IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received March 17, 2022, accepted April 5, 2022, date of publication April 13, 2022, date of current version April 20, 2022.

Digital Object Identifier 10.1109/ACCESS.2022.3167163

Distributed Fusion Tracking Estimation Under Range-Only Measurement

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ABSTRACT This paper is concerned with the distributed fusion estimation problem of range-only target tracking system with unknown but bounded noises, where the linear and nonlinear motion models are both considered. Particularly, a kind of nonlinear transformation is used to convert the nonlinear distance measurement model into a linear one, which eliminates the corresponding linearization errors in the design of estimation error system. In spite of the transformed measurement noise becomes more complicated, while it is still bounded. Moreover, for the nonlinear target motion model, the state linearization error caused by the Taylor expansion is modeled by the state dependent matrix with uncertainty bounded matrix. In this case, based on the bounded recursive optimization algorithm, two kinds of convex optimization problems are established to determine the gains of the local/fusion estimators, and the stability of the designed estimators also can be guaranteed. Finally, two different range-only target tracking systems are presented to show the effectiveness and advantages of the proposed methods.

INDEX TERMS Target tracking, range-only measurement, nonlinear transformation, bounded recursive optimization, distributed fusion estimation.

I. INTRODUCTION

As an essential application of multi-sensor fusion estimation, target location and tracking not only attracted significant interest in the military field initially, but also has been a research hotspot in the civilian field over the past decades, such as emergency location [1], intelligent transport systems [2], cellular mobile station location [3]. Generally, the time-of-arrival (TOA), time-difference-of-arrival (TDOA), angle-of-arrival (AOA) and received signal strength (RSS) are conventional measurement-based location and tracking methods [4]–[8], where AOA is based on the angle of the target, RSS is based on the signal strength, TOA and TDOA are based on the time information. However, among these location methods, AOA requires the sensor to have angle-measuring capability, TOA and TDOA require the clock synchronization between the sensors, and RSS requires sensors to identify the strength of the signal. In fact, the target tracking problem also can be viewed as a nonlinear state estimation problem for the range-only tracking systems. Therefore, different from above conventional location methods, this paper will focus on distributed fusion estimation problem of the target tracking systems, which only requires to know the range-only measurement.

Since the target tracking problem is essentially a nonlinear state estimation problem, a variety of nonlinear filters have been developed in [9]–[13] to track the mobile target. Specifically, the extended Kalman filter (EKF) [14] usually linearized the nonlinear process and measurement model by Taylor series expansion, then the classical Kalman filter (KF) can be used to estimate the target's position. However, the linearization errors were inevitably introduced by using the Taylor expansion, which might causing the nonlinear filtering was unstable. For example, a stability condition with respect to the linearization errors should be given to ensure the stability of the estimator in [11]. Though the second-order or higher-order EKF [15] can reduced the linearization errors, the Jacobian matrix or Hessian matrix were difficult to be calculated and thus the computation burden was increased as well. To better overcome this limitation, the unscented Kalman filter (UKF) [16] was developed to estimate the nonlinear system, where the sigma points were sampled to approximate the probability distribution of the nonlinear system, and the estimation precious can up to third-order

The associate editor coordinating the review of this manuscript and approving it for publication was Dongxiao Yu.

Taylor expansion. However, the UKF performed a sensitive ability to nonpositive definite covariances, thus the key parameters α, β, κ also should be adjusted reasonably in practice. Then, a cubature Kalman filter (CKF) [17] was emerged to reduce the sampling points, thus the computation time can be reduced while keeping a fairly estimation performance and a better stability. Unfortunately, when the prediction error was larger, the UKF or CKF methods might be performed an unsatisfactory estimation performance that introduced by the higher orders. Therefore, for the target tracking systems with range-only measurements, a kind of nonlinear transformation [18] is employed in this paper to transfer the nonlinear measurement into a linear form, which can avoids the linearization errors caused by above linearization approaches.

Notice that, whether the above single-sensor tracking estimation methods, or multi-sensor fusion tracking methods [19]–[22], the system uncertainty should be effectively handled. Particularly, for the most existing distributed estimation works, a saturated-constrained distributed filtering algorithm was developed in [23] under uncertain missing probabilities, and the boundedness and monotonicity were also discussed. In [24], a recursive and distributed Kalman filtering algorithm was developed for interconnected dynamic systems, where each local estimator was designed by its own and neighbors' information. Additionally, a generalized nonlinear weighted measurement fusion UKF (WMF-UKF) algorithm was developed in [25], and the asymptotic global optimality has also been demonstrated. However, the above methods were all assumed that the noise covariances of systems should be have a prior information, while the process and measurement may contain various noises without statistical properties in practice [26]. In this case, a noise estimator was proposed in [27] for estimating the noises, and then eliminated the estimated noises to obtain a desirable performance. Although a nonlinear transformation developed in [18] can transformed the nonlinear distance measurement to a linear one [28], the converted noises were no longer Gaussian even if the original system noises assumed to be Gaussian. Then, by using a similar nonlinear transformation, the dimension of the augmented measurement in [29] was reduced by the Gauss–Markov measurement fusion, and an adaptive factor was introduced by the hypothesis test to deal with the non-Gaussian noises. Recently, a bounded recursive optimization algorithm was developed in [30] to solve the uncertainty caused by unknown bounded noises, while the stability of the designed nonlinear estimator was not discussed. Meanwhile, a distributed nonlinear estimation problem was considered in [31] based on the recursive optimization approach for rang-only target tracking system. Then, a decoupling strategy with sequential-structure was proposed in [32] to solve the interconnected terms, where the designed algorithm can implemented fully distributed estimation. In spite of [33] presented the stability condition of the nonlinear estimator without noise statistical characteristics, the distance measurement function or the linearization errors of the range-only measurement were not easily judged as bounded, thus the stability condition was useless in the range-only tracking systems.

From the above analysis, the design problem of stable fusion tracking estimators for range-only target tracking systems will be considered in this paper. The contributions of this paper can be summarized as follows: (1) A nonlinear transformation is employed to transform the distance measurement into a linear measurement model, which can avoid the linearization errors caused by the conventional linearization approaches; (2) The linearization error of the nonlinear state model is modeled by the bounded uncertainty matrix with state-dependent matrix, and a bounded recursive optimization scheme is constructed to establish different convex optimization problems, which can deal with the transformed measurement noises without statistical characteristics. Then, the stable local estimator gains and fusion criterion can be determined by the standard software packages. Finally, two different target tracking examples are given to verify the effectiveness and advantages of the proposed methods.

Notations: $\mathbb{R}^{m\times n}$ denotes the set of $m \times n$ real matrices, the superscript "T" denotes the matrix transpose, and $A < 0$ denotes a negative definite matrix. O and I are zero and identity matrix with appropriate dimension, respectively, the symmetric terms of a symmetric matrix are denoted by "$*$". Moreover, $||\cdot||_2$ represents the 2-norm, and $\lambda_{\max}(A)$ is the maximum eigenvalue of the A. col$\{b_1, \ldots, b_n\}$ denotes a column vector whose elements are $b_1, \ldots, b_n$, while diag $\{\cdot\}$ denotes a block diagonal matrix.

II. PROBLEM FORMULATION

Consider a multi-sensor range-only target tracking system (see Fig. 1), where the motion process of the target can be described by the following nonlinear state model:

$$x(t+1) = f(x(t)) + B(t)w(t) \tag{1}$$

where the $x(t) \in \mathbb{R}^n$ denotes the target's motion state in the planar coordinate, $f(x(t))$ is the nonlinear function that assume to be continuously differentiable. $B(t) \in \mathbb{R}^{n\times r}$ is time-varying matrix, $w(t) \in \mathbb{R}^{r\times 1}$ is the system noise satisfy:

$$\|w(t)\|_2 \leq \sigma_w \tag{2}$$

where σ_w is a unknown upper bound.

Moreover, when considering only the distance between the target and each sensor can be measured by the sensors, and there are $q_i \geq 3$ sensor nodes within each sensor group. Then, the measurement model of each sensor can be given by

$$\begin{aligned}
d_1^i(t) &= \sqrt{(x_p(t) - x_1^i)^2 + (y_p(t) - y_1^i)^2} + v_1^i(t)\\
d_2^i(t) &= \sqrt{(x_p(t) - x_2^i)^2 + (y_p(t) - y_2^i)^2} + v_2^i(t)\\
&\vdots\\
d_{q_i}^i(t) &= \sqrt{(x_p(t) - x_{q_i}^i)^2 + (y_p(t) - y_{q_i}^i)^2} + v_{q_i}^i(t)
\end{aligned} \tag{3}$$

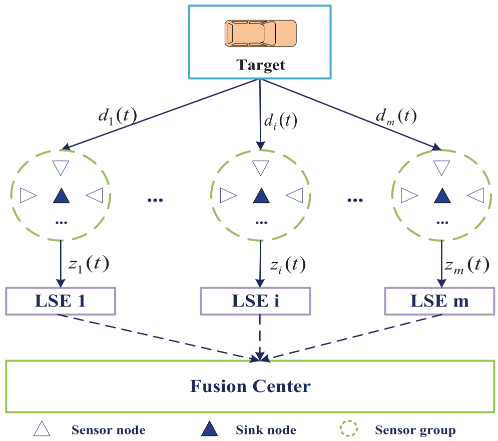

FIGURE 1. Distributed multi-sensor fusion tracking structure.

where $i = 1, 2, \ldots, m$ and m is the number of the sensor groups, $(x_p(t), y_p(t))$ and $(x^i_{q_i}, y^i_{q_i})$ are the target's position and the sensor's position in the $\mathrm{X}-\mathrm{Y}$ planar, respectively, and $v^i_{q_i}(t)$ is the sensor measurement noise satisfy:

$$\|v^i_{q_i}(t)\|_2 \leq \sigma^i_{q_i} \tag{4}$$

where $\sigma^i_{q_i}$ is also a unknown upper bound.

Generally, the sink nodes collecting the measurements from each sensor of its corresponding sensor groups, then the measurement $y_i(t) \in \mathbb{R}^{q_i}$ can be written as:

$$y_i(t) = h_i(x(t)) + v_i(t) \tag{5}$$

where $y_i(t) \triangleq \mathrm{col}\{d^i_1(t), d^i_2(t), \ldots, d^i_{q_i}(t)\}$ is a nonlinear measurement. In order to reduce the influence of linearization errors in the nonlinear state estimation problem, a kind of nonlinear transformation, which has been developed in [18], with respect to range-only information of target tracking system is introduced to convert $y_i(t)$ as a linear model. Specifically, rearranging and squaring both sides of the equations in (3) to yield

$$\begin{aligned}
(d^i_1(t) - v^i_1(t))^2 &= (x_p(t) - x^i_1)^2 + (y_p(t) - y^i_1)^2 \\
(d^i_2(t) - v^i_2(t))^2 &= (x_p(t) - x^i_2)^2 + (y_p(t) - y^i_2)^2 \\
&\vdots \\
(d^i_{q_i}(t) - v^i_{q_i}(t))^2 &= (x_p(t) - x^i_{q_i})^2 + (y_p(t) - y^i_{q_i})^2
\end{aligned} \tag{6}$$

Then, taking the 1-th equation in (6) as a benchmark, and subtracting the 1-th equation from others, one has

$$z_i(t) = C_i(t)x(t) + \bar{v}_i(t) \tag{7}$$

where

$$C_i(t) = 2\begin{bmatrix} (x^i_1 - x^i_2) & (y^i_1 - y^i_2) \\ (x^i_1 - x^i_3) & (y^i_1 - y^i_3) \\ \vdots & \vdots \\ (x^i_1 - x^i_{q_i}) & (y^i_1 - y^i_{q_i}) \end{bmatrix} \tag{8}$$

$$\bar{v}_i(t) = \begin{bmatrix} 2d^i_2(t)v^i_2(t) - 2d^i_1(t)v^i_1(t) \\ 2d^i_3(t)v^i_3(t) - 2d^i_1(t)v^i_1(t) \\ \vdots \\ 2d^i_{q_i}(t)v^i_{q_i}(t) - 2d^i_1(t)v^i_1(t) \end{bmatrix} + \begin{bmatrix} (v^i_1(t))^2 - (v^i_2(t))^2 \\ (v^i_1(t))^2 - (v^i_3(t))^2 \\ \vdots \\ (v^i_1(t))^2 - (v^i_{q_i}(t))^2 \end{bmatrix} \tag{9}$$

$$z_i(t) = \begin{bmatrix} (x^i_1)^2 - (x^i_2)^2 + (y^i_1)^2 - (y^i_2)^2 \\ (x^i_1)^2 - (x^i_3)^2 + (y^i_1)^2 - (y^i_3)^2 \\ \vdots \\ (x^i_1)^2 - (x^i_{q_i})^2 + (y^i_1)^2 - (y^i_{q_i})^2 \end{bmatrix} + \begin{bmatrix} (d^i_2(t))^2 - (d^i_1(t))^2 \\ (d^i_3(t))^2 - (d^i_1(t))^2 \\ \vdots \\ (d^i_{q_i}(t))^2 - (d^i_1(t))^2 \end{bmatrix} \tag{10}$$

Under this case, the measurements $\{z_i(1), z_i(2), \ldots, z_i(t)\}$ are employed to design nonlinear local state estimate (LSE) $\hat{x}^n_i(t)$ as follows:

$$\begin{cases} \hat{x}^p_i(t) = f(\hat{x}^n_i(t-1)) \\ \hat{x}^n_i(t) = \hat{x}^p_i(t) + \mathrm{K}^n_i(t)[z_i(t) - C_i(t)\hat{x}^p_i(t)] \end{cases} \tag{11}$$

where $\hat{x}^p_i(t)$ denotes one-step prediction, and $\mathrm{K}^n_i(t)$ is the nonlinear local gain to be determined in the next section.

Next, the LSEs $\hat{x}^n_i(t)(i = 1, 2, \ldots, m)$ are used to design nonlinear distributed fusion estimate (DFE) $\hat{x}_n(t)$ as follows:

$$\hat{x}_n(t) = \sum_{i=1}^{m} \mathrm{W}^n_i(t)\hat{x}^n_i(t) \tag{12}$$

where the distributed weighting fusion matrices satisfy $\sum_{i=1}^{m} \mathrm{W}^n_i(t) = I$.

Notice that, when the nonlinear state model (1) reduces to a linear state space:

$$x(t+1) = A(t)x(t) + \Gamma(t)w(t) \tag{13}$$

where $A(t)$ is the state transition matrix with appropriate dimensions. Then, the linear LSEs $\hat{x}^l_i(t)(i = 1, 2, \ldots, m)$ for linear systems (13) and (7) can be designed as:

$$\begin{aligned} \hat{x}^l_i(t) = {} & A(t-1)\hat{x}^l_i(t-1) \\ & + \mathrm{K}^l_i(t)[z_i(t) - C_i(t)A(t-1)\hat{x}^l_i(t-1)] \end{aligned} \tag{14}$$

where $\mathrm{K}^l_i(t)$ is the linear local gain to be determined.

Similarly, based on the LSEs $\hat{x}^l_i(t)(i = 1, 2, \ldots, m)$, the linear DFE $\hat{x}_l(t)$ is given by:

$$\hat{x}_l(t) = \sum_{i=1}^{m} \mathrm{W}^l_i(t)\hat{x}^l_i(t) \tag{15}$$

where the weighting fusion matrices satisfy $\sum_{i=1}^{m} \mathrm{W}^l_i(t) = I$.

Consequently, the main objectives of this paper are to solve the following problems:

- The first is to design local gains $\mathrm{K}_i^n(t)$ in (11) and $\mathrm{K}_i^l(t)$ in (14) such that the local linear/nonlinear estimators are stable, respectively, while the upper bound of square errors (SEs) of the corresponding LSEs are minimal at each time.
- The second is to design distributed weighting fusion matrices $\mathrm{W}_i^n(t)$ in (12) and $\mathrm{W}_i^l(t)$ in (15) such that the fusion estimators are stable, while the upper bound of SEs of the corresponding DFEs are also minimal at each time.

Remark 1: Notice that, the nonlinear raw measurement $y_i(t)$ in (5) are usually linearized by the Taylor series expansion and unscented transformation, while the linearization errors will inevitably be introduced. This is because the Taylor series usually expanding the nonlinear function in first or second order, while the higher-order terms are neglected on the designing of filter, and the neglected higher-order terms also affect the accuracy of the estimation. At the same time, the calculate of Jacobian or Hessian matrices will increase the computation burden of the designed estimation algorithms. Though the unscented transformation by using the sigma points to approximate the probability distribution, which avoids solving the Jacobian or Hessian matrices and the estimation precision can up to the third order Taylor expansion, it still might be given a undesirable estimation error caused by the higher orders or prediction errors. In this case, a nonlinear transformation is introduced in this paper to convert the nonlinear measurement model (3) to a linear one (7), which can avoid the linearization errors caused by above linearization approaches and improve the stability of the designed estimators.

Remark 2: It can be seen from (9) that the measurement noise $v_i(t)$ in (5) after nonlinear transformation is no longer Gaussian and cross uncorrelated, even assuming that the measurement noises in (3) obey mutually independent Gaussian distribution, thus the various extension Kalman filtering methods cannot be directly employed to track the target. Though it increases the difficulty of estimator design, the developed estimator (11) in this paper are not require to have a prior knowledge of the noise statistical characteristics, which not only can performs an effective estimation performance, but also reduces the computation complexity of the noise covariances as compared with [29].

III. MAIN RESULTS

In this section, the distributed linear/nonlinear fusion estimation algorithms will be proposed. Firstly, an useful Lemma is introduced for deriving the main results as follows.

Lemma 1 ([36]): Let $P_1^{\mathrm{T}} = P_1$, P_2 and P_3 be real matrices of appropriate dimensions with $F(t)$ satisfying $F^{\mathrm{T}}(t)F(t) \leq I$. Then

$$P_1 + P_3F(t)P_2 + P_2^{\mathrm{T}}F^{\mathrm{T}}(t)P_3^{\mathrm{T}} < 0$$

if and only if there exists a positive scalar $\epsilon > 0$ such that

$$\begin{bmatrix} -\epsilon I & \epsilon P_2 & 0 \\ * & P_1 & P_3 \\ * & * & -\epsilon I \end{bmatrix} < 0$$

A. LINEAR DISTRIBUTED FUSION ESTIMATION ALGORITHM DESIGN

For the linear target motion model (13), based on the designed local estimator (14) and fusion criterion (15), the local estimator gain $\mathrm{K}_i^l(t)$ and weighting fusion matrix $\mathrm{W}_i^l(t)$ will be given in Theorem 1.

Define

$$\begin{cases} A_{C_i}^{\mathrm{K}}(t) \triangleq (I - \mathrm{K}_i^l(t)C_i(t))A(t-1) \\ \Gamma_{C_i}^{\mathrm{K}}(t) \triangleq (I - \mathrm{K}_i^l(t)C_i(t))\Gamma(t-1) \\ A_{\mathrm{W}}^{L}(t) \triangleq \mathrm{W}_l(t)\mathrm{diag}\{A_{C_1}^{\mathrm{K}}(t), \ldots, A_{C_m}^{\mathrm{K}}(t)\} \\ \Gamma_{\mathrm{W}}^{L}(t) \triangleq \mathrm{W}_l(t)\mathrm{col}\{\Gamma_{C_1}^{\mathrm{K}}(t), \ldots, \Gamma_{C_m}^{\mathrm{K}}(t)\} \\ \mathrm{K}_{\mathrm{W}}^{L}(t) \triangleq \mathrm{W}_l(t)\mathrm{diag}\{-\mathrm{K}_1^l(t), \ldots, -\mathrm{K}_m^l(t)\} \\ \mathrm{W}_l(t) \triangleq [\mathrm{W}_1^l(t), \ldots, \mathrm{W}_{m-1}^l(t), I - \sum_{i=1}^{m-1} \mathrm{W}_i^l(t)] \end{cases} \tag{16}$$

Theorem 1: Each local estimator gain $\mathrm{K}_i^l(t)$ for linear systems (13) and (7) can be determined by solving the following convex optimization problem:

$$\min_{\substack{\Theta_i(t)>0,\ \Upsilon_i(t)>0,\\ \Xi_i(t)>0, \mathrm{K}_i^l(t), \chi_i(t)}} \mathrm{Tr}\{\Upsilon_i(t)\} + \mathrm{Tr}\{\Xi_i(t)\}$$

$$\text{s.t. :} \begin{cases} \begin{bmatrix} -I & A_{C_i}^{\mathrm{K}}(t) & \Gamma_{C_i}^{\mathrm{K}}(t) & -\mathrm{K}_i^l(t) \\ * & -\Theta_i(t) & O & O \\ * & * & -\Upsilon_i(t) & O \\ * & * & * & -\Xi_i(t) \end{bmatrix} < 0 \\ \Theta_i(t) - \chi_i(t)I < I \\ \chi_i(t) \leq 1 \end{cases} \tag{17}$$

where $A_{C_i}^{\mathrm{K}}(t)$ and $\Gamma_{C_i}^{\mathrm{K}}(t)$ have been defined in (16). In this case, the local estimation error (LEE) $\tilde{x}_i^l(t)$ will be bounded, that is

$$\lim_{t\to\infty} [\tilde{x}_i^l(t)]^{\mathrm{T}}\tilde{x}_i^l(t) < \gamma_i \tag{18}$$

where $\gamma_i > 0$ is a scalar. Next, the distributed weighting fusion matrix $\mathrm{W}_i^l(t)$ can be solved by the following convex optimization problem:

$$\min_{\substack{\Theta(t)>0, \Upsilon(t)>0, \Xi(t)>0,\\ \Theta_1(t), \Theta_2(t), \Upsilon_1(t), \mathrm{W}(t)}} \mathrm{Tr}\{\Theta(t)\} + \mathrm{Tr}\{\Upsilon(t)\} + \mathrm{Tr}\{\Xi(t)\}$$

$$\text{s.t. :} \left\{ \begin{bmatrix} -I & A_{\mathrm{W}}^{L}(t) & \Gamma_{\mathrm{W}}^{L}(t) & \mathrm{K}_{\mathrm{W}}^{L}(t) \\ * & -\Theta(t) & -\Theta_1(t) & -\Theta_2(t) \\ * & * & -\Upsilon(t) & -\Upsilon_1(t) \\ * & * & * & -\Xi(t) \end{bmatrix} < 0 \right. \tag{19}$$

where $A_{\mathrm{W}}^{L}(t)$, $\Gamma_{\mathrm{W}}^{L}(t)$, $\mathrm{K}_{\mathrm{W}}^{L}(t)$ are defined in (16).

Proof: Define the local estimation error $\tilde{x}_i^l(t) = x(t) - \hat{x}_i^l(t)$. Then, from (13) and (14), one has

$$\tilde{x}_i^l(t) = A_{C_i}^{\mathrm{K}}(t)\tilde{x}_i^l(t-1) + \Gamma_{C_i}^{\mathrm{K}}(t)w(t-1) - \mathrm{K}_i^l(t)\bar{v}_i(t) \tag{20}$$

where $A_{C_i}^{\mathrm{K}}(t)$ and $\Gamma_{C_i}^{\mathrm{K}}(t)$ are defined in (16).

Next, in order to construct an upper bound of $[\tilde{x}_i^l(t)]^{\mathrm{T}}\tilde{x}_i^l(t)$, it gives that

$$\begin{aligned} [\tilde{x}_i^l(t)]^{\mathrm{T}}\tilde{x}_i^l(t) < {} & [\tilde{x}_i^l(t-1)]^{\mathrm{T}}\Theta_i(t)\tilde{x}_i^l(t-1) \\ & + w^{\mathrm{T}}(t-1)\Upsilon_i(t)w(t-1) \\ & + \bar{v}_i^{\mathrm{T}}(t)\Xi_i(t)\bar{v}_i(t) \end{aligned} \tag{21}$$

where $\Theta_i(t) > 0$, $\Upsilon_i(t) > 0$ and $\Xi_i(t) > 0$. Subsequently, substituting (20) into (21), one has

$$\begin{bmatrix} \tilde{x}_i^l(t) \\ w(t-1) \\ \bar{v}_i(t) \end{bmatrix}^{\mathrm{T}} \underbrace{\begin{bmatrix} J_1^i(t) & J_1^i(t) & J_3^i(t) \\ * & J_4^i(t) & J_5^i(t) \\ * & * & J_6^i(t) \end{bmatrix}}_{\mathrm{J}_i(t)} \begin{bmatrix} \tilde{x}_i^l(t) \\ w(t-1) \\ \bar{v}_i(t) \end{bmatrix} < 0 \quad (22)$$

where

$$\begin{cases} J_1^i(t) \triangleq (A_{C_i}^{\mathrm{K}}(t))^{\mathrm{T}} A_{C_i}^{\mathrm{K}}(t) - \Theta_i(t) \\ J_2^i(t) \triangleq (A_{C_i}^{\mathrm{K}}(t))^{\mathrm{T}} \Gamma_{C_i}^{\mathrm{K}}(t) \\ J_3^i(t) \triangleq (A_{C_i}^{\mathrm{K}}(t))^{\mathrm{T}} \mathrm{K}_i^l(t) \\ J_4^i(t) \triangleq (\Gamma_{C_i}^{\mathrm{K}}(t))^{\mathrm{T}} \Gamma_{C_i}^{\mathrm{K}}(t) - \Upsilon_i(t) \\ J_5^i(t) \triangleq (\Gamma_{C_i}^{\mathrm{K}}(t))^{\mathrm{T}} \mathrm{K}_i^l(t) \\ J_6^i(t) \triangleq (\mathrm{K}_i^l(t))^{\mathrm{T}} \mathrm{K}_i^l(t) - \Xi_i(t) \end{cases} \quad (23)$$

To ensure the right term of (21) can be viewed as the upper bound of $[\tilde{x}_i^l(t)]^{\mathrm{T}} \tilde{x}_i^l(t)$, the condition $\mathrm{J}_i(t) < 0$ must be held. Therefore, by using the Schur complement lemma [36], the first inequality in (17) is equivalent to $\mathrm{J}_i(t) < 0$. Furthermore, when the second and third inequalities in (17) are satisfied, it can be obtained from the similar derivation in [30, Th.1] that the bounded condition (18) of local estimator holds.

Let $\vartheta_i(t-1) \triangleq \mathrm{col}\{w(t-1), \bar{v}_i(t)\}$, notice that

$$\begin{aligned} &\vartheta_i^{\mathrm{T}}(t-1) \begin{bmatrix} \Upsilon_i(t) & 0 \\ * & \Xi_i(t) \end{bmatrix} \vartheta_i(t-1) \\ &\leq \lambda_{\max}\left(\vartheta_i(t-1)\vartheta_i^{\mathrm{T}}(t-1)\right)(\mathrm{Tr}\{\Upsilon_i(t)\} \\ &\quad +\mathrm{Tr}\{\Xi_i(t)\}) \end{aligned} \quad (24)$$

Then, it can be concluded that

$$\begin{aligned} [\tilde{x}_i^l(t)]^{\mathrm{T}} \tilde{x}_i^l(t) &< \chi_i(t)[\tilde{x}_i^l(t-1)]^{\mathrm{T}} \tilde{x}_i^l(t-1) \\ &\quad +\lambda_{\max}\left(\vartheta_i(t-1)\vartheta_i^{\mathrm{T}}(t-1)\right)(\mathrm{Tr}\{\Upsilon_i(t)\} \\ &\quad +\mathrm{Tr}\{\Xi_i(t)\}) \end{aligned} \quad (25)$$

Similarly, the right term of (25) also can be taken as an upper bound. Thereby, the "min(Tr$\{\Upsilon_i(t)\}$+Tr$\{\Xi_i(t)\}$)" is selected as an optimization objective to minimize this upper bound, and then the optimization problem (17) is formulated to determine the $\mathrm{K}_i^l(t)$ based on the designed stable estimator (14).

Next, let $\tilde{x}_l(t) \triangleq x(t) - \hat{x}_l(t)$ be the fusion estimation error (FEE), it follows from (13) and (15) that

$$\tilde{x}_l(t) = \sum_{i=1}^{m} \mathrm{W}_i^l(t)\tilde{x}_i^l(t) \quad (26)$$

Then, based on the LEE (20) and FEE (26), an fusion error system is formulated to determine each distributed weighting fusion matrix $\mathrm{W}_i^l(t)$ as follows

$$\tilde{x}_l(t) = A_{\mathrm{W}}^L(t)\tilde{x}_m(t-1) + \Gamma_{\mathrm{W}}^L(t)w(t-1) + \mathrm{K}_{\mathrm{W}}^L(t)\bar{v}(t) \quad (27)$$

where $\tilde{x}_m(t-1) \triangleq \mathrm{col}\{\tilde{x}_1^l(t-1), \ldots, \tilde{x}_m^l(t-1)\}$ and $\bar{v}(t) \triangleq \mathrm{col}\{\bar{v}_1(t), \ldots, \bar{v}_m(t)\}$, and $A_{\mathrm{W}}^L(t)$, $\Gamma_{\mathrm{W}}^L(t)$, $\mathrm{K}_{\mathrm{W}}^L(t)$ have been defined in (16).

Moreover, define $\varphi(t) \triangleq \mathrm{col}\{\tilde{x}_m(t), w(t), \bar{v}(t+1)\}$, and introducing some unknown matrices such that

$$\tilde{x}_l^{\mathrm{T}}(t)\tilde{x}_l(t) < \varphi^{\mathrm{T}}(t-1)\begin{bmatrix} \Theta(t) & \Theta_1(t) & \Theta_2(t) \\ * & \Upsilon(t) & \Upsilon_1(t) \\ * & * & \Xi(t) \end{bmatrix}\varphi(t-1) \quad (28)$$

By using the similar derivation in [30, Th.1], the (28) holds when the inequality in (19) is held. Then, it follows from (28) that

$$\begin{aligned} \tilde{x}_l^{\mathrm{T}}(t)\tilde{x}_l(t) \leq\ & \lambda_{\max}(\varphi(t-1)\varphi^{\mathrm{T}}(t-1)) \\ & \times(\mathrm{Tr}\{\Theta(t)\} + \mathrm{Tr}\{\Upsilon(t)\} + \mathrm{Tr}\{\Xi(t)\}) \end{aligned} \quad (29)$$

Therefore, the right term of (29) can be regarded as an upper bound, and "min(Tr$\{\Theta(t)\}$+Tr$\{\Upsilon(t)\}$+Tr$\{\Xi(t)\}$)" is selected as an optimization objective to construct the optimization problem (19). This completes the proof. ■

Based on Theorem 1, the computation procedures for the linear LSEs $\hat{x}_i^l(t)(i = 1, \cdots, m)$ and the DFE $\hat{x}_l(t)$ are summarized in Algorithm 1.

Algorithm 1 Distributed Fusion Estimation Based on Range-Only Measurement for Linear Target Dynamic

1: Initialization state $x(0)$ and $\hat{x}_i^l(0)$;
2: **for** $i := 1$ **to** m **do**
3: Solve the optimization problems (17) and (19) by the "*mincx*" function of MATLAB LMI Toolbox;
4: Determine local linear estimator gain $\mathrm{K}_i^l(t)$ and distributed weighting fusion matrix $\mathrm{W}_i^l(t)$;
5: Calculate linear LSE $\hat{x}_i^l(t)$ by (14) and DFE $\hat{x}_l(t)$ by (15);
6: **end for**
7: Return to Step 2 and implement Steps 2-6 for determining $\hat{x}_i^l(t+1)$ and $\hat{x}_l(t+1)$.

Remark 3: By using the idea of bounded recursive optimization approach [30], the convex optimization problems (17) and (19) are established in terms of linear matrix inequalities (LMIs), which can be solved by using the function "*mincx*" of MATLAB LMI Toolbox [36]. Moreover, when considering the linear LSEs $\hat{x}_i^l(t)(i = 1, 2, \ldots, m)$ determined by the stable estimators (14), then the designed fusion estimator (15) is stable, that is, the SE of the DFE for the linear systems (13) and (7) is bounded at each time.

B. NONLINEAR DISTRIBUTED FUSION ESTIMATION ALGORITHM DESIGN

When considering the nonlinear target dynamic process (1), according to the proposed nonlinear local estimator (11) and fusion criterion (12), the nonlinear local estimator gain $\mathrm{K}_i^n(t)$ and weighting fusion matrix $\mathrm{W}_i^n(t)$ are given in Theorem 2.

Define

$$\begin{cases} A^N_{\mathrm{K}_i}(t) \triangleq (I - \mathrm{K}^n_i(t)C_i(t))A_{n_i}(t-1) \\ B^N_{\mathrm{K}_i}(t) \triangleq (I - \mathrm{K}^n_i(t)C_i(t))B(t-1) \\ E^N_{\mathrm{K}_i}(t) \triangleq (I - \mathrm{K}^n_i(t)C_i(t))E_{n_i}(t) \\ \bar{E}^N_{\mathrm{K}_i}(t) \triangleq \begin{bmatrix} (E^N_{\mathrm{K}_i}(t))^{\mathrm{T}} & O & O & O \end{bmatrix}^{\mathrm{T}} \\ A^N_{\mathrm{E}_i}(t) \triangleq (I - \mathrm{K}^n_i(t)C_i(t))(A_{n_i}(t-1) + E_{n_i}(t)F_{n_i}(t)) \\ O^I_i(t) \triangleq \begin{bmatrix} O & \epsilon_i(t)I & O & O \end{bmatrix} \\ \Delta_i(t) \triangleq \begin{bmatrix} -I & A^N_{\mathrm{K}_i}(t) & B^N_{\mathrm{K}_i}(t) & -\mathrm{K}^n_i(t) \\ * & -\Pi_i(t) & O & O \\ * & * & -\Psi_i(t) & O \\ * & * & * & -\Phi_i(t) \end{bmatrix} \end{cases} \tag{30}$$

Theorem 2: Each nonlinear local estimator gain $\mathrm{K}^n_i(t)$ for nonlinear systems (1) and (7) can be determined by solving the following convex optimization problem:

$$\min_{\substack{\Pi_i(t)>0,\Psi_i(t)>0,\epsilon_i(t) \\ \Phi_i(t)>0,\ \mathrm{K}^n_i(t),\ \zeta_i(t)}} \mathrm{Tr}\{\Psi_i(t)\} + \mathrm{Tr}\{\Phi_i(t)\}$$

$$\text{s.t. :} \begin{cases} \begin{bmatrix} -\epsilon_i(t)I & O^I_i(t) & O \\ * & \Delta_i(t) & \bar{E}^N_{\mathrm{K}_i}(t) \\ * & * & -\epsilon_i(t)I \end{bmatrix} < 0 \\ \Pi_i(t) - \zeta_i(t)I < I \\ \zeta_i(t) \le 1 \end{cases} \tag{31}$$

where $O^I_i(t)$ and $\Delta_i(t)$ have been defined in (30). In this case, the nonlinear LEE $\tilde{x}^n_i(t)$ will be bounded, that is

$$\lim_{t\to\infty} [\tilde{x}^n_i(t)]^{\mathrm{T}}\tilde{x}^n_i(t) < \alpha_i \tag{32}$$

where $\alpha_i > 0$ is a scalar. Then, the distributed weighting fusion matrix $\mathrm{W}^n_i(t)$ can be solved by the following convex optimization problem:

$$\min_{\substack{\Pi(t)>0,\Psi(t)>0,\Phi(t)>0,\epsilon(t) \\ \Pi_1(t),\ \Pi_2(t),\ \Psi_1(t),\ \mathrm{W}(t)}} \mathrm{Tr}\{\Pi(t)\} + \mathrm{Tr}\{\Psi(t)\} + \mathrm{Tr}\{\Phi(t)\}$$

$$\text{s.t. :} \left\{ \begin{bmatrix} -\epsilon(t)I & O_I(t) & O \\ * & \Delta(t) & \bar{E}^N_{\mathrm{W}}(t) \\ * & * & -\epsilon(t)I \end{bmatrix} < 0 \right. \tag{33}$$

where $O_I(t)$, $\Delta(t)$, $E^N_{\mathrm{W}}(t)$ are defined by:

$$\begin{cases} A^{\mathrm{N}}_{\mathrm{W}}(t) \triangleq \mathrm{W}_n(t)\mathrm{diag}\{A^N_{\mathrm{K}_1}(t), \ldots, A^N_{\mathrm{K}_m}(t)\} \\ B^{\mathrm{N}}_{\mathrm{W}}(t) \triangleq \mathrm{W}_n(t)\mathrm{col}\{B^N_{\mathrm{K}_1}(t), \ldots, B^N_{\mathrm{K}_m}(t)\} \\ \mathrm{K}^N_{\mathrm{W}}(t) \triangleq \mathrm{W}_n(t)\mathrm{diag}\{-\mathrm{K}^n_1(t), \ldots, -\mathrm{K}^n_m(t)\} \\ E^{\mathrm{N}}_{\mathrm{W}}(t) \triangleq \mathrm{W}_n(t)\mathrm{diag}\{E^N_{\mathrm{K}_1}(t), \ldots, E^N_{\mathrm{K}_m}(t)\} \\ \bar{E}^{\mathrm{N}}_{\mathrm{W}}(t) \triangleq \begin{bmatrix} (E^{\mathrm{N}}_{\mathrm{W}}(t))^{\mathrm{T}} & O & O & O \end{bmatrix}^{\mathrm{T}} \\ O_I(t) \triangleq \begin{bmatrix} O & \epsilon(t)I & O & O \end{bmatrix} \\ \Delta(t) \triangleq \begin{bmatrix} -I & A^N_{\mathrm{W}}(t) & B^N_{\mathrm{W}}(t) & \mathrm{K}^N_{\mathrm{W}}(t) \\ * & -\Pi(t) & -\Pi_1(t) & -\Pi_2(t) \\ * & * & -\Psi(t) & -\Psi_1(t) \\ * & * & * & -\Phi(t) \end{bmatrix} \\ \mathrm{W}_n(t) \triangleq [\mathrm{W}^n_1(t), \ldots, \mathrm{W}^n_{m-1}(t), I - \sum_{i=1}^{m-1} \mathrm{W}^n_i(t)] \end{cases} \tag{34}$$

Proof: Firstly, define the nonlinear estimation error $\tilde{x}^n_i(t) \triangleq x(t) - \hat{x}^n_i(t)$, thus, it follows from (1) and (11) that

$$\begin{aligned} \tilde{x}^n_i(t) &= f(x(t-1)) + B(t-1)w(t-1) \\ &\quad - f(\hat{x}^n_i(t-1)) - \mathrm{K}^n_i(t)(z_i(t) - C_i(t)f(\hat{x}^n_i(t-1))) \end{aligned} \tag{35}$$

Then, by using the Taylor expansion to linearize the nonlinear function $f(x(t-1))$ near the $\hat{x}^n_i(t-1)$, one has

$$\begin{aligned} f(x(t-1)) &= f(\hat{x}^n_i(t-1)) \\ &\quad + A_{n_i}(t-1)\tilde{x}^n_i(t-1) + \mathcal{O}_f([\tilde{x}^n_i(t-1)]^2) \end{aligned} \tag{36}$$

where $A_{n_i}(t-1) = \partial f(x(t-1))/\partial x(t-1)|_{x(t-1)=\hat{x}^n_i(t-1)}$, and $\mathcal{O}_f([\tilde{x}^n_i(t-1)]^2)$ represents the high order-terms of Taylor series expansion.

In order to further analyze the stability of the designed estimators, the linearization errors $\mathcal{O}_f([\tilde{x}^n_i(t-1)]^2)$ in (36) can be modeled by state-dependent scaling matrix with uncertain matrix [35], one obtains

$$\mathcal{O}_f([\tilde{x}^n_i(t-1)]^2) \triangleq E_{n_i}(t)F_{n_i}(t)\tilde{x}^n_i(t-1) \tag{37}$$

where $E_{n_i}(t)$ is the state-dependent scaling matrix, and $F_{n_i}(t)$ is a unknown matrix with norm constraint, i.e.

$$\|F_{n_i}(t)\|_2 < 1$$

Then, substituting (36) and (37) into (35) yields

$$\tilde{x}^n_i(t) = A^N_{\mathrm{E}_i}(t)\tilde{x}^n_i(t-1) + B^N_{\mathrm{K}_i}(t)w(t-1) - \mathrm{K}^n_i(t)\bar{v}_i(t) \tag{38}$$

where $A^N_{\mathrm{E}_i}(t)$ and $B^N_{\mathrm{K}_i}(t)$ have been defined in (30). Since (38) has a similar form of (20), thus to ensure $[\tilde{x}^n_i(t)]^{\mathrm{T}}\tilde{x}^n_i(t)$ has an upper bound, the following inequality is held from the similar derivation in Theorem 1:

$$\begin{bmatrix} -I & A^N_{\mathrm{E}_i}(t) & B^N_{\mathrm{K}_i}(t) & -\mathrm{K}^n_i(t) \\ * & -\Pi_i(t) & O & O \\ * & * & -\Psi_i(t) & O \\ * & * & * & -\Phi_i(t) \end{bmatrix} < 0 \tag{39}$$

Then, it can be derived from Lemma 1 that the first inequality in (31) holds. Moreover, the optimization problems (31) and (33) can be formulated by using the similar proof in [34, Th.1,Th.2], the detailed derivation is omitted here. ■

Based on Theorem 2, the computation procedures for the nonlinear LSEs $\hat{x}^n_i(t)(i = 1, \cdots, m)$ and the DFE $\hat{x}_n(t)$ can be summarized in Algorithm 2.

Remark 4: Though the range-only measurement model (3) can avoid the linearization errors by using a nonlinear transformation, it cannot be used for the nonlinear state model (1). In this case, the Taylor expansion has been used in [30] to linearize nonlinear function $f(x)$, while the higher-order terms were modeled by bounded noises, thus the linearization errors were still be ignored and the stability of the estimator was not discussed as well. In fact, the estimation performance will be affected by the linearization errors, thus the

Algorithm 2 Distributed Fusion Estimation Based on Range-Only Measurement for Nonlinear Target Dynamic

1: Initialization state $x(0)$ and $\hat{x}_i^n(0)$;
2: **for** $i := 1$ **to** m **do**
3: Solve the optimization problems (31) and (33) by the "*mincx*" function of MATLAB LMI Toolbox;
4: Determine local linear estimator gain $\mathrm{K}_i^n(t)$ and distributed weighting fusion matrix $\mathrm{W}_i^n(t)$;
5: Calculate nonlinear LSE $\hat{x}_i^n(t)$ by (11) and DFE $\hat{x}_n(t)$ by (12);
6: **end for**
7: Return to Step 2 and implement Steps 2-6 for determining $\hat{x}_i^n(t+1)$ and $\hat{x}_n(t+1)$.

$\mathcal{O}_f([\tilde{x}_i^n(t-1)]^2)$ in (36) is modeled by a state-dependent matrix with unknown bounded matrix, which can further to analyze the characteristic of the linearization errors. Then, the stability of the designed nonlinear local/fusion estimators can be guaranteed, and the estimation performance also can be further improved.

Remark 5: Notice that the computational complexity of Algorithm 1 and Algorithm 2 depends on solving different convex optimization problems, while these optimization problems on the design of the local estimators and distributed fusion criteria are established in terms of linear matrix inequalities (LMIs). In this case, the calculation amount of the proposed algorithms are mainly determined by following aspects: one is the dimension of the LMIs, and the other is the number of introduced parameters for these optimization problems. For example, the dimension of the first LMI in (17) is $\mathcal{D}_l = 2n+r+q_i$ and the number of the introduced unknown parameters is $\mathcal{S}_l = n^2 + r^2 + q_i^2$, then the computational complexity of optimization problem (17) can be assumed to be $\mathcal{O}(\mathcal{D}_l^2\mathcal{S}_l)$. Consequently, dwindling the dimension of the LMIs and decreasing the number of the introduced unknown parameters are feasible schemes to reduce the search space for these optimization problems, then the solution time can be shortened. Moreover, the computational time will also have a significant impact on the online running of Algorithm 1 and Algorithm 2, while the calculating speed can be improved with the rapid development of computer software and hardware technology.

IV. SIMULATION EXAMPLES

In this section, two kinds of target tracking systems are presented to show the effectiveness and advantages of the proposed fusion estimation algorithms.

A. LINEAR TARGET MOTION SYSTEM

Consider a single target tracking example in X-Y planar, where only the distance information can be collected by each sensor. Generally, the position and velocity of the target are taken as the state, i.e. $x(t) \triangleq \mathrm{col}\{x_p(t), \dot{x}_p(t), y_p(t), \dot{y}_p(t)\}$, and the motion process of the target can be modeled by (13), where the specific parameters are given as follows [38]

$$A(t) = \begin{bmatrix} 1 & \dfrac{\sin(\omega T_0)}{\omega} & 0 & \dfrac{\cos(\omega T_0)-1}{\omega} \\ 0 & \cos(\omega T_0) & 0 & -\sin(\omega T_0) \\ 0 & \dfrac{1-\cos(\omega T_0)}{\omega} & 1 & \dfrac{\sin(\omega T_0)}{\omega} \\ 0 & \sin(\omega T_0) & 0 & \cos(\omega T_0) \end{bmatrix} \tag{40}$$

$$\Gamma(t) = \begin{bmatrix} T_0 & 1 & 0 & 0 \\ 0 & 0 & T_0 & 1 \end{bmatrix}^{\mathrm{T}} \tag{41}$$

where angular velocity ω is taken as $\pi/15$ rad, and sampling period T_0 is given by 0.5. It is assumed that 12 sensors are installed to obtain the measurements, which can be divided into 3 sensor groups. Then, the position of sensors are taken as $(x_{11}, y_{11}) = (80, 5)$, $(x_{12}, y_{12}) = (95, 10)$, $(x_{13}, y_{13}) = (75, -8)$, $(x_{14}, y_{14}) = (88, -5)$; $(x_{21}, y_{21}) = (-6, 85)$, $(x_{22}, y_{22}) = (12, 88)$, $(x_{23}, y_{23}) = (-10, 70)$, $(x_{24}, y_{24}) = (5, 75)$; $(x_{31}, y_{31}) = (70, 85)$, $(x_{32}, y_{32}) = (75, 70)$, $(x_{33}, y_{33}) = (86, 90)$, $(x_{34}, y_{34}) = (88, 72)$. Here, the process noise $w(t)$ in (13) and measurements v_{i,q_i} $(i = 1, 2, 3, q_i = 1, 2, 3, 4)$ in (3) are set as

$$\begin{cases} w(t) = \mathrm{col}\{0.3\varrho_1(t) - 0.1, 0.2\varrho_2(t) - 0.1\}; \\ v_{11}(t) = 0.05\ell_{11}(t) - 0.02, \ v_{12}(t) = 0.02\ell_{12}(t) - 0.01, \\ v_{13}(t) = 0.07\ell_{13}(t) - 0.02, \ v_{14}(t) = 0.02\ell_{14}(t) - 0.01; \\ v_{21}(t) = 0.06\ell_{21}(t) - 0.03, \ v_{22}(t) = 0.01\ell_{22}(t) - 0.01, \\ v_{23}(t) = 0.08\ell_{23}(t) - 0.03, \ v_{24}(t) = 0.03\ell_{24}(t) - 0.02; \\ v_{31}(t) = 0.05\ell_{31}(t) - 0.01, \ v_{32}(t) = 0.02\ell_{32}(t) - 0.01, \\ v_{33}(t) = 0.07\ell_{33}(t) - 0.03, \ v_{34}(t) = 0.01\ell_{34}(t) - 0.01. \end{cases} \tag{42}$$

where $\varrho_\kappa(t)(\in [0, 1])(\kappa = 1, 2)$ and $\ell_{ij}(t)(\in [0, 1])(i = 1, 2, 3, j = 1, 2, 3, 4)$ are random variables. The initial state is assumed to be known, which is taken as $x(0) = \hat{x}_i^l(0) = [10, 0.5, 10, 0.5]^{\mathrm{T}}$.

By implementing the range-only measurement fusion estimation method in Algorithm 1, the tracking effect of the local estimators and fusion estimator are drawn in Fig. 2. It is seen from the subfigure (a-c) of Fig. 2 that each local estimator can tracking the target's motion trajectory well, and subfigure (d) shows the distributed fusion estimator also tracking the target's trajectory well, which indicate that the proposed fusion estimation Algorithm 1 performing a effective tracking performance, that is the designed LSE (14) and DFE (15) in this paper are useful. Moreover, the mean square error (MSE) is employed to further assess the tracking capabilities, which is defined by

$$\mathrm{MSE}(t) \triangleq \frac{1}{N}\sum_{j=1}^{N}(x(t) - \hat{x}_j(t))^{\mathrm{T}}(x(t) - \hat{x}_j(t)) \tag{43}$$

where $\hat{x}_j(t)$ denotes the corresponding local or fusion estimates at time t, and N is the number of Monte Carlo simulations. Then, the MSEs over an average of 100 runs of Monte Carlo method [9] for LSEs and DFE are drawn in

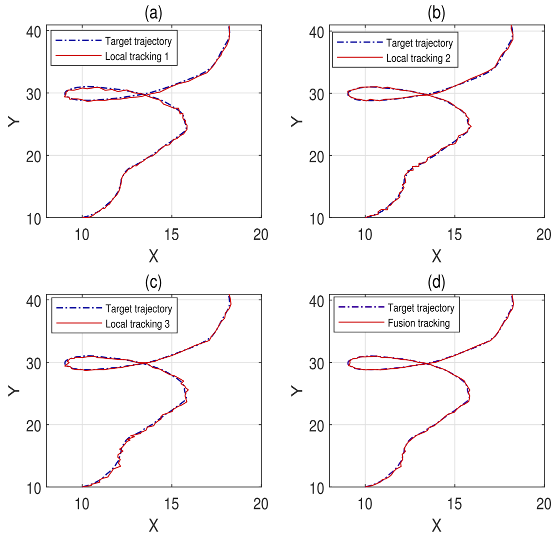

FIGURE 2. (a-c) The tracking trajectory of the three local estimators, respectively; (d) The target's tracking trajectory of the designed linear distributed fusion estimator.

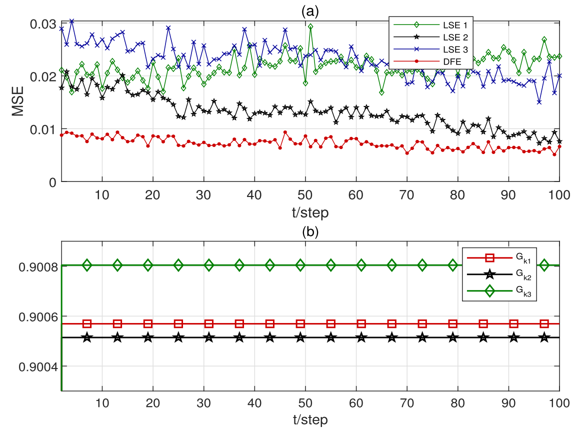

FIGURE 3. (a) The MSEs of local estimators and fusion estimator; (b) The stability judgement conditions $G_{Ki}(t)(i = 1, 2, 3)$.

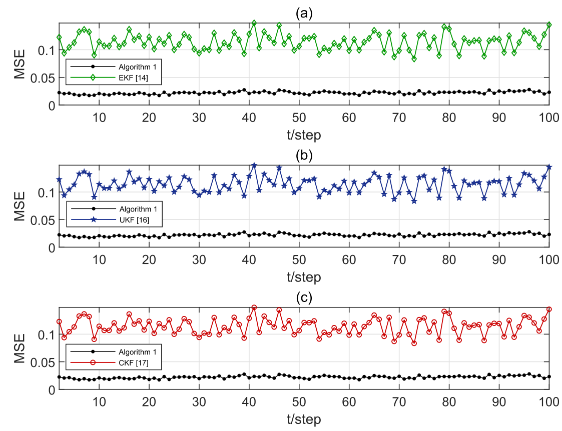

FIGURE 4. (a-c) The comparison of tracking performance among the estimation Algorithm 1, EKF [14], UKF [16] and CKF [17].

Fig. 3 (a), respectively. It can be found that the MSE of the DFE is smaller than that of each LSE, which implies that the tracking capabilities can be further improved by the designed fusion estimation algorithm. This is because for the DFE (15), an optimization problem (19) is established with respect to each LSE, and then a better tracking performance can be obtained based on the local estimates.

Moreover, the stability condition of the designed estimators has been addressed in the theoretical analysis section, then it can be concluded from Theorem 1 that (18) holds when the optimization problem (17) can be solvable at each time. In this case, the following condition must be held:

$$\mathrm{G}_{\mathrm{K}i}(t) \triangleq \|(I - \mathrm{K}_i^l(t)C_i(t))A(t-1)\|_2 < 1 \tag{44}$$

which has been verified by Fig. 3 (b), and then the stability of the local estimators (14) can be guaranteed. Since the distributed fusion estimator is a linear combination of local estimators, thus the designed fusion estimator (15) is also stable.

To further show the advantages of the Algorithm 1 for tracking the motion target, the tracking performance of proposed local estimator, EKF [14], UKF [16] and CKF [17] methods are presented in Fig. 4. It is seen that the MSE of LSE (14) is smaller than EKF, UKF and CKF with known covariances $Q_w = \mathrm{diag}\{3, 1\} \times 10^{-2}$, $Q_v = \mathrm{diag}\{5, 6, 7, 3\} \times 10^{-3}$, which implies that the developed local estimators in this paper have an better tracking capabilities. The main reason for this situation is that the statistical characteristics of noises are difficult to be accurately obtained in the target motion process, while the normal filtering methods are required a prior information of the noises. Thus, for the unknown bounded noises (42), the designed tracking algorithm with independent of the noise statistical information has a better tracking performance.

B. NONLINEAR INTELLIGENT VEHICLE TRACKING SYSTEM

In this example, consider a nonlinear intelligent vehicle tracking system, where also only the distance information from the vehicle to sensors can be obtained. When taking the vehicle's state as $x(t) \triangleq \mathrm{col}\left\{x_p(t), y_p(t), \theta(t)\right\}$, where $x_p(t)$ and $y_p(t)$ are vehicle's position in the X and Y axis of plane coordinates, respectively, and $\theta(t)$ is the angular orientation. Then the motion trajectory of the vehicle can be modeled by (1), and the parameters can be set as [37]:

$$\begin{cases} f(x(t)) \triangleq \begin{bmatrix} x_p(t) + \frac{c_t}{c_r}\cos\left(\theta(t) + \frac{t_0 c_r}{2}\right) \\ y_p(t) + \frac{c_t}{c_r}\sin\left(\theta(t) + \frac{t_0 c_r}{2}\right) \\ \theta(t) + t_0 c_r \end{bmatrix} \\ w(t) \triangleq \mathrm{col}\left\{ \dfrac{\hat{c}_t}{\hat{c}_r}\cos(\theta(t) + \dfrac{t_0\hat{c}_r}{2}) - \dfrac{c_t}{c_r}\cos(\theta(t) + \dfrac{t_0 c_r}{2}), \right. \\ \qquad \dfrac{\hat{c}_t}{\hat{c}_r}\sin(\theta(t) + \dfrac{t_0\hat{c}_r}{2}) - \dfrac{c_t}{c_r}\sin(\theta(t) + \dfrac{t_0 c_r}{2}), \\ \qquad t_0 w_r(t)\} \\ B(t) = \mathrm{diag}\{1, 1, t_0\} \end{cases} \tag{45}$$

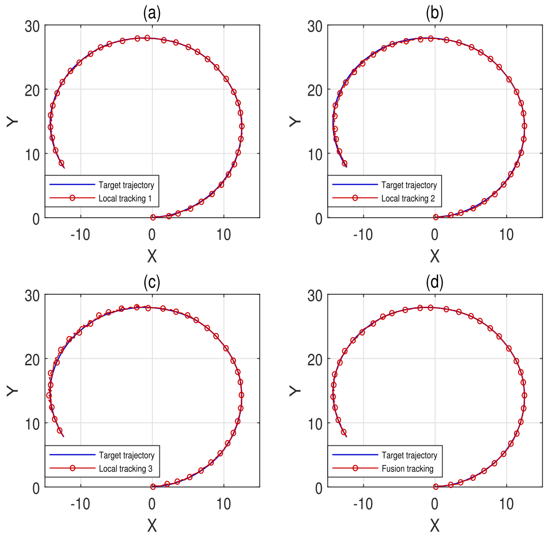

FIGURE 5. (a-c) The tracking trajectory of the local estimators, respectively; (d) The target's tracking trajectory of the nonlinear distributed fusion estimator.

where $t_0 = 0.5$ is the sampling period, $w(t)$ is the process noise, $c_t = 0.2$ and $c_r = 0.8$ are the control translational and rotational velocity, respectively. Similarly, there are 12 sensors that are divided into 3 groups to obtain the measurements, and the position of sensors are taken as $(x_{11}, y_{11}) = (5, 20)$, $(x_{12}, y_{12}) = (9, 23)$, $(x_{13}, y_{13}) = (0, 15)$, $(x_{14}, y_{14}) = (7, 16)$; $(x_{21}, y_{11}) = (18, 21)$, $(x_{22}, y_{22}) = (21, 25)$, $(x_{23}, y_{23}) = (12, 16)$, $(x_{24}, y_{24}) = (20, 18)$; $(x_{31}, y_{31}) = (20, 6)$, $(x_{32}, y_{32}) = (24, 12)$, $(x_{33}, y_{33}) = (12, 3)$, $(x_{34}, y_{34}) = (21, 5)$. Here, $E_{n_i}(i = 1, 2, 3)$ and the noises $w_p(t)$, $w_r(t)$ are set as

$$\begin{cases} E_{n_1} = \mathrm{diag}\{0.01, 0.01, 0.01\} \\ E_{n_2} = \mathrm{diag}\{0.03, 0.03, 0.03\} \\ E_{n_3} = \mathrm{diag}\{0.02, 0.02, 0.02\} \\ w_p(t) = 0.2\rho_1(t) - 0.1,\ w_r(t) = 0.3\rho_2(t) - 0.1; \end{cases} \tag{46}$$

where $\rho_\kappa(t)(\in [0, 1])(\kappa = 1, 2)$ are random variables, and v_{i,q_i} are given in (42). Meanwhile, the initial state is given by $x(0) = \hat{x}_i^n(0) = [0.1, 0.05, 0.08]^{\mathrm{T}}$. In addition, the matrix $A_{n_i}(t-1)$ can be calculated by

$$A_{n_i}(t-1) = \begin{bmatrix} 1 & 0 & -\frac{c_t}{c_r}\sin(\theta(t) + \frac{t_0 c_r}{2}) \\ 0 & 1 & \frac{c_t}{c_r}\cos(\theta(t) + \frac{t_0 c_r}{2}) \\ 0 & 0 & 1 \end{bmatrix}_{x(t-1)=x^*} \tag{47}$$

Then, implementing the fusion estimation Algorithm 2, the tracking effect of the LSEs (11) and DFE (11) are drawn in Fig. 5. As shown in subfigure (a-d) that all nonlinear local estimators and distributed nonlinear fusion estimator can tracking the target's motion trajectory well, it verified that the proposed nonlinear estimation Algorithm 2 addressing a good tracking ability, and the designed nonlinear estimators (11) and (12) are also effective. Then, the MSEs of the LSEs and the DFE with 100 Monte Carlo runs are performed in Fig. 6 (a), respectively. It shows that the tracking capability of the nonlinear DFE is better than that of each LSE, thus the tracking performance of this tracking system can be further improved by the fusion estimation Algorithm 2. Particularly, Fig. 6 (b) shows the condition $\|A_{\mathrm{E}_i}^N(t)\|_2 < 1$ holds, when the optimization problem (31) is solvable. Then, it can be concluded from Theorem 2 that the stability of local/fusion nonlinear estimators is held. Notice that, the linearization errors of the $f(x(t))$ in (45) are not easily judged as bounded errors, thus the stability condition (C.2) of [33, Th.2] is useless for this example. However, this paper modeling the linearization errors by the state-dependent scaling matrices with unknown bounded matrices, which can modified the stability conditions of [33, Th.2] and ensured the designed nonlinear estimators are stable.

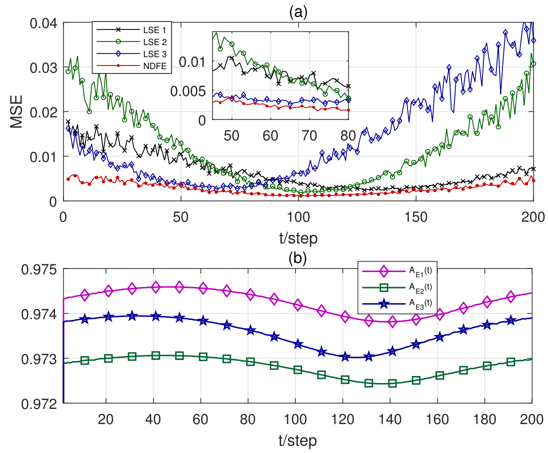

FIGURE 6. (a) The estimation performance comparison between the local estimators and fusion estimator; (b) The stability judgement conditions $A_{Ei}(t)(i = 1, 2, 3)$.

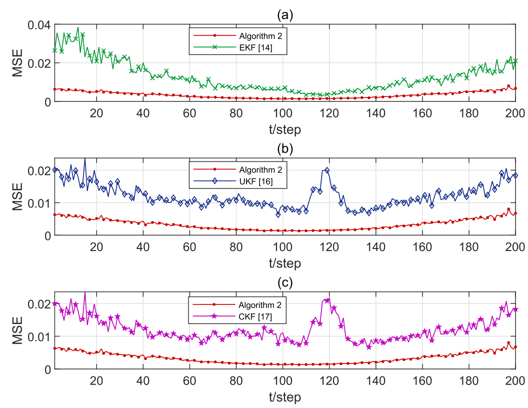

FIGURE 7. (a-c) The comparison of tracking performance among the nonlinear estimation Algorithm 2, EKF [14], UKF [16] and CKF [17].

Similarly, when considering the unknown bounded noises (42) and (46), the MSEs of local estimator proposed in Algorithm 2, EKF, UKF and CKF methods are plotted in Fig. 7, respectively. From Fig. 7, it shows that the proposed

LSE (11) in this article has a better tracking performance than that of EKF, UKF and CKF with known covariances $Q_w = \text{diag}\{3, 1, 2\} \times 10^{-2}$, $Q_v = \text{diag}\{2, 1, 2, 3\} \times 10^{-3}$ as well. This is because these nonlinear tracking methods usually require to know the covariances of Gaussian noises, but the statistical properties of noises in (42) and (46) are still without knowledge. Therefore, it follows from the analysis of previous example that the developed nonlinear tracking Algorithm 2 is more suitable for practical tracking system.

V. CONCLUSION

In this article, a distributed fusion tracking problem with range-only measurement has been investigated. For the nonlinear distance measurement model, which was converted to a linear model in regard to the position of the target by using a nonlinear transformation. When considering the nonlinear motion dynamic of the target, a uncertainty bounded matrix combined with sate-dependent matrix were introduced to describe the state linearization error caused by the Taylor expansion. Though the measurement noises were more complicated and the uncertainty parameters have been introduced by the corresponding nonlinear transformation, a robust designed approach based on the bounded recursive optimization scheme was developed in this article, which can avoided the instability of nonlinear estimator. Then, the stable local estimator gains and fusion criterion were determined by the constructed convex optimization problems. Finally, two range-only target tracking examples were addressed to verify the effectiveness of the proposed tracking algorithms.

Furthermore, the target tracking and location problems over the WSNs are also give a significant research direction. Particularly, the time-delay, packet-dropouts, resource constraints and cyber-attacks problems in the networked structure, which are widely existed. In this case, based on the developed dimensionality reduction approach, quantization approach and event-triggered scheme, how to design the stable estimator for WSNs-based target tracking systems will be our future works.

REFERENCES

[1] L. A. M. Hernandez, S. P. Arteaga, G. S. Perez, A. L. S. Orozco, and L. J. G. Villalba, "Outdoor location of mobile devices using trilateration algorithms for emergency services," *IEEE Access*, vol. 7, pp. 52052–52059, 2019.

[2] Y. Zhao, "Mobile phone location determination and its impact on intelligent transportation systems," *IEEE Trans. Intell. Transp. Syst.*, vol. 1, no. 1, pp. 55–64, Mar. 2000.

[3] M. A. Spirito, "On the accuracy of cellular mobile station location estimation," *IEEE Trans. Veh. Technol.*, vol. 50, no. 3, pp. 674–685, May 2001.

[4] K. W. Cheung, H. C. So, W. K. Ma, and Y. T. Chan, "Least squares algorithms for time-of-arrival-based mobile location," *IEEE Trans. Signal Process.*, vol. 52, no. 4, pp. 1121–1128, Apr. 2004.

[5] F. Liu, H. Chen, L. Zhang, and L. Xie, "Time-difference-of-arrival-based localization methods of underwater mobile nodes using multiple surface beacons," *IEEE Access*, vol. 9, pp. 31712–31725, 2021.

[6] B. Li, K. Zhao, and X. Shen, "Dilution of precision in positioning systems using both angle of arrival and time of arrival measurements," *IEEE Access*, vol. 8, pp. 192506–192516, 2020.

[7] H. C. So and L. Lin, "Linear least squares approach for accurate received signal strength based source localization," *IEEE Trans. Signal Process.*, vol. 59, no. 8, pp. 4035–4040, Aug. 2011.

[8] X. Shi, G. Mao, B. D. O. Anderson, Z. Yang, and J. Chen, "Robust localization using range measurements with unknown and bounded errors," *IEEE Trans. Wireless Commun.*, vol. 16, no. 6, pp. 4065–4078, Jun. 2017.

[9] Y. Bar-Shalom, X. R. Li, and T. Kirubarajan, *Estimation With Applications to Tracking and Navigation*. Hoboken, NJ, USA: Wiley, 2001.

[10] X. Yang, W.-A. Zhang, and L. Yu, "A bank of decentralized extended information filters for target tracking in event-triggered WSNs," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 50, no. 9, pp. 3281–3289, Sep. 2020.

[11] M. Boutayeb and D. Aubry, "A strong tracking extended Kalman observer for nonlinear discrete-time systems," *IEEE Trans. Autom. Control*, vol. 44, no. 8, pp. 1550–1556, Aug. 1999.

[12] W. F. Leven and A. D. Lanterman, "Unscented Kalman filters for multiple target tracking with symmetric measurement equations," *IEEE Trans. Autom. Control*, vol. 54, no. 2, pp. 370–375, Feb. 2009.

[13] Y. Zhai, P. Song, Z. Mou, X. Chen, and X. Liu, "Occlusion-aware correlation particle filter target tracking based on RGBD data," *IEEE Access*, vol. 6, pp. 50752–50764, 2018.

[14] S. Julier and J. Uhlmann, "A new extension of the Kalman filter to nonlinear systems," *Proc. SPIE*, vol. 3068, pp. 182–193, Jul. 1997.

[15] T. H. Kerr, "Streamlining measurement iteration for EKF target tracking," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 27, no. 2, pp. 408–421, Mar. 1991.

[16] S. J. Julier and J. K. Uhlmann, "Unscented filtering and nonlinear estimation," *Proc. IEEE*, vol. 92, no. 3, pp. 401–422, Mar. 2004.

[17] I. Arasaratnam and S. Haykin, "Cubature Kalman filters," *IEEE Trans. Autom. Control*, vol. 54, no. 6, pp. 1254–1269, May 2009.

[18] B. Friedlander, "A passive localization algorithm and its accuracy analysis," *IEEE J. Ocean. Eng.*, vol. OE-12, no. 1, pp. 234–245, Jan. 1987.

[19] J. A. Roecker and C. D. McGillem, "Comparison of two-sensor tracking methods based on state vector fusion and measurement fusion," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-24, no. 4, pp. 447–449, Jul. 1988.

[20] T. Li, H. Fan, J. Garcia, and J. M. Corchado, "Second-order statistics analysis and comparison between arithmetic and geometric average fusion: Application to multi-sensor target tracking," *Inf. Fusion*, vol. 51, pp. 233–243, Nov. 2019.

[21] X. He, T. Wang, W. Liu, and T. Luo, "Measurement data fusion based on optimized weighted least-squares algorithm for multi-target tracking," *IEEE Access*, vol. 7, pp. 13901–13916, 2019.

[22] M. Xu, Y. Zhang, D. Zhang, and B. Chen, "Distributed robust dimensionality reduction fusion estimation under DoS attacks and uncertain covariances," *IEEE Access*, vol. 9, pp. 10328–10337, 2021.

[23] J. Hu, J. Li, Y. Kao, and D. Chen, "Optimal distributed filtering for nonlinear saturated systems with random access protocol and missing measurements: The uncertain probabilities case," *Appl. Math. Comput.*, vol. 418, pp. 1–18, Apr. 2022.

[24] Y. Zhang, B. Chen, L. Yu, and D. W. C. Ho, "Distributed Kalman filtering for interconnected dynamic systems," *IEEE Trans. Cybern.*, early access, Jun. 16, 2021, doi: 10.1109/TCYB.2021.3072198.

[25] G. Hao, S.-L. Sun, and Y. Li, "Nonlinear weighted measurement fusion unscented Kalman filter with asymptotic optimality," *Inf. Sci.*, vol. 299, pp. 85–98, Apr. 2015.

[26] Y. Dan, J. Hongbing, and G. Yongchan, "A robust D–S fusion algorithm for multi-target multi-sensor with higher reliability," *Inf. Fusion*, vol. 47, pp. 32–44, May 2019.

[27] W. Gao, J. Li, G. Zhou, and Q. Li, "Adaptive Kalman filtering with recursive noise estimator for integrated SINS/DVL systems," *J. Navigat.*, vol. 68, no. 1, pp. 142–161, Aug. 2014.

[28] B.-C. Liu, K.-H. Lin, and J.-C. Wu, "Analysis of hyperbolic and circular positioning algorithms using stationary signal-strength-difference measurements in wireless communications," *IEEE Trans. Veh. Technol.*, vol. 55, no. 2, pp. 499–509, Mar. 2006.

[29] X. Yang, W.-A. Zhang, A. Liu, and L. Yu, "Linear fusion estimation for range-only target tracking with nonlinear transformation," *IEEE Trans. Ind. Informat.*, vol. 16, no. 10, pp. 6403–6412, Oct. 2020.

[30] B. Chen, G. Hu, D. W. C. Ho, and L. Yu, "A new approach to linear/nonlinear distributed fusion estimation problem," *IEEE Trans. Autom. Control*, vol. 64, no. 3, pp. 1301–1308, Mar. 2019.

[31] Y. Teng, B. Chen, and S. Sheng, "Distributed nonlinear estimation: A recursive optimization approach," *Circuits, Syst., Signal Process.*, vol. 41, no. 4, pp. 2397–2410, Apr. 2022, doi: 10.1007/s00034-021-01884-6.

[32] B. Chen, G. Hu, D. W. C. Ho, and L. Yu, "Distributed estimation and control for discrete time-varying interconnected systems," *IEEE Trans. Autom. Control*, early access, Apr. 27, 2021, doi: 10.1109/TAC.2021.3075198.

[33] B. Chen and G. Hu, "Nonlinear state estimation under bounded noises," *Automatica*, vol. 98, pp. 159–168, Dec. 2018.
[34] R. Wang, B. Chen, and L. Yu, "Distributed nonlinear fusion estimation without knowledge of noise statistical information: A robust design approach," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 57, no. 5, pp. 3107–3117, Oct. 2021.
[35] G. Calafiore, "Reliable localization using set-valued nonlinear filters," *IEEE Trans. Syst., Man, Cybern., A, Syst. Hum.*, vol. 35, no. 2, pp. 189–197, Mar. 2005.
[36] S. P. Boyd, L. E. Chaoui, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*. Philadelphia, PA, USA: SIAM, 1994.
[37] D. Zhu, B. Chen, Z. Hong, and L. Yu, "Networked nonlinear fusion estimation under DoS attacks," *IEEE Sensors J.*, vol. 21, no. 5, pp. 7058–7066, Mar. 2021.
[38] Y. Hu, Z. Jin, and Y. Wang, "State fusion estimation for networked stochastic hybrid systems with asynchronous sensors and multiple packet dropouts," *IEEE Access*, vol. 6, pp. 10402–10409, 2018.

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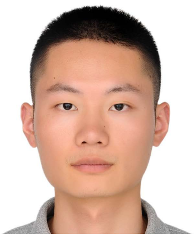

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