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A Leader-Follower Control Strategy for Input-Delay Stochastic Systems

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ABSTRACT This paper designs an optimal open-loop leader-follower control strategy of input-delay stochastic systems. The leader-follower control problem is described as an optimal problem based on Stackelberg strategies. A control strategy of the leader is given to minimize the function of H_2 norm, while a control strategy of the follower in worst case maximizing the function of H_∞ norm is applied to the systems. Then the optimal leader-follower control strategy is obtained by establishing the information relationship of the forward and the backward variables with augmentation of the variables and solving several Riccati equations. At last, an example illustrates the effectiveness of the proposed strategy.

INDEX TERMS Leader-follower control, stochastic systems, input delay, open-loop control.

I. INTRODUCTION

As we all know, several dynamic game approaches have been widely used to research control problems, such as maximum entropy, convex optimization, Nash equilibrium, and Stackelberg strategy. In addition, as one kind of dynamic games, the leader-follower game has been paid more attention because of its hierarchical structure in decision making between leader and follower. The leader-follower problem is attractive for both theory and applications. The problem was originated in the context of a static economic competition [1]. Since then, the dynamic games have been extensively explored with the academic development [2]–[4]. Over the past decades, the leader-follower problem is widely applied in economic, technology, social science, and political science [5]–[10]. For example, in order to describe the demand response management and model the interactive relationship between insurer and reinsurer, the leader-follower game is respectively applied to the smart grid in [11] and the optimal reinsurance in [12].

In practice, when we deal with the leader-follower problem, systems maybe appear stochastic disturbance or information delay in the process of information transmission. The Brownian motion makes stochastic continuous systems exist uncertainty and interference factors, besides, delay input may make the stability of the systems weak. The leader-follower problem of the stochastic systems with Brownian motion and input delay is necessary to be explored. Fortunately, up to now, researchers have only proposed several solutions to solve the difficulties caused by Brownian motion or input delay. For example, the paper [13] is concerned with leader-follower games of systems with time delay. It solves the control problem caused by delay, but does not consider the influence of the Brownian motion. A leader-follower differential games of stochastic dynamics is considered in [14]. The paper only introduces Brownian motion to the system. Inspired by the literature above, we tend to discuss the leader-follower problem of the system with Brownian motion and input delay, which makes control problem more complicated. The cost functions are chosen as same as in [15].

For the leader-follower game problem of stochastic systems with delay and Brownian motion, resolving the non-causal variables relationship of leader and follower produced by delay and processing stochastic information brought by Brownian motion are the main difficulties. To our best knowledge, there are few papers taking into account leader-follower control problems on both of delay and Brownian motion.

Motivated by the above studies, we are interested in a leader-follower controller design of the stochastic systems with Brownian motion and delay time. We will see that there are three interesting challenges to dealing with our problem. They are (1) Applying maximum principle and Itô formula to solve optimization problem of the complex systems with Brownian motion and time delay; (2) Constructing one

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new state and two co-states to deal with past and future information respectively; (3) Establishing the relationship of the new state and co-states.

In this paper, our aim is to develop a leader-follower control strategy of the input-delay systems with Brownian motion. Based on the existing work, a leader-follower controller is proposed for the input-delay systems with Brownian motion. The contributions of this paper may be shown as: (1) The method of games is proposed to consider the optimal leader-follower control problems in order; (2) The information relationship of the forward and the backward variables of leader and follower is established for the optimal control problems respectively; (3) A unique leader-follower control scheme is obtained through the solutions of several Riccati equations.

In the paper, R^n is denoted as n-dimensional Euclidean space, 0 means zero vector or matrix with appropriate dimensions. A^{-1} and A' denote the inverse and transpose of matrix A. $||z||^2$ is the 2-norm of z. I is the appropriate dimension of identity matrix. $\mathbf{E}[\cdot]$ represents the expectation.

II. A LEAD-FOLLOWER PROBLEM

Let $(\Omega, F, \{\mathcal{F}_s\}_{s\geq 0}, P)$ be a probability space. Information structure is built by a filtration $\{\mathcal{F}_s\}_{s\geqslant 0}$ generated by $\beta(s)$, a one-dimensional standard Brownian motion and augmented by all the P-null sets. Random process $x(s)$ of leader-follower dynamic control systems is considered as follows

$$\begin{cases} dx(s) = [Ax(s) + Bu_1(s-l) + Cu_2(s)]ds \\ \qquad\qquad +[\bar{A}x(s) + \bar{B}u_1(s-l) + \bar{C}u_2(s)]d\beta(s), \\ x(0) = x_0, \quad u_1(t) = u_0(t), \quad t \in [-l, 0), \end{cases} \quad (1)$$

$$z_j(s) = \begin{bmatrix} C_j x(s) \\ D_{j1}u_1(s-l) \\ D_{j2}u_2(s) \end{bmatrix}, \quad j = 1, 2, \quad (2)$$

where $x(s) \in R^n$ represents the state; $u_1(s-l) \in R^m$ represents control of the leader; $u_2(s) \in R^k$ represents control of the follower. $z_1(s) \in R^r$ and $z_2(s) \in R^h$ represent outputs of leader and follower. $A, \bar{A}, B, \bar{B}, C, \bar{C}, C_j$ and $D_{ji}(i = 1, 2; j = 1, 2)$ are constant matrices with appropriate dimensions. $x_0 \in R^n$ and $u_0(t) \in R^m$ are the given initial values. n, m, k, r and h are finite positive integers, and the constant $l > 0$ represents time delay.

Define two output energy functions

$$J_2(u_1, u_2) = \frac{1}{2}\mathbf{E}\left[\int_0^T \|z_1(s)\|^2 ds\right] \quad (3)$$

and

$$J_\infty(u_1, u_2) = \frac{1}{2}\mathbf{E}[\int_0^T (\|z_2(s)\|^2 - \gamma^2\|u_2(s)\|^2)ds] \quad (4)$$

where γ is a given disturbance attenuation level, T is a positive finite integer representing the time horizon.

We want to find a unique leader-follower control strategy (u_1, u_2) of system (1)-(2) with the initial state x_0 such that the output energy (3) of the leader is minimized while the worst-case u_2 of the follower maximizing the output energy (4) is applied to the systems. To obtain the unique leader-follower control strategy (u_1, u_2), the same statement is adopted as in [16].

III. MAIN RESULTS

The unique leader-follower open-loop control strategy is obtained in the section from the following three subsections.

A. OPTIMIZATION OF THE FOLLOWER'S CONTROL

We firstly consider the optimization problem of the follower's control. For given control u_1 of the leader, an optimal control u_2 of the follower is solved in this subsection.

Problem (T_0):

$$\max_{u_2} J_\infty(u_1, u_2) \quad s.t. \quad (1)$$

Lending the idea of [17], [18], and [19], we are going to present a solution of the problem (T_0) as follows.

Theorem 1: For Problem (T_0), if the open-loop strategy u_2 of the follower exists and is unique, $u_2(s)$ satisfies

$$u_2(s) = -\delta^{-1}(s)\left\{[\bar{C}'L_1(s)\bar{A} + C'L_1(s)]x(s) + C'\rho_1(s) + \bar{C}'L_1(s)\bar{B}u_1(s-l) + \bar{C}'\bar{\rho}_1(s)\right\} \quad (5)$$

where $L_1(s)$ is the solution of the Riccati equation

$$\begin{aligned} -\dot{L}_1(s) &= \bar{A}'L_1(s)\bar{A} + A'L_1(s) + L_1(s)A + C_2'C_2 \\ &\quad -[\bar{A}'L_1(s)\bar{C} + L_1(s)C][\bar{C}'L_1(s)\bar{C} + \lambda]^{-1} \\ &\quad \times[\bar{C}'L_1(s)\bar{A} + C'L_1(s)], \ L_1(T) = 0, \\ &\quad \bar{C}'L_1(s)\bar{C} + \lambda > 0, \end{aligned} \quad (6)$$

$\rho_1(s)$ is a new co-state satisfying

$$d\rho_1(s) = -[\bar{\bar{D}}_1'(s)u_1(s-l) + D_1'(s)\rho_1(s) + \bar{D}_1'\bar{\rho}_1(s)]ds + \bar{\rho}_1(s)d\beta(s), \ \rho_1(T) = 0, \quad (7)$$

and

$$\begin{aligned} \delta(s) &= \lambda + \bar{C}'L_1(s)\bar{C}, \quad \lambda = D_{22}'D_{22} - \gamma^2 I, \\ D_1(s) &= A - C\delta^{-1}(s)[\bar{C}'L_1(s)\bar{A} + C'L_1(s)], \\ \bar{D}_1(s) &= \bar{A} - \bar{C}\delta^{-1}(s)[\bar{C}'L_1(s)\bar{A} + C'L_1(s)], \\ \bar{\bar{D}}_1(s) &= \bar{B}'L_1(s)\left\{\bar{A} - \bar{C}\delta^{-1}(s)[\bar{C}'L_1(s)\bar{A} + C'L_1(s)]\right\} \\ &\quad +B'L_1(s). \end{aligned}$$

Proof: According to the maximum principle obtained in [15], the optimal solution satisfies that

$$\begin{aligned} dp_1(s) &= -[A'p_1(s) + \bar{A}'q_1(s) + C_2'C_2x(s)]ds \\ &\quad +q_1(s)d\beta(s), \quad p_1(T) = L_1(T)x(T), \\ 0 &= \bar{C}'q_1(s) + C'p_1(s) + \lambda u_2(s). \end{aligned} \quad (8)$$

By observing (8), the nonhomogeneous relationship of $x(s)$ and $p_1(s)$ is denoted by a new co-state $\rho_1(s)$ satisfying

$$\rho_1(s) = p_1(s) - L_1(s)x(s) \quad (9)$$

where $\rho_1(s)$ is to be determined, and $L_1(s)$ is associated with the Generalized differential Riccati equation (GDRE) (6). We assume that

$$d\rho_1(s) = S(s)ds + \bar{\rho}_1(s)d\beta(s). \tag{10}$$

Applying (10) and Itô formula to (9), we have

$$\begin{aligned} &dp_1(s)\\ &= d\left[\rho_1(s) + L_1(s)x(s)\right]\\ &= \{\dot{L}_1(s)x(s) + L_1(s)[Ax(s) + Bu_1(s-l) + Cu_2(s)]\}ds\\ &\quad +L_1(s)[\bar{A}x(s) + \bar{B}u_1(s-l) + \bar{C}u_2(s)]d\beta(s)\\ &\quad +S(s)ds + \bar{\rho}_1(s)d\beta(s). \end{aligned} \tag{11}$$

Compared (11) with the first equation of (8), we obtain

$$q_1(s) = L_1(s)[\bar{B}u_1(s-l) + \bar{A}x(s) + \bar{C}u_2(s)] + \bar{\rho}_1(s) \tag{12}$$

and

$$\begin{aligned} &\dot{L}_1(s)x(s) + L_1(s)[Ax(s) + Bu_1(s-l) + Cu_2(s)] + S(s)\\ &\quad = -[A'L_1(s)x(s) + A'\rho_1(s) + \bar{A}'q_1(s) + C_2'C_2x(s)]. \end{aligned} \tag{13}$$

Substituting (9) and (12) into the second equation of (8), we have

$$\begin{aligned} 0 &= C'\left[L_1(s)x(s) + \rho_1(s)\right] + \bar{C}'\bar{\rho}_1(s) + \lambda u_2(s)\\ &\quad +\bar{C}'L_1(s)[\bar{A}x(s) + \bar{B}u_1(s-l) + \bar{C}u_2(s)]\\ &= [C'L_1(s) + \bar{C}'L_1(s)\bar{A}]x(s) + \bar{C}'L_1(s)\bar{B}u_1(s-l)\\ &\quad +\bar{C}'\bar{\rho}_1(s) + C'\rho_1(s) + [\lambda + \bar{C}'L_1(s)\bar{C}]u_2(s)\\ &= [C'L_1(s) + \bar{C}'L_1(s)\bar{A}]x(s) + \bar{C}'L_1(s)\bar{B}u_1(s-l)\\ &\quad +\delta(s)u_2(s) + C'\rho_1(s) + \bar{C}'\bar{\rho}_1(s). \end{aligned} \tag{14}$$

According to (6), (14), and $\delta(s) > 0$, (5) can be obtained.

Substituting (5) and (12) into the equation (13), we have

$$\begin{aligned} &\dot{L}_1(s)x(s) + L_1(s)Ax(s)\\ &\quad -[L_1(s)C + \bar{A}'L_1(s)\bar{C}]\delta^{-1}(s)[\bar{C}'L_1(s)\bar{A} + C'L_1(s)]x(s)\\ &\quad +[A'L_1(s) + \bar{A}'L_1(s)\bar{A} + C_2'C_2]x(s)\\ &\quad -[L_1(s)C + \bar{A}'L_1(s)\bar{C}]\delta^{-1}(s)C'\rho_1(s)\\ &\quad -[L_1(s)C + \bar{A}'L_1(s)\bar{C}]\delta^{-1}(s)\bar{C}'L_1(s)\bar{B}u_1(s-l)\\ &\quad -[L_1(s)C + \bar{A}'L_1(s)\bar{C}]\delta^{-1}(s)\bar{C}'\bar{\rho}_1(s) + S(s)\\ &\quad +\bar{A}'L_1(s)\bar{B}u_1(s-l) + A'\rho_1(s) + \bar{A}'\bar{\rho}_1(s)\\ &\quad +L_1(s)Bu_1(s-l) = 0. \end{aligned} \tag{15}$$

Simplifying (15), and substituting (6) into (15), then we obtain that

$$\begin{aligned} S(s) &= [\bar{A}'L_1(s)\bar{C} + L_1(s)C]\delta^{-1}(s)\bar{C}'L_1(s)\bar{B}u_1(s-l)\\ &\quad -[L_1(s)B + \bar{A}'L_1(s)\bar{B}]u_1(s-l)\\ &\quad -\left\{A' - [\bar{A}'L_1(s)\bar{C} + L_1(s)C]\delta^{-1}(s)C'\right\}\rho_1(s)\\ &\quad -\left\{\bar{A}' - [\bar{A}'L_1(s)\bar{C} + L_1(s)C]\delta^{-1}(s)\bar{C}'\right\}\bar{\rho}_1(s)\\ &= -[\bar{\bar{D}}_1'(s)u_1(s-l) + D_1'(s)\rho_1(s) + \bar{D}_1'\bar{\rho}_1(s)]. \end{aligned} \tag{16}$$

Then (7) is obtained from (10) and (16). The proof is completed.

B. OPTIMIZATION FOR THE LEADER

Now we discuss the optimization for control u_1 of the leader based on the purpose of the paper.

According to Theorem 1 and (5), (1) can be rewritten as below.

$$\begin{aligned} dx(s) &= [D_2(s)u_1(s-l) + D_1(s)x(s) + F_1(s)\rho_1(s)\\ &\quad +F_2(s)\bar{\rho}_1(s)]ds + [\bar{D}_2(s)u_1(s-l) + \bar{D}_1(s)x(s)\\ &\quad +F_2{}'(s)\rho_1(s) + \bar{F}_2(s)\bar{\rho}_1(s)]d\beta(s) \end{aligned} \tag{17}$$

where

$$\begin{aligned} D_2(s) &= B - C\delta^{-1}(s)\bar{C}'L_1(s)\bar{B},\\ \bar{D}_2(s) &= \bar{B} - \bar{C}\delta^{-1}(s)\bar{C}'L_1(s)\bar{B},\\ F_1(s) &= -C\delta^{-1}(s)C',\\ F_2(s) &= -C\delta^{-1}(s)\bar{C}',\\ \bar{F}_2(s) &= -\bar{C}\delta^{-1}(s)\bar{C}'. \end{aligned}$$

Then we would choose control u_1 of the leader to deal with the following stochastic control problem.

Problem (T_1):

$$\min_{u_1} J_2(u_1, u_2) \quad s.t. \quad (7) \quad and \quad (17)$$

Lemma 1: The new co-state ρ_2 is given as

$$\begin{aligned} &d\rho_2(s)\\ &= -\Big\{[A_1'(s)\Upsilon(s)\bar{C}'L_1(s)\bar{B} + L_2(s)D_2(s)\\ &\quad +\bar{D}_1'(s)L_2(s)\bar{D}_2(s)]u_1(s-l) + [L_2(s)F_1(s)\\ &\quad +A_1'(s)\Upsilon(s)C' + \bar{D}_1'(s)L_2(s)F_2'(s)]\rho_1(s)\\ &\quad +[L_2(s)F_2(s) + A_1'(s)\Upsilon(s)\bar{C}' + \bar{D}_1'(s)L_2(s)\bar{F}_2(s)]\bar{\rho}_1(s)\\ &\quad +D_1'(s)\rho_2(s) + \bar{D}_1'(s)\bar{\rho}_2(s)\Big\}\, ds + \bar{\rho}_2(s)d\beta(s),\\ &\quad \rho_2(T) = 0 \end{aligned} \tag{18}$$

where $\Upsilon(s) = \delta^{-1}(s)D_{12}'D_{12}\delta^{-1}(s), A_1(s) = C'L_1(s) + \bar{C}'L_1(s)\bar{A}$, and $L_2(s)$ satisfties the following equation

$$\begin{aligned} -\dot{L}_2(s) &= L_2(s)D_1(s) + D_1'(s)L_2(s) + \bar{D}_1'(s)L_2(s)\bar{D}_1(s)\\ &\quad +C_1'C_1 + A_1'(s)\Upsilon(s)A_1(s),\\ L_2(T) &= 0. \end{aligned} \tag{19}$$

Proof: Applying the maximum principle again and combining with the approach in [20], we have

$$\begin{aligned} dp_2(s) &= -\Big\{A_1'(s)\Upsilon(s)[\bar{C}'L_1(s)\bar{B}u_1(s-l) + C'\rho_1(s)\\ &\quad +\bar{C}'\bar{\rho}_1(s)] + A_1'(s)\Upsilon(s)A_1(s)x(s)\\ &\quad +C_1'C_1x(s) + D_1'(s)p_2(s) + \bar{D}_1'(s)q_2(s)\Big\}\, ds\\ &\quad +q_2(s)d\beta(s),\\ p_2(T) &= L_2(T)x(T), \end{aligned} \tag{20}$$

and

$$\begin{aligned} &d\xi(s)\\ &= \{C\Upsilon(s)[\bar{C}'L_1(s)\bar{B}u_1(s-l) + A_1(s)x(s) + \bar{C}'\bar{\rho}_1(s)]\\ &\quad +C\Upsilon(s)C'\rho_1(s) + F_1(s)p_2(s) + F_2(s)q_2(s)\\ &\quad +D_1(s)\xi(s)\}ds \end{aligned}$$

$$
\begin{aligned}
&+\{\bar{C}\Upsilon(s)[A_1(s)x(s)+\bar{C}'L_1(s)\bar{B}u_1(s-l)+\bar{C}'\bar{\rho}_1(s)]\\
&+\bar{C}\Upsilon(s)C'\rho_1(s)+F_2'(s)p_2(s)+\bar{F}_2(s)q_2(s)\\
&+\bar{D}_1(s)\xi(s)\}d\beta(s),\\
&\xi(0)=0, \qquad (21)
\end{aligned}
$$

$$
\begin{aligned}
&D_{11}'D_{11}u_1(s-l)+\bar{B}'L_1(s)\bar{C}\Upsilon(s)\bar{C}'L_1(s)\bar{B}u_1(s-l)\\
&+\mathbf{E}\Big[\bar{B}'L_1(s)\bar{C}\Upsilon(s)[A_1(s)x(s)+\bar{C}'\bar{\rho}_1(s)+C'\rho_1(s)]\\
&+D_2'(s)p_2(s)+\bar{D}_2'(s)q_2(s)+\bar{\bar{D}}_1(s)\xi(s)|\mathcal{F}_{s-l}\Big]=0. \qquad (22)
\end{aligned}
$$

The nonhomogeneous relationship of $x(s)$ and $p_2(s)$ is denoted by the other new co-state ρ_2 satisfying

$$
p_2(s)=L_2(s)x(s)+\rho_2(s) \qquad (23)
$$

where $\rho_2(s)$ is to be determined, and $L_2(s)$ is associated with the equation (19).

Denote $d\rho_2(s)=\Gamma(s)dt+\bar{\rho}_2(s)d\beta(s)$. Applying (20) and Itô formula to (23), we get

$$
\begin{aligned}
dp_2(s)=&-\Big\{A_1'(s)\Upsilon(s)[C'\rho_1(s)+\bar{C}'L_1(s)\bar{B}u_1(s-l)\\
&+\bar{C}'\bar{\rho}_1(s)]+A_1'(s)\Upsilon(s)A_1(s)x(s)+C_1'C_1x(s)\\
&+D_1'(s)p_2(s)+\bar{D}_1'(s)q_2(s)\Big\}ds\\
&+q_2(s)d\beta(s). \qquad (24)
\end{aligned}
$$

Thus

$$
\begin{aligned}
&[\dot{L}_2(s)+A_1'(s)\Upsilon(s)A_1(s)+L_2(s)D_1(s)\\
&\quad+C_1'C_1+D_1'(s)L_2(s)+\bar{D}_1'(s)L_2(s)\bar{D}_1(s)]x(s)\\
&\quad+[L_2(s)D_2(s)+A_1'(s)\Upsilon(s)\bar{C}'L_1(s)\bar{B}\\
&\quad+\bar{D}_1'(s)L_2(s)\bar{D}_2(s)]u_1(s-l)\\
&\quad+[L_2(s)F_1(s)+A_1'(s)\Upsilon(s)C'+\bar{D}_1'(s)L_2(s)F_2'(s)]\rho_1(s)\\
&\quad+[L_2(s)F_2(s)+A_1'(s)\Upsilon(s)\bar{C}'+\bar{D}_1'(s)L_2(s)\bar{F}_2(s)]\bar{\rho}_1(s)\\
&\quad+\Gamma(s)+D_1'(s)\rho_2(s)+\bar{D}_1'(s)\bar{\rho}_2(s)=0, \qquad (25)\\
&q_2(s)\\
&=L_2(s)[\bar{D}_2(s)u_1(s-l)+\bar{D}_1(s)x(s)\\
&\quad+F_2'(s)\rho_1(s)+\bar{F}_2(s)\bar{\rho}_1(s)]+\bar{\rho}_2(s). \qquad (26)
\end{aligned}
$$

Noticing (19) and (25), we have

$$
\begin{aligned}
\Gamma(s)=&-[L_2(s)D_2(s)+A_1'(s)\Upsilon(s)\bar{C}'L_1(s)\bar{B}\\
&+\bar{D}_1'(s)L_2(s)\bar{D}_2(s)]u_1(s-l)\\
&-[L_2(s)F_1(s)+A_1'(s)\Upsilon(s)C'\\
&+\bar{D}_1'(s)L_2(s)F_2'(s)]\rho_1(s)\\
&-[L_2(s)F_2(s)+\bar{D}_1'(s)L_2(s)\bar{F}_2(s)\\
&+A_1'(s)\Upsilon(s)\bar{C}']\bar{\rho}_1(s)\\
&-D_1'(s)\rho_2(s)-\bar{D}_1'(s)\bar{\rho}_2(s).
\end{aligned}
$$

Then we can obtain that the formula of $\rho_2(s)$ is exactly (18). The proof is completed.

In order to solve the optimal strategy of Problem (T_1) based on (22), we would construct an extended state space model and get the expression of the strategy from equilibrium conditions (8) and (22).

We first introduce new state equations by augmenting variables. According to (23) and (26), the new state equations are created by combining (7) and (18), (21) and (17)

$$
\begin{aligned}
d\mu(s)=&-[M'(s)\mu(s)+\bar{M}'(s)\bar{\mu}(s)+N_2(s)u_1(s-l)]ds\\
&+\bar{\mu}(s)d\beta(s),\\
\mu(T)=&\,0
\end{aligned}
$$

and

$$
\begin{aligned}
d\nu(s)=&[M(s)\nu(s)+S_1(s)\mu(s)+S_2(s)\bar{\mu}(s)\\
&+N_1(s)u_1(s-l)]ds+[\bar{M}(s)\nu(s)+S_2'(s)\mu(s)\\
&+\bar{S}_2(s)\bar{\mu}(s)+\bar{N}_1(s)u_1(s-l)]d\beta(s),\\
\nu(0)=&\begin{bmatrix}0 & x_0\end{bmatrix}'
\end{aligned}
$$

with the augmentation of the variables $\nu(s), M(s), S_1(s)$, $\bar{M}(s)$, $N_2(s)$, $S_2(s)$, $N_1(s)$, and $\bar{N}_1(s)$, as shown at the bottom of the page, where the two co-states $\rho_1(s)$, $\rho_2(s)$ and the new state $\xi(s)$ are shown in (7), (18) and (21). Then (22) can also be rewritten as

$$
\begin{aligned}
0=\mathbf{E}\Big[N_1'(s)\mu(s)+N_2'(s)\nu(s)+\bar{N}_1'(s)\bar{\mu}(s)|\mathcal{F}_{s-l}\Big]&\\
+\tau(s)u_1(s-l)& \qquad (27)
\end{aligned}
$$

where

$$
\tau(s)=D_{11}'D_{11}+\bar{B}'L_1(s)\bar{C}\Upsilon(s)\bar{C}'L_1(s)\bar{B}+\bar{D}_2'(s)L_2(s)\bar{D}_2(s).
$$

Next, we could illustrate a relationship of $\nu(s)$ and $\mu(s)$ inspired by the idea of [15].

Define

$$
\begin{aligned}
&-\dot{P}(s)\\
&=\begin{cases}
P(s)M(s)+P(s)S_1(s)P(s)+M'(s)P(s)\\
+[\bar{M}'(s)+P(s)S_2(s)][I-P(s)\bar{S}_2(s)]^{-1}P(s)\\
\times\bar{M}(s)+[\bar{M}'(s)+P(s)S_2(s)][I-P(s)\bar{S}_2(s)]^{-1}\\
\times P(s)S_2'(s)P(s),\ \ s\geq T-l,\\
P(s)M(s)+M'(s)P(s)+P(s)S_1(s)P(s)\\
+[\bar{M}'(s)+P(s)S_2(s)][I-P(s)\bar{S}_2(s)]^{-1}P(s)\bar{M}(s)\\
+[\bar{M}'(s)+P(s)S_2(s)][I-P(s)\bar{S}_2(s)]^{-1}\\
\times P(s)S_2'(s)P(s)-Y(s,s+l),\ s\leq T-l,
\end{cases}\\
&\quad\det[I-P(s)\bar{S}_2(s)]\neq 0, P(T)=0, \qquad (28)
\end{aligned}
$$

and

$$
\begin{aligned}
&-\frac{\partial}{\partial s}Y(s,\theta)\\
&=\Big\{P(s)S_1(s)+M'(s)+[P(s)S_2(s)+\bar{M}'(s)]\\
&\quad\times[I-P(s)\bar{S}_2(s)]^{-1}P(s)S_2'(s)\Big\}Y(s,\theta)\\
&\quad+Y(s,\theta)\{M(s)+S_2(s)[I-P(s)\bar{S}_2(s)]^{-1}P(s)\bar{M}(s)\}\\
&\quad+Y(s,\theta)\Big\{S_1(s)+S_2(s)[I-P(s)\bar{S}_2(s)]^{-1}P(s)S_2'(s)\Big\}\\
&\quad\times P(s)-\int_{\theta}^{\min(T,s+l)}Y(s,\theta)\Big\{S_1(s)
\end{aligned}
$$

$$+S_2(s)[I-P(s)\bar{S}_2(s)]^{-1}P(s)S_2'(s)\big\}Y(s,\tau)d\tau$$
$$-\int_{\theta}^{\min(T,s+l)}Y(s,\tau)\big\{S_1(s)+S_2(s)[I-P(s)\bar{S}_2(s)]^{-1}$$
$$\times P(s)S_2'(s)\big\}Y(s,\theta)d\tau \quad (29)$$

where $s\in[0,T]$, $\theta\in(s,\min(T,s+l)]$. Then the conclusion is presented.

Lemma 2 ([15]): Provided that equations (28)-(29) admit solutions $P(s)$ and $Y(s,\theta)$, it holds that

$$\mu(s)=P(s)\nu(s)-\int_{s}^{\min(T,s+l)}Y(s,\theta)\hat{\nu}(s|\theta-l)d\theta \quad (30)$$

where $\hat{\nu}(s|\theta-l)=\mathbf{E}\big[\nu(s)|\mathcal{F}_{\theta-l}\big]$, $s\le\theta\le\min(T,s+l)$.

At last, we obtain the expression of the optimal leader control strategy. The following theorem is obtained.

Theorem 2: Consider Problem (T_1), if there exist solutions of (19) and (28)-(29), and $\det\Pi(s)\neq 0$, then the leader control of Problem (T_1) is computed by

$$u_1(s-l)=-\Pi^{-1}(s)\Phi(s)\hat{\nu}(s|s-l) \quad (31)$$

where

$$\Pi(s)=\tau(s)+\bar{N}_1'(s)[I-P(s)\bar{S}_2(s)]^{-1}P(s)\bar{N}_1(s),$$
$$\Phi(s)=\big\{N_1'(s)+\bar{N}_1'(s)[I-P(s)\bar{S}_2(s)]^{-1}P(s)S_2'(s)\big\}$$
$$\times\Big[P(s)-\int_{s}^{\min(T,s+l)}Y(s,\theta)d\theta\Big]+N_2'(s)$$
$$+\bar{N}_1'(s)[I-P(s)\bar{S}_2(s)]^{-1}P(s)\bar{M}(s). \quad (32)$$

Proof: From Lemma 2, we denote

$$\Delta(s)=-\int_{s}^{\min(T,s+l)}Y(s,\theta)\hat{\nu}(s|\theta-l)d\theta$$
$$=\mu(s)-P(s)\nu(s), \quad (33)$$

and assume $\Delta(s)$ satisfies

$$d\Delta(s)=\Delta_1(s)ds+\bar{\Delta}(s)d\beta(s). \quad (34)$$

By applying Itô formula to (33) and noticing (34), on the one hand, it can be obtained that

$$d\Delta(s)=-\big\{\dot{P}(s)\nu(s)+M'(s)\mu(s)+\bar{M}'(s)\bar{\mu}(s)$$
$$+N_2(s)u_1(s-l)+P(s)[N_1(s)u_1(s-l)$$
$$+M(s)\nu(s)+S_1(s)\mu(s)+S_2(s)\bar{\mu}(s)]\big\}ds$$
$$+\big\{\bar{\mu}(s)-P(s)[S_2'(s)\mu(s)+\bar{M}(s)\nu(s)$$
$$+\bar{N}_1(s)u_1(s-l)+\bar{S}_2(s)\bar{\mu}(s)]\big\}d\beta(s),$$

and

$$\bar{\Delta}(s)=\bar{\mu}(s)-P(s)[S_2'(s)\mu(s)+\bar{M}(s)\nu(s)$$
$$+\bar{N}_1(s)u_1(s-l)+\bar{S}_2(s)\bar{\mu}(s)],$$

thus

$$\bar{\mu}(s)=[I-P(s)\bar{S}_2(s)]^{-1}\big\{\bar{\Delta}(s)+P(s)[\bar{M}(s)\nu(s)$$
$$+S_2'(s)\mu(s)+\bar{N}_1(s)u_1(s-l)]\big\}. \quad (35)$$

By substituting (35) into (27), it is obtained that

$$u_1(s-l)$$
$$=-\Pi^{-1}(s)\mathbf{E}\Big[\big\{N_2'(s)+\bar{N}_1'(s)[I-P(s)\bar{S}_2(s)]^{-1}P(s)$$
$$\times\bar{M}(s)\big\}\nu(s)+\big\{N_1'(s)+\bar{N}_1'(s)[I-P(s)\bar{S}_2(s)]^{-1}$$
$$\times P(s)S_2'(s)\big\}[P(s)\nu(s)+\Delta(s)]$$
$$+\bar{N}_1{}'(s)[I-P(s)\bar{S}_2(s)]^{-1}\bar{\Delta}(s)\Big|\mathcal{F}_{s-l}\Big] \quad (36)$$

where

$$\Pi(s)=\tau(s)+\bar{N}_1'(s)[I-P(s)\bar{S}_2(s)]^{-1}P(s)\bar{N}_1(s),$$
$$\det\Pi(s)\neq 0.$$

$$\nu(s)=\begin{bmatrix}\xi(s)\\x(s)\end{bmatrix},\ \mu(s)=\begin{bmatrix}\rho_1(s)\\\rho_2(s)\end{bmatrix},\quad \bar{\mu}(s)=\begin{bmatrix}\bar{\rho}_1(s)\\\bar{\rho}_2(s)\end{bmatrix},$$
$$M(s)=\begin{bmatrix}D_1(s) & F_1(s)L_2(s)+F_2(s)L_2(s)\bar{D}_1(s)+C\Upsilon(s)A_1(s)\\0 & D_1(s)\end{bmatrix},$$
$$S_1(s)=\begin{bmatrix}F_2(s)L_2(s)F_2'(s)+C\Upsilon(s)C' & F_1(s)\\F_1(s) & 0\end{bmatrix},$$
$$\bar{M}(s)=\begin{bmatrix}\bar{D}_1(s) & F_2'(s)L_2(s)+\bar{F}_2(s)L_2(s)\bar{D}_1(s)+\bar{C}\Upsilon(s)A_1(s)\\0 & \bar{D}_1(s)\end{bmatrix},$$
$$N_2(s)=\begin{bmatrix}\bar{D}_1'(s)\\L_2(s)D_2(s)+A_1'(s)\Upsilon(s)\bar{C}'L_1(s)\bar{B}+\bar{D}_1'(s)L_2(s)\bar{D}_2(s)\end{bmatrix},$$
$$S_2(s)=\begin{bmatrix}F_2(s)L_2(s)\bar{F}_2(s)+C\Upsilon(s)\bar{C}' & F_2(s)\\F_2(s) & 0\end{bmatrix},$$
$$\bar{S}_2(s)=\begin{bmatrix}\bar{F}_2(s)L_2(s)\bar{F}_2(s)+\bar{C}\Upsilon(s)\bar{C}' & \bar{F}_2(s)\\\bar{F}_2(s) & 0\end{bmatrix},$$
$$N_1(s)=\begin{bmatrix}F_2(s)L_2(s)\bar{D}_2(s)+C\Upsilon(s)\bar{C}'L_1(s)\bar{B}\\D_2(s)\end{bmatrix},$$
$$\bar{N}_1(s)=\begin{bmatrix}\bar{F}_2(s)L_2(s)\bar{D}_2(s)+\bar{C}\Upsilon(s)\bar{C}'L_1(s)\bar{B}\\\bar{D}_2(s)\end{bmatrix}$$

On the other hand, by observing the equality

$$d\Delta(s) = -d\Big[\int_s^{\min(T,s+l)} Y(s,\theta)\hat{\nu}(s|\theta-l)d\theta\Big],$$

it is shown that $\bar{\Delta}(s) = 0$ and

$$\begin{aligned}
&u_1(s-l)\\
&= -\Pi^{-1}(s)\mathbf{E}\Big[\big\{N_2'(s) + \bar{N}_1'(s)[I - P(s)\bar{S}_2(s)]^{-1}P(s)\\
&\quad \times\bar{M}(s)\big\} + \big\{N_1'(s) + \bar{N}_1{}'(s)[I - P(s)\bar{S}_2(s)]^{-1}P(s)\\
&\quad \times S_2'(s)\big\}[P(s) - \int_s^{\min(T,s+l)} Y(s,\theta)d\theta]\Big]\hat{\nu}(s|s-l).
\end{aligned}$$

Then

$$u_1(s-l) = -\Pi^{-1}(s)\Phi(s)\hat{\nu}(s|s-l)$$

where

$$\begin{aligned}
\Phi(s) = &\Big\{N_1'(s) + \bar{N}_1{}'(s)[I - P(s)\bar{S}_2(s)]^{-1}P(s)S_2'(s)\Big\}\\
&\times\Big[P(s) - \int_s^{\min(T,s+l)} Y(s,\theta)d\theta\Big] + N_2'(s)\\
&+\bar{N}_1'(s)[I - P(s)\bar{S}_2(s)]^{-1}P(s)\bar{M}(s).
\end{aligned}$$

Therefore the proof is completed.

C. A LEADER-FOLLOWER CONTROL STRATEGY

We shall give the optimal leader-follower control of the input-delay stochastic systems.

Theorem 3: Consider (1)-(2) and (3)-(4), if the equations (6), (19), (28), and (29) exist unique solution $L_1(s)$, $L_2(s)$, $P(s)$ and $Y(s,\theta)$, then a unique open-loop leader-follower control strategy (u_1, u_2) satisfying that the cost function (3) is minimized while the worst-case u_2 maximizing (4) is proposed as below

$$u_1(s-l) = -\Pi^{-1}(s)\Phi(s)\hat{\nu}(s|s-l), \tag{37}$$

$$u_2(s) = G_1(s)\nu(s) + G_3(s)\hat{\nu}(s|s-l) + \int_s^{\min(T,s+l)} G_2(s,\theta)\hat{\nu}(s|\theta-l)d\theta \tag{38}$$

where

$$\begin{aligned}
G_1(s) &= -\delta^{-1}(s)\big\{[0\ \ \bar{C}'L_1(s)\bar{A} + C'L_1(s)]\\
&\quad +[C'\ \ 0]P(s) + [\bar{C}'\ \ 0][I - P(s)\bar{S}_2(s)]^{-1}\\
&\quad \times P(s)[S_2'(s)P(s) + \bar{M}(s)]\big\},\\
G_2(s) &= \delta^{-1}(s)\big\{[C'\ \ 0]Y(s,\theta)\\
&\quad +[\bar{C}'\ \ 0][I - P(s)\bar{S}_2(s)]^{-1}P(s)S_2'(s)Y(s,\theta)\big\},\\
G_3(s) &= \delta^{-1}(s)\big\{[\bar{C}'\ \ 0][I - P(s)\bar{S}_2(s)]^{-1}P(s)\bar{N}_1(s)\\
&\quad +\bar{C}'L_1(s)\bar{B}\big\}\Pi^{-1}(s)\Phi(s).
\end{aligned} \tag{39}$$

And the optimal costs J_∞ and J_2 are obtained as

$$\begin{aligned}
J_\infty = &\frac{1}{2}\mathbf{E}\Big[x'(0)\rho_1(0) + x'(0)L_1(0)x(0)\\
&+ \int_0^T u_1'(s-l)\big\{\big[0\ B'L_1(s) + \bar{B}'L_1(s)\bar{A}\big]\nu(s)\\
&+\big[B'\ \ \ 0\big]\mu(s) + \big[\bar{B}'\ \ \ 0\big]\bar{\mu}(s) + \bar{B}'L_1(s)\bar{C}u_2(s)\\
&+[\bar{B}'L_1(s)\bar{B} + D_{21}'D_{21}]u_1(s-l)\big\}ds\Big],\\
J_2 = &\frac{1}{2}\mathbf{E}\Big[x'(0)\rho_2(0) + x'(0)L_2(0)x(0)\\
&+ \int_0^T u_1'(s-l)\big\{N_2'(s)\nu(s) + \Pi(s)u_1(s-l)\\
&+N_1'(s)\mu(s) + \bar{N}_1'(s)[I - P(s)\bar{S}_2(s)]^{-1}\\
&\times P(s)[\bar{M}(s)\nu(s) + S_2'(s)\mu(s)]\big\}ds\Big]
\end{aligned} \tag{40}$$

where $u_2(s)$, $u_1(s-l)$ are formulated as (37) and (38).

Proof: From [16] and the conditions mentioned as above, we can obtain that the leader-follower control strategy (u_1, u_2) exists and is unique. The equation (37) has been obtained from (31). On the other hand, from the formulas of (30) and (35), we make a simple matrixing method to deal with (5). Then we have

$$\begin{aligned}
&u_2(s)\\
&= -\delta^{-1}(s)\Big\{[0\ \ \bar{C}'L_1(s)\bar{A} + C'L_1(s)]\nu(s)\\
&\quad +[C'\ \ 0]P(s)\nu(s) + [\bar{C}'\ \ 0][I - P(s)\bar{S}_2(s)]^{-1}\\
&\quad \times\{P(s)[S_2'(s)P(s) + \bar{M}(s)]\nu(s) + P(s)\bar{N}_1(s)u_1(s-l)\\
&\quad -P(s)S_2'(s)\int_s^{\min(T,s+l)} Y(s,\theta)\hat{\nu}(s|\theta-l)d\theta\}\\
&\quad +\bar{C}'L_1(s)\bar{B}u_1(s-l)\\
&\quad -[C'\ \ 0]\int_s^{\min(T,s+l)} Y(s,\theta)\hat{\nu}(s|\theta-l)d\theta\Big\}.
\end{aligned} \tag{41}$$

Plugging (37) into (41), and making the simplest formula, we have

$$\begin{aligned}
&u_2(s)\\
&= -\delta^{-1}(s)\Big\{\big([0\ \ C'L_1(s) + \bar{C}'L_1(s)\bar{A}] + [C'\ \ 0]P(s)\\
&\quad +[\bar{C}'\ \ 0][I - P(s)\bar{S}_2(s)]^{-1}P(s)[\bar{M}(s) + S_2'(s)P(s)]\big)\nu(s)\\
&\quad -\int_s^{\min(T,s+l)} \big([C'\ \ 0]Y(s,\theta) + [\bar{C}'\ \ 0]\\
&\quad \times[I - P(s)\bar{S}_2(s)]^{-1}P(s)S_2'(s)Y(s,\theta)\big)\hat{\nu}(s|\theta-l)d\theta\\
&\quad -\big(\bar{C}'L_1(s)\bar{B} + [\bar{C}'\ \ 0][I - P(s)\bar{S}_2(s)]^{-1}P(s)\bar{N}_1(s)\big)\\
&\quad \times\Pi^{-1}(s)\Phi(s)\hat{\nu}(s|s-l)\Big\}.
\end{aligned}$$

Let $G_1(s)$, $G_2(s)$ and $G_3(s)$ be written as in (39), (38) is obtained directly.

In order to obtain the optimal cost J_∞, we pay attention to $p_1'(s)x(s)$ firstly. According to [17], combining with (1) and (8), we have

$$d[p_1'(s)x(s)] = dp_1'(s)\cdot x(s) + dp_1'(s)\cdot dx(s) + p_1'(s)\cdot dx(s).$$

By taking integration from 0 to T and expectation, it is obtained that

$$\begin{aligned}
&\mathbf{E}\Big[\int_0^T d[p_1'(s)x(s)]\Big]\\
&= \mathbf{E}\Big[\int_0^T \{-[A'p_1(s) + \bar{A}'q_1(s) + C_2'C_2x(s)]'x(s)
\end{aligned}$$

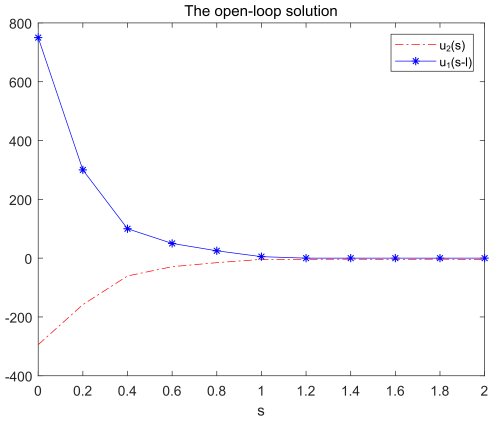

FIGURE 1. The curves of the open-loop solutions.

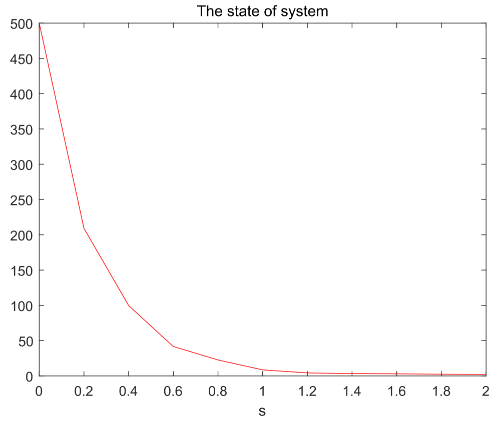

FIGURE 2. The curve of the state.

$$
\begin{aligned}
&+p_1'(s)[Ax(s)+Bu_1(s-l)+Cu_2(s)]\\
&+q_1'(s)[\bar{A}x(s)+\bar{B}u_1(s-l)+\bar{C}u_2(s)]\big\}ds\Big]\\
=\ &\mathbf{E}\Big[\int_0^T\big\{-x'(s)C_2'C_2x(s)+u_1'(s-l)[B'p_1(s)\\
&+\bar{B}'q_1(s)]+u_2'(s)[C'p_1(s)+\bar{C}'q_1(s)]\big\}ds\Big].
\end{aligned}
$$

In addition, we know

$$
\int_0^T d[p_1'(s)x(s)] = p_1'(T)x(T)-p_1'(0)x(0)
$$

with $p_1'(T)=0$. Therefore, it is easy to obtain that

$$
\begin{aligned}
&2J_\infty\\
&=\mathbf{E}\Big[x'(0)p_1(0)+\int_0^T\{u_1'(s-l)[B'p_1(s)+\bar{B}'q_1(s)]\\
&\quad+u_2'(s)[C'p_1(s)+\bar{C}'q_1(s)]+u_1'(s-l)D_{21}'D_{21}u_1(s-l)\\
&\quad+u_2'(s)(D_{22}'D_{22}-\gamma^2I)u_2(s)\}ds\Big].
\end{aligned}
$$

Combining with (8), we have

$$
\begin{aligned}
J_\infty &= \frac{1}{2}\mathbf{E}\Big[x'(0)p_1(0)+\int_0^T\{u_1'(s-l)[B'p_1(s)+\bar{B}'q_1(s)]\\
&\quad+u_1'(s-l)D_{21}'D_{21}u_1(s-l)\}ds\Big]\\
&=\frac{1}{2}\mathbf{E}\Big[x'(0)\rho_1(0)+x'(0)L_1(0)x(0)\\
&\quad+\int_0^T u_1'(s-l)\big\{\left[0\ \ B'L_1(s)+\bar{B}'L_1(s)\bar{A}\right]\nu(s)\\
&\quad+\left[B'\ \ 0\right]\mu(s)+\left[\bar{B}'\ \ 0\right]\bar{\mu}(s)+\bar{B}'L_1(s)\bar{C}u_2(s)\\
&\quad+[\bar{B}'L_1(s)\bar{B}+D_{21}'D_{21}]u_1(s-l)\big\}ds\Big]
\end{aligned}
$$

where $u_1(s-l)$ and $u_2(s)$ are formulated as (37) and (38). We can obtain the expression of J_2 using a similar method. Then (40) is given. The proof is completed.

D. NUMERICAL EXAMPLE

Here, we provide a simulation example to verify the effectiveness of the designed controllers.

Consider the systems (1)-(2) and energy functions (3)-(4) with the following parameters: $A = -0.02, B = -0.5, C = 2, \bar{A} = -1, \bar{B} = -1.5, \bar{C} = -2, C_1 = 1.8, D_{11} = 0.1, D_{12} = 1, C_2 = 1.2, D_{21} = 1, D_{22} = 2$ and $\gamma^2 = 2$. Let $T = 2s$ (seconds) and $x_0 = 500$, inspired by [21], we use the Euler's method proposed in [22] to deal with our explored question, and we obtain the following results.

FIGURE 1 illustrates the performance of the controllers. The plot in red exhibits the dynamics of the follower's controller. The blue one exhibits the dynamics of the leader's controller. The plots show that the change of the controllers become stable even if there exist time delay and Brownian motion in the system after a transient period. The plot in FIGURE 2 illustrates the change of the state x employed by the open-loop leader-follower control strategy $(u_1(s-l), u_2(s))$. It shows that the dynamic system tends to zero asymptotically as expected.

IV. CONCLUSION

The paper researches the leader-follower control problem for input delay stochastic systems. The optimal leader-follower control scheme is obtained by building some new states and exploring a nonhomogeneous relationship between them. The used approach here will make a contribution to the other important systems with input delay and Brownian motion. To get the explicit solution of some complicated problems is our future work.

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