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A Gradient-Based Event-Driven MPC for Nonlinear Systems With Additive Disturbances Using State-Dependent Thresholds

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ABSTRACT In this article, we propose a gradient-based event-driven model predictive control (GEMPC) algorithm with a state-dependent threshold for nonlinear systems with additive disturbances and input and state constraints. Firstly, a novel gradient-based event-driven strategy is constructed in the light of the error gradient between the optimal prediction of the state and the real one, which could ensure the Zeno-free property via a positive triggering interval. Subsequently, the novel triggering mechanism and the dual-mode control are combined to establish a GEMPC framework, to further reduce the computing burden and communication transmission especially when the computational resources are limited. Additionally, the feasibility of the GEMPC algorithm and the input-to-state practical stability (ISpS) property of the considered system have been strictly proved in theory. Finally, the simulation comparison results on control of a perturbed nonlinear system are utilized to show the validity of the GEMPC algorithm.

INDEX TERMS Event-driven control, model predictive control (MPC), gradient-based mechanism, constrained systems, state-dependent threshold.

I. INTRODUCTION

With the development of wireless technologies and network communications, the cyber-physical system (CPS) has received significant and increasing attention in recent years which includes industrial control systems, smart grid systems, intelligent transportation systems, telemedicine systems, and so on [1]–[4]. In view of the wide application of CPS, how to effectively control the networked system has become of great importance.

In the field of control, model predictive control (MPC) is an exceedingly efficient advanced algorithm with effective control effect, due to the advantage of considering the prospective system actions while effectively dealing with system constraints [5]–[7]. Nevertheless, the traditional MPC controller that addresses the optimal control problem (OCP) at every sampling time may not be tractable because of the limited communication resources and computing power of the practical systems. Event-driven control is a state-feedback control strategy obtaining growing attention nowadays. Unlike the time-driven control which needs to update the control action periodically at each sampling node, the event-driven control only acts when the expected control performance cannot be guaranteed, which could significantly save the energy consumption and communication resources [8]–[10].

On account of the above-mentioned statement, there is an efficient and resource-saving control mode generated by combining MPC and event-driven control, viz. event-driven MPC (EMPC), see [6], [11], [12] and references therein. The authors of [6] reported an intuitive EMPC scheme for nonlinear systems with disturbances where the triggering condition was constructed in the light of the error gradient between the predicted state and the actual one, with a time-varying robust constraint in the OCP. Furthermore, the triggering condition and robust state constraint were both optimized in [11], to yield a larger disturbance margin than [6]; a new triggering condition based on state integration was adopted in [12], to further reduce the number of times of solving the OCP. For perturbed nonlinear affine systems, via analyzing the closed-

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loop theoretical characteristic of the system, a self-triggered MPC (SMPC) based on a zero-order holder was designed in [13], additionally, Reference [14] extended this method to a first-order holder and reduced the deviation between the reconstructed control signal and the optimal control signal. Different from the aforementioned strategies that took the optimal cost as the Lyapunov function, the authors in [15] determined a new EMPC algorithm to solve the OCP, in which the controller selected a sampling mode for the transmission, making this method more suitable for practice. For perturbed nonlinear discrete-time systems, an adaptive SMPC was proposed in [16], in which the optimal control sequence and a prediction time gradually decreasing with time were simultaneously given by solving the OCP, reducing the average sampling frequency while ensuring a low level of performance loss. In [17], a dynamic EMPC framework without terminal constraints was proposed, in which the prediction horizon shrunk with the system state approached to the terminal region by addressing the relationship between triggering rate and measurement frequency. To reduce the computational burden in MPC, the authors of [18] proposed a control scheme with the adaptive transmission intervals via a sample-and-hold way for nonholonomic systems with multiple constraints. A co-design technique was developed for event-triggered control and MPC in [19], where triggering conditions and control inputs were jointly designed to achieve potentially less conservative results. In [20], two EMPCs guaranteed that the system state of the closed-loop system remained in a robust control invariant set and the recursive feasibility of terminal constraints, respectively, and in addition, the sample-hold approach was used to avoid the transmission of continuous predictive control input trajectories.

Note that for the perturbed systems, input-to-state practical stability (ISpS) [21] theory is an important mean to discuss the robust stability of systems, and actually, it is an extended version of input-to-state stability (ISS) [22]. For the nonlinear systems with input constraints and additive disturbances, a quasi-min-max MPC control method was proposed in [23], and the iterative feasibility of the control algorithm was realized combined with ISS theory. A class of bounded economic penalty functions were considered in [24], in which the ISpS with the upper bound of economic performance was established by optimizing the weighted economic and tracking objective functions. The authors in [25] studied an EMPC problem for perturbed linear systems with constraints, in which the ISpS property was guaranteed using the compressed uncertain parameter set technique. The ISS characteristic of discrete-time perturbed nonlinear systems with EMPC scheme was studied in [26], where an event-driven scheduling scheme was established in a dual-mode MPC framework. Based on it, the authors extended the previous work in [27], in which the EMPCs with fixed and mixed thresholds were developed respectively, and their ISpS characteristics were analyzed theoretically.

Based on the above review for the design of EMPC, the main challenging problems are twofold: the first challenge is the design of a proper triggering condition in a reasonable form, which should not only be easy to use but also could save more computing and communication resources without affecting the control performance; The second challenge is the theoretical analysis regarding the inter-event time for EMPC, which could specify the amount of saved computing and communication resources explicitly for the proposed EMPC method.

Note that the existing EMPC algorithms did not fully consider the changing trend of state error, which could reflect the speed of system state change and reduce the influence of state mutation. In this paper, a gradient-based EMPC (GEMPC) is suggested for perturbed nonlinear systems that could alleviate the computational and communication burden while guaranteeing control performance. In particular, there is a state-dependent triggering threshold in the control framework, which could solve the problem that a fixed triggering threshold may be too conservative as the actual state approaches or enters the terminal set. The main contribution and novelty of our proposal could be summarized as

- Considering nonlinear systems with additive disturbances, a GEMPC algorithm is proposed, which determines the controller update in the light of the error gradient between the optimal prediction of the state and the actual one.
- A triggering mechanism containing a state-dependent threshold is introduced, which could remove the limit on the upper bound of inter-event time in the conventional event-driven control, to economize more computation and communication resources.
- The sufficient conditions are derived to guarantee the feasibility of the GEMPC framework and the ISpS characteristic of the considered system. In addition, the inter-event time allows a lower bound to guarantee the Zeno-free property.

Note that the main motivation of this work is to improve the performance of the existing event-triggered MPC by developing a novel event triggering condition. More specifically, compared with the state of the art event triggered mechanisms, e.g., [17]–[20], which are almost all based on the error information at a single time instant, introducing the gradient of the state error between two consecutive sampling instants could better characterize the dynamic change of the controlled system under the event-driven sampling mechanism, effectively avoid the impact of state mutation on system performance and quicken the dynamic response speed such that the closed-loop performance, as well as the update frequency of the EMPC system, could be significantly improved; and this has also been illustrated on the simulation experiments shown in the Simulation section. Besides, adding the state-dependent threshold in the proposed EMPC triggering condition is aimed to eliminate the upper limit of time between events in traditional event-driven control and further saves more computing and communication resources, which becomes another main contribution of the proposed method over its existing counterparts.

II. PRELIMINARIES

A. NOTATIONS

$\mathbb{N}$ ($\mathbb{N}_{\geq 0}$) and $\mathbb{R}$ ($\mathbb{R}_{\geq 0}$) are the (non-negative) natural integers and the (non-negative) real numbers sets. Given a matrix E, E^{T} represents its transpose, and its maximum and minimum real parts of eigenvalues are defined as $\lambda_{\max}(E)$ and $\lambda_{\min}(E)$, separately. For a column vector n, its P-weighted norm is expressed as $\|n\|_P = \sqrt{n^{\mathrm{T}}Pn}$, and Euclidean norm is $\|n\| = \sqrt{n^{\mathrm{T}}n}$. Given two sets $\mathcal{S}_1 \subset \mathbb{R}^n$, $\mathcal{S}_2 \subset \mathbb{R}^n$, $\mathcal{S}_1 \backsim \mathcal{S}_2$ means $\{s \in \mathbb{R}^n | s + s_2 \in \mathcal{S}_1, \forall s_2 \in \mathcal{S}_2\}$, where set operator " $\backsim$" denotes the Pontryagin difference.

B. SYSTEM DESCRIPTION

For the following perturbed nonlinear system:

$$\dot{x}(t) = h\left(x(t), u(t)\right) + \delta\left(t\right), \ t \geq 0, \tag{1}$$

with $x(t_0) = x_0$, $t_0 \geq 0$, where $u(t) \in \mathbb{U} \subset \mathbb{R}^l$, $x(t) \in \mathbb{X} \subset \mathbb{R}^k$ are the control input and the system state, and $\delta(t) \in \mathbb{W} \subset \mathbb{R}^m$ is the additive disturbances with the maximum $\bar{\delta} = \max_{\delta(t)\in\mathbb{W}} \|\delta(t)\|$. $\mathbb{X}$, $\mathbb{U}$, $\mathbb{W}$ are compact sets comprising the original point as an inner point.

The corresponding nominal system is described as

$$\dot{x}(t) = h\left(x(t), u(t)\right), \tag{2}$$

with the twice continuously differentiable function $h: \mathbb{R}^k \times \mathbb{R}^l \rightarrow \mathbb{R}^k$.

Under the above conditions, system (2) could be linearized as:

$$\dot{x}(t) = Mx(t) + Nu(t), \tag{3}$$

in which $M = (\partial h(x, u)/\partial x)|_{(0,0)}$, $N = (\partial h(x, u)/\partial x)|_{(0,0)}$. It should be noted that the controller is devised using the linearized system (3) is to design the local control gain K in Lemma 1 later, which is only utilized when the states enter the terminal set, and when the states are outside the set, the control signal is obtained by solving the MPC optimization problem considering the nonlinear system.

Definition 1: A continuous-time function $f(x) : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$ is identified as a $\mathcal{K}$-function if: (1) $f(0) = 0$; (2) it is monotonically increasing. $f(x)$ is a $\mathcal{K}_\infty$-function if it satisfies that $f(x) \in \mathcal{K}$ and $f(x) \rightarrow \infty$ as $x \rightarrow \infty$.

Furthermore, a continuous-time function $h(p, q) : \mathbb{R}_{\geq 0} \times \mathbb{N}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$ is a $\mathcal{KL}$ function if: (1) for a constant $p \geq 0$, $h(p, q)$ is strictly decreasing, viz., $h(p, q) \rightarrow 0$ as $q \rightarrow \infty$; (2) for a constant $q \geq 0$, $h(p, q) \in \mathcal{K}$.

Definition 2: The nonlinear system $\dot{x}(t) = h\left(x(t), \delta(t)\right)$ is considered to be ISpS, if there are functions $\beta \in \mathcal{K}$, $\gamma \in \mathcal{KL}$ and a constant $c \geq 0$ satisfy

$$\|x(t)\| \leq \gamma\left(\|x_0\|, t\right) + \beta(\bar{\delta}) + c, \ \forall t \geq 0. \tag{4}$$

A standard assumption [5] is provided for (3) as follows.

Assumption 1: There exists a state feedback matrix $K \in \mathbb{R}^{n\times m}$ to stabilize the system (3).

Since such property can be satisfied for controllable systems, which are very common, and thus Assumption 1 is not strong and has been used in a large number of existing works regarding EMPC design [11], [12], [15], [16]. Furthermore, once it is ensured that the considered system is controllable, the pole placement and linear optimal control techniques can directly be used to obtain the state feedback matrix K.

Based on the above discussions, a conventional lemma [6] is shown as follows:

Lemma 1: Supposing that Assumption (1) is valid for system (1), given matrices R, $Q > 0$, there are a matrix P, and a constant ι, such that:

1) $Q^* + (\iota I + M + NK)^{\mathrm{T}}P + P^{\mathrm{T}}(\iota I + M + NK) \leq 0$ permits a unique solution $P > 0$ and $\iota < -\lambda_{\max}(M + NK)$, where $Q^* = Q + K^{\mathrm{T}}RK$;
2) $\Omega \triangleq \{x \in \mathbb{R}^n \mid \|x(t)\| \leq \alpha\}$ is a invariant set with $\alpha > 0$;
3) $\dot{F}(x(t)) \leq -\|x(t)\|^2_{Q^*}$ holds for $x(t) \in \Omega$, with $F(x(t)) = \|x(t)\|^2_P$.

Remark 1: Note that the main purpose of Lemma 1 is to obtain a number of preliminary properties of the considered system to facilitate the theoretical analysis of the proposed GEMPC algorithm, and more specifically, the purpose of each term could be summarized as follows: The first term provides a systematic method that can solve offline for the weight matrix P and will be used to prove the closed-loop asymptotic stability of the proposed GEMPC algorithm; The second term defines the terminal invariant set with a radius α, indicating that the system satisfies the state constraint $x(t) \in \mathbb{X}$ in the terminal set and the linear state feedback control law $u = Kx$ satisfies the control constraint $u(t) \in \mathbb{U}$; The third term guarantees the Lyapunov function of the system in the terminal set is decreasing, which indicates that the system is asymptotically stable after the state enters the terminal set.

For the nominal system (2), there is a standard assumption [6], [28] to analyze the ISpS properties of aperiodic control.

Assumption 2: $h(x, u)$ with $x \in \mathbb{X} \times \mathbb{U}$, has a Lipschitz constant L_h and meanwhile, $h(x(t), Kx(t))$ with $x \in \Omega \times \mathbb{U}$ has a Lipschitz constant L_k, such that

$$\|h(x_1, u) - h(x_2, u)\| \leq L_h \|x_1 - x_2\|; \tag{5a}$$

$$\|h(x_1, Kx_1) - h(x_2, Kx_2)\| \leq L_k \|x_1 - x_2\|. \tag{5b}$$

Remark 2: Note that the nonlinear system $h(x, u)$ is Lipschitz continuous both inside and outside the terminal region with the Lipschitz constants L_h and L_k respectively. Since the theoretically analysis of MPC is normally conducted within the MPC region of attraction [11], [27], and given such a limited region for the state, the aforementioned Lipschitz constants can normally be directly calculated following the method proposed in [27]. Therefore, (5a)-(5b) are standard assumptions for the nominal system, and could be found in [11]–[14] and [26]–[28] and references therein.

III. GEMPC FRAMEWORK

A. OCP FOR GEMPC

For the nonlinear system (1), $\{t_\zeta\}$, $\zeta \in \mathbb{N}$ is a time sequence set at which the OCP is handled, and the OCP is constructed as:

$$\hat{u}^*(t|t_\zeta) = \underset{\hat{u}(t|t_\zeta)}{\arg\min}\, J\left(\hat{x}(t|t_\zeta),\ \hat{u}(t|t_\zeta),\ H\right), \tag{6}$$

$$\text{s.t. } \dot{\hat{x}}(t|t_\zeta) = f\left(\hat{x}(t|t_\zeta), \hat{u}(t|t_\zeta)\right),$$

$$\hat{x}(t_\zeta|t_\zeta) = x(t_\zeta); \tag{7a}$$

$$\hat{x}(t|t_\zeta) \in \mathbb{X}_{t-t_\zeta},\ \forall t \in \left[t_\zeta, t_\zeta + H\right]; \tag{7b}$$

$$\hat{x}(t_\zeta + H \mid t_\zeta) \in \Theta; \tag{7c}$$

$$u(t|t_\zeta) \in \mathbb{U},\ \forall t \in \left[t_\zeta, t_\zeta + H\right], \tag{7d}$$

in which H is the prediction horizon. $\hat{u}(t|t_\zeta)$, $\hat{x}(t|t_\zeta)$ in system (7a) are the input trajectory and state signal at instant t predicted at t_ζ, $\forall t \in \left[t_\zeta, t_\zeta + H\right]$. $\Theta \subset \Omega$ is denoted for the terminal set which will be defined in Section IV-A, and the tightening constraint set $\mathbb{X}_{t-t_\zeta} = \mathbb{X} \backsim \mathbb{S}_{t-t_\zeta}$, where $\mathbb{S}_{t-t_\zeta} \triangleq \left\{x(t) \in \mathbb{R}^k : \ \|x(t)\| \le 2\bar{\delta}\exp\{L_h(t - t_\zeta)\}/L_h\right\}$, with $t \in [t_\zeta, t_\zeta + H]$. Given such a definition, $\mathbb{S}_{t-t_\zeta}$ can be obtained based on the following steps: (1) calculate the Lipschitz constant L_h via the method mentioned in Remark 2; (2) determine the disturbance maximum $\bar{\delta}$ following Theorem 2; (3) obtain the time interval $t - t_\zeta$ according to the specific time t and then obtain the required $\mathbb{S}_{t-t_\zeta}$. The goal of solving the OCP is finding $\hat{u}^*(t|t_\zeta)$ that minimize the penalty function:

$$J\left(\hat{x}(t|t_\zeta), \hat{u}(t|t_\zeta), H\right) \triangleq \left\|\hat{x}(t+H|t_\zeta)\right\|_P^2 + \int_{t_\zeta}^{t_\zeta+H} \left\|\hat{x}(t|t_\zeta)\right\|_Q^2 + \left\|\hat{u}(t|t_\zeta)\right\|_R^2 dt, \tag{8}$$

in which $\left\|\hat{x}(t+H|t_\zeta)\right\|_P^2$ and $\left\|\hat{x}(t|t_\zeta)\right\|_Q^2 + \left\|\hat{u}(t|t_\zeta)\right\|_R^2$ are terminal and running cost terms, respectively.

B. GRADIENT-BASED EVENT-DRIVEN TRIGGERING MECHANISM

Inspired by the studies in [29] and [11] and our preliminary work [30], a state-dependent triggering mechanism is proposed in this part, namely the gradient-based event-driven triggering mechanism. In general, due to the additive disturbances, the real state $x(t|t_\zeta)$ and the optimal state $\hat{x}^*(t|t_\zeta)$ cannot match with each other. Based on this fact, the error gradient between the two state trajectories is installed as the triggering threshold, and different from [6], [11], it has a state-dependent term. Considering $e(t) = \hat{x}^*(t|t_\zeta) - x(t|t_\zeta)$, the triggering instant could be chosen as:

$$t_{\zeta+1} = \inf_{t > t_\zeta + \gamma} \{t : \|e(t) - e(t-\xi)\| = \max\{\mu\left(\|x(t)\|\right),\ \rho\}\}, \tag{9}$$

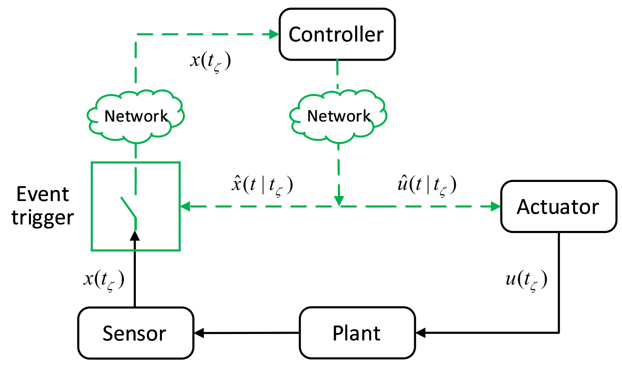

FIGURE 1. The schematic diagram of GEMPC.

where $\mu\left(\|x(t)\|\right) \in \mathcal{K}_\infty$, $\rho = 2\bar{\delta}\exp\{L_h\theta H\}/L_h$ with a scaling constant $\theta \in (0,\ 1]$, and a triggering parameter $\xi \in [\theta H,\ H]$. Note that the proposed triggering condition (9) means that the next triggering instant $t_{\zeta+1}$ is determined by the maximum value between the constant threshold ρ and the state-dependent threshold $\mu(\|x(t)\|)$, and in order to facilitate the implementation of the triggering condition, the upper bound of $\|x(t)\|$ is derived offline for the comparison such that the online computation of $\|x(t)\|$ could be effectively reduced. Given such a design, the theoretical analysis is then carried out based on the upper bound of $\|x(t)\|$. The GEMPC framework described in Fig. 1 has the following basic principle: the sensor periodically measures the system state and the event trigger determines whether to send the state data to the remote controller. At each triggering instant, the controller calculates the predictive control and state sequences at the same time, and then transmits them to the actuator and event trigger respectively. Then, when the event trigger receives a new prediction state sequence at the triggering time, the triggering condition is updated. Finally, the actuator aperiodically updates the control input and transfers it to the controlled plant. Given such an update strategy, there is no need to solve the optimal control problem or send the control signal between triggering instants, and thus, is capable of reducing the online computation and data transmission of the MPC system.

To ensure the Zeno-free property [31] of the event-driven control, it is essential to guarantee that there is a positive interval between inter-event time, which will be discussed via the following theorem.

Theorem 1: If the triggering instant is obtained via (9), then the triggering interval will not be less than θH.

Proof: Based on the triggering condition with $x(t) \in \mathbb{X}$, the theorem will be carried out from two cases.

Case i : When $\rho > \mu\left(\|x(t)\|\right)$, the triggering condition (9) is converted to $t_{\zeta+1} = \inf_{t > t_\zeta + \xi} \{\|e(t) - e(t-\xi)\| = \rho\}$. By means of the trigonometric inequality and Lipschitz continuity in Assumption 2, we can obtain: $\|e(t)\| \le \bar{\delta}(t - t_\zeta) + \int_{t_\zeta}^{t} L_h \|e(t)\| \, dt$, and then $\|e(t) - e(t-\xi)\| \le 2\bar{\delta}\exp\{L_h(t - t_\zeta)\}/L_h$ could be got via Gronwall-Bellman inequality [32]. Obviously, we can get $t_{\zeta+1} - t_\zeta \ge \theta H$

for $t = t_{\zeta+1}$, viz., the triggering interval is not less than θH.

Case ii : When $\rho \leq \mu\left(\|x(t)\|\right)$, according to the situation in *Case i*, $\rho \leq \mu\left(\|x(t)\|\right) \leq 2\bar{\delta}\exp\{L_h(t_{\zeta+1}-t_\zeta)\}$ for $t = t_{\zeta+1}$ obviously holds, viz., the triggering interval is not less than θH.

To sum up, the threshold is selected as (9) such that the minimum time interval is θH to ensure the Zeno-free property. ■

Remark 3: Unlike earlier EMPC approaches that only consider the disturbance-based triggering threshold, such as [6], [11], [12], the state-dependent triggering term $\mu(\|x(t)\|)$ is introduced in this paper. It will make significant sense since the proposed triggering mechanism could remove the limit on the upper bound of inter-event time in the conventional event-driven control, i.e., the intervals may be larger than H, so as to further economize more computation and communication resources.

C. GEMPC ALGORITHM

According to the analyses of the aforementioned OCP and the gradient-based event-driven triggering mechanism, the dual-mode [33] strategy is adopted to design the GEMPC algorithm. That is to say, if state does not enter the terminal set Θ, the OCP is solved to get the optimal series; otherwise, the state-feedback control law is utilized for brevity, and the whole control process is summarized in Algorithm 1.

Algorithm 1 GEMPC With a State-Dependent Threshold

Require: Terminal set Θ; weighting matrices P, Q, R; prediction horizon H; penalty function J; state feedback matrix K; triggering thresholds μ & ρ.

```
while x(t) ∉ Θ do
    if ζ == 0 then
        Get û*(t|t_ζ) via solving the OCP in (6);
    end if
    while t_{ζ+1} is not met do
        Utilize û*(t|t_ζ) to (1);
    end while
    Get û*(t|t_ζ), t ∈ [t_ζ, t_ζ + H] via solving the OCP;
    ζ = ζ + 1;
end while
Apply the local controller K x̂(t|t_ζ) to (1);
```

IV. ANALYSIS

The main theoretical results are given in detailed in this section, i.e., the feasibility of the GEMPC framework and the ISpS characteristic of the considered system.

A. FEASIBILITY ANALYSIS

A standard assumption about the terminal set Θ needs to be presented as a preliminary knowledge before the feasibility theorem.

Assumption 3 [26]: For $x(t) \in \Omega$, the system (2) will be steered to the terminal set $\Theta \triangleq \{\hat{x}(\omega|t) \in \mathbb{R}^n \mid \|\hat{x}(\omega|t)\| \leq \alpha_f\}$ at $\omega \geq t + \theta H$ by the state feedback matrix K defined in Assumption 1, where ω represents the time instant when the state enters the terminal set Θ with a radius of α_f.

Remark 4: Note that Assumption 3 is not exceedingly strict, and on the contrary, it is conventional in many literature, such as, for discrete systems [26], [34] and for continuous systems [15], [27]. In fact, it is an extension of the local controller defined in Assumption 1, and further explains the relationship between Ω and $\mathbb{X}_{t-t_\zeta}$, i.e., $\Omega \subset \mathbb{X}_{t-t_\zeta}$, $\forall t \in [t_\zeta,\ t_\zeta + H]$.

The feasibility is analyzed as follows.

Theorem 2: For system (1), supposing that the OCP is feasible at t_0, and Assumptions 1-3 hold. Then Algorithm 1 is feasible and $\hat{x}(t_\zeta + H|t) \in \Omega$, $\forall t \in [t_\zeta, t_{\zeta+1}]$ satisfied, if the maximum value of disturbance satisfies

$$\bar{\delta} \leq \min\left\{\bar{\delta}_1,\ \bar{\delta}_2\right\} \tag{10}$$

with

$$\bar{\delta}_1 = \frac{L_h\left(\alpha - \alpha_f\right)}{\exp\left\{L_h H\left(1 + \max\left\{\theta_\mu,\ \theta\right\}\right)\right\} - \exp\{L_h H\}}, \tag{11a}$$

$$\bar{\delta}_2 = L_k\left(\alpha - \alpha_f\right)/\exp\left\{L_k H \max\left\{\theta_\mu,\ \theta\right\}\right\}, \tag{11b}$$

in which $\mu(d_x) = 2\bar{\delta}\exp\{L_h\theta_\mu H\}/L_h$, with a constant $d_x \geq \|x(t)\|$ for $x(t) \in \mathbb{X}$.

Proof: A feasible control trajectory candidate $\tilde{u}(\eta|t)$ is established as follows:

for $t \in (t_\zeta, t_\zeta + H]$,

$$\tilde{u}(\eta|t) = \begin{cases} \hat{u}^*(\eta|t_\zeta), & \eta \in [t, t_\zeta + H] \\ K\hat{x}(\eta|t), & \eta \in [t_\zeta + H, t + H] \end{cases}; \tag{12}$$

for $t > t_\zeta + H$,

$$\tilde{u}(\eta|t) = K\hat{x}(\eta|t),\ \eta \in [t, t + H]. \tag{13}$$

To prove the theorem, the OCP with a constructed input candidate $\tilde{u}(\eta|t)$ needs to satisfy the state tightening constraint $\hat{x}(\eta|t) \in \mathbb{X}_{\eta-t}$, the terminal constraint $\hat{x}(t+H|t) \in \Theta$ and the input constraint $\tilde{u}(\eta|t) \in \mathbb{U}$, for $\eta \in [t, t + H]$.

Firstly, we will prove that the $\hat{x}(\eta|t)$ meets the tightening constraint $\hat{x}(\eta|t) \in \mathbb{X}_{\eta-t}$, $\forall \eta \in [t, t + H]$.

$\left\|x(\eta|t) - \hat{x}^*(\eta|t_\zeta)\right\| \leq \bar{\delta}(\exp\{L_h(\eta - t_\zeta)\} - \exp\{L_h(\eta - t)\})/L_h$ could be got via Gronwall-Bellman inequality. Constructing that $\chi_1 = x(\eta|t) - \hat{x}^*(\eta|t_\zeta) + \chi_2$, $\chi_2 \in \mathbb{S}_{\eta-t}$, then we easily have $\|\chi_1\| \leq \bar{\delta}(\exp\{L_h(\eta - t_\zeta)\} + \exp\{L_h(\eta - t)\})/L_h \leq 2\bar{\delta}\exp\{L_h(\eta - t_\zeta)\}/L_h$, so $\chi_1 \in \mathbb{S}_{\eta-t_\zeta}$. Since $\hat{x}^*(\eta|t_\zeta) \in \mathbb{X}_{\eta-t_\zeta}$, then we obtain $x(\eta|t) + \chi_2 = \chi_1 + \hat{x}^*(\eta|t_\zeta) \in \mathbb{X}$, i.e., $x(\eta|t) \in \mathbb{X}_{\eta-t}$, $\forall \eta \in [t, t + H]$.

Secondly, the terminal constraint satisfaction $\hat{x}(t + H|t) \in \Theta$ will be proved from the following two cases.

Case i $\left(t_{\zeta+1} \leq t_\zeta + H\right)$: It could be deduced that

$$\begin{aligned}&\left\|\hat{x}(t_\zeta + H|t) - \hat{x}^*(t_\zeta + H|t_\zeta)\right\| \\ &\qquad \leq \left\|x(t) - \hat{x}^*(t|t_\zeta)\right\| \exp\{L_h(t_\zeta + H - t)\}.\end{aligned} \tag{14}$$

Then via trigonometric inequality and the condition (11a), we get

$$\begin{aligned}\left\|\hat{x}(t_\zeta+H|t)\right\| &\leq \exp\{L_h(t_\zeta+H-t)\}\left\|x(t)-\hat{x}^*(t|t_\zeta)\right\|+\alpha_f \\ &\leq \alpha. \end{aligned} \tag{15}$$

Then (15) yields that $\hat{x}(t+H|t)\in\Omega,\ \forall t\in[t_\zeta,t_\zeta+\theta H]$. Considering Assumption 3, $\hat{x}(t+H|t)\in\Theta,\ \forall t\in[t_\zeta+\theta H,t_\zeta+H]$ can be got.

Case ii $\left(t_{\zeta+1}>t_\zeta+H\right)$: It has been proven that $\hat{x}(t+H|t)\in\Theta,\ \forall t\in[t_\zeta+\theta H,t_\zeta+H]$. For $t\in[t_\zeta+H,t_{\zeta+1}]$, we first analyze the relation between $x(t)$ and $\hat{x}(t|t_\zeta)$. Noting that $\left\|x(t)-\hat{x}(t|t_\zeta)\right\|\leq L_k\int_{t_\zeta+H}^{t}\left\|x(t)-\hat{x}(t|t_\zeta)\right\|dt+\left\|x(t_\zeta+H)-\hat{x}(t_\zeta+H|t_\zeta)\right\|$, so we get $\|x(t)\|\leq\left\|\hat{x}(t|t_\zeta)\right\|+\delta\exp\{L_kH\max\{\theta_\mu,\ \theta\}\}/L_k\leq\alpha$ with the help of Gronwall-Bellman inequality and condition (11b), and a similar corollary is described in detail in [12]. Thus, we yield that $\hat{x}(t+H|t)\in\Omega,\ \forall t\in[t_\zeta,t_\zeta+\theta H]$ and $\hat{x}(t+H|t)\in\Theta$, $\forall t\in[t_\zeta+\theta H,t_\zeta+H]$ based on Assumption 3.

Thirdly, $\tilde{u}(\eta|t)\in\mathbb{U},\ \forall\eta\in[t,t+H]$ could be guaranteed directly since the definition of $\hat{u}^*(\eta|t_\zeta)$ for $\eta\in[t,t_\zeta+H]$ and $K\hat{x}(\eta|t)$ in Assumption 3.

The proof is completed. ■

B. STABILITY ANALYSIS

We will prove the ISpS characteristic of the system under the proposed GEMPC controller in detail in this section.

The penalty function $J(x(t),\tilde{u}(\eta|t))$ is specified as a Lyapunov function $\Xi(x(t))$. Before proving the main content of stability, a theorem about two Lyapunov functions $\Xi(x(a;t))$, $\Xi(x(b;t))$ is proposed as an essential precondition, in which $x(a;t),\ x(b;t)$ are two state with $a,\ b\in[t_\zeta,t_{\zeta+1}]$ and $x(a;t_\zeta)=x(b;t_\zeta)$.

Theorem 3: For system (2), given two Lyapunov functions $\Xi(x(a;t)),\ \Xi(x(b;t))$ satisfying $\hat{x}(a;t_\zeta+H|t),\ \hat{x}(b;t_\zeta+H|t)\in\Omega$ for $t\in[t_\zeta,t_\zeta+H]$ and $\hat{x}(a;t),\ \hat{x}(b;t)\in\Omega$ for $t\in[t_\zeta+H,t_{\zeta+1}]$, then the following inequality could be derived:

$$\Xi(x(a;t))-\Xi(x(b;t))\leq\epsilon\left(\|x(a;t)-x(b;t)\|\right), \tag{16}$$

with $\mathcal{K}-$functions

$$\begin{aligned}&\epsilon(\eta)\\ &=\max\{\epsilon_1(\eta),\epsilon_2(\eta)\}, &(17a)\\ &\epsilon_1(\eta)\\ &=(\frac{2d_x\lambda_{\max}(Q)\left(\exp\{L_h(t_\zeta+H-t)\}-1\right)}{L_h} &(17b)\\ &\quad+\frac{2\alpha\lambda_{\max}(Q^*)\exp\{L_h(t_\zeta+H-t)\}\left(\exp\{L_k(t-t_\zeta)\}-1\right)}{L_k}\\ &\quad+2\alpha\exp\{L_h(t_\zeta+H-t)+L_k(t-t_\zeta)\})\eta,\\ &\epsilon_2(\eta)\\ &=(\frac{2\alpha\lambda_{\max}(Q^*)\left(\exp\{L_kH\}-1\right)}{L_k}+2\alpha\exp\{L_kH\})\eta. &(17c)\end{aligned}$$

Proof: This theorem will be proved in two cases.

Case i $\left(t_{\zeta+1}\leq t_\zeta+H\right)$: According to (8), we know that

$$\begin{aligned}&\Xi(x(a;t))-\Xi(x(b;t))\\ &\quad\leq\int_t^{t_\zeta+H}\left(\left\|\hat{x}(a;\eta|t)\right\|_Q^2-\left\|\hat{x}(b;\eta|t)\right\|_Q^2\right)d\eta\\ &\quad\quad+\int_{t_\zeta+H}^{t+H}\left(\left\|\hat{x}(a;\eta|t)\right\|_{Q^*}^2-\left\|\hat{x}(b;\eta|t)\right\|_{Q^*}^2\right)d\eta\\ &\quad\quad+\left\|\hat{x}(a;t+H|t)\right\|_P^2-\left\|\hat{x}(b;t+H|t)\right\|_P^2.\end{aligned} \tag{18}$$

For $\eta\in[t,t_\zeta+H]$, it could be derived that

$$\begin{aligned}&\int_t^{t_\zeta+H}\left(\left\|\hat{x}(a;\eta|t)\right\|_Q^2-\left\|\hat{x}(b;\eta|t)\right\|_Q^2\right)d\eta\\ &\leq 2d_x\lambda_{\max}(Q)\int_t^{t_\zeta+H}\exp\{L_h(\eta-t)\}d\eta\left\|x(a;t)-x(b;t)\right\|\\ &\leq\frac{2d_x\lambda_{\max}(Q)\left(\exp\{L_h(t_\zeta+H-t)\}-1\right)}{L_h}\\ &\quad\times\left\|x(a;t)-x(b;t)\right\|,\end{aligned} \tag{19}$$

in which d_x is defined in Theorem 2.

For $\eta\in[t_\zeta+H,t+H]$, according to (14) we have that

$$\begin{aligned}&\int_{t_\zeta+H}^{t+H}\left(\left\|\hat{x}(a;\eta|t)\right\|_{Q^*}^2-\left\|\hat{x}(b;\eta|t)\right\|_{Q^*}^2\right)d\eta\\ &\leq 2\alpha\lambda_{\max}(Q^*)\exp\{L_h(t_\zeta+H-t)\}\\ &\quad\times\int_{t_\zeta+H}^{t+H}\exp\{L_k(\eta-t_\zeta-t)\}d\eta\left\|x(a;t)-x(b;t)\right\|\\ &\leq\frac{2\alpha\lambda_{\max}(Q^*)\exp\{L_h(t_\zeta+H-t)\}\left(\exp\{L_k(t-t_\zeta)\}-1\right)}{L_k}\\ &\quad\times\left\|x(a;t)-x(b;t)\right\|.\end{aligned} \tag{20}$$

For the third term, according to $\hat{x}(t_\zeta+H|t)\in\Omega$, we have

$$\begin{aligned}&\left\|\hat{x}(a;t+H|t)\right\|_P^2-\left\|\hat{x}(b;t+H|t)\right\|_P^2\\ &\leq 2\alpha\exp\{L_h(t_\zeta+H-t)+L_k(t-t_\zeta)\}\left\|x(a;t)-x(b;t)\right\|.\end{aligned} \tag{21}$$

Combining (19)-(21), we know that ϵ_1 could be presented as (17b).

Case ii $\left(t_{\zeta+1}>t_\zeta+H\right)$: The difference between $\Xi(x(a;t))$ and $\Xi(x(b;t))$ could be derived as:

$$\begin{aligned}&\Xi(x(a;t))-\Xi(x(b;t))\\ &\quad\leq\int_t^{t+H}\left(\left\|\hat{x}(a;\eta|t)\right\|_{Q^*}^2-\left\|\hat{x}(b;\eta|t)\right\|_{Q^*}^2\right)d\eta\\ &\quad\quad+\left\|\hat{x}(a;t+H|t)\right\|_P^2-\left\|\hat{x}(b;t+H|t)\right\|_P^2.\end{aligned} \tag{22}$$

For $\eta\in[t,t+H]$, we know

$$\begin{aligned}&\int_t^{t+H}\left(\left\|\hat{x}(a;\eta|t)\right\|_{Q^*}^2-\left\|\hat{x}(b;\eta|t)\right\|_{Q^*}^2\right)d\eta\\ &\leq 2\alpha\lambda_{\max}(Q^*)\int_t^{t+H}\exp\{L_k(\eta-t)\}d\eta\left\|x(a;t)-x(b;t)\right\|\\ &\leq\frac{2\alpha\lambda_{\max}(Q^*)\left(\exp\{L_kH\}-1\right)}{L_k}\left\|x(a;t)-x(b;t)\right\|.\end{aligned} \tag{23}$$

Similar to (21), we get

$$\left\|\hat{x}(a;t+H|t)\right\|_P^2-\left\|\hat{x}(b;t+H|t)\right\|_P^2 \le 2\alpha\exp\{L_kH\}\left\|x(a;t)-x(b;t)\right\|. \quad (24)$$

Then (17c) can be derived via (23) and (24).

According to the aforementioned two cases, Theorem 3 has been proven. ■

The ISpS characteristic of the system under the proposed GEMPC controller is described via the following theorem.

Theorem 4: Supposing that Assumptions 1-3 and the conditions in Theorem 2 are valid, then the system (1) is ISpS, i.e., there are a $\mathcal{KL}-$function γ, a $\mathcal{K}-$function β and a constant $c\ge 0$ satisfy

$$\|x(t)\|\le\gamma\left(\|x_0\|,t\right)+\beta(\bar{\delta})+c,\ \forall t\ge 0. \quad (25)$$

Proof: For Lyapunov function $\Xi(x(t))$, we know that

$$\Xi(x(t))\ge\gamma_1\left(\|x(t)\|\right), \quad (26)$$

where $\gamma_1\left(\|x(t)\|\right)=\lambda_{\min}(Q)\left(\|x(t)\|^2\right)$, $x(t)\in\mathbb{X}$ is a $\mathcal{K}_\infty-$ function with $\|x(t)\|$.

According to Theorem 3, we get that

$$J\left(\hat{x}(t),K\hat{x}(t),H\right)\le\|x(t)\|^2,\ x(t)\in\Theta. \quad (27)$$

Considering (26), (27) and the boundedness of set $\mathbb{X}$, it can be concluded that

$$\Xi(x(t))\le\gamma_2\left(\|x(t)\|\right)+c_1 \quad (28)$$

in which $\gamma_2\left(\|x(t)\|\right)=\frac{\alpha_v^2}{\alpha_f^2}\|x(t)\|^2$ with a constant $\alpha_v>\alpha_f$ is a $\mathcal{K}_\infty-$function with $\|x(t)\|$ and $c_1=\sup_{x(t)\in\mathbb{X}}|\Xi(x(t))-J\left(\hat{x}(t),K\hat{x}(t),H\right)|$.

Consider two Lyapunov functions $\Xi(\hat{x}(t))$ and $\Xi(\hat{x}(t+\Delta t))$.

Case i: If $[t,t+\Delta t]\subset[t_\zeta,t_\zeta+H]$, we get

$$\Xi(\hat{x}^*(t+\Delta t|t_\zeta))-\Xi(\hat{x}^*(t|t_\zeta))\le-\int_t^{t+\Delta t}\left\|\hat{x}^*(\eta|t_\zeta)\right\|_Q^2d\eta, \quad (29)$$

and then we know

$$D^+\Xi(\hat{x}^*(t|t_\zeta))=\lim_{\Delta t\to 0^+}\frac{\Xi(\hat{x}^*(t+\Delta t|t_\zeta))-\Xi(\hat{x}^*(t|t_\zeta))}{\Delta t} \le-\frac{\lambda_{\min}(Q)\alpha_f^2}{\alpha_v^2}\left(\Xi(\hat{x}^*(t|t_\zeta))+c_1\right). \quad (30)$$

Case ii: If $[t,t+\Delta t]\subset[t_\zeta+H,t_{\zeta+1}]$, we get

$$\Xi(\hat{x}(t+\Delta t|t_\zeta))-\Xi(\hat{x}(t|t_\zeta))\le-\int_t^{t+\Delta t}\left\|\hat{x}(\eta|t_\zeta)\right\|_{Q^*}^2d\eta, \quad (31)$$

and we know

$$D^+\Xi(\hat{x}(t|t_\zeta))\le-\lambda_{\min}(Q^*)\Xi(\hat{x}(t|t_\zeta)). \quad (32)$$

Based on the comparison principle [35] with (30) and (32), we know that

$$\Xi(\hat{x}(t|t_\zeta)) \le\exp\{-\min\{\frac{\lambda_{\min}(Q)\alpha_f^2}{\alpha_v^2},\lambda_{\min}(Q^*)\}(t-t_\zeta)\}\Xi(x(t_\zeta))+c_1. \quad (33)$$

Combining Theorem 3 and (33), we obtain

$$\Xi(x(t))\le\exp\{-\min\{\frac{\lambda_{\min}(Q)\alpha_f^2}{\alpha_v^2},\lambda_{\min}(Q^*)\}t\}\Xi(x_0)+c_1 +\epsilon\left(\|e(t)\|\right). \quad (34)$$

According to (9), (26) and (33), we know that

$$\|x(t)\|\le\gamma\left(\|x_0\|,t\right)+\beta(\bar{\delta})+c, \quad (35)$$

in which

$$\gamma\left(\|x_0\|,t\right) =\gamma_1^{-1}(\exp\{-\min\{\frac{\lambda_{\min}(Q)\alpha_f^2}{\alpha_v^2},\lambda_{\min}(Q^*)\}t\}\gamma_2\left(\|x(t)\|\right)); \quad (36a)$$

$$\beta(\bar{\delta}) =\gamma_1^{-1}\left(\epsilon\left(2\bar{\delta}\exp\{L_hH\max\{\theta,\theta_\mu\}\}/L_h\right)\right); \quad (36b)$$

$$c =\gamma_1^{-1}\left(c_1\right). \quad (36c)$$

According to Definition 2, system (1) is ISpS in $\mathbb{X}$. ■

V. SIMULATION VERIFICATION

In this section, two examples are given to verify the effectiveness of the proposed algorithm.

A. EXAMPLE 1

The simulation of the proposed GEMPC framework on a classical nonlinear system [5], [11], [36], and the comparison with traditional MPC as well as EMPC [11] are provided. The nonlinear system with additive disturbances is shown as:

$$\begin{aligned}\dot{x}_1(t)&=x_2(t)+u(t)\left(\vartheta+(1-\vartheta)x_1(t)\right)+\delta(t),\\ \dot{x}_2(t)&=x_1(t)+u(t)\left(\vartheta-4(1-\vartheta)x_2(t)\right),\end{aligned} \quad (37)$$

where the parameter $\vartheta\in(0,1)$ is chosen as 0.6. The input constraint is $u(t)\in[-2,2]$ and the state constraint is $x_1(t),x_2(t)\in[-1.2,1.2]$. The weight matrices in (8) are chosen as $R=0.5$, $Q=\begin{bmatrix}0.2&0\\0&0.2\end{bmatrix}$, and P is calculated as $\begin{bmatrix}3.8546&1.8546\\1.8546&3.8546\end{bmatrix}$ based on the feedback matrix $K=[1.5570\ 1.5570]$ in the Theorem 3. The Lipschitz constants are set as $L_h=1.5$, $L_k=3.0$. The parameters of invariant set and terminal set are $\alpha=0.3435$, $\alpha_f=0.1223$. The parameters related to the GEMPC controller are set to $H=\xi=1.0s$, $\theta=0.2$, $\theta_\mu=0.4$. The total simulation time is $10s$, and the sampling period is set to $0.05\ s$. The initial state

TABLE 1. Disturbance upper bounds comparison.

Control methods	$\bar{\delta}_1$(feasibility)	$\bar{\delta}_2$(stability)
EMPC [11]	0.0040	0.0018
Algorithm 1	0.0090	-

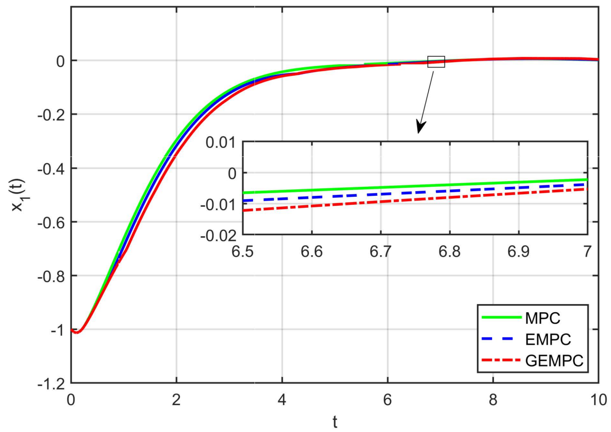

FIGURE 2. Comparison of $x_1(t)$.

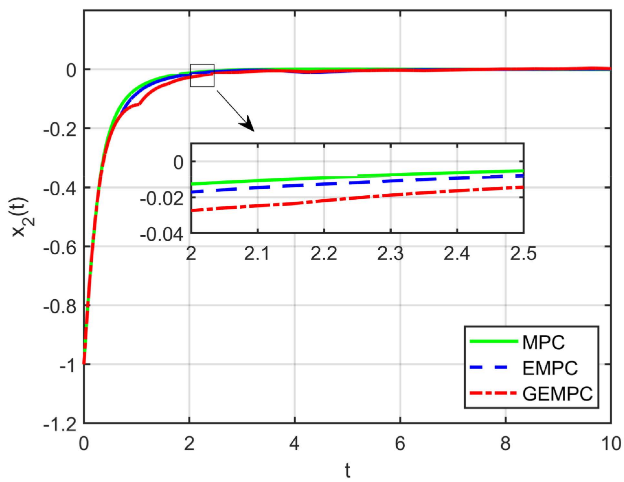

FIGURE 3. Comparison of $x_2(t)$.

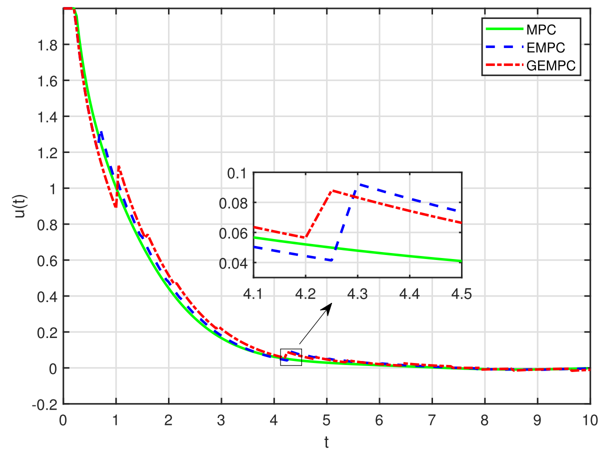

FIGURE 4. Comparison of $u(t)$.

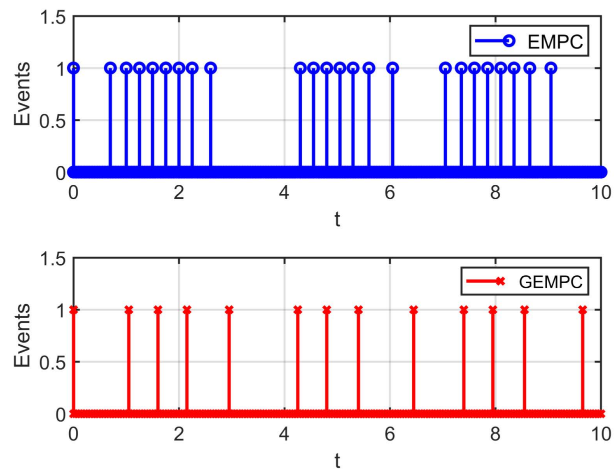

FIGURE 5. Comparison of triggering instants.

$x_0 = (-1, -1)$, and the disturbance $\delta(t) = 0.009 \times \sin(t)$ with the upper bound $\bar{\delta} = 0.0090$ which is calculated via Theorem 2.

Note that the disturbance upper bounds are listed in Table 1 under the same parameter configurations, indicating that Algorithm 1 has a greater anti-disturbance margin than [11]. In all figures, standard MPC, EMPC in [11] and Algorithm 1 are represented by green, blue and red lines, respectively. The comparison of state trajectories $x_1(t)$ and $x_2(t)$ are described via Fig. 2 and Fig. 3. Besides, Fig. 4 expresses the control signal under three control strategies. It is observed that, under the control of GEMPC algorithm, the state signal $x(t)$ and the control input $u(t)$ can satisfy the constraints just as the existing works. It is also worth mentioning that due to the state-dependent threshold and the consideration of the gradient of the state error in the proposed event triggering condition, the state mutation is effectively reduced and therefore the fluctuation of the input signal could relatively smaller than that of the existing methods. Fig. 5 reveals the comparison of triggering instants between EMPC [11] and Algorithm 1 where the time when the ordinate is 1 indicates that the OCP needs to be solved again. It is reflected that our proposal has a larger triggering interval and a fewer triggering time than EMPC, which is clearly illustrated by the data in Table 2, i.e. Algorithm 1 reduces the optimization times by 93.5% compared with periodic MPC and 45.8% compared with EMPC [11]. It could be concluded that our proposal can save quite a few computing and communication resources while not negatively affecting control performance.

B. EXAMPLE 2

The cart-damper-spring system is the most common mechanical vibration system, which is widely used in real life, such as buffer in automobile damping device, damper in building seismic device, and so on. To make the validation more practically convincible, a cart-damper-spring system [6],

TABLE 2. Comparison of solving times of OCPs.

Control methods	Solving times	Improvement
MPC	200	-
EMPC [11]	24	88%
Algorithm 1	13	93.5% (45.8%[1])

[1] The advancement of the GEMPC over EMPC in [11].

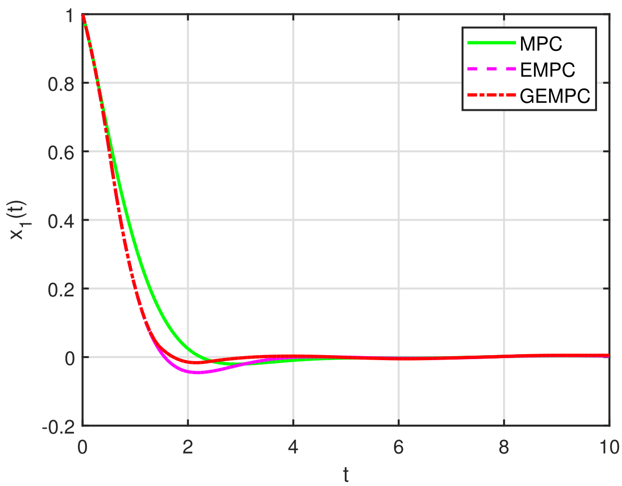

FIGURE 6. Comparison of displacements under MPC, EMPC [12] and GEMPC.

[12], [37] is considered in this section, whose dynamic equation is shown as follows:

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = -\dfrac{k}{m} e^{-x_1(t)} x_1(t) - \dfrac{\nu}{m} x_2(t) + \dfrac{u(t)}{m} + \dfrac{\delta(t)}{m}, \end{cases} \tag{38}$$

where $x_1(t)$, $x_2(t)$ are the displacement and velocity of the cart, and the mess is $m = 1.25kg$. $u(t)$ is the input signal bounded by $[-1.5, \ 1.5]$. The nonlinear spring is with a factor $k = 0.9N/m$, and damper parameter is $\nu = 0.42N \cdot s/m$. The Lipschitz constants are computed as $L_h = 1.4$ and $L_k = 2.6$. Given the weight matrices $Q = [0.2 \ 0; 0 \ 0.2]$ and $R = 0.5$, it can be concluded that $K = [-0.4454, \ -1.0932]$ and $P = [0.2927 \ 0.2026; 0.2026 \ 0.3565]$ following Lemma 1. The invariant set and the terminal set are set as $\Omega = \{x \in \mathbb{R}^n \mid \|x(t)\| \leq 0.5320\}$ and $\Theta = \{x \in \mathbb{R}^n \mid \|x(t)\| \leq 0.2725\}$, i.e., $\alpha = 0.5320$, $\alpha_f = 0.2725$. The parameters are set as $\xi = 1s$, $\theta = 0.2$, $\theta_\mu = 0.4$, $\delta(t) = 0.005 \times \sin(t)$. The prediction horizon is chosen to $H = 1.6s$ and the total simulation time is $10s$. The starting point is $x_0 = [1, -0.6]$ and the target is the origin.

Based on the above parameters, the comparison results of the time-driven MPC, EMPC [12] and the proposed GEMPC are provided via Figs. 6-9. Fig. 6 and Fig. 7 show the comparisons of the displacements and velocities of the cart, and Fig. 8 describes the comparison of control signals. It can be seen that the control performance of the GEMPC is basically comparable to that of the other two MPC strategies. Besides, according to Fig. 9, we know that with similar control performance, the number of times

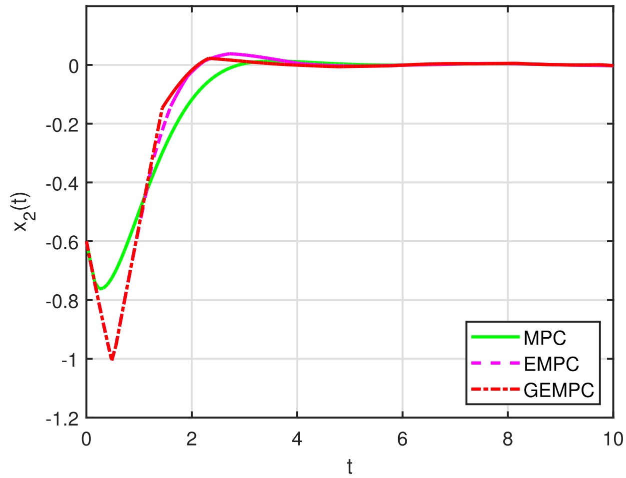

FIGURE 7. Comparison of velocities under MPC, EMPC [12] and GEMPC.

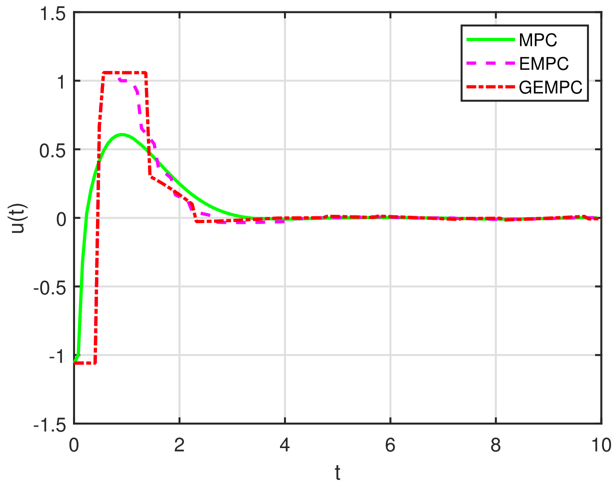

FIGURE 8. Comparison of control signals under MPC, EMPC [12] and GEMPC.

TABLE 3. Comparison of times of solving OCPs.

Methods	Solving times	Improvement
MPC	125	-
EMPC [12]	19	84.8%
Algorithm 1	9	93.5% (52.6%[1])

[1] The percentage advancement of the GEMPC over EMPC in [12].

GEMPC solves the OCP is substantially reduced relative to EMPC in [12]. In order to visualize the advantages of the proposed algorithm, the relevant data are presented in Table 3, indicating that compared with the time-driven MPC and EMPC in [12], the updated time of Algorithm 1 could be reduced by 93.5% and 52.6% respectively.

Furthermore, please note that compared with other types of computational efficient controller, e.g., [38]–[41], the proposed method may not achieve the best performance in terms of the computing burden, but as the proposed GEMPC is able to handle explicitly the input and state constraints, it would still be of certain significance for some

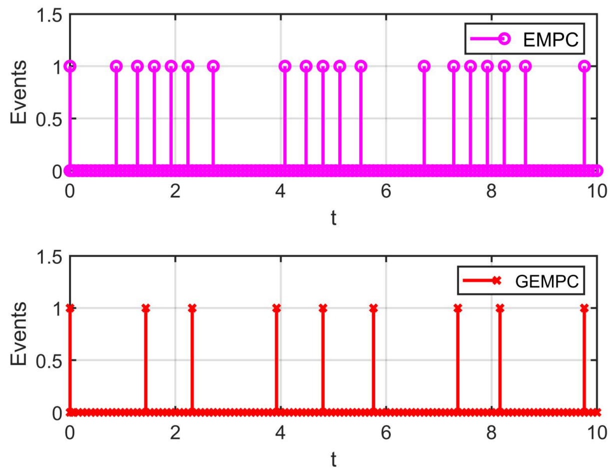

FIGURE 9. Comparison of triggering instants under EMPC [12] and GEMPC.

control systems with physical constraints, e.g., quadruple tank process systems, mobile robot systems and continuous stirred tank reactors.

VI. CONCLUSION

In this work, we suggest a GEMPC algorithm for nonlinear systems with additive disturbances, in which the error gradient between the optimal prediction of the state and the actual one is specified as the triggering condition. Moreover, the triggering threshold is designed with a state-dependent item rather than a fixed value one, to further alleviate the computational and communication burden. Subsequently, the Zeno-free property, the algorithm feasibility and the ISpS property of the considered system are studied theoretically. In the end, the simulation and comparison results demonstrate the effectiveness of the proposed GEMPC framework. The potential future research directions mainly involve extending the proposed method to handle other more complex systems, e.g., distributed systems subject to additive noise. Besides, as cyber security problem of event-based control system becomes more and more important recently, extending the proposed method to tackle with different cyber-attacks, e.g., FDI attack, is another promising future research direction.

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