Direct Data Driven Model Reference Control for Flight Simulation Table

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ABSTRACT We present a direct data driven model reference control framework for flight simulation table from both the theoretical analysis and engineering application. Sate of the art direct driven model reference control designs the unknown controller based on the input-output data and guarantees the actual model converge to the reference model, under a case of no any priori knowledge for the unknown plant. We improve this direct data driven model reference control strategy by considering its stability validation and synthesis analysis for nonlinear controller. The resulting direct data driven model reference control scheme can be implemented to ensure flight simulation table rotate accuracy, resulting in improved performance index. As the main technical contribution, we show that the proposed direct data driven model reference control framework also ensures closed loop stability and suitable extension for nonlinear controller.

INDEX TERMS Model reference control, direct data driven, flight simulation table, synthesis analysis.

I. INTRODUCTION

There exists the growing research about data drive method, applied optimization and the classical consideration of engineering mathematics and mathematical physics. Data driven discovery is currently revolutionizing how we model, predict and control complex systems. The most pressing scientific and engineering problems in the modern era are not amenable to empirical models or derivations based on first principles. Therefore, researchers are studying data driven approach for a diverse range of complex systems, such as turbulence, the brain, climate, finance, robotics and autonomy. These systems are typically nonlinear, dynamic, multi scale in space and time, high dimensional, with dominant underlying patters that would be characterized and modeled for the eventual goal of sensing, prediction, estimation and control. With modern mathematical methods, enabled by unprecedented availability of data and computational resources, we are now able to handle some unattainable challenge problems. Driving modern data science is the availability of vast and increasing quantities of data, enabled by remarkable innovations in low cost sensors, orders of magnitudes increases in computational power, such vast quantities of data are affording engineers and scientists across all disciplines new opportunities or data driven discovery, which has been referred to as the fourth paradigm of scientific discovery.

Dynamical systems provide a mathematical framework to describe the world around us, modeling the rich interactions between quantities that co-evolve in time. Formally, dynamical systems concern the analysis, prediction, and understanding of the behavior of systems of differential equations or iterative mappings that describe the evolution of the state of a system. This formulation is general enough to encompass a staggering range of phenomena, including those observed in classical mechanical system, electrical circuit, turbulent fluid. Generally the main task of data driven idea is to extract some useful information from the data, collecting through some sensors. It means the collected data, i.e. input and output data, are applied for the next processes directly,such as system identification, fault detection, controller design and statistical analysis, etc, so in this paper we always use this concept-direct data driven to emphasis the importance of data. By the way, the direct idea tells us those useful information are obtained from the collected data directly, not through other intermediate process. As now we are in data era and considering more research about data science, this idea of direct data driven is more widely used in research fields.

Due to the property of extracting useful information from data in the framework of data driven, this nice idea can be combined with other interesting subjects to yield new fields, for example, direct data driven identification, direct data

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driven control, direct data driven learning, and reinforcement learning, etc. The first direct data driven identification corresponds to the classical system identification, proposed in 1960s to construct the mathematical model for the unknown plant through statistical analysis of the collected input-output data. Based on this direct data driven idea, we are thinking can it be combined with control strategy to form direct data driven control? The reply is to design the unknown controller, whatever feedback controller and feed-forward controller, on the basis of the collected or measured input-output data. From the theoretical perspective, data includes mots important information or knowledge for the unknown plant and unknown controller simultaneously, our mission is to construct the mathematical model for the unknown plant and design the controller for he unknown controller respectively, which are related with direct data driven identification and direct data driven control. As there are lots of references on the first direct data driven identification, but few about direct data driven control, so in these recent years, more researchers start to study direct data driven control deeply from the point of theoretical perspective.

Related work:Due to the wide consideration of direct data driven control and the combination with other different control strategies, direct data driven model predictive control was proposed to design two degrees of freedom controllers for an unknown plant based on input-output measurement [1]. To relax the strict probabilistic description on disturbance in analyzing stability and robustness, the dissipativity properties were analyzed in the constrained optimal control [2], where a special computational approach was proposed to achieve the robustness guarantees. A data driven method to design reference tracking controllers was introduced for nonlinear system in [3], where it delivered directly a time varying state feedback controller by combining an online and an offline scheme. In reference [4], a robust data driven model predictive control was proposed to control linear time invariant systems and behavioral systems theory or past measured trajectories were used as implicit model description. A quasi-infinite horizon nonlinear predictive control was presented for tracking of generic reference trajectories [5], and similarly a nonlinear robust model predictive control was considered for general state and input dependent disturbance [6], which used an online constructed tube to tighten the nominal constraints. As a consequence, based on reference [7], one linear state space form with control input and input-output noises was considered, not the linear rational function. In case of these measurement noises, described as set membership uncertainty, the idea of dynamic programming is introduced to achieve the goal of state estimation. During this year, our contributions about this direct data driven control are formulated here. Reference [8] combines system identification, direct data driven control and optimal algorithm in designing two controllers for one cascade control system, i.e. the inner controller and outer controller without any knowledge of the unknown plant. Then iterative operation is introduced with direct data driven control to yield the so called iterative data

driven control [9], which is benefit for stability analysis in one model matching problem. Data driven model predictive control is considered to adjust the varying coupling conditions among the different parts of the system, then through adding the inequality constraint to the constructed model predictive control, one persistently exciting data driven model predictive control is obtained in [10]. After substituting the obtained system parameter into the prediction output to construct one cost function, reference [11] takes the derivative of the above cost function with respect to control input to achieve one optimal input. Set membership identification can be not only applied in MC, but also in stochastic adaptive control, where a learning theory -kernel is introduced to achieve the approximation for nonlinear function or system. Based on the bounded noise, many parameters are also included in known intervals in prior, then robust optimal control with adjustable uncertainty sets are studied in [12], where robust optimization is introduced to consider uncertain noise and uncertain parameter simultaneously. To solve the expectation operation with dependence on the uncertainty, sample size of random convex programs is considered to replace the expectation by finite sum. Generally, many practical problems in systems and control, such as controller synthesis and state estimation, are often formulated as optimization problems. In many cases the cost function incorporates variables that are used to model uncertainty, in addition to optimization variables, and reference [13] employs uncertainty described as probabilistic variables.

In recent years, more novel ways ar explored to develop model predictive control, for example, the idea of data driven, mentioned above, is combined with model predictive control to yield a new control strategy- data driven model predictive control. In [14], data driven model predictive control is applied to design the classical PID for a deterministic continuous time system. For the case of switching controllers in some industries, data driven model predictive control is also benefit in regulating the switching rule [15]. Consider the uncertain factors exist in the closed loop situation, one robust data driven model predictive control is proposed to alleviate and suppress the bad effect, coming from these uncertainties [16]. Further to be convenient for the use of data driven model predictive control, some existed softwares are produced for researchers, such as in python package [17] and in its intelligent form to control the heat treatment electric furnace [18].

To show the closed relations among identification, control and optimization, here one example of our ongoing studied data driven control is used to illustrate the importance of the optimization theory. As the application of data driven approach widely in control field, and the similar point between data driven approach and system identification, we call their combination as identification for control, i.e. system identification for direct data driven control. More specifically, we describe a concise introduction or contribution on system identification for direct data driven control, which belongs to data driven approach. In case of the unknown

but bounded noise, one bounded error identification is proposed to identify the unknown systems with time varying parameters. Then one feasible parameter set is constructed to include the unknown parameter with a given probability level. In [19], the feasible parameter set is replaced by one confidence interval, as this confidence interval can accurately describe the actual probability that the future predictor will fall into the constructed confidence interval. The problem about how to construct this confidence interval is solved by a linear approximation/programming approach [20], which can identify the unknown parameter only for linear regression model. According to the obtained feasible parameter set or confidence interval, the midpoint or center can be deemed as the final parameter estimation, further a unified framework for solving the center of the confidence interval is modified to satisfy the robustness. This robustness corresponds to other external noises, such as outlier, unmeasured disturbance [21]. The above mentioned identification strategy, used to construct one set or interval for unknown parameter, is called as set membership identification, dealing with the unknown but bounded noise. There are two kinds of descriptions on external noise, one is probabilistic description, the other is deterministic description, corresponding to the unknown but bounded noise here [22]. For the probabilistic description on external noise, the noise is always assumed to be one white noise, and its probabilistic density function (PDF) is known in advance. On the contrary for deterministic description on external noise, the only information about noise is bound, so this deterministic description can relax the strict assumption on probabilistic description. In reality or practice, bounded noise is more common than white noise. Within the deterministic description on external noise, set membership identification is adjusted to design controllers with two degrees of freedom [23], it corresponds to direct data driven control or set membership control. Set membership control is applied to design feedback control in a closed loop system with nonlinear system in [24], where the considered system is identified by set membership identification, and the obtained system parameter will be benefit for the prediction output. After substituting the obtained system parameter into the prediction output to construct one cost function, reference [25] takes the derivative of the above cost function with respect to control input to achieve one optimal input. Set membership identification can be not only applied in MC, but also in stochastic adaptive control [26], where a learning theory -kernel is introduced to achieve the approximation for nonlinear function or system. Based on the bounded noise, many parameters are also included in known intervals in prior, then robust optimal control with adjustable uncertainty sets are studied in [27], where robust optimization is introduced to consider uncertain noise and uncertain parameter simultaneously. To solve the expectation operation with dependence on the uncertainty, sample size of random convex programs is considered to replace the expectation by finite sum [28]. Generally, many practical problems in systems and control, such as controller synthesis and state estimation, are often

formulated as optimization problems [29]. In many cases the cost function incorporates variables that are used to model uncertainty, in addition to optimization variables, and reference [30] employs uncertainty described as probabilistic variables. Reference [31] studies data driven output feedback controllers for nonlinear system, and applies event triggered mode to analyze the robust stability. Data driven estimation is used to achieve hybrid system identification [32], whose nonlinearity is described by one kernel function. During these recent years, the first author studies this direct data driven control too, for example, the closed relation between system identification and direct data driven control [33], and data driven model predictive control [34]. Based on above descriptions on direct data driven control and our existed research about system identification, model predictive control, direct data driven control and convex optimization theory etc, our mission in this paper is to combine our previous results and apply them in practical engineering. During these two years, a new interesting subject about persistently of excitation is studied again in data driven control and model predictive control. Willem's fundamental lemma from [35]gives a data based parametrization of trajectories for one linear time invariant system. Based on this Willem's fundamental lemma, one parametrization of linear closed loop system is derived to pave a way to study important controller design problems [36]. As the number of references on model predictive control is very vast and here we can not list all of them, so we wish the above detailed descriptions on model predictive control mean model predictive control is truly worth to be studied deeply from different points, such as theoretical analysis and practical engineering. Then the mission of this paper is to combine them together. The above descriptions about the combinations of the direct data driven control and other control strategy concern on the direct data driven model predictive control, moreover the idea of direct data driven can be related with other control strategy, such as direct data driven model reference control, which is the main work in this new paper.

Contribution: Model reference control means that after given one reference model or reference performance function, we need to design the unknown controller to guarantee the output approach to that reference performance function. It means the process of controller design is to make the error value between the real output and reference value as small as possible. To the best of our knowledge, every control strategy has its own control task. As model reference control was firstly proposed in MIT, and then applied in some engineering problems during these years, so it is still worth to study from different new points. This paper introduces direct data driven into model reference control to yield one novel direct data driven model reference control strategy. The detailed direct data driven model reference control and its synthesis are described, then its application in flight simulation table is given as an example to achieve the practical engineering. Flight simulation table is key key hardware device for semi-physical simulation and test experiments in aviation,

aerospace and other defense fields, and plays an important role in the development of aircraft. It can simulate the various attitude angle movements of the aircraft in the air, such as pitch, roll, and yaw, and reproduce various dynamic characteristics during its motion. Furthermore, it can repeatedly test the performance of the aircraft's guidance system, control system and components. In order to obtain sufficient experimental data, the system is redesigned and improved according to the obtained data to meet the performance index requirements of the overall design of the aircraft. Therefore, the performance of the flight simulation table is directly related to the reliability and confidence of the simulation test, and is the basis for ensuring the accuracy and performance of aerospace models and weapon systems. It not only speeds up the development process of aerospace model products and modern weapon systems, but also greatly reduces development costs, so it has a very important significance in the development of the defense industry. The above is the reason about why we apply our proposed direct data driven model reference control into flight simulation table.

The main contributions of this new paper are formulated as follows.

(1) To avoid the identification process, the model reference control and direct data driven idea are combined to form a new control strategy-direct data driven model reference control.

(2) Synthesis analysis for nonlinear controller with the idea of direct data driven is yielded to extend one optimal condition.

(3) To connect the theoretical research and engineering practice, our considered direct data driven model predictive control is applied in flight simulation table, and the detailed application process in also given.

Outline: This paper is organized as follows. In section 2, the direct data driven model reference control is described in detail. After reviewing some basic knowledge about classical model reference control, then the novel direct data driven model reference control and its extended synthesis analysis are given to show our new recent contributions. The detailed introduction and analysis about the application for flight simulation table are proposed in section 3, where basis knowledge for flight simulation table, control structure description and the process of direct data driven model reference control are all completed. Section 4 gives the simulation example to prove our proposed theories. Finally, conclusions and comments about next work are presented in section 5. Roughly speaking, this paper reviews our new contributions on direct data driven model reference control and merges it for flight simulation table, while achieving the combination with theory and practice.

II. DIRECT DATA DRIVEN MODEL REFERENCE CONTROLLER DESIGN

The commonly used closed loop system with one unit feedback loop is considered in the following Figure 1, where $r(t)$ is the external input signal, chosen by the researcher, error value $e(t) = r(t) - y(t)$ is the input signal for that

unknown controller $C(z)$. The feedback property is to turn back the output signal $y(t)$ for the whole closed loop system to the input interface, and use this error signal $e(t)$ to affect the unknown controller $C(z)$. $\{u(t), y(t)\}$ correspond to the input-output signal with respect to the unknown plant $P(z)$, $d(t)$ is the external disturbance or noise in statistical framework or deterministic case. $M(z)$ is one reference model or expected reference value, being known in prioričň, z is the backward shift operator.
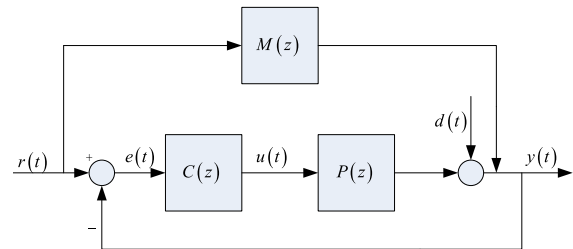

FIGURE 1. Closed loop system with unit feedback.

A. MODEL REFERENCE CONTROL

From Figure 1, as $M(z)$ is one expected reference model, then it holds that

$$y(t) = M(z)r(t) \tag{1}$$

Equation (1) means after chosen the approximated external input signal $r(t)$ and expected reference model $M(z)$, we can easily compute the closed loop output value $y(t)$. For clarity of explanation, this output $y(t) = M(z)r(t)$ is named as the ideal or expected output value.

Observing the lower part of Figure 1, the following equations hold.

$$\begin{aligned}
y(t) &= P(z)r(t) + d(t) \\
&= P(z)C(z)[r(t) - y(t)] + d(t) \\
&= P(z)C(z)r(t) - P(z)C(z)y(t) + d(t) \\
y(t) &= \frac{P(z)C(z)}{1 + P(z)C(z)}r(t) + \frac{1}{1 + P(z)C(z)}d(t) \tag{2}
\end{aligned}$$

In the framework of model reference control, that unknown controller $C(z)$ is parametrized by one unknown parameter vector θ, so we always denote it as $C(z, \theta)$. As a consequence, equation (2) is rewritten as that.

$$y(t, \theta) = \frac{P(z)C(z, \theta)}{1 + P(z)C(z, \theta)}r(t) + \frac{1}{1 + P(z)C(z, \theta)}d(t) \tag{3}$$

The parametrized controller $C(z, \theta)$ is always seen in practice, such as the PID controller. As only three controller parameters exist in the PID controller, so the problem of controller design is turned to identify or choose three parameter estimates, while satisfying the researchers' control mission. Base on this parametrized controller $C(z, \theta)$, the main process for model reference control is to design the unknown parameters, while guaranteeing the error value between $y(t)$ and $y(t, \theta)$ be as small as possible, i.e. the ideal case corresponds

to zero error, which means that following approximations hold.

$$y(t, \theta) \rightarrow y(t)$$
$$\frac{P(z)C(z, \theta)}{1 + P(z)C(z, \theta)}r(t) \rightarrow M(z)r(t) \quad (4)$$

Using the property of L2 norm, the above approximation relation is measured through the L2 norm, i.e.

$$\left\|\frac{P(z)C(z, \theta)}{1 + P(z)C(z, \theta)}r(t) - M(z)r(t)\right\|_2^2 \quad (5)$$

where $\|.\|_2^2$ is the L2 norm.

Then the problem of controller design is changed as one number optimization problem, whose decision variable is that unknown parameter vector θ.

$$\hat{\theta} = \arg\min_\theta \left\|\frac{P(z)C(z, \theta)}{1 + P(z)C(z, \theta)}r(t) - M(z)r(t)\right\|_2^2 \quad (6)$$

where $\hat{\theta}$ is the parameter estimate.

Observing the optimization problem (6), when to identify the parameter estimate $\hat{\theta}$, we need sone knowledge about the plant $P(z)$, as there $P(z)$ and $C(z, \theta)$ are all unknown, so the first principle or system identification are firstly proposed to identify that unknown plant $P(z)$, it is the drawback for model reference control. For the purpose of illustration, assume there exists one true parameter vector θ_0, such that.

$$\frac{P(z)C_0(z, \theta_0)}{1 + P(z)C_0(z, \theta_0)}r(t) = M(z)r(t) \quad (7)$$

substituting equation (7) into (6), then model reference control is reduced to

$$\hat{\theta} = \arg\min_\theta \left\|\frac{P(z)C(z, \theta)}{1 + P(z)C(z, \theta)}r(t)\right.$$
$$\left. - \frac{P(z)C_0(z, \theta_0)}{1 + P(z)C_0(z, \theta_0)}r(t)\right\|_2^2 \quad (8)$$

Making use of the Taylor series expansion on that error value to get.

$$\frac{P(z)C(z, \theta)}{1 + P(z)C(z, \theta)} - \frac{P(z)C_0(z, \theta_0)}{1 + P(z)C_0(z, \theta_0)}$$
$$= P(z)\left[\frac{C(z, \theta)}{1 + P(z)C(z, \theta)} - \frac{C_0(z, \theta_0)}{1 + P(z)C_0(z, \theta_0)}\right]$$
$$= P(z)\frac{1}{[1 + P(z)C_0(z, \theta_0)]^2}[C(z, \theta) - C_0(z, \theta_0)]$$
$$= P(z)[1 - M(z)]^2[C(z, \theta) - C_0(z, \theta_0)] \quad (9)$$

where the following equity is used in deriving equation (9).

$$\frac{P(z)C_0(z, \theta_0)}{1 + P(z)C_0(z, \theta_0)} = M(z)$$
$$1 - M(z) = \frac{1}{1 + P(z)C_0(z, \theta_0)} \quad (10)$$

Combining equation (6), (8) and (9), that cost function in equation (8) is formulated as that.

$$\left\|\left[\frac{P(z)C(z, \theta)}{1 + P(z)C(z, \theta)} - M(z)\right]r(t)\right\|_2^2$$

$$= \left\|\left[\frac{P(z)C(z, \theta)}{1 + P(z)C(z, \theta)} - \frac{P(z)C_0(z, \theta_0)}{1 + P(z)C_0(z, \theta_0)}\right]r(t)\right\|_2^2$$
$$= \|P(z)[1 - M(z)]^2[C(z, \theta) - C_0(z, \theta_0)]r(t)\|_2^2 \quad (11)$$

The ideal case is that for all $t = 1, 2, \cdots N$, where N is the total number of the time instant, we have

$$P(z)[1 - M(z)]^2[C(z, \theta) - C_0(z, \theta_0)]r(t) = 0 \quad (12)$$

More specifically, consider the linearly parametrized controller $C(z, \theta)$, i.e.

$$C(z, \theta) = \alpha(z)\theta, \quad C_0(z, \theta_0) = \alpha(z)\theta_0 \quad (13)$$

where $\alpha(z)$ is the basis functions, for example, $\alpha(z) = [1\ z\ z^2\cdots]^T$.

substituting the linearly parametrized controller into equation (12) to get.

$$P(z)[1 - M(z)]^2\alpha(z)[\theta - \theta_0]r(t) = 0$$
$$P(z)[1 - M(z)]^2\alpha(z)r(t) = P(z)[1 - M(z)]^2$$
$$\times C_0(z, \theta_0)r(t) \quad (14)$$

Define the following variables to simplify the expression for the parameter estimate as

$$\varphi_1(t) = P(z)[1 - M(z)]^2\alpha(z)r(t)$$
$$r_1(t) = P(z)[1 - M(z)]^2C_0(z, \theta_0)r(t) \quad (15)$$

Then we have

$$r_1(t) = \varphi_1^T(t)\theta \quad (16)$$

i.e. the parameter estimate $\hat{\theta}$ is solved by the least squares method.

$$\hat{\theta} = \left[\sum_{t=1}^N \varphi_1^T(t)\varphi_1(t)\right]^{-1}\left[\sum_{t=1}^N \varphi_1^T(t)r_1(t)\right] \quad (17)$$

Furthermore, after taking expectation operation on both sides of equation (17), we see $E[\hat{\theta}] = \theta_0$ easily, it means the least squares parameter estimate $\hat{\theta}$ is unbiased. Generally, from above description about model reference control, when $\{P(z), M(z)\}$ are known in advance, then the controller $C(z)$ is designed to make the approximated error be zero, i.e. $\frac{P(z)C(z, \theta)}{1 + P(z)C(z, \theta)} = M(z)$.

B. DIRECT DATA DRIVEN MODEL REFERENCE CONTROL

To avoid the identification process for that unknown plant $P(z)$ in above model reference control, the idea of direct data driven is introduced to yield our novel direct data driven model reference control strategy. Observing Figure 1 again, data $\{r(t), y(t)\}_{t=1}^N$ are used in model reference control, but now we do not regard $\{r(t), y(t)\}_{t=1}^N$ as out considered data, on the contrary, data $\{u(t), y(t)\}_{t=1}^N$ are concerned. Through comparing these two data sequences $\{r(t), y(t)\}_{t=1}^N$ and $\{u(t), y(t)\}_{t=1}^N$, we see the first data correspond to the whole closed loop system, but the second are for the unknown controller $C(z)$. More specifically, the input signal for the unknown controller $C(z)$ is $e(t) = r(t) - y(t)$, and its output

signal is $u(t)$, so in this special case we consider the data $\{r(t) - y(t), u(t)\}_{t=1}^{N}$.

From Figure 1, the relation between input $e(t) = r(t) - y(t)$ and output $u(t)$ is that.

$$u(t) = C(z.\theta)e(t) = C(z.\theta)[r(t) - y(t)] \quad (18)$$

substituting equity $y(t) = M(z)r(t)$ and using the inverse property about the expected model $M(z)$, we have

$$u(t) = C(z.\theta)[M^{-1}(z)y(t) - y(t)]$$
$$= C(z.\theta)[M^{-1}(z) - 1]y(t) \quad (19)$$

Summing up from $t = 1$ to N, direct data driven model reference control is formulated as the following optimization problem with respect to the unknown parameter vector θ.

$$\hat{\theta} = \arg\min_\theta \frac{1}{N} \sum_{t=1}^{N} [u(t) - C(z.\theta)[M^{-1}(z) - 1]y(t)]^2 \quad (20)$$

The main difference between adaptive control and our considered control is in equation (20), as no any information about the plant is needed. But for the classical adaptive control in equation (8), information $P(z)$ is needed.

After given the expected model $M(z)$ and collected data $\{u(t), y(t)\}_{t=1}^{N}$, only $C(z, \theta)$ is unknown in cost function of equation (20). Similarly, consider the linearly parametrized controller $C(z, \theta) = \alpha(z)\theta$, the least squares parameter estimate is obtained.

$$\hat{\theta} = [\sum_{t=1}^{N} \varphi_2^T(t)\varphi_2(t)]^{-1}[\sum_{t=1}^{N} \varphi_2^T(t)u(t)]$$
$$\varphi_2(t) = [M^{-1}(z) - 1]y(t)\alpha(z) \quad (21)$$

Furthermore, to increase the computational speed, its recursive least squares algorithm is that.

$$P^{-1}(t) = \sum_{t=1}^{N} \varphi_2^T(t)\varphi_2(t)$$
$$L(t) = \frac{P(t-1)\varphi_2(t)}{1 + \varphi_2^T(t)P(t-1)\varphi_2(t)}$$
$$P(t) = P(t-1) - \frac{P(t-1)\varphi_2^T(t)\varphi_2(t)P(t-1)}{1 + \varphi_2^T(t)P(t-1)\varphi_2(t)}$$
$$\hat{\theta}(t) = \hat{\theta}(t-1) + L(t)[u(t) - \varphi_2(t)(t)\hat{\theta}(t-1)]$$

where $\hat{\theta}(t)$ and $\hat{\theta}(t-1)$ are the parameter estimations at time instant t and $t-1$ respectively.

To start the above recursive least squares algorithm, the initial parameter value $\hat{\theta}(0)$ can be chosen as

$$\hat{\theta}(0) = \frac{1}{2}I$$

where I is a column vector with element 1.

To analyze the convergence performance about equation (20) or (21), the following Assumption 1 is needed, i.e.

Assumption 1: There exists a continuously twice differentiable Lyapunov function $V_1(\theta)$ satisfying the following conditions.

(1) Its second derivative is bounded;

(2) $V_1(\theta) > 0, \forall \theta \neq \theta_0; V_1(\theta_0) = 0$ *and* $V_1(\theta) \to \infty$ *as* $\|\theta\| \to \infty$

(3) For any $\epsilon > 0$ there is a β_ϵ such that

$$\sup_{\|\theta - \theta_0\| > \epsilon} V_{1\theta}(\theta)[u(t) - \varphi_2(t)(t)\hat{\theta}(t-1)] = -\beta_\epsilon < 0$$

where $V_{1\theta}(\theta)$ denotes the gradient of $V_1(\theta)$. Then on the basis of above assumption, the convergence analysis of equation (20) or (21) is formulated as follows.

Proposition 1: Assume assumptions 1 hold. Then for any initial value, $\hat{\theta}_k$ given by equation (20) or (21), converges to the zero θ_0 of the cost function $f_2(\theta)$ as $k \to \infty$.

Proof of Proposition 1 can be seen our previous paper [9].

Equation (21) corresponds to the parameter estimate for the parametrized controller, but for the non-parametrized controller $C(z)$, an very easy way to design the controller exists. Without loss of generality, the ideal case for equation (20) is that.

$$u(t) - C(z)[M^{-1}(z) - 1]y(t) = 0$$
$$u(t) = C(z)[M^{-1}(z) - 1]y(t) \quad (22)$$

Multiplying $y(t)$ and taking expectation operation on both sides of equation (21) to get

$$E[u(t)y^T(t)] = C(z)[M^{-1}(z) - 1]E[y(t)y^T(t)]$$
$$\phi_{uy}(w) = C(z)[M^{-1}(z) - 1]\phi_y(w) \quad (23)$$

where $\phi_{uy}(w)$ and $\phi_y(w)$ are cross spectral and auto-spectral. From equation (22), we have

$$C(z) = \frac{\phi_{uy}(w)}{\phi_y(w)} \frac{M(z)}{1 - M(z)} \quad (24)$$

Consider the controller $C(z)$ in equation (23), $\{u(t), y(t)\}_{t=1}^{N}$ are collected to obtain their own spectral estimates $\{\phi_{uy}(w), \phi_y(w)\}$. Reference model $M(z)$ is given in priori, then term $\frac{M(z)}{1-M(z)}$ is also obtained easily. Based on these two spectral estimates $\{\phi_{uy}(w), \phi_y(w)\}$ and reference model $M(z)$, a rough controller $C(z)$ is chosen from expression (23). According to computing the two spectral estimates $\{\phi_{uy}(w), \phi_y(w)\}$ on the basis of input-output data $\{u(t), y(t)\}_{t=1}^{N}$, then existed Matlab programs can be utilized directly without wasting any time.

C. STABILITY VALIDATION

The closed loop system, designed by the proposed direct data driven model reference control, must satisfy the closed loop stability, or this obtained closed loop system is rejected. That about whether to accept or reject the obtained closed loop system is dependent on probabilistic method, i.e. the input-output observed data $\{u(t), y(t)\}_{t=1}^{N}$ are combined with prediction error identification to estimate that plant $P(z)$.

Assume that plant $P(z)$ is parametrized by one parameter vector η as one parametrized model $P(z, \eta)$, and one real or true parameter vector η_0 exists to satisfy that $P(z) = P(z, \eta_0)$,

where this real parameter vector η_0 is unknown, but it can be estimated through prediction error identification too.

From the asymptotic analysis with the framework of probabilistic theory, parameter estimation $\hat{\eta}$ has an asymptotic normal distribution around that real parameter η_0, i.e.

$$\hat{\eta} : N(\eta_0, \Sigma_\eta)$$

where $N(\eta_0, \Sigma_\eta)$ denotes the asymptotic normal distribution with mean value η_0 and variance Σ_η, being estimated by the observed data too.

Then the criterion for validating the designed controller is formulated as follows, if the observed controller $C(z, \theta)$ can guarantee that when plant $P(z, \eta)$ changes within uncertain set, then closed loop system output will always track one desired output value, so the closed loop system is stable, and then the designed controller $C(z, \theta)$ is accepted. Assume the probabilistic density function of the real parameter η_0, i.e.

$$\eta_0 : P(\eta_0)$$

Through the Bayesian rule, the conditional probabilistic density function is yielded as that

$$P(\hat{\eta}/\eta_0) = N(\eta_0, \Sigma_\eta)$$

It means

$$P(/\eta_0/\hat{\eta}) = P(\hat{\eta}/\eta_0)P(\eta_0)/P(\hat{\eta})$$

when $P(\eta_0) = N(m_{\eta_0}, \Sigma_{\eta_0})$, $P(\hat{\eta}/\eta_0) = N(\eta_0, \Sigma_{\hat{\eta}})$, we have

$$P(/\eta_0/\hat{\eta}) = N(m_{\eta/\hat{\eta}}, \Sigma_{\eta/\hat{\eta}})$$

where

$$\Sigma_{\eta/\hat{\eta}} = (\Sigma_{\hat{\eta}}^{-1} + \Sigma_{\eta_0}^{-1})^{-1}$$
$$m_{\eta/\hat{\eta}} = \Sigma_{\eta/\hat{\eta}}(\Sigma_{\hat{\eta}}^{-1}\hat{\eta} + \Sigma_{\eta_0}^{-1}m_{\eta_0})$$

Choosing the posterior distribution as the priori distribution for the all parameter space, it holds that

$$\lim_{\Sigma_{\eta_0}^{-1} \to \infty} P(/\eta_0/\hat{\eta}) = N(\hat{\eta}, \Sigma_{\hat{\eta}})$$

The probability that controller $C(z, \theta)$ destabilizes $P(z, \eta_0)$ is that

$$P = \int I(\eta_0)P(/\eta_0/\hat{\eta})d\eta_0 = E[I(\eta_0/\hat{\eta})]$$

where $I(\eta_0)$ is that

$$I(\eta_0) = \begin{cases} 1 & \text{if } C(z, \theta) \text{ destabilizes } P(z, \eta_0) \\ 0 & \text{otherwise} \end{cases}$$

Computing P and one given threshold $\bar{P}$, if $P < \bar{P}$, then accept that designed controller $C(z, \theta)$, or reject it.

D. SYNTHESIS ANALYSIS FOR NONLINEAR CONTROLLER

Controller (21) and (23) are for parametrized controller and non-parametrized controller respectively. Here we concern on how to consider the special case of nonlinear controller, i.e. the unknown controller $C(z)$ shows one nonlinear form in Figure 2.

Nonlinear controller exists in practice more widely then linearly parametrized controller. As linear system theory is mature, and lots of existed results about linear system theory can be used directly, so nonlinear system theory is always approximated by its equivalent linear system theory through Taylor series expansion or linearized process around one equilibrium point. When nonlinear controller $C(z)$ exists in closed loop system, we consider to design the optimal linear approximation to the formal nonlinear controller.
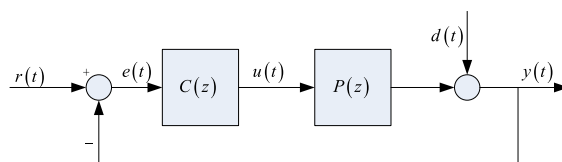

FIGURE 2. Nonlinear closed loop.

Due to the input-output signal for the nonlinear controller $C(z)$ be $\{r(t) - y(t), u(t)\} = \{[M^{-1}(z) - 1]y(t), u(t)\}$, its relation is written as.

$$u(t) = C(z)e(t) = C(e(t), z)$$
$$e(t) = r(t) - y(t) = M^{-1}(z)y(t) - y(t)$$
$$= [M^{-1}(z) - 1]y(t) \tag{25}$$

Now we think to optimally approximate the nonlinear controller for a given reference input $e_0(t)$ by the output of a linear controller. The class of linear controller, denoted by W, which we will consider for optimal approximation are represented in convolution form as integral operator. Thus for one input $e_0(t)$, then output of the linear controller W is given by

$$u_l(t) = (W(e_0))(t) = \int_{-\infty}^{\infty} w(t)e_0(t - \tau)d\tau \tag{26}$$

with the understanding that $e(t) = 0$ for all $t \leq 0$, the convolution kernel is chosen to minimize the main squared error, defined by.

$$\epsilon(w) = \lim_{T \to \infty} \frac{1}{T} \int_0^T [We_0(\tau) - Ce_0(\tau)]^2 d\tau \tag{27}$$

As before for all T, the integral above exists absolutely. The main result about optimal linear approximation to nonlinear controller is formulated in Theorem 1.

Theorem 1: There exists an optimal causal, stable linear controller approximate solving the optimal problem, with convolution kernel $w^*()$, then $w^*()$ satisfies.

$$\phi_{w^*}(\tau) = \phi_C(\tau) \tag{28}$$

where ϕ_{w^*} is short hand for the cross correlation between $e_0()$ and $W^*(e_0())$, i.e.

$$\phi_{w^*}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T [e_0(t-\tau)]W^*e_0 dt$$

$$\phi_C(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T [e_0(t-\tau)]C(z)e_0 dt \quad (29)$$

where $\phi_C(\tau)$ is the correlation between $e_0()$ and $C(z)e_0(\tau)$.

Equation (27) signifies that.

$$\lim_{T \to \infty} \frac{1}{T} \int_0^T e_0(t-\tau)[W^*e_0 - C(z)e_0(\tau)]dt \quad (30)$$

Proof: Set $W_1(z)$ is another linear time invariant controller, let

$$W_1(z)e_0(t) = \int_0^t w_1(t-\tau)e_0(\tau)d\tau \quad (31)$$

and similarly we also have

$$W(z)y(t) = \int_0^t w(t-\tau)e_0(\tau)d\tau \quad (32)$$

Define

$$\epsilon(t) = w_1(t) - w(t)$$

Then

$$\epsilon(z)e_0(t) = w_1(z)e_0(t) - w(z)e_0(t)$$
$$= \int_0^t \epsilon(t-\tau)e_0(\tau)d\tau$$
$$= \int_0^t \epsilon(\tau)e_0(t-\tau)d\tau \quad (33)$$

Then using the defined error criterion to get

$$E[W_1(z)] - E[W(z)]$$
$$= \lim_{N \to \infty} \int_0^T \frac{1}{T}[W_1(z)e_0(t) - C(e_0(t))]^2$$
$$- [W(z)e_0(t) - C(e_0(t))]^2 dt$$
$$= \lim_{N \to \infty} \int_0^T \frac{1}{T}[\epsilon(z)e_0(t)]^2$$
$$+ 2\epsilon(z)e_0(t)(W(z)e_0(t) - C(e_0(t))dt \quad (34)$$

As $W(z)$ minimizes the error criterion $E[W(z)]$ if and only is $E[W_1(z)] \geq E[W(z)]$ for all linear time invariant controller $W_1(z)$. The right hand side is nonnegative for all linear time invariant controller $W_1(z)$ if and only if the linear term in $\epsilon(z)e_0(t)$ is identically zero, i.e.

$$\lim_{N \to \infty} \int_0^T \frac{1}{T}\epsilon(z)e_0(t)(W(z)e_0(t) - C(e_0(t))dt = 0, \quad \forall \epsilon(z) \quad (35)$$

substituting $\epsilon(z)e_0(t)$ and interchanging the order of integration gives

$$0 = \lim_{N \to \infty} \frac{1}{T} \int_0^T \int_0^t \epsilon(\tau)e_0(t-\tau)$$
$$\times (W(z)e_0(t) - C(e_0(t))d\tau dt$$

$$= \lim_{N \to \infty} \frac{1}{T} \int_0^T [\int_\tau^T e_0(t-\tau)$$
$$\times (W(z)e_0(t) - C(e_0(t))dt]d\tau \quad (36)$$

Since $\epsilon(t)$ is an arbitrary impulse response, the coefficient of $\epsilon(t)$ must be identically zero. Then after interchanging the order of integration with respect to τ and taking the limit with respect to T, the optimality condition becomes.

$$0 = \lim_{N \to \infty} \frac{1}{T} \int_\tau^T e_0(t-\tau)(W(z)e_0(t) - C(e_0(t))dt \quad (37)$$

It means

$$0 = \phi_{y,Cy-f}(\tau) = \phi_{e_0,We_0}(\tau) - \phi_{e_0,C}(\tau), \quad \forall \tau \geq 0 \quad (38)$$

Equation (38) is the same as

$$\phi_{e_0,We_0}(\tau) = \phi_{e_0,C}(\tau), \quad \forall \tau \geq 0$$

Which completes the proof of the Theorem 1.

Theorem 1 gives one optimal condition, that is satisfied for the optimal linear controller. It can be also applied in non-linear system identification, so that the unknown nonlinear system is approximated by one simple linear system, then lots of results about linear system identification are applied directly.

III. APPLICATION FOR FLIGHT SIMULATION TABLE

To be convenient in understanding the latter application for flight simulation table with respect to our considered direct data driven model reference control, some basic knowledge for flight simulation table and its corresponding control structure are given here.

A. BASIC KNOWLEDGE FOR FLIGHT SIMULATION TABLE

Flight simulation table is a high precision servo control system. On the basis of meeting the basic requirements of the control system for stability, accuracy and speed, it also has its own control requirements, such as high frequency response, low speed, wide speed regulation and high precision. High frequency response means that the table must has the ability to track fast changing signals, thus requiring the control system has a wide frequency band. The physical plot about flight simulation table can be seen in Figure 3, where the table is approximated as a linear time invariant system, and some nonlinear factors such as dead zone and friction are all ignored.

PID controller series compensation is adopted for flight simulation table. The main idea of PID controller is to use a PID controller to construct a compensation link to improve the gain of the system, and to eliminate the large inertia link of the physical structure of the table, thus improving the stability and dynamic characteristics. The important control system for flight simulation table uses typical three-loop control structure, shown in Figure 4. This three-loop control structure corresponds to current loop, velocity loop and position loop, all of which use PID correction links to achieve their own tasks respectively. However, when the performance index is

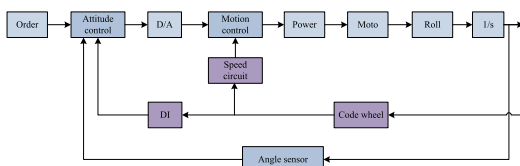**FIGURE 3.** Flight simulation table.

FIGURE 4. Three loop structure for flight simulation table.

very demanding, then the nonlinear factors and time varying characteristics of the table will affect the control system only by the linear control. These nonlinear factors must be considered in practice, as they are inevitable.

Observing the three-loop control structure in Figure 4, there are not only series-connected controllers, but also three feedback controllers. From our popularly used control strategies, some improved control strategies are still in consideration, for example, iterative sliding mode control, robust adaptive control and neural network control, etc. The unknown controller parameters, existing in the controller, can be estimated through an intelligent optimization algorithm- ant colony algorithm. These improved control strategies combine some nice properties of intelligent control, while making up the deficiency of a single control.

B. CONTROL STRUCTURE DESCRIPTION

For some high precision flight simulation tables, it is often required to run smoothly at very low speed. In this case, the control signal, added to the two ends of the motor, is very small, and the additional disturbance torque, coming from the cogging effect, is not neglected. At this time, the output angular velocity and angular position of the table will produce the periodic fluctuations under the action of the disturbance torque, so the stability of the low speed operation is destroyed. More specifically, the combination with position loop and velocity loop is shown in Figure 5.

where all physical variables in Figure 5 are the same with those variables in our previous paper [8]. If readers are interesting about this, please refer to that reference.
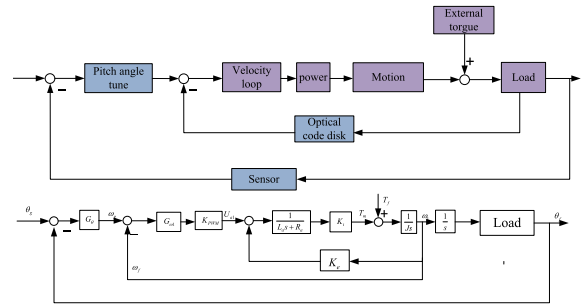

FIGURE 5. Combination with position loop and velocity loop.

C. DIRECT DATA DRIVEN MODEL REFERENCE CONTROLLER

Because the flight simulation table simulates the various attitude angle movements of the aircraft, and reproduces the various dynamic characteristics of its movement. Given a desired rotation trajectory for a certain attitude angle, such as sine wave and cosine wave, we need to design a controller to ensure that the servo turntable rotates the sine and cosine curves in a form that meets the performance index, and always maintains the performance during the rotation process. The design of this controller is consistent with the idea of direct data driven model reference control. Figure 6 describes this control block diagram for the flight simulation table.
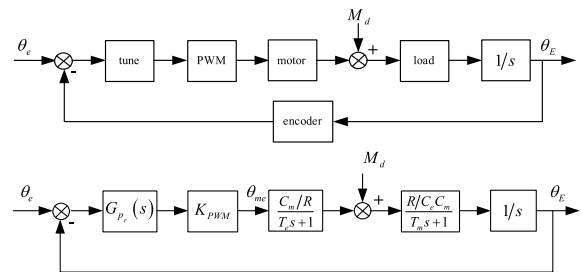

FIGURE 6. Block diagram for direct data driven model reference control.

where in Figure 6, direct data driven model reference control is adopted from the inside to the outside to design three feedback controllers. Firstly, the signals at the left and right ends of the current loop are collected, and the given expected trajectory is used as the reference model. Then the error index is minimized to construct the controller. For optimization problems, game theory is used to determine the reasonable distribution of collected data, and dynamic programming is used to solve the optimal controller in the current loop. Secondly, we take the solved current loop and the speed loop as a whole part, and collect the signals at the left and right ends of the speed loop to design the optimal controller in the speed loop according to our proposed direct data driven model reference control. Finally, as the current loop and the speed loop are all known, in conjunction with the outermost position loop, the controller of the outermost position loop is generated from the perspective of reference trajectory. The entire direct data driven model reference control is to ensure

that the flight simulation table rotates around a predetermined desired trajectory.

IV. SIMULATION EXAMPLE

Based on our theoretical analysis for direct data driven model reference control in section 2, and the detailed introduction about flight simulation table, this section starts to give our simulation results.

FIGURE 7. Simulation environment.

(1). The simulation environment is established in Figure 7, where in the whole simulation process, we use C language to write DSP digital controller software. The digital control program, implemented in the DSP, mainly collects the speed and position numbers of the flight simulation table, and receives the control command signal from the monitor. Then our simulation is to realize direct data driven model reference control and the data exchange and communication between the DSP and the monitor. The program of the monitoring machine is to implement a good human-machine interface interface, which is convenient for real-time monitoring of the operation of the flight simulation table. At the same time, it analyzes and processes various signal data fed back during the operation of the turntable, being convenient for the performance debugging and testing of the table. The real-time system frame about simulating the flight simulation table is plotted in Figure 8.

In Figure 5, the electrical and mechanical links of the motor and load are given as the following transfer function forms.

$$\frac{1}{L_a z + R_a} = \frac{1}{0.0076 z + 3.62};$$
$$\frac{1}{J_m z + B_m} = \frac{1}{0.0486 z + 0.224} \quad (39)$$

where L_r os the input signal for the motor. The controller in the position loop is chosen as one PID controller.

$$C(z, \theta) = K_p + \frac{K_i z}{z - 1} + K_d \frac{z - 1}{z}$$
$$\theta = [K_p \ K_i \ K_d]^T \quad (40)$$

Regarding the motor and load as one whole plant, and collecting this whole plant's left and right signal as the input-output
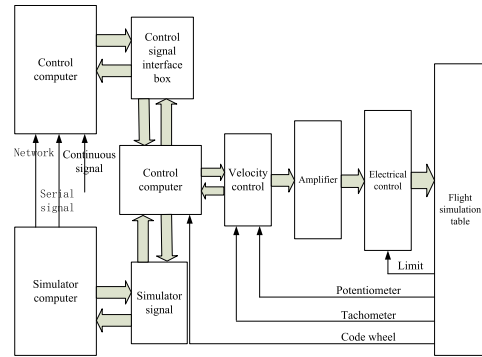

FIGURE 8. Real-time system frame.

data $\{L_r, L\}$. L_g is computed from the collected output signal and the given reference model $M(z)$, i.e. $L_g = M^{-1}(z)L$, where $M(z)$ is defined as that.

$$M(z) = \frac{1414.51}{z^2 + 58.67 z + 1414.51} \quad (41)$$

According to the practical experience, those three PID controller parameters are chosen as

$$\theta = [K_p \ K_i \ K_d]^T = [17 \ 3.8 \ 2.9]^T \quad (42)$$

Collecting the above input-output data $\{L_r, L\}$ and setting the sampling time interval as $0.002s$. The step response curve of the given reference model $M(z)$ is plotted in Figure 9.
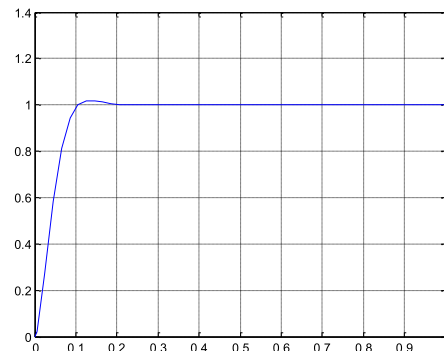

FIGURE 9. Step response curve of the given reference model.

Remark: The reason of some overshoots is that some disturb or noise exists, and the noise can not be avoided, so one filter is used to alleviate the noise. From 0.2 s, the curve converges to value because of the parameter adjustments or algorithm optimization. For the problem of estimating those three unknown PID controller parameters during the framework of our proposed direct data driven model reference control strategy, the least squares algorithm in equation (21) is used to obtain the three parameter estimates iteratively. Three iterative convergence curves for the PID controller parameters are shown in Figure 10, where the final controller parameters are yielded as that.
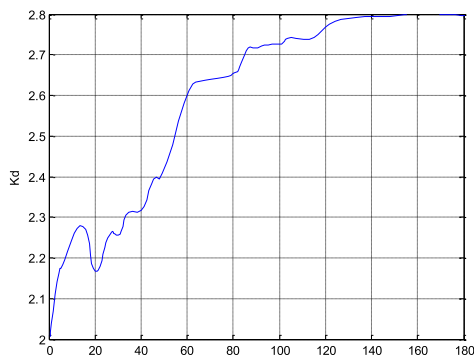
$$\theta = [K_p \ K_i \ K_d]^T = [17 \ 3.8 \ 2.8]^T \quad (43)$$
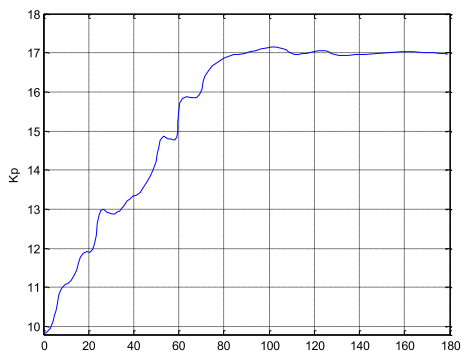
(1) Iterative convergence curve for controller parameter k_p
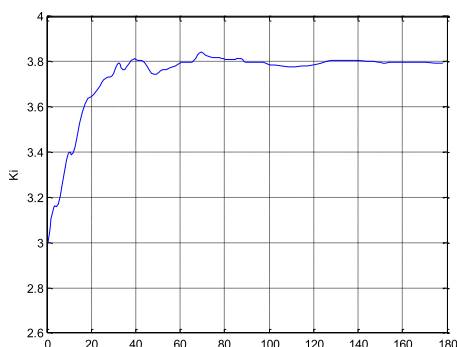
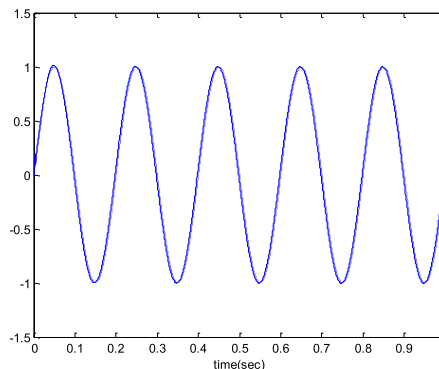(2) Iterative convergence curve for controller parameter k_i

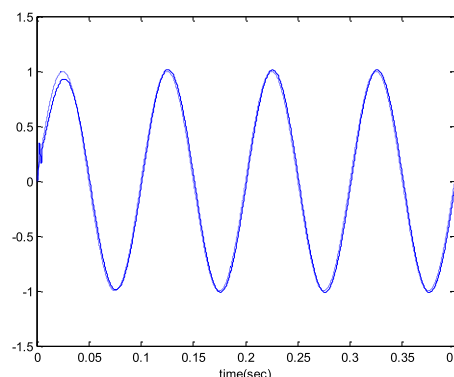(3) Iterative convergence curve for controller parameter k_d

FIGURE 10. Iterative convergence curves for three PID parameters.

(1) Sine wave tracking response with f=3Hz

(2) Sine wave tracking response with f=5Hz

FIGURE 11. Sine wave tracking response with different frequencies.

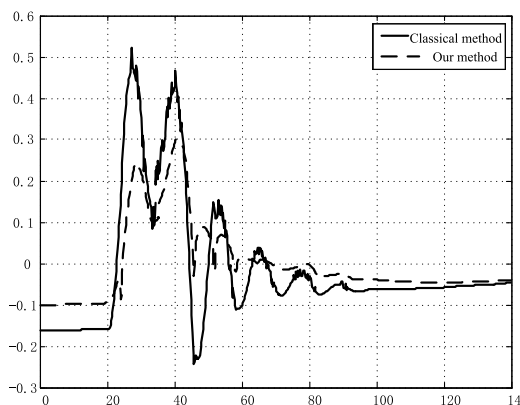**FIGURE 12.** Comparison the cost functions with the step.

Based on the designed PID controller, when the command input signal is a sine wave, the position tracking response curve recorded at different frequencies are shown in Figure 11, where the controller designed by direct data driven model reference control has a good position tracking response, and no distortion waveform exists, while the closed loop has rapid tracking and strong rapidity.

To compare our direct data driven model reference control and classical model reference control, the main difference between then are equation (6) and (20), where plant is needed only for model reference control, but not for our method. Figure 12 shows the optimization curves for equation (6) and (20) respectively, corresponding to our method and classical method, i.e. the cost functions vary with the time increases.From Figure 12, we see the iterative cost functions will all converge to zero with the iterative step.

More specially, consider the classical model reference control, more errors exist during interval [0, 80] due to the inaccurate plant. As our method is independent of the plant, so the cost function converges to zero more quickly.

Figure 12 compares the cost functions with steps from the optimization theory. To testify the efficiency of our method for flight simulation table from the practice, the same step signal is used to excite the position loop for flight simulation table with different control methods, i.e. our proposed method and classical model reference control method. Two step responses are collected in Figure 13, where the green curve is for our proposed method and red curve corresponds
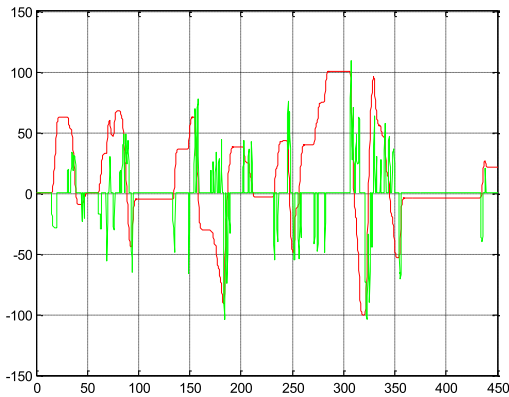
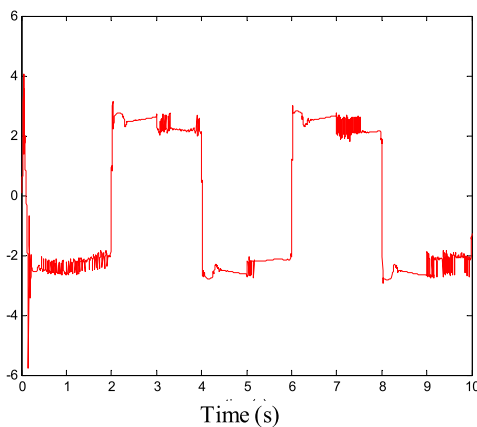**FIGURE 13.** Comparison the step responses for flight simulation table.

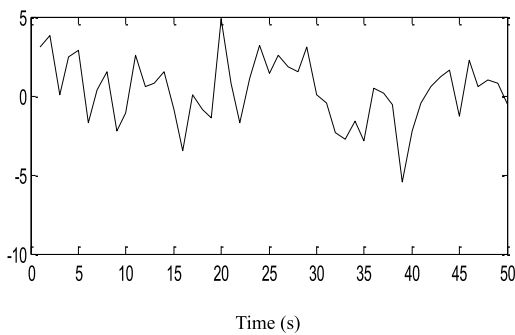**FIGURE 14.** The applied input signal.

FIGURE 15. The observed output signal.

to classical model reference control method. The practical results in Figure 13 are the same with Figure 12, where after 350 seconds two step responses are coincide with each other, but during 0-350 second, more flutters or derivations exist for classical method.

(2). Observing that considered system with controller $C(z)$ (40) and reference model $M(z)$ (41), we impose one kind of input signal to this system and collect it output signal. Specifically, the applied input signal is given in Figure 14, and we measure the output signal $y(t)$ by some measuring
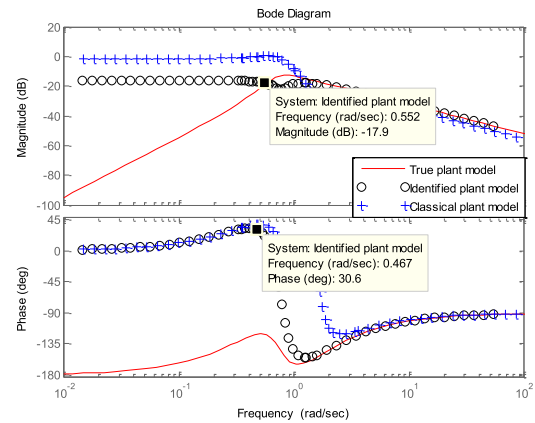

FIGURE 16. Bode plot for plant.

devices, where observed output signal is plotted in Figure 15. When using direct data driven model reference control to design the unknown controller parameters, if estimation error $\|\hat{\theta}(t) - \theta\|/\|\theta\|$ is less than one very small value 0.005, then terminate the recursive methods.

To verify the efficiency of the designed controller $C(\hat{\theta}_N)$ by our derived results and make sure that this designed controller can be used to replace the real controller, we compare the Bode responses among real controller $C_0(z)$, our designed controller $C(\hat{\theta}_N)$ and classical model reference control strategy,respectively in Figure 16. From Figure 16, we see that these three Bode response curves coincide with each other, and our designed controller $C(\hat{\theta}_N)$ can converge to its real controller quickly than classical model reference control strategy. This means that our model error $\tilde{C}(z)$ will converge to zero with time increases very quickly.

Generally from above theoretical analysis and practical engineering, the merits of our considered direct data driven model reference control are formulated as follows.

(a) In identifying the unknown controller parameters, the initial parameter values are chosen from the engineering experience.

(b) The tracking performance is minimized through adjusting the controller parameters.

(c) This control strategy is independent of the unknown plant, as the controller parameters are extracted for data.

V. CONCLUSION

This paper connects direct data driven and model reference control to design the unknown controller without any knowledge of the unknown plant, thus avoiding the identification process. The proposed direct data driven model predictive control is to extract some information about the unknown controller from the input-output data directly, while achieving the perfect tracking performance with a given reference model. Then this novel control strategy is applied to design the three-loop controllers for flight simulation table, after introducing some basic knowledge about this simulation device in aircraft engineering. As this is our primarily

analysis for direct data driven model reference control, that about how to merge game theory and direct data driven model reference control is our next ongoing work.

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