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Robust Widely Linear Beamforming With Multiple Uncertainty Sets

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ABSTRACT The widely linear minimum variance distortionless response (WL-MVDR) beamformer has a better performance than the conventional MVDR beamformer when the received signals are potentially noncircular. However, it is sensitive to the extended steering vector (ESV) mismatches which can be caused by direction of arrival (DOA) errors, noncircularity coefficient errors, array imperfections and so on. To improve the robustness against large ESV mismatches, a robust WL beamformer based on the worst-case performance optimization (WCPO) method with multiple uncertainty sets is proposed. The resultant beamformer has the mathematical form of a nonconvex optimization problem, which can be converted into a semidefinite programming (SDP) problem and solved iteratively. Simulation results show that, in comparison with several representative robust WL beamformers, the proposed method can achieve better performances, especially under large ESV mismatch conditions.

INDEX TERMS Robust adaptive beamforming, noncircular signal, worst-case performance optimization, multiple constraints.

I. INTRODUCTION

Adaptive beamforming plays an important role in array signal processing and has been widely applied in various fields, such as radar, sonar, wireless communications, and so on [1]–[3]. Conventional beamformers such as the well-known minimum variance distortionless response (MVDR) beamformer mainly consider signals to be second-order (SO) circular. Nevertheless, there are many SO noncircular signals used in the areas of radio communication or satellite communication, such as amplitude-shift keying (ASK), phase-shift keying (BPSK), and unbalanced quaternary phase-shift keying (UQPSK) signals [4]. For noncircular signals, conventional beamformers become suboptimal since they ignore the correlation information between the real and imaginary parts of the noncircular signals. And the optimal beamformers are proved to be widely linear (WL), which use both the noncircular signal and its conjugate [5], [6].

The WL-MVDR beamformer was first proposed in [7] but remains suboptimal since it only exploits the SO noncircularity of interferences. In [8], the optimal WL-MVDR beamformer was proposed, which takes into account both the noncircularity of desired signals and interferences. And the superior performance of the optimal WL-MVDR beamformer was analyzed in [9], [10].

In practice, conventional beamformers suffer from severe performance degradation in the presence of steering vector (SV) mismatches. To improve the robustness against SV mismatches, numerous robust adaptive beamforming methods have been proposed, such as linearly constrained minimum variance (LCMV) methods [11]–[13], diagonal loading (DL) methods [14]–[17], subspace-based methods [18], [19], and interference-plus-noise covariance matrix reconstruction methods [20]–[23]. With the application of convex optimization theory to array signal processing, robust adaptive beamforming methods based on the uncertainty set constraints have been developed [16], [17], [24]–[28]. In these methods, the desired SV is constrained in a possible uncertainty set, then the optimal weight vector or the actual SV can be found by solving optimization problems. Many of these optimization problems are nonconvex and will be transformed into tractable convex forms, which can be solved

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by suitable numerical techniques such as the interior point algorithm [29], the newly proposed IAIDNN method [30], and so on.

To improve the robustness against extended steering vector (ESV) mismatches, some robust methods for WL beamformers have also been proposed [31]–[35] in recent years. In [31], Xu et.al proposed a classical estimator of noncircularity coefficient and extended the diagonal loading method to the WL beamformer. In [32], a robust WL beamforming method based on the spatial spectrum of noncircularity coefficient was presented, but it heavily depends on the precision of desired signal's SV. [33] and [34] extended the robust Capon beamforming (RCB) method to WL beamformers. And a robust algorithm combining the iterative adaptive method and interference-plus-noise covariance matrix reconstruction was proposed in [35]. These robust WL beamformers perform well when the ESV mismatches are small. However, if a large mismatch occurs, the robustness of these beamformers cannot be guaranteed.

In this paper, a robust WL beamforming design based on the worst-case performance optimization (WCPO) method is proposed. To improve the robustness against large ESV mismatches, multiple uncertainty sets are used in the constraint to characterize the entire large uncertainty region instead of a single one. The resultant beamformer has the mathematical form of a nonconvex optimization problem which can be transformed into a tractable form by semidefinite relaxation (SDR) and solved iteratively. Simulation results demonstrate that, in comparison with several representative robust WL beamformers, the proposed method is more robust against kinds of large ESV mismatches caused by DOA errors, noncircularity coefficient errors, and array imperfections.

The rest of this paper is organized as follows. The definition for noncircularity signals and the widely linear beamformer are given in Section II. In section III, the WL-WCPO robust beamforming design with multiple uncertainty sets is proposed. In section IV, simulation results are presented to illustrate the performance of the proposed method. Finally, the conclusions are drawn in Section V.

II. SIGNAL MODEL AND PROBLEM FORMULATION

A. SIGNAL MODEL

Considering a uniform linear array (ULA) with M omnidirectional sensors that receives one desired signal and P interferences, the observation vector at the nth snapshot can be written as

$$\mathbf{x}(n) = \mathbf{a}_0 s_0(n) + \sum_{i=1}^{P} \mathbf{j}_i s_i(n) + \mathbf{n}(n)$$
$$\triangleq \mathbf{a}_0 s_0(n) + \mathbf{v}(n), \tag{1}$$

where $s_0(n)$ and $s_i(n), i = 1, 2, \ldots, P$ are the complex envelope of the desired signal and interferences, assumed zero-mean and potentially noncircular. All of them are far-field narrowband signals and uncorrelated with each other. $\mathbf{a}_0$ and $\mathbf{j}_i, i = 1, 2, \ldots, P$ are the steering vectors of the desired signal and interferences, respectively. $\mathbf{n}(n)$ is the additive white Gaussian noise vector and $\mathbf{v}(n)$ represents the whole interference-plus-noise (IPN) vector.

To describe the noncircularity of the desired signal, the noncircularity coefficient of $s_0(n)$ is defined as [6]

$$\gamma_0 = \frac{\langle E\left[s_0^2(n)\right]\rangle}{\langle E\left[|s_0(n)|^2\right]\rangle} = |\gamma_0|\, e^{j\varphi_0}, \quad 0 \le |\gamma_0| \le 1, \tag{2}$$

where $\langle\cdot\rangle$ denotes the time-averaging operation over the observation window, $|\gamma_0|$ and φ_0 are called the noncircularity rate and noncircularity phase of $s_0(n)$. When $\gamma_0 \neq 0$, $s_0(n)$ is a noncircular signal.

The noncircularity coefficient γ_0 can be regarded as a measure of the correlation between $s_0(n)$ and $s_0^*(n)$. When $\gamma_0 \neq 0$, $s_0^*(n)$ is correlated with $s_0(n)$ and can be orthogonally decomposed as

$$s_0^*(n) = \gamma_0^* s_0(n) + [\sigma_0^2(1 - |\gamma_0|^2)]^{1/2} s_0'(n), \tag{3}$$

where $\sigma_0^2 = \langle E[|s_0(n)|^2]\rangle$, $s_0'(n)$ is the component orthogonal to $s_0(n)$, thus $\langle E[s_0(n)s_0'(n)^*]\rangle = 0$ and $\langle E[|s_0'(n)|^2]\rangle = 1$ [8].

Then, define the extended observation vector $\tilde{\mathbf{x}}(n) \triangleq \left[\mathbf{x}^T(n), \mathbf{x}^H(n)\right]^T$. Combining the result in (3), $\tilde{\mathbf{x}}(n)$ can be described as

$$\tilde{\mathbf{x}}(n) = s_0(n)\tilde{\mathbf{a}}_0 + \tilde{\mathbf{v}}(n), \tag{4}$$

where $\tilde{\mathbf{a}}_0 = \left[\mathbf{a}_0^T, \gamma_0^* \mathbf{a}_0^H\right]^T$ is defined as the ESV of desired signal, $\tilde{\mathbf{v}}(n) = [\mathbf{v}^T(n), \mathbf{v}^H(n) + s_0'(n)[\sigma_0^2(1 - |\gamma_0|^2)]^{1/2}\mathbf{a}_0^H]^T$ is the extended interference-plus-noise vector.

B. WIDELY LINEAR BEAMFORMER

Given that $\tilde{\mathbf{w}}$ denotes the $2M \times 1$ weight vector, the output of the WL beamformer can be expressed as

$$y(n) = \tilde{\mathbf{w}}^H \tilde{\mathbf{x}}(n) = s_0(n)\tilde{\mathbf{w}}^H \tilde{\mathbf{a}}_0 + \tilde{\mathbf{w}}^H \tilde{\mathbf{v}}(n). \tag{5}$$

According to [8], the optimal WL-MVDR beamformer is given by solving

$$\min_{\tilde{\mathbf{w}}} \ \tilde{\mathbf{w}}^H \mathbf{R}_{\tilde{\mathbf{v}}} \tilde{\mathbf{w}} \quad \text{s.t. } \tilde{\mathbf{w}}^H \tilde{\mathbf{a}}_0 = 1, \tag{6}$$

where $\mathbf{R}_{\tilde{\mathbf{v}}} = \left\langle E\left[\tilde{\mathbf{v}}(n)\tilde{\mathbf{v}}^H(n)\right]\right\rangle$ is the extended interference-plus-noise covariance matrix and the optimal solution can be written as

$$\tilde{\mathbf{w}}_{\text{MVDR}} = \left[\tilde{\mathbf{a}}_0^H \mathbf{R}_{\tilde{\mathbf{v}}}^{-1} \tilde{\mathbf{a}}_0\right]^{-1} \mathbf{R}_{\tilde{\mathbf{v}}}^{-1} \tilde{\mathbf{a}}_0. \tag{7}$$

The output signal to interference-plus-noise ratio (SINR) at $\tilde{\mathbf{w}}$ is defined by

$$\text{SINR}[\tilde{\mathbf{w}}] \triangleq \frac{\sigma_0^2 \left|\tilde{\mathbf{w}}^H \tilde{\mathbf{a}}_0\right|^2}{\tilde{\mathbf{w}}^H \mathbf{R}_{\tilde{\mathbf{v}}} \tilde{\mathbf{w}}}. \tag{8}$$

In practice, $\mathbf{R}_{\tilde{\mathbf{v}}}$ and $\tilde{\mathbf{a}}_0$ are typically unavailable. Therefore, $\mathbf{R}_{\tilde{\mathbf{v}}}$ is usually replaced by the extended sampled covariance matrix

$$\hat{\mathbf{R}}_{\tilde{\mathbf{x}}} = \frac{1}{N} \sum_{n=1}^{N} \tilde{\mathbf{x}}(n)\tilde{\mathbf{x}}^H(n), \tag{9}$$

where N is the number of snapshots. And the actual ESV of desired signal $\tilde{\mathbf{a}}_0$ is replaced by the nominal ESV $\bar{\tilde{\mathbf{a}}}_0$. Due to the existence of various errors, such as the DOA estimation errors, noncircularity coefficient estimation errors, array imperfections and so on, there are mismatches between the nominal ESV $\bar{\tilde{\mathbf{a}}}_0$ and the actual one $\tilde{\mathbf{a}}_0$. In the presence of significant ESV mismatches, the optimal WL-MVDR beamformer will mistake the desired signal from $\tilde{\mathbf{a}}_0$ as interference and suppress it, resulting in severe degradation of beamforming performance.

III. PROPOSED METHOD

In this section, the WL-WCPO beamformer with multiple uncertainty sets is proposed. Firstly, we derive the expression of the ESV error and extend the WCPO method to the WL beamformer. Then, multiple small uncertainty sets are exploited in the constraint and an iterative semidefinite programming method is developed to solve the resultant nonconvex problem. Finally, the computational complexity of the proposed method is analyzed.

A. ROBUST WIDELY LINEAR BEAMFORMER BASED ON THE WCPO METHOD

According to the WCPO method, it is assumed that the SV error $\mathbf{e}_0 = \mathbf{a}_0 - \bar{\mathbf{a}}_0$ belongs to an uncertainty set that describes various possible mismatches. And the norm of $\mathbf{e}_0$ has a given upper bound ε. Consequently, the actual SV $\mathbf{a}_0$ belongs to an uncertainty set A, whose radius is ε and center is determined by the nominal SV $\bar{\mathbf{a}}_0$, i.e.

$$A \triangleq \{\mathbf{p} | \mathbf{p} = \bar{\mathbf{a}}_0 + \mathbf{e}, \|\mathbf{e}\| \leq \varepsilon\}, \tag{10}$$

where p is any vector in A, $\mathbf{e}$ is the corresponding error vector of p.

Similarly, the ESV error can be described as

$$\begin{aligned}\tilde{\mathbf{e}}_0 = \tilde{\mathbf{a}}_0 - \bar{\tilde{\mathbf{a}}}_0 &= \begin{bmatrix} \mathbf{a}_0 - \bar{\mathbf{a}}_0 \\ \gamma_0^* \mathbf{a}_0^* - \hat{\gamma}_0^* \bar{\mathbf{a}}_0^* \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{e}_0 \\ \gamma_0^* \mathbf{e}_0^* + \gamma_\Delta^* (\bar{\mathbf{a}}_0 + \mathbf{e}_0) \end{bmatrix},\end{aligned} \tag{11}$$

where γ_0 and $\hat{\gamma}_0$ are the actual and estimated noncircularity coefficient of the designed signal respectively. And the estimation error is denoted as $\gamma_\Delta = \gamma_0 - \hat{\gamma}_0$, which is also assumed to have an given upper bound ε_γ.

The common estimator of $\hat{\gamma}_0$ is given by [31]

$$\hat{\gamma}_0 = -\frac{\bar{\mathbf{a}}_0^H \mathbf{E} \bar{\mathbf{a}}_0^*}{\bar{\mathbf{a}}_0^H \mathbf{D} \bar{\mathbf{a}}_0} \cdot \frac{\bar{\mathbf{a}}_0^H \bar{\mathbf{a}}_0}{\bar{\mathbf{a}}_0^H (\mathbf{I}_M - \hat{\sigma}_n^2 \mathbf{R}_x^{-1}) \bar{\mathbf{a}}_0}, \tag{12}$$

where

$$\mathbf{D} \triangleq (\mathbf{R}_x - \mathbf{C}_x \mathbf{R}_x^{*-1} \mathbf{C}_x^*)^{-1}, \tag{13}$$

$$\mathbf{E} \triangleq -\mathbf{D} \mathbf{C}_x \mathbf{R}_x^{*-1}, \tag{14}$$

where $\mathbf{R}_x = \langle E[\mathbf{x}(n)\mathbf{x}^H(n)] \rangle$, $\mathbf{C}_x = \langle E[\mathbf{x}(n)\mathbf{x}^T(n)] \rangle$, $\hat{\sigma}_n^2$ can be estimated by the minimum eigenvalue of $\mathbf{R}_x$.

Therefore, the upper bound of the ESV error is calculated by

$$\begin{aligned}\|\tilde{\mathbf{e}}_0\|^2 &= \tilde{\mathbf{e}}_0^H \tilde{\mathbf{e}}_0 = \|\mathbf{e}_0\|^2 + \|\hat{\gamma}_0 \mathbf{e}_0 + \gamma_\Delta \bar{\mathbf{a}}_0 + \gamma_\Delta \mathbf{e}_0\|^2 \\ &\leq \|\mathbf{e}_0\|^2 + \left(\|\hat{\gamma} \mathbf{e}_0\| + \|\gamma_\Delta \bar{\mathbf{a}}_0\| + \|\gamma_\Delta \mathbf{e}_0\|\right)^2 \\ &\leq \underbrace{\varepsilon^2 + (|\hat{\gamma}_0| \varepsilon + \varepsilon_\gamma \sqrt{M} + \varepsilon_\gamma \varepsilon)^2}_{\tilde{\varepsilon}^2}.\end{aligned} \tag{15}$$

Thus, $\tilde{\varepsilon}$ is the radius of the ESV's uncertainty set and the uncertainty set of the ESV can finally be expressed as

$$\tilde{A} \triangleq \left\{\tilde{\mathbf{p}} | \tilde{\mathbf{p}} = \bar{\tilde{\mathbf{a}}}_0 + \tilde{\mathbf{e}}, \|\tilde{\mathbf{e}}\| \leq \tilde{\varepsilon}\right\}, \tag{16}$$

where $\tilde{\mathbf{p}}$ is any vector in $\tilde{A}$ and $\tilde{\mathbf{e}}$ is the corresponding error vector of $\tilde{\mathbf{p}}$.

Since the actual ESV $\bar{\tilde{\mathbf{a}}}_0$ can be any vector in $\tilde{A}$, a constraint is imposed that for all vectors belonging to $\tilde{A}$, the absolute value of the array response should not be smaller than one. Thus, the WL-WCPO beamformer with single uncertainty set (expressed as WL-WCPO-S in the following) can be described as

$$\begin{aligned}&\min_{\tilde{\mathbf{w}}} \ \tilde{\mathbf{w}}^H \hat{\mathbf{R}}_{\tilde{\mathbf{x}}} \tilde{\mathbf{w}} \\ &\text{s.t.} \ \left|\tilde{\mathbf{w}}^H \tilde{\mathbf{p}}\right| \geq 1, \quad \forall \tilde{\mathbf{p}} \in \tilde{A}.\end{aligned} \tag{17}$$

The constraints in (17) guarantee that the distortionless response will be maintained in the worst case, and improve the beamforming robustness against kinds of ESV mismatches that satisfy $\|\tilde{\mathbf{e}}\| \leq \tilde{\varepsilon}$.

Problem (17) is a semi-infinite nonconvex quadratic programming problem and can be equivalently expressed as [24]

$$\begin{aligned}&\min_{\tilde{\mathbf{w}}} \ \tilde{\mathbf{w}}^H \hat{\mathbf{R}}_{\tilde{\mathbf{x}}} \mathbf{w} \\ &\text{s.t.} \ \tilde{\mathbf{w}}^H \bar{\tilde{\mathbf{a}}}_0 \geq \varepsilon \|\tilde{\mathbf{w}}\| + 1 \\ &\quad \text{Im}(\tilde{\mathbf{w}}^H \bar{\tilde{\mathbf{a}}}_0) = 0,\end{aligned} \tag{18}$$

which is a second order cone programming (SOCP) problem and can be solved by the interior point method [36] directly.

Note: According to [17], the WCPO method has an equivalent solution to the robust Capon beamformer (RCB) [16]. Similarly, the WL-WCPO-S method can also have an equivalent solution to the WL-RCB method in [33].

B. WL-WCPO BEAMFORMER WITH MULTIPLE UNCERTAINTY SETS

As mentioned above, the WL-WCPO-S method can be robust against kinds of ESV mismatches that satisfy $\|\tilde{\mathbf{e}}\| \leq \tilde{\varepsilon}$. However, due to the movement of signal sources and kinds of non-ideal factors, such as array imperfections, source wave-front distortions and so on, it is common to find large DOA mismatches along with other small errors. In this case, the ESV mismatches will increase, and a large uncertainty set is required accordingly for the WL-WCPO-S method. If the size of the uncertainty set is too large, this method will be too conservative and the ability to suppress the noise and

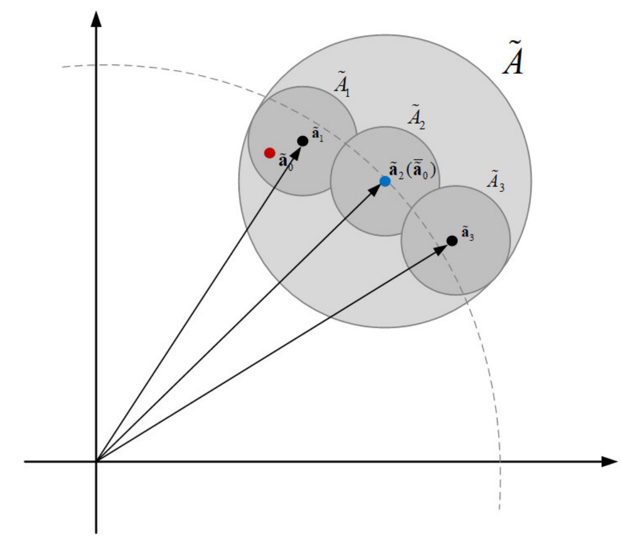


FIGURE 1. Illustration with three small uncertainty sets.

interferences can not be guaranteed. But if the size is not large enough, the actual ESV of the designed signal may not be included in the uncertainty set and the performance will deteriorate dramatically.

To alleviate such problems, we use multiple small uncertainty sets rather than a single large one to characterize the entire large uncertainty region. Considering large DOA estimation errors, some ESVs corresponding to the directions $\bar{\theta}_k, k = 1, 2, \ldots, K$ around the desired signal's estimated direction $\hat{\theta}_0$ are selected as the centers of these small uncertainty sets. Thus, the constraint region can finally be described as

$$\tilde{A}_k = \left\{\tilde{\mathbf{p}} | \tilde{\mathbf{p}} = \tilde{\mathbf{a}}_k + \tilde{\mathbf{e}}, \left\|\tilde{\mathbf{e}}\right\| \le \tilde{\varepsilon}_k\right\}, \quad k = 1, 2, \ldots, K, \tag{19}$$

where $\tilde{\mathbf{a}}_k = \left[\mathbf{a}_k^T, \hat{\gamma}_k^* \mathbf{a}_k^H\right]^T$ is the kth ESV that is presumed, $\left\|\tilde{\mathbf{a}}_k\right\| = M\sqrt{1 + \left|\hat{\gamma}_k\right|^2}$. $\mathbf{a}_k = \mathbf{a}(\bar{\theta}_k)$ is the presumed SV of $\bar{\theta}_k$, and $\hat{\gamma}_k$ is estimated with $\mathbf{a}_k$ in (12). $\tilde{\varepsilon}_k$ is the radius for the kth uncertainty set and K is the number of the uncertainty sets. The illustration with three small uncertainty sets $\tilde{A}_k, k = 1, 2, 3$ compared with the large one $\tilde{A}$ is shown in Fig.1. By eliminating unnecessary regions in the large uncertainty set $\tilde{A}$, the interference and noise suppression ability of the beamformer is guaranteed.

Then the WL-WCPO beamformer with multiple uncertainty sets can be expressed as

$$\begin{aligned} &\min_{\tilde{\mathbf{w}}} \ \tilde{\mathbf{w}}^H \hat{\mathbf{R}}_{\tilde{\mathbf{x}}} \tilde{\mathbf{w}} \\ &\text{s.t.} \ \left|\tilde{\mathbf{w}}^H \tilde{\mathbf{p}}\right| \ge 1, \quad \forall \tilde{\mathbf{p}} \in \tilde{A}_k, \ k = 1, 2, \ldots, K. \end{aligned} \tag{20}$$

It's also a semi-infinite nonconvex constraints optimization problem and can be transformed into a finite constraints problem similarly

$$\begin{aligned} &\min_{\tilde{\mathbf{w}}} \ \tilde{\mathbf{w}}^H \hat{\mathbf{R}}_{\tilde{\mathbf{x}}} \tilde{\mathbf{w}} \\ &\text{s.t.} \ \left|\tilde{\mathbf{w}}^H \tilde{\mathbf{a}}_k\right| - \tilde{\varepsilon}_k \left\|\tilde{\mathbf{w}}\right\| \ge 1, \quad k = 1, 2, \ldots, K. \end{aligned} \tag{21}$$

It is noted that only if $\text{Im}\left(\tilde{\mathbf{w}}^H \tilde{\mathbf{a}}_k\right) = 0, k = 1, 2, \ldots, K$, thus $\left|\tilde{\mathbf{w}}^H \tilde{\mathbf{a}}_k\right| = \tilde{\mathbf{w}}^H \tilde{\mathbf{a}}_k$, this problem can be a convex problem. But it is hard to find a $\tilde{\mathbf{w}}$ that satisfies all these K constraints.

C. SOLUTION TO PROPOSED METHOD

Since the resulting optimization problem (21) is nonconvex, in this section it will be approximately transformed into a convex form by SDR and solved in an iterative way.

Firstly, K auxiliary variables β_k, $k = 1, 2, \ldots, K$ are introduced and let

$$\left|\tilde{\mathbf{w}}^H \tilde{\mathbf{a}}_k\right| = \sqrt{\beta_k}, \quad k = 1, 2, \ldots, K. \tag{22}$$

Then the constraints in (21) can be rewritten as

$$\tilde{\varepsilon}_k \left\|\tilde{\mathbf{w}}\right\| \le \left|\tilde{\mathbf{w}}^H \tilde{\mathbf{a}}_k\right| - 1 = \sqrt{\beta_k} - 1, \quad k = 1, 2, \ldots, K. \tag{23}$$

The problem (21) is transformed into

$$\begin{aligned} &\min_{\tilde{\mathbf{w}}, \beta_k} \ \tilde{\mathbf{w}}^H \hat{\mathbf{R}}_{\tilde{\mathbf{x}}} \tilde{\mathbf{w}} \\ &\text{s.t.} \begin{cases} \left|\tilde{\mathbf{w}}^H \tilde{\mathbf{a}}_k\right| = \sqrt{\beta_k} \\ \tilde{\varepsilon}_k \left\|\tilde{\mathbf{w}}\right\| \le \sqrt{\beta_k} - 1 \\ k = 1, 2, \ldots, K. \end{cases} \end{aligned} \tag{24}$$

According to the SDR method [37], defining the rank one symmetric positive semidefinite (PSD) matrix $\mathbf{W} \triangleq \tilde{\mathbf{w}}\tilde{\mathbf{w}}^H$ and taking square on both sides of the constraints, problem (24) can be rewritten as

$$\begin{aligned} &\min_{\mathbf{W}, \beta_k} \text{tr}(\hat{\mathbf{R}}_{\tilde{\mathbf{x}}} \mathbf{W}) \\ &\text{s.t.} \begin{cases} \text{tr}(\tilde{\mathbf{a}}_k \tilde{\mathbf{a}}_k^H \mathbf{W}) = \beta_k \\ \tilde{\varepsilon}_k^2 \text{tr}(\mathbf{W}) \le \beta_k - 2\sqrt{\beta_k} + 1 \mathbf{W} \succeq 0 \\ \text{rank}(\mathbf{W}) = 1 \\ k = 1, 2, \ldots, K. \end{cases} \end{aligned} \tag{25}$$

where $\text{tr}(\cdot)$ is the trace operator, $\text{rank}(\cdot)$ is the rank of the matrix.

Dropping the rank-one constraint in (25), the relaxed problem is given by

$$\begin{aligned} &\min_{\mathbf{W}, \beta_k} \text{tr}(\hat{\mathbf{R}}_{\tilde{\mathbf{x}}} \mathbf{W}) \\ &\text{s.t.} \begin{cases} \text{tr}(\tilde{\mathbf{a}}_k \tilde{\mathbf{a}}_k^H \mathbf{W}) = \beta_k \\ \tilde{\varepsilon}_k^2 \text{tr}(\mathbf{W}) \le \beta_k - 2\sqrt{\beta_k} + 1 \mathbf{W} \succeq 0 \\ k = 1, 2, \ldots, K. \end{cases} \end{aligned} \tag{26}$$

Due to the existence of the nonlinear terms $\sqrt{\beta_k}, k = 1, 2, \ldots, K$, problem (26) is still nonconvex. To solve this problem, an iterative method is proposed in the following.

Assumed that $(\mathbf{W}_{n-1}, \beta_{1,n-1}, \beta_{2,n-1}, \ldots, \beta_{K,n-1})$ is a feasible solution to (26) which is obtained in the $(n-1)$th

TABLE 1. **WL-WCPO method with multiple uncertainty sets.**

Step1:	Give a_k, $\tilde{\varepsilon}_k, k = 1, 2, ..., K$ and ξ.
Step2:	Calculate $\hat{\gamma}_k$ and $\beta_{k,\min}$ by (12) and (31) to get $\tilde{\mathbf{a}}_k$ and $\beta_{k,0}, k = 1, 2, ..., K$; Set $n = 1$.
Step3:	Obtain $\mathbf{W}_n$ and $\beta_{k,n}, k = 1, 2, ..., K$ by solving (27).
Step4:	If $\|\boldsymbol{\beta}_n - \boldsymbol{\beta}_{n-1}\| \leq \xi$, go to **Step5**. Otherwise, let $n = n+1$, go to **Step3**.
Step5:	Let the approximate optimal solution $\hat{\mathbf{W}} = \mathbf{W}_n$.
Step6:	Calculate the maximum eigenvalue λ_1 and the corresponding eigenvector $\mathbf{u}_1$ of $\hat{\mathbf{W}}$, obtain the final weight vector $\tilde{\mathbf{w}} = \sqrt{\lambda_1}\mathbf{u}_1$.

iteration, it is also a feasible solution to the problem (27)

$$\min_{\mathbf{W},\beta_k} \operatorname{tr}(\hat{\mathbf{R}}_{\tilde{\mathbf{x}}}\mathbf{W})$$
$$\text{s.t.} \begin{cases} \operatorname{tr}(\tilde{\mathbf{a}}_k\tilde{\mathbf{a}}_k^H\mathbf{W}) = \beta_k \\ \tilde{\varepsilon}_k^2\operatorname{tr}(\mathbf{W}) \leq \beta_k - \dfrac{\beta_k + \beta_{k,n-1}}{\sqrt{\beta_{k,n-1}}} + 1 \\ \mathbf{W} \succeq 0 \\ k = 1, 2, \ldots, K. \end{cases} \tag{27}$$

Noted that for given $\beta_{k,n-1}, k = 1, 2, \ldots K$, (27) is a semidefinite programming (SDP) problem and can be solved via the interior point method efficiently. Considering $(\mathbf{W}_n, \beta_{1,n}, \beta_{2,n}, \ldots, \beta_{K,n})$ is the optimal solution to (27), it is obvious that

$$\operatorname{tr}\left(\hat{\mathbf{R}}_{\tilde{\mathbf{x}}}\mathbf{W}_n\right) \leq \operatorname{tr}\left(\hat{\mathbf{R}}_{\tilde{\mathbf{x}}}\mathbf{W}_{n-1}\right). \tag{28}$$

According to the relationship $\beta_k + \beta_{k,n-1} \geq 2\sqrt{\beta_k\beta_{k,n-1}}$, we can get

$$\beta_k - \frac{\beta_k + \beta_{k,n-1}}{\sqrt{\beta_{k,n-1}}} + 1 \leq \beta_k - 2\sqrt{\beta_k} + 1. \tag{29}$$

Namely, the feasible domain in (27) is a subset of that in (26). Therefore, $(\mathbf{W}_n, \beta_{1,n}, \beta_{2,n}, \ldots, \beta_{K,n})$ is another feasible solution to (26) and has a better performance than $(\mathbf{W}_{n-1}, \beta_{1,n-1}, \beta_{2,n-1}, \ldots, \beta_{K,n-1})$.

If given initial variables $\beta_{k,0}, k = 1, 2, \ldots, K$, the iteration can be performed as aforementioned until the following ending condition holds

$$\left\|\boldsymbol{\beta}_n - \boldsymbol{\beta}_{n-1}\right\| \leq \xi, \tag{30}$$

where $\boldsymbol{\beta}_n = \left[\beta_{1,n}, \beta_{2,n}, \ldots, \beta_{K,n}\right]^T$. Thus, the approximate optimal solution $(\hat{\mathbf{W}}, \hat{\beta}_1, \hat{\beta}_2, \ldots, \hat{\beta}_K)$ is obtained.

According to [28], the initial variables $\beta_{k,0}, k = 1, 2, \ldots, K$ can be given by

$$\beta_{k,0} = \beta_{k,\min} = \frac{1}{\left(1 - \tilde{\varepsilon}_k \Big/ \sqrt{\lambda_{\max}\left\{\tilde{\mathbf{a}}_k\tilde{\mathbf{a}}_k^H\right\}}\right)^2}, \tag{31}$$

where $\lambda_{\max}\{\cdot\}$ represents the maximum eigenvalue of the matrix.

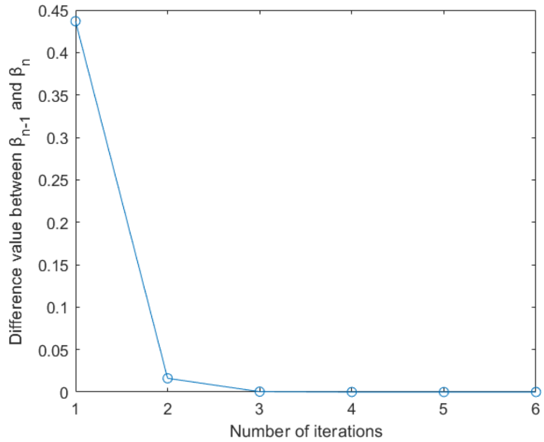


FIGURE 2. **The convergence rate of the proposed method.**

Then the approximate optimal solution $\hat{\mathbf{W}}$ to (27) should be converted into a feasible solution $\tilde{\mathbf{w}}$ to (24). According to [37], if $\hat{\mathbf{W}}$ is of rank one, $\hat{\mathbf{W}}$ can be written as $\hat{\mathbf{W}} = \tilde{\mathbf{w}}\tilde{\mathbf{w}}^H$ and $\tilde{\mathbf{w}}$ will be a feasible–in fact optimal solution to the original problem. On the other hand, if the rank of $\hat{\mathbf{W}}$ is larger than one, a rank-one approximation should be extracted from $\hat{\mathbf{W}}$ to obtain the feasible solution $\tilde{\mathbf{w}}$ to (24).

An efficient way to obtain the rank-one approximation is the eigenvalue-decomposition method. Let

$$\hat{\mathbf{W}} = \sum_{i=1}^{r} \lambda_i\mathbf{u}_i\mathbf{u}_i^H, \tag{32}$$

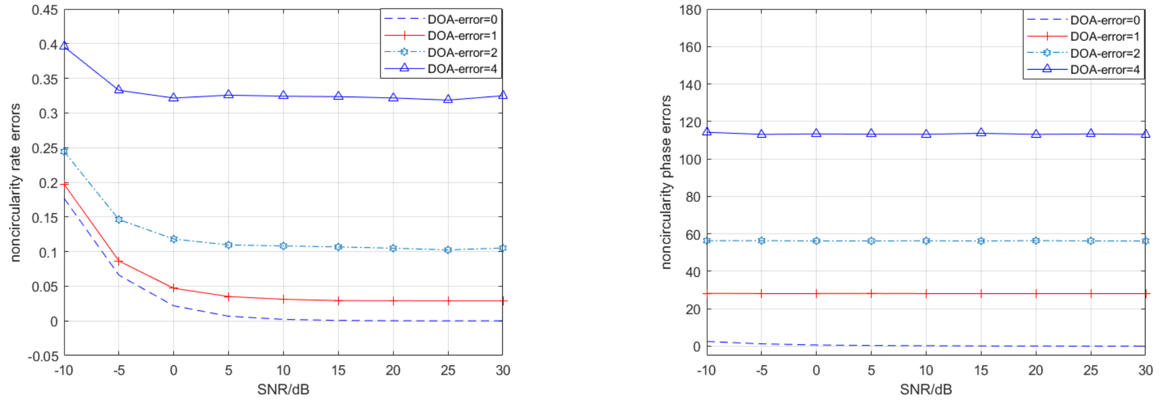
where $r = \operatorname{rank}(\hat{\mathbf{W}})$, $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_r > 0$ are the eigenvalues of $\hat{\mathbf{W}}$ and $\mathbf{u}_i, i = 1, 2, \ldots, r$ are the respective eigenvectors.

In the least squares sense, $\hat{\mathbf{W}}_1 = \lambda_1\mathbf{u}_1\mathbf{u}_1^H$ is the best rank-one approximation of $\hat{\mathbf{W}}$ and the weight vector can finally be written as

$$\tilde{\mathbf{w}} = \sqrt{\lambda_1}\mathbf{u}_1. \tag{33}$$

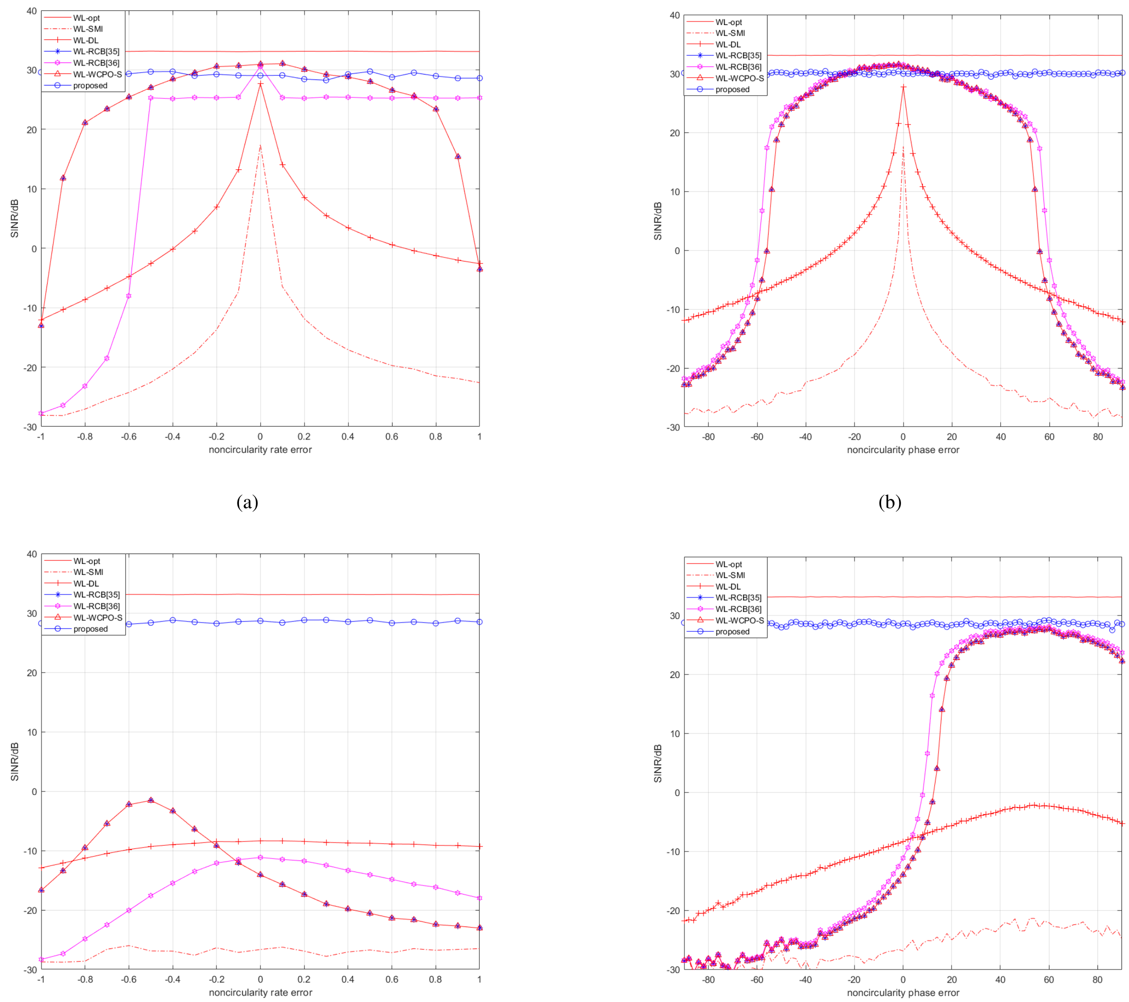
A summary of the proposed method is given in Table 1.

Computational Complexity: The complexity of the proposed method mainly lies in solving the SDP problem (27) in each iteration. According to [37], the SDP can be solved with a worst-case complexity of $O(max(M, N_c)^4 N_c^{1/2} log(1/\delta))$, where M is the number of the antenna elements, N_c is the number of constraints and $\delta > 0$ is a given solution accuracy. For WL beamformers, the virtual array aperture is extended to $2M$. Consequently, the worst-case computational complexity of the proposed method is $O(N_I \cdot max(2M, N_c)^4 N_c^{1/2} log(1/\delta))$, where N_I is the number of iterations. Fig.2 illustrates the convergence rate of the proposed method under the same condition as Section IV-B with $\Delta\theta = 2°$. And it can be noticed that this iterative approach can converge in less than 10 iterations.



(a) (b)

FIGURE 3. Influence of different DOA errors and SNR on the estimation error of noncircularity coefficient. (a) Noncircularity rate errors versus input SNR with different DOA errors; (b) Noncircularity phase errors versus input SNR with different DOA errors.



(a) (b) (c) (d)

FIGURE 4. Robustness of the compared beamformers against noncircularity coefficient error in different scenarios. SINR versus noncircularity rate error when (a) $\Delta\theta = 0°$, (c) $\Delta\theta = 2°$; SINR versus noncircularity phase error when (b) $\Delta\theta = 0°$, (d) $\Delta\theta = 2°$.

IV. SIMULATION RESULTS

In the simulations, a uniform linear array (ULA) with $M = 10$ omnidirectional sensors spaced half a wavelength is considered. The desired signal comes from the direction $\theta_0 = 5°$ and two interferences impinge from the directions $\theta_1 = -30°$, $\theta_2 = 40°$. All of them are BPSK signals

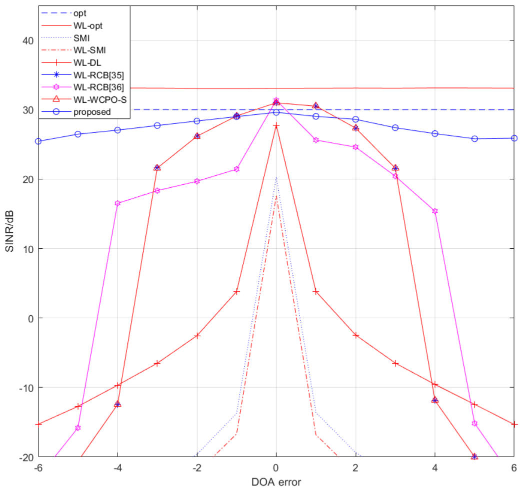


FIGURE 5. **Output SINR versus the DOA error that varies from −6° to 6° with SNR = 20dB.**

with a noncircularity rate of 1. And the noncircularity phases of them are 0, $\pi/3$, $\pi/4$, respectively. The interference-to-noise ratio (INR) of each sensor is 20dB, and the number of snapshots is fixed at 800. In each scenario, 200 Monte Carlo simulations are carried out.

The proposed method is compared with the SMI beamformer, the WL-SMI beamformer with an estimated noncircularity coefficient, the robust WL beamformer based on diagonal loading [31], the WL-RCB beamformer in [33], and the WL-RCB beamformer in [34]. And the WL-WCPO method with single uncertainty set (WL-WCPO-S) is also presented for comparison. Besides, the optimal SINRs for conventional beamformers and WL beamformers are shown as performance benchmarks. The diagonal loading factor ξ is assumed to be $10\hat{\sigma}_n^2$ for [31]. The parameters ε_s, ε_γ are 1.2 and 0.1 for [34]. For the beamformer in [33], the WL-WCPO-S method and the proposed method, the parameters ε_γ, $\tilde{\varepsilon}$ and $\tilde{\varepsilon}_k$ are set to be 0.1, 2.5 and 0.8 respectively. The number of constraint points in the proposed method is set to be 3. And the convex optimization problems are solved by the convex optimization toolbox CVX [36].

A. ROBUSTNESS AGAINST NONCIRCULARITY COEFFICIENT ERROR

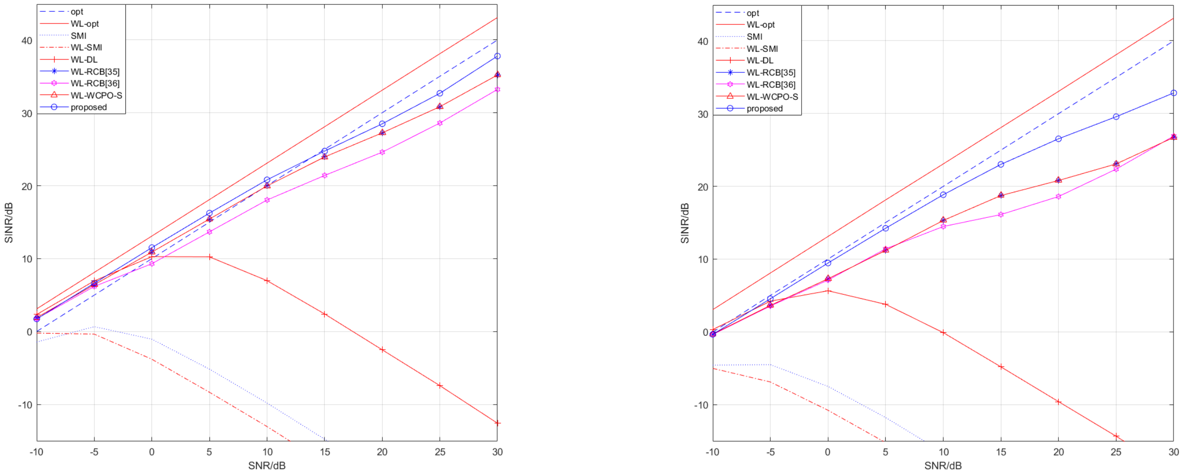
In this section, the robustness against noncircularity coefficient errors is discussed. As mentioned above, the estimation method of the noncircularity coefficient used in this paper is described in [31]. At first, the influence of different factors on the estimation error of the noncircularity rate (the modulus of noncircularity coefficient) and noncircularity phase is discussed. Fig.3(a) shows the absolute value of the noncircularity rate error versus the input SNR with different DOA errors. And Fig.3(b) is the absolute value of the noncircularity phase error under the same circumstances. It can be observed that, in the absence of the DOA error, both the estimation errors of the noncircularity rate and noncircularity phase are close to zero at high SNRs. And when the DOA error increases, these estimated results get worse apparently. Additionally, as input SNR decreases, the estimation error of the noncircularity rate increases, but the effect on estimation of the noncircularity phase remains negligible.

Then the performance of compared beamformers with different noncircularity coefficient errors is shown in Fig.4. In this simulation, the noncircularity coefficient error is assumed to be independent of the DOA error. Fig.4(a) and Fig.4(c) illustrate the output SINR versus different noncircularity rate errors when the input SNR is fixed at 20dB and the DOA error is 0° and 2°, respectively. And Fig.4(b) and Fig.4(d) illustrate the output SINR versus different noncircularity phase errors in the same condition. We can see that, compared with other beamformers, the proposed method is able to hold satisfying robustness against the noncircularity coefficient errors under all these conditions. In the cases where noncircularity coefficient errors are small and the DOA error is close to zero, the WL-WCPO-S and WL-RCB beamformers can outperform the proposed method slightly. But when DOA errors exist, even if they are small, the proposed method has a better performance than the other beamformers. Additionally, as mentioned in section III, the WL-WCPO-S beamformer has the same performance as the WL-RCB beamformer in [33].

B. ROBUSTNESS AGAINST DIFFERENT DOA ERRORS

In this simulation, the robustness against different DOA errors is verified. Fig.5 considers the scenario where the DOA error varies from −6° to 6° and the input SNR is fixed at 20dB. The output SINR curves of compared beamformers are shown in this figure. It is observed that the optimal SINR of the WL beamformer is 3 dB higher than that of conventional beamformers in ideal conditions due to the utilization of noncircularity in received signals. However, when a DOA error occurs, the desired signal coming from the actual DOA is suppressed and the performances of WL-SMI and SMI beamformers degraded severely. Compared with SMI and WL-SMI beamformers, these robust beamformers can maintain better performance. And proposed method outperforms all these beamformers, especially when the DOA error is large.

Fig.6 demonstrates the output SINR curves versus the input SNR with different DOA errors. In Fig.6(a), we assume that the DOA error $\Delta\theta = 2°$, the nominal DOA $\bar{\theta}_0 = 7°$, and the constraint points are at angles 5°, 7°, 9°. It can be seen that when SNR $\leq$ 15dB, the proposed method, the WL-WCPO-S beamformer and the WL-RCB beamformer in [33] almost have the same output SINRs. As the SNR increases, the proposed method is better than other competitors. In Fig.6(b), the DOA error $\Delta\theta = 4°$, $\bar{\theta}_0 = 9°$ and the constraint points are at angles 5°, 9°, 13°. It is observed that the proposed method has a higher output SINR than the other beamformers over



(a) (b)

FIGURE 6. Output SINR versus input SNR that varies form 10dB to 30dB in the presence of different DOA errors. (a) the DOA error $\Delta\theta = 2°$; (b) the DOA error $\Delta\theta = 4°$.

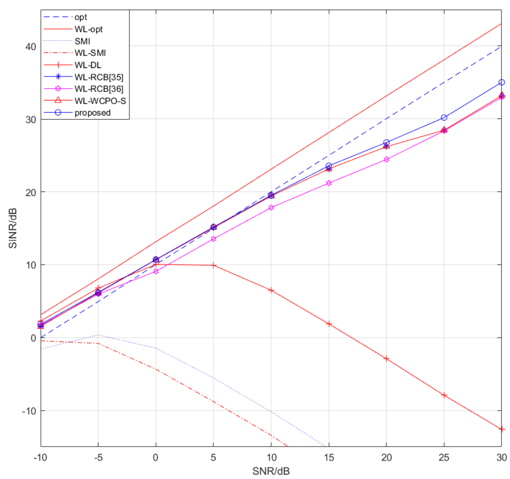


FIGURE 7. Output SINR versus input SNR that varies form 10dB to 30dB with amplitude and phase perturbations.

a wide range of input SNR. And the performance difference grows with the increase of SNR.

C. ROBUSTNESS AGAINST AMPLITUDE AND PHASE PERTURBATIONS

In this simulation, we investigate the robustness against amplitude and phase perturbations of compared beamformers. It is supposed that the amplitude error and phase error of each sensor is derived from the random generator $N(1, 0.1^2)$ and $N(1, (0.25\pi)^2)$, respectively. The DOA error is fixed at 2° and the SNR still varies from -10dB to 30dB. As shown in Fig.7, the proposed method, the WL-WCPO-S beamformer, and the WL-RCB beamformers in [33], [34] have similar performances and all of them have good robustness against amplitude and phase perturbations.

V. CONCLUSION

The WL-WCPO beamformer with multiple uncertainty sets is proposed in this paper. To improve the robustness against large ESV errors, the WCPO method is introduced to the WL beamformer and multiple uncertainty sets of the ESV are used in the constraint. The resultant beamformer is presented mathematically as a nonconvex optimization problem that can be transformed into a tractable form by SDR and solved by an iterative method. Simulation results demonstrate that the proposed method is robust to various types of ESV mismatches and performs better than other compared beamformers, especially under large ESV mismatch conditions.

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