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Event-Triggered Safe Control for the Zero-Sum Game of Nonlinear Safety-Critical Systems With Input Saturation

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ABSTRACT In this paper, a novel adaptive dynamic programming (ADP)-based event-triggered safe control method is proposed to solve the zero-sum game problem of nonlinear safety-critical systems with safety constraints and input saturation. First, the barrier function-based system transformation, the zero-sum game problem with safety constraints and input saturation is transformed into an equivalent input saturation zero-sum game problem, so as to guarantee that the system does not violate the safety constraints. Furthermore, the non-quadratic utility function is introduced into the performance function to solve input saturation. Then, a critic neural network (NN) is constructed to approximate the optimal safety value function. Subsequently, a novel event-triggered scheme is developed to determine the update instant of the control law and the disturbance law. Therefore, the proposed ADP-based event-triggered safe control method can ensure that the states of nonlinear safety-critical systems satisfy the safety constraints, while greatly reducing the amount of calculation and saving communication resources. In addition, during the learning process, the concurrent learning is used to relax the persistence of excitation (PE) condition. According to the Lyapuov theory, it is proved that the weight estimation error of the critic neural network and the states are uniformly ultimately bounded (UUB), and the Zeno behavior is excluded. Finally, a simulation example verifies the effectiveness of the proposed method.

INDEX TERMS Adaptive dynamic programming, barrier function, event-triggered control, input saturation, safety constraints, zero-sum game.

I. INTRODUCTION

Nowadays, with the development of safety-critical systems such as self-driving cars [1], [2] and intelligent robots [3]–[5], the safety of the system has attracted more and more attention. Generally, when the state of the system is always evolving within a user-defined safety range, the system is said to be safe, otherwise, we call that the system is unsafe [6], [7]. In practical projects, a safe system is what we expect [8]. Therefore, how to design a controller that satisfies the safety constraints is a major challenge. In recent years, many safe controller design methods have been proposed [9]–[11]. In [9], the method based on quadratic programming was used to design the safe controller. Although the scheme can guarantee local safety within each time step, too small step size will lead to redundant calculations [10], while too large step size will cause unsafe behavior, which makes it difficult to guarantee the safety of the system. In addition, calculation methods such as sum of squares [11] can also be used to design safe controllers. However, these methods can only be used in the case of polynomial systems and constraints. Therefore, it is extremely urgent to find a suitable safe controller design scheme for safety-critical systems.

In recent years, the barrier function (BF) has proven to be an effective tool to ensure the safety of the safety-critical

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system [12], [13]. The barrier function has been used to transform a system with safety constraints into an equivalent system without safety constraints, and then a safe controller was designed to ensure the safety of the system [14]–[17]. Reference [14] had incorporated state in reinforcement learning framework used penalty function and BF-based state transformation. And then, this method was extended to the case with input saturation [15]. A safe reinforcement learning method was shown in [16], in which a control barrier function was added into the cost function, which ensured safety and optimality of the system. In [17], a safe non-policy reinforcement learning method was presented to design a safe optimal controller for safety-critical systems with dynamic uncertainty. However, the above methods do not considered the external disturbance, which inspired our research.

In actual projects, the external disturbance must be considered [18]–[22]. The H_∞ control is widely used as one of the powerful control method to reduce the influence of the external disturbance in dynamic systems [18], [19]. The H_∞ control problem is usually regarded as a zero-sum game (ZSG) problem [20].It is well known that solving the ZSG problem is equivalent to solving the Nash equilibrium solution of the Hamilton-Jacobi-Isaacs (HJI) equation. However, because of its own partial differential properties, it is incredibly tough, even literally impossible to obtain analytical solution of the HJI equation for many nonlinear systems. Fortunately, the adaptive dynamic programming technology has become a favorable method to solve the approximate solution of the ZSG problem in recent years [21]–[28]. For instance, in [24], an ADP-based method was developed to solve the two-player ZSG problem of discrete systems with partially unknown dynamics. In [25], the online adaptive algorithm was used to approximate the Nash equilibrium solution of the two-player ZSG problem for the continuous-time nonlinear system with completely unknown dynamics. In [26], an actor-critic-disturbance NN framework was used to solve the two-player ZSG problem for discrete-time affine nonlinear systems. In [27] and [28], the iterative learning method and the model-free global dual heuristic dynamic programming method were proposed to solve the multi-player ZSG problem for continuous-time systems and the discrete-time nonlinear ZSG problem, respectively. However, it should be noted that the above-mentioned methods and technologies are all developed under the time-triggered control (TTC) mechanism, which may cause a heavy computational burden and waste of communication resources.

In order to overcome the heavy computational burden and waste of communication resources by TTC mechanism, the event-triggered control (ETC) mechanism has begun to attract much attention [29]–[31]. Compared with the TTC mechanism, the ETC mechanism only updates the controller when the event-triggered condition is violated, which can reduce the amount of calculation and save communication resources [32]–[35]. Therefore, many event-triggered ADP methods are proposed to solve the ZSG problem. In [36], the event-triggered ADP method was proposed to solve the partially unknown continuous-time nonlinear ZSG problem. In [37], for the nonlinear continuous-time system ZSG problem, an event-triggered adaptive controller was designed to achieve the system's anti-disturbance ability. However, the above methods do not take into account the safety constraints and input saturation of the system, which has a great risk in practical applications.

Inspired by the above methods, in this paper, a new ADP-based event-triggered safe control method is proposed to solve the ZSG problem of nonlinear safety-critical systems with input saturation and safety constraints. The main contributions of this paper include the following three aspects:

1) By using the BF function, the nonlinear safety-critical system with input saturation and safety constraints is transformed into an equivalent system, which satisfies the user-defined safety constraints. And the H_∞ control problem of the transformed system is described as a two-player ZSG problem with input saturation.
2) To reduce the amount of calculation and save communication resources, the ETC mechanism is introduced and an event-triggered condition is derived for the nonlinear safety-critical system, so that the safety control law and the safety disturbance law are updated only when the event-triggered condition is destroyed. In addition, the Zeno behavior is excluded.
3) In this paper, an event-triggered safe control method is proposed and in the implementation process, the concurrent learning method is used to design the critic neural network to approximate the optimal safety value function for the nonlinear safety-critical system.

The rest of this paper is organized as follows. The problem statement and problem transformation are shown in section II. In section III, an event-triggered safe controller design scheme is presented. In section IV, the ADP-based event-triggered safe control method is presented, including its derivation, proof and implementation. Section V demonstrates the effectiveness of the proposed method through a simulation example. Finally, conclusions are given in section VI.

II. PRELIMINARIES

A. PROBLEM STATEMENT

Consider the nonlinear safety-critical system with input saturation described by equation of the form,

$$\dot{x} = f(x) + g(x)u(t) + k(x)d(t), \quad |u_i(t)| \leq \kappa, \tag{1}$$

where $x = [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^n$ is the state vector, and $u(t) \in \mathbb{R}^m$ is the control input required to satisfy $|u_i(t)| \leq \kappa, i = 1, 2, \ldots, m$, and κ is the input boundary. $d(t) \in \mathbb{R}^q$ is the external disturbance. Moreover, $f(x) \in \mathbb{R}^n$ is the drift system dynamics, $g(x) \in \mathbb{R}^{n\times m}$ is the input dynamics, and $k(x) \in \mathbb{R}^{n\times q}$ is the disturbance matrix. Assume that the functions $f(x)$, $g(x)$, $k(x)$ are Lipschitz continuous. In addition, for the system (1), note that its state

$x = [x_1, x_2, \ldots, x_n]^T$ satisfy the following safety constraints:

$$\begin{aligned} x_1 &\in (a_1, A_1), \\ x_2 &\in (a_2, A_2), \\ &\vdots \\ x_n &\in (a_n, A_n). \end{aligned} \tag{2}$$

For the nonlinear safety-critical system with safety constraints (2), due to the input saturation and the external disturbance, the zero-sum differential game framework is introduced [20]. Therefore, the following infinite horizon performance function is defined as

$$J\big(x(0), u, d\big) = \int_0^\infty \Theta(x, u, d)dt, \tag{3}$$

where $\Theta(x, u, d) = H(x) + Z(u) - \gamma^2 d^T d$ with $H(x)$ and $Z(u)$ are positive definite functions. Then, the ZSG problem with input saturation and safety constraints can be expressed as follows.

Problem 1: The infinite horizon performance function and the nonlinear safety-critical system are given by (3) *and* (1)*, respectively, find the Nash equilibrium solution* (u^*, d^*) *of the closed-loop system with safety constraints* (2)*.*

In order to ensure that the system state is always within the safety constraints. Next, we introduce some definitions of the barrier function.

Definition 1 (Barrier Function [14], [15]): The function $B(\cdot) : \mathbb{R} \to \mathbb{R}$ *defined on the interval* (a, A) *is referred to as barrier function if*

$$B(z; a, A) = log(\frac{A(a-z)}{a(A-z)}), \quad \forall z \in (a, A), \tag{4}$$

where $a < 0$ *and* $A > 0$*. In addition, on the interval* (a, A)*, the invertibility of the barrier function is reflected, that is,*

$$B^{-1}(y; a, A) = \frac{aA(e^{\frac{y}{2}} - e^{-\frac{y}{2}})}{ae^{\frac{y}{2}} - Ae^{-\frac{y}{2}}}, \quad \forall y \in \mathbb{R}. \tag{5}$$

Furthermore, the derivative of (5) *is,*

$$\frac{dB^{-1}(y; a, A)}{dy} = \frac{Aa^2 - aA^2}{a^2e^y - 2aA + Ae^{-y}}. \tag{6}$$

Remark 1: To ensure that the safety-critical system (1) *always satisfies the safety constraints, the barrier function in the Definition 1 has the following characteristics [14], [15].*

1) *The barrier function takes a finite value, if the its parameters satisfy the safety constraints.*
2) *When the system state is close to the safety constraint boundary, the barrier function changes as follows,*

$$\lim_{z \to a^+} B(z; a, A) = -\infty, \ \lim_{z \to A^-} B(z; a, A) = +\infty. \tag{7}$$

3) *When the system state reaches the equilibrium, the barrier function loses its effect, that is,*

$$B(0; a, A) = 0, \quad \forall a < A. \tag{8}$$

For the safety-critical system (1) with the safety constraints (2), we will use the barrier function to perform equivalent transformation. Therefore, the following transformations are performed

$$\begin{aligned} s_i &= B(x_i; a_i, A_i), \\ x_i &= B^{-1}(s_i; a_i, A_i), \end{aligned} \tag{9}$$

where $i = 1, 2, \ldots, n$. Then, the derivative of x_i with respect to t is $\frac{dx_i}{dt} = \frac{dx_i}{ds_i}\frac{ds_i}{dt}$, and after using Definition 1, we can get

$$\begin{aligned} \dot{s}_i &= \frac{a_{i+1}A_{i+1}(e^{\frac{s_{i+1}}{2}} - e^{-\frac{s_{i+1}}{2}})}{a_{i+1}e^{\frac{s_{i+1}}{2}} - A_{i+1}e^{-\frac{s_{i+1}}{2}}} \frac{a_i^2e_i^s - 2a_iA_i + A_ie^{-s_i}}{A_ia_i^2 - a_iA_i^2} \\ &= F_i(s_i, s_{i+1}), \quad i = 1, 2, \ldots, n-1. \end{aligned} \tag{10}$$

$$\begin{aligned} \dot{s}_n &= \dot{x}\frac{a_n^2e_n^s - 2a_nA_n + A_ne^{-s_n}}{A_na_n^2 - a_nA_n^2} \\ &= F_n(s) + g_n(s)u(t) + k_n(s)d(t), \end{aligned} \tag{11}$$

where

$$\begin{aligned} F_n(s) &= \frac{a_n^2e_n^s - 2a_nA_n + A_ne^{-s_n}}{A_na_n^2 - a_nA_n^2} f([B_1^{-1}(s_1)\ldots B_n^{-1}(s_n)]), \\ g_n(s) &= \frac{a_n^2e_n^s - 2a_nA_n + A_ne^{-s_n}}{A_na_n^2 - a_nA_n^2} g([B_1^{-1}(s_1)\ldots B_n^{-1}(s_n)]), \\ k_n(s) &= \frac{a_n^2e_n^s - 2a_nA_n + A_ne^{-s_n}}{A_na_n^2 - a_nA_n^2} k([B_1^{-1}(s_1)\ldots B_n^{-1}(s_n)]). \end{aligned} \tag{12}$$

Then, the system (11) can be rewritten as

$$\dot{s} = F(s) + G(s)u(t) + K(s)d(t), \quad |u_i(t)| \leq \kappa, \tag{13}$$

where $F(s) = \big[F_1(s_1, s_2), \ldots, F_n(s)\big]^T$, $G(s) = \big[0, \ldots, g_n(s)\big]^T$ and $K(s) = \big[0, \ldots, k_n(s)\big]^T$.

The following assumption exist for the transformed system (13).

Assumption 1 ([32], [37]): Assume that $F(s)$ *is Lipschitz continuous with* $F(0) = 0$*, i.e.,* $\|F(s)\| \leq f_m\|s\|$*, where* f_m *is positive constant.* $G(s)$ *and* $K(s)$ *are upper-bounded by* $\|G(s)\| \leq g_m$*, and* $\|K(s)\| \leq k_m$*, where* g_m*,* k_m *are positive constants. Furthermore, the transformed system* (13) *is controllable, and* $s = 0$ *is an equilibrium of* (13)*.*

B. BARRIER-FUNCTION-BASED ZSG PROBLEM TRANSFORMATION

For the transformed system (13), similar to (3), the infinite horizon performance function is introduced as follows

$$V\big(s_0, u(t), d(t)\big) = \int_0^\infty r(s, u, d)dt, \tag{14}$$

where $r(s, u, d) = Q(s) + Z(u) - \gamma^2 d^T d$, with $Q(s) = s^T Qs$, Q is user-defined positive-definite matrix, and γ is a positive constant. Moreover, $s_0 = s(0)$ denotes the initial state, $Z(u)$ is a non-quadratic utility function to solve input saturation, and $Z(u) \geq 0$ with $Z(0) = 0$. Inspired by [31], [39], $Z(u)$ can be defined as

$$Z(u) = 2\kappa \int_0^{u(s)} tanh^{-1}\Big(\frac{\xi}{\kappa}\Big)^T Rd\xi, \tag{15}$$

where $R = diag\{l_1, l_2, \ldots, l_m\}$ is a positive definite diagonal matrix. Besides, the performance function (14) satisfies zero-state observability. Therefore, for the transformed system (13), the following input saturation ZSG problem is considered as follows.

Problem 2: For the transformed system (13), *and the performance function is given by* (14), *then, find the Nash equilibrium u^* and d^* of the input saturation ZSG problem of the safety-critical system with the safety constraints.*

The safe value function is defined as follows:

$$V\big(s(t), u(t), d(t)\big) = \int_t^{\infty} r\big(s(\nu), u(\nu), d(\nu)\big)d\nu. \tag{16}$$

For the safe value function (16), if it is continuous and differentiable, then, the following nonlinear Lyapunov equation can be derived

$$r(s, u, d) + \big(\nabla V(s)\big)^T \big(F(s) + G(s)u + K(s)d\big) = 0, \tag{17}$$

where $\nabla V(s)$ is the partial derivative of $V(s, u, d)$ with respect to s.

Then, the safe Hamiltonian associated with (17) is presented by

$$H\big(s, u, d, \nabla V(s)\big) = r(s, u, d) + \big(\nabla V(s)\big)^T \big(F(s) + G(s)u + K(s)d\big). \tag{18}$$

According to the Bellman's optimal theory, the optimal safety value function $V^*(s)$ satisfies the following safe HJI equation

$$\begin{aligned} 0 &= \min_u \max_d H\big(s, u, d, \nabla V^*(s)\big) \\ &= \min_d \max_u H\big(s, u, d, \nabla V^*(s)\big). \end{aligned} \tag{19}$$

The safe control pair (u^*, d^*) satisfies following conditions:

$$\frac{\partial H\big(s, u, d, \nabla V^*(s)\big)}{\partial u} = 0, \tag{20}$$

$$\frac{\partial H\big(s, u, d, \nabla V^*(s)\big)}{\partial d} = 0. \tag{21}$$

According to (19), (20) and (21), the optimal safety control law and the worst safety disturbance law can be obtained

$$u^*(s) = -\kappa tanh\Big(\frac{1}{2\kappa}R^{-1}G^T(s)\nabla V^*(s)\Big), \tag{22}$$

$$d^*(s) = \frac{1}{2\gamma^2}K^T(s)\nabla V^*(s). \tag{23}$$

Substituting (22) and (23) into (19), the safe HJI equation is written as

$$\begin{aligned} 0 = &\int_0^{-\kappa tanh(\frac{1}{2\kappa}R^{-1}G^T(s)\nabla V^*(s))} 2\kappa tanh^{-1}\Big(\frac{\xi}{\kappa}\Big)^T Rd\xi \\ &+ \nabla V^{*T}(s)F(s) + s^T Qs \\ &- \nabla V^{*T}(s)G(s)\kappa tanh\Big(\frac{1}{2\kappa}R^{-1}G^T(s)\nabla V^*(s)\Big) \\ &+ \frac{1}{4\gamma^2}\nabla V^{*T}(s)K(s)K^T(s)\nabla V^*(s), \end{aligned} \tag{24}$$

$$0 = V^*(0). \tag{25}$$

Considering the BF-based system transformation, the ZSG problem 1 can be transformed into the ZSG problem 2. Next, the following lemma is discussed, which guarantee the equivalence between the ZSG problem 1 and the ZSG problem 2.

Lemma 1: Suppose that Assumption 1 is satisfied and that the Nash equilibrium solution u^* and d^* solve the ZSG problem 2 for transformed system (13) *with the performance function* (14). *Then, the following hold*:

1) *The premise of the closed-loop system satisfies the safety constraints* (2) *is that the initial state x_0 of the nonlinear safety-critical system* (1) *is within the interval (a_i, A_i), $\forall i = 1, 2, \ldots, n$.*
2) *The performance function described by* (14) *is equivalent to the performance function* (3) *on the premise that the function $H(x)$ and $Q(x)$ satisfies $H(x) = Q(B(x)) = Q(s)$.*

Proof: 1. According to the zero-state observability of the performance function (14) and Assumption 1, this guarantees the existence of the safe optimal value function $V^*(s)$. Based on (18), we can get $\nabla V^*(t) \leq 0$, therefore, for all $t \geq 0$, $V^*(s(t)) \leq V^*(s(0))$ can be obtained. Therefore, it can be known from Remark 1 that if the initial state x_0 of the system (1) satisfies safety constraints (2), $V^*(s(0))$ is bounded, then, $V^*(s(t))$ is also bounded. Finally, we can get

$$x_i(t) \in (a_i, A_i), \quad i = 1, 2, \ldots, n. \tag{26}$$

Thus, the given Nash equilibrium solution u^* and d^* satisfy the constraints of ZSG problem 1.

2. By using the BF-based state transformation (9), it can be obtained that if the system state $x(t)$ satisfies the safety constraints (2), then each element in $s = [B_1(x_1), \ldots, B_n(x_n)]$ is finite. Then, by comparing performance functions (3) and (14), the following equivalence relation can be obtained

$$J\big(x(0), u, d\big) = V\big(s(0), u, d\big) \tag{27}$$

provided that $H(x) = Q(s)$. This completes the proof. □

Remark 2: For the safe HJI equation (21), *we can use ADP-based time-triggered methods to solve it [24]–[28]. However, these time-triggered methods will cause a lot of calculations, and resources such as storage space and computation bandwidth are limited in practical projects. Therefore, we will introduce the event-triggered mechanism to overcome this trouble.*

III. EVENT-TRIGGERED SAFE CONTROLLER DESIGN

In this section, the event-triggered mechanism is introduced. Before this, we need to consider a sequence of triggering instants $\{\tau_j\}_{j=0}^{\infty}$, where τ_j represents the *jth* triggering instant, and for all $j \in \mathbb{N}$, there is $\tau_{j+1} > \tau_j$. Then, the sampled state vector is expressed as $s(\tau_j) = s_j, t \in [\tau_j, \tau_{j+1}), j \in \mathbb{N}$. In addition, the gap between the sampled state and the current state is called the event-triggered error, and the event-triggered error $e_j(t)$ is expressed by the following equation

$$e_j(t) = s_j - s(t), \quad \forall t \in [\tau_j, \tau_{j+1}). \tag{28}$$

By using (28), the optimal safety control law and the worst safety disturbance law can be expressed by $u(s_j) = u\big(s(t) + e_j(t)\big)$ and $d(s_j) = d\big(s(t) + e_j(t)\big)$, respectively. Meanwhile, under event-triggered framework, the transformed system (13) is rewritten by

$$\begin{aligned}\dot{s} &= F(s) + G(s)u\big(s(t) + e_j(t)\big) + K(s)d\big(s(t) + e_j(t)\big) \\ &= F(s) + G(s)u(s_j) + K(s)d(s_j), |u_i(t)| \le \kappa. \end{aligned} \tag{29}$$

Then, according to (22) and (23), the optimal event-triggered safety control law and the worst event-triggered safety disturbance law can be represented as

$$u^*(s_j) = -\kappa tanh\Big(\frac{1}{2\kappa}R^{-1}G^T(s_j)\nabla V^*(s_j)\Big) \tag{30}$$

$$d^*(s_j) = \frac{1}{2\gamma^2}K^T(s_j)\nabla V^*(s_j). \tag{31}$$

Substituting (30), (31) into (19), the safe event-triggered HJI equation is presented by

$$\begin{aligned}0 = &\int_0^{-\kappa tanh(\frac{1}{2\kappa}R^{-1}G^T(s_j)\nabla V^*(s_j))} 2\kappa tanh^{-1}(\frac{\xi}{\kappa})^T R d\xi \\ &+ \nabla V^{*T}(s)F(s) + s^TQs \\ &- \nabla V^{*T}(s)G(s)\kappa tanh\Big(\frac{1}{2\kappa}R^{-1}G^T(s_j)\nabla V^*(s_j)\Big) \\ &+ \frac{1}{2\gamma^2}\nabla V^{*T}(s)K(s)K^T(s_j)\nabla V^*(s_j) \\ &- \frac{1}{4\gamma^2}\nabla V^{*T}(s_j)K(s_j)K^T(s_j)\nabla V^*(s_j), \end{aligned} \tag{32}$$

$$0 = V^*(0). \tag{33}$$

Remark 3: In this paper, a zero-sum game problem of the nonlinear safety-critical system with input saturation is considered. It should be pointed out that the event-triggered mechanism is not only used to the control law, and the disturbance law is still based on the event-triggered mechanism.

Now, we define $D^*(s) = \frac{1}{2\kappa}R^{-1}G^T(s)\nabla V^*(s)$, then, a necessary assumption related to $D^*(s)$ is given as follows.

Assumption 2: $D^*(s)$ is Lipschitz continuous with respect to the event-triggered error e_j, and there exists following inequality

$$\big\|D^*(s_j) - D^*(s)\big\| \le K_u\|s_j - s\| = K_u\|e_j\|, \tag{34}$$

where K_u is a positive constant.

Theorem 1: For the transformed system (13)*, the optimal event-triggered safety control law and the worst event-triggered safety disturbance law are given by (30) and (31), respectively. When the closed-loop system 13 is asymptotically stable, the triggering condition is given as follows:*

$$\|e_j\|^2 \le \frac{(1-\eta^2)\lambda_{min}(Q)\|s\|^2 + Z(u^*(s_j)) - \gamma^2\|d^*(s_j)\|^2}{\kappa^2K_u^2\|R\|}, \tag{35}$$

where

$$Z(u^*(s_j)) = 2\kappa\int_0^{u^*(s_j)} tanh^{-1}\Big(\frac{\xi}{\kappa}\Big)^T R d\xi, \tag{36}$$

and $\eta \in (0, 1)$ is a user-defined design parameter, $\lambda_{min}(Q)$ is the minimal eigenvalue of Q.

Proof: Select $V^*(s)$ as the Lyapunov candidate function. With the optimal event-triggered safety control law (30) and the worst event-triggered safety disturbance law (31), we take the derivative of $V^*(s)$ along the trajectory of the event-triggered system (29), and we have

$$\begin{aligned}\dot{V}^*(s) &= \nabla V^{*T}(s)\big(F(s) + G(s)u^*(s_j) + K(s)d^*(s_j)\big) \\ &= \nabla V^{*T}(s)F(s) + \nabla V^{*T}(s)G(s)u^*(s_j) \\ &\quad + \nabla V^{*T}(s)K(s)d^*(s_j). \end{aligned} \tag{37}$$

According to (24), we can get

$$\begin{aligned}V^{*T}(s)F(s) = &-\int_0^{-\kappa tanh(D^*(s))} 2\kappa tanh^{-1}\Big(\frac{\xi}{\kappa}\Big)^T R d\xi \\ &- s^TQs + \nabla V^{*T}(s)G(s)\kappa tanh\big(D^*(s)\big) \\ &- \frac{1}{4\gamma^2}\nabla V^{*T}(s)K(s)K^T(s)\nabla V^*(s). \end{aligned} \tag{38}$$

In addition, from the optimal safety control law (22) and the worst safety disturbance law (23), we have

$$\nabla V^{*T}(s)G(s) = -2\kappa tanh^{-1}\Big(\frac{u^*(s)}{\kappa}\Big)^T R, \tag{39}$$

$$\nabla V^{*T}(s)K(s) = 2\gamma^2\big(d^*(s)\big)^T. \tag{40}$$

Substituting (38), (39) and (40) into (37), we have

$$\begin{aligned}\dot{V}^*(s) = &-s^TQs - \int_0^{-\kappa tanh(D^*(s))} 2\kappa tanh^{-1}(\frac{\xi}{\kappa})^T R d\xi \\ &+ \nabla V^{*T}(s)G(s)\kappa tanh\big(D^*(s)\big) \\ &- \frac{1}{4\gamma^2}\nabla V^{*T}(s)K(s)K^T(s)\nabla V^*(s) \\ &- 2\kappa tanh^{-1}\Big(\frac{u^*(s)}{\kappa}\Big)^T Ru^*(s_j) \\ &+ 2\gamma^2\big(d^*(s)\big)^T d^*(s_j). \end{aligned} \tag{41}$$

In addition, according to [31], [39], (15) can be expressed as

$$\begin{aligned}Z(u) &= 2\kappa\int_0^{u(s)} tanh^{-1}\Big(\frac{\xi}{\kappa}\Big)^T R d\xi \\ &= 2\kappa u^T R tanh^{-1}\Big(\frac{u}{\kappa}\Big) + \kappa^2\bar{R}ln\big[1 - \frac{u^2}{\kappa^2}\big], \end{aligned} \tag{42}$$

where $\bar{R} = [l_1, l_2, \dots, l_m] \in \mathbb{R}^{1\times m}$, $1 = [1, 1, \dots, 1]^T \in \mathbb{R}^m$. And substituting (42) into (41), we have

$$\begin{aligned}\dot{V}^*(s) = &-s^TQs - \kappa^2\bar{R}ln\big[1 - \frac{u^{*2}}{\kappa^2}\big] \\ &- 2\kappa tanh^{-1}\Big(\frac{u^*(s)}{\kappa}\Big)^T Ru^*(s_j) \\ &- \gamma^2\big(d^*(s)\big)^T d^*(s) + 2\gamma^2\big(d^*(s)\big)^T d^*(s_j). \end{aligned} \tag{43}$$

Then, according to (42), the second term in equation (43) can be rewritten as

$$\begin{aligned}\kappa^2\bar{R}ln\Big[1-\Big(\frac{u^{*2}}{\kappa^2}\Big)\Big] &= 2\kappa u^{*T}Rtanh^{-1}\Big(\frac{u^*}{\kappa}\Big) \\ &\quad +\kappa^2\bar{R}ln\Big[1-\frac{u^{*2}}{\kappa^2}\Big] \\ &\quad -2\kappa u^{*T}Rtanh^{-1}\Big(\frac{u^*}{\kappa}\Big) \\ &\quad -2\kappa\int_0^{u^*(s_j)} tanh^{-1}\Big(\frac{\xi}{\kappa}\Big)^T Rd\xi \\ &\quad +2\kappa\int_0^{u^*(s_j)} tanh^{-1}\Big(\frac{\xi}{\kappa}\Big)^T Rd\xi \\ &= 2\kappa\int_{u^*(s_j)}^{u^*(s)} tanh^{-1}\Big(\frac{\xi}{\kappa}\Big)^T Rd\xi \\ &\quad -2\kappa u^{*T}Rtanh^{-1}\Big(\frac{u^*}{\kappa}\Big) \\ &\quad +Z\big(u^*(s_j)\big). \end{aligned} \tag{44}$$

The third term in equation (43) becomes

$$\begin{aligned} &2\kappa tanh^{-1}\Big(\frac{u^*(s)}{\kappa}\Big)^T Ru^*(s_j) \\ &= \int_{u^*(s)}^{u^*(s_j)} 2\kappa D^{*T}(s)Rd\xi \\ &\quad -\nabla V^{*T}(s)G(s)\kappa tanh(D^*(s)). \end{aligned} \tag{45}$$

Substituting (44) and (45) into (43), we have

$$\begin{aligned}\dot{V}^*(s) &= -s^TQs - Z(u^*(s_j)) \\ &\quad +\int_{u^*(s)}^{u^*(s_j)} 2\kappa\Big[tanh^{-1}\Big(\frac{\xi}{\kappa}\Big)+D^*(s)\Big]^T Rd\xi \\ &\quad -\gamma^2\big(d^*(s)\big)^T d^*(s)+2\gamma^2\big(d^*(s)\big)^T d^*(s_j). \end{aligned} \tag{46}$$

Let $\xi = -\kappa tanh(\nu)$, and based on Assumption 1, then the third term in (46) can be expressed as

$$\begin{aligned} &\int_{u^*(s)}^{u^*(s_j)} 2\kappa\Big[tanh^{-1}\Big(\frac{\xi}{\kappa}\Big)+D^*(s)\Big]^T Rd\xi \\ &\le \int_{D^*(s)}^{D^*(s_j)} 2\kappa^2(\nu-D^*(s))^T Rd\nu \\ &= \kappa^2\big(D^*(s_j)-D^*(s)\big)^T R\big(D^*(s_j)-D^*(s)\big) \\ &\le \kappa^2K_u^2\|R\|\|e_j\|^2. \end{aligned} \tag{47}$$

Therefore, substituting (47) into (46), we can obtain

$$\begin{aligned}\dot{V}^*(s) &\le -s^TQs - Z(u^*(s_j)) + \kappa^2K_u^2\|R\|\|e_j\|^2 \\ &\quad +\gamma^2\big(d^*(s_j)\big)^T d^*(s_j) \\ &\le -\eta^2\lambda_{min}(Q)\|s\|^2+(\eta^2-1)\lambda_{min}(Q)\|s\|^2 \\ &\quad -Z(u^*(s_j)) \\ &\quad +\kappa^2K_u^2\|R\|\|e_j\|^2+\gamma^2\|d^*(s_j)\|. \end{aligned} \tag{48}$$

According to the above proof, we can know that if the triggering condition (35) is satisfied, then, for any $s(t) \neq 0$, there is $\dot{V}^*(s) \le -\eta^2\lambda_{min}(Q)\|s\|^2 < 0$. Therefore, according to Lyapunov theory, the closed-loop system (13) is asymptotically stable. The proof is completed. □

Remark 4: For the safe event-triggered HJI equation (32)*, due to its partial differential characteristics, its analytical solution is almost impossible to obtain. Therefore, in the next section, an ADP-based event-triggered safe control method is presented to solve the approximate optimal solution of the safe event-triggered HJI equation* (32).

IV. APPROXIMATE SOLUTION OF ADP-BASED EVENT-TRIGGERED SAFE CONTROL

In this section, an ADP-based event-triggered safe control method is proposed for the zero-sum game problem of safety-critical systems with input saturation. First, a critic NN is developed to approximate the optimal safety value function. Then, according to the Lyapunov theory, the stability of the nonlinear safety-critical system is analyzed. Finally, the Zeno behavior is excluded.

A. CRITIC NN DESIGN

According to the Weierstrass approximation theorem [40], the optimal safety value function $V^*(s)$ can be approximated by a critic NN as follows:

$$V^*(s) = W_c^T\phi(s)+\varepsilon(s), \tag{49}$$

where $W_c \in \mathbb{R}^n$ denotes the ideal weight vector, $\phi(s) \in \mathbb{R}^n$ denotes the activation function, n is the number of hidden neurons and $\varepsilon(s) \in \mathbb{R}$ is the reconstruction error of NN. The derivative of (49) with respect to s is

$$\nabla V^*(s) = \nabla\phi^T(s)W_c+\nabla\varepsilon(s), \tag{50}$$

where $\nabla V^*(s)$, $\nabla\phi(s)$ and $\nabla\varepsilon(s)$ mean the derivative of $V^*(s)$, $\phi(s)$ and $\varepsilon(s)$ with respect to s, respectively.

According to (30), (31) and (50), the optimal event-triggered safety control law and the worst event-triggered safety disturbance law can be written as

$$u^*(s_j) = -\kappa tanh\Big(\frac{1}{2\kappa}R^{-1}G^T(s_j)\big(\nabla\phi^T(s_j)W_c+\nabla\varepsilon(s_j)\big)\Big), \tag{51}$$

$$d^*(s_j) = \frac{1}{2\gamma^2}K^T(s_j)\big(\nabla\phi^T(s_j)W_c+\nabla\varepsilon(s_j)\big). \tag{52}$$

Because the ideal weight W_c is not known, the optimal event-triggered safety control law and the worst event-triggered safety disturbance law cannot be applied to the control process. Therefore, $V^*(s)$ is approximated as follows:

$$\hat{V}(s) = \hat{W}_c^T\phi(s), \tag{53}$$

where $\hat{W}_c \in \mathbb{R}^n$ is an approximation of the ideal weight W_c. The derivative of (53) with respect to s is

$$\nabla\hat{V}(s) = \nabla\phi^T(s)\hat{W}_c. \tag{54}$$

Then, the optimal event-triggered safety control law and the worst event-triggered safety disturbance law can be approximated as

$$\hat{u}(s_j) = -\kappa tanh\Big(\frac{1}{2\kappa}R^{-1}G^T(s_j)\nabla\phi^T(s_j)\hat{W}_c\Big), \tag{55}$$

$$\hat{d}(s_j) = \frac{1}{2\gamma^2}K^T(s_j)\nabla\phi^T(s_j)\hat{W}_c. \tag{56}$$

Substituting (55), (56) into (18), the approximate safe event-triggered Hamiltonian is rewritten as

$$\begin{aligned}\hat{H}\big(s, \hat{u}(s_j), \hat{d}(s_j), \hat{W}_c\big) &= r\big(s(t), \hat{u}(s_j), \hat{d}(s_j)\big) \\ &\quad + \hat{W}_c^T \nabla\phi(s)\big(F(s) + G(s)\hat{u}(s_j) \\ &\quad + K(s)\hat{d}(s_j)\big) \\ &= e_c, \end{aligned} \tag{57}$$

where $r\big(s(t), \hat{u}(s_j), \hat{d}(s)\big) = s^T Q s + Z\big(\hat{u}(s_j)\big) - \gamma^2 \hat{d}^T(s_j)\hat{d}(s_j)$. For the critic NN learning, it is desired to get $\hat{W}_c$ to minimize the objective function $E = (1/2)e_c^T e_c$ through the gradient descent method [41]. Therefore, the weight vector of critic NN is updated with

$$\begin{aligned}\dot{\hat{W}}_c &= -a_c \frac{\sigma}{(\sigma^T\sigma + 1)^2} e_c^T \\ &= -a_c \frac{\sigma}{(\sigma^T\sigma + 1)^2}\big[r(s(t), \hat{u}(s_j), \hat{d}(s_j)) + \sigma^T \hat{W}_c\big],\end{aligned} \tag{58}$$

where $\sigma = \nabla\phi(s)\big[F(s) + G(s)\hat{u}(s_j) + K(s)\hat{d}(s_j)\big]$ and $a_c > 0$ is the learning rate. Let the weight estimation error as $\tilde{W}_c = W_c - \hat{W}_c$, and substituting $\tilde{W}_c$ into (58), we have

$$\dot{\tilde{W}}_c = -a_c \frac{\sigma\sigma^T}{(\sigma^T\sigma + 1)^2}\tilde{W}_c + a_c \frac{\sigma}{(\sigma^T\sigma + 1)^2} e_H. \tag{59}$$

where $e_H = -\nabla\varepsilon^T(s)[F(s) + G(s)\hat{u}(s_j) + K(s)\hat{d}(s_j)]$.

However, for the update rule (58), according to [42]–[45], we know that the PE condition of $\sigma/(\sigma^T\sigma + 1)$ is required to ensure convergence of $\hat{W}_c$ to W_c. Nevertheless, due to the existence of hidden-layers in term $\sigma/(\sigma^T\sigma + 1)$, the PE condition is difficult to verify. Besides, to obtain the persistence of the excitation signal, a common way is to add exploration noise, which may cause the system to violate safety constraints, or even instability. This creates obstacles to the learning of critic NN.

Therefore, to overcome the above problem, the concurrent learning method is adopted [42]–[45], the critic NN weights can be updated by past data and current data. Define the residual equation error at time t_k as

$$e_c(t_k) = r\big(s(t_k), \hat{u}(s_j), \hat{d}(s_j)\big) + \hat{W}_c^T \sigma_k, \tag{60}$$

where $k \in \{1, 2 \ldots, l\}$ is the index of a past data $s(t_k)$, $t_k \in [\tau_j, \tau_{j+1})$, $j \in \mathbb{N}$, and l is the number of past samples. Then, $r\big(s(t_k), \hat{u}(s_j), \hat{d}(s_j)\big) = s(t_k)^T Q s(t_k) + Z\big(\hat{u}(s_j)\big) - \gamma^2 \hat{d}^T(s_j)\hat{d}(s_j)$, $\sigma_k = \nabla\phi\big(s(t_k)\big)\big[F(s(t_k)) + G(s(t_k))\hat{u}(s_j) + K(s(t_k))\hat{d}(s_j)\big]$, and $e_H(t_k) = -\nabla\varepsilon^T(s(t_k))[F(s(t_k)) + G(s(t_k))\hat{u}(s_j) + K(s(t_k))\hat{d}(s_j)]$. Similarly, for the critic NN learning, we have defined a new objective function as follow:

$$E_k = \frac{1}{2}e_c^T e_c + \frac{1}{2}\sum_{k=1}^{l} e_c^T(t_k)e_c(t_k). \tag{61}$$

Therefore, a novel weight update rule for the critic NN is given by

$$\dot{\hat{W}}_c = -a_c \frac{\sigma}{(\sigma^T\sigma + 1)^2} e_c - a_c \sum_{k=1}^{l} \frac{\sigma(t_k)e_c(t_k)}{\big(\sigma^T(t_k)\sigma(t_k) + 1\big)^2}. \tag{62}$$

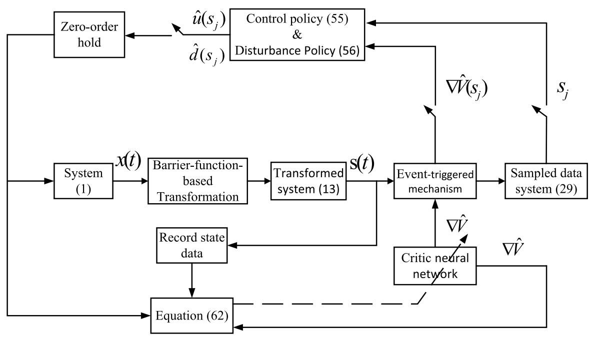

FIGURE 1. The flowchart of the ADP-based event-triggered safe control method.

Substituting the weight estimation error $\tilde{W}_c = W_c - \hat{W}_c$ into (62), we have

$$\begin{aligned}\dot{\tilde{W}}_c &= -a_c\big[\frac{\sigma\sigma^T}{(\sigma^T\sigma + 1)^2} + S_k\big]\tilde{W}_c \\ &\quad + a_c\big[\frac{\sigma}{(\sigma^T\sigma + 1)^2}e_H + S_p e_H(t_k)\big],\end{aligned} \tag{63}$$

where $S_k = \sum_{k=1}^{l} \frac{\sigma(t_k)\sigma(t_k)^T}{\big(\sigma^T(t_k)\sigma(t_k)+1\big)^2}$, and $S_p = \sum_{k=1}^{l} \frac{\sigma(t_k)}{\big(\sigma^T(t_k)\sigma(t_k)+1\big)^2}$.

Remark 5: In this paper, the flowchart of the ADP-base event-triggered safe control method is shown in Figure 1. It should be noted that both the approximate optimal safety control law and the approximate worst safety disturbance law are updated based on the event-triggered mechanism. This improves the utilization efficiency of communication resources and greatly reduces the amount of calculation.

Remark 6: In the update rule (62) *of the critic NN, the effect of the first term is similar to that of* (58)*, that is, it is used to minimize the objective function. On the other hand, since the traditional PE condition is difficult to verify, we adopt the method of concurrent learning, in which current data and past data are used to update the weights of the critic NN. That is the effect of the second term in* (62).

B. STABILITY ANALYSIS

Next, the main theorem of this paper is given, in which the stability of the system is analysed by using Lyapunov theory. Before this, we need the following necessary assumptions [30]–[32], [36], [37].

Assumption 3: The derivative of the critic NN activation function and the control dynamics $G(s)$ are Lipschitz continuous, i.e., $\|\nabla\phi(s) - \nabla\phi(s_j)\| \le B_\phi\|e_j(t)\|$, $\|G(s) - G(s_j)\| \le A_g\|e(s_j)\|$, where B_ϕ and A_g are positive constants.

Assumption 4: There are known positive constants b_{e_c}, b_ε, and l_ϕ, have $\|e_H\| \le b_{e_c}$, $\|\nabla\varepsilon(s)\| \le b_\varepsilon$, and $\|\nabla\phi(s)\| \le l_\phi$.

Theorem 2: Suppose that Assumptions 1, 2, 3, 4 are satisfied. Consider the transformed system (13)*, the weights of critic NN are updated by* (62)*, the approximate optimal event-triggered safety control law $\hat{u}(s_j)$ is given by* (55)*, and the approximate worst event-triggered safety disturbance law*

$\hat{d}(s_j)$ is given by (56). Then, the system is asymptotically stable, and the critic NN weight estimation error $\tilde{W}_c$ is guaranteed to be UUB, if the condition

$$\|e_j\|^2 \leq \frac{(1-\eta^2)\lambda_{min}(Q)\|s\|^2 + Z(\hat{u}(s_j)) - \gamma^2\|\hat{d}(s_j)\|}{\Lambda^2\|R\|^{-1}\|\hat{W}_c\|^2} \equiv \|e_T\|^2 \tag{64}$$

and the inequality of the weight estimation error $\tilde{W}_c$

$$\|\tilde{W}_c\| > \sqrt{\frac{2\|R\|^{-1}g_m^2 b_\varepsilon^2 + \sum_{k=1}^{l+1} b_{e_c}^2}{\lambda_{min}[\psi(\omega,\omega(t_k))] - 2\|R\|^{-1}g_m^2 l_\phi^2}} \tag{65}$$

are satisfied, where $\Lambda^2 = A_g^2 l_\phi^2 + B_\phi^2 g_m^2$, and $\lambda_{min}[\psi(\omega,\omega(t_k))]$ denotes the minimal eigenvalue of $\psi(\omega,\omega(t_k))$.

Proof: The Lyapunov candidate function is given by,

$$L(t) = L_1(t) + L_2(t) + L_3(t), \tag{66}$$

where $L_1(t) = V^*(s(t))$, $L_2(t) = V^*(s_j)$ and $L_3(t) = (1/2)\tilde{W}_c^T a_c^{-1}\tilde{W}_c$. The stability analysis includes the following two cases.

Case 1: When $t \in [\tau_j, \tau_{j+1})$, events are not triggered. The derivative of (66) can be obtain as follows,

$$\begin{cases} \dot{L}_1(t) = (\nabla V^*(s))^T(F(s) + G(s)\hat{u}(s_j) + K(s)\hat{d}(s_j)), \\ \dot{L}_2(t) = 0, \\ \dot{L}_3(t) = \tilde{W}_c^T a^{-1}\dot{\tilde{W}}_c. \end{cases} \tag{67}$$

Substituting (38), (39) and (40) into $\dot{L}_1(t)$, we can obtain

$$\begin{aligned} \dot{L}_1(t) &= \nabla V^{*T}(s)F(s) + \nabla V^{*T}(s)G(s)\hat{u}(s_j) \\ &\quad + \nabla V^{*T}(s)K(s)\hat{d}(s_j) \\ &= -s^TQs - Z(\hat{u}(s_j)) \\ &\quad + \int_{u^*(s)}^{\hat{u}(s_j)} 2\kappa\Big[tanh^{-1}\Big(\frac{\xi}{\kappa}\Big) + D^*(s)\Big]^T R d\xi \\ &\quad - \gamma^2\big(d^*(s)\big)^T d^*(s) + 2\gamma^2\big(d^*(s)\big)^T\hat{d}(s_j). \end{aligned} \tag{68}$$

For the third term in (68), according to Assumption 1 and (47), we can get

$$\begin{aligned} &\int_{u^*(s)}^{\hat{u}(s_j)} 2\kappa\Big[tanh^{-1}\Big(\frac{\xi}{\kappa}\Big) + D^*(s)\Big]^T R d\xi \\ &\leq \int_{D^*(s)}^{\hat{D}(s_j)} 2\kappa(\nu - D^*(s))^T R d\nu \\ &= \kappa^2\big(\hat{D}(s_j) - D^*(s)\big)^T R\big(\hat{D}(s_j) - D^*(s)\big) \\ &\leq \kappa^2\|R\|\|D^*(s) - \hat{D}(s_j)\|^2, \end{aligned} \tag{69}$$

where $\hat{D}(s_j) = \frac{1}{2\kappa}R^{-1}G^T(s_j)\nabla\phi^T(s_j)\hat{W}_c$, and substituting (50) into $D^*(s)$, we have

$$D^*(s) = \frac{1}{2\kappa}R^{-1}G^T(s_j)\big(\nabla\phi^T(s_j)W_c + \nabla\varepsilon(s)\big). \tag{70}$$

According to (70), $\hat{D}(s_j)$ and the weight estimation error $\tilde{W}_c = W_c - \hat{W}_c$, we can obtain

$$\begin{aligned} &\|D^*(s) - \hat{D}(s_j)\|^2 \\ &= \Big\|\frac{1}{2\kappa}R^{-1}G^T(s)\big[\nabla\phi^T(s)(\tilde{W}_c + \hat{W}_c) + \nabla\varepsilon(s)\big] \\ &\quad - \frac{1}{2\kappa}R^{-1}G^T(s_j)\nabla\phi^T(s_j)\hat{W}_c\Big\|^2 \\ &= \frac{1}{4\kappa^2\|R\|^2}\Big\|G^T(s)\big(\nabla\phi^T(s)\tilde{W}_c + \nabla\varepsilon(s)\big) \\ &\quad + \big[G^T(s)\nabla\phi^T(s) - G^T(s_j)\nabla\phi^T(s_j)\big]\hat{W}_c\Big\|^2. \end{aligned} \tag{71}$$

According to Assumptions 2, 3, 4, and the Young's inequality $\|c+d\|^2 \leq 2\|c\|^2 + 2\|d\|^2$, we have

$$\begin{aligned} &\|G^T(s)\nabla\phi^T(s) - G^T(s_j)\nabla\phi^T(s_j)\|^2 \\ &= \Big\|\big[\nabla\phi(s)\big(G(s) - G(s_j)\big)\big]^T \\ &\quad + \big[\big(\nabla\phi(s) - \nabla\phi(s_j)\big)G(s_j)\big]^T\Big\|^2 \\ &\leq 2\big\|\nabla\phi(s)\big(G(s) - G(s_j)\big)\big\|^2 \\ &\quad + 2\big\|\big(\nabla\phi(s) - \nabla\phi(s_j)\big)G(s_j)\big\|^2 \\ &\leq 2(A_g^2 l_\phi^2 + B_\phi^2 g_m^2)\|e_j(t)\|^2. \end{aligned} \tag{72}$$

Substituting (72) into (71), we have

$$\begin{aligned} &\|D^*(s) - \hat{D}(s_j)\|^2 \\ &\leq \frac{1}{\kappa^2\|R\|^2}\|G^T(s)\nabla\phi^T(s)\tilde{W}_c\|^2 \\ &\quad + \frac{1}{\kappa^2\|R\|^2}\|G^T(s)\nabla\varepsilon(s)\|^2 \\ &\quad + \frac{1}{\kappa^2\|R\|^2}(A_g^2 l_\phi^2 + B_\phi^2 g_m^2)\|e_j(t)\|^2\|\hat{W}_c\|^2 \\ &\leq \frac{1}{\kappa^2\|R\|^2}g_m^2 l_\phi^2\|\tilde{W}_c\|^2 + \frac{1}{\kappa^2\|R\|^2}g_m^2 b_\varepsilon^2 \\ &\quad + \frac{1}{\kappa^2\|R\|^2}(A_g^2 l_\phi^2 + B_\phi^2 g_m^2)\|e_j(t)\|^2\|\hat{W}_c\|^2. \end{aligned} \tag{73}$$

Therefore, substituting (73) into (69), we can get

$$\begin{aligned} &\int_{u^*(s)}^{\hat{u}(s_j)} 2\kappa\Big[tanh^{-1}\Big(\frac{\xi}{\kappa}\Big) + D^*(s)\Big]^T R d\xi \\ &\leq \kappa^2\|R\|\|D^*(s) - \hat{D}(s_j)\|^2 \\ &\leq \|R\|^{-1}g_m^2 l_\phi^2\|\tilde{W}_c\|^2 + \|R\|^{-1}g_m^2 b_\varepsilon^2 \\ &\quad + \|R\|^{-1}(A_g^2 l_\phi^2 + B_\phi^2 g_m^2)\|e_j(t)\|^2\|\hat{W}_c\|^2. \end{aligned} \tag{74}$$

Similarly, for the last two terms in (68), using the inequality $2cd \leq c^2 + d^2$, we have

$$-\gamma^2\big(d^*(s)\big)^T d^*(s) + 2\gamma^2\big(d^*(s)\big)^T\hat{d}(s_j) \leq \gamma^2\hat{d}^T(s_j)\hat{d}(s_j). \tag{75}$$

Finally, substituting (74) and (75) into $\dot{L}_1(t)$, we have

$$\begin{aligned} \dot{L}_1(t) &\leq -s^TQs - Z\big(\hat{u}(s_j)\big) + \gamma^2\hat{d}^T(s_j)\hat{d}(s_j) \\ &\quad + \|R\|^{-1}g_m^2 l_\phi^2\|\tilde{W}_c\|^2 + \|R\|^{-1}g_m^2 b_\varepsilon^2 \\ &\quad + \|R\|^{-1}(A_g^2 l_\phi^2 + B_\phi^2 g_m^2)\|e_j(t)\|^2\|\hat{W}_c\|^2 \end{aligned}$$

$$\leq -\eta^2\lambda_{min}(Q)\|s\|^2 + (\eta^2-1)\lambda_{min}(Q)\|s\|^2 - Z(\hat{u}(s_j)) + \gamma^2\|\hat{d}(s_j)\|^2 + \|R\|^{-1}g_m^2 l_\phi^2\|\tilde{W}_c\|^2 + \|R\|^{-1}g_m^2 b_\varepsilon^2 \quad (76)$$
$$+ \|R\|^{-1}(A_g^2 l_\phi^2 + B_\phi^2 g_m^2)\|e_j(t)\|^2\|\hat{W}_c\|^2. \quad (77)$$

On the other side, substituting (63) into $\dot{L}_3$, we can obtain

$$\dot{L}_3(t) = \tilde{W}_c^T a_c^{-1}\dot{\tilde{W}}_c = -\tilde{W}_c^T[\frac{\sigma\sigma^T}{(\sigma^T\sigma+1)^2} + S_k]\tilde{W}_c + \tilde{W}_c^T[\frac{\sigma}{(\sigma^T\sigma+1)^2}e_H + S_p e_H(t_k)]. \quad (78)$$

Next, we define $\omega = \frac{\sigma}{(\sigma^T\sigma+1)}$, $\varpi = \frac{\sigma}{(\sigma^T\sigma+1)^2}$, $\omega(t_k) = \frac{\sigma(t_k)}{(\sigma(t_k)^T\sigma(t_k)+1)}$, and $\varpi(t_k) = \frac{\sigma(t_k)}{(\sigma^T(t_k)\sigma(t_k)+1)^2}$. Then, $\dot{L}_3$ can be written as

$$\dot{L}_3(t) = -\tilde{W}_c^T[\omega\omega^T + S_k]\tilde{W}_c + \tilde{W}_c^T\varpi e_H + \tilde{W}_c^T S_p e_H(t_k). \quad (79)$$

According to the Young's inequality $a^T b \leq \frac{a^T a}{2} + \frac{b^T b}{2}$, the second term of (79) is rewritten as

$$\tilde{W}_c^T\varpi e_H \leq \frac{1}{2}\tilde{W}_c^T\varpi\varpi^T\tilde{W}_c + \frac{1}{2}e_H^T e_H \leq \frac{1}{2}\tilde{W}_c^T\omega\omega^T\tilde{W}_c + \frac{1}{2}e_H^T e_H. \quad (80)$$

Similarly, for the third term of (79), we have

$$\tilde{W}_c^T S_p e_H(t_k) \leq \frac{1}{2}\tilde{W}_c^T\sum_{k=1}^{l}\varpi(t_k)\varpi^T(t_k)\tilde{W}_c + \frac{1}{2}\sum_{k=1}^{l}e_H(t_k)^T e_H(t_k) \leq \frac{1}{2}\tilde{W}_c^T\sum_{k=1}^{l}\omega(t_k)\omega^T(t_k)\tilde{W}_c + \frac{1}{2}\sum_{k=1}^{l}e_H(t_k)^T e_H(t_k). \quad (81)$$

Substituting (80) and (81) into (79), we have

$$\dot{L}_3(t) \leq -\tilde{W}_c^T[\omega\omega^T + S_k]\tilde{W}_c + \frac{1}{2}\tilde{W}_c^T\omega\omega^T\tilde{W}_c + \frac{1}{2}e_H^T e_H + \frac{1}{2}\tilde{W}_c^T\sum_{k=1}^{l}\omega(t_k)\omega^T(t_k)\tilde{W}_c + \frac{1}{2}\sum_{k=1}^{l}e_H(t_k)^T e_H(t_k)$$
$$\leq -\tilde{W}_c^T[\omega\omega^T + S_k]\tilde{W}_c + \frac{1}{2}\tilde{W}_c^T[\omega\omega^T + S_k]\tilde{W}_c + \frac{1}{2}[\sum_{k=1}^{l}e_H(t_k)^T e_H(t_k) + e_H^T e_H]$$
$$\leq -\frac{1}{2}\tilde{W}_c^T[\omega\omega^T + \sum_{k=1}^{l}\omega(t_k)\omega^T(t_k)]\tilde{W}_c + \frac{1}{2}\sum_{k=1}^{l+1}b_{e_c}^2$$
$$\leq -\frac{1}{2}\lambda_{min}[\psi(\omega,\omega(t_k))]\|\tilde{W}_c\|^2 + \frac{1}{2}\sum_{k=1}^{l+1}b_{e_c}^2, \quad (82)$$

where $\psi(\omega,\omega(t_k)) = \omega\omega^T + \sum_{k=1}^{l}\omega(t_k)\omega^T(t_k)$, and $\lambda_{min}[\psi(\omega,\omega(t_k))]$ denotes the minimal eigenvalue of $\psi(\omega,\omega(t_k))$.

Finally, substituting $\dot{L}_1(t)$, $\dot{L}_2(t)$ and $\dot{L}_3(t)$ into $\dot{L}(t)$, we can get

$$\dot{L}(t) \leq -\eta^2\lambda_{min}(Q)\|s\|^2 + (\eta^2-1)\lambda_{min}(Q)\|s\|^2 - Z(\hat{u}(s_j)) + \gamma^2\|\hat{d}(s_j)\|^2 + \|R\|^{-1}g_m^2 l_\phi^2\|\tilde{W}_c\|^2 + \|R\|^{-1}(A_g^2 l_\phi^2 + B_\phi^2 g_m^2)\|e_j(t)\|^2\|\hat{W}_c\|^2 + \|R\|^{-1}g_m^2 b_\varepsilon^2 - \frac{1}{2}\lambda_{min}[\psi(\omega,\omega(t_k))]\|\tilde{W}_c\|^2 + \frac{1}{2}\sum_{k=1}^{l+1}b_{e_c}^2. \quad (83)$$

Therefore, if conditions (64) and (65) are satisfied, we can easily get $\dot{L}(t) \leq -\eta^2\lambda_{min}(Q)\|s\|^2 < 0$ for all $s(t) \neq 0$.

Case 2: When $t = \tau_{j+1}$, events are triggered. Consider the difference operation of $L(t)$, we have

$$\Delta L(t) = L(s_{j+1}) - L(s(\tau_{j+1}^-)) = \Delta L_1 + \Delta L_2 + \Delta L_3. \quad (84)$$

In case 1, when events are not triggered, we can know that $\dot{L}(t) < 0$ for all $t \in [\tau_j, \tau_{j+1})$. Here, $s(\tau_{j+1}^-)$ is given by the definition of the derivative, namely, $s(t_{j+1}^-) = \lim_{\Delta t\to 0} s(\tau_{j+1} - \Delta t)$. Then, the following conclusions can be obtained,

$$\Delta L_1(t) = V^*(s_{j+1}) - V^*(s(\tau_{j+1}^-)) \leq 0, \quad (85)$$
$$\Delta L_2(t) = V^*(s_{j+1}) - V^*(s_j), \quad (86)$$
$$\Delta L_3(t) = \frac{1}{2}\big[\tilde{W}_c^T(s_{j+1})a_c^{-1}\tilde{W}_c(s_{j+1}) - \tilde{W}_c^T(s(\tau_{j+1}^-))a_c^{-1}\tilde{W}_c(s(\tau_{j+1}^-))\big] \leq 0. \quad (87)$$

Substituting $\Delta L_1(t)$, $\Delta L_2(t)$ and $\Delta L_3(t)$ into (84), we can obtain

$$\Delta L(t) = V^*(s_{j+1}) - V^*(s(\tau_{j+1}^-)) + V^*(s_{j+1}) - V^*(s_j) + \frac{1}{2}[\tilde{W}_c^T(s_{j+1})a_c^{-1}\tilde{W}_c(s_{j+1}) - \tilde{W}_c^T(s(\tau+1_{j+1}^-))a_c^{-1}\tilde{W}_c(s(\tau_{j+1}^-))]$$
$$\leq V^*(s_{j+1}) - V^*(s_j) \leq -\mathcal{K}(\|e_{j+1}(t_j)\|). \quad (88)$$

where $\mathcal{K}(\cdot)$ denotes a class $\mathcal{K}$ function, and $e_{j+1}(\tau_j) = s_{j+1} - s_j$. According to the characters of the class $\mathcal{K}$ function, the $\Delta L(t)$ is continuously decreasing for all $t = \tau_{j+1}$.

Based on the proof of above two cases, it can be obtained when the condition (64) and the inequality related to $\tilde{W}_c$

(65) are satisfied, the closed-loop system is asymptotically stable and the weight estimation error of the critic NN $\tilde{W}_c$ is guaranteed to be UUB. The proof is completed. □

Remark 7: Due to the weight estimation error $\tilde{W}_c$ in the NN approximation process, then, two different event-triggered conditions are obtained, i.e., the event-triggered condition (35) *in Theorem 1 and the event-triggered condition* (64) *in Theorem 2. In the implementation process, it is more appropriate to use the condition* (62)*, where* Λ *is obtained through many experiments.*

C. MINIMUM SAMPLING PERIOD ANALYSIS

For the transformed system (13) with the optimal event-triggered safety control law (22) and the worst event-triggered safety disturbance law (23), if there exists the minimal intersampling time $\Delta t_{min} = min_{j\in\mathbb{N}}\{\tau_{j+1} - \tau_j\} = 0$, which will cause the Zeno behavior to occur [30], [31]. Then, the following Theorem 3 will be given to ensure that Zeno behavior is excluded.

Theorem 3: Consider the event-triggered condition is given by (35)*, and the sampled data system* (29)*. Suppose that Assumptions 1, 4 are satisfied. Then, the minimal intersampling time* $\Delta t_{min} = min_{j\in\mathbb{N}}\{\tau_{j+1} - \tau_j\}$ *has a lower bound that is not zero, that is,*

$$\Delta t_{min} \geq \frac{1}{f_m} ln\Big(1 + \frac{\|e_j\|}{D_m}\Big) > 0, \tag{89}$$

where f_m *and* $D_m = \frac{f_m\|s_j\|+D}{f_m}$ *are positive constants.*

Proof: According to (28), the derivative of the event-triggered error $e_j(t)$ with respect to t is

$$\frac{de_j(t)}{dt} = \dot{e}_j(t) = \dot{s}_j - \dot{s} = -\dot{s}, \quad \forall t \in [\tau_j, \tau_{j+1}). \tag{90}$$

By using the approximate optimal event-triggered safety control law (55), and the approximate worst event-triggered safety disturbance law (56), we have

$$\begin{aligned}\|\dot{e}_j\| &= \|\dot{s}_j - \dot{s}\| = \|\dot{s}\| \\ &= \big\|F(s) + G(s)\hat{u}(s_j) + K(s)\hat{d}(s_j)\big\| \\ &\leq \|F(s)\| + \|G(s)\hat{u}(s_j)\| \\ &\quad + \|\frac{1}{2\gamma^2}K(s)K^T(s_j)\nabla\phi^T(s_j)\hat{W}_c\|. \end{aligned}\tag{91}$$

Since the control input satisfies $u_i(t) \leq \kappa$, then, we can deduce $\hat{u}(s_j) \leq \kappa$. In addition, based on Assumptions 1, 4, we can get

$$\begin{aligned}\|\dot{e}_j\| &\leq \|F(s)\| + \|G(s)\hat{u}(s_j)\| \\ &\quad + \|\frac{1}{2\gamma^2}K(s)K^T(s_j)\nabla\phi^T(s_j)\hat{W}_c\| \\ &\leq f_m\|s\| + g_m\kappa + \frac{1}{2\gamma^2}k_m^2 l_\phi\|\hat{W}_c\| \\ &\leq f_m\|s\| + D \\ &\leq f_m\|s_j - e_j\| + D \\ &\leq f_m\|s_j\| + f_m\|e_j\| + D, \end{aligned}\tag{92}$$

where $D = g_m\kappa + \frac{1}{2\gamma^2}k_m^2 l_\phi\|\hat{W}_c\|$.

TABLE 1. Parameters of the single link robot arm.

Parameter	Description	Value
M	Mass of the payload	10kg
g	Acceleration of gravity	$9.81m/s^2$
l	Length of the arm	0.5m
$\tilde{D}$	Viscous friction	2N
$\tilde{G}$	Moment of inertia	$10\text{kg}\cdot m^2$
κ	Input boundary	$\|u\| \leq \kappa = 7$

According to [30], [31], we can get the following inequality from (92)

$$\|e_j\| \leq \frac{f_m\|s_j\| + D}{f_m}\Big(e^{f_m(t-\tau_j)} - 1\Big) \tag{93}$$

for all $t \in [\tau_j, \tau_{j+1})$. Then, we can obtain that the jth intersampling time satisfies

$$\tau_{j+1} - \tau_j \geq \frac{1}{f_m} ln\Big(1 + \frac{\|e_j\|}{D_m}\Big) > 0, \tag{94}$$

where $D_m = \frac{f_m\|s_j\|+D}{f_m}$. To sum up, the intersampling time $\Delta t_{min} > 0$. Consequently, the Zeno behavior is excluded. The proof is finished. □

V. SIMULATION

In this section, a single link robot arm system simulation example is given to demonstrate the effectiveness of the proposed method. Consider the single link robotic arm system represented by

$$\ddot{\theta}(t) = -\frac{Mgl}{\tilde{G}} sin(\theta(t)) - \frac{\tilde{D}}{\tilde{G}}\dot{\theta}(t) + \frac{1}{\tilde{G}}u(t) + kd(t), \tag{95}$$

where $\theta(t)$ and $\dot{\theta}(t)$ represent angle position and angle velocity, respectively. Other parameters of the single link robotic arm system are shown in the Table 1. In addition, let $x = [x_1, x_2]^T = [\theta, \dot{\theta}]^T$, and the initial state is selected as $x_0 = \big[x_0(1), x_0(2)\big]^T = [2, 2]^T$. Then, (95) can be written as

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix} x_2 \\ -\frac{Mgl}{\tilde{G}} sin(x_1) - \frac{\tilde{D}}{\tilde{G}}x_2\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{\tilde{G}}\end{bmatrix}u + \begin{bmatrix}0\\ k\end{bmatrix}d.$$

In addition, the states of the single link robot arm system satisfy the following safety constraints,

$$x_1 \in (-1.5, 3), \quad x_2 \in (-3, 3). \tag{96}$$

Therefore, in order to deal with safety constraints, the barrier-function-based system transformation (9) is used to obtain the following transformed system without safety constraints,

$$\dot{s} = F(s) + G(s)u + K(s)d, \tag{97}$$

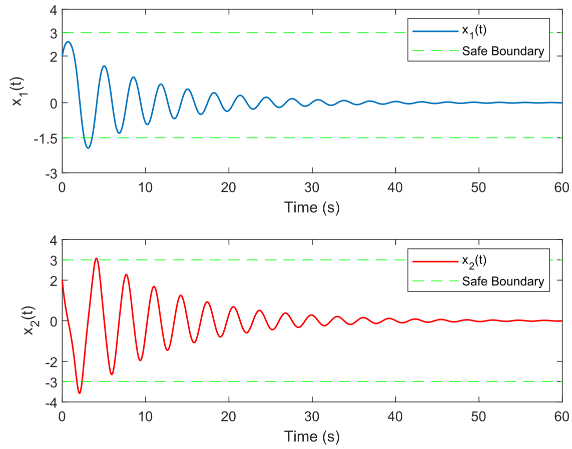

FIGURE 2. **Evolution of the state $x(t)$ without using the proposed method in this paper.**

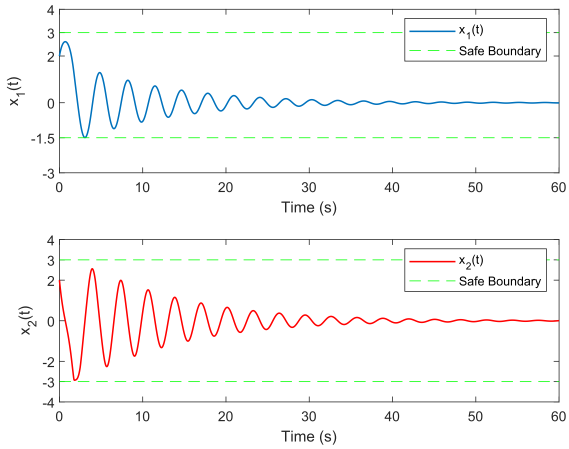

FIGURE 3. **Evolution of the state $x(t)$ by using the proposed method in this paper.**

where

$$F(s) = \begin{bmatrix} \frac{a_2A_2\left(e^{\frac{s_2}{2}}-e^{-\frac{s_2}{2}}\right)}{a_2e^{\frac{s_2}{2}}-A_2e^{-\frac{s_2}{2}}} \frac{a_1^2e^{s_1}-2a_1A_1+A_1e^{-s_1}}{A_1a_1^2-a_1A_1^2} \\ f\left(B^{-1}(s)\right)\frac{a_2^2e^{s_2}-2a_2A_2+A_2e^{-s_2}}{A_2a_2^2-a_2A_2^2} \end{bmatrix},$$

$$G(s) = \begin{bmatrix} 0 \\ \frac{1}{\bar{G}}\frac{a_2^2e^{s_2}-2a_2A_2+A_2e^{-s_2}}{A_2a_2^2-a_2A_2^2} \end{bmatrix},$$

$$K(s) = \begin{bmatrix} 0 \\ k\frac{a_2^2e^{s_2}-2a_2A_2+A_2e^{-s_2}}{A_2a_2^2-a_2A_2^2} \end{bmatrix}. \tag{98}$$

For the transformed single link robot arm system (97), the initial state is selected as $s_0 = \left[s_0(1), s_0(2)\right]^T = \left[B\left(x_0(1); a_1, A_1\right), B\left(x_0(2); a_2, A_2\right)\right]^T$, and the learning rate of the critic NN is $a_c = 0.1$. The important parameters of triggering condition are chosen as $R = 1, Q = 2I, \gamma = 4$, and $|u| \leq \kappa = 7$ is the input boundary. On the other hand, the activation function of the critic NN is given by $\phi(s) = \left[\, s_1^2 \;\; s_1s_2 \;\; s_2^2 \,\right]^T$, and the sampling time is set to 0.05s.

The simulation results of the proposed method are displayed in Figures 2-10. In Figure 2, by using the classical event-triggered control method, the system states can quickly converge to zero, however, the system states cannot satisfy the given safety constraints. Compared with Figure 2, in Figure 3, the event-triggered safe control method can ensure that the system states quickly converge to zero, while still satisfying the given safety constraints. In Figure 4, based on the event-triggered safe control method, the evolution process of the state $s(t)$ is given. Figure 5 depicts that the convergence process of the critic NN weights, and finally converges to

$$\hat{W}_c = \left[\, -3.3073 \quad 1.6670 \quad -2.0424 \,\right]^T. \tag{99}$$

In Figure 6 and Figure 7, we can see the convergence process of the optimal event-triggered control law and the worst event-triggered disturbance law, respectively.

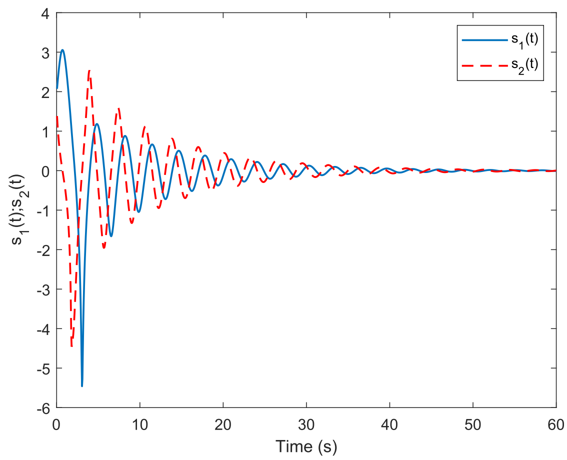

FIGURE 4. **Evolution of the state $s(t)$ by using the proposed method in this paper.**

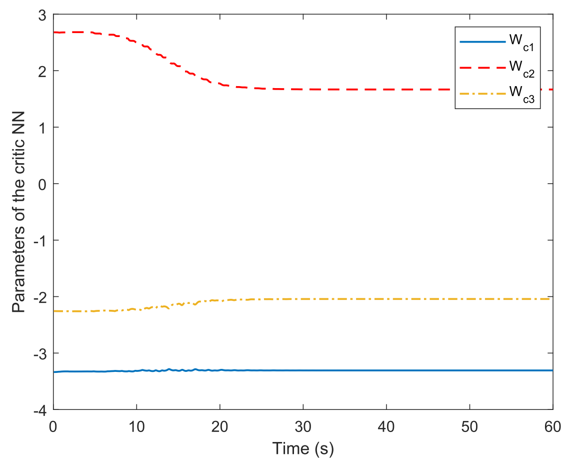

FIGURE 5. **Convergence of the critic NN weights.**

The evolution process of the event-triggered threshold e_T and the event-triggered condition $e_j(t)$ is displayed in Figure 8. As shown in Figure 9, the sampling period is given, and the

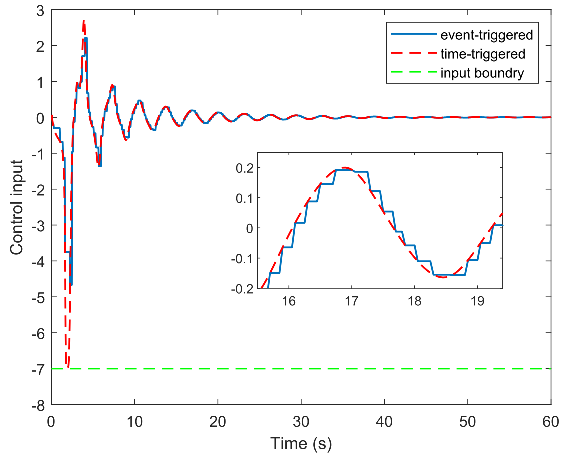

FIGURE 6. Comparisons of the evolution disturbance input by the event-triggered and the time-triggered.

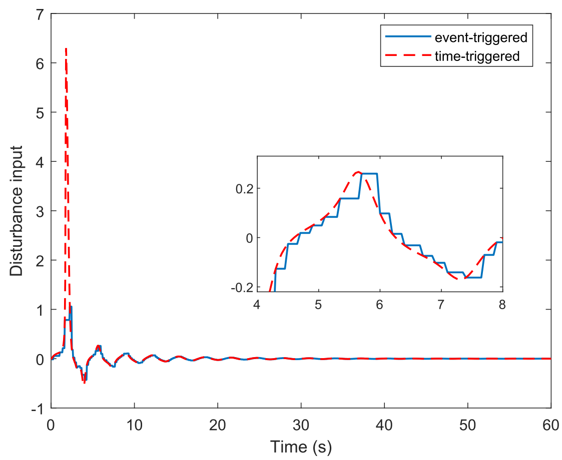

FIGURE 7. Comparisons of the evolution disturbance input by the event-triggered and the time-triggered.

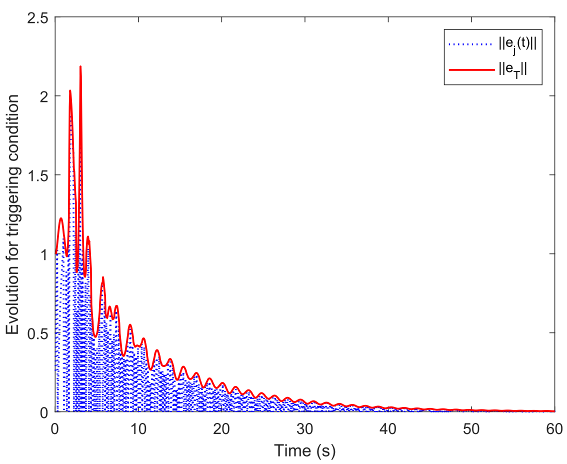

FIGURE 8. Development of the event-triggering condition.

minimum sampling period is 0.15s. Finally, in Figure 10, the result means that the event-triggered safe control method only uses 260 samples, which is in sharp contrast to the time-

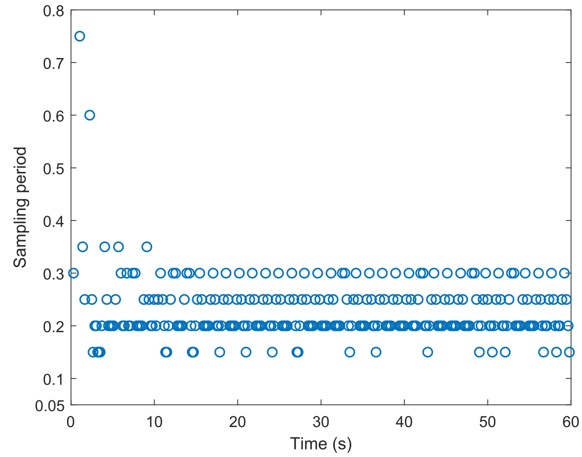

FIGURE 9. Sampling period during the learning process.

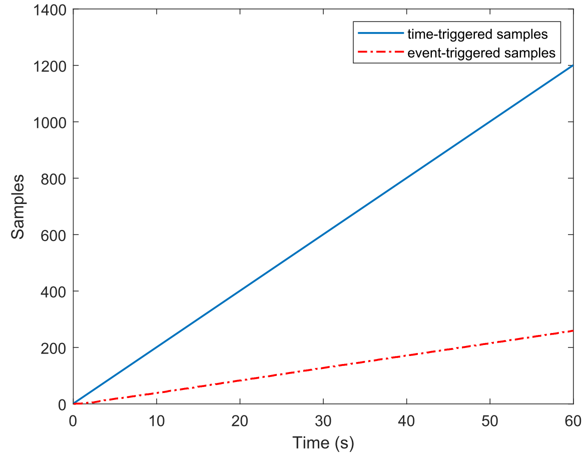

FIGURE 10. Sample numbers.

triggered method that requires 1200 samples. Furthermore, it greatly reduces the amount of calculation, and saves communication costs.

VI. CONCLUSION

In this paper, an event-triggered safe control method was proposed for a class of nonlinear safety-critical systems with safety constraints and input saturation by using adaptive dynamic programming. First, by using the barrier function to transformed the system with safety constraints and input saturation, and a system with only input saturation was obtained. Moreover, the non-quadratic utility function was introduced into the performance function to solve input saturation. Then, for relieving the computation pressure and saving communication cost, the event-triggered mechanism was introduced into the transformed system, and a new event-triggered condition was presented, in which the states was sampled and updated when the triggering condition was broken. Then, the optimal safety value function was approximated by a critic neural network, the approximate optimal safety control law and the approximate worst safety disturbance law can be obtainedand. During the learning process, the past data and current data were used to relax the persistence of excitation condition. Then, the stability of the proposed method was analyzed by the Lyapunov theory. Finally, the simulation results showed the feasibility of our proposed method. In the future, we will extend the proposed method to multi-agent systems.

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