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Research on Hover Control of AUV Uncertain Stochastic Nonlinear System Based on Constructive Backstepping Control Strategy

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ABSTRACT The paper is concerned with station keeping control problem via backstepping for an autonomous underwater vehicle (AUV) systems with nonlinear continuous function. This paper is the first time to using backstepping control method for stochastic nonlinear systems. Under nonlinear growth conditions, when the AUV motion affected the driving force, resistance of ocean and the force generated by current the difficulty arising from the underwater robot system output feedback makes the station keeping problems challenging and forward-looking. There exist two problems to research the fast response time and good station keeping, respectively. There a new control method is proposed for the first time to construct a backstepping stabilizing controller. In this paper, a concept of randomness to study the station keeping control problem of output feedback of stochastic nonlinear systems, remove the original harsher growth conditions, make it meet the more general function growth conditions. In order to deal with the station error of system converges to arbitrarily small domains. Under all the states of the system meet boundedness, a coordinate transformation is proposed. A useful technical theorem is proposed in the stability analysis to show that combined with the backstepping method to cleverly construct a set of Lyapunov functions, and obtain the output controller to ensure that the system is asymptotically probabilistic in the global scope. Finally, through the ocean library in the Simulation X simulation software and the Hardware-in-the-loop simulation, the controller design results are imported into the AUV actuator model to verify the effectiveness of the controller design.

INDEX TERMS Station keeping, complex uncertain nonlinear systems, system growth conditions, feedback control with backstepping.

I. INTRODUCTION

The operation of AUV is known as one of the most challenging problems in the field of nonlinear control, and they are playing a crucial role in exploration and utilization of resources located at deep oceanic environments [1]–[3]. In recent decades, station keeping, tracking control and path planning of underwater robots have gained the attention of researchers. Therefore, the control theory and engineering fields urgently need to study the output feedback control theory and related technologies, which are suitable for complex systems or systems affected by various factors (such as nonlinearity, randomness, time delay) [4]–[6]. This project mainly studies the output feedback control of the system under the action of the above-mentioned various factors, and at the same time tries to make the studied system more in line with the actual system, relaxes and removes the strict assumptions attached to the system, and makes the final conclusion more accurate. Generality, and on this basis, the theoretical results are applied to an appropriate practical engineering system—the underwater electro-hydraulic actuator system [7], [8].

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As we all know, the control theory system established under the framework of deterministic system is no longer applicable to stochastic system [9]–[11]. Therefore, it is a big challenge for the control theory and engineering community to solve the output feedback control problem of stochastic nonlinear systems with the help of stochastic control theory and related mathematical tools [12]–[14]. For a system, its input and output are likely to be random signals, the probability of this happening is very high in practice, and the system combining nonlinear characteristics with random characteristics is a research field that the theoretical circle is more concerned about. Up to now, the research on the design of stochastic nonlinear systems is still very incomplete. There are still many problems to be solved in the research on the design ideas of feedback controllers based on inverse inference and constructive establishment, such as how remove stochastic nonlinear systems and form a relatively complete stochastic nonlinear control theory system based on the inverse method or the constructive controller design method [15]–[17]. In addition, in the actual system, there is a lag of data measurement and actuators, some time delays do not change with time, some time delays change with time, which increases the complexity of the system and makes the research on the output feedback control of the system-like system is full of necessity and practical application value.

Previous studies limitations of are as follows:

- Underwater vehicle is considered as fully actuated,
- Limited numbers of attempts are considered underwater currents and their effects.
- Same papers which are used for tracking control are used for station keeping, therefore time of converge to zero is long and controller design is complex in nature.

It is particularly worth noting for the field of deep-sea electro-hydraulic actuator system development and the station keeping of the underwater robot, due to the impact of water flow and the harsh environment of the deep sea [18]–[21], the system model of AUV in the process of performing tasks presents relative uncertainty and nonlinear characteristics, at the same time, the hovering control of the underwater robot system is the most critical part [22]. Based on the above analysis, the system of AUV engineering problems include nonlinearity, uncertainty, piecewise linearity, etc. And the hover control for underwater robot determines the stability and reliability of the operation of the entire underwater equipment, which is related to the safety of ocean generation and development.

In the existing research, the hover control strategy for AUV is mainly based on linearizing the system or linearizing the original nonlinear system. For example, [20], [24], in the paper [20] simplifies the AUV system, which is a linear system. In this way, multiple controller switching can be performed for a simple controlled system, but the system cannot reflect the complex characteristics of the actual system. Literature [24] uses the PID algorithm to realize the stable hovering of the AUV. However, the PID control does not have the complex characteristic description inside the complex control object and does not require an accurate control object description, which will directly lead to inaccurate, unstable and non-persistent control effects. Different from the existing research results, this paper conducts detailed model identification for the AUV controlled system. Nonlinear systems is constructed with random perturbations and uncertain parameters. This system will achieve fine hover control of the AUV. The contributions of this paper are characterized by the following features:

(1) To the best of author's knowledge, Two papers (Guo and Zhou, 2021; Guo and Ni, 2021) are the newest achievement to consider AUV systems hovering control. It should be pointed out that, This articles are mainly based on the research of hover control with the stable special working point of the AUV controlled object. The overall control through the analysis of several operating points will produce large errors, and this paper is based on the analysis and controller design of a continuous nonlinear system with uncertain parameters.

(2) This paper is the first time to use backstepping control method for stochastic nonlinear systems. The backstepping control method is used to solve the problem of AUV hovering, which the nonlinear vector terms depend on the unmeasurable states besides the measurable output, and it is satisfying the more general growth conditions.

(3) For the first time, constraints of the system are studied in the AUV hovering technology, which makes the constraints more relaxed and the AUV system's underwater operation more complex with the actual situation and disturbance. By finding the optimal controller as the goal and minimizing the conservatism as the condition, the hovering of the system is asymptotically stable in the sense of probability under random disturbance.

(4) The high-order gain observer technique is applied to the backstepping control for stochastic nonlinear systems for the first time.

The paper is organized as follows. Section 2 provides the description of dynamic modelling of robot, In Section 3 is the station keeping control design, Section 4 is the underwater robot transportation model example and Section 5 is the concluding and future prospects.

II. DESCRIPTION OF DYNAMIC MODELLING OF ROBOT

In this section, we consider the output feedback control problem for a well-known underactuated underwater robot system model. As shown in Figure 1.

Whether on the surface or underwater, an underwater robot cannot be seen as operating in an ideal state without interference. There are many kinds of interference, such as the most common surface gravitational waves [23]. This kind of interference will attenuate quickly when the operating depth is deep; then there is the ocean current, which is also a disturbance factor. Due to various reasons such as thermal radiation, evaporation and precipitation in the seawater, the seawater will produce various inconveniences, water masses of the same density. Ocean currents can not only appear in shallow waters, but also exist in deep waters far away

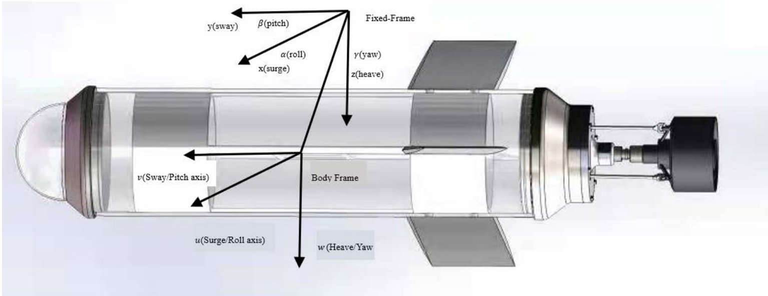

FIGURE 1. Body-fixed frame and earth-fixed reference frame for underwater robot.

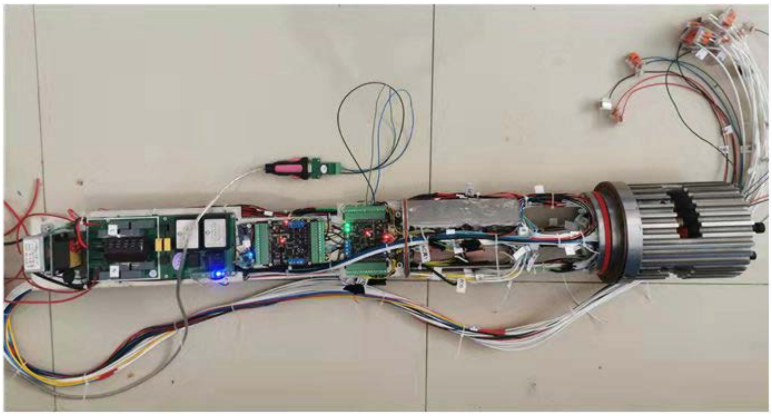

FIGURE 2. Hardware-in-the-loop simulation device with underwater robot.

from the surface, which is its biggest feature. At the same time, it also shows that the ocean current is more likely to have an impact on the dynamics and control performance of the underwater robot, and the impact of the ocean current should be fully considered for the control problem [24], [25]. Usually, we assume that the displacement of the underwater robot is measurable from the ground, and consider the ocean current as an external disturbance to the dynamics of the underwater robot in the coordinate system.

The underwater kinematics equation of the underactuated AUV can be expressed:

$$M\dot{\nu} + C(\nu)\nu + D(\nu)\nu + g(\epsilon) = F \tag{1}$$

where,

M are the inertia matrix, which contains AUV additional quality. $C(\nu)$ are Coriolis centripetal force representing underwater vehicles. $D(\nu)$ stands for underwater vehicle fluid damping. $g(\epsilon)$ represents the gravity of the underwater vehicle in operation and the restoring force (moment) generated by the buoyancy. $F = [F_x \quad F_y \quad F_z \quad F_k \quad F_m \quad F_n]^T$ are the resultant forces and moments. $\dot{\nu} = [\dot{u} \quad \dot{\nu} \quad \dot{\omega} \quad \dot{p} \quad \dot{q} \quad \dot{r}]^T$ are the linear and angular accelerations in surge, sway and heave direction respectively with respect to body(moving) frame, $\nu = [u \quad \nu \quad \omega \quad p \quad q \quad r]^T$ are linear and angular velocity with respect to body(moving)frame, u - surge velocity, ν - sway velocity, ω - heave velocity, p - roll rate, q - pitch rate, r - yaw rate. $\epsilon = [x \quad y \quad z \quad \alpha \quad \beta \quad \gamma]^T$ are position and orientations with respect to inertial(fixed) frame, x - surge position, y - sway position, z - heave position, α - roll angle, β - pitch angle, γ - yaw angle.

It can be seen that, because of its special properties, if the controller is designed with the above mention six degree of freedom model, the controller will be very complicated and difficult to realize. Therefore, when the AUV moves at underwater for designing the controller, In the body frame, the velocity can generally be decomposed into two motion models, linear velocity variables $\nu_1 = [u \quad \nu \quad \omega]^T$ and angular velocity variables $\nu_2 = [p \quad q \quad r]^T$. In the earth-fixed frame, the velocity can generally be decomposed into two motion models, vehicle position $\epsilon_1 = [x \quad y \quad z]^T$ and vehicle orientation $\epsilon_2 = [\alpha \quad \beta \quad \gamma]^T$, which can greatly simplify the AUV model. In this paper only consider the velocity variables $\nu_1 = [u \quad \nu \quad \omega]^T$ and linear motion $\epsilon_1 = [x \quad y \quad z]^T$ of the AUV underwater.

The relationship between body frame and fixed frame in linear velocity is given by:

$$\dot{\epsilon}_1 = J(\epsilon)\nu_1 \tag{2}$$

where, $J(\epsilon)$ is the kinematic transformation matrix and it is in the following form:

$$J(\epsilon) = \begin{pmatrix} c\gamma c\beta & -s\gamma c\alpha + c\gamma s\beta s\alpha & s\gamma s\alpha + c\gamma c\beta s\alpha \\ s\gamma c\beta & c\gamma c\alpha + s\gamma s\beta s\alpha & -c\gamma s\alpha + s\gamma s\beta c\alpha \\ -s\beta & c\beta s\alpha & c\beta c\alpha \end{pmatrix}$$

where, $s = sin$ and $c = cos$. Another, $\epsilon_2 = [\alpha \quad \beta \quad \gamma]^T$ is the angle in surge, sway and heave direction with respect to earth frame, respectively. This transformation is undefined for $\beta = \pm 90°$ and to overcome this singularity, a quaternion approach must be considered. However, most of the robots are designed to operate at pitch angles well below $\pm 90°$ and hence this limitation has no major significance here.

For the better understanding and good detailed analysis, it is preferred to investigate the system with respect to the earth fixed frame of reference in order to maintain every state to a single reference frame. For this, the coordinate transformation $(\epsilon_1, \nu_1) \xrightarrow{\mu} (\epsilon_1, \dot{\epsilon}_1)$ is performed using Eq.(2), which yields:

$$\begin{pmatrix} \epsilon_1 \\ \dot{\epsilon}_1 \end{pmatrix} = \begin{bmatrix} I & 0 \\ 0 & J(\epsilon) \end{bmatrix} \begin{pmatrix} \epsilon_1 \\ \nu_1 \end{pmatrix}$$

The coordinate transformation μ is a global diffromorphism, analogous to a similarity transformation in linear system. The robot dynamic model with respect to the earth fixed frame of reference becomes:

$$M_{\epsilon_1}\ddot{\epsilon}_1 + C_{\epsilon_1}\dot{\epsilon}_1 + D_{\epsilon_1}\dot{\epsilon}_1 + g_{\epsilon_1} = F_{\epsilon_1} \tag{3}$$

where,

$$\begin{aligned} M_{\epsilon_1} &= J(\epsilon)^{-T} M J(\epsilon)^{-1} \\ C_{\epsilon_1} &= J(\epsilon)^{-T}\left(C(\nu) - MJ(\epsilon)^{-1}\dot{J}(\epsilon)\right)J(\epsilon)^{-1} \\ D_{\epsilon_1} &= J(\epsilon)^{-T} D(\nu) J(\epsilon)^{-1} \\ g_{\epsilon_1} &= J(\epsilon)^{-T} g(\epsilon) \\ F_{\epsilon_1} &= J(\epsilon)^{-T} F \end{aligned} \tag{4}$$

The following assumptions are needed.

Assumption 1: The article only consider the velocity of between self position and fixed frame, which is $\dot{l} = (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{\frac{1}{2}}$. Assumption the system origin station $\epsilon_1(0) = [0 \quad 0 \quad 0]$ and $\dot{\epsilon}_1(0) = [0 \quad 0 \quad 0]$.

Assumption 2: In the paper. The velocity $\epsilon_1 = [x \quad y \quad z]$ and the angle $\epsilon_2 = [\alpha \quad \beta \quad \gamma]$ is known by using sensors.

According to the characteristics of underwater robot system, we generalize the dynamic model to obtain a broad mathematical model with unknown control directions as follows.

$$\begin{aligned} dl_1 &= l_2 dt + f_1(t, l(t), u)dt + g_1(l)d\omega \\ dl_2 &= udt + f_2(t, l(t), u)dt + g_2(l)d\omega \\ y &= l_1 \end{aligned} \tag{5}$$

where, $l = (l_1, l_2)^T \in R^2, u \in R, y \in R$, are the states, input and output of system; corresponding to this. Here, we introduce a stochastic process for the system: ω is an m - dimensional standard Wiener process defined on the complete probability space (Ω, Γ, P) with Ω being a sample space, Γ being a filtration, and P being a probability measure. Observable state l_2 is not measurable. $f_i : \mathbb{R}^n \to \mathbb{R}$ and $g_i : \mathbb{R}^n \to \mathbb{R}^r, i = 1, 2$ is satisfied a linear growth of the following conditions and is local Lipschitz. And $f_i(0) = 0$, $g_i(0) = 0$.

Definition 1: for any given function $V \in C^2(\mathbb{R}^2; \mathbb{R})$ associated (5), the differential operator $\$$ is defined as

$$\$V(l) = \frac{\partial V(l)}{\partial l} f(l, t) + \frac{1}{2} Tr\{g^T(l, t)\frac{\partial^2 V(l)}{\partial l^2} g(l, t)\} \tag{6}$$

Definition 2: If there is a $\mathcal{K}_\infty$ function γ, its derivative γ' exists and is also a $\mathcal{K}_\infty$ function, a matrix-valued function $R_2(l)$ that satisfies $R_2(l) = R_2^T(l) > 0$ for all l, and there is a feedback control function $u = \alpha_1$ that is continuous except for the origin $\alpha(0) = 0$. If the feedback control function can ensure that the equilibrium point $l = 0$ is globally asymptotically stable for the probability, and the performance function

$$J(u) = E\left(\int_0^\infty \left[s(x) + \gamma(|R_2(l)^{1/2}u|)\right] d\tau\right)$$

can be guaranteed to take the minimum value, it is said that the inverse optimal stabilization problem for the probability of the system (5) is to be solvable.

We give the solution to the probabilistic inverse optimal stabilization problem as follows:

Lemma 1: Considering the control law:

$$u = \alpha(l) = -R_2^{-1}(L_{g2}V)^T \frac{\ell\gamma(|L_{g2}VR_2^{-1/2}|)}{|L_{g2}VR_2^{-1/2}|^2} \tag{7}$$

where $V(l)$ is a preselected Lyapunov function, $\gamma(\cdot)$ is a $\mathcal{K}_\infty$ function, and its derivative γ' exists and is also a $\mathcal{K}_\infty$ function, and $R_2(l)$ is a matrix-valued function that satisfies $R_2(l) = R_2^T(l) > 0$. If the control law (7) makes the system (5) is globally asymptotically stable according to the probability about $V(l)$, then the control law

$$u^* = \alpha^*(l) = -\frac{\beta}{2} R_2^{-1}(L_{g2}V)^T \frac{(\gamma')^{-1}(|L_{g2}VR_2^{-1/2}|)}{|L_{g2}VR_2^{-1/2}|}, \quad \beta \geq 2 \tag{8}$$

can make the performance function

$$J(u) = E\left(\int_0^\infty \left[s(l) + \beta^2\gamma\left(\frac{2}{\beta}|R_2(l)^{1/2}u|\right)\right] d\tau\right) \tag{9}$$

to solve the probabilistic inverse optimal stabilization problem of system (5) by minimizing where $s(x)$ is a positive definite radially unbounded function satisfying

$$\begin{aligned} s(l) = 2\beta\left[\ell\gamma(|L_{g2}VR_2^{-1/2}|) - L_f V - \frac{1}{2}Tr\left(g_1^T \frac{\partial^2 V}{\partial l^2} g_1\right)\right] \\ + \beta(\beta - 2)\ell\gamma(|L_{g2}VR_2^{-1/2}|) \end{aligned} \tag{10}$$

Assumption 3: there exists known positive constant $C_f, C_g \geq 0$ such that the following inequality holds

$$\begin{aligned} |f_i(l)| &\leq C_f(|l_1| + \cdots + |l_i|) \\ |g_i(l)| &\leq C_g(|l_1| + \cdots + |l_i|) \end{aligned} \tag{11}$$

Young inequality: for any two real vector has the same dimension l and y, $l^T y \leq \frac{\lambda^p}{p}|l|^p + \frac{1}{q\lambda^q}|y|^q$, where $\lambda > 0$, $p > 1, q > 1$, and $p^{-1} + q^{-1} = 1$.

Based on this assumption, for the system (5) to design a smooth output feedback controller, enable closed-loop system at the origin is the probability of global asymptotic stability, and the probability of inverse optimization problems are solvable.

III. STATION KEEPING CONTROL DESIGN

The controller design of nonlinear system (5) is divided into two parts. The first part is to design a state observer for the system; the second part uses the backstepping method to design an output feedback controller for system (5) to meet the corresponding control performance indicators.

A. HIGH-ORDER GAIN OBSERVER DESIGN

Since the system is not measurable except for l_1 which is a measurable state, a set of high-order gain observers is first constructed:

$$\begin{aligned}\dot{\hat{l}}_1 &= \hat{l}_2 + H^1 K_1 \left(l_1 - \hat{l}_1\right),\\ \dot{\hat{l}}_2 &= u + H^2 K_2 \left(l_1 - \hat{l}_1\right),\end{aligned} \tag{12}$$

In this set of observers, the $H > 0$ is a unknown high-gain parameter and real number $K_i > 0$ is a known constant such that the matrix $A = \begin{pmatrix} -K_1 & 1 \\ -K_2 & 0 \end{pmatrix}$ is asymptotically stable, thus there exists a positive definite matrix P that satisfies $A^T P + PA = -I$. we define the error equation of system (5) as

$$\tilde{l} = \left(\tilde{l}_1, \tilde{l}_2\right)^T, \tilde{l}_i = \frac{l_i - \hat{l}_i}{H^{i-1}}, i = 1, 2 \tag{13}$$

According to (5) and (12), the system error model can be obtained as

$$d\tilde{l} = HA\tilde{l}dt + F(l)dt + G(l)d\omega \tag{14}$$

where

$$\begin{aligned}F(l) &= \left(f_1(l), \frac{1}{H} f_2(l)\right)^T,\\ G(l) &= \left(g_1^T(l), \frac{1}{H} g_2^T(l)\right)^T\end{aligned} \tag{15}$$

Choosing $V_0(\tilde{l}) = 3\tilde{l}^T P \tilde{l}$, applying $|\frac{1}{H^{i-1}} l_i| \le |\frac{1}{H^{i-1}} \hat{l}_i| + |\tilde{l}_i|$, $\left(\sum_{i=1}^2 a_i\right)^2 \le 2\sum_{i=1}^2 a_i^2, H > 0$, Definition 1 and Assumption 3, we can obtain

$$\begin{aligned}\$V_0 &= 3H\tilde{l}^T \left(A^T P + PA\right) \tilde{l} + 6\tilde{l}^T PF(l)\\ &\quad + 3Tr\{G^T(l) PG(l)\}\\ &\le -3H|\tilde{l}|^2 + 3|\tilde{l}^T P|^2\\ &\quad + 3\left(\sum_{i=1}^2 |\frac{f_i(l)}{H^{i-1}}|^2 + \lambda_{max}(P) \sum_{i=1}^2 |\frac{g_i(l)}{H^{i-1}}|^2\right)\\ &\le -3H|\tilde{l}|^2 + 3\|P\|^2|\tilde{l}|^2\\ &\quad + 3\left(\left(\sum_{i=1}^2 |\frac{f_i(l)}{H^{i-1}}|\right)^2 + \lambda_{max}(P)\left(\sum_{i=1}^2 |\frac{g_i(l)}{H^{i-1}}|\right)^2\right)\\ &\le -3H|\tilde{l}|^2 + 3\|P\|^2|\tilde{l}|^2\\ &\quad + 3C_f^2\left(\sum_{i=1}^2 \frac{1}{H^{i-1}}\right)^2 \left(|l_1| + \frac{|l_2|}{H} + \cdots + \frac{|l_2|}{H^1}\right)^2\\ &\quad + 3\lambda_{max}(P) C_f^2 \left(\sum_{i=1}^2 \frac{1}{H^{i-1}}\right)^2\\ &\quad \times \left(|l_1| + \frac{|l_2|}{H} + \cdots + \frac{|l_2|}{H^1}\right)^2\\ &= -3H|\tilde{l}|^2 + 3\|P\|^2|\tilde{l}|^2 + c_1\left(|l_1| + \frac{|l_2|}{H} + \cdots + \frac{|l_2|}{H^1}\right)^2\\ &\le -3H|\tilde{l}|^2 + 3\|P\|^2|\tilde{l}|^2 + c_1\left(\sum_{i=1}^2 |\frac{1}{H^{i-1}}\hat{l}_i| + \sum_{i=1}^2 |\tilde{l}_i|\right)^2\\ &\le -3H|\tilde{l}|^2 + 3\|P\|^2|\tilde{l}|^2\\ &\quad + 2c_1\left(\left(\sum_{i=1}^2 |\frac{1}{H^{i-1}}\hat{l}_i|\right)^2 + \left(\sum_{i=1}^2 |\tilde{l}_i|\right)^2\right)\\ &\le -3H|\tilde{l}|^2 + 3\|P\|^2|\tilde{l}|^2\\ &\quad + 4c_1\left(\sum_{i=1}^2 \left(|\frac{1}{H^{i-1}}\hat{l}_i|\right)^2 + \sum_{i=1}^2 \left(|\tilde{l}_i|\right)^2\right)\\ &= -\left(3H - 3\|P\|^2 - 4c_1\right)|\tilde{l}|^2 + 4c_1\left(\hat{l}_1^2 + \frac{\hat{l}_2^2}{H^2}\right)\end{aligned} \tag{16}$$

where

$$\begin{aligned}c_1 &= 3(C_f^2 + \lambda_{max}(P) C_g^2)\left(\sum_{i=1}^2 \frac{1}{H^{i-1}}\right)^2\\ &= \frac{3(C_f^2 + \lambda_{max}(P) C_g^2)}{H^2}\left(\sum_{i=0}^1 (i+1)H^i + (3-i)H^2\right)\end{aligned} \tag{17}$$

B. OUTPUT-FEEDBACK CONTROLLER DESIGN

In this subsection, we give the backstepping controller design procedure. Firstly, introduce a series of coordinate transformation

$$z_1 = \tilde{l}_1, z_2 = \tilde{l}_2 - \alpha_1\left(\hat{l}_{[11]}\right) \tag{18}$$

where $\alpha_1\left(\hat{l}_{[11]}\right)$ is the virtual control laws to be designed.

step 1: definiting

$$V_1(\tilde{l}, z_1) = V_0(\tilde{l}) + \frac{1}{2} z_1^2 \tag{19}$$

by Definite 1, system (12), (16)-(19) and Young's inequality

$$\begin{aligned}\$V_1 &\le -\left(3H - 3\|P\|^2 - 4c_1\right)|\tilde{l}|^2 + 4c_1\left(\hat{l}_1^2 + \frac{\hat{l}_2^2}{H^2}\right)\\ &\quad + z_1(\hat{l}_2 + Hl_1\tilde{l}_1)\\ &\le -\left(3H - 3\|P\|^2 - 4c_1\right)|\tilde{l}|^2 + 4c_1 z_1^2 + 4c_1\frac{\hat{l}_2^2}{H^2}\\ &\quad + z_1\alpha_1 + z_1 z_2 + H\tilde{l}_1^2 + \frac{H}{4} l_1^2 z_1^2\end{aligned} \tag{20}$$

Using the (18) and $(a+b)^2 \leq 2a^2+2b^2$, choosing $H \geq 8c_1$, we can obtain

$$4c_1 z_1^2 \leq \frac{H}{2}z_1^2, 4c_1\frac{1}{H^2}\tilde{l}_2^2 \leq 8c_1\frac{1}{H^2}z_2^2 + 8c_1\frac{1}{H^2}\alpha_1^2 \quad (21)$$

Therefore, according the equation (20) and equation (21), we can get the first virtual controller as

$$\alpha_1(\hat{l}_1) = -Hb_1 z_1, b_1 = \frac{1}{2} + \frac{l_1^2}{4} + 2 \quad (22)$$

renders

$$\begin{aligned}\$V_1 \leq& -\left(3H - 3\|P\|^2 - 4c_1\right)|\tilde{l}|^2 + H\left(\frac{1}{2} + \frac{l_1^2}{4}\right)z_1 \\ &+ 8c_1\frac{1}{H^2}z_2^2 + 8c_1\frac{1}{H^2} + z_1\alpha_1 + z_1 z_2 + H\tilde{l}_1^2 \\ \leq& -\left(2H - 3\|P\|^2 - 4c_1\right)|\tilde{l}|^2 - (2H - 8c_1 b_1^2)z_1^2 \\ &+ 8c_1\frac{1}{H^2}z_2^2 + z_1 z_2 \end{aligned} \quad (23)$$

In the last step through backstepping theory, select the controller

$$\begin{aligned} z_2 &= \hat{l}_2 + Hb_1\hat{l}_1 \\ dz_2 &= \left(u + H^2 d_{20}\tilde{l}_1 + H^2 d_{21}z_1 + H^2\bar{d}_{2,1}z_1 + H^2 d_{22}z_2\right)dt \end{aligned} \quad (24)$$

where

$$\begin{aligned} d_{i0} &= l_i + b_{i-1}l_{i-1} + b_{i-1}b_{i-2}l_{i-2} + \cdots + b_{i-1}b_{i-2}\ldots b_1 l_1 \\ d_{ij} &= \prod_{k=j-1}^{i-1} b - k - b_j\prod_{k=j}^{i-1} b_k, \quad j=1,\ldots,i-2,\ b_0=0 \\ \bar{d}_{i,i-1} &= b_{i-1}b_{i-2} - b_{i-1}^2, \quad d_{ii} = b_{i-1} \end{aligned} \quad (25)$$

By (24), Choosing control law

$$\begin{aligned} u\left(\hat{l}_{[2]}\right) &= -Hb_2 z_2 = -M(\hat{l})z_2 \\ &= -\sum_{i=1}^{2} H^i\left(\prod_{j=2-(i-1)}^{2} b_j\right)\hat{l}_{2-(i-1)} \end{aligned} \quad (26)$$

renders

$$\begin{aligned}\$V_2 \leq& -\left(H - 3\|P\|^2 - 4c_1\right)|\tilde{l}|^2 \\ &- \sum_{j=1}^{i}\frac{1}{H^{2j-2}}\left(H - 8c_1 b_j^2\right)z_j^2 - \frac{1}{H^2}Hz_2^2 \end{aligned} \quad (27)$$

where

$$V_2(\tilde{l}, z_{[2]}) = 3\tilde{l}^T P\tilde{l} + \sum_{j=1}^{2}\frac{1}{2H^{2(j-1)}}z_j^2 \quad (28)$$

$M(\hat{l}) = Hb_2, b_2 > 0$ is a real number that satisfies (22) and does not depend on H

C. STABILITY ANALYSIS

Theorem 1: For a stochastic nonlinear system (5) satisfying the Assumptions 3, there must be constants $H_1^* \geq 0$, and for any $H > H_1^*$, controller (8) and (26) can get as follows:

Conclusion:

(1) For any initial value $(l_0, \hat{l}_0)$, the solution process of the closed-loop system (5), (8) and (26) is almost everywhere and unique.

(2) The equilibrium point of the closed-loop system at the origin is globally asymptotically stable according to probability.

(3) Control law

$$u^* = \alpha^*(\hat{l}) = -\beta Hb_2 z_2, \quad \beta \geq 2 \quad (29)$$

makes the closed-loop system inverse optimal stabilization according to the probability and the cost function

$$J(u) = E\left(\int_0^\infty\left[s(\tilde{l}, \hat{l}) + \frac{1}{H^2}M^{-1}(\hat{l})u^2\right]d\tau\right) \quad (30)$$

is minimum, where $s(\tilde{l}, \hat{l})$ and (10) have the same definition, and $\hat{l} = (\hat{l}_1, \hat{l}_2)^T, \bar{f}(\tilde{l}, \hat{l}) = ((HA\tilde{l})^T + F^T(l), Hs_1\tilde{l}_1 + \hat{l}_2, H^2 s_2\tilde{l})^T, \bar{g}_1(\tilde{l}, \hat{l}) = (G^T(l), 0), \bar{g}_2(\tilde{l}, \hat{l}) = (0, 1)^T, V = V_2$

Proof: According to $d_{ii} = b_{i-1}$, (22) and (25), which can get $b_i > b_{i-1}, b_0 = 0, b_i > 1 (i = 1, 2)$. Considering $max\{4c_1, 8c_1, 8c_1 b_1^2\} = 8c_1 b_1^2$. If

$$H > 8c_1 b_1^2 + 3\|P\|^2 \quad (31)$$

is holds, According to (27), (28) and Theorem 1, the conclusion (1) and (2) is clearly established.

When $i = 1$, by analyzing (31), (17) and (22), the article is easy to get c_1 is dependent on H and b_1 is not dependent on H. Where

$$H > \frac{24(C_f^2 + \lambda_{max}(P)C_g^2)}{H^2} \times \left(\sum_{i=0}^{1}(i+1)H^i + (3-i)H^2\right)b_1^2 + 3\|P\|^2$$

is equivalent to the following.

$$\begin{aligned} H^3 > 24(C_f^2 + \lambda_{max}(P)C_g^2)b_1^2\left(\sum_{i=0}^{i}(i+1)H^i + (3-i)H^2\right) \\ + 3\|P\|^2 H^2 \end{aligned} \quad (32)$$

renders

$$H^3 + \sum_{i=0}^{2} a_i H^i > 0 \quad (33)$$

Meantime, $a_0 = -\Delta, a_1 = -2\Delta, a_2 = -\Delta - 3\|P\|^2, \Delta = 24(C_f^2 + \lambda_{max}(P)C_g^2)b_1^2$ Based on the real coefficient of the polynomial Factorization theorem. Now, we discuss the choice of H_1^* in two cases:

(i) If there is at least one positive real number in H_1, H_2, H_3, choose $H_1^* = max_{1\leq i\leq 3}\{H_i\}$ to be the largest real root.

$$\begin{aligned} H &= 8c_1 b_1^2 + 3\|P\|^2 \\ &= 24(C_f^2 + \lambda_{max}(P)C_g^2)b_1^2(1 + \frac{1}{H})^2 + 3\|P\|^2 \end{aligned} \quad (34)$$

(ii) otherwise, choosing $H_1^* = 0$. Therefore, there must be $H_1^* \geq 0$, so that for any $H > H_1^*$ (31) holds. next step, we will to prove conclusion (3). (12) and (14) can be written

$$\begin{pmatrix} d\tilde{l} \\ d\hat{l} \end{pmatrix} = \bar{f}(\tilde{l}, \hat{l})dt + \bar{g}_1(\tilde{l}, \hat{l})d\omega + \bar{g}_2(\tilde{l}, \hat{l})udt \tag{35}$$

By choosing $\gamma(r) = \frac{1}{2H^2}r^2$, we can get $(\gamma')^{-1}(r) = H^2 r$ and $\ell\gamma(r) = \frac{1}{2}H^2r^2$, According to (7), (35) and $V = V_2$ can get

$$\begin{aligned} u = \alpha(\hat{l}) &= -R_2^{-1}(\hat{l})\frac{1}{H^2}z_2\frac{1}{2}H^2 \\ &= -\frac{1}{2}R_2^{-1}(\hat{l})z_2 \end{aligned} \tag{36}$$

choosing $R_2(\hat{l}) = (2M(\hat{l}))^{-1} = \frac{1}{2Hb_2}$, we can get $u = -M(\hat{l})z_2$, it has same form with (26). Considering conclusion (2) and Lemma 1, the inverse optimal controller

$$\begin{aligned} u^* = \alpha^*(\hat{l}) &= -\frac{\beta}{2}R_2^{-1}(\hat{l})\frac{1}{H^2}z_2\frac{1}{2}H^2 \\ &= -\frac{\beta}{2}R_2^{-1}(\hat{l})z_2 = \beta\alpha(\hat{l}), \quad \beta \geq 2 \end{aligned} \tag{37}$$

minimizes the performance function

Remark 1: There have been some researches on this type of system and some results have been obtained. For example, in the [26], the random impulse control problem of the system under Hölder conditions is analyzed. By comparing previous studies, the described nonlinear system conditions have a stronger scope of application in this paper. On the basis of nonlinear system analysis method, the problem of robust sliding mode controller design is considered for piecewise linear discrete time-delay systems with state lag and uncertain parameters, piecewise linear time-delay systems is applying sliding mode control theory, which can make the system state moving along a given "sliding mode" motion trajectory, and the sliding mode variable structure control theory is to applied the underwater robot states keeping control system, which has practical significance in the control theory and application of the piecewise linear system.

Remark 2: On the basis of designing observers for nonlinear systems, scholars have launched the observer-based controller design. Among them, the controller design of the observer combining the Lyapunov stability theorem and the LMIs method has achieved certain research results. In [27], [28] the author studied the robust controller design of the state observer for uncertain time-delay systems. From the above research results, it can be obtained that robust control has a wide range of applications, and it also reflects the degree of attention paid to the research of uncertain nonlinear systems with time delays. But on the other hand, most of the research results mentioned at present are the study of time-delay nonlinear systems, state-uncertain nonlinear systems, or both time-delay characteristics and parameter-uncertain nonlinear systems. There are relatively few researches on observers of time-delay, state uncertainty and time-delay uncertainty, as well as observers of nonlinear discrete systems include state variables and nonlinear functions of time-delay variables and robust control based on observers.

Remark 3: In the control design program, the design problem of the controller is transformed into a parameter construction problem by introducing the appropriate coordinate transformation, and the forward push and saturation control design method based on the iterative program is not used. The controller gain obtained by the iterative design method may become too small during multiple iterations. The obtained controller is theoretically effective, but it is difficult to achieve in engineering practice, when the dimensionality of the system is relatively high. The multiple iteration procedures will greatly increase the difficulty and complexity of the design. The controller designed in this project has a simple form and moderate dynamic/static gain strength, so it is easier to apply to engineering practice. In this article, we take the underwater robot system as an example, and the robust controller designed is applied above to verify the effectiveness of the controller.

IV. UNDERWATER ROBOT TRANSPORTATION MODEL EXAMPLE

In order to investigate the effect of the proposed output feedbacking control method on the performance of the station keeping of underwater robot, the output feedbacking controller is simply adopted in this section, it describes the design of the nonlinear station controller for the AUV to maintain a given reference coordinates using the output feedbacking control law.

Consider the following stochastic nonlinear system:

$$\begin{aligned} \dot{l}_1 &= l_2 + \frac{1}{10}l_1l_2 + \frac{1}{10}l_1l_2\dot{\omega} \\ \dot{l}_2 &= u + \frac{1}{10}l_2 sinl_1 + \frac{1}{10}l_2 sinl_1\dot{\omega} \\ y &= l_1 \end{aligned} \tag{38}$$

Obviously, Assumption 3 was established. here select $C_f = \frac{1}{10}$ and $C_g = \frac{1}{10}$, and select the observer as

$$\begin{aligned} \dot{\hat{l}}_1 &= \hat{l}_2 + H(l_1 - \hat{l}_1) \\ \dot{\hat{l}}_2 &= u + H^2(l_1 - \hat{l}_1) \end{aligned} \tag{39}$$

The parameters are selected as $K_1 = K_2 = 1$, the real number H > 0 is the unknown gain parameter to be designed. Define the $\tilde{l}_1 = l_1 - \hat{l}_1$ and $\tilde{l}_2 = \frac{l_2 - \hat{l}_2}{H}$, the following is obtained:

$$\begin{aligned} d\tilde{l} &= HA\tilde{l}dt + F(l)dt + G(l)d\omega, \tilde{l} = (\tilde{l}_1, \tilde{l}_2)^T \\ F(l) &= \left(\frac{1}{10}l_2 sinl_1, \frac{1}{10}(l_1l_2)^{\frac{1}{2}}\right)^T \\ G(l) &= \left(\frac{1}{10}l_2 sinl_1, \frac{1}{10}(l_1l_2)^{\frac{1}{2}}\right)^T \end{aligned} \tag{40}$$

Introduction,

$$\begin{aligned} z_1 &= \hat{l}_1 \\ z_2 &= \hat{l}_2 - \alpha_1(\hat{l}_1) \end{aligned} \tag{41}$$

By using the stability theorem and controller design method proposed in Section 3, which can be obtained the output feedback controller.

$$\alpha_1(\hat{l}_1) = -Hb_1z_1, b_1 = \frac{1}{2} + \frac{1}{4} + 2 = 2.75, \quad H \geq 8c_1 \tag{42}$$

$$u = -Hb_2z_2, \quad b_2 = \frac{d_{20}^2}{4} + \frac{d_{21}^2}{4} + d_{22} + 2 = 19.03 \tag{43}$$

Meantime

$$c_1 = \frac{3}{100}\left(1 + \lambda_{max}(P)\right)\left(1 + \frac{1}{H}\right)^2 \tag{44}$$

$P = \begin{pmatrix} 0 & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{pmatrix}$, $\|P\| = \frac{\sqrt{6}}{2}$, $\lambda_{max}(P) = \frac{1+\sqrt{2}}{2}$, $d_{20} = 1 + b_1 = 3.75$, $d_{21} = 1 - b_1^2 = -6.56$, $d_{22} = b_1 = 2.75$.

In this section, we apply Theorem 1 to a specific the underwater electro-hydraulic compound valve position control system described in Figure 2. Hence, by using Theorem 1, we can design an output-feedback controller as follows. The related control laws are implemented in the following form: $u = -Hb_2z_2$. The parameters are selected as $H > H_1^* = 9.9609$.

A. SIMPLE STATION KEEPING RESULTS IN THE PRESENCE OF UNDERWATER CURRENT

According to the analysis above, we simulate a group to prove the effectiveness of our proposing strategy. Simulation results are shown in Figures 3 and 4. The initial conditions are $[l_1(0), l_2(0), \hat{l}_1(0), \hat{l}_2(0)]^T = [0, 10, 0, 0]^T$.

B. HARDWARE-IN-THE-LOOP SIMULATION RESULTS IN THE PRESENCE OF UNDERWATER CURRENT

In order to investigate the performance of the proposed scheme, typical simulations are conducted and results are presented in this section. For confirming the effectiveness of the proposed scheme, hardware-in-the-loop (HIL) simulation is also carried out, where the control outputs are given as the inputs of the actual actuators and the actuators responses are feedback to the simulation model. For numerical and HIL simulations, the test device underwater robot Figure 2 is considered.

The same previous operating condition has taken for the HIL simulation to confirm the performance of the proposed scheme in the real time. The results of the experiment are given in Figures 5 and 6, which shows that the proposed system can effectively keep the given desired coordinates in the presence of uncertainties such as disturbances (underwater current). The simulation results error of HILS and numerical simulations are given in Figures 7 and 8, which confirmed that the proposed controller effectiveness can compensate the underwater currents effectively. The settling time or steady state time (from HIL values), corresponding to position errors of robot are shows in Figures 7 and 8, respectively. HIL values are showing similar behaviour of the numerical simulation values with disturbance effects. These disturbance effects are mainly due to the actuator response delay, instrument underwave and mechanical vibrations of the robot - actuator frame.

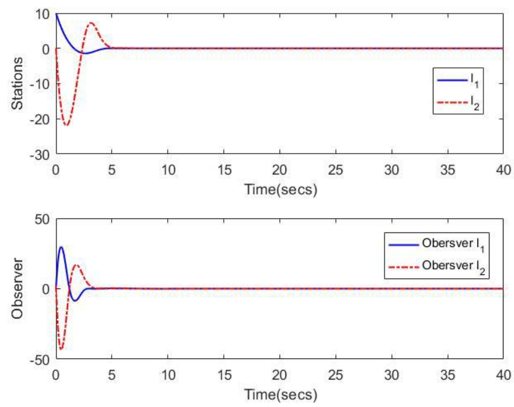

FIGURE 3. Station keeping results for the proposed control method with underwater current.

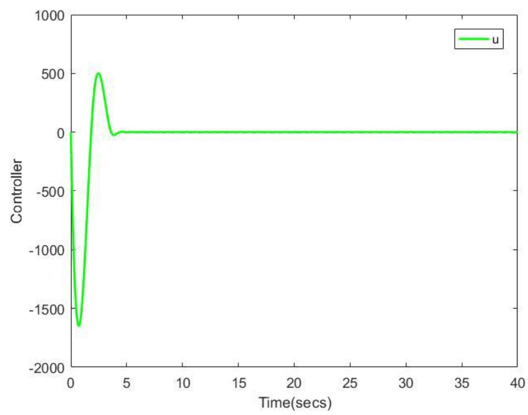

FIGURE 4. The controller u trajectory of underwater robot system.

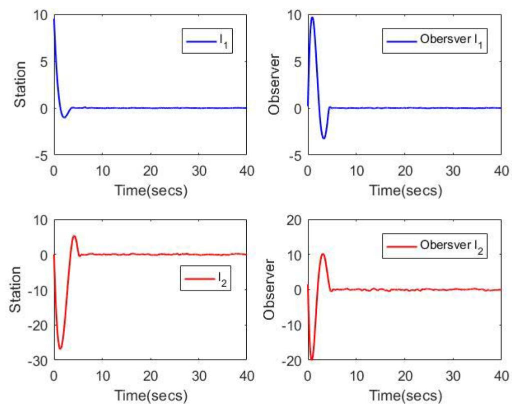

FIGURE 5. HIL simulation results for the proposed configuration with underwater current.

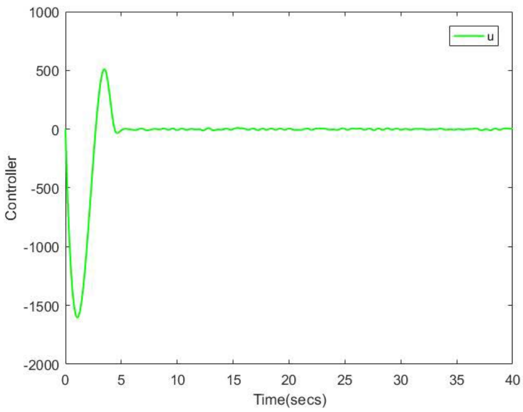

FIGURE 6. HIL simulation results for the controller u trajectory of underwater robot system.

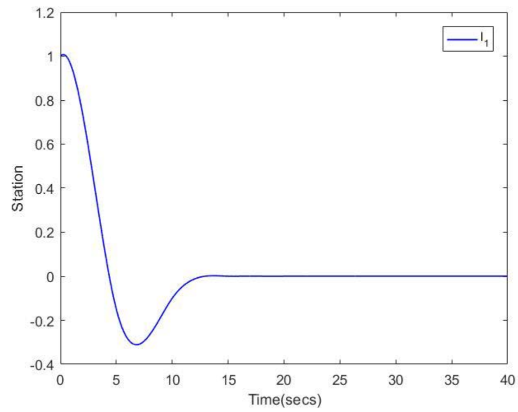

FIGURE 9. Station keeping results for the PID method with underwater current.

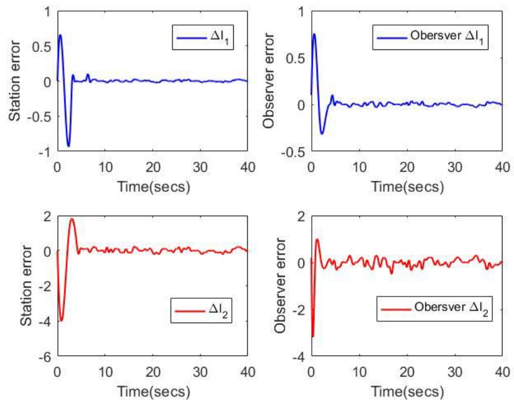

FIGURE 7. Results error of numerical and HIL simulation for the proposed configuration with underwater current.

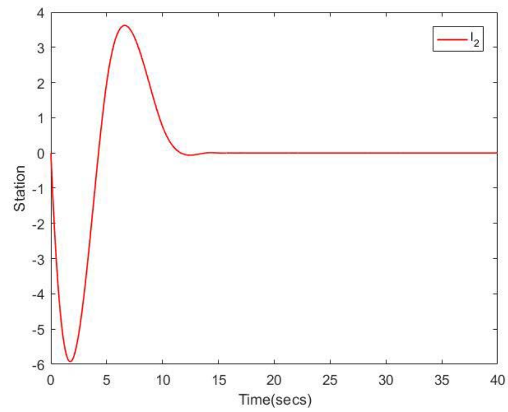

FIGURE 10. Station keeping results for the PID method with underwater current.

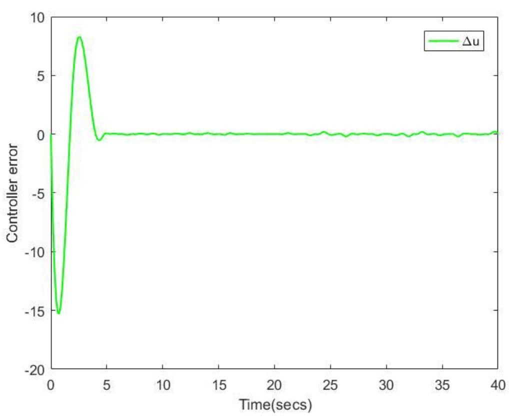

FIGURE 8. Results error of numerical and HIL simulation for the controller u trajectory of underwater robot system.

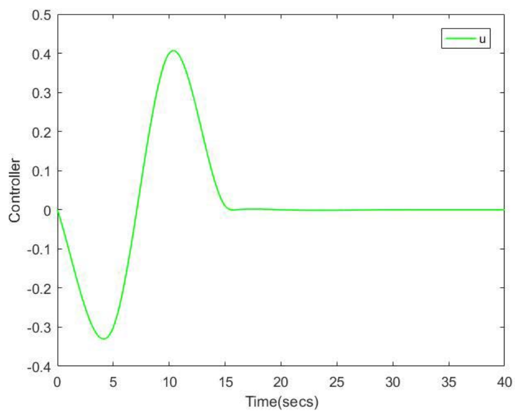

FIGURE 11. The PID controller u trajectory of underwater robot system.

C. PID SIMULATION RESULTS IN THE PRESENCE OF UNDERWATER CURRENT

According to the PID simulation results, simulation results are shown in Figures 9 and 11. The initial conditions are $[l_1(0), l_2(0)]^T = [0, 1]^T$. We compare results of the backstepping and the PID. By the designing backstepping controller method, the initial value is more large, the method can reduce the vibration time of converge to zero for the underwater robot and reduce the vibration amplitude of the robot when it moves underwater. We can confirm the effectiveness of our proposing strategy.

V. CONCLUDING AND FUTURE PROSPECTS

In this paper, we study the design of the system observer under the condition that the state of the system is unmeasurable, and the system controller is designed based on the observer. The system has uncertain disturbances and nonlinear terms. By introducing the error between the observation value and the actual value, combining the error equation and the closed loop of the observer-based control system, the Lyapunov method for stability analysis is used. Finally, the relevant theorems obtained from the stability analysis to design the observer and the controller is combined. The Simulation X is used to solve the backstepping controller under the given parameters; finally, the AUV position control model and the robot position of its actuator through the controller designed are established and analyzed in this paper, respectively. In this paper, we study the design of system observers when state variables are unmeasurable, and implement observer and controller design are based on the stability theorem. The nonlinear system studied includes uncertain disturbances and nonlinear terms.

However, in the actual research process, there is a class of nonlinear systems that can be piecewise linearized. The system model is a piecewise linear model, that is, a nonlinear system is composed of finite or infinite linear subsystems. Piecewise linear systems are widely used in practice. In the life and production process, the piecewise linear system is also an important approximation method for the nonlinear system. The nonlinear system can be described by the piecewise linear system. Therefore, it is necessary to study the control problem of the piecewise linear system.

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CONFLICT OF INTEREST

The author declares that there is no conflict of interest regarding the publication of this paper.

DATA AVAILABILITY STATEMENT

All data generated or analyzed during this research are included in this paper.

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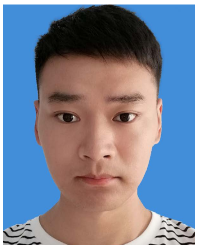

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