynomial form in $x(t)$. If $h(x(t))$ is a SOS then it infers that $h(x(t)) \geq 0$.

Proposition 2 ([39]): Consider a polynomial matrix $\Gamma(x)$ in x, and the vector v is independent on x. If $v^T\Gamma(x)v$ is a SOS then it concludes that $\Gamma(x) \geq 0$.

Remark 1: The nonlinear large-scale system with polynomial forms that merely contains the non-negative integer exponentiation of variables such as $x^2, x^3, x^4 \ldots$. While the nonlinear large-scale system with non-polynomial forms that consist of not only the polynomial terms but also the non-polynomial terms such as $\sin(x)$, $\cos(x)$, $tang(x)$, square roots of the variables, and so on. With the existence of the non-polynomial terms, it is unable to apply the linear polynomial system framework to model the nonlinear large-scale system as in Eq. (2). Thus, in this paper, the non-polynomial terms are put in time-varying non-polynomial matrices $\tilde{A}^l(x_l)$ and $\tilde{B}^l(x_l)$ and then the new approach for designing a controller to eliminate the effects of these matrices is proposed in the consequent sections.

Remark 2: It should be noted that the nonlinear large-scale system is complex. There exists the method only applied for the large-scale nonlinear system with strick feedback form [5], otherwise, it is failed to synthesize to design a controller for this system. Several methods apply the linearization methods [8], [17], [18] to transform into a large-scale linear system. However, linearizing the nonlinear large-scale system will cause modeling errors that lead to reduce the accuracy of the control system. Another method proposed in papers [25]–[37] is that the nonlinear large-scale system is represented by a large-scale T-S fuzzy system. But the drawback of this method is that if the number of subsystems of the large-scale nonlinear system increase, then the number of fuzzy rules will exponentially grow up. Thus, both complexity and computational load will considerably increase.

III. CONTROLLER SYNTHESIS FOR THE LARGE-SCALE NONLINEAR SYSTEM WITHOUT NON-POLYNOMIAL TERMS

In this section, a decentralized polynomial controller will be designed for the nonlinear large-scale system in scenario 1 which is modeled in the system (2).

Consider the decentralized polynomial controller form for the system (2) as follows

$$u_l(t) = -K^l(x_l)x_l \tag{4}$$

Substituting (4) into (2), the closed-loop polynomial large-scale system is obtained

$$\dot{x}_l = A^l(x_l)x_l - B^l(x_l)K^l(x_l)x_l + \sum_{m=1,m\neq l}^{N} M^{lm}(x_l)x_m\} \tag{5}$$

Theorem 1: The system (2) with the controller (4) is stabilized if there exists the symmetric matrix P_l, polynomial matrix $K^l(x_l)$, and constant α_l satisfying the following conditions:

$$v_{1l}^T\{P_l - \varepsilon_{1l}I\}v_{1l} \text{ is SOS} \tag{6}$$

$$-v_{2l}^T\{\Xi_l(x_l) + \varepsilon_{2l}(x_l)I\}v_{2l} \text{ is SOS} \tag{7}$$

where

$$\Xi_l(x_l) = \begin{bmatrix} \Upsilon_l(x_l) + \alpha_l\left(\sum_{m=1,m\neq l}^{N}(N-1)(M^{ml}(x_l))^T M^{ml}(x_l)\right) & P_l \\ P_l & -\alpha_l I \end{bmatrix} \tag{8}$$

$$\Upsilon_l = (A^l(x_l))^T P_l + P_l A^l(x_l) - (K^l(x_l))^T (B^l(x_l))^T P_l - P_l B^l(x_l)K^l(x_l) \tag{9}$$

v_{1l}, v_{2l} are the vector dependent on x_l, $\varepsilon_{1l} > 0$ is constant, and $\varepsilon_{2l}(x_l) > 0$.

Proof: Choose the Lyapunov function as follows

$$V(t) = \sum_{l=1}^{N} V_l(t) = \sum_{l=1}^{N} x_l^T P_l x_l \tag{10}$$

Condition (6) of Theorem 1 implies that $P_l > 0$, hence, $V(t) > 0$.

Taking the derivative of (10) yields

$$\dot{V}(t) = \sum_{l=1}^{N} \dot{V}_l(t) = \sum_{l=1}^{N} [\dot{x}_l^T P_l x_l + x_l^T P_l \dot{x}_l] \tag{11}$$

Substituting (5) into (11) obtains

$$\begin{aligned}\dot{V}(t) = \sum_{l=1}^{N} \{[A^l(x_l)x_l - B^l(x_l)K^l(x_l)x_l \\ + \sum_{m=1,m\neq l}^{N} M^{lm}(x_l)x_m]^T \\ \times P_l x_l + x_l^T P_l[A^l(x_l)x_l - B^l(x_l)K^l(x_l)x_l \\ + \sum_{m=1,m\neq l}^{N} M^{lm}(x_l)x_m]\}\end{aligned} \quad (12)$$

$$\begin{aligned}\dot{V}(t) = \sum_{l=1}^{N} \{x_l^T[((A^l(x_l))^T P_l + P_l A^l(x_l)) \\ - (K^l(x_l))^T(B^l(x_l))^T P_l - P_l B^l(x_l)K^l(x_l))]x_l \\ + \sum_{l=1}^{N}\{[\sum_{m=1,m\neq l}^{N} M^{lm}(x_l)x_m]^T P_l x_l \\ + x_l^T P_l[\sum_{m=1,m\neq l}^{N} M^{lm}(x_l)x_m]\}\end{aligned} \quad (13)$$

Let us define

$$\delta_l(\circ) = \sum_{m=1,m\neq l}^{N} M^l(x_l)x_m \quad (14)$$

From (13) and (14), it infers that

$$\begin{aligned}\dot{V}(t) &= \sum_{l=1}^{N}\{[x_l^T[((A^l(x_l))^T P_l + P_l A^l(x_l)) - (K^l(x_l))^T(B^l(x_l))^T \\ &\quad \times P_l - P_l B^l(x_l)K^l(x_l)]x_l + \delta_l^T(x)P_l x_l + x_l^T P_l\delta_l(x)\} \\ &= \sum_{l=1}^{N}\{\begin{bmatrix} x_l \\ \delta_l(\circ)\end{bmatrix}^T \Xi_l(x_l)\begin{bmatrix} x_l \\ \delta_l(\circ)\end{bmatrix}\}\end{aligned} \quad (15)$$

where

$$\Xi_l(x_l) = \begin{bmatrix}\Upsilon_l(x_l) & P_l \\ P_l & 0\end{bmatrix}$$
$$\Upsilon_l(x_l) = (A^l(x_l))^T P_l + P_l A^l(x_l) - (K^l(x_l))^T(B^l(x_l))^T P_l - P_l B^l(x_l)K^l(x_l)$$

From (15), it is seen that if condition (7) of Theorem 1 holds, then $\dot{V}(t) < 0$. However, because there exists element zero in the right bottom corners of $\Xi_l(x_l)$, it means that $\Xi_l(x_l)$ could not be a negative definite matrix. In order to cope with this difficulty, the following steps are needed.

From (14), we have

$$\begin{aligned}&\sum_{l=1}^{N} \delta_l^T(\circ)\delta_l(\circ) \\ &= \sum_{l=1}^{N}\left\{\left[\sum_{m=1,m\neq l}^{N} M^{lm}(x_l)x_m\right]^T\left[\sum_{m=1,m\neq l}^{N} M^{lm}(x_l)x_m\right]\right\} \\ &= \sum_{l=1}^{N}\left\{\left[\sum_{m=1,m\neq l}^{N} M^{ml}(x_l)x_l\right]^T\left[\sum_{m=1,m\neq l}^{N} M^{ml}(x_l)x_l\right]\right\}\end{aligned} \quad (16)$$

Based on Lemma 1, (16) becomes

$$\begin{aligned}&\sum_{l=1}^{N} \delta_l^T(\circ)\delta_l(\circ) \\ &\leq \sum_{l=1}^{N}\left\{x_l^T\left[(N-1)\sum_{m=1,m\neq l}^{N}(M^{ml}(x_l))^T(M^{ml}(x_l))\right]x_l\right\}\end{aligned} \quad (17)$$

From (17), it infers that

$$\begin{aligned}&\sum_{l=1}^{N} \delta_l^T(\circ)\delta_l(\circ) \\ &\quad - \sum_{l=1}^{N}\left\{x_l^T\left[(N-1)\sum_{m=1,m\neq l}^{N}(M^{ml}(x_l))^T(M^{ml}(x_l))\right]x_l\right\} \\ &= \sum_{l=1}^{N}\left\{\left\{\begin{bmatrix} x_l \\ \delta_l(\circ)\end{bmatrix}^T \Omega_l(x_l)\begin{bmatrix} x_l \\ \delta_l(\circ)\end{bmatrix}\right\}\right\} \leq 0\end{aligned} \quad (18)$$

where

$$\Omega_l(x_l) = \begin{bmatrix} -\left((N-1)\sum_{m=1,m\neq l}^{N}(M^{ml}(x_l))^T(M^{ml}(x_l))\right) & 0 \\ 0 & I\end{bmatrix}$$

From (15) and (18), on the basis of Lemma 2 (S-Procedure), if there exists a positive constant α_l satisfying

$$\begin{aligned}&\sum_{l=1}^{N}\{\begin{bmatrix} x_l \\ \delta_l(\circ)\end{bmatrix}^T\begin{bmatrix}\Upsilon_l(x_l) & P_l \\ P_l & 0\end{bmatrix}\begin{bmatrix} x_l \\ \delta_l(\circ)\end{bmatrix}\} \\ &\quad - \alpha_l\sum_{l=1}^{N}\left\{\begin{bmatrix} x_l \\ \delta_l(\circ)\end{bmatrix}^T \Omega_l(x_l) \\ &\quad \times \begin{bmatrix} x_l \\ \delta_l(\circ)\end{bmatrix}\right\} \\ &= \sum_{l=1}^{N}\{\begin{bmatrix} x_l \\ \delta_l(\circ)\end{bmatrix}^T \\ &\quad \times \begin{bmatrix}\Upsilon_l(x_l) + \alpha_l\left(\sum_{m=1,m\neq l}^{N}(N-1)(M^{ml}(x_l))^T M^{ml}(x_l)\right) & P_l \\ P_l & -\alpha_l I\end{bmatrix}\begin{bmatrix} x_l \\ \delta_l(\circ)\end{bmatrix}\} < 0\end{aligned} \quad (19)$$

then

$$\dot{V}(t) \leq \sum_{l=1}^{N}\{\begin{bmatrix} x_l \\ \delta_l(\circ)\end{bmatrix}^T\begin{bmatrix}\Upsilon_l(x_l) & P_l \\ P_l & 0\end{bmatrix}\begin{bmatrix} x_l \\ \delta_l(\circ)\end{bmatrix}\} < 0 \quad (20)$$

From (19), it is obvious that if the condition (7) of Theorem 1 holds, then (19) is ensured, it means that (20) is guaranteed and $\dot{V}(t) < 0$. Hence, the proof is completed.

It should be noted that condition (7) of Theorem 1 is PBMI (Polynomial Bilinear Matrix Inequality) which is difficult to solve in Matlab. Therefore, it must be transformed into PLMI (Polynomial Linear Matrix Inequality) in the following theorem.

Theorem 2: The system (2) with the controller (4) is stabilized if there exist the symmetric P_l, polynomial matrix $K_l(x_l)$ and constant α_l such that the following conditions hold

$$v_{1l}^T\{W_l - \varepsilon_{1l}I\}v_{1l} \text{ is SOS} \tag{21}$$

$$-v_{2l}^T\{\Delta_l(x_l) + \varepsilon_{2l}(x_l)I\}v_{2l} \text{ is SOS} \tag{22}$$

where

$$\Delta_l(x_l) = \begin{bmatrix} \Phi_l(x_l) & W_l(\bar{M}^l(x_l))^T & \rho_l I \\ \bar{M}^l(x_l)W_l & -\rho_l I & 0 \\ \rho_l I & 0 & -\rho_l I \end{bmatrix} \tag{23}$$

$$\Phi_l(x_l) = W_l(A^l(x_l))^T + A^l(x_l)W_l - (R^l(x_l))^T(B^l(x_l))^T - B^l(x_l)R^l(x_l) \tag{24}$$

$$\bar{M}^l(x_l) = \left[\sqrt{(N-1)}M^{1l}(x_l)\ \sqrt{(N-1)}M^{2l}(x_l) \ldots \times \sqrt{(N-1)}M^{ml}(x_l) \ldots \sqrt{(N-1)}M^{Nl}(x_l)\right]^T \tag{25}$$

$$\rho_l = 1/\alpha_l \tag{26}$$

$$W_l = P_l^{-1} \tag{27}$$

v_{1l}, v_{2l} are the vector dependent on x_l, $\varepsilon_{1l} > 0$ is constant, and $\varepsilon_{2l}(x_l) > 0$.

The controller gains are computed as follows

$$K_l(x_l) = R_l(x_l)P_l \tag{28}$$

Proof: From (7), it infers that

$$\begin{bmatrix} \Upsilon_l(x_l)+\alpha_l\left(\sum_{m=1,m\neq l}^{N}(N-1)(M^{ml}(x_l))^T M^{ml}(x_l)\right) & P_l \\ P_l & -\alpha_l I \end{bmatrix} < 0 \tag{29}$$

where

$$\Upsilon_l(x_l) = (A^l(x_l))^T P_l + P_l A^l(x_l) - (K^l(x_l))^T(B^l(x_l))^T P_l - P_l B^l(x_l)K^l(x_l)$$

Define $W_l = P_l^{-1}$.

Pre and post-multiplying (29) with $\begin{bmatrix} W_l & 0 \\ 0 & I\end{bmatrix}$ yields

$$\begin{bmatrix} W_l\Upsilon_l(x_l)W_l+\alpha_l W_l\left(\sum_{m=1,m\neq l}^{N}(N-1)\times(M^{ml}(x_l))^T M^{ml}(x_l)\right)W_l & P_l W_l \\ W_l P_l & -\alpha_l I \end{bmatrix} < 0 \tag{30}$$

Because $W_l = P_l^{-1}$ then (30) becomes

$$\begin{bmatrix} \Phi_l(x_l) + \alpha_l W_l\left((\bar{M}^l(x_l))^T\bar{M}^l(x_l)\right)W_l & I \\ I & -\alpha_l I \end{bmatrix} < 0 \tag{31}$$

where

$$\begin{aligned}\Phi_l(x_l) &= W_l\Upsilon_l(x_l)W_l \\ &= W_l(A^l(x_l))^T + A^l(x_l)W_l - W_l(K^l(x_l))^T(B^l(x_l))^T - B^l(x_l)K^l(x_l)W_l \\ \bar{M}^l(x_l) &= \left[\sqrt{(N-1)}M^{1l}(x_l)\ \sqrt{(N-1)}M^{2l}(x_l) \ldots \times \sqrt{(N-1)}M^{ml}(x_l) \ldots \sqrt{(N-1)}M^{Nl}(x_l)\right]^T\end{aligned} \tag{32}$$

Pre and post-multiplying (31) with $\begin{bmatrix} I & 0 \\ 0 & 1/\alpha_l\end{bmatrix}$ results in

$$\begin{bmatrix} \Phi_l(x_l) + \alpha_l W_l\left((\bar{M}^l(x_l))^T\bar{M}^l(x_l)\right)W_l & 1/\alpha_l I \\ 1/\alpha_l I & -1/\alpha_l I \end{bmatrix} < 0 \tag{33}$$

Define

$$R^l(x_l) = K^l(x_l)W_l \tag{34}$$

Substituting (34) into (32) yields

$$\Phi_l(x_l) = W_l(A^l(x_l))^T + A^l(x_l)W_l - (R^l(x_l))^T(B^l(x_l))^T - B^l(x_l)R^l(x_l) \tag{35}$$

Employing Schur complement for (33), it is rewritten as follows

$$\begin{bmatrix} \Phi_l(x_l) & W_l(\bar{M}^l(x_l))^T & 1/\alpha_l I \\ \bar{M}^l(x_l)W_l & -1/\alpha_l I & 0 \\ 1/\alpha_l I & 0 & -1/\alpha_l I \end{bmatrix} < 0 \tag{36}$$

where

$$\Phi_l(x_l) = W_l(A^l(x_l))^T + A^l(x_l)W_l - (R^l(x_l))^T(B^l(x_l))^T - B^l(x_l)R^l(x_l)$$

Let us denote $\rho_l = 1/\alpha_l$, then (36) becomes

$$\begin{bmatrix} \Phi_l(x_l) & W_l(\bar{M}^l(x_l))^T & \rho_l I \\ \bar{M}^l(x_l)W_l & -\rho_l I & 0 \\ \rho_l I & 0 & -\rho_l I \end{bmatrix} < 0 \tag{37}$$

where

$$\Phi_l(x_l) = W_l(A^l(x_l))^T + A^l(x_l)W_l - (R^l(x_l))^T(B^l(x_l))^T - B^l(x_l)R^l(x_l)$$

It is seen that (37) is equivalent to the condition (22) of Theorem 2 and they are the PLMI, therefore, it is successful to transform PBMI (7) of Theorem 1 into PLMI (22) of Theorem 2. Thus, the proof is completed.

IV. CONTROLER SYNTHESIS FOR THE LARGE-SCALE SYSTEM WITH NON-POLYNOMIAL TERMS

In this section, a controller is synthesized for the nonlinear large-scale system which is mentioned in Scenario 2. The nonlinear system (1) is modeled under the framework of the large-scale polynomial system (3), therefore, we will design the controller for system (3) instead of the system (1).

To synthesize the polynomial controller for the system (3), the following assumption is necessary.

Assumption 1 ([46]): Assume that the non-polynomial terms of a system (3) satisfy the matching condition $\tilde{A}^l(x_l) = E_a^l(x_l)a^l(x_l)F_a^l(x_l)$, $\tilde{B}_b^l(x_l) = E_b^l(x_l)b^l(x_l)F_b^l(x_l)$, where $a^l(x_l)$ and $b^l(x_l)$ compose of the non-polynomial terms and fulfill the bounded constraint $\|a^l(x_l)\| \le \gamma_a^l(x_l)$, $\|b^l(x_l)\| \le \gamma_b^l(x_l)$.

The decentralized polynomial controller for the system (3) is expressed as follows

$$u_l(t) = -K^l(x_l)x_l \tag{38}$$

Under Assumption 1, substituting (38) into (3) obtains

$$\begin{aligned}\dot{x}_l &= [A^l(x_l) + E_a^l(x_l)a^l(x_l)F_a^l(x_l)]x_l \\ &\quad - [B_b^l(x_l) + E_b^l(x_l)b^l(x_l)F_b^l(x_l)]K^l(x_l)x_l \\ &\quad + \sum_{m=1,m\neq l}^{N} M^{lm}(x_l)x_m\}\end{aligned} \tag{39}$$

Theorem 3 : The system (3) with the controller (38) is stable if there exist the symmetric Q_l, polynomial matrix $K^l(x_l)$, and constant scalar β_l such that the following conditions hold

$$v_{1l}^T\{Q_l - \varepsilon_{1l}I\}v_{1l} \text{ is SOS} \tag{40}$$

$$-v_{2l}^T\left\{\Psi^l(x_l) + \varepsilon_{2l}(x_l)I\right\}v_{2l} \text{ is SOS} \tag{41}$$

where

$$\Psi^l(x_l) = \begin{bmatrix}\Omega^l(x_l) + \beta_l\Gamma^l(x_l) & Q_l \\ Q_l & -\beta_l I\end{bmatrix} \tag{42}$$

$$\begin{aligned}\Omega^l(x_l) &= (A^l(x_l))^TQ_l + Q_lA^l(x_l) - (K^l(x_l))^T(B^l(x_l))^TQ_l \\ &\quad - Q_lB^l(x_l)K^l(x_l) + Q_lE_a^l(x_l)(E_a^l(x_l))^TQ_l \\ &\quad + \gamma_a^l(x_l)(F_a^l(x_l))^TF_a^l(x_l) + Q_lE_b^l(x_l)(E_b^l(x_l))^TQ_l \\ &\quad + \gamma_b^l(x_l)(K^l(x_l))^T(F_b^l(x_l))^TF_b^l(x_l)K^l(x_l)\end{aligned} \tag{43}$$

$$\Gamma^l(x_l) = \left(\sum_{m=1,m\neq l}^{N}(N-1)(M^{ml}(x_l))^TM^{ml}(x_l)\right) \tag{44}$$

v_{1l}, v_{2l} are the vectors which are independent on x_l, $\varepsilon_{1l} > 0$ is constant, and $\varepsilon_{2l}(x_l) > 0$, $i, j = 1, 2, \ldots, r_l$, $l = 1, 2, \ldots, N$

Proof:

The Lyapunov function is chosen as follows

$$V(t) = \sum_{l=1}^{N}V_l(t) = \sum_{l=1}^{N}x_l^TQ_lx_l \tag{45}$$

From (40), it implies that $Q_l > 0$, it means that $V(t) > 0$.

Taking the derivative of (45), we have

$$\dot{V}(t) = \sum_{l=1}^{N}\dot{V}_l(t) = \sum_{l=1}^{N}[\dot{x}_l^TQ_lx_l + x_l^TQ_l\dot{x}_l] \tag{46}$$

Substituting (39) into (46), one obtains

$$\begin{aligned}\dot{V}(t) &= \sum_{l=1}^{N}\{A^l(x_l)x_l + E_a^l(x_l)a^l(x_l) \\ &\quad \times F_a^l(x_l)x_l - B^l(x_l)K^l(x_l)x_l \\ &\quad - E_b^l(x_l)b^l(x_l)F_b^l(x_l)K^l(x_l)x_l \\ &\quad + \sum_{m=1,m\neq l}^{N}M^{lm}(x_l)x_m]^TQ_lx_l + x_l^TQ_l[A^l(x_l)x_l \\ &\quad + E_a^l(x_l)a^l(x_l)F_a^l(x_l)x_l - B^l(x_l)K^l(x_l)x_l \\ &\quad - E_b^l(x_l)b^l(x_l)F_b^l(x_l)K^l(x_l)x_l + \sum_{m=1,m\neq l}^{N}M^{lm}(x_l)x_m]\} \\ &= \sum_{l=1}^{N}\{x_l^T[((A^l(x_l))^TQ_l + Q_lA^l(x_l)) \\ &\quad - ((K^l(x_l))^T(B^l(x_l))^TQ_l + Q_lB^l(x_l)K^l(x_l)) \\ &\quad + ((F_a^l(x_l))^T(a^l(x_l))^T(E_a^l(x_l))^TQ_l \\ &\quad + Q_lE_a^l(x_l)a^l(x_l)F_a^l(x_l)) - ((K^l(x_l))^T(F_b^l(x_l))^T \\ &\quad \times (b^l(x_l))^T(E_b^l(x_l))^TQ_l + Q_lE_b^l(x_l)b^l(x_l) \\ &\quad \times F_b^l(x_l)K^l(x_l))]\}x_l \\ &\quad + \sum_{l=1}^{N}\{[\sum_{m=1,m\neq l}^{N}M^{lm}(x_l)x_m]^TQ_lx_l \\ &\quad + x_l^TQ_l[\sum_{m=1,m\neq l}^{N}M^{lm}(x_l)x_m]\}\end{aligned} \tag{47}$$

Employing Lemma 1 and Assumption 1, it can be found that

$$\begin{aligned}&(F_a^l(x_l))^T(a^l(t))^T(E_a^l(x_l))^TQ_l + Q_lE_a^l(x_l)a^l(t)F_a^l(x_l) \\ &\quad \le Q_lE_a^l(x_l)(E_a^l(x_l))^TQ_l + (F_a^l(x_l))^T(a^l(x_l))^Ta^l(x_l)F_a^l(x_l) \\ &\quad \le Q_lE_a^l(x_l)(E_a^l(x_l))^TQ_l + \gamma_a^l(x_l)(F_a^l(x_l))^TF_a^l(x_l)\end{aligned} \tag{48}$$

$$\begin{aligned}&-(K^l(x_l))^T(F_b^l(x_l))^T(b^l(x_l))^T(F_b^l(x_l))^TQ_l - Q_lE_b^l(x_l)b^l(x_l) \\ &\quad \times F_b^l(x_l)K^l(x_l) \le Q_lE_b^l(x_l)(E_b^l(x_l))^TQ_l \\ &\quad + (K^l(x_l))^T(F_b^l(x_l))^T \\ &\quad \times (b^l(x_l))^Tb^l(x_l)F_b^l(x_l)K^l(x_l) \\ &\quad \le Q_lE_b^l(x_l)(E_b^l(x_l))^TQ_l + \gamma_b^l(x_l)(K^l(x_l))^T(F_b^l(x_l))^T \\ &\quad \times F_b^l(x_l)K^l(x_l)\end{aligned} \tag{49}$$

Let us denote

$$\delta_l(\circ) = \sum_{m=1,m\neq l}^{N}M^{lm}(x_l)x_m \tag{50}$$

Combining (47), (48), (49) and (50), (47) is equivalent to

$$\begin{aligned}\dot{V}(t) &\le \sum_{l=1}^{N}\{x_l^T[((A^l(x_l))^TQ_l + Q_lA^l(x_l)) - (K^l(x_l))^T \\ &\quad \times (B^l(x_l))^TQ_l - Q_lB^l(x_l)K^l(x_l) \\ &\quad + Q_lE_a^l(x_l)(E_a^l(x_l))^TQ_l \\ &\quad + \gamma_a^l(x_l)(F_a^l(x_l))^TF_a^l(x_l) + Q_lE_b^l(x_l)(E_b^l(x_l))^TQ_l \\ &\quad + \gamma_b^l(x_l) \\ &\quad \times (K^l(x_l))^T(F_b^l(x_l))^TF_b^l(x_l)K^l(x_l)]x_l + \delta_l^T(\circ)Q_lx_l \\ &\quad + x_l^TQ_l\delta(\circ)\} \\ &= \sum_{l=1}^{N}\{\begin{bmatrix}x_l \\ \delta_l(\circ)\end{bmatrix}^T\begin{bmatrix}\Omega^l(x_l) & Q_l \\ Q_l & 0\end{bmatrix}\begin{bmatrix}x_l \\ \delta_l(\circ)\end{bmatrix}\}\end{aligned} \tag{51}$$

where

$$\Omega^l(x_l) = (A^l(x_l))^T Q_l + Q_l A^l(x_l) - (K^l(x_l))^T (B^l(x_l))^T Q_l - Q_l \times B^l(x_l) K^l(x_l) + Q_l E_a^l(x_l)(E_a^l(x_l))^T Q_l + \gamma_a^l(x_l)(F_a^l(x_l))^T F_a^l(x_l) + Q_l E_b^l(x_l)(E_b^l(x_l))^T Q_l + \gamma_b^l(x_l)(K^l(x_l))^T \times (F_b^l(x_l))^T F_b^l(x_l) K^l(x_l) \tag{52}$$

From (51), it is obvious that $\dot{V}(t) < 0$ if only if

$$\Theta^l(x_l) = \begin{bmatrix} \Omega^l(x_l) & Q_l \\ Q_l & 0 \end{bmatrix} < 0 \tag{53}$$

However, similarly to the problem in Section 3, the right bottom corner of (53) also consists of zero elements, hence, it is infeasible to obtain the suitable polynomial matrices $K^l(x_l)$ and symmetric positive matrix Q_l when solving (53). Hence, we need further steps as below to transform (53) to other forms which can be solved in SOS tools easily.

Similar steps (16)-(17) in Section 3, one obtains

$$\sum_{l=1}^{N} \delta_l^T(\circ)\delta_l(\circ) - \sum_{l=1}^{N} \Big\{ x_l^T \, [(N-1) \times \sum_{m=1, m\neq l}^{N} (M^{ml}(x_l))^T M^{ml}(x_l) x_m] x_l \} = \sum_{l=1}^{N} \{ \begin{bmatrix} x_l \\ \delta_l(\circ) \end{bmatrix}^T \Xi(x_l) \begin{bmatrix} x_l \\ \delta_l(\circ) \end{bmatrix} \} \leq 0 \tag{54}$$

where

$$\Xi_l(x_l) = \begin{bmatrix} -\left((N-1) \sum_{m=1, m\neq l}^{N} (M^{ml}(x_l))^T M^{ml}(x_l) \right) & 0 \\ 0 & I \end{bmatrix}$$

Relied on S-Procedure in Lemma 3, and combining (53) and (54), if there exist a positive constant β_1 such that the following condition holds

$$\sum_{l=1}^{N} \{ \begin{bmatrix} x_l \\ \delta_l(\circ) \end{bmatrix}^T \begin{bmatrix} \Omega^l(x_l) & Q_l \\ Q_l & 0 \end{bmatrix} \begin{bmatrix} x_l \\ \delta_l(\circ) \end{bmatrix} \} - \beta_l \sum_{l=1}^{N} \Big\{ \begin{bmatrix} x_l \\ \delta_l(\circ) \end{bmatrix}^T \times \begin{bmatrix} -\left(\sum_{m=1, m\neq l}^{N} (N-1)(M^{ml}(x_l))^T M^{ml}(x_l) \right) & 0 \\ 0 & I \end{bmatrix} \times \begin{bmatrix} x_l \\ \delta_l(\circ) \end{bmatrix} \Big\}$$
$$= \sum_{l=1}^{N} \{ \begin{bmatrix} x_l \\ \delta_l(\circ) \end{bmatrix}^T \begin{bmatrix} \Omega^l(x_l) + \beta_l \Gamma^l(x_l) & Q_l \\ Q_l & -\beta_l I \end{bmatrix} \begin{bmatrix} x_l \\ \delta_l(\circ) \end{bmatrix} \}$$
$$= \sum_{l=1}^{N} \{ \begin{bmatrix} x_l \\ \delta_l(\circ) \end{bmatrix}^T \Psi^l(x_l) \begin{bmatrix} x_l \\ \delta_l(\circ) \end{bmatrix} \} < 0 \tag{55}$$

where

$$\Psi^l(x_l) = \begin{bmatrix} \Omega^l(x_l) + \beta_l \Gamma^l(x_l) & Q_l \\ Q_l & -\beta_l I \end{bmatrix} \Gamma^l(x_l) = \left(\sum_{m=1, m\neq l}^{N} (N-1)(M^{ml}(x_l))^T M^{ml}(x_l) \right)$$

then, we can conclude that

$$\dot{V}(t) \leq \sum_{l=1}^{N} \{ \begin{bmatrix} x_l \\ \delta_l(\circ) \end{bmatrix}^T \begin{bmatrix} \Omega^l(x_l) & Q_l \\ Q_l & 0 \end{bmatrix} \begin{bmatrix} x_l \\ \delta_l(\circ) \end{bmatrix} \} < 0 \tag{56}$$

It is seen that if the condition (41) of Theorem 3 holds, then (55) is satisfied, this means that the equation (56) holds and $\dot{V}(t) < 0$ The proof is completed.

However, condition (41) of Theorem 3 is PBMI which is hard to resolve in Matlab. Thus, Theorem 4 is necessary to convert PBMI (41) to PLMI which can easily obtain the solutions by using the SOS tool of Matlab.

Theorem 4: The system (2) with the controller (3) is stabilized if there exist the symmetric W_l, polynomial matrix $K^l(x_l)$, and constant β_l such that the following conditions hold

$$v_{3l}^T \{W_l - \varepsilon_{1l} I\} v_{3l} \text{ is SOS} \tag{57}$$

$$-v_{4l}^T \left\{ \Gamma^l(x_l) + \varepsilon_{2l}(x_l) I \right\} v_{4l} \text{ is SOS} \tag{58}$$

where

$$\Gamma^l(x_l) = \begin{bmatrix} \Pi^l(x_l) & \gamma_a^l(x_l) W_l (F_a^l(x_l))^T & \gamma_b^l(x_l)(R^l(x_l))^T (F_b^l(x_l))^T & W_l(\bar{M}^l(x_l))^T & \lambda_l I \\ F_a^l(x_l) W_l & -I & 0 & 0 & 0 \\ F_b^l(x_l) R^l(x_l) & 0 & -I & 0 & 0 \\ \bar{M}^l(x_l) W_l & 0 & 0 & -\lambda_l I & 0 \\ \lambda_l I & 0 & 0 & 0 & -\lambda_l I \end{bmatrix} \tag{59}$$

$$\Pi^l(x_l) = W_l (A^l(x_l))^T + A^l(x_l) W_l - (R^l(x_l))^T (B^l(x_l))^T - B^l(x_l) R^l(x_l) + E_a^l(x_l)(E_a^l(x_l))^T + E_b^l(x_l)(E_b^l(x_l))^T \tag{60}$$

$$\bar{M}^l(x_l) = \left[\sqrt{(N-1)} M^{1l}(x_l) \ \sqrt{(N-1)} M^{2l}(x_l) \ \ldots \times \sqrt{(N-1)} M^{ml}(x_l) \ \ldots \ \sqrt{(N-1)} M^{Nl}(x_l) \right]^T \tag{61}$$

$$\lambda_l = {}^1/_{\beta_l} \tag{62}$$

$$W_l = Q_l^{-1} \tag{63}$$

v_{3l}, v_{4l} are the vectors which are independent of x_l, $\varepsilon_{1l} > 0$ is constant, and $\varepsilon_{2l}(x_l) > 0$.

The controller gains are computed as follows

$$K^l(x_l) = R^l(x_l) Q_l. \tag{64}$$

Proof:

From (57), it implies that

$$\begin{bmatrix} \Omega^l(x_l)+\beta_l\Gamma^l(x_l) & Q_l \\ Q_l & -\beta_l I \end{bmatrix} < 0 \tag{65}$$

where

$$\begin{aligned}\Omega^l(x_l) &= (A^l(x_l))^T Q_l + Q_l A^l(x_l) \\ &\quad - (K^l(x_l))^T (B^l(x_l))^T Q_l - Q_l \\ &\quad \times B^l(x_l)K^l(x_l) + Q_l E_a^l(x_l)(E_a^l(x_l))^T Q_l \\ &\quad + \gamma_a^l(x_l)(F_a^l(x_l))^T F_a^l(x_l) \\ &\quad + Q_l E_b^l(x_l)(E_b^l(x_l))^T Q_l + \gamma_b^l(x_l)(K^l(x_l))^T \\ &\quad \times (F_b^l(x_l))^T F_b^l(x_l)K^l(x_l)\end{aligned}$$

$$\Gamma^l(x_l) = \left((N-1)\sum_{m=1,m\neq l}^{N} (M^{ml}(x_l))^T M^{ml}(x_l)\right)$$

Let us define $W_l = Q_l^{-1}$ and pre and post-multiplying (65) with $\begin{bmatrix} W_l & 0 \\ 0 & I\end{bmatrix}$ yields

$$\begin{aligned}&\begin{bmatrix} W_l & 0 \\ 0 & I\end{bmatrix}\begin{bmatrix} \Omega^l(x_l)+\beta_l\Gamma^l(x_l) & Q_l \\ Q_l & -\beta_l I \end{bmatrix}\begin{bmatrix} W_l & 0 \\ 0 & I\end{bmatrix} \\ &= \begin{bmatrix} W_l\Omega^l(x_l)W_l + \beta_l W_l\Gamma^l(x_l)W_l & Q_l W_l \\ W_l Q_l & -\beta_l I\end{bmatrix} < 0 \end{aligned}\tag{66}$$

Due to $W_l = Q_l^{-1}$, (66) is rewritten

$$\begin{bmatrix} \Sigma^l(x_l)+\beta_l W_l\Gamma^l(x_l)W_l & I \\ I & -\beta_l I \end{bmatrix} < 0 \tag{67}$$

in which

$$\begin{aligned}\Sigma^l(x_l) &= W_l\Omega^l(x_l)W_l \\ &= W_l(A^l(x_l))^T Q_l W_l + W_l Q_l A^l(x_l) W_l \\ &\quad - W_l(K^l(x_l))^T(B^l(x_l))^T Q_l W_l \\ &\quad - W_l Q_l B^l(x_l)K^l(x_l)W_l + W_l Q_l E_a^l(x_l) \\ &\quad \times (E_a^l(x_l))^T Q_l W_l \\ &\quad + \gamma_a^l(x_l)W_l(F_a^l(x_l))^T F_a^l(x_l)W_l \\ &\quad + W_l Q_l E_b^l(x_l)(E_b^l(x_l))^T Q_l W_l \\ &\quad + \gamma_b^l(x_l)W_l(K^l(x_l))^T(F_b^l(x_l))^T F_b^l(x_l)K^l(x_l)W_l \\ &= W_l(A^l(x_l))^T + A^l(x_l)W_l - W_l(K^l(x_l))^T(B^l(x_l))^T \\ &\quad - B^l(x_l)K^l(x_l) \\ &\quad \times W_l + E_a^l(x_l)(E_a^l(x_l))^T + \gamma_a^l(x_l)W_l(F_a^l(x_l))^T \\ &\quad \times F_a^l(x_l)W_l + E_b^l(x_l) \\ &\quad \times (E_b^l(x_l))^T + \gamma_b^l(x_l)W_l(K^l(x_l))^T(F_b^l(x_l))^T \\ &\quad \times F_b^l(x_l)K^l(x_l)W_l \end{aligned}\tag{68}$$

Let us define

$$\begin{aligned}\bar{M}^l(x_l) = \Big[&\sqrt{(N-1)}M^{1l}(x_l)\ \sqrt{(N-1)}M^{2l}(x_l)\ \ldots \\ &\times\ \sqrt{(N-1)}M^{ml}(x_l)\ \ldots\ \sqrt{(N-1)}M^{Nl}(x_l)\Big]^T\end{aligned}$$

Then (68) is equivalent to

$$\begin{bmatrix} \Sigma^l(x_l)+\beta_l W_l\left((\bar{M}^l(x_l))^T\bar{M}^l(x_l)\right)W_l & I \\ I & -\beta_l I \end{bmatrix} < 0 \tag{69}$$

Pre and post-multiplying (69) with $\begin{bmatrix} I & 0 \\ 0 & {}^1/_{\beta_l}\end{bmatrix}$ yields

$$\begin{bmatrix} \Sigma^l(x_l)+\beta_l W_l\left((\bar{M}^l(x_l))^T\bar{M}^l(x_l)\right)W_l & {}^1/_{\beta_l} I \\ {}^1/_{\beta_l} I & -{}^1/_{\beta_l} I \end{bmatrix} < 0 \tag{70}$$

Let us denote

$$R^l(x_l) = K^l(x_l)W_l \tag{71}$$

Substituting (71) into (70) yields

$$\begin{aligned}&\Sigma^l(x_l) \\ &= W_l(A^l(x_l))^T + A^l(x_l)W_l - (R^l(x_l))^T(B^l(x_l))^T - B^l(x_l) \\ &\quad \times R^l(x_l) + E_a^l(x_l)(E_a^l(x_l))^T + \gamma_a^l(x_l)W_l(F_a^l(x_l))^T F_a^l(x_l)W_l \\ &\quad + E_b^l(x_l)(E_b^l(x_l))^T + \gamma_b^l(x_l)(R^l(x_l))^T(F_b^l(x_l))^T F_b^l(x_l)R^l(x_l)\end{aligned}\tag{72}$$

Applying Schur complement for (72) and denoting $\lambda_l = {}^1/_{\beta_l}$, (72) is equivalent to

$$\begin{aligned}&\Phi^l(x_l) \\ &= \begin{bmatrix} \Pi^l(x_l) & \gamma_a^l(x_l)W_l(F_a^l(x_l))^T & \gamma_b^l(x_l)(R^l(x_l))^T(F_b^l(x_l))^T & W_l(\bar{M}^l(x_l))^T & \lambda_l I \\ F_a^l(x_l)W_l & -I & 0 & 0 & 0 \\ F_b^l(x_l)R^l(x_l) & 0 & -I & 0 & 0 \\ \bar{M}^l(x_l)W_l & 0 & 0 & -\lambda_l I & 0 \\ \lambda_l I & 0 & 0 & 0 & -\lambda_l I \end{bmatrix}\end{aligned}\tag{73}$$

where

$$\begin{aligned}\Pi^l(x_l) &= W_l(A^l(x_l))^T + A^l(x_l)W_l - (R^l(x_l))^T(B^l(x_l))^T \\ &\quad - B^l(x_l)R^l(x_l) + E_a^l(x_l)(E_a^l(x_l))^T + E_b^l(x_l)(E_b^l(x_l))^T\end{aligned}$$

It is obvious that (40) is equivalent to the condition (25) and this is a PLMI that can be solved easily in Matlab by SOS TOOL. Thus, the PBMI (6) of Theorem 1 has been transformed into PLMI (25) of Theorem

V. RESULTS AND DISCUSSION

In this section, two numerical examples are provided to illustrate the successes of the proposed methods. In Example 1, a decentralized polynomial controller is synthesized for the nonlinear large-scale system which does not consist of non-polynomial terms. While designing a decentralized polynomial controller for the large-scale nonlinear system which includes the non-polynomial terms is presented in Example 2.

Example 1: Consider the large-scale nonlinear system with three interconnection terms as follows

$$S^1 : \begin{cases} \dot{x}_{11} = -3x_{11} + (x_{21})^2 + 0.1x_{12} + 0.3x_{13} + u \\ \dot{x}_{21} = 4x_{11}x_{21} - 2x_{21} + 0.3x_{12}x_{21} + 0.1x_{21}x_{13} + u \end{cases} \tag{74}$$

$$S^2 : \begin{cases} \dot{x}_{12} = -2x_{12} + (x_{22})^2 + 0.4x_{11} + 0.1x_{13} + u \\ \dot{x}_{22} = 4x_{12} - x_{22} + 0.06x_{22}x_{11} - 0.2x_{22}x_{13} + u \end{cases} \tag{75}$$

$$S^3 : \begin{cases} \dot{x}_{13} = -2x_{23}x_{13} + x_{23} + 0.1x_{11} + 0.1x_{23}x_{12} + u \\ \dot{x}_{23} = 5x_{13}x_{23} - 3x_{23} + 0.03x_{11}x_{23} + 0.03x_{12}x_{23} + u \end{cases} \tag{76}$$

The large-scale nonlinear system (74)-(76) is represented under the framework of the large-scale polynomial linear system as follows

$$\dot{x}_l = A^l(x_l)x_l + B^l(x_l)u_l(t) + \sum_{m=1, m\neq l}^{N} M^{lm}(x_l)x_m \quad l, m = 1, 2, 3. \tag{77}$$

where

$$x_1 = \begin{bmatrix} x_{11} \\ x_{21} \end{bmatrix}, \quad x_2 = \begin{bmatrix} x_{12} \\ x_{22} \end{bmatrix}, \; x_3 = \begin{bmatrix} x_{13} \\ x_{23} \end{bmatrix}$$

$$A^1(x_1) = \begin{bmatrix} -3 & x_{21} \\ 4x_{21} & -2 \end{bmatrix} \quad B^1(x_1) = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

$$M^{12}(x_1) = \begin{bmatrix} 0.1 & 0 \\ 0.3x_{21} & 0 \end{bmatrix}, \quad A^2(x_2) = \begin{bmatrix} -2 & x_{22} \\ 3 & -1 \end{bmatrix}$$

$$M^{13}(x_1) = \begin{bmatrix} 0.3 & 0 \\ 0.1x_{21} & 0 \end{bmatrix}, \quad B^2(x_2) = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

$$M^{21}(x_2) = \begin{bmatrix} 0.4 & 0 \\ 0.06x_{22} & 0 \end{bmatrix}, \quad M^{23}(x_2) = \begin{bmatrix} 0.1 & 0 \\ -0.2x_{22} & 0 \end{bmatrix},$$

$$A^3(x_3) = \begin{bmatrix} -2x_{23} & 1 \\ 5x_{23} & -3 \end{bmatrix}, \quad B^3(x_3) = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

$$M^{31}(x_3) = \begin{bmatrix} 0.1 & 0 \\ 0.03x_{23} & 0 \end{bmatrix}, \quad M^{32}(x_2) = \begin{bmatrix} 0.1x_{23} & 0 \\ 0.03x_{23} & 0 \end{bmatrix}$$

Solving the conditions of Theorem 2 by the SOS tool obtains the controller gains as follows

$$\begin{aligned} K^1(x_1) &= \left[K_{11}(x_1) \; K_{12}(x_1) \right] \\ K_{11}(x_1) &= 457.436x_{21}^2 + 40.888x_{21} + 633.7616 \\ K_{12}(x_1) &= 114.322x_{21}^2 + 30.413x_{21} - 113.232 \\ K^2(x_2) &= \left[K_{21}(x_2) \; K_{22}(x_2) \right] \\ K_{21}(x_2) &= 1.464x_{22}^2 + 50.838x_{22} + 82.312 \\ K_{22}(x_2) &= 518.944x_{22}^2 - 58.692x_{22} + 722.127 \\ K^3(x_3) &= \left[K_{31}(x_2) \; K_{32}(x_2) \right] \\ K_{31}(x_2) &= 791.772x_{23}^2 - 4.957x_{23} + 2943.981 \\ K_{32}(x_2) &= 0.0012x_{23}^2 + 0.0245x_{23} - 2.666 \end{aligned}$$

Simulating the system in Simulink of Matlab, the simulation results are obtained in Figs. 2-7 as follows.

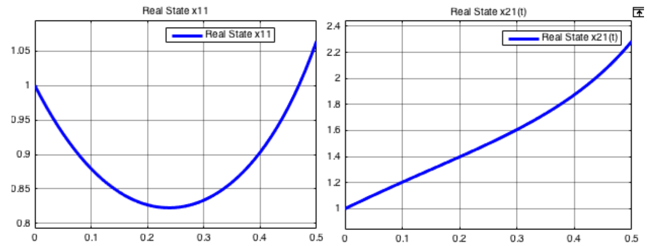

FIGURE 2. States $x_{11}(t)$ and $x_{21}(t)$ without the controller.

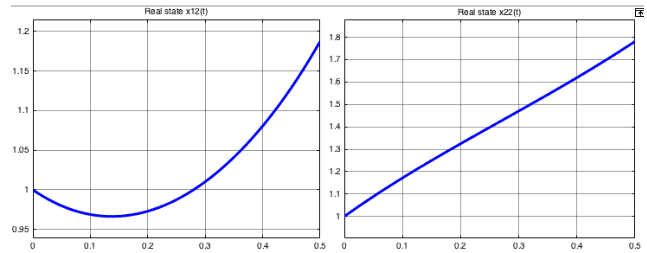

FIGURE 3. States $x_{12}(t)$ and $x_{22}(t)$ without the controller.

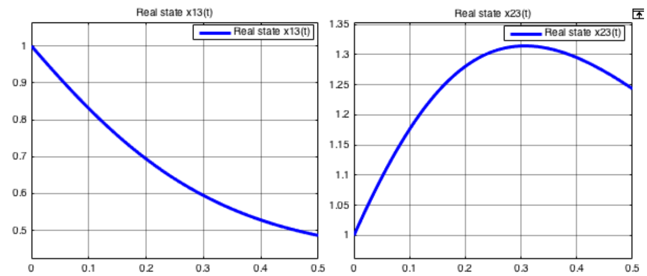

FIGURE 4. States $x_{13}(t)$ and $x_{23}(t)$ without the controller.

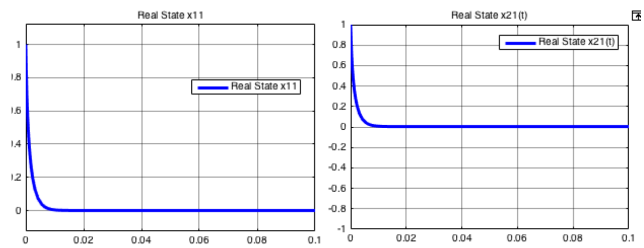

FIGURE 5. States $x_{11}(t)$ and $x_{21}(t)$ with the controller.

The simulation results are illustrated in Figs. 2-10. Figs. 2-4 demonstrate the state response of the open-loop system, Figs. 5-7 show the state variables of the closed-loop system and Figs. 8-10 present the control signals of the three decentralized controllers. From Figs. 2-3, it is easily seen that all states x_{11}, x_{21}, x_{12}, x_{22}, x_{13}, and x_{23} of the open-loop system (2) are divergent, it means that the system (2) is unstable when the controller is not used. When the proposed controller is synthesized for the system (2), the responses of all states are demonstrated in Figs. 4-7. The simulation results in Figs. 4-7 show that all states x_{11}, x_{21}, x_{12}, x_{22}, x_{13}, and x_{23} approach zero asymptotically. Therefore, we can conclude that the proposed method in this work is successful to synthesize a controller for stabilizing the system (2).

Remark 3: It is seen that the large-scale nonlinear system consists of three nonlinear terms: x_{21}, x_{22}, and x_{23}. If we apply the controller design methods in [8], [17], [18], these nonlinear terms must be linearized to transform the nonlinear subsystems into the linear subsystems. However, it is obvious that when the linearization is carried out, there exists a modeling error between the new model and the original nonlinear large-scale system. This modeling error will degrade the performance of the nonlinear large-scale system. While, in this paper, the polynomial system is employed to represent the nonlinear large-scale system in which the nonlinear terms are put inside the system matrices, thus, the modeling error does not exist in this work.

Example 2: Consider the large-scale nonlinear system with three interconnection terms as follows

$$S^1: \begin{cases} \dot{x}_{11} = -2(x_{21}+1)x_{11} + 0.5x_{11}x_{21} \\ \quad + 1.05x_{11}x_{21}\sin(2x_{11}) \\ \quad - 0.105x_{21}^2\sin(2x_{11}) + x_{12} \\ \quad + x_{13} + (1 - 0.02x_{21}\cos(x_{21}))u \\ \times\dot{x}_{21} = x_{11}x_{21} - 0.575x_{21} + 0.1x_{11}\sin(2x_{11}) \\ \quad - 0.01x_{21}\sin(2x_{11}) + 0.5x_{12}x_{21} \\ \quad + 0.5x_{21}x_{13} + (1 + 0.02\cos(x_{21}))u \end{cases} \tag{78}$$

$$S^2: \begin{cases} \dot{x}_{12} = -2x_{12}^2 + 0.75(x_{22})^2 \\ \quad + 0.015x_{12}x_{22}\cos(0.5x_{22}) \\ \quad + 0.005(x_{22})^2\cos(0.5x_{22}) + 0.4x_{11}x_{22} \\ \quad + 0.4x_{13}x_{22} \\ \quad + (1 + 0.012x_{22}\sin(2x_{12}))u \\ \dot{x}_{22} = 0.5x_{12} - 0.8x_{22}^2 + 0.063x_{12}\cos(0.5x_{22}) \\ \quad + 0.021x_{22} \\ \quad \times \cos(0.5x_{22}) + 0.6x_{22}x_{11} + 0.6x_{21}x_{23} \\ \quad + (1 + 0.06\sin(2x_{12})u \end{cases} \tag{79}$$

$$S^3: \begin{cases} \dot{x}_{13} = -x_{13}^2 + 0.3x_{23}^2 + (0.008x_{13}x_{23} \\ \quad + 0.04x_{23}^2)\sin(x_{13}) \\ \quad \times \cos(2x_{23}) + 0.2x_{11} + 0.2x_{12} \\ \quad + (1 + 0.036x_{23}\cos(x_{23}))u \\ \dot{x}_{23} = x_{13} - 0.775x_{23}^2 + (0.02x_{13} \\ \quad + 0.1x_{23})\sin(x_{13})\cos(2x_{23}) \\ \quad + 0.05x_{21}x_{23} + 0.05x_{22}x_{23} \\ \quad + (1 + 0.06\cos(x_{23}))u \end{cases} \tag{80}$$

It should be noted that the large-scale nonlinear system (78)-(80) contains non-polynomial terms such as cosine and sine terms, therefore, it is unable to model this system under the polynomial system (2) and apply the method in Section 3 to design a controller for the system (78)-(80). To overcome this challenge, the nonlinear large-scale system (78)-(80) is represented under the form in (3).

$$\dot{x}_l = [A^l(x_l) + \tilde{A}^l(x_l)]x_l + [B^l(x_l) + \tilde{B}^l(x_l)]u_l(t) + \sum_{m=1,m\neq l}^{N} M^{lm}(x_l)x_m, \quad l = 1,2,3. \tag{81}$$

where the equation can be derived, as shown at the bottom of next page.

Under Assumption 1, $\tilde{A}^1(x_1)$, $\tilde{A}^2(x_2)$, $\tilde{A}^3(x_3)$, $\tilde{B}^1(x_1)$, $\tilde{B}^2(x_2)$, and $\tilde{B}^3(x_3)$ are decomposed and we obtain the following polynomial matrices:

$$E_a^1(x_1) = \begin{bmatrix} 1.05x_{21} \\ 0.1 \end{bmatrix}, \quad F_a^1(x_1) = \begin{bmatrix} 1 & -0.1 \end{bmatrix}, \quad \gamma_a^1(x_1) = 1$$

$$E_b^1(x_1) = \begin{bmatrix} -0.1x_{21} \\ 0.1 \end{bmatrix}, \quad F_b^1(x_1) = 0.2, \ \gamma_b^1(x_1) = 1$$

$$E_a^2(x_2) = \begin{bmatrix} 0.05x_{22} \\ 0.21 \end{bmatrix}, \quad F_a^2(x_2) = \begin{bmatrix} 0.3 & 0.1 \end{bmatrix}, \ \gamma_a^2(x_2) = 1$$

$$E_b^2(x_2) = \begin{bmatrix} 0.04x_{22} \\ 0.21 \end{bmatrix}, \quad F_b^2(x_2) = 0.3, \ \gamma_b^2(x_2) = 1$$

$$E_a^3(x_3) = \begin{bmatrix} 0.04x_{23} \\ 0.1 \end{bmatrix}, \quad F_a^3(x_3) = \begin{bmatrix} 0.2 & 0.1 \end{bmatrix}, \ \gamma_a^3(x_3) = 1$$

$$E_b^3(x_3) = \begin{bmatrix} 0.06x_{23} \\ 0.1 \end{bmatrix}, \quad F_b^3(x_3) = 0.6, \ \gamma_b^3(x_3) = 1$$

Solving the conditions in Theorem 4 by employing the SOS tool of Matlab, the controller gains are obtained as follows

$$K^1(x_1) = \begin{bmatrix} K_{11}(x_1) & K_{12}(x_1) \end{bmatrix},$$
$$K^2(x_2) = \begin{bmatrix} K_{21}(x_2) & K_{22}(x_2) \end{bmatrix},$$
$$K^3(x_3) = \begin{bmatrix} K_{31}(x_3) & K_{32}(x_3) \end{bmatrix},$$

where

$$K_{11}(x_1) = 14.45x_{11}^2 + 0.074x_{11}x_{21} - 0.163x_{11} + 14.25x_{21}^2 + 0.155x_{21} + 46.97$$
$$K_{12}(x_1) = 0.552x_{11}^2 - 0.011x_{11}x_{21} - 0.122x_{11} + 0.60x_{21}^2 - 0.079x_{21} + 2.00$$
$$K_{21}(x_2) = 2.04x_{12}^2 + 0.038x_{12}x_{22} + 0.937x_{12} + 1.945x_{22}^2 - 0.597x_{22} + 6.945$$
$$K_{22}(x_2) = 5.152x_{12}^2 - 0.047x_{12}x_{22} - 0.749x_{12} + 5.25x_{22}^2 + 0.649x_{22} + 16.77$$
$$K_{31}(x_3) = 1.150x_{13}^2 - 0.0014x_{13}x_{23} + 0.255x_{13} + 1.1388x_{23}^2 - 0.043x_{23} + 3.933$$
$$K_{32}(x_3) = 2.7688x_{13}^2 - 0.0065x_{13}x_{23} - 0.1422x_{13} + 2.782x_{23}^2 + 0.1564x_{23} + 9.437$$

With the above controller gains, carrying simulation in Simulink of Matlab, the obtained results are shown in Figs. 8-13.

Firstly, we simulate the large-scale nonlinear system (78)-(80) without the controller and the responses of the states x_{11}, x_{21}, x_{12}, x_{22}, x_{13}, and x_{23} are illustrated in Figs. 11-13.

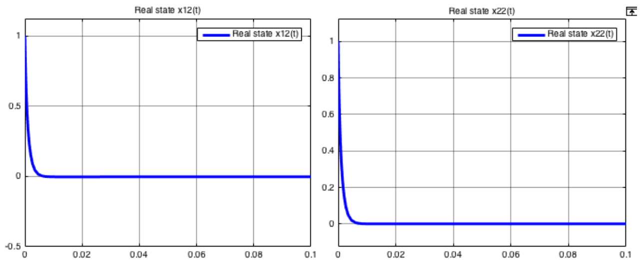

FIGURE 6. States $x_{12}(t)$ and $x_{22}(t)$ with the controller.

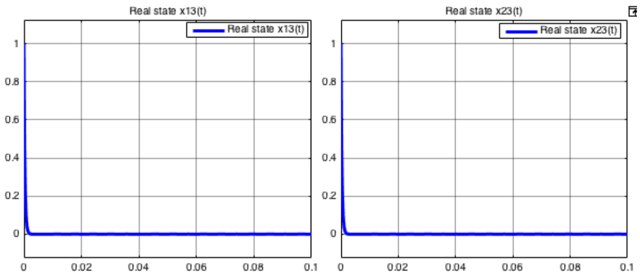

FIGURE 7. States $x_{13}(t)$ and $x_{23}(t)$ with the controller.

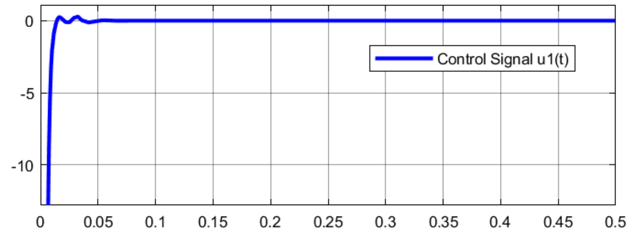

FIGURE 8. Control signal $u_1(t)$.

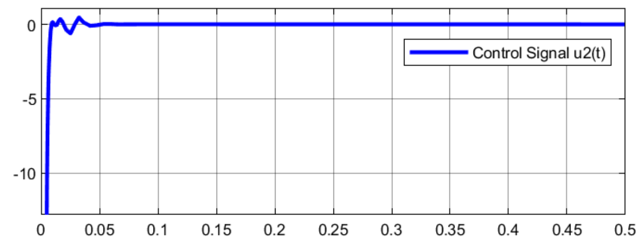

FIGURE 9. Control signal $u_2(t)$.

From these figures, it is seen that all states of the system are divergent, it means that the system is unstable. After that, we apply the method in Section IV to design the controller for this system and carry out simulating in Matlab; the obtained results are shown in Figs. 14-16. These figures show that, with a controller, all states of the system are approach zero asymptotically. The control signals $u_1(t)$, $u_2(t)$, and $u_3(t)$,

$$
\begin{aligned}
x_1 &= \begin{bmatrix} x_{11} \\ x_{21} \end{bmatrix}, \quad x_2 = \begin{bmatrix} x_{12} \\ x_{22} \end{bmatrix}, \; x_3 = \begin{bmatrix} x_{13} \\ x_{23} \end{bmatrix}, \\
B^2(x_2) &= \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \\
A^1(x_1) &= \begin{bmatrix} -2(x_{21}+1) & 0.5x_{11} \\ x_{21} & -0.575 \end{bmatrix}, \quad B^1(x_1) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \\
B^3(x_3) &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \tilde{A}^1(x_1) = \begin{bmatrix} 1.05x_{21}\sin(2x_{11}) & -0.105x_{21}\sin(2x_{11}) \\ 0.1\sin(2x_{11}) & -0.01\sin(2x_{11}) \end{bmatrix}, \\
\tilde{B}^1(x_1) &= \begin{bmatrix} -0.02x_{21}\cos(x_{21}) \\ 0.02\cos(x_{21}) \end{bmatrix}, \quad M^{12}(x_1) = \begin{bmatrix} 1 & 0 \\ 0.5x_{21} & 0 \end{bmatrix} \\
M^{13}(x_1) &= \begin{bmatrix} 1 & 0 \\ 0.5x_{21} & 0 \end{bmatrix}, \quad A^2(x_2) = \begin{bmatrix} -2x_{12} & 0.75x_{22} \\ 0.5 & -0.8x_{22} \end{bmatrix}, \\
\tilde{A}^2(x_2) &= \begin{bmatrix} 0.015x_{22}\cos(0.5x_{22}) & 0.005x_{22}\cos(0.5x_{22}) \\ 0.063x_{12}\cos(0.5x_{22}) & 0.021x_{22}\cos(0.5x_{22}) \end{bmatrix} \\
\tilde{B}^2(x_2) &= \begin{bmatrix} 0.012x_{22}\sin(2x_{12}) \\ 0.06\sin(2x_{12}) \end{bmatrix}, \quad M^{21}(x_2) = \begin{bmatrix} 0.4x_{22} & 0 \\ 0.6x_{22} & 0 \end{bmatrix} \\
M^{23}(x_2) &= \begin{bmatrix} 0.4x_{22} & 0 \\ 0.6x_{22} & 0 \end{bmatrix}, \quad A^3(x_3) = \begin{bmatrix} -x_{13} & 0.3x_{23} \\ 1 & -0.775x_{23} \end{bmatrix} \\
\tilde{A}^3(x_3) &= \begin{bmatrix} 0.008x_{23}\sin(x_{13})\cos(x_{23}) & 0.04x_{23}\sin(x_{13})\cos(x_{23}) \\ 0.02\sin(x_{13})\cos(x_{23}) & 0.1\sin(x_{13})\cos(x_{23}) \end{bmatrix} \\
\tilde{B}^3(x_3) &= \begin{bmatrix} 0.036x_{23}\cos(x_{23}) \\ 0.06\cos(x_{23}) \end{bmatrix} \\
M^{31}(x_3) &= \begin{bmatrix} 0.2 & 0 \\ 0.05x_{23} & 0 \end{bmatrix}, \quad M^{32}(x_3) = \begin{bmatrix} 0.2 & 0 \\ 0.05x_{23} & 0 \end{bmatrix}
\end{aligned}
$$

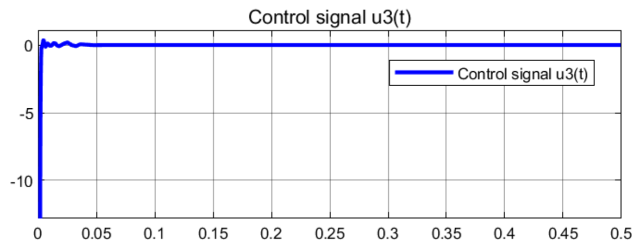

FIGURE 10. Control signal $u_3(t)$.

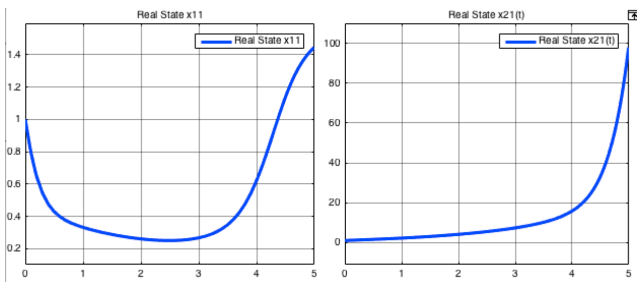

FIGURE 11. States x_{11}(t) and x_{21}(t) without the controller.

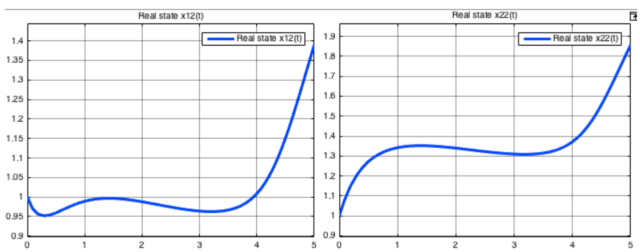

FIGURE 12. States $x_{12}(t)$ and $x_{22}(t)$ without the controller.

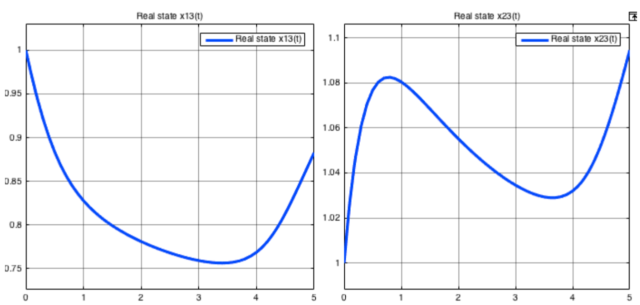

FIGURE 13. States x_{13}(t) and x_{23}(t) without the controller.

are shown in Fig. 17, 18, and 19, respectively. With these simulation results, it concludes that the proposed method in this paper is successful to synthesize the controller for the nonlinear large-scale system (78)-(80).

Remark 4: It is obvious that in Example 2, there exist the 12 nonlinear terms in the large-scale nonlinear system (78)-(90). Therefore, if the previous methods in papers [8], [17], [18] are employed to design the controller, we have to linearize many nonlinear terms such as x_{11}, x_{21}, $sin(2x_{21})$, $cos(x_{21})$, x_{12}, x_{22}, $cos(0.5x_{22})$, $cos(x_{21})$, x_{13}, x_{23}, $sin(x_{13})$, $cos(x_{23})$ that

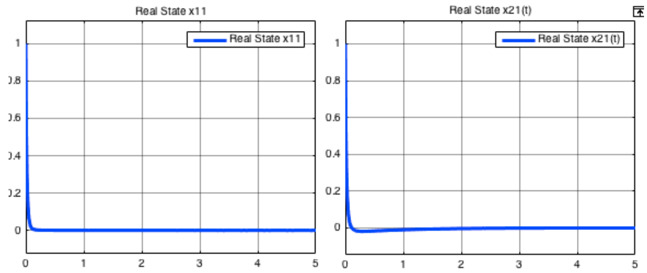

FIGURE 14. States x_{11}(t) and x_{21}(t) with the controller.

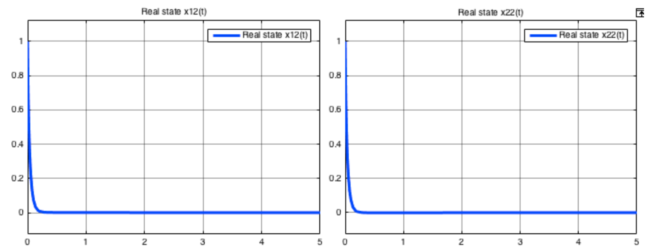

FIGURE 15. States x_{12}(t) and x_{22}(t) with the controller.

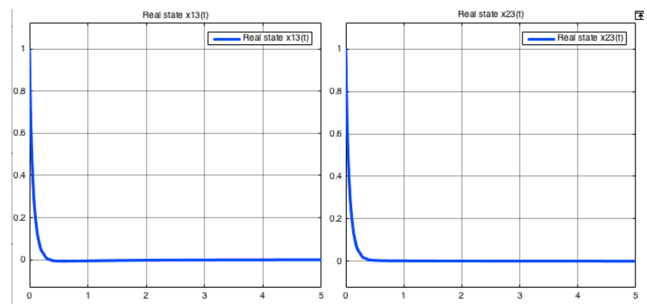

FIGURE 16. States x_{13}(t) and x_{23}(t) with the controller.

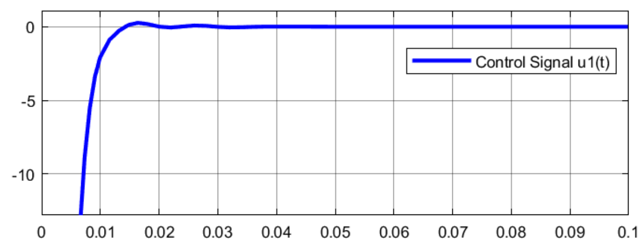

FIGURE 17. Control signal $u_1(t)$.

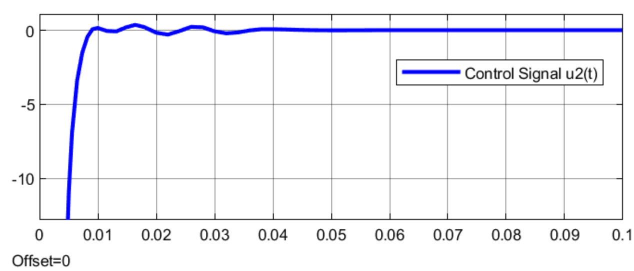

FIGURE 18. Control signal $u_2(t)$.

will cause the modeling error and make the performance of the control system degrade. Moreover, if we apply the T-S

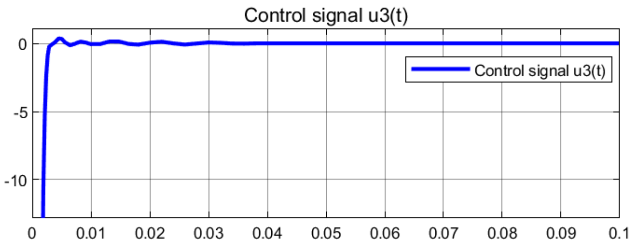

FIGURE 19. Control signal $u_3(t)$.

fuzzy model in [25]–[37] to model the large-scale nonlinear system (78)-(80) then the number of the fuzzy rule are 2^{12} *rules* that will make the controller design procedure much more complicated and the computational burden significantly increase. While, with the proposed method, the controller gain can be obtained easily by solving the conditions in Theorem 2 by using the SOS tool of Matlab.

VI. CONCLUSION

This paper studies the method to design the decentralized polynomial controller for the nonlinear large-scale system. The large-scale nonlinear system in this work is modeled in terms of the polynomial system. The advantage of the proposed method is that the nonlinear terms of the large-scale nonlinear system are put inside the system matrices and unnecessary to linearize, therefore the complexity and computational load are significantly reduced in comparison with previous studies. Both large-scale nonlinear systems with non-polynomial and polynomial terms are considered in this paper. By resolving SOS conditions in the main theorems, the parameters of the controller are obtained. The simulation results of two illustrative examples show the success of the proposed method. However, there still exist many problems of designing decentralized controller for this system such as the system with the existence of the faults, uncertainties, or signal dropout and so on. Thus, these issues will be investigated in my future work.

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