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Joint Maintenance Decision Based on Remaining Useful Lifetime Prediction Using Accelerated Degradation Data

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ABSTRACT Existing joint maintenance decision research typically ignores remaining useful lifetime (RUL) predictions for the accelerated degradation of equipment. A joint maintenance decision method for the replacement and spare-parts ordering strategy based on RUL prediction using accelerated degradation data for equipment is proposed in this paper. First, an RUL prediction model under accelerated stress is built by considering the proportional relationship between the drift coefficient and diffusion coefficient in the Wiener process. Second, based on the principle of step-by-step estimation, accelerated degradation test (ADT) data of the equipment are used to estimate the a priori unknown parameters. Finally, based on the RUL prediction results, a joint optimization model for the replacement and spare-parts ordering strategy is developed. Through example verification and cost parameter sensitivity analysis, the proposed method is shown to effectively improve the accuracy of RUL prediction and the scientific value of the joint optimization plan for equipment replacement and spare-part ordering, which is important to many engineering applications.

INDEX TERMS Accelerated degradation test, proportional relationship, Wiener process, remaining useful lifetime prediction, joint maintenance decision.

I. INTRODUCTION

With the increasingly complex battlefield environment of modern war, the technological level of weapons and equipment has improved markedly in recent decades. As a result, stricter requirements for the reliability of equipment as well as the maintenance and support capability of troops have been proposed. To improve the reliability of equipment, as well as the maintenance and support capability of troops, prognostics and health management (PHM) have been proposed and have gained wide public attention. This technology could effectively improve the maintenance support efficiency of equipment and represents the future development direction of equipment support [1]–[4].

The essence of PHM is to obtain the status information of weapons and equipment via advanced sensor technology and then to predict their performance evolution trends and failure states, thereby obtaining RUL prediction information. Thus, scientific maintenance support methods have been developed to effectively improve the maintenance and support efficiency of equipment. Narrowly defined PHM technology primarily consists of two components: 1) predicting the RUL of equipment; and 2) making maintenance decisions for equipment based on RUL prediction information.

The specificity of the immediate task makes the weapons and equipment generally exhibit high reliabilities and long lifetimes, which makes it difficult to determine sufficient lifetimes or degradation data via conventional life and degradation testing. To address the shortcomings of traditional methods, the accelerated degradation test (ADT) was used in this study for the RUL prediction of equipment with high reliability and long lifetimes, and good results were achieved [5], [6]. Tang *et al.* [7] analyzed the accelerated degradation data of lasers and constructed an accelerated degradation model based on the Wiener process to predict the RUL. However, the fact that the diffusion coefficient varies with stress was not considered; thus, the accuracy of RUL prediction decreased. Based on Tang's research, Liu *et al.* [8]

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believed that accelerated stress affects the drift coefficient and the diffusion coefficient to some extent. Based on this assumption, the effectiveness of the method was validated via accelerometer degradation data by Liu *et al.* [8]. However, this method can only predict the RUL via the accelerated degradation data of equipment, and the on-site monitoring data are more general in the real operating environment. Accelerated degradation data and on-site monitoring data can improve the accuracy of RUL prediction. Thus, a method to update the parameters of the accelerated degradation model of equipment using the Bayes principle was proposed by Cai *et al.* [9], and the online RUL prediction of equipment based on the fusion data was realized. However, in this method, the effect of accelerated stress on the diffusion coefficient was neglected, and thus, the accuracy promotion of RUL prediction was limited because only the drift coefficient was updated. To synchronously update the drift and diffusion coefficients, Wang *et al.* [10] used Bayes' principle to online update the drift and diffusion coefficients based on the assumption that these coefficients obey a specific conjugate prior distribution. However, prediction was poor when the drift and diffusion coefficients did not meet the specific conjugate prior distribution hypothesis. Based on [10], Wang *et al.* [11] proposed a method in which the Bayesian statistical inference of the drift coefficient and diffusion coefficient was conducted based on a no conjugate prior distribution. In combination with the accelerated degradation data and on-site test data, the RUL of the target equipment was predicted, which extends the application scope of the method. However, to develop the degradation model, the default drift/diffusion coefficient in the Wiener process obeys the conjugate prior distribution similar to the normal-gamma distribution. However, when the drift and diffusion coefficients do not obey the distribution type, the RUL prediction result obtained by this method lacks credibility. Also, considering the acceleration factor constant principle in [12], Wang *et al.* [13] and Wang *et al.* [14] showed that the drift and diffusion coefficients both change based on stress and satisfy the proportional relationship. Based on these conclusions, Wang *et al.* [15] proposed a proportional relationship between the drift and diffusion coefficients under accelerated stress. Thus, an accelerated degradation model is constructed, which improves the effectiveness of the RUL prediction method. However, this model cannot be applied to accelerated-stress situations and does not consider the effects of individual differences and measurement errors on degradation modeling.

Scientific maintenance decisions can effectively improve the operating state of equipment and reduce the maintenance cost during their life cycle, which is important in military and economic considerations. In daily maintenance, the maintenance strategy that only considers equipment cannot effectively reduce maintenance costs during equipment life cycles. A well-developed spare-parts management strategy can reduce costs, thus promoting the transformation from a single maintenance strategy to joint optimization of the maintenance strategy and a spare-parts management strategy. The joint optimization between the preventive replacement strategy and spare parts production strategy was performed to achieve the lowest maintenance cost by Aghezzaf *et al.* [16]. The combined optimization of the preventive replacement strategy and spare parts inventory strategy was also conducted by Zequeiraa *et al.* [17]. Overall, the RUL prediction information of equipment was not considered in the literature, which has reduced the scientific nature of maintenance decisions to a certain extent. Currently, few studies have investigated the joint optimization of maintenance and spare-parts ordering strategies based on RUL prediction. The RUL prediction was obtained by Elwany *et al.* [18] using the linear and exponential degradation model. Thus, the maintenance cost was reduced by establishing the sequential optimization model of product replacement and spare-parts inventory. To expand the applicability of this method, Wang *et al.* [19] constructed a degradation model based on the Wiener process, and the maintenance decision model was established according to the RUL prediction of the equipment. Thus, the optimal decision of the equipment replacement strategy and spare-parts ordering strategy was achieved. However, the spare-parts ordering decision was made based on the optimal replacement strategy, which may lead to local optima of the decision result and affect its effectiveness. Jiang *et al.* [20] effectively enhanced the scientific nature of the results and ensured the rationality of the maintenance scheme via the joint optimization of the equipment replacement strategy and spare parts ordering strategy. However, this method can only update the drift coefficient online during RUL prediction but fails to update the diffusion coefficient synchronously, which restricts the accuracy enhancement of RUL prediction and is not conducive to the realization of scientific maintenance decisions.

According to the problems with current joint optimization for equipment replacement and the spare-parts ordering strategy based on the RUL prediction information of accelerated degradation, the synchronous effect of accelerating stress on the drift coefficient and diffusion coefficient is analyzed in this study, where the relationship between the drift coefficient and diffusion coefficient is assumed to be proportional. Also, the drift and diffusion coefficients were updated synchronously using a Kalman filter and on-site monitoring data of target equipment, which can effectively reduce the uncertainty of prediction while ensuring RUL prediction accuracy. Thus, this study develops a joint optimization model of replacement and spare-parts ordering strategy that considers RUL prediction information based on the renewal reward theory and determines the optimal average cost ratio of an equipment operation cycle via joint optimization with the equipment replacement time and spare-parts ordering time.

II. RUL PREDICTION MODEL

In different situations, Wiener process models can be generally divided into linear drift models, logarithmic change models, time scale transformation models, and general Wiener process models [21], [22], among which the time-scale transformation model is primarily used in the process of accelerated degradation modeling and can be expressed as:

$$X(t) = X(0) + \alpha\Lambda(t\,|\boldsymbol{v}) + \beta B\left(\Lambda(t\,|\boldsymbol{v})\right) \tag{1}$$

where $X(t)$ is the performance degradation for the equipment at t time; $\Lambda(t\,|\boldsymbol{v})$ is the function of t, $\boldsymbol{v}$ is the unknown parameter; α is the drift coefficient, and $\alpha \sim N(\mu_\alpha, \sigma_\alpha^2)$; β is the diffusion coefficient; $B(t)$ is the standard Brownian motion item, and $B(\Lambda(t\,|\boldsymbol{v})) \sim N(0, \Lambda(t\,|\boldsymbol{v}))$.

To shorten the time and reduce the cost of highly reliable product degradation tests, a high-stress level is often used to accelerate equipment degradation. Considering the effect of acceleration stress on the drift coefficient and diffusion coefficient, an accelerated degradation model can be established as follows:

$$X(t) = X(0) + \alpha(S\,|\boldsymbol{\theta})\Lambda(t\,|\boldsymbol{v}) + \beta(S\,|\boldsymbol{\eta})B\left(\Lambda(t\,|\boldsymbol{v})\right) \tag{2}$$

where $\alpha(S\,|\boldsymbol{\theta})$ and $\beta(S\,|\boldsymbol{\eta})$ are the drift coefficient and diffusion coefficient of the equipment under stress S; and $\boldsymbol{\theta}$ and $\boldsymbol{\eta}$ are unknown parameters. If the stress in Equation (2) is S_0 and Equation (1) equals Equation (2), the degradation model under constant stress is a special form of the accelerated degradation model.

[12] demonstrated that the accelerated degradation test should satisfy the principle of a constant acceleration factor, which can be expressed as follows:

$$A_{S_1,S_2} = \frac{\alpha(S_1\,|\boldsymbol{\theta})}{\alpha(S_2\,|\boldsymbol{\theta})} = \frac{\beta(S_1\,|\boldsymbol{\eta})^2}{\beta(S_2\,|\boldsymbol{\eta})^2} \tag{3}$$

where S_1 and S_2 are the accelerated stress levels; and A_{S_1,S_2} is the accelerated factor of S_2 to S_1, and its value only depends on S_1 and S_2.

Equation (3) implies that:

$$\frac{\alpha(S_1\,|\boldsymbol{\theta})}{\beta(S_1\,|\boldsymbol{\eta})^2} = \frac{\alpha(S_2\,|\boldsymbol{\theta})}{\beta(S_2\,|\boldsymbol{\eta})^2} = \frac{1}{g} \tag{4}$$

where g is a constant. Equation 4 implies that the drift and diffusion coefficients of the equipment degradation model have a fixed proportion relationship under any acceleration stress, the ratio is independent of the stress level under the circumstance that the failure mechanism does not change, and $\alpha(S\,|\boldsymbol{\theta})/\beta(S\,|\boldsymbol{\eta})^2 = 1/g$. Substituting this proportion relation into Equation (2), the proportional accelerated degradation model can be obtained:

$$X(t) = X(0) + \alpha(S\,|\boldsymbol{\theta})\Lambda(t\,|\boldsymbol{v}) + \sqrt{g\alpha(S\,|\boldsymbol{\theta})}B\left(\Lambda(t\,|\boldsymbol{v})\right) \tag{5}$$

For ease of analysis, the Arrhenius acceleration model and step-accelerated degradation test are used as examples in this study. The combination of other acceleration models and the accelerated degradation test types is similar to the abovementioned analysis and will not be described here. The Arrhenius model can be expressed as follows:

$$\alpha(S\,|\boldsymbol{\theta}) = a\exp(-b/S) \tag{6}$$

where $\boldsymbol{\theta} = \{a, b\}$; S is the thermodynamic temperature; $a \sim N(\mu_a, \sigma_a^2)$; $\mu_\alpha = \mu_a\exp(-b/S)$; and $\sigma_\alpha^2 = \sigma_a^2\exp(-2b/S)$.

We assume that the test data of the on-site equipment are $\boldsymbol{Y}_{1:k} = [Y_1, Y_2, \cdots, Y_k]$, and its corresponding performance degradation data are $\boldsymbol{X}_{1:k} = [X_1, X_2, \cdots, X_k]$. According to Equation (5) and Equation (6), the degradation model of equipment under constant stress is:

$$\begin{aligned} X(t) = {} & X(0) + a\exp(-b/S_0) \\ & \cdot\Lambda(t\,|\boldsymbol{v}) + \sqrt{g\exp(-b/S_0)a}B\left(\Lambda(t\,|\boldsymbol{v})\right) \end{aligned} \tag{7}$$

In this study, the unknown parameters in the degradation model are updated online based on the Kalman filtering principle. The state-space model of the equipment under constant stress is:

$$\begin{cases} X_k = X_{k-1} + a_{k-1}\exp(-b/S_0)\Delta\Lambda(t_k\,|\boldsymbol{v}) \\ \qquad + \sqrt{ga_{k-1}\exp(-b/S_0)}B\left(\Delta\Lambda(t_k\,|\boldsymbol{v})\right) \\ a_k = a_{k-1} \\ Y_k = X_k + \varepsilon \end{cases} \tag{8}$$

where $\Delta\Lambda(t_k\,|\boldsymbol{v}) = \Lambda(t_k\,|\boldsymbol{v}) - \Lambda(t_{k-1}\,|\boldsymbol{v})$; $t_0 = 0$; $B\left(\Delta\Lambda(t_k\,|\boldsymbol{v})\right) = B\left(\Lambda(t_k\,|\boldsymbol{v})\right) - B\left(\Lambda(t_{k-1}\,|\boldsymbol{v})\right)$; ε is the error term; and $\varepsilon \sim N(0, \sigma_\varepsilon^2)$.

Also, Equation (8) can be converted into the standard form of the Kalman filter:

$$\begin{cases} \boldsymbol{Z}_k = \boldsymbol{A}_k\boldsymbol{Z}_{k-1} + \boldsymbol{W}_{k-1} \\ Y_k = \boldsymbol{L}\boldsymbol{Z}_{k-1} + \varepsilon \end{cases} \tag{9}$$

where

$$\begin{aligned} \boldsymbol{Z}_k &= [X_k, a_k]^{\mathrm{T}}, \\ \boldsymbol{W}_k &= [\sqrt{ga_{k-1}\exp(-b/S_0)}B(\Delta\Lambda(t_k\,|\boldsymbol{v})), \end{aligned}$$

and $\boldsymbol{L} = [1, 0]$, and $\boldsymbol{A}_k = \begin{bmatrix} 1 & \exp(-b/S_0)\Delta\Lambda(t_k\,|\boldsymbol{v}) \\ 0 & 1 \end{bmatrix}$.

The iterative process of Kalman filtering can be expressed as:

$$\hat{\boldsymbol{Z}}_{k|k} = \hat{\boldsymbol{Z}}_{k|k-1} + \boldsymbol{K}_k(Y_k - \boldsymbol{L}\hat{\boldsymbol{Z}}_{k|k-1}) \tag{10}$$

$$\boldsymbol{P}_{k|k} = \boldsymbol{P}_{k|k-1} - \boldsymbol{K}_k\boldsymbol{L}\boldsymbol{P}_{k|k-1} \tag{11}$$

$$\hat{\boldsymbol{Z}}_{k|k-1} = \boldsymbol{A}_k\hat{\boldsymbol{Z}}_{k-1|k-1} \tag{12}$$

$$\boldsymbol{K}_k = \boldsymbol{P}_{k|k-1}\boldsymbol{L}^{\mathrm{T}}(\boldsymbol{L}\boldsymbol{P}_{k|k-1}\boldsymbol{L}^{\mathrm{T}} + \hat{\sigma}_\varepsilon^2)^{-1} \tag{13}$$

$$\begin{aligned} \boldsymbol{P}_{k|k-1} = {} & \boldsymbol{A}_k\boldsymbol{P}_{k-1|k-1}\boldsymbol{A}_k^{\mathrm{T}} \\ & + \begin{bmatrix} E\left(a_{k-1}\,|\boldsymbol{Y}_{1:k-1}\right)\hat{g}\exp(-\hat{b}/S_0)\Delta\Lambda(t_k\,|\hat{\boldsymbol{v}}) & 0 \\ 0 & 0 \end{bmatrix} \end{aligned} \tag{14}$$

where:

$$\hat{\boldsymbol{Z}}_{k|k} = \begin{bmatrix} E\left(X_k\,|\boldsymbol{Y}_{1:k}\right) \\ E\left(a_k\,|\boldsymbol{Y}_{1:k}\right) \end{bmatrix} \tag{15}$$

$$\boldsymbol{P}_{k|k} = \begin{bmatrix} D\left(X_k\,|\boldsymbol{Y}_{1:k}\right) & \mathrm{Cov}\left(X_k, a_k\,|\boldsymbol{Y}_{1:k}\right) \\ \mathrm{Cov}\left(X_k, a_k\,|\boldsymbol{Y}_{1:k}\right) & D\left(a_k\,|\boldsymbol{Y}_{1:k}\right) \end{bmatrix} \tag{16}$$

$$\hat{\boldsymbol{Z}}_{k|k-1} = \begin{bmatrix} E\left(X_k\,|\boldsymbol{Y}_{1:k-1}\right) \\ E\left(a_k\,|\boldsymbol{Y}_{1:k-1}\right) \end{bmatrix} \tag{17}$$

$$\boldsymbol{P}_{k|k-1} = \begin{bmatrix} D(X_k|\boldsymbol{Y}_{1:k-1}) & \mathrm{Cov}(X_k, a_k|\boldsymbol{Y}_{1:k-1}) \\ \mathrm{Cov}(X_k, a_k|\boldsymbol{Y}_{1:k-1}) & D(a_k|\boldsymbol{Y}_{1:k-1}) \end{bmatrix} \tag{18}$$

$$\hat{\boldsymbol{Z}}_{0|0} = \begin{bmatrix} 0 \\ \hat{\mu}_a \end{bmatrix}, \quad \boldsymbol{P}_{0|0} = \begin{bmatrix} 0 & 0 \\ 0 & \hat{\sigma}_a^2 \end{bmatrix} \tag{19}$$

In [23], the probability distribution function of RUL corresponding to the degradation model is as follows:

$$f_{L_k}(l_k|a_k, X_k, S_0, \boldsymbol{Y}_{1:k}) \cong \frac{\psi(l_k)}{\int_0^{+\infty}\psi(l_k)\mathrm{d}l_k} \tag{20}$$

where l_k is the RUL of the equipment under constant stress and $\psi(l_k)$ is the function of l_k. More information is available in [23].

A Kalman filter was used to update the process, where $\boldsymbol{Z}$ is a two-dimensional normal random variable, and the distribution coefficient of a_k 与 X_k can be obtained. The integral representation of Equation (20) is as follows:

$$\begin{aligned} &f_{L_k|S_0}(l_k|S_0, \boldsymbol{Y}_{1:k}) \\ &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f_{L_k}(l_k|a_k, X_k, S_0, \boldsymbol{Y}_{1:k})P(X_k|a_k, \boldsymbol{Y}_{1:k}) \\ &\quad \cdot P(a_k|\boldsymbol{Y}_{1:k})\mathrm{d}X_k\mathrm{d}a_k \end{aligned} \tag{21}$$

III. PRIOR PARAMETER ESTIMATION

To determine the specific situation of the probability distribution of the RUL of the equipment, it is necessary to estimate the unknown parameters contained in Equation (21). In this paper, the unknown parameters in the RUL prediction model under constant stress were estimated using the degradation data of similar equipment under accelerated stress. Due to the complexity of the model considered in this study, the traditional prior parameter estimation method based on the maximum likelihood principle and EM principle has difficulty establishing the likelihood function and performing the iterative calculation. Therefore, a new step-by-step maximum likelihood estimation (MLE) method is proposed to estimate model parameters.

In this study, $\boldsymbol{\Theta} = \{\mu_a, \sigma_a^2, b, \boldsymbol{v}, \sigma_\varepsilon^2, g\}$ represents all the unknown parameters of the RUL prediction model. Assuming that the accelerated degradation test contains N samples, and each sample undergoes M accelerated stress, then if $t_{i,j,k}$ is the k_{th} observed time of the i_{th} sample under the j_{th} stress, $Y_{i,j,k} = Y(t_{i,j,k})$ is its measured value of degradation value, S_j is the accelerated stress, $i = 1, 2, \cdots N, j = 1, 2, \cdots M$, and $k = 1, 2, \cdots L_{i,j}$. $\boldsymbol{Y}_{i,j}^{\mathrm{T}} = [Y_{i,j,1}, Y_{i,j,2}, \cdots Y_{i,j,L_{i,j}}]$ represents all the degradation data of the i_{th} sample under the S_j stress. If $\Delta Y_{i,j,k} = Y(t_{i,j,k}) - Y(t_{i,j,k-1})$ and $\Delta\boldsymbol{Y}_{i,j}^{\mathrm{T}} = [\Delta Y_{i,j,1}, \Delta Y_{i,j,2}, \cdots \Delta Y_{i,j,L_{i,j}}]$. $\boldsymbol{Y}_j = [\boldsymbol{Y}_{1,j}, \boldsymbol{Y}_{2,j}, \cdots, \boldsymbol{Y}_{N,j}]$ represents all the degradation data under the stress of j, and all the data from the accelerated degradation test can be expressed as $\boldsymbol{Y} = \{\boldsymbol{Y}_1, \boldsymbol{Y}_2, \cdots, \boldsymbol{Y}_M\}$.

Based on this analysis, $\Delta\boldsymbol{Y}_{i,j} \sim N(\boldsymbol{\mu}_{i,j}, \boldsymbol{\Sigma}_{i,j})$, $\boldsymbol{\mu}_{i,j}$ is the expectancy matrix, and $\boldsymbol{\Sigma}_{i,j}$ is the covariance matrix, which can be expressed as follows:

$$\boldsymbol{\mu}_{i,j} = \alpha_{i,j}\Delta\boldsymbol{T}_{i,j} \tag{22}$$

$$\boldsymbol{\Sigma}_{i,j} = \beta_{i,j}^2\boldsymbol{D}_{i,j} + \sigma_\varepsilon^2\boldsymbol{F}_{i,j} \tag{23}$$

$$\Delta\boldsymbol{T}_{i,j}^{\mathrm{T}} = [\Delta T_{i,j,1}, \Delta T_{i,j,2}, \cdots \Delta T_{i,j,L_{i,j}}] \tag{24}$$

$$\Delta T_{i,j,k} = \Lambda(t_{i,j,k}|\boldsymbol{v}) - \Lambda(t_{i,j,k-1}|\boldsymbol{v}) \tag{25}$$

$$\boldsymbol{D}_{i,j} = \begin{pmatrix} \Delta t_{i,j,1} & 0 & \cdots & 0 \\ 0 & \Delta t_{i,j,k} & \vdots & \vdots \\ \vdots & \cdots & \ddots & 0 \\ 0 & \cdots & 0 & \Delta t_{i,j,k} \end{pmatrix}_{L_{i,j}\times L_{i,j}} \tag{26}$$

$$\Delta t_{i,j,k} = t_{i,j,k} - t_{i,j,k-1} \tag{27}$$

$$\boldsymbol{F}_{i,j} = \begin{pmatrix} 1 & -1 & 0 & \cdots & 0 \\ -1 & 2 & -1 & \cdots & \vdots \\ 0 & -1 & 2 & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & -1 \\ 0 & 0 & \cdots & -1 & 2 \end{pmatrix}_{L_{i,j}\times L_{i,j}} \tag{28}$$

Based on this analysis, the contour logarithmic likelihood function corresponding to the accelerated degradation data is as follows:

$$\begin{aligned} &\ln L(\boldsymbol{Y}|\boldsymbol{\Theta}) \\ &= -\frac{\ln 2\pi}{2}\sum_{i=1}^{N}\sum_{j=1}^{M}L_{i,j} - \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{M}\ln\left|\boldsymbol{\Sigma}_{i,j}\right| \\ &\quad -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{M}(\Delta\boldsymbol{Y}_{i,j} - \boldsymbol{\mu}_{i,j})^T\boldsymbol{\Sigma}_{i,j}^{-1}(\Delta\boldsymbol{Y}_{i,j} - \boldsymbol{\mu}_{i,j}) \end{aligned} \tag{29}$$

If $\tilde{\boldsymbol{\Sigma}}_{i,j} = \boldsymbol{\Sigma}_{i,j}/\beta_{i,j}^2, \tilde{\sigma}_\varepsilon^2 = \sigma_\varepsilon^2/\beta_{i,j}^2$, Equation (29) can be described by follows:

$$\begin{aligned} &\ln L(\boldsymbol{Y}|\boldsymbol{\Theta}) \\ &= -\frac{\ln 2\pi}{2}\sum_{i=1}^{N}\sum_{j=1}^{M}L_{i,j} \\ &\quad -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{M}\ln\left|\tilde{\boldsymbol{\Sigma}}_{i,j}\right| - \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{M}L_{i,j}\ln\beta_{i,j}^2 \\ &\quad -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{M}\frac{1}{\beta_{i,j}^2}(\Delta\boldsymbol{Y}_{i,j} - \alpha_{i,j}\Delta\boldsymbol{T}_{i,j})^T \\ &\quad \cdot\tilde{\boldsymbol{\Sigma}}_{i,j}^{-1}(\Delta\boldsymbol{Y}_{i,j} - \alpha_{i,j}\Delta\boldsymbol{T}_{i,j}) \end{aligned} \tag{30}$$

Taking the partial derivatives with respect to and for Equation (30) and making them equal to zero, we can obtain:

$$\hat{\alpha}_{i,j} = \frac{\Delta\boldsymbol{T}_{i,j}^{\mathrm{T}}\tilde{\boldsymbol{\Sigma}}_{i,j}^{-1}\Delta\boldsymbol{Y}_{i,j}}{\Delta\boldsymbol{T}_{i,j}^{\mathrm{T}}\tilde{\boldsymbol{\Sigma}}_{i,j}^{-1}\Delta\boldsymbol{T}_{i,j}} \tag{31}$$

$$\hat{\beta}_{i,j}^2 = \frac{(\Delta\boldsymbol{Y}_{i,j} - \alpha_{i,j}\Delta\boldsymbol{T}_{i,j})^T\tilde{\boldsymbol{\Sigma}}_{i,j}^{-1}(\Delta\boldsymbol{Y}_{i,j} - \alpha_{i,j}\Delta\boldsymbol{T}_{i,j})}{L_{i,j}} \tag{32}$$

Substituting Equations (31) and (32) into Equation (30), we can obtain:

$$\ln L(\boldsymbol{Y}\,|\boldsymbol{\Theta}) = -\frac{1+\ln 2\pi}{2}\sum_{i=1}^{N}\sum_{j=1}^{M} L_{i,j} - \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{M}\ln\left|\tilde{\boldsymbol{\Sigma}}_{i,j}\right| - \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{M} L_{i,j}\ln\hat{\beta}_{i,j}^{2} \tag{33}$$

The covariance matrix $\tilde{\boldsymbol{\Sigma}}_{i,j}$ contains unknown parameters $\boldsymbol{\nu}$ and σ_{ε}^{2}; the maximum value of Equation (33) can be obtained; and an estimate of the parameters $\hat{\boldsymbol{\nu}}$ and $\hat{\sigma}_{\varepsilon}^{2}$ can be obtained. Substituting $\hat{\boldsymbol{\nu}}$ and $\hat{\sigma}_{\varepsilon}^{2}$ into Equation (31) and Equation (32), and combining the degradation data of the equipment, the parameters $\hat{\alpha}_{i,j}$ and $\hat{\beta}_{i,j}^{2}$, which represent the estimated drift coefficient and diffusion coefficient for the i^{th} sample under the j^{th} stress, can be determined. The set of the total drift coefficient and the estimated diffusion coefficient is $\hat{\boldsymbol{\alpha}}$ and $\hat{\boldsymbol{\beta}}^{2}$. Based on the assumption of a random distribution of the drift coefficient and the proportional relationship above, we can obtain:

$$\begin{aligned}\ln L(\hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\beta}}^{2}\,|\boldsymbol{\Theta}) &= -\sum_{i=1}^{N}\sum_{j=1}^{M}\ln 2\pi - \sum_{i=1}^{N}\sum_{j=1}^{M}\ln\sigma_{a}^{2} + \sum_{i=1}^{N}\sum_{j=1}^{M} 2b/S_j \\ &\quad - \frac{1}{2\sigma_{a}^{2}}\sum_{i=1}^{N}\sum_{j=1}^{M}\frac{\left(\hat{\alpha}_{i,j}-\mu_{a}e^{-b/S_j}\right)^{2}+\left(\hat{\beta}_{i,j}^{2}/g-\mu_{\alpha}e^{-b/S_j}\right)^{2}}{e^{-2b/S_j}}\end{aligned} \tag{34}$$

$$\hat{\mu}_{\alpha} = \frac{\sum_{i=1}^{N}\sum_{j=1}^{M}\hat{\alpha}_{m,i} + \sum_{i=1}^{N}\sum_{j=1}^{M}\hat{\beta}_{i,j}^{2}/\hat{g}}{2\sum_{i=1}^{N}\sum_{j=1}^{M} e^{-2b/S_j}} \tag{35}$$

$$\hat{\sigma}_{a}^{2} = \frac{1}{2NM}\sum_{i=1}^{N}\sum_{j=1}^{M}\frac{\left(\hat{\alpha}_{i,j}-\hat{\mu}_{a}e^{-b/S_j}\right)^{2}+\left(\hat{\beta}_{i,j}^{2}/\hat{g}-\hat{\mu}_{\alpha}e^{-b/S_j}\right)^{2}}{e^{-2b/S_j}} \tag{36}$$

$$\hat{g} = \frac{\sum_{i=1}^{N}\sum_{j=1}^{M}\left(\hat{\beta}_{i,j}^{2}e^{b/S_j}\right)^{2}}{\hat{\mu}_{a}\sum_{i=1}^{N}\sum_{j=1}^{M}\hat{\beta}_{i,j}^{2}e^{b/S_j}} \tag{37}$$

Substituting Equations (35)~(37) into Equation (34) yields:

$$\ln L(\hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\beta}}^{2}\,|\boldsymbol{\Theta}) = -MN - \sum_{i=1}^{N}\sum_{j=1}^{M}\ln 2\pi - \sum_{i=1}^{N}\sum_{j=1}^{M}\ln\hat{\sigma}_{a}^{2} + \sum_{i=1}^{N}\sum_{j=1}^{M} 2b/S_j \tag{38}$$

The parameter estimation $\hat{b}$ can be obtained by determining the maximum value of Equation (38). $\hat{b}$ Then, is substituted into Equations (35)~(37), and, $\hat{\mu}_{\alpha}$, $\hat{\sigma}_{a}^{2}$, and $\hat{g}$ can be obtained.

IV. JOINT OPTIMIZATION MODEL OF THE REPLACEMENT AND SPARE PARTS PURCHASING STRATEGY

Replacement and spare-parts ordering are the key factors that restrict the efficient operation of equipment. The reasonable replacement and spare-parts strategy can effectively reduce the cost in the life cycle of the equipment. Currently, the renewal reward theorem has been widely used in the study of equipment maintenance decisions [24]. With this theorem, the mathematical relationship between equipment maintenance cost per unit time and maintenance decision variables can be easily expressed, which could lay a solid foundation for the forthcoming study.

In this study, the RUL prediction information, in combination with the renewal reward theorem, was used to develop the joint optimization model of equipment replacement and spare-parts purchasing strategy to minimize the cost of one life cycle. The detailed information can be expressed as follows:

$$\min c(t_p, t_s) = \frac{E\left(C(\tau)\right)}{E(\tau)} \tag{39}$$

where t_p is the preventive replacement time; t_s is the spare part procurement time; and $c(t_p, t_s)$ is the average cost ratio of the equipment operating cycle under the circumstance of (t_p, t_s). $C(\tau)$ is the total cost of running the equipment for one life cycle; τ is one life cycle of the equipment; and $E(\cdot)$ represents the expectation.

Also, the basic assumptions of the joint optimization model can be given:

Assumption 1: The target equipment runs under normal working stress and exhibits single performance degradation, and its degradation process satisfies the nonlinear Wiener process.

Assumption 2: The equipment begins to run at the initial moment, and there is no spare-part inventory at the initial moment, but at most one spare part is purchased or stored at any time thereafter.

Assumption 3: There is a fixed delivery period for spare parts from the beginning of procurement to the arrival of the goods.

Assumption 4: The time required to replace the equipment can be neglected compared to the running time. If the equipment can still run properly after the spare parts arrive, preventive replacement is performed at time t_p, and the replacement cost is C_p. The storage cost of spare parts per unit time is H_1. If the equipment fails before the spare parts arrive, the replacement cost is C_f, and the unit time loss caused by downtime is H_2.

For Assumption 1, the Wiener process can accurately describe monotonic and nonmonotonic degradation processes, and its universal applicability makes degradation modeling more general. For Assumption 2, taking single

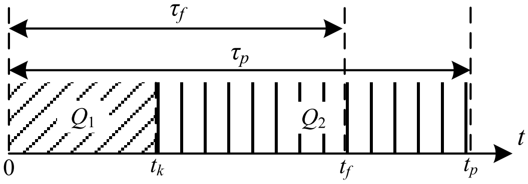


FIGURE 1. Replacement strategy.

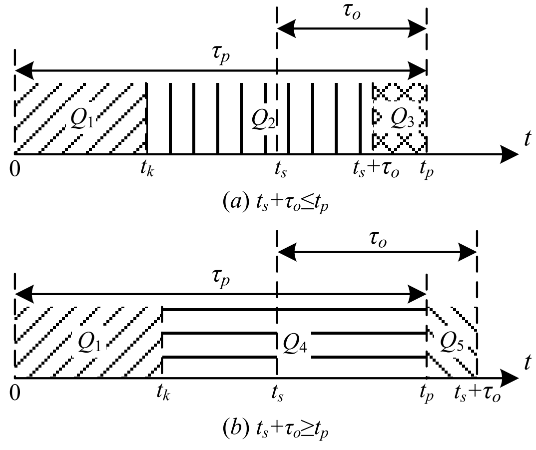


FIGURE 2. Spare parts ordering strategy.

equipment as the research object is representative, and similar ideas can be used for modeling and analysis under multiple equipment conditions. Assumptions 3 and 4 are set based on the real process of spare parts ordering and equipment maintenance support.

In this study, progressive analytical thinking was used to analyze the equipment replacement strategy model and spare parts ordering strategy model, and then, the combined optimization model was obtained.

A. REPLACEMENT STRATEGY MODEL

The replacement strategy model is primarily used to determine the optimal preventive replacement time to balance the costs of preventive replacement and failure replacement of the equipment. A schematic diagram of the replacement strategy is shown in Figure 1.

Figure 1 shows the possible replacement process of the equipment in a life cycle, and the maintenance costs incurred during a replacement cycle may include preventive replacement costs or inoperative replacement costs.

If the equipment does not fail before the preventive replacement time, preventative replacement can proceed under time t_p, and the total cost is C_p. If the equipment fails between the current operating time t_k and the preventive replacement time t_p, fault replacement can be performed immediately under time t_f, and the total cost is C_f. Thus, the total cost expectation corresponding to the replacement strategy model is:

$$E\left(C(\tau)\right) = C_p + (C_f - C_p)F_{L_k|S_0}(t_p - t_k\,|S_0\,,\boldsymbol{Y}_{1:k}) \quad (40)$$

Figure 1 shows that the running time of equipment in the replacement strategy model can be divided into two areas of Q_1 and Q_2, where Q_1 is the area before the current running time, in which the equipment is running properly, and Q_2 is the area in the future running time, in which the equipment is running properly. Thus, the expectation of the operation period corresponding to the replacement strategy model is as follows:

$$\begin{aligned} E(\tau) &= E(\tau_f) + E(\tau_p) = \tau_f P(\tau_f) + \tau_p P(\tau_p) \\ &= t_p - \int_0^{t_p - t_k} F_{L_k|S_0}(l_k\,|S_0\,,\boldsymbol{Y}_{1:k})\mathrm{d}l_k \end{aligned} \quad (41)$$

See Appendix A for the details on the solution process for Equation (40) and Equation (41).

B. SPARE PARTS ORDERING STRATEGY MODEL

The model of the spare parts purchasing strategy is primarily used to determine the optimal spare-parts ordering time to balance the cost caused by spare-parts shortage and spare parts storage. Figure 2 shows a schematic diagram of the spare parts ordering strategy.

Figure 2 shows the procurement process of spare parts in a life cycle, and the related costs incurred in a procurement cycle may include the storage cost of spare parts or the loss of spare parts shortage. If $t_s + \tau_o \leq t_p$, then the spare parts arrived before preventive replacement of equipment, and the spare parts were in storage; if $t_s+\tau_o \geq t_p$, then the spare parts could not arrive before preventive replacement of equipment. If τ^+ and τ^- represent the storage time and shortage time of spare parts, $\tau^+ = t_p - t_s - \tau_o$, $\tau^- = t_s + \tau_o - t_p$. Then, the total cost expectation corresponding to the spare parts ordering can be written as:

$$\begin{aligned} E\left(C(\tau)\right) &= H_1 E(\tau^+) + H_2 E(\tau^-) \\ &= H_1 \int_{t_s+\tau_o-t_k}^{t_p-t_k} \left(1 - F_{L_k|S_0}(l_k\,|S_0\,,\boldsymbol{Y}_{1:k})\right)\mathrm{d}l_k \\ &\quad + H_2 \int_{t_s-t_k}^{t_s+\tau_o-t_k} F_{L_k|S_0}(l_k\,|S_0\,,\boldsymbol{Y}_{1:k})\mathrm{d}l_k \end{aligned} \quad (42)$$

Figure 2 shows that if $t_s + \tau_o \leq t_p$, preventive replacement is performed when the equipment is in good condition, and its operation cycle is τ_p. If $t_s + \tau_o \geq t_p$, the operating period of the equipment is prolonged due to the waiting for spare parts, $\tau_{p+}\tau^-$. Thus, the expectation of the spare parts ordering strategy model corresponding to the equipment operation cycle is as follows:

$$\begin{aligned} E(\tau) &= E(\tau_p) + E(\tau^-) \\ &= t_k + \int_0^{t_p-t_k} \left(1 - F_{L_k|S_0}(l_k\,|S_0\,,\boldsymbol{Y}_{1:k})\right)\mathrm{d}l_k \\ &\quad + \int_{t_s-t_k}^{t_s+\tau_o-t_k} F_{L_k|S_0}(l_k\,|S_0\,,\boldsymbol{Y}_{1:k})\mathrm{d}l_k \end{aligned} \quad (43)$$

See Appendix B for the details on the solution process for Equation (42) and Equation (43).

C. JOINT OPTIMIZATION MODEL

Considering the entire process of replacement and spare-parts ordering during the life cycle, a joint optimization model of replacement and spare-parts ordering strategy was developed to determine the optimal spare-parts ordering time and preventive replacement time to achieve the lowest average cost rate of the equipment operation cycle.

Based on this analysis, the total cost in the joint optimization model includes the cost generated by equipment replacement and the cost generated by spare parts ordering, which can be described as follows:

$$\begin{aligned} &E(C(\tau)) \\ &= C(\tau_f)P(\tau_f) + C(\tau_p)P(\tau_p) + H_1E(\tau^+) + H_2E(\tau^-) \\ &= C_p + (C_f - C_p)F_{L_k|S_0}(t_p - t_k\,|S_0\,, \boldsymbol{Y}_{1:k}) \\ &\quad + H_1\int_{t_s+\tau_o-t_k}^{t_p-t_k}\left(1 - F_{L_k|S_0}(l_k\,|S_0\,, \boldsymbol{Y}_{1:k})\right)\mathrm{d}l_k \\ &\quad + H_2\int_{t_s-t_k}^{t_s+\tau_o-t_k} F_{L_k|S_0}(l_k\,|S_0\,, \boldsymbol{Y}_{1:k})\mathrm{d}l_k \end{aligned} \tag{44}$$

Considering that any replacement moment is equivalent to the end of the current running cycle or the beginning of the next running cycle, comparing the expression of equipment operation cycle expectation between the replacement strategy model and the spare parts ordering model, the expected operating cycle of the equipment in the easily obtained joint optimization model is as follows:

$$\begin{aligned} E(\tau) &= E(\tau_p) + E(\tau^-) \\ &= t_k + \int_0^{t_p-t_k}\left(1 - F_{L_k|S_0}(l_k\,|S_0\,, \boldsymbol{Y}_{1:k})\right)\mathrm{d}l_k \\ &\quad + \int_{t_s-t_k}^{t_s+\tau_o-t_k} F_{L_k|S_0}(l_k\,|S_0\,, \boldsymbol{Y}_{1:k})\mathrm{d}l_k \end{aligned} \tag{45}$$

Considering the derivation of the spare-parts ordering strategy model, the optimal replacement time (t_p) in the joint optimization model should not be less than $t_s + \tau_o$.

V. CASE STUDY

Micro-electro mechanical system (MEMS) gyroscopes are the core equipment of modern navigation and positioning systems; have high service reliabilities and long effective life cycles; and have been widely used in aviation, aerospace, and equipment. In this study, the RUL of a target piece of equipment is predicted based on the step-stress accelerated degradation data and field monitoring data of a certain MEMS gyroscope, and the joint optimization decision of equipment replacement and spare parts ordering strategy is made accordingly. The step-stress accelerated degradation test included 4 samples and 3 groups of stress levels ($S_1 = 40°\text{C}$, $S_2 = 70°\text{C}$, $S_3 = 100°\text{C}$), and 50 samples were taken at an interval of 10 h under each group of stress conditions. Fifty samples were taken at an interval of 10 h. The field monitoring data included all degradation data of the target equipment operating under normal stress ($S_0 = 25°\text{C}$) for 180 days, and the specific degradation process is shown in Figures 3 and 4.

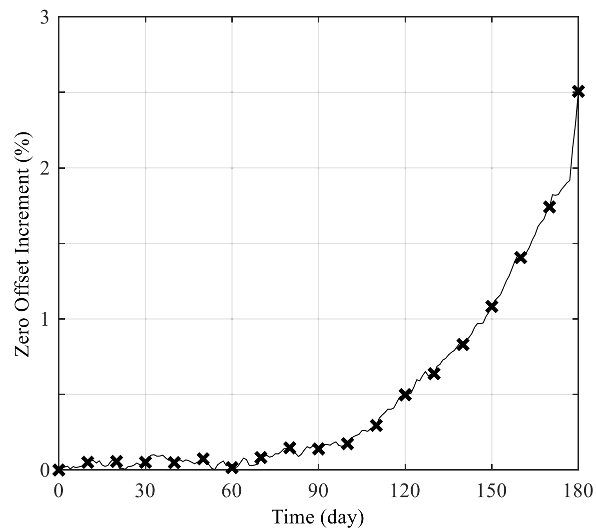


FIGURE 3. Field monitoring data.

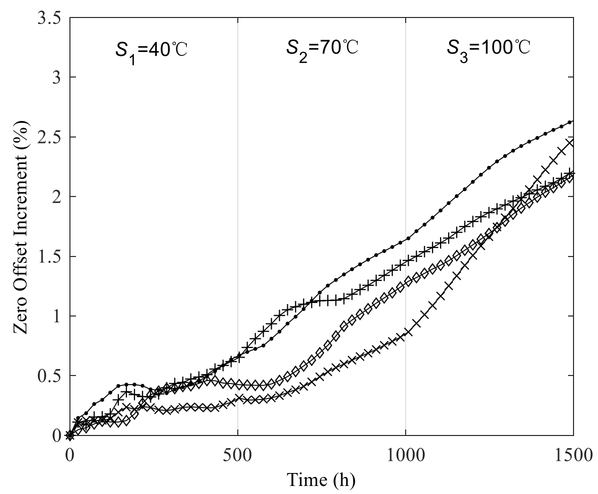


FIGURE 4. Accelerated degradation data.

To verify the accuracy of the parameter estimation method, we simulate the degradation data of the equipment under normal stress (S_0) for different $\boldsymbol{\Theta}$ and calculate the estimated value $\hat{\boldsymbol{\Theta}}$ using the proposed step-by-step MLE estimation method. The parameter estimation results are shown in TABLE 1.

TABLE 1 shows that the relative errors between the estimated and real values of the different parameters are all less than 30%, which indicates that the step-by-step MLE estimation method can accurately estimate the parameters.

The degradation process of the MEMS gyroscope is nonlinear under normal stress conditions. We let

TABLE 1. Comparison of parameter estimation.

	$\boldsymbol{\Theta}$	$\hat{\boldsymbol{\Theta}}$	Relative error
1	0.5	0.521	4.20%
	2×10-6	1.7161×10-6	14.20%
	2	1.8664	6.70%
	0.02	0.0216	7.80%
	1×10-8	1.2122×10-8	21.20%
	0.001	1.0340×10-3	3.40%
2	0.3	0.3096	3.20%
	5×10-6	4.0951×10-6	18.10%
	1.5	1.3845	7.70%
	0.01	0.0109	8.80%
	2×10-8	2.6142×10-8	29.70%
	0.0015	1.5362×10-3	2.40%
3	1	1.0921	9.20%
	3×10-6	2.4243×10-6	19.20%
	4	3.6523	8.70%
	0.03	0.0323	7.70%
	3×10-8	3.8488×10-8	27.30%
	0.0005	5.3206×10-4	6.40%

TABLE 2. Priori parameter estimation.

Parameter	Units	Parameter estimation
μ_a	--	0.1173
σ_a^2	--	2.0170×10^{-6}
b	K	3.7276
v	day^{-1}	1.5669×10^{-2}
σ_ε^2	--	1.1802×10^{-8}
g	--	0.0045

$\Lambda(t\,|\boldsymbol{\nu}) = \exp(\nu t) - 1$ in this study. Using the acceleration degradation data of the MEMS gyroscope in Figure 4 and the a priori parameter estimation method proposed above, the a priori parameter estimates can be described as follows.

The prior parameter estimation results in TABLE 2 are calculated from accelerated degradation data (hour to day). Also, on-site monitoring data were used to update the state based on the Kalman filter method. The detailed update process is shown in Figure 5.

Generally, when the zero offset increment of the MEMS gyroscope exceeds 2.5% of the initial value, it can be considered to fail, and the failure threshold value $D = 2.5$. Therefore, the target equipment fails at 180 days; thus, the real life of the target piece of equipment is 180 days. Based on this analysis, the RUL online prediction method was used to perform the online prediction of RUL of target equipment under the constant stress condition. For the ease of comparative analysis, the joint optimization model proposed in this paper is referred to as M0, and the joint optimization model proposed in [14] is referred to as M1. The online prediction method of RUL proposed in [11] is applied to the joint optimization model of replacement and spare-parts ordering, which can be marked as M2. The RUL prediction result, mean squared error (MSE) and 95% confidence interval of RUL prediction corresponding to different methods are shown in TABLE 3.

The MSE of RUL can be calculated by Equation (46):

$$\mathrm{MSE} = \int_0^\infty (l_k - T + t_k)^2 f_{L_k|S_0}(l_k\,|S_0\,, \boldsymbol{Y}_{1:k})\mathrm{d}l_k \quad (46)$$

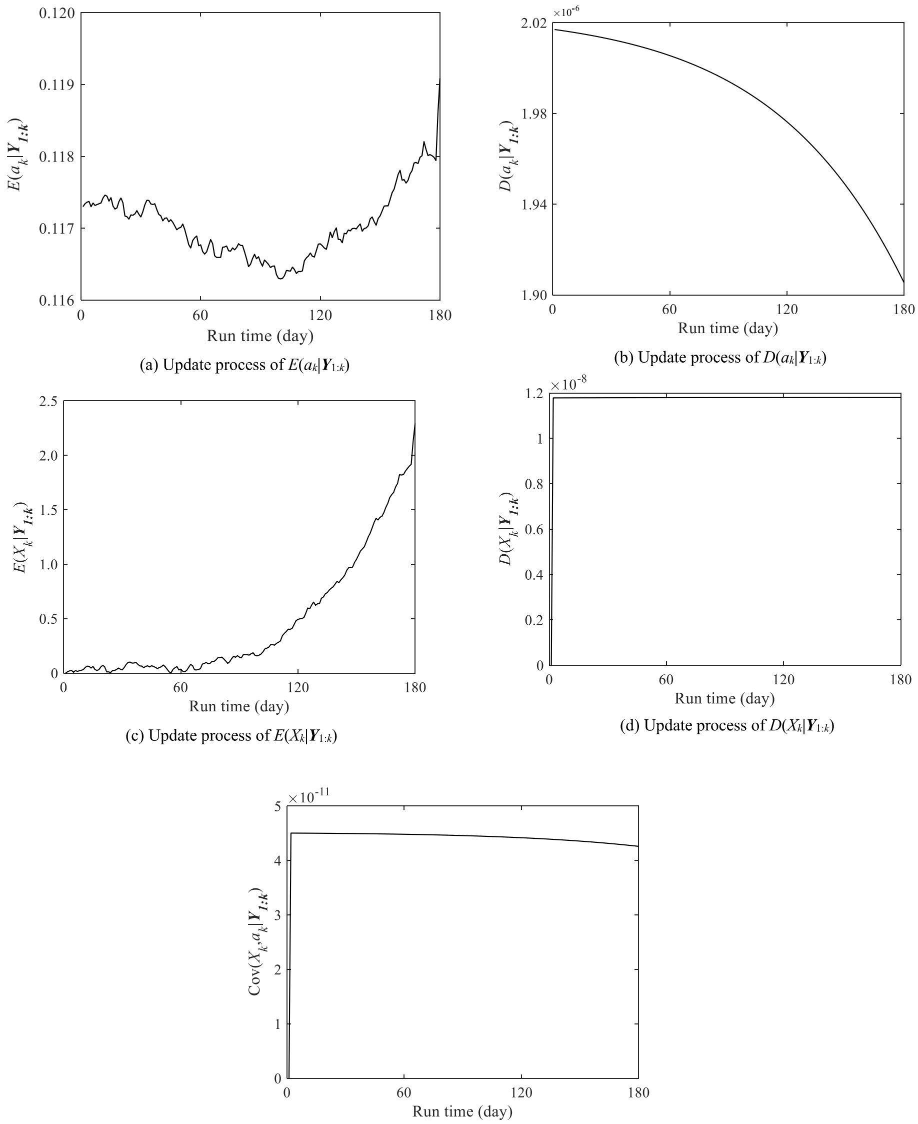
where T is the lifetime of the piece of equipment.

TABLE 3 shows that M0 has more accurate RUL prediction results and smaller MSE than M1 and M2 at different state monitoring times. The 95% confidence interval of RUL corresponding to M0 can completely contain the real RUL of the target equipment, which indicates that the M0 model can predict the RUL of equipment more accurately and will have a positive impact on the subsequent joint optimization decision of the replacement and spare parts ordering strategy. Further analysis of TABLE 3 shows that M1 has the narrowest confidence interval width compared to M0 and M2, which shows that the uncertainty of RUL prediction of M1 is small and the prediction accuracy is high. The primary cause of this result is that M1 only updated the drift coefficient as a random variable online, while M0 and M2 updated the drift and diffusion coefficients synchronously, increasing prediction uncertainty. However, M1 can achieve a high prediction accuracy based on the loss of precision, which keeps the confidence interval from covering the real RUL of the target piece of equipment, which is not conducive to scientific maintenance decisions. Comparing M0 and M2, the prediction results of M0 and M2 are found to be similar, but the width of the confidence interval under M0 is narrower than that under M2, which shows that M0 has a lower uncertainty and better performance based on ensuring the accuracy of the RUL prediction.

To compare and analyze the advantages and disadvantages of M0, M1 and M2 in the joint optimization decision of replacement and spare-parts ordering, the variation curves of the average cost ratio of the equipment operation cycle corresponding to different models are calculated when it is 40, 80 and 120 days, as shown in Figure 6, where $t_p = t_s + \tau_o + \tau_p$, $C_P = 50$ RMB, $C_f = 100$ RMB, $H_1 = 1.5$ RMB/day, $H_2 = 150$ RMB/day, and $\tau_o = 20$ day.

The optimal spare-parts ordering time and preventive replacement time with different monitoring times is shown in Figure 6, and the corresponding results are shown in TABLE 4.

TABLE 4 shows that the average cost ratio of M0 in the operation cycle is lower than that of M1 and M2 at different monitoring times, which demonstrates that the proposed method is better than the traditional method. The primary reason for this result is that the proposed RUL prediction method performs better than M1 and M2; thus, the preventive replacement time and spare-parts ordering time can be determined more scientifically. Based on the abovementioned theory, RUL prediction results have a strong



(a) Update process of $E(a_k|\boldsymbol{Y}_{1:k})$

(b) Update process of $D(a_k|\boldsymbol{Y}_{1:k})$

(c) Update process of $E(X_k|\boldsymbol{Y}_{1:k})$

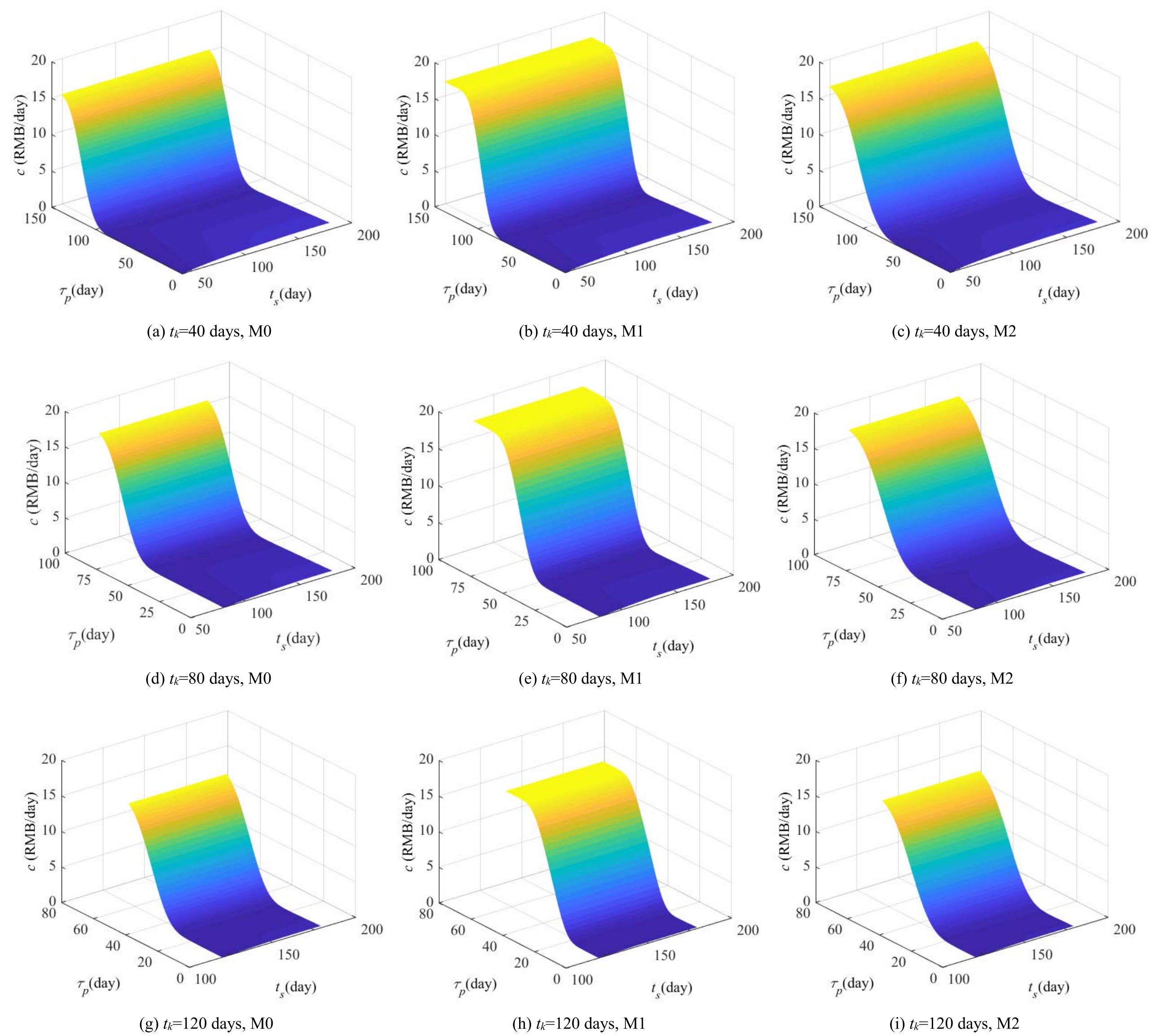
(d) Update process of $D(X_k|\boldsymbol{Y}_{1:k})$

(e) Update process of $\mathrm{Cov}(X_k,a_k|\boldsymbol{Y}_{1:k})$

FIGURE 5. Degradation state update process.

TABLE 3. RUL prediction.

	Condition monitoring time (day)	True RUL (day)	MSE of RUL (day^2)	Predicted mean value of RUL (day)	95% confidence interval for RUL (day)	Width of confidence interval (day)
M0	40	140	217.1	128.8	[110.0,144.0]	34.0
	80	100	101.8	92.7	[77.5,105.8]	28.3
	120	60	040.8	57.1	[42.3, 67.3]	25.0
	160	20	22.8	21.8	[13.3, 30.0]	16.7
M1	40	140	944.6	110.5	[96.5,122.3]	25.8
	80	100	613.4	75.4	[64.5, 85.5]	21.0
	120	60	294.7	42.9	[35.3, 50.3]	15.0
	160	20	31.2	14.6	[9.5, 19.8]	10.3
M2	40	140	711.7	119.2	[88.0,142.5]	54.5
	80	100	316.3	86.2	[63.5,105.0]	41.5
	120	60	96.2	53.9	[39.0, 68.0]	29.0
	160	20	20.8	21.4	[13.1, 30.2]	17.1



(a) t_k=40 days, M0
(b) t_k=40 days, M1
(c) t_k=40 days, M2
(d) t_k=80 days, M0
(e) t_k=80 days, M1
(f) t_k=80 days, M2
(g) t_k=120 days, M0
(h) t_k=120 days, M1
(i) t_k=120 days, M2

FIGURE 6. Cost curves.

TABLE 4. Joint optimization results.

t_k (day)	t_s (day)			t_p (day)			c (RMB/day)		
	M0	M1	M2	M0	M1	M2	M0	M1	M2
40	118	107	93	138	127	113	0.3716	0.4002	0.4580
80	128	118	113	148	138	133	0.3433	0.3676	0.3835
120	138	130	131	158	150	151	0.3215	0.3373	0.3344

influence on the result of the joint decision of replacement and spare-parts ordering.

Based on this analysis, the control variable method was used to analyze the parameter sensitivity of the proposed joint optimization model, where the condition monitoring time $t_k = 80$ days; the maintenance cost $C_p \in [1, 100]$ RMB, $C_f \in [50, 500]$ RMB; the spare-parts ordering cost $H_1 \in [1, 10]$ RMB/day, $H_2 \in [100, 200]$ RMB/day, and $\tau_o \in [1, 50]$ day. The quantitative relationship between each parameter and the optimal joint decision result is shown in Figures 7 through 11.

Figure 7 shows that the average cost ratio in one operating cycle is strongly influenced by the cost of preventive replacement. The average cost ratio increases linearly with C_p, and its corresponding growth rate is 0.0067 day^{-1}. The optimal spare-parts purchase time t_s and preventive

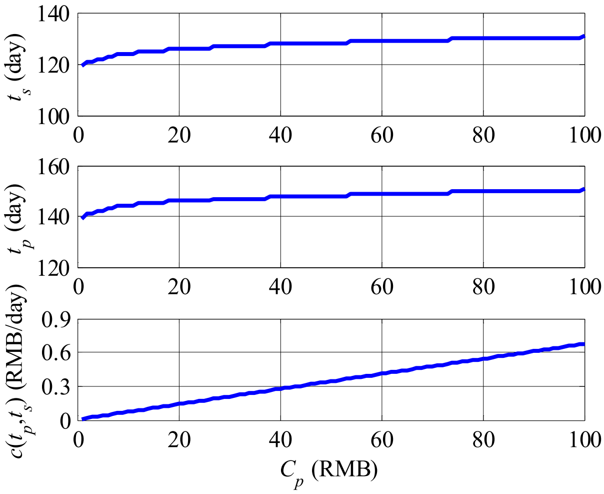


FIGURE 7. Sensitivity analysis of C_p.

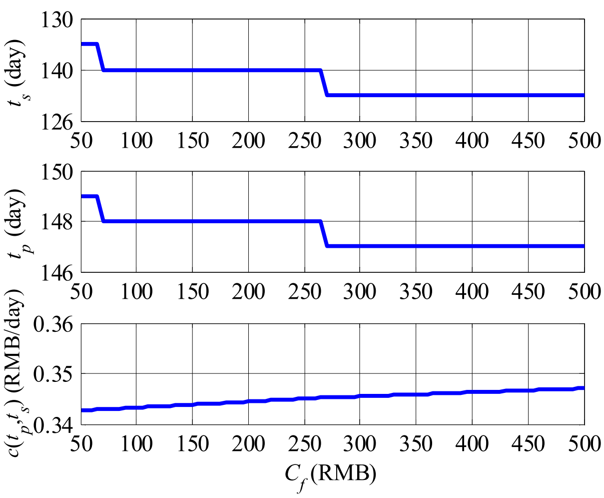


FIGURE 8. Sensitivity analysis of C_f.

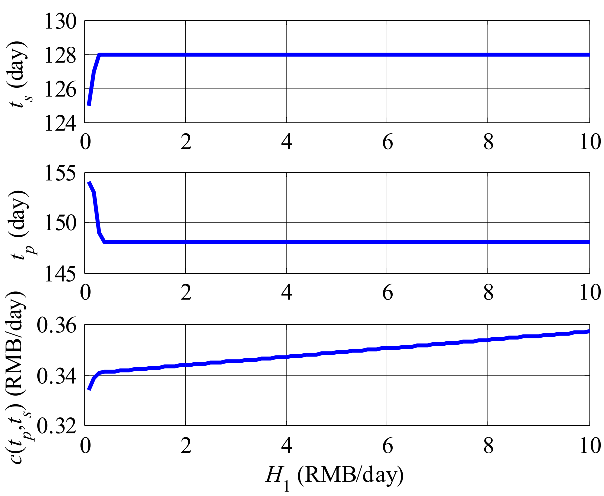


FIGURE 9. Sensitivity analysis of H_1.

replacement time t_p; t_s and t_p show a stepwise growth with the increase of C_p; and the corresponding increment is 12 days.

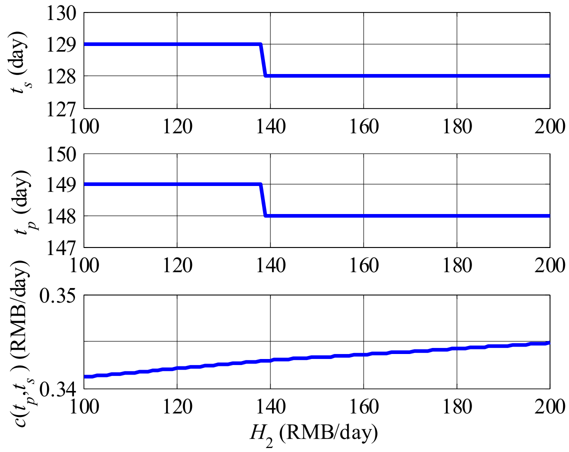


FIGURE 10. Sensitivity analysis of H_2.

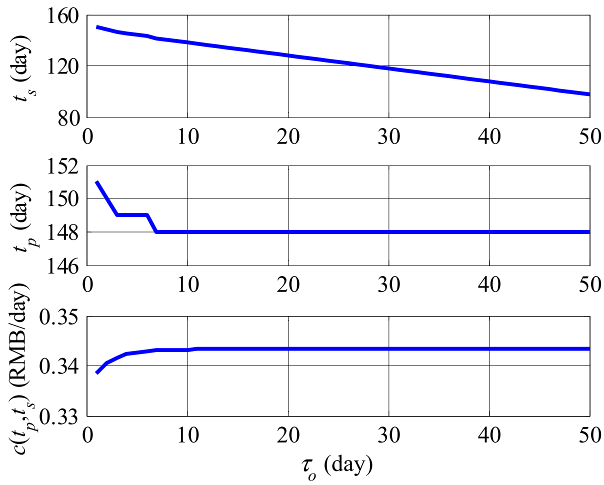


FIGURE 11. Sensitivity analysis of τ_o.

Results show that the optimal spare parts ordering time and preventive replacement time are sensitive to the change in preventive replacement cost. With further analysis, the precision of the optimal joint decision increases with the increase of C_p, in which the higher preventive replacement costs can lead to marked increases in the total operating costs. Therefore, the increase in t_s and t_p could increase the expected operation time of the equipment.

Figure 8 shows that the average cost ratio of the equipment operation cycle increases with increasing failure replacement cost, but this increase is relatively small, which indicates that the average cost ratio of the equipment operating cycle is not sensitive to the change. This result demonstrates that the probability of failure replacement will markedly decrease, thus reducing the impact of the failure replacement fee on the total operating cost when the RUL of equipment can be accurately predicted. Similarly, the variance of C_f has a negligible effect on the optimal spare parts ordering time and preventive replacement time. When t_s and t_p are 65 RMB and 265 RMB, a sudden increase occurs.

Figure 9 shows that the average cost ratio in one operation cycle is highly sensitive to the storage cost of the spare parts and exhibits a linear trend except for the point of 0.4 RMB/day. When the increment of H_1 is 1 RMB/day, the average cost ratio in one operation cycle is 0.0017 RMB/day. H_1 has a negligible influence on the optimal spare parts purchase time t_s and preventive replacement time t_p, which is primarily because when t_s and t_p remain constant, the cost of the equipment maintenance (replacement) remains constant, resulting in the total operating cost of the equipment being primarily affected by the related cost of spare parts. When H_2 remains constant, H_1 has a negligible influence on $c(t_p, t_s)$.

Figure 10 shows that the average cost ratio in one operation cycle changes marginally with the change in the spare parts shortage loss, and t_s and t_p remain stable. Therefore, the sensitivity of the optimal joint decision pairs is low.

Figure 11 shows that the average cost ratio and preventive replacement time of the equipment operation cycle remain constant in different spare parts ordering periods. The spare parts purchasing time decreases linearly with increasing τ_o, and the rate of decline was approximately 1.0612. Further analysis shows that the optimal (t_s, t_p) and τ_o obey the equation. $\tau_o + t_s = t_p$, which demonstrates that when the spare parts ordering strategy perfectly fits the equipment maintenance strategy, the optimal solution of the joint optimization model can be realized.

VI. CONCLUSION

In this study, the online prediction of RUL is achieved by integrating accelerated degradation data and on-site monitoring data. The optimal replacement and spare parts ordering strategies are obtained using the RUL prediction information.

1) To predict the RUL of the accelerated degradation equipment, it is necessary to update the drift coefficient and diffusion coefficient of the equipment simultaneously, which can effectively improve RUL prediction accuracy.

2) The accelerated degradation model with the proportional relationship can more intuitively describe the true degenerate features of the equipment, achieves better model fitting, and reduces the uncertainty of RUL prediction.

3) The higher the accuracy and precision of the RUL prediction method are, the more helpful it is to obtain the optimal maintenance strategy. Higher accuracies can also achieve lower cost consumptions and effectively improve maintenance efficiencies.

In this study, the time-scale transformation model is used, and the application of this model has certain limitations. In future research, we intend to test a more general nonlinear Wiener degradation model, such as in [25], to expand the applicability of the proposed method.

APPENDIX A

Based on the basic principles of probability theory, we can obtain:

$$\begin{aligned}
&E\left(C(\tau)\right)\\
&= C(\tau_f)P(\tau_f) + C(\tau_p)P(\tau_p)\\
&= C_f P(\tau_f) + C_p P(\tau_p)\\
&= C_f P(t_k < \tau \le t_p) + C_p P(\tau \ge t_p)\\
&= C_f P(0 < \tau - t_k \le t_p - t_k) + C_p P(\tau - t_k \ge t_p - t_k)\\
&= C_f P(0 < l_k \le t_p - t_k) + C_p P(l_k \ge t_p - t_k)\\
&= C_f \int_0^{t_p - t_k} f_{L_k|S_0}(l_k\,|S_0\,, \boldsymbol{Y}_{1:k})\mathrm{d}l_k\\
&\quad + C_p \int_{t_p - t_k}^{+\infty} f_{L_k|S_0}(l_k\,|S_0\,, \boldsymbol{Y}_{1:k})\mathrm{d}l_k\\
&= C_f \int_0^{t_p - t_k} f_{L_k|S_0}(l_k\,|S_0\,, \boldsymbol{Y}_{1:k})\mathrm{d}l_k\\
&\quad + C_p \left(1 - \int_0^{t_p - t_k} f_{L_k|S_0}(l_k\,|S_0\,, \boldsymbol{Y}_{1:k})\mathrm{d}l_k\right)\\
&= C_p + (C_f - C_p)F_{L_k|S_0}(t_p - t_k\,|S_0\,, \boldsymbol{Y}_{1:k})
\end{aligned} \tag{A1}$$

From the basic property of opposite events, we know that:

$$P(\tau_f) + P(\tau_p) = 1 \tag{A2}$$

Based on Equation (A2), $E(\tau)$ can be calculated as follows:

$$\begin{aligned}
&E(\tau)\\
&= \tau_f P(\tau_f) + \tau_p P(\tau_p)\\
&= t_k + (\tau_f - t_k)P(\tau_f) + (\tau_p - t_k)P(\tau_p)\\
&= t_k + \int_0^{t_p - t_k} l_k f_{L_k|S_0}(l_k\,|S_0\,, \boldsymbol{Y}_{1:k})\mathrm{d}l_\mathrm{k}\\
&\quad + (t_p - t_k)\int_{t_p - t_k}^{+\infty} f_{L_k|S_0}(l_k\,|S_0\,, \boldsymbol{Y}_{1:k})\mathrm{d}l_k\\
&= t_k + \int_0^{t_p - t_k} l_k \mathrm{d}F_{L_k|S_0}(l_k\,|S_0\,, \boldsymbol{Y}_{1:k})\\
&\quad + (t_p - t_k)\left(1 - F_{L_k|S_0}(t_p - t_k\,|S_0\,, \boldsymbol{Y}_{1:k})\right)\\
&= t_k + (t_p - t_k)F_{L_k|S_0}(t_p - t_k\,|S_0\,, \boldsymbol{Y}_{1:k})\\
&\quad - \int_0^{t_p - t_k} F_{L_k|S_0}(l_k\,|S_0\,, \boldsymbol{Y}_{1:k})\mathrm{d}l_k\\
&\quad + (t_p - t_k)\left(1 - F_{L_k|S_0}(t_p - t_k\,|S_0\,, \boldsymbol{Y}_{1:k})\right)\\
&= t_p - \int_0^{t_p - t_k} F_{L_k|S_0}(l_k\,|S_0\,, \boldsymbol{Y}_{1:k})\mathrm{d}l_k =
\end{aligned} \tag{A3}$$

APPENDIX B

Based on the basic principles of probability theory, we can obtain:

$$\begin{aligned}
&E(\tau^-)\\
&= \int_0^{t_s - t_k} \tau_o f_{L_k|S_0}(l_k\,|S_0\,, \boldsymbol{Y}_{1:k})\mathrm{d}l_\mathrm{k}\\
&\quad + \int_{t_s - t_k}^{t_s + \tau_o - t_k} (t_s + \tau_o - t_p) f_{L_k|S_0}(l_k\,|S_0\,, \boldsymbol{Y}_{1:k})\mathrm{d}l_\mathrm{k}\\
&= \tau_o F_{L_k|S_0}(t_s - t_k\,|S_0\,, \boldsymbol{Y}_{1:k})\\
&\quad + \int_{t_s - t_k}^{t_s + \tau_o - t_k} (t_s + \tau_o - t_k - l_k) f_{L_k|S_0}(l_k\,|S_0\,, \boldsymbol{Y}_{1:k})\mathrm{d}l_\mathrm{k}
\end{aligned}$$

$$
\begin{aligned}
&= \tau_o F_{L_k|S_0}(t_s - t_k |S_0 , \boldsymbol{Y}_{1:k}) \\
&\quad +(t_s + \tau_o - t_k)F_{L_k|S_0}(t_s + \tau_o - t_k |S_0 , \boldsymbol{Y}_{1:k}) \\
&\quad -(t_s + \tau_o - t_k)F_{L_k|S_0}(t_s - t_k |S_0 , \boldsymbol{Y}_{1:k}) \\
&\quad -\int_{t_s-t_k}^{t_s+\tau_o-t_k} l_k f_{L_k|S_0}(l_k |S_0 , \boldsymbol{Y}_{1:k})\mathrm{dl_k} \\
&= \int_{t_s-t_k}^{t_s+\tau_o-t_k} F_{L_k|S_0}(l_k |S_0 , \boldsymbol{Y}_{1:k})\mathrm{d}l_k
\end{aligned} \tag{B1}
$$

$$
\begin{aligned}
&E(\tau^+) \\
&= \int_{t_s+\tau_o-t_k}^{t_p-t_k} (t_p - t_s - \tau_o) f_{L_k|S_0}(l_k |S_0 , \boldsymbol{Y}_{1:k})\mathrm{dl_k} \\
&\quad +(t_p - t_s - \tau_o)\int_{t_p-t_k}^{+\infty} f_{L_k|S_0}(l_k |S_0 , \boldsymbol{Y}_{1:k})\mathrm{d}l_k \\
&= \int_{t_s+\tau_o-t_k}^{t_p-t_k} (l_k + t_k - t_s - \tau_o) f_{L_k|S_0}(l_k |S_0 , \boldsymbol{Y}_{1:k})\mathrm{dl_k} \\
&\quad +(t_p - t_s - \tau_o)\int_{t_p-t_k}^{+\infty} f_{L_k|S_0}(l_k |S_0 , \boldsymbol{Y}_{1:k})\mathrm{d}l_k \\
&= \int_{t_s+\tau_o-t_k}^{t_p-t_k} l_k f_{L_k|S_0}(l_k |S_0 , \boldsymbol{Y}_{1:k})\mathrm{d}l_k \\
&\quad -\int_{t_s+\tau_o-t_k}^{t_p-t_k} (t_s + \tau_o - t_k) f_{L_k|S_0}(l_k |S_0 , \boldsymbol{Y}_{1:k})\mathrm{dl_k} \\
&\quad +(t_p - t_s - \tau_o)\left(1 - F_{L_k|S_0}(t_p - t_k |S_0 , \boldsymbol{Y}_{1:k})\right) \\
&= (t_p - t_k)F_{L_k|S_0}(t_p - t_k |S_0 , \boldsymbol{Y}_{1:k}) \\
&\quad -(t_s + \tau_o - t_k)F_{L_k|S_0}(t_s + \tau_o - t_k |S_0 , \boldsymbol{Y}_{1:k}) \\
&\quad -\int_{t_s+\tau_o-t_k}^{t_p-t_k} F_{L_k|S_0}(l_k |S_0 , \boldsymbol{Y}_{1:k})\mathrm{dl_k} \\
&\quad -(t_s + \tau_o - t_k) f_{L_k|S_0}(t_p - t_k |S_0 , \boldsymbol{Y}_{1:k}) \\
&\quad +(t_s + \tau_o - t_k) f_{L_k|S_0}(t_s + \tau_o - t_k |S_0 , \boldsymbol{Y}_{1:k}) \\
&\quad +(t_p - t_s - \tau_o) - (t_p - t_s - \tau_o)F_{L_k|S_0}(t_p - t_k |S_0 , \boldsymbol{Y}_{1:k}) \\
&= t_p - t_s - \tau_o - \int_{t_s+\tau_o-t_k}^{t_p-t_k} F_{L_k|S_0}(l_k |S_0 , \boldsymbol{Y}_{1:k})\mathrm{dl_k} \\
&= \int_{t_s+\tau_o-t_k}^{t_p-t_k} \left(1 - F_{L_k|S_0}(l_k |S_0 , \boldsymbol{Y}_{1:k})\right)\mathrm{d}l_k
\end{aligned} \tag{B2}
$$

$E(\tau_p)$ can be calculated as follows:

$$
\begin{aligned}
&E(\tau_p) \\
&\quad = t_k + \int_0^{t_p-t_k} l_k f_{L_k|S_0}(l_k |S_0 , \boldsymbol{Y}_{1:k})\mathrm{dl_k} \\
&\qquad +(t_p - t_k)\int_{t_p-t_k}^{+\infty} f_{L_k|S_0}(l_k |S_0 , \boldsymbol{Y}_{1:k})\mathrm{d}l_k \\
&\quad = t_p - \int_0^{t_p-t_k} F_{L_k|S_0}(l_k |S_0 , \boldsymbol{Y}_{1:k})\mathrm{d}l_k \\
&\qquad + = t_k + \int_0^{t_p-t_k} \left(1 - F_{L_k|S_0}(l_k |S_0 , \boldsymbol{Y}_{1:k})\right) \quad \mathrm{d}l_k
\end{aligned} \tag{B3}
$$

Based on Equation (B1) and Equation (B3), Equation (43) can be obtained.

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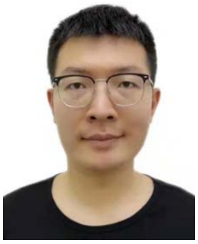

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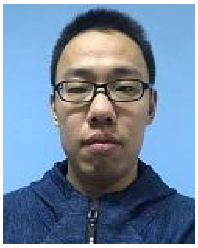

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