

A Comparison of Promethee and TOPSIS Techniques Based on Bipolar Soft Covering-Based Rough Sets

FAIZA TUFAIL¹, MUHAMMAD SHABIR¹, AND EL-SAYED A. ABO-TABL^{2,3}

¹Department of Mathematics, Quaid-i-Azam University Islamabad, Islamabad 45320, Pakistan

²Department of Mathematics, Faculty of Science, Assiut University, Assiut 71516, Egypt

³Department of Mathematics, College of Science and Arts, Qassim University, Buridah 51931, Saudi Arabia

Corresponding author: Faiza Tufail (faizatufail85@gmail.com)

ABSTRACT The uncertainty in the data is an obstacle in decision-making problems. In order to solve problems with a variety of uncertainties a number of useful mathematical approaches together with fuzzy sets, rough sets, soft sets, bipolar soft sets have been developed. The rough set theory is an effective technique to study the uncertainty in data, while bipolar soft sets have the ability to handle the vagueness, as well as bipolarity of the data in a variety of situations. This study develops a new methodology, which we call the theory of Bipolar soft covering-based rough sets (BSCB-RSs), which will be used to propose a new technique to solve decision-making problems. The idea introduced in this study has never been discussed earlier. Furthermore, this concept has been explored by means of a detailed study of the structural properties. By combining the BSCB-RSs model with two traditional decision-making methods (the PROMETHEE-II method and the TOPSIS method), we introduce a novel method for addressing multi-criteria group decision-making (MCGDM) problems. We give an application in multi-criteria group decision making (MCGDM) to show that the proposed technique can be successfully applied to some real world problems including uncertainty, namely, the selection of site for renewable energy project (Earth Dam). The effectiveness of the proposed method is validated by comparing it with existing methods. The showed techniques exhibit the practicability, feasibility and sustainability of Site selection. Both MCGDM methods give one Site as conclusion.

INDEX TERMS Approximation operator, bipolar soft neighborhood, bipolar soft covering, bipolar soft covering-based rough set, decision-making application.

I. INTRODUCTION

Many complicated problems in business, social sciences, engineering, management sciences, military, medical sciences, economics and many other fields involve uncertain data. These problems cannot be solved using classical mathematical methods. The classical mathematical model is rational model of decision making which is based on the assumption that managers have access to complete information and are capable to an optimal decision by weighting every alternatives. Because of that, the mathematical model is too complex, the exact solution cannot be found.

To overcome this difficulty, a number of researchers are attempting to determine some appropriate approaches and a

number of mathematical theories to cope with uncertainty in data, such as Fuzzy Set Theory, Rough Set Theory, Interval Mathematical Theory, Vague Set Theory, Graph Theory, Automata Theory, Decision-Making Theory etc., are formulated to solve such problems, and have been found only partially successful. These theories reduced the distance between the classical mathematical designs and the vague real-world data.

In 1965, fuzzy set theory [84] was suggested to model fuzzy data by Zadeh. However, in this theory, determining of membership function is rather difficult sometimes. Therefore, in 1999, Molodtsov [46] proposed the notion of soft set as a completely new approach for modelling uncertainty, free from this difficulty. Unlike classical mathematics, where exact solution of a mathematical model is required, soft set theory instead requires an approximate description of an

The associate editor coordinating the review of this manuscript and approving it for publication was Christian Pilato.

object as its initial point. The choice of adequate parameterization tools such as words, real number, functions etc., make soft set theory very convenient and easy to apply in practice. Many interesting applications of soft set theory can be seen in [5], [8], [17], [51].

The rough set theory [58], [59], is another successful mathematical tool for dealing uncertainties. In this theory, uncertainty is represented by a boundary region of a set. Pawlak used the upper and lower approximations of a collection of objects to investigate how close the objects are to the information attached to them. Feng *et al.* [27], [28], proposed the relationships among soft sets, rough sets and fuzzy sets, obtaining three types of hybrid models: rough soft sets, soft rough sets, and soft-rough fuzzy sets. Shabir *et al.* [62] redefined a version of soft rough set known as modified soft rough set (MRS-set).

Soft set theory [46] and Rough set theory [58] are regarded as effective mathematical approaches to address uncertainty. In 2011, Feng *et al.* [28] established a relationship among these two theories and introduced the concept of a new hybrid version of the soft rough sets (SRSs), that can give better approximations over Pawlak's RS theory in some cases ([28], Example 4.7). This approach can be viewed as a generalization of RS theory.

The idea of covering based rough sets was proposed by Zakowski [85]. Then Pomykala [60] introduced several additional approximation operators by using coverings, inclusive of two pairs of dual approximations. Some researchers researched the covering based rough sets and the general covering based rough sets in [77], [103], [106]. Yao [81] in particular examined the two pairs of dual operators by using coverings induced by binary relations. Couso and Dubois [15] studied the two pairs in the framework of incomplete information. In particular in 1998, Bonikowski *et al.* [11] put forth a covering based rough set model based on the notion of minimal description. Likewise Zhu [105] proposed several covering based rough set models and discussed their relationships. Tsang *et al.* [66] and Xu and Zhang [77] proposed additional covering based rough set models. Liu and Sai [40] compared Zhu's covering based rough set models and Xu and Zhang's covering based rough set models. Some recent important properties of covering based rough set models have appeared in [43], [82], [107]. An expanded overview of the advances about covering based rough sets appeared in some recently published articles like Yao and Yao [82] and D'eer *et al.* [19], [20].

In numerous sorts of data analysis, the bipolarity of the data is a key component to be taken into consideration while developing mathematical models for some issues. Bipolarity discusses the positive and negative aspects of the data. The positive data addresses what is assumed to be possible, while the negative data addresses what is not possible or certainly false. The concept that lies behind the presence of bipolar information is that a wide assortment of human decision-making depends on bipolar judgemental thinking. For example, sweetness and sourness of food, participation

and rivalry, friendship and hostility, effects and side effects of drugs are the two different aspects of information in decision-making and coordination. The soft sets, the fuzzy sets, and the rough sets are not appropriate tools to handle this bipolarity.

Based on the need of presenting both positive and negative sides of data, notion of bipolar soft set and its operations such as union, intersection and complement were first defined by Shabir and Naz [65]. After this research, BSSs have become increasingly popular with researchers. Karaaslan and Karatas [32] redefined bipolar soft sets with a new approximation providing opportunity to study on topological structures of bipolar soft sets. Also Naz and Shabir [54] proposed the concept of fuzzy bipolar soft sets and investigated their algebraic structures. Bipolar soft rough sets were firstly introduced by Karaaslan and Cagman [31] to handle roughness of bipolar soft set, which is a combination of RS theory and BSSs. They also provide applications of BSRs in decision making. Malik and Shabir [53] introduced the idea of rough fuzzy bipolar soft sets in 2019.

Multi-criteria decision-making (MCDM) method is referred as a method used for scoring or ranking a finite number of alternatives by considering multiple criteria attached to the alternatives. MCDM concerns with evaluating and selecting alternatives that fit with the goals and necessity. The preference ranking organization method for enrichment evaluation (PROMETHEE) and the techniques for the order of preference by similarity to positive ideal solution (TOPSIS) are the two most known techniques developed to handle multi-criteria decision making problems. The biggest difference between PROMETHEE and other MCDM methods is the inner relationship of PROMETHEE during the decision-making process [50]. It is well adapted to the decision problems where a finite set of alternatives is to be outranked subjected to multiple conflicting criteria [6], [10], [67]. The PROMETHEE method is based on pairwise comparisons of alternatives with respect to each criterion. According to [70], the PROMETHEE has at least three advantages. The first advantage is its user-friendly outranking method. The second advantage is the success of PROMETHEE in applications to real-life planning problems. Another advantage of PROMETHEE lies on completeness of ranking. The PROMETHEE I and PROMETHEE II allow partial and complete ranking of alternatives, respectively. The PROMETHEE I is used to obtain partial ranking while PROMETHEE II is used for complete ranking. These two methods were developed in [9], [12]. On the other hand, the main TOPSIS concept measures the distance between each alternative and ideal solution. Hwang and Yoon [26] implemented the multiple criteria decision making method and applications. Such methods were based on crisp knowledge and could not accommodate information that was imprecise. In 2000, fuzzy version of TOPSIS method was suggested in Chen's [14] research work. Several TOPSIS related approaches have since been suggested and applied to various multiple criteria decision making problems. Chen and

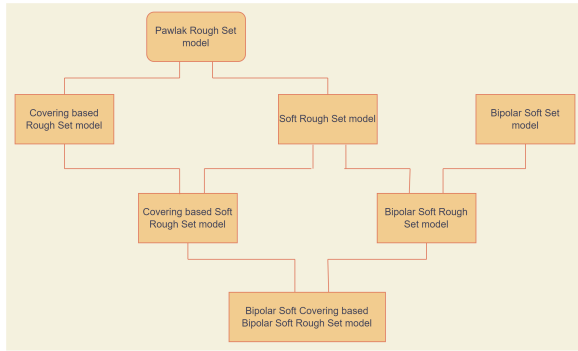


FIGURE 1. Relationship among the generalized rough set models.

Tsao [16] suggested interval valued fuzzy set based method of TOPSIS.

A. MOTIVATION

Based on the above descriptions for MCDM and the basic principle of bipolar soft rough sets, this paper attempts to propose a novel approach to multi-criteria group decision making (MCGDM) problems by combining the bipolar soft covering based rough set with two traditional decision-making methods. In particular, a summary of motivations of this paper is provided as follows:

(1) If we recap all of the preceding arguments, we can see that bipolar soft sets can deal with the bipolarity of information about specific objects using two mappings. The positivity of the information is handled by one mapping, while the negative is measured by the other. Given the link between rough sets and bipolar soft sets, one attempts to investigate the roughness of bipolar soft sets has been made: by Karaaslan and Caman [31]. This is the primary motivation for us to present and investigate a novel approach to bipolar soft set roughness using Bipolar soft covering-based rough sets (BSCB-RSs), as well as to discuss their application in the decision-making and able to describe the best and worst side in decision making. Fig A shows how our new rough set model is generalization of some existing models.

(2) To address the issue of data processing in decision-making, the superior performance of rough set theory has been demonstrated. Liang *et al.* [37] investigated a decision-making approach that combines the TOPSIS method with a decision-theoretic rough set. Zhang *et al.* [101] recently applied covering based fuzzy rough set models to solve the issue of company recruitment decision-making. In addition, several MCDM approaches focused on covering based fuzzy rough set theory have been investigated in the literature [99], [102]. This paper extends the PROMETHEE and TOPSIS approaches using covering based bipolar soft rough sets and applies it to the optimal earth dam power plant site selection problem in order to widen the application ranges of covering based fuzzy rough set theory in MCDM.

(3) The proposed method not only considers the opinions of key decision-makers but also incorporates the past experiences by CB-BSR-approximations in actual scenarios.

B. AIM OF THE PROPOSED STUDY

The main goal of this study is to present another interesting and novel version of bipolar soft rough sets by utilizing BSCB-RSs.

We highlight the article by the following pioneering work:

- A novel concept known as BSCB-RSs is proposed.
- Some important structural properties of BSCB-RSs are investigated in detail.
- Two comprehensive MCGDM methods in the framework of BSCB-RSs is introduced and the validity of these approaches is also verified by a practical example.
- The effectiveness of the proposed method is validated by comparing it with existing methods.

C. OUTLINE OF THE PAPER

The article has been organized in the following manner. Section 2 gives an overview of some basic ideas, which are required for the understanding of our research work. Section 3 starts by characterizing some bipolar soft covering-based bipolar soft operators. Further, we discuss the relationship between these operators and their properties. Moreover, based upon these operators, we proposed the notion of BSCB-RSs. The notion is further investigated by considering its important structural properties in detail. Section 4 proposes a new decision-making method to MCGDM problems based on the PROMETHEE method and the TOPSIS method. After that, we give an illustrative example of the proposed decision making technique to show that the technique can be effectively applied to some real-life problems in section 5. In section 6, a comparison analysis is made between the proposed model and some other well-known decision making techniques. At the last, section 7 concludes with a summary of the present work and a suggestion for further research.

II. PERLIMINARIES

In this section, we recall some essential notions related to rough set, soft set, soft rough set, bipolar soft set, bipolar soft rough sets and Soft covering based soft rough sets that would be accommodating in the upcoming discussion. Throughout this paper, we will use \mathfrak{S} for an initial universe, \hat{E} for set of parameters, C for a non-empty subset of the parameters set \hat{E} and $P(\mathfrak{S})$ for the power set of \mathfrak{S} , unless stated otherwise.

Definition 1 [58]: Let \mathfrak{S} be a non-empty finite universe, and R be an equivalence relation over \mathfrak{S} . Then the pair $(\mathfrak{S}; R)$ is said to be Pawlak approximation space.

If $A \subseteq \mathfrak{S}$, then A may or may not be written as a union of some equivalence classes of \mathfrak{S} . If A is written as a union of some equivalence classes, then A is called R -definable; otherwise it is called R -undefinable. If A is R -undefinable then, it can be approximated with the help of the following

two definable subsets:

$$\underline{R}(A) = \{a \in \mathfrak{S} \mid [a]_R \subseteq A\}, \tag{1}$$

$$\overline{R}(A) = \{a \in \mathfrak{S} \mid [a]_R \cap A \neq \emptyset\}. \tag{2}$$

Equations (1) and (2) are called lower and upper approximations of A with respect to the equivalence relation R , respectively, where the equivalence class $[a]_R$ of an element $a \in \mathfrak{S}$ is the set consists of all objects $b \in \mathfrak{S}$ such that $(a, b) \in R$, that is,

$$[a]_R = \{b \in \mathfrak{S} \mid (a, b) \in R\}.$$

Moreover, the boundary region (area of uncertainty) of rough set is defined as:

$$Bnd_R(A) = \overline{R}(A) - \underline{R}(A).$$

Definition 2 [46]: Let \mathfrak{S} be a set of objects called the universe, C be a non-empty subset of parameters (attributes). Then a pair (\widehat{F}, C) is said to be a soft set over \mathfrak{S} , where \widehat{F} is a mapping given by $\widehat{F} : C \rightarrow P(\mathfrak{S})$.

Thus, a soft set over \mathfrak{S} gives a parameterized family of subsets of the universe \mathfrak{S} . For $\tilde{\varepsilon} \in C$, $\widehat{F}(\tilde{\varepsilon})$ is considered to be a set of $\tilde{\varepsilon}$ -approximate elements of \mathfrak{S} by the soft set (\widehat{F}, C) . Thus

$$(\widehat{F}, C) = \{\widehat{F}(\tilde{\varepsilon}) \in P(\mathfrak{S}) \mid \tilde{\varepsilon} \in C \subseteq \widehat{E}\}.$$

Feng et al. established a link between soft set and rough set and introduced the idea of a new hybrid model of “soft rough set” based on a different granulation structure known as “soft approximation space”.

Definition 3 [28]: Let $P = (\widehat{F}, C)$ be a soft set over \mathfrak{S} . Then the pair $P^* = (\mathfrak{S}, P)$ is called a soft approximation space. The lower and upper soft rough approximations of any set $A \subseteq \mathfrak{S}$ is defined as follows, respectively:

$$\underline{apr}_{P^*}(A) = \bigcup_{\tilde{\varepsilon} \in C} \{\widehat{F}(\tilde{\varepsilon}) \mid \widehat{F}(\tilde{\varepsilon}) \subseteq A\},$$

$$\overline{apr}_{P^*}(A) = \bigcup_{\tilde{\varepsilon} \in C} \{\widehat{F}(\tilde{\varepsilon}) \mid \widehat{F}(\tilde{\varepsilon}) \cap A \neq \emptyset\}.$$

If $\underline{S}_{P^*}(A) = \overline{S}_{P^*}(A)$, A is said to be soft P^* -definable; otherwise A is called a soft P^* -rough set.

Definition 4 [47]: Let C be a set of parameters. Then, NOT set of C , denoted by $\neg C$, is defined by $\neg C = \{\neg \tilde{\varepsilon} : \tilde{\varepsilon} \in C\}$ where $\neg \tilde{\varepsilon} = \text{not } \tilde{\varepsilon}$ for $\tilde{\varepsilon} \in C$.

Definition 5 [65]: The triplet $\psi = (\widehat{F}, \widehat{H}, C)$ is called a bipolar soft set over a universe \mathfrak{S} , in which \widehat{F}, \widehat{H} are mappings given by $\widehat{F} : C \rightarrow P(\mathfrak{S}), \widehat{H} : \neg C \rightarrow P(\mathfrak{S})$ such that $\widehat{F}(\tilde{\varepsilon}) \cap \widehat{H}(\neg \tilde{\varepsilon}) = \emptyset$.

Thus, a bipolar soft set over \mathfrak{S} gives two parameterized families of subsets of the universe \mathfrak{S} and the condition $\widehat{F}(\tilde{\varepsilon}) \cap \widehat{H}(\neg \tilde{\varepsilon}) = \emptyset$, for all $\tilde{\varepsilon} \in C, \neg \tilde{\varepsilon} \in \neg C$, is imposed as a consistency constraint.

From now onward, set of all bipolar soft sets over the universe \mathfrak{S} will be referred to by $BS^{\mathfrak{S}}$

Definition 6 [65]: Let $(\widehat{F}, \widehat{H}, C) \in BS^{\mathfrak{S}}$. Then, the complement of $(\widehat{F}, \widehat{H}, C)$, denoted by $(\widehat{F}, \widehat{H}, C)^c$, is defined by

$(\widehat{F}, \widehat{H}, C)^c = (\widehat{F}^c, \widehat{H}^c, C)$ where \widehat{F}^c and \widehat{H}^c are mappings given by $\widehat{F}^c(\tilde{\varepsilon}) = \widehat{H}(\neg \tilde{\varepsilon})$ and $\widehat{H}^c(\neg \tilde{\varepsilon}) = \widehat{F}(\tilde{\varepsilon})$ for all $\tilde{\varepsilon} \in C$.

Karaaslan and Çağman [31] presented the concept of bipolar soft rough set, which is a combination of rough set and bipolar soft set.

Definition 7 [31]: Let $(\widehat{F}, \widehat{H}, C) \in BS^{\mathfrak{S}}$. Then $\varphi = (\mathfrak{S}, (\widehat{F}, \widehat{H}, C))$ is said to be a bipolar soft approximation space. Based on ρ , the following four operators are defined for any $A \subseteq \mathfrak{S}$:

$$\underline{apr}_{\rho^+}(A)$$

$$= \{t \in \mathfrak{S} : \exists \tilde{\varepsilon} \in C, [t \in \widehat{F}(\tilde{\varepsilon}) \subseteq A]\},$$

$$\underline{apr}_{\rho^-}(A)$$

$$= \{t \in \mathfrak{S} : \exists \tilde{\varepsilon} \in \neg C, [t \in \widehat{H}(\tilde{\varepsilon}), \widehat{H}(\tilde{\varepsilon}) \cap A^c \neq \emptyset]\},$$

$$\overline{apr}_{\rho^+}(A)$$

$$= \{t \in \mathfrak{S} : \exists \tilde{\varepsilon} \in C, [t \in \widehat{F}(\tilde{\varepsilon}), \widehat{F}(\tilde{\varepsilon}) \cap A \neq \emptyset]\},$$

$$\overline{apr}_{\rho^-}(A)$$

$$= \{t \in \mathfrak{S} : \exists \tilde{\varepsilon} \in \neg C, [t \in \widehat{H}(\tilde{\varepsilon}) \subseteq A^c]\}.$$

Which are called soft ρ -lower positive approximation, soft ρ -lower negative approximation, soft ρ -upper positive approximation and soft ρ -upper negative approximation of A , respectively.

Definition 8 [83]: A soft set (\widehat{F}, C) over \mathfrak{S} is called a full soft set if $\bigcup_{\tilde{\varepsilon} \in C} \widehat{F}(\tilde{\varepsilon}) = \mathfrak{S}$.

Definition 9: A full soft set $P = (\widehat{F}, C)$ over \mathfrak{S} is called a covering soft set if $\widehat{F}(\tilde{\varepsilon}) \neq \emptyset$, for all $\tilde{\varepsilon} \in C$.

Yüksel et al. [83] proposed soft covering based rough sets, which is a fusion of soft set and covering based rough set.

Definition 10 [83]: Let $\mathfrak{R}_P = (\widehat{F}, C)$ be a covering soft set over \mathfrak{S} . Then the pair $(\mathfrak{S}, \mathfrak{R}_P)$ is called a soft covering approximation space.

Definition 11 [83]: Let $(\mathfrak{S}, \mathfrak{R}_P)$ be a soft covering approximation space and $t \in \mathfrak{S}$. Then the soft minimal description of t is defined as follows:

$$\begin{aligned} Md_{\rho}(t) &= \{\widehat{F}(\tilde{\varepsilon}_1) : \tilde{\varepsilon}_1 \in C \wedge t \in \widehat{F}(\tilde{\varepsilon}_1) \\ &\wedge (\forall \tilde{\varepsilon}_2 \in C \wedge t \in \widehat{F}(\tilde{\varepsilon}_2) \subseteq \widehat{F}(\tilde{\varepsilon}_1) \\ &\Rightarrow \widehat{F}(\tilde{\varepsilon}_1) = \widehat{F}(\tilde{\varepsilon}_2))\}. \end{aligned}$$

We only need the basic properties of an object to describe it, not all of them. The goal of the minimal description notion is to achieve this.

Definition 12 [83]: Let $\rho = (\mathfrak{S}, \mathfrak{R}_P)$ be a soft covering approximation space. For a set $A \subseteq \mathfrak{S}$, soft covering lower and upper approximations are, respectively, defined as:

$$\underline{S}_{\rho}(A) = \bigcup_{\tilde{\varepsilon} \in C} \{\widehat{F}(\tilde{\varepsilon}) : \widehat{F}(\tilde{\varepsilon}) \subseteq A\},$$

$$\overline{S}_{\rho}(A) = \bigcup \{Md_{\rho}(t) : t \in A\}.$$

Moreover, the sets

$$Pos_S(A) = \underline{S}_{\rho}(A),$$

$$Neg_S(A) = \mathfrak{S} - \overline{S}_{\rho}(A),$$

$\text{Bnd}_\rho(A) = \overline{S}_\rho(A) - \underline{S}_\rho(A)$ are called the soft covering positive, negative, and boundary regions of A , respectively. In addition, if $\overline{S}_\rho(A) \neq \underline{S}_\rho(A)$, then A is said to be soft covering based rough set; otherwise A is called soft covering based definable.

Example 1: Let $\mathfrak{S} = \{t_1, t_2, t_3, t_4, t_5\}$ be universe and (\widehat{F}, C) a covering soft set over \mathfrak{S} , where $\widehat{F}(\tilde{\varepsilon}_1) = \{t_1, t_2\}$, $\widehat{F}(\tilde{\varepsilon}_2) = \{t_1, t_2, t_3, t_4\}$, $\widehat{F}(\tilde{\varepsilon}_3) = \{t_3, t_4\}$, $\widehat{F}(\tilde{\varepsilon}_4) = \{t_2, t_3, t_4, t_5\}$, $\widehat{F}(\tilde{\varepsilon}_5) = \{t_1, t_2, t_4, t_5\}$. Then $\rho = (\mathfrak{S}, \mathfrak{R}_\rho)$ be a soft covering approximation space. For $A_1 = \{t_1, t_2\} \subseteq \mathfrak{S}$, we have

$$\underline{S}_\rho(A_1) = \bigcup_{\tilde{\varepsilon} \in C} \{\widehat{F}(\tilde{\varepsilon}) : \widehat{F}(\tilde{\varepsilon}) \subseteq A_1\} = \{t_1, t_2\},$$

$$\overline{S}_\rho(A_1) = \bigcup \{Md_\rho(t) : t \in A_1\} = \{t_1, t_2\}.$$

Thus, $\overline{S}_\rho(A) = \underline{S}_\rho(A)$, so A_1 is soft covering based definable. For $A_2 = \{t_1, t_2, t_3\} \subseteq \mathfrak{S}$, we have

$$\underline{S}_\rho(A_2) = \bigcup_{\tilde{\varepsilon} \in C} \{\widehat{F}(\tilde{\varepsilon}) : \widehat{F}(\tilde{\varepsilon}) \subseteq A_2\} = \{t_1, t_2\},$$

$$\overline{S}_\rho(A_2) = \bigcup \{Md_\rho(t) : t \in A_2\} = \{t_1, t_2, t_3, t_4\}.$$

Thus, $\overline{S}_\rho(A) \neq \underline{S}_\rho(A)$, so A_2 is soft covering based rough set.

The properties satisfied by soft covering lower and upper approximations can be found in [83].

III. BIPOLAR SOFT COVERING BASED SOFT ROUGH SETS

From the concept of bipolar soft set, we know that a bipolar soft set is determined by the two set-valued mappings, one from a set of parameters to the power set of the universe and the other from a not set of parameters to the power set of the universe. In this section, we use a special kind of bipolar soft covering with rough set and establish a bipolar soft covering approximation space and present its basic properties.

Definition 13: A bipolar soft set $(\widehat{F}, \widehat{H}, C)$ over \mathfrak{S} is called a full bipolar soft set if $\bigcup_{\tilde{\varepsilon} \in C} \widehat{F}(\tilde{\varepsilon}) = \mathfrak{S}$ and $\bigcap_{\neg \tilde{\varepsilon} \in \neg C} \widehat{H}(\neg \tilde{\varepsilon}) = \emptyset$.

Remark 1: By using Definition 5 of the bipolar soft set, we have $\widehat{F}(\tilde{\varepsilon}) \cap \widehat{H}(\neg \tilde{\varepsilon}) = \emptyset$. Which implies that $\widehat{F}(\tilde{\varepsilon}) \subseteq \widehat{H}^c(\neg \tilde{\varepsilon})$, and $\bigcup_{\tilde{\varepsilon} \in C} \widehat{F}(\tilde{\varepsilon}) \subseteq \bigcup_{\neg \tilde{\varepsilon} \in \neg C} \widehat{H}^c(\neg \tilde{\varepsilon})$. By definition 8, we have $\mathfrak{S} \subseteq \bigcup_{\neg \tilde{\varepsilon} \in \neg C} \widehat{H}^c(\neg \tilde{\varepsilon})$. Now by taking complement both side, $(\mathfrak{S})^c \supseteq (\bigcup_{\neg \tilde{\varepsilon} \in \neg C} \widehat{H}^c(\neg \tilde{\varepsilon}))^c \implies \emptyset \supseteq \bigcap_{\neg \tilde{\varepsilon} \in \neg C} \widehat{H}(\neg \tilde{\varepsilon})$ and it is obvious that $\emptyset \subseteq \bigcap_{\neg \tilde{\varepsilon} \in \neg C} \widehat{H}(\neg \tilde{\varepsilon})$, so $\bigcap_{\neg \tilde{\varepsilon} \in \neg C} \widehat{H}(\neg \tilde{\varepsilon}) = \emptyset$. Thus for full bipolar soft set it is sufficient that $\bigcup_{\tilde{\varepsilon} \in C} \widehat{F}(\tilde{\varepsilon}) = \mathfrak{S}$.

Definition 14: A bipolar full soft set $\wp = (\widehat{F}, \widehat{H}, C)$ over \mathfrak{S} is called a bipolar soft covering if $\widehat{F}(\tilde{\varepsilon}) \neq \emptyset$ and $\widehat{H}(\neg \tilde{\varepsilon}) \neq \mathfrak{S}$, for all $\tilde{\varepsilon} \in C$.

Example 2: As an illustration, let $\wp = (\widehat{F}, \widehat{H}, C)$ be a full bipolar soft set over \mathfrak{S} , where $\mathfrak{S} = \{t_1, t_2, t_3, t_4, t_5\}$, $C = \{\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \tilde{\varepsilon}_3\}$ and $\neg C = \{\neg \tilde{\varepsilon}_1, \neg \tilde{\varepsilon}_2, \neg \tilde{\varepsilon}_3\}$. The mappings \widehat{F} and \widehat{H} are given as below:

$$\widehat{F} : C \longrightarrow P(\mathfrak{S}), \quad \widehat{H} : \neg C \longrightarrow P(\mathfrak{S})$$

$$\tilde{\varepsilon} \longmapsto \begin{cases} \{t_1, t_2\} & \text{if } \tilde{\varepsilon} = \tilde{\varepsilon}_1 \\ \{t_3, t_4\} & \text{if } \tilde{\varepsilon} = \tilde{\varepsilon}_2 \\ \{t_5\} & \text{if } \tilde{\varepsilon} = \tilde{\varepsilon}_3 \end{cases}$$

$$\neg \tilde{\varepsilon} \longmapsto \begin{cases} \{t_4, t_5\} & \text{if } \neg \tilde{\varepsilon} = \neg \tilde{\varepsilon}_1 \\ \{t_2\} & \text{if } \neg \tilde{\varepsilon} = \neg \tilde{\varepsilon}_2 \\ \{t_1\} & \text{if } \neg \tilde{\varepsilon} = \neg \tilde{\varepsilon}_3 \end{cases}$$

Now, according to Definition 14, we can easily see that bipolar full soft set \wp is a bipolar soft covering over \mathfrak{S} .

Definition 15: Let $\mathfrak{R}_\wp = (\widehat{F}, \widehat{H}, C)$ be a bipolar soft covering over \mathfrak{S} . Then the pair $(\mathfrak{S}, \mathfrak{R}_\wp)$ is called a bipolar soft covering approximation space.

Definition 16: Let $\rho = (\mathfrak{S}, \mathfrak{R}_\rho)$ be a bipolar soft covering approximation space and $t \in \mathfrak{S}$. Then the bipolar soft minimal description of t is defined as follows:

$$Md_{\rho^+}(t) = \{\widehat{F}(\tilde{\varepsilon}_1) : \tilde{\varepsilon}_1 \in C \wedge t \in \widehat{F}(\tilde{\varepsilon}_1) \wedge (\forall \tilde{\varepsilon}_2 \in C \wedge t \in \widehat{F}(\tilde{\varepsilon}_2) \subseteq \widehat{F}(\tilde{\varepsilon}_1) \implies \widehat{F}(\tilde{\varepsilon}_1) = \widehat{F}(\tilde{\varepsilon}_2))\},$$

$$Md_{\rho^-}(t) = \{\widehat{H}(\neg \tilde{\varepsilon}_1) : \neg \tilde{\varepsilon}_1 \in \neg C \wedge t \in \widehat{H}(\neg \tilde{\varepsilon}_1) \wedge (\forall \neg \tilde{\varepsilon}_2 \in \neg C \wedge t \in \widehat{H}(\neg \tilde{\varepsilon}_2) \subseteq \widehat{H}(\neg \tilde{\varepsilon}_1) \implies \widehat{H}(\neg \tilde{\varepsilon}_1) = \widehat{H}(\neg \tilde{\varepsilon}_2))\}.$$

We only need the basic properties of an object to describe it, not all of them. So we use the minimal description concept for this purpose.

Definition 17: Let $\rho = (\mathfrak{S}, \mathfrak{R}_\rho)$ be a bipolar soft covering approximation space. For a set $A \subseteq \mathfrak{S}$, based on ρ , bipolar soft covering lower and upper approximations are, respectively, defined as:

$$\underline{BS}_\rho^+(A) = \bigcup_{\tilde{\varepsilon} \in C} \{\widehat{F}(\tilde{\varepsilon}) : \tilde{\varepsilon} \in C \wedge \widehat{F}(\tilde{\varepsilon}) \subseteq A\},$$

$$\underline{BS}_\rho^-(A) = \bigcup \{Md_{\rho^-}(t) : t \in A^c\},$$

$$\overline{BS}_\rho^+(A) = \bigcup \{Md_{\rho^+}(t) : t \in A\},$$

$$\overline{BS}_\rho^-(A) = \bigcup_{\neg \tilde{\varepsilon} \in \neg C} \{\widehat{H}(\neg \tilde{\varepsilon}) : \widehat{H}(\neg \tilde{\varepsilon}) \subseteq A^c\}.$$

Which are called bipolar soft covering ρ -lower positive approximation, bipolar soft covering ρ -lower negative approximation, bipolar soft covering ρ -upper positive approximation and bipolar soft covering ρ -upper negative approximation of A , respectively. Generally, the two pairs given as:

$$\underline{BS}_\rho(A) = (\underline{S}_\rho^+(A), \underline{S}_\rho^-(A)),$$

$$\overline{BS}_\rho(A) = (\overline{S}_\rho^+(A), \overline{S}_\rho^-(A))$$

are called bipolar soft covering rough approximations of $A \subseteq \mathfrak{S}$ with respect to ρ . Moreover, if $\underline{BS}_\rho(A) \neq \underline{BS}_\rho(A)$, then A is called bipolar soft covering based bipolar soft rough set, otherwise A is called bipolar soft ρ -definable. In addition, bipolar soft covering positive region and negative region of A is defined as, respectively:

$$BS \text{ Pos}(A) = \underline{BS}_\rho(A),$$

$BS \text{ Neg}(A) = \mathfrak{S} - \overline{BS}_\rho(A)$. Further, the boundary region (or area of uncertainty) of bipolar soft covering based rough set is defined as:

$$BS \text{ Bnd}(A) = \overline{BS}_\rho(A) - \underline{BS}_\rho(A) = (\overline{S}_{\rho+}(A) - \underline{S}_{\rho+}(A), \overline{S}_{\rho-}(A) - \underline{S}_{\rho-}(A)).$$

Example 3 (Continued From Example 2): As an illustration, according to Definition 17, For $A_1 = \{t_1, t_3, t_5\} \subseteq \mathfrak{S}$, bipolar soft covering ρ -lower positive approximation, bipolar soft covering ρ -lower negative approximation, bipolar soft covering ρ -upper positive approximation and bipolar soft covering ρ -upper negative approximation of A_1 , respectively, can be calculated as:

$$\underline{BS}_{\rho+}(A_1) = \{t_5\}, \underline{BS}_{\rho-}(A_1) = \{t_2, t_4, t_5\}, \\ \overline{BS}_{\rho+}(A_1) = \{t_1, t_2, t_3, t_4, t_5\}, \overline{BS}_{\rho-}(A_1) = \{t_2\}.$$

So, the lower and upper approximations of A_1 given as:

$$\underline{BS}_\rho(A_1) = (\{t_5\}, \{t_2, t_4, t_5\}), \\ \overline{BS}_\rho(A_1) = (\{t_1, t_2, t_3, t_4, t_5\}, \{t_2\}).$$

Since $\underline{BS}_\rho(A_1) \neq \overline{BS}_\rho(A_1)$, A_1 is a bipolar soft covering based bipolar soft rough set.

For $A_2 = \{t_1, t_2\} \subseteq \mathfrak{S}$, we have

$$\underline{BS}_{\rho+}(A_2) = \{t_1, t_2\}, \underline{BS}_{\rho-}(A_2) = \{t_4, t_5\}, \\ \overline{BS}_{\rho+}(A_2) = \{t_1, t_2\}, \overline{BS}_{\rho-}(A_2) = \{t_4, t_5\}.$$

So, the lower and upper approximations of A_2 given as:

$$\underline{BS}_\rho(A_2) = (\{t_1, t_2\}, \{t_4, t_5\}), \\ \overline{BS}_\rho(A_2) = (\{t_1, t_2\}, \{t_4, t_5\}).$$

Since $\underline{BS}_\rho(A_2) = \overline{BS}_\rho(A_2)$, A_2 is a bipolar soft covering based definable set.

Now, we investigate some properties of the bipolar soft covering lower and upper approximations.

Theorem 1: Let $\wp = (\widehat{F}, \widehat{H}, C)$ be a bipolar soft covering over \mathfrak{S} , $\rho = (\mathfrak{S}, \mathfrak{K}_\wp)$ be a bipolar soft covering approximation space and $A, B \subseteq \mathfrak{S}$. Then the bipolar soft covering lower and upper approximations have the following properties:

- 1) $\underline{BS}_\rho(\mathfrak{S}) = \overline{BS}_\rho(\mathfrak{S}) = \mathfrak{S}$
- 2) $\underline{BS}_\rho(\emptyset) = \overline{BS}_\rho(\emptyset) = \emptyset$
- 3) $\underline{BS}_\rho(A) \subseteq A \subseteq \overline{BS}_\rho(A)$
- 4) $A \subseteq B \Rightarrow \underline{BS}_\rho(A) \subseteq \underline{BS}_\rho(B),$

Proof 1: From Definition 17, we can easily prove the properties 1, 2 and 3.

- 4) Since $A \subseteq B$, for all $t \in \underline{BS}_{\rho+}(A)$, there exists $\tilde{\varepsilon} \in C$ such that $t \in \widehat{F}(\tilde{\varepsilon})$ and $\widehat{F}(\tilde{\varepsilon}) \subseteq A \subseteq B$. According to Definition 17, $\widehat{F}(\tilde{\varepsilon}) \subseteq \underline{BS}_{\rho+}(B)$, so $t \in \underline{BS}_{\rho+}(B)$. Hence $\underline{BS}_{\rho+}(A) \subseteq \underline{BS}_{\rho+}(B)$. Since $A \subseteq B$, by Definition of complements, we have $B^c \subseteq A^c$. By Definition 17, $\underline{BS}_{\rho-}(B) = \bigcup\{Md_\rho(t) : t \in B^c\} \subseteq \bigcup\{Md_\rho(t) : t \in A^c\} = \underline{BS}_{\rho-}(A)$. Therefore, $\underline{BS}_{\rho-}(A) \supseteq \underline{BS}_{\rho-}(B)$. So, by using two inequalities $\underline{BS}_{\rho+}(A) \subseteq \underline{BS}_{\rho+}(B)$

and $\underline{BS}_{\rho-}(A) \supseteq \underline{BS}_{\rho-}(B)$, we conclude that $\underline{BS}_\rho(A) \subseteq \underline{BS}_\rho(B)$.

Hence $A \subseteq B \Rightarrow \underline{BS}_\rho(A) \subseteq \underline{BS}_\rho(B)$.

Theorem 2: Let $\wp = (\widehat{F}, \widehat{H}, C)$ be a bipolar soft covering over \mathfrak{S} , $\rho = (\mathfrak{S}, \mathfrak{K}_\wp)$ be a bipolar soft covering approximation space and $A, B \subseteq \mathfrak{S}$. Then the bipolar soft covering lower and upper approximations have the following properties:

- 1) $\underline{BS}_{\rho+}(A \cap B) \subseteq \underline{BS}_{\rho+}(A) \cap \underline{BS}_{\rho+}(B), \underline{BS}_{\rho-}(A \cap B) = \underline{BS}_{\rho-}(A) \cup \underline{BS}_{\rho-}(B)$
- 2) $\underline{BS}_{\rho+}(A \cup B) \supseteq \underline{BS}_{\rho+}(A) \cup \underline{BS}_{\rho+}(B), \underline{BS}_{\rho-}(A \cup B) \subseteq \underline{BS}_{\rho-}(A) \cap \underline{BS}_{\rho-}(B)$
- 3) $\underline{BS}_{\rho+}(A \cap B) \subseteq \overline{BS}_{\rho+}(A) \cap \overline{BS}_{\rho+}(B), \overline{BS}_{\rho-}(A \cap B) \supseteq \overline{BS}_{\rho-}(A) \cup \overline{BS}_{\rho-}(B)$
- 4) $\overline{BS}_{\rho+}(A \cup B) = \overline{BS}_{\rho+}(A) \cup \overline{BS}_{\rho+}(B), \overline{BS}_{\rho-}(A \cup B) = \overline{BS}_{\rho-}(A) \cap \overline{BS}_{\rho-}(B)$

Proof 2:

- 1) Firstly we prove that $\underline{BS}_{\rho+}(A \cap B) \subseteq \underline{BS}_{\rho+}(A) \cap \underline{BS}_{\rho+}(B)$. Let $u \in \underline{BS}_{\rho+}(A \cap B)$, by using Definition of bipolar soft covering ρ -lower positive approximation, we have $u \in \bigcup_{\tilde{\varepsilon} \in C} \{\widehat{F}(\tilde{\varepsilon}) : \tilde{\varepsilon} \in C \wedge \widehat{F}(\tilde{\varepsilon}) \subseteq A \cap B\}$
 $\Rightarrow u \in \bigcup_{\tilde{\varepsilon} \in C} \{\widehat{F}(\tilde{\varepsilon}) : \tilde{\varepsilon} \in C \wedge \widehat{F}(\tilde{\varepsilon}) \subseteq A\} \cap \bigcup_{\tilde{\varepsilon} \in C} \{\widehat{F}(\tilde{\varepsilon}) : \tilde{\varepsilon} \in C \wedge \widehat{F}(\tilde{\varepsilon}) \subseteq B\}$
 $\Rightarrow u \in \underline{BS}_{\rho+}(A) \cap \underline{BS}_{\rho+}(B)$
 Therefore, $\underline{BS}_{\rho+}(A \cap B) \subseteq \underline{BS}_{\rho+}(A) \cap \underline{BS}_{\rho+}(B)$. Next we prove that $\underline{BS}_{\rho-}(A \cap B) = \underline{BS}_{\rho-}(A) \cup \underline{BS}_{\rho-}(B)$. Let $u \in \underline{BS}_{\rho-}(A \cap B)$, by using Definition of bipolar soft covering ρ -lower negative approximation, we have $u \in \underline{BS}_{\rho-}(A \cap B) = \bigcup\{Md_\rho(t) : t \in (A \cap B)^c\}$
 $\iff u \in \bigcup\{Md_\rho(t) : t \in A^c \cup B^c\}$
 $\iff u \in \bigcup\{Md_\rho(t) : t \in A^c\} \cup u \in \bigcup\{Md_\rho(t) : t \in B^c\}$
 $\iff u \in \underline{BS}_{\rho-}(A) \cup \underline{BS}_{\rho-}(B)$
 $\underline{BS}_{\rho-}(A \cap B) = \underline{BS}_{\rho-}(A) \cup \underline{BS}_{\rho-}(B)$.
 2) Firstly we prove that $\underline{BS}_{\rho+}(A \cup B) \subseteq \underline{BS}_{\rho+}(A) \cup \underline{BS}_{\rho+}(B)$ and then give a counter example for reverse inclusion. Let $u \in \underline{BS}_{\rho+}(A) \cup \underline{BS}_{\rho+}(B)$, by using Definition of bipolar soft covering ρ -lower positive approximation, we have $u \in \underline{BS}_{\rho+}(A) \cup \underline{BS}_{\rho+}(B) = \bigcup_{\tilde{\varepsilon} \in C} \{\widehat{F}(\tilde{\varepsilon}) : \tilde{\varepsilon} \in C \wedge \widehat{F}(\tilde{\varepsilon}) \subseteq A\} \cup \bigcup_{\tilde{\varepsilon} \in C} \{\widehat{F}(\tilde{\varepsilon}) : \tilde{\varepsilon} \in C \wedge \widehat{F}(\tilde{\varepsilon}) \subseteq B\}$
 $\Rightarrow u \in \bigcup_{\tilde{\varepsilon} \in C} \{\widehat{F}(\tilde{\varepsilon}) : \tilde{\varepsilon} \in C \wedge \widehat{F}(\tilde{\varepsilon}) \subseteq A \cup B\}$
 $\Rightarrow u \in \underline{BS}_{\rho+}(A \cup B)$
 Therefore, $\underline{BS}_{\rho+}(A \cup B) \supseteq \underline{BS}_{\rho+}(A) \cup \underline{BS}_{\rho+}(B)$. Next we prove that $\underline{BS}_{\rho-}(A \cup B) = \underline{BS}_{\rho-}(A) \cap \underline{BS}_{\rho-}(B)$. Let $u \in \underline{BS}_{\rho-}(A \cup B)$, by using Definition of bipolar soft covering ρ -lower negative approximation, we have $u \in \underline{BS}_{\rho-}(A \cup B) = \bigcup\{Md_\rho(t) : t \in (A \cup B)^c\}$
 $\Rightarrow u \in \bigcup\{Md_\rho(t) : t \in A^c \cap B^c\}$
 $\Rightarrow u \in \bigcup\{Md_\rho(t) : t \in A^c\} \cap u \in \bigcup\{Md_\rho(t) : t \in B^c\}$

$$\begin{aligned} & t \in B^c \\ \Rightarrow & u \in \underline{BS}_{\rho^+}(A) \cup \underline{BS}_{\rho^+}(B) \\ \underline{BS}_{\rho^-}(A \cup B) & \subseteq \underline{BS}_{\rho^-}(A) \cap \underline{BS}_{\rho^-}(B). \end{aligned}$$

- 3) The proof of this assertion is similar to the proof of (1).
- 4) The proof of this assertion is similar to the proof of (2).

Example 4: Let $\wp = (\widehat{F}, \widehat{H}, C)$ be a bipolar soft covering over \mathfrak{S} , $\rho = (\mathfrak{S}, \mathfrak{K}_{\rho})$ be a bipolar soft covering approximation space and $A, B \subseteq \mathfrak{S}$. Then the bipolar soft covering lower and upper approximations have not the following properties:

- Theorem 3:
- 1) $\underline{BS}_{\rho^+}(A \cap B) \subseteq \underline{BS}_{\rho^+}(A) \cap \underline{BS}_{\rho^+}(B)$
 - 2) $\underline{BS}_{\rho^+}(A \cup B) \supseteq \underline{BS}_{\rho^+}(A) \cup \underline{BS}_{\rho^+}(B)$
 - 3) $\underline{BS}_{\rho^-}(A \cup B) \subseteq \underline{BS}_{\rho^-}(A) \cap \underline{BS}_{\rho^-}(B)$

(1). The following example show that the reverse inclusion of inequality mentioned above does not hold:

As an illustration, let $\wp = (\widehat{F}, \widehat{H}, C)$ be a full bipolar soft set over \mathfrak{S} , where $\mathfrak{S} = \{t_1, t_2, t_3, t_4, t_5, t_6\}$, $C = \{\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \tilde{\varepsilon}_3, \tilde{\varepsilon}_4\}$ and The mapping $\widehat{F} : C \rightarrow P(\mathfrak{S})$,

$$\tilde{\varepsilon} \mapsto \begin{cases} \{t_1, t_2, t_3\} & \text{if } \tilde{\varepsilon} = \tilde{\varepsilon}_1 \\ \{t_2, t_3, t_4\} & \text{if } \tilde{\varepsilon} = \tilde{\varepsilon}_2 \\ \{t_4, t_5\} & \text{if } \tilde{\varepsilon} = \tilde{\varepsilon}_3 \\ \{t_1, t_5, t_6\} & \text{if } \tilde{\varepsilon} = \tilde{\varepsilon}_4 \end{cases}$$

For $A = \{t_1, t_2, t_3, t_4\} \subseteq \mathfrak{S}$, $B = \{t_4, t_5\} \subseteq \mathfrak{S}$, we have $\underline{BS}_{\rho^+}(A) = \{t_1, t_2, t_3, t_4\}$, $\underline{BS}_{\rho^+}(B) = \{t_4, t_5\}$, so $\underline{BS}_{\rho^+}(A) \cap \underline{BS}_{\rho^+}(B) = \{t_4\}$, but $\underline{BS}_{\rho^+}(A \cap B) = \emptyset$. Which implies that $\underline{BS}_{\rho^+}(A \cap B) \not\supseteq \underline{BS}_{\rho^+}(A) \cap \underline{BS}_{\rho^+}(B)$.

(2). The following example show that the reverse inclusion of inequality mentioned above does not hold:

As an illustration, let $\wp = (\widehat{F}, \widehat{H}, C)$ be a full bipolar soft set over \mathfrak{S} , where $\mathfrak{S} = \{t_1, t_2, t_3, t_4, t_5, t_6\}$, $C = \{\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \tilde{\varepsilon}_3, \tilde{\varepsilon}_4\}$ and The mapping $\widehat{F} : C \rightarrow P(\mathfrak{S})$,

$$\tilde{\varepsilon} \mapsto \begin{cases} \{t_1, t_2, t_3\} & \text{if } \tilde{\varepsilon} = \tilde{\varepsilon}_1 \\ \{t_2, t_3, t_4\} & \text{if } \tilde{\varepsilon} = \tilde{\varepsilon}_2 \\ \{t_4, t_5\} & \text{if } \tilde{\varepsilon} = \tilde{\varepsilon}_3 \\ \{t_1, t_5, t_6\} & \text{if } \tilde{\varepsilon} = \tilde{\varepsilon}_4 \end{cases}$$

For $A = \{t_1, t_2\} \subseteq \mathfrak{S}$, $B = \{t_3, t_4\} \subseteq \mathfrak{S}$, we have $\underline{BS}_{\rho^+}(A) = \{\}$, $\underline{BS}_{\rho^+}(B) = \{\}$, so $\underline{BS}_{\rho^+}(A) \cup \underline{BS}_{\rho^+}(B) = \{\}$, but $\underline{BS}_{\rho^+}(A \cup B) = \{t_1, t_2, t_3, t_4\}$. Which implies that $\underline{BS}_{\rho^+}(A \cup B) \not\subseteq \underline{BS}_{\rho^+}(A) \cup \underline{BS}_{\rho^+}(B)$.

(3) The following example show that the reverse inclusion of inequality mentioned above does not hold:

As an illustration, let $\wp = (\widehat{F}, \widehat{H}, C)$ be a full bipolar soft set over \mathfrak{S} , where $\mathfrak{S} = \{t_1, t_2, t_3, t_4, t_5, t_6\}$, $C = \{\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \tilde{\varepsilon}_3, \tilde{\varepsilon}_4\}$ and The mapping

$$\begin{aligned} & \widehat{H} : C \rightarrow P(\mathfrak{S}), \\ & \tilde{\varepsilon} \mapsto \begin{cases} \{t_1\} & \text{if } \tilde{\varepsilon} = \tilde{\varepsilon}_1 \\ \{t_2, t_3\} & \text{if } \tilde{\varepsilon} = \tilde{\varepsilon}_2 \\ \{t_4, t_5\} & \text{if } \tilde{\varepsilon} = \tilde{\varepsilon}_3 \\ \{t_6\} & \text{if } \tilde{\varepsilon} = \tilde{\varepsilon}_4 \end{cases} \end{aligned}$$

For $A = \{t_1, t_2\} \subseteq \mathfrak{S}$, $B = \{t_3, t_4\} \subseteq \mathfrak{S}$, we have $\underline{BS}_{\rho^-}(A) = \{t_2, t_3, t_4, t_5, t_6\}$, $\underline{BS}_{\rho^-}(B) = \{t_1, t_2, t_3, t_4, t_5, t_6\}$, so $\underline{BS}_{\rho^-}(A) \cap \underline{BS}_{\rho^-}(B) = \{t_2, t_3, t_4, t_5, t_6\}$, but $\underline{BS}_{\rho^-}(A \cup B) = \{t_4, t_5, t_6\}$. Which implies that $\underline{BS}_{\rho^-}(A \cup B) \not\supseteq \underline{BS}_{\rho^-}(A) \cap \underline{BS}_{\rho^-}(B)$.

IV. A NEW PROPOSAL FOR MULTI-ATTRIBUTE GROUP DECISION-MAKING USING BSCB-RSs HYBRID WITH PROMETHEE METHOD AND TOPSIS METHOD

A. MULTI-ATTRIBUTE GROUP DECISION MAKING BASED ON BSCB-RSS USING PROMETHEE TECHNIQUE

In this section, a multi-criteria group decision-analysis (MCGDA) approach, based on the promethee method combined with bipolar soft covering based rough set is presented to solve multi-criteria decision-making problems. Promethee is a rapid, flexible and progressive method for pair-wise comparison in MCDM. This method considers the outranking flows for evaluating alternatives. The concept is built on pairwise comparison between alternatives and calculates two outranking flows for each alternative, namely positive and negative outranking flows. The positive outranking flow gives a measure of how the alternative outranks all the other, while the negative outranking flow gives a measure of how the alternative is outranked by all the others. The higher $\phi^+(a)$ is the better alternative when $\phi^+(a)$ represents the power of a . On the other hand, the smaller $\phi^-(a)$ is the better alternative when $\phi^-(a)$ represents the weakness of a .

In the following, we present an algorithm over the bipolar soft covering based rough set hybrid with Promethee. We apply this algorithm for selection of most optimal site for earth dam. Let $\mathfrak{S} = \{t_1, t_2, \dots, t_n\}$ be the finite universe of objects, $C = \{\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \dots, \tilde{\varepsilon}_m\}$ be the set of all possible parameters and $\wp = (\widehat{F}, \widehat{H}, C)$ be a bipolar soft set over \mathfrak{S} . Suppose that $G = \{p_1, p_2, \dots, p_k\}$ is a set of expert persons, Y_1, Y_2, \dots, Y_k are non-empty subsets of \mathfrak{S} , represent results of primary evaluations of expert persons p_1, p_2, \dots, p_k , respectively and bipolar soft set D_1, D_2, \dots, D_r are the actual result that previously obtained for problems in different places or different times.

Definition 18: Let $\underline{BS}_{D_q}(Y_j) = (Y_{j\rho^+}, Y_{j\rho^-})$ and $\overline{BS}_{D_q}(Y_j) = (\overline{Y}_{j\rho^+}, \overline{Y}_{j\rho^-})$ be lower and upper approximations of bipolar soft set Y_j ; ($j = 1, 2, \dots, k$) related to D_q ; ($q = 1, 2, \dots, r$). Then

$$\begin{aligned} & [BS]_{\rho^+, \rho^-} \\ & = \begin{bmatrix} \{Y_1^{1+}, Y_1^{1-}\} & \{Y_2^{1+}, Y_2^{1-}\} & \dots & \{Y_k^{1+}, Y_k^{1-}\} \\ \{Y_1^{2+}, Y_1^{2-}\} & \{Y_2^{2+}, Y_2^{2-}\} & \dots & \{Y_k^{2+}, Y_k^{2-}\} \\ \vdots & \vdots & \ddots & \vdots \\ \{Y_1^{r+}, Y_1^{r-}\} & \{Y_2^{r+}, Y_2^{r-}\} & \dots & \{Y_k^{r+}, Y_k^{r-}\} \end{bmatrix} \end{aligned} \tag{3}$$

and

$$[\overline{BS}]_{\rho^+, \rho^-}$$

$$= \begin{bmatrix} \{\overline{Y}_{1\rho^+}^{1+}, \overline{Y}_{1\rho^-}^{1-}\} \{\overline{Y}_{2\rho^+}^{1+}, \overline{Y}_{2\rho^-}^{1-}\} \dots \{\overline{Y}_{k\rho^+}^{1+}, \overline{Y}_{k\rho^-}^{1-}\} \\ \{\overline{Y}_{1\rho^+}^{2+}, \overline{Y}_{1\rho^-}^{2-}\} \{\overline{Y}_{2\rho^+}^{2+}, \overline{Y}_{2\rho^-}^{2-}\} \dots \{\overline{Y}_{k\rho^+}^{2+}, \overline{Y}_{k\rho^-}^{2-}\} \\ \vdots \\ \{\overline{Y}_{1\rho^+}^{r+}, \overline{Y}_{1\rho^-}^{r-}\} \{\overline{Y}_{2\rho^+}^{r+}, \overline{Y}_{2\rho^-}^{r-}\} \dots \{\overline{Y}_{k\rho^+}^{r+}, \overline{Y}_{k\rho^-}^{r-}\} \end{bmatrix} \quad (4)$$

are said to be bipolar soft lower approximation matrix and bipolar soft upper approximation matrix, respectively. Here

$$\underline{Y}_{j\rho^+}^{q+} = (t_{1j}^{q+}, t_{2j}^{q+}, \dots, t_{nj}^{q+}) \quad (5)$$

$$\underline{Y}_{j\rho^-}^{q-} = (t_{1j}^{q-}, t_{2j}^{q-}, \dots, t_{nj}^{q-}) \quad (6)$$

$$\overline{Y}_{j\rho^+}^{q+} = (\overline{t}_{1j}^{q+}, \overline{t}_{2j}^{q+}, \dots, \overline{t}_{nj}^{q+}) \quad (7)$$

$$\overline{Y}_{j\rho^-}^{q-} = (\overline{t}_{1j}^{q-}, \overline{t}_{2j}^{q-}, \dots, \overline{t}_{nj}^{q-}) \quad (8)$$

where

$$\underline{t}_{1j}^{q+} = \begin{cases} 1 & \text{if } t_i \in Y_{j\rho^+} \\ 0 & \text{if } t_i \notin Y_{j\rho^+} \end{cases} \quad (9)$$

$$\underline{t}_{1j}^{q-} = \begin{cases} -0.5 & \text{if } t_i \in Y_{j\rho^+} \\ 0 & \text{if } t_i \notin Y_{j\rho^+} \end{cases} \quad (10)$$

$$\overline{t}_{1j}^{q+} = \begin{cases} 0.5 & \text{if } t_i \in \overline{Y}_{j\rho^+} \\ 0 & \text{if } t_i \notin \overline{Y}_{j\rho^+} \end{cases} \quad (11)$$

$$\overline{t}_{1j}^{q-} = \begin{cases} -1 & \text{if } t_i \in \overline{Y}_{j\rho^-} \\ 0 & \text{if } t_i \notin \overline{Y}_{j\rho^-} \end{cases} \quad (12)$$

Definition 19: Let $[BS]_{\rho^+,\rho^-}$ and $[\overline{BS}]_{\rho^+,\rho^-}$ be bipolar soft lower approximation matrix and bipolar soft upper approximation matrix, respectively. Then

$$[W]_{\rho^+,\rho^-} = [BS]_{\rho^+,\rho^-} + [\overline{BS}]_{\rho^+,\rho^-} = \begin{bmatrix} u_{11} & u_{12} & u_{1k} \\ u_{21} & u_{22} & u_{2k} \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ u_{r1} & u_{r2} & u_{rk} \end{bmatrix} = [u_{ij}]_{r \times k} \quad (13)$$

is called weighted covering based parameter matrix, where each entry is of the form

$$u_{ij} = \{u_{ij}^{\rho^+}, u_{ij}^{\rho^-}\} = \{Y_{j\rho^+} \oplus \overline{Y}_{j\rho^+}, Y_{j\rho^-} \oplus \overline{Y}_{j\rho^-}\}.$$

Here the operation represent the vector addition.

Definition 20: Let $[W]_{\rho^+,\rho^-}$ be the weighted covering based parameter matrix. Then

$$[S]_{\rho^+,\rho^-} = [\{s_{ij}^{\rho^+}, s_{ij}^{\rho^-}\}]_{r \times k} \quad (14)$$

is called the standardized covering based decision matrix,

where $s_{ij}^{\rho^+} = \sum_{i=1}^k u_{ij}^{\rho^+}$ and $s_{ij}^{\rho^-} = \sum_{i=1}^k u_{ij}^{\rho^-}$; $j = 1, 2, \dots, r$.

Definition 21: Let $[S]_{\rho^+,\rho^-}$ be the standardized covering based decision matrix. Then the corresponding normalized covering based decision matrix is defined as:

$$[N]_{\rho^+,\rho^-} = \begin{bmatrix} \eta_{11} & \eta_{12} & \dots & \eta_{1k} \\ \eta_{21} & \eta_{22} & \dots & \eta_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \eta_{r1} & \eta_{r2} & \dots & \eta_{rk} \end{bmatrix} = [\eta_{ij}]_{r \times k}$$

where each entry $\eta_{ij} = \{\eta_{ij}^{\rho^+}, \eta_{ij}^{\rho^-}\}$ with the following conditions:

$$\eta_{ij}^{\rho^+} = \frac{s_{ij}^{\rho^+}}{\sqrt{\sum_{l=1}^r (s_{lj}^{\rho^+})^2}} \quad (15)$$

and

$$\eta_{ij}^{\rho^-} = \frac{s_{ij}^{\rho^-}}{\sqrt{\sum_{l=1}^r (s_{lj}^{\rho^-})^2}} \quad (16)$$

Definition 22: Let $[S]_{\rho^+,\rho^-}$ be the normalized covering based decision matrix. Then the corresponding average weighted normalized covering based decision matrix is defined as:

$$[U]_{\rho^+,\rho^-} = \begin{bmatrix} u_{11} & u_{12} & \dots & u_{1k} \\ u_{21} & u_{22} & \dots & u_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ u_{r1} & u_{r2} & \dots & u_{rk} \end{bmatrix} = [u_{ij}]_{r \times k}$$

where each entry

$$u_{ij} = \left| \eta_{ij}^{\rho^+} + \eta_{ij}^{\rho^-} \right|. \quad (17)$$

Definition 23: Let $[U]_{\rho^+,\rho^-} = [u_{ij}]_{r \times k}$ be the average weighted normalized covering based decision matrix. Then determine the deviation by pairwise comparison by using the following equation

$$d_j(a, b) = g_j(a) - g_j(b) \quad (18)$$

Where

$$g_j = \frac{[u_{ij} - \min(u_{ij})]}{[\max(u_{ij}) - \min(u_{ij})]}$$

and $d_j(a, b)$ denotes the difference between the evaluations of a and b on each criterion.

Let $\xi = (\xi_1, \xi_2, \dots, \xi_l)^T$ be the weight vector of the attributes, where $0 \leq \xi_i \leq 1, i = 1, 2, \dots, l$, and satisfies $\sum_{i=1}^l \xi_i = 1$, where ξ_i is the weight for i th criterion.

Definition 24: Let $d_j(a, b)$ denotes the difference between the evaluations of a and b on each criterion. Then we define the preference function

$$P_j(a, b) = F_j[d_j(a, b)] \quad (19)$$

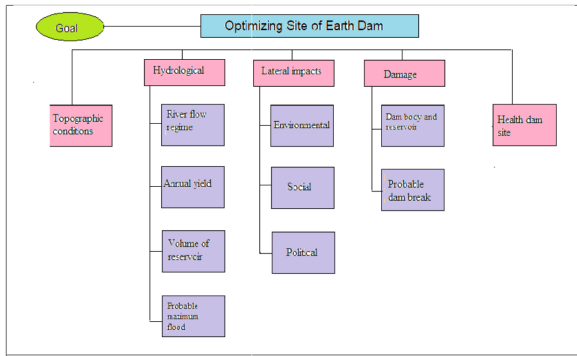


FIGURE 2. Criteria for selection of appropriate dam site.

$P_j(a, b)$ represent the function of difference between the evaluations of alternative a regarding alternative b on each criterion into a degree ranging from 0 to 1. Preference function is to replace the negative function by 0 and retains the positive values.

Definition 25: Let $P_j(a, b)$ represent the preference function, determine the multi-criteria preference index by the following Equation

$$\pi(a, b) = \sum_{j=1}^k p(a, b)\xi_j \quad (20)$$

Definition 26: Let $P_j(a, b)$ be the multi-criteria preference index. Then we define the positive outranking flow $\psi^+(a)$ and negative outranking flow $\psi^-(a)$ by the following formula, respectively.

$$\psi^+(a) = \frac{1}{n-1} \sum_{b=1}^n \pi(a, b), (a \neq b)$$

and $\psi^-(a) = \frac{1}{n-1} \sum_{b=1}^n \pi(b, a), (a \neq b) \quad (21)$

Definition 27: Let $\psi^+(a)$ and $\psi^-(a)$ be the positive outranking flow and negative outranking flow, respectively. Then we calculate the net flow values

$$\psi(a) = \psi^+(a) - \psi^-(a).$$

1) PROPOSED ALGORITHM

In this section, we present the algorithm for the established method of considered multi criteria group decision making problem in section 4.1.

Step 1: Take primary evaluations Y_1, Y_2, \dots, Y_k of experts P_1, P_2, \dots, P_k .

Step 2: Construct D_1, D_2, \dots, D_r bipolar soft covering based Soft sets using the real results.

Step 3: Compute $BS_{D_q}^{\rho^+}(Y_j)$, $BS_{D_q}^{\rho^-}(Y_j)$, $\overline{BS}_{D_q}^{\rho^+}(Y_j)$ and $\overline{BS}_{D_q}^{\rho^-}(Y_j)$ for each $j = 1, 2, \dots, k$ and $q = 1, 2, \dots, r$.

Step 4: Construct covering bipolar soft lower approximation matrix $[BS]_{\rho^+, \rho^-}$, covering bipolar soft upper approximation matrix $[\overline{BS}]_{\rho^+, \rho^-}$.

Step 5: Compute weighted covering based parameter matrix $[W]_{\rho^+, \rho^-}$.

Step 6: Compute standardized covering based decision matrix $[S]_{\rho^+, \rho^-}$.

Step 7: Compute normalized covering based decision matrix $[N]_{\rho^+, \rho^-}$.

Step 8: Compute average weighted normalized covering based decision matrix $[U]_{\rho^+, \rho^-}$.

Step 9: Determination of deviation by pairwise comparison.

Step 10: Determine the multi-criteria preference index.

Step 11: Calculate the net flow values and rank accordingly.

2) CASE STUDY

In this subsection, the bipolar soft covering based rough set model for selection of appropriate Dam site is applied in a numerical example. It is shown in fig 1. A decision maker group formed for this reason, consisting of a geographer, an energy engineer and a map engineer. Let the sites $t_1, t_2, t_3, t_4, t_5, t_6$ be selected as the alternative for Earth dam site location. The decision maker group evaluate these alternatives and for selection of a suitable alternative we will use selection criterion. In order to determine effective factors in selecting an appropriate site, extensive studies were conducted and the most effective attributes (criteria and subcriteria) were selected. These attributes are shown in Fig 1.

A brief explanation about the attributes is presented:

$\tilde{\epsilon}_1$) Topographical Conditions:

It is critical to have a secondary valley or stone abutments with proper topography around the main river while building a dam spillway. In addition, because the main river is U or S shaped, the length of tunnels, channels, and other water transfer systems to divert or transfer water from upstream to downstream during dam building and afterward is limited. In general, the best location for a dam reservoir and its body is where a vast valley with high walls connects to a narrow canyon with tenacious walls.

$\tilde{\epsilon}_2$) Hydrological: This criteria consists of four subcriterion, which is presented below.

SC₁) River flow regime: At the dam location, the river's permanent or seasonal flow regime is critical. Seasonal rivers convey more silt and have poorer water quality, making water resource management more difficult owing to inaccurate water delivery into reservoirs. As a result, it is suggested that the flow be maintained indefinitely.

SC₂) Annual yield: The yearly yield is the annual volume of water that passes through the cross section of the river in the dam site, and it plays a vital part in determining where the dam should be built.

SC₃) Volume of reservoir: When the reservoir generated after dam building has larger volume, the surface area of the reservoir water increases, which has a greater impact on the climate, but it also increases the possibility for evaporation and water pollution. On the other hand, if the dam is built in a location where the surface area of the reservoir water

will not be considerably affected by raising the volume of the reservoir water, (Because of the valley's steep slope), the height and hydrostatic pressure of the water will rise, which will benefit energy generation and downstream water transfer. However, the dam's body would be subjected to more force, and the structure would need to be raised and strengthened. As a result, the dam should be built in such a way that the reservoir capacity is determined by the aforementioned factors, such as leakage and other losses are optimal.

SC₄) Probable maximum flood: The largest volume of water produced by thawing snow and ice or other atmospheric precipitation that is likely to occur in rivers is known as the probable maximum flood.

$\tilde{\varepsilon}_3$) Lateral Impacts:

There are three subcriterion in this criteria, which are listed below.

SC₁) Environmental impacts: Other factors that play a part in determining the dam site include changing weather conditions, vegetation, and wildlife.

SC₂) Social impacts: The social consequences of population centre displacement and integration of different ethnic cultures as a result of the demolition of residential areas for dam construction, reservoir dewatering, and downstream dam water use should all be considered.

SC₃) Political impacts: Dam construction purposes for decreasing political tensions, such as water supply for a community, preventing grievances, and immigration of people of a border city, should all be taken into account.

$\tilde{\varepsilon}_4$) Damage: This criteria has two sub-criteria, which are listed below.

SC₁) Dam body and reservoir: Environmental damages, such as the destruction of mines, historical monuments, agricultural fields, and residential areas; road, railway, and power line displacement; and changes in the path of oil and gas pipelines, telecommunication facilities, among other things, should be addressed.

SC₂) Probable dam break: Material and moral damages caused by a possible dam collapse are essential factors to consider when choosing a dam site, and the dam should be built in an area where the amount of harm caused by a possible dam break is minimal.

$\tilde{\varepsilon}_5$) Health Dam Site: The dam location must be in an area with few seams and tracks, as well as a low risk of tectonic activity such as earthquakes, landslides, and subsidence. Furthermore, greater results will be realised in the dam location with reduced permeability and liquefaction properties of soil and natural materials. Furthermore, the region's soil mechanical qualities (compaction, consolidation, and so on) as well as the type of geological layers in the region have an impact on reservoir water quality.

Then NOT set of parameters of C is $\lrcorner C$, $\lrcorner \tilde{\varepsilon} \in \lrcorner C$.

Step 1: Primary evaluations of experts persons (geographer, energy engineer and map engineer) p_1, p_2 and p_3 are:

$$Y_1 = \{t_1, t_3, t_5\}, \quad Y_2 = \{t_1, t_2, t_3, t_4\}$$

$$\text{and } Y_3 = \{t_3, t_5\}.$$

Step 2: Real results in five different periods are represented as bipolar soft covering over $\mathfrak{S}, D_1 = (\widehat{F}_1, \widehat{H}_1, C), D_2 = (\widehat{F}_2, \widehat{H}_2, C), D_3 = (\widehat{F}_3, \widehat{H}_3, C), D_4 = (\widehat{F}_4, \widehat{H}_4, C)$ and $D_5 = (\widehat{F}_5, \widehat{H}_5, C)$ as follows:

The real result in D_1 period choose the set of parameters as: $\widehat{E}_1 = \{\tilde{\varepsilon}_1, \tilde{\varepsilon}_4, \tilde{\varepsilon}_5\}, \widehat{F}_1 : \widehat{E}_1 \rightarrow P(\mathfrak{S})$ by

$$\tilde{\varepsilon} \mapsto \begin{cases} \{t_1, t_3, t_5\} & \text{if } \tilde{\varepsilon} = \tilde{\varepsilon}_1 \\ \{t_3, t_5\} & \text{if } \tilde{\varepsilon} = \tilde{\varepsilon}_4 \\ \{t_1, t_2, t_4\} & \text{if } \tilde{\varepsilon} = \tilde{\varepsilon}_5 \end{cases}$$

and $\widehat{H}_1 : \lrcorner \widehat{E}_1 \rightarrow P(\mathfrak{S})$ by

$$\lrcorner \tilde{\varepsilon} \mapsto \begin{cases} \{t_2\} & \text{if } \tilde{\varepsilon} = \lrcorner \tilde{\varepsilon}_1 \\ \{t_1, t_4\} & \text{if } \tilde{\varepsilon} = \lrcorner \tilde{\varepsilon}_4 \\ \{t_3, t_5\} & \text{if } \tilde{\varepsilon} = \lrcorner \tilde{\varepsilon}_5 \end{cases}$$

The real result in D_2 period choose the set of parameters as: $\widehat{E}_2 = \{\tilde{\varepsilon}_1, \tilde{\varepsilon}_3\}, \widehat{F}_2 : \widehat{E}_2 \rightarrow P(\mathfrak{S})$ by

$$\tilde{\varepsilon} \mapsto \begin{cases} \{t_2, t_4, t_5\} & \text{if } \tilde{\varepsilon} = \tilde{\varepsilon}_1 \\ \{t_1, t_3\} & \text{if } \tilde{\varepsilon} = \tilde{\varepsilon}_3 \end{cases}$$

and $\widehat{H}_2 : \lrcorner \widehat{E}_2 \rightarrow P(\mathfrak{S})$ by

$$\lrcorner \tilde{\varepsilon} \mapsto \begin{cases} \{t_3\} & \text{if } \tilde{\varepsilon} = \lrcorner \tilde{\varepsilon}_1 \\ \{t_2, t_5\} & \text{if } \tilde{\varepsilon} = \lrcorner \tilde{\varepsilon}_3 \end{cases}$$

The real result in D_3 period choose the set of parameters as: $\widehat{E}_3 = \{\tilde{\varepsilon}_2, \tilde{\varepsilon}_3, \tilde{\varepsilon}_4\}, \widehat{F}_3 : \widehat{E}_3 \rightarrow P(\mathfrak{S})$ by

$$\tilde{\varepsilon} \mapsto \begin{cases} \{t_1, t_2\} & \text{if } \tilde{\varepsilon} = \tilde{\varepsilon}_2 \\ \{t_3, t_4\} & \text{if } \tilde{\varepsilon} = \tilde{\varepsilon}_3 \\ \{t_5\} & \text{if } \tilde{\varepsilon} = \tilde{\varepsilon}_4 \end{cases}$$

and $\widehat{H}_3 : \lrcorner \widehat{E}_3 \rightarrow P(\mathfrak{S})$ by

$$\lrcorner \tilde{\varepsilon} \mapsto \begin{cases} \{t_4, t_5\} & \text{if } \tilde{\varepsilon} = \lrcorner \tilde{\varepsilon}_2 \\ \{t_2\} & \text{if } \tilde{\varepsilon} = \lrcorner \tilde{\varepsilon}_3 \\ \{t_1\} & \text{if } \tilde{\varepsilon} = \lrcorner \tilde{\varepsilon}_4 \end{cases}$$

The real result in D_4 period choose the set of parameters as: $\widehat{E}_4 = \{\tilde{\varepsilon}_1, \tilde{\varepsilon}_2\}, \widehat{F}_4 : \widehat{E}_4 \rightarrow P(\mathfrak{S})$ by

$$\tilde{\varepsilon} \mapsto \begin{cases} \{t_1, t_2, t_3, t_4\} & \text{if } \tilde{\varepsilon} = \tilde{\varepsilon}_1 \\ \{t_5\} & \text{if } \tilde{\varepsilon} = \tilde{\varepsilon}_2 \end{cases}$$

and $\widehat{H}_4 : \lrcorner \widehat{E}_4 \rightarrow P(\mathfrak{S})$ by

$$\lrcorner \tilde{\varepsilon} \mapsto \begin{cases} \{t_5\} & \text{if } \tilde{\varepsilon} = \lrcorner \tilde{\varepsilon}_1 \\ \{t_1, t_2\} & \text{if } \tilde{\varepsilon} = \lrcorner \tilde{\varepsilon}_2 \end{cases}$$

The real result in D_5 period choose the set of parameters as: $\widehat{E}_5 = \{\tilde{\varepsilon}_2, \tilde{\varepsilon}_5\}, \widehat{F}_5 : \widehat{E}_5 \rightarrow P(\mathfrak{S})$ by

$$\tilde{\varepsilon} \mapsto \begin{cases} \{t_1, t_3\} & \text{if } \tilde{\varepsilon} = \tilde{\varepsilon}_2 \\ \{t_2, t_3, t_4, t_5\} & \text{if } \tilde{\varepsilon} = \tilde{\varepsilon}_5 \end{cases}$$

$\widehat{H}_5 : \widehat{E}_5 \rightarrow P(\mathfrak{S})$ by

$$\uparrow \widetilde{\varepsilon} \mapsto \begin{cases} \{t_2, t_4\} & \text{if } \widetilde{\varepsilon} = \uparrow \widetilde{\varepsilon}_2 \\ \{t_1\} & \text{if } \widetilde{\varepsilon} = \uparrow \widetilde{\varepsilon}_5 \end{cases}$$

Step 3: Using the Definition 17, to calculate the operators $\underline{BS}_{D_j}(Y_j)$, $\overline{BS}_{D_j}(Y_j)$, for $j = 1, 2, 3$ and $q = 1, 2, \dots, 5$.

$$\begin{aligned} \underline{BS}_{D_1}(Y_1) &= (\{t_1, t_3, t_5\}, \{t_1, t_2, t_4\}) \\ \underline{BS}_{D_2}(Y_1) &= (\{t_1, t_3\}, \{t_2, t_5\}) \\ \underline{BS}_{D_3}(Y_1) &= (\{t_5\}, \{t_2, t_4, t_5\}) \\ \underline{BS}_{D_4}(Y_1) &= (\{t_5\}, \{t_1, t_2\}) \\ \underline{BS}_{D_5}(Y_1) &= (\{t_1, t_3\}, \{t_2, t_4\}) \end{aligned}$$

and

$$\begin{aligned} \overline{BS}_{D_1}(Y_1) &= (\{t_1, t_3, t_5\}, \{t_2\}) \\ \overline{BS}_{D_2}(Y_1) &= (\{t_1, t_2, t_3, t_4, t_5\}, \{\}) \\ \overline{BS}_{D_3}(Y_1) &= (\{t_1, t_2, t_3, t_4, t_5\}, \{t_2\}) \\ \overline{BS}_{D_4}(Y_1) &= (\{t_1, t_2, t_3, t_4, t_5\}, \{\}) \\ \overline{BS}_{D_5}(Y_1) &= (\{t_1, t_2, t_3, t_4, t_5\}, \{t_2, t_4\}) \end{aligned}$$

Similarly,

$$\begin{aligned} \underline{BS}_{D_1}(Y_2) &= (\{t_1, t_2, t_3, t_4\}, \{t_3, t_5\}), \\ \underline{BS}_{D_2}(Y_2) &= (\{t_1, t_3\}, \{t_2, t_5\}), \\ \underline{BS}_{D_3}(Y_2) &= (\{t_1, t_2, t_3, t_4\}, \{t_4, t_5\}), \\ \underline{BS}_{D_4}(Y_2) &= (\{t_1, t_2, t_3, t_4\}, \{t_5\}), \\ \underline{BS}_{D_5}(Y_2) &= (\{t_1, t_3\}, \{\}), \end{aligned}$$

and

$$\begin{aligned} \overline{BS}_{D_1}(Y_2) &= (\{t_1, t_2, t_3, t_4, t_5\}, \{\}) \\ \overline{BS}_{D_2}(Y_2) &= (\{t_1, t_2, t_3, t_4, t_5\}, \{\}) \\ \overline{BS}_{D_3}(Y_2) &= (\{t_1, t_2, t_3, t_4\}, \{\}) \\ \overline{BS}_{D_4}(Y_2) &= (\{t_1, t_2, t_3, t_4\}, \{t_5\}) \\ \overline{BS}_{D_5}(Y_2) &= (\{t_1, t_2, t_3, t_4, t_5\}, \{\}) \end{aligned}$$

Also,

$$\begin{aligned} \underline{BS}_{D_1}(Y_3) &= (\{t_3, t_5\}, \{t_1, t_2, t_4\}) \\ \underline{BS}_{D_2}(Y_3) &= (\{\}, \{t_2, t_5\}), \\ \underline{BS}_{D_3}(Y_3) &= (\{t_5\}, \{t_1, t_2, t_4, t_5\}) \\ \underline{BS}_{D_4}(Y_3) &= (\{t_5\}, \{t_1, t_2\}) \\ \underline{BS}_{D_5}(Y_3) &= (\{\}, \{t_1, t_2, t_4\}), \end{aligned}$$

and

$$\begin{aligned} \overline{BS}_{D_1}(Y_3) &= (\{t_3, t_5\}, \{t_1, t_2, t_4\}) \\ \overline{BS}_{D_2}(Y_3) &= (\{t_1, t_2, t_3, t_4, t_5\}, \{\}) \\ \overline{BS}_{D_3}(Y_3) &= (\{t_3, t_4, t_5\}, \{t_1, t_2\}) \\ \overline{BS}_{D_4}(Y_3) &= (\{t_1, t_2, t_3, t_4, t_5\}, \{t_1, t_2\}) \\ \overline{BS}_{D_5}(Y_3) &= (\{t_2, t_3, t_4, t_5\}, \{t_1, t_2, t_4\}) \end{aligned}$$

Step 4: Covering based bipolar soft lower approximation matrix $[\underline{BS}]_{\rho^+, \rho^-}$ and covering based bipolar soft upper approximation matrix $[\overline{BS}]_{\rho^+, \rho^-}$, respectively, are obtained as follows:

$$\begin{aligned} [\underline{BS}]_{\rho^+, \rho^-} &= \begin{bmatrix} \{(1, 0, 1, 0, 1), (0, -0.5, 0, -0.5, 0)\} \\ \{(1, 0, 1, 0, 0), (0, -0.5, 0, 0, -0.5)\} \\ \{(0, 0, 0, 0, 1), (0, -0.5, 0, -0.5, -0.5)\} \\ \{(0, 0, 0, 0, 1), (-0.5, -0.5, 0, 0, 0)\} \\ \{(1, 0, 1, 0, 0), (0, -0.5, 0, -0.5, 0)\} \\ \{(1, 1, 0, 1, 0), (0, 0, -0.5, 0, -0.5)\} \\ \{(1, 0, 1, 0, 0), (0, -0.5, 0, 0, -0.5)\} \\ \{(1, 1, 1, 1, 0), (0, 0, 0, -0.5, -0.5)\} \\ \{(1, 1, 1, 1, 0), (0, 0, 0, 0, -0.5)\} \\ \{(1, 0, 1, 0, 0), (0, 0, 0, 0, 0)\} \\ \{(0, 0, 1, 0, 1), (-0.5, -0.5, 0, -0.5, 0)\} \\ \{(0, 0, 0, 0, 0), (0, -0.5, 0, 0, -0.5)\} \\ \{(0, 0, 0, 0, 1), (-0.5, -0.5, 0, -0.5, -0.5)\} \\ \{(0, 0, 0, 0, 1), (0, 0, 0, 0, 0)\} \\ \{(0, 0, 0, 0, 0), (-0.5, -0.5, 0, -0.5, 0)\} \end{bmatrix} \\ [\overline{BS}]_{\rho^+, \rho^-} &= \begin{bmatrix} \{(0.5, 0, 0.5, 0, 0.5), (0, -1, 0, 0, 0)\} \\ \{(0.5, 0.5, 0.5, 0.5, 0.5), (0, 0, 0, 0, 0)\} \\ \{(0.5, 0.5, 0.5, 0.5, 0.5), (0, -1, 0, 0, 0)\} \\ \{(0.5, 0.5, 0.5, 0.5, 0.5), (0, 0, 0, 0, 0)\} \\ \{(0.5, 0.5, 0.5, 0.5, 0.5), (0, -1, 0, -1, 0)\} \\ \{(0.5, 0.5, 0.5, 0.5, 0.5), (0, 0, 0, 0, 0)\} \\ \{(0.5, 0.5, 0.5, 0.5, 0.5), (0, 0, 0, 0, 0)\} \\ \{(0.5, 0.5, 0.5, 0.5, 0), (0, 0, 0, 0, 0)\} \\ \{(0.5, 0.5, 0.5, 0.5, 0.5), (0, 0, 0, 0, -1)\} \\ \{(0.5, 0.5, 0.5, 0.5, 0.5), (0, 0, 0, 0, 0)\} \\ \{(0, 0, 0.5, 0, 0.5), (-1, -1, 0, -1, 0)\} \\ \{(0.5, 0.5, 0.5, 0.5, 0.5), (0, 0, 0, 0, 0)\} \\ \{(0, 0, 0.5, 0.5, 0.5), (-1, -1, 0, 0, 0)\} \\ \{(0.5, 0.5, 0.5, 0.5, 0.5), (-1, -1, 0, 0, 0)\} \\ \{(0, 0.5, 0.5, 0.5, 0.5), (-1, -1, 0, -1, 0)\} \end{bmatrix} \end{aligned}$$

Step 5: Now we construct weighted covering based parameter matrix $[W]_{\rho^+, \rho^-}$ by using Equation (13), which is given as:

$$\begin{aligned} [W]_{\rho^+, \rho^-} &= \begin{bmatrix} \{(1.5, 0, 1.5, 0, 1.5), (0, -1.5, 0, -0.5, 0)\} \\ \{(1.5, 0.5, 1.5, 0.5, 0.5), (0, -0.5, 0, 0, -0.5)\} \\ \{(0.5, 0.5, 0.5, 0.5, 1.5), (0, -1.5, 0, -0.5, -0.5)\} \\ \{(0.5, 0.5, 0.5, 0.5, 1.5), (-0.5, -0.5, 0, 0, 0)\} \\ \{(1.5, 0.5, 1.5, 0.5, 0.5), (0, -1.5, 0, -1.5, 0)\} \\ \{(1.5, 1.5, 0.5, 1.5, 0.5), (0, 0, -0.5, 0, -0.5)\} \\ \{(1.5, 0.5, 1.5, 0.5, 0.5), (0, -0.5, 0, 0, -0.5)\} \\ \{(1.5, 1.5, 1.5, 1.5, 0), (0, 0, 0, -0.5, -0.5)\} \\ \{(1.5, 1.5, 1.5, 1.5, 0), (0, 0, 0, 0, -1.5)\} \\ \{(1.5, 0.5, 1.5, 0.5, 0.5), (0, 0, 0, 0, 0)\} \\ \{(0, 0, 0.5, 0, 0.5), (-1.5, -1.5, 0, -1.5, 0)\} \\ \{(0.5, 0.5, 0.5, 0.5, 0.5), (0, -0.5, 0, 0, -0.5)\} \\ \{(0, 0, 0.5, 0.5, 1.5), (-1.5, -1.5, 0, -0.5, -0.5)\} \\ \{(0.5, 0.5, 0.5, 0.5, 1.5), (-1, -1, 0, 0, 0)\} \\ \{(0, 0.5, 0.5, 0.5, 0.5), (-1.5, -1.5, 0, -1.5, 0)\} \end{bmatrix} \end{aligned}$$

Step 6: Compute standardized covering based decision matrix $[S]_{\rho^+, \rho^-}$ by using Equation (14), we have

$$[S]_{\rho^+, \rho^-} = \begin{bmatrix} \{3, -1.5\} & \{1.5, -3\} & \{2.5, -0.5\} \\ \{3.5, 0\} & \{1.5, -1.5\} & \{3.5, 0\} \\ \{2, -1.5\} & \{2, -2\} & \{2.5, 0\} \\ \{2.5, -1.5\} & \{2.5, -1.5\} & \{2.5, 0\} \\ \{3, -1.5\} & \{1.5, -3\} & \{3.5, 0\} \\ & \{1.5, -2\} & \{2.5, -0.5\} \\ & \{1.5, 0\} & \{1.5, -1.5\} \\ & \{2.5, -1.5\} & \{3, -1.5\} \\ & \{2.5, 0\} & \{3, -1.5\} \\ & \{1.5, -3\} & \{1.5, 0\} \end{bmatrix}$$

Step 7: We construct normalized covering-based decision matrix $[N]_{\rho^+, \rho^-}$ by using Equations (15) and (16), which is given as:

$$[N]_{\rho^+, \rho^-} = \begin{bmatrix} \{0.472, -0.5\} & \{0.364, -0.583\} & \{0.379, -1\} \\ \{0.55, 0\} & \{0.364, -0.29\} & \{0.53, 0\} \\ \{0.31, -0.5\} & \{0.485, -0.388\} & \{0.379, 0\} \\ \{0.39, -0.5\} & \{0.607, -0.29\} & \{0.379, 0\} \\ \{0.472, -0.5\} & \{0.364, -0.583\} & \{0.53, 0\} \\ & \{0.342, -0.512\} & \{0.466, -0.189\} \\ & \{0.342, 0\} & \{0.279, -0.566\} \\ & \{0.569, -0.384\} & \{0.599, -0.566\} \\ & \{0.569, 0\} & \{0.599, -0.566\} \\ & \{0.342, -0.767\} & \{0.279, 0\} \end{bmatrix}$$

Step 8: Now we construct average weighted normalized covering-based decision matrix $[V]_{\rho^+, \rho^-}$ by using Equations (17), which is given as:

	$\tilde{\epsilon}_1$	$\tilde{\epsilon}_1$	$\tilde{\epsilon}_1$	$\tilde{\epsilon}_1$	$\tilde{\epsilon}_1$
t_1	0.028	0.55	0.19	0.11	0.028
t_2	0.219	0.074	0.097	0.317	0.219
t_3	0.621	0.53	0.379	0.379	0.53
t_4	0.17	0.342	0.185	0.569	0.425
t_5	0.277	0.287	0.033	0.033	0.279

Step 9: We calculate the deviation by pairwise comparison by using Formula (18), which is given below.

	$\tilde{\epsilon}_1$	$\tilde{\epsilon}_1$	$\tilde{\epsilon}_1$	$\tilde{\epsilon}_1$	$\tilde{\epsilon}_1$
t_1	0	1	0.454	0.144	0
t_2	0.322	0	0.185	0.53	0.38
t_3	1	0.958	1	0.646	1
t_4	0.239	0.563	0.439	1	0.791
t_5	0.42	0.447	0	0	0.5

Step 10: Next, we calculate the multi-criteria preference index by using Formula (20).

	t_1	t_2	t_3	t_4	t_5
t_1	0	0.245	0.009	0.094	0.217
t_2	0.236	0	0.201	0.025	0.178
t_3	0.587	0.386	0	0.399	0.633
t_4	0.375	0.316	0.103	0	0.391
t_5	0.161	0.131	0	0.054	0

Step 11: Finally, we calculate the net flow values and rank accordingly.

Alternative	t_1	t_2	t_3	t_4	t_5
Pos. outranking flow	0.141	0.16	0.501	0.296	0.09
Neg. outranking flow	0.34	0.27	0.078	0.143	0.36
Net flow	-0.199	-0.11	0.423	0.153	0.27

Ranking the preference order is: $t_3 > t_4 > t_2 > t_1 > t_5$. Which indicate that Site t_3 is the best site for earth dam.

B. MULTI-CRITERIA GROUP DECISION MAKING BASED ON BSCB-RSS USING TOPSIS TECHNIQUE

TOPSIS is a useful multi-criteria group decision making (MCGDM) technique for ranking of design alternatives and selection of the best alternative in concept evaluation process through computation of Euclidean distances. The aggregating function calculated in TOPSIS represents “closeness to ideal solution”. TOPSIS uses vector normalization to make criteria of same units. The basic principle of TOPSIS is that the alternative that has been chosen as the best, should have the shortest distance from the positive ideal solution (PIS) and the farthest from the negative ideal solution (NIS).

In this subsection, we apply bipolar soft TOPSIS method to solve proposed problems to make a comparison with bipolar soft Promethee method.

The procedure of TOPSIS technique under bipolar soft covering based rough sets environment is explained as follows:

As in the subsection of bipolar soft Promethee, Steps 1–8 have already been done in subsection (A-2). So we move on step 9–11.

Definition 28: Let $[U]_{\rho^+, \rho^-} = [u_{ij}]_{r \times k}$ be the average weighted normalized covering based decision matrix. Then the expressions

$$PIS = \{\mu_1^\top, \mu_2^\top, \dots, \mu_k^\top\} = \{\vee u_{ij} \mid i \in I_r\} \quad (22)$$

and

$$NIS = \{\mu_1^\perp, \mu_2^\perp, \dots, \mu_k^\perp\} = \{\wedge u_{ij} \mid i \in I_r\} \quad (23)$$

are called Positive ideal solution and negative ideal solution, respectively.

Definition 29: Let PIS be positive ideal solution and NIS be negative ideal solution. Then the separation measurements of each alternative to PIS is calculated as:

$$S_i^\top = \sqrt{\sum_{j=1}^k (u_{ij} - u_j^\top)^2}; \quad i = 1, 2, \dots, r. \quad (24)$$

The separation measurements of each alternative to NIS is calculated as:

$$S_i^\perp = \sqrt{\sum_{j=1}^k (u_{ij} - u_j^\perp)^2}; \quad i = 1, 2, \dots, r. \quad (25)$$

Definition 30: Let S_i^\top and S_i^\perp be the separation measurements of the positive ideal solution and the negative ideal

solution, respectively. Then the relative closeness of alternatives to ideal solutions (represented as \wp_i) is defined as:

$$\wp_i = \frac{S_i^\perp}{S_i^\top + S_i^\perp}; \quad 0 \leq \wp_i \leq 1; \quad i = 1, 2, \dots, r. \tag{26}$$

1) PROPOSED ALGORITHM

In this section, we present the algorithm for the established method of considered multi criteria group decision making problem in section 4.2.

Step 1-8: These steps have already been done in subsection (A-2).

Step 9: Find positive ideal solution (PIS) and negative ideal solution (NIS).

Step 10: Calculate separation measurements of PIS S_i^\top and NIS S_i^\perp for each alternative.

Step 11: Calculate relative closeness \wp_i of alternatives to ideal solution and rank accordingly.

2) NUMERICAL EXAMPLE

In Sect. IV-A2, the decision-making problems have presented using bipolar soft Promethee method. Here, we present these applications using bipolar soft TOPSIS method to take into account the comparison of bipolar soft Promethee method and bipolar soft TOPSIS method. Steps 1–8 have already been done in Sect. IV-A2. So we move on step 9–11.

Step 9: The positive ideal solution (PIS) and negative ideal solution (NIS) by using the Equations (22) and (23) are obtained as:

$$PIS = \{0.621, 0.55, 0.379, 0.569, 0.53\};$$

$$NIS = \{0.028, 0.074, 0.097, 0.033, 0.028\}.$$

Step 10: The separation measurements of PIS and NIS for each parameter by using the Equations (24) and (25) are:

$$S_1^\top = 0.922 \quad S_1^\perp = 0.419$$

$$S_2^\top = 0.792 \quad S_2^\perp = 0.371$$

$$S_3^\top = 0.192 \quad S_3^\perp = 1.005$$

$$S_4^\top = 0.542 \quad S_4^\perp = 0.738$$

$$S_5^\top = 0.812 \quad S_5^\perp = 0.417$$

Step 11: The relative closeness of alternatives to the ideal solution by using Equation (26) are

$$\wp_1 = 0.347$$

$$\wp_2 = 0.319$$

$$\wp_3 = 0.840$$

$$\wp_4 = 0.577$$

$$\wp_5 = 0.339$$

Ranking the preference order is: $t_3 > t_4 > t_1 > t_5 > t_2$. which indicate that Site t_3 is the best site for earth dam.

Figure 2 illustrates the visual representation of the site rankings.

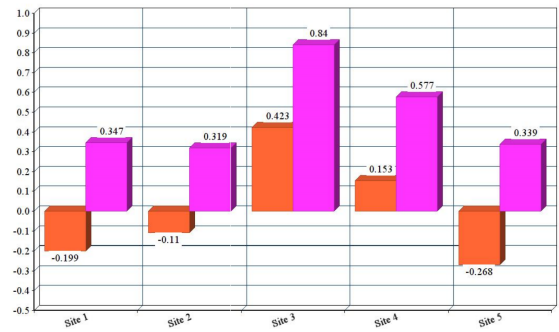


FIGURE 3. Ranking of sites by using Promethee and TOPSIS!

V. DISCUSSION AND COMPARATIVE ANALYSIS

In this section, we address validity of the proposed method, advantages, and disadvantages, as well as a comparison of the proposed techniques to several existing techniques.

A. VALIDITY OF THE PROPOSED MODEL

1) As we all know, aggregation is a vital stage in classical group decision making approaches for gathering the preferences or opinions of all decision-makers. In our proposed decision making approaches, every decision-maker expressed their opinion as a Bipolar Soft set, and afterward, all opinions given by decision-makers are aggregated through the usage the Bipolar soft Covering based approximations, and then a compromise optimal proposal is acquired. So, the Bipolar soft covering based rough sets approach to MCGDM provides a different strategy to aggregate the preferences of decision-makers. Therefore, the proposed decision making approaches (Promethee and TOPSIS) are valid and offer a novel technique and perspective to investigate GDM problems in real life. The basic idea of these both techniques (Promethee and TOPSIS) is given below:

i) Promethee (Preference Ranking Organization Method for enrichment evaluations) methods are family of outranking methods including Promethee I, II, III, IV, V and VI. Promethee I is partial outranking method, Promethee III to VI are actually having the fundamental basics of Promethee II with the little variations in assumption and methodology. In this article, we have used the Promethee II technique which is a complete outranking method. This method compare the alternatives pairwise for each criterion, finding the strength of preferring one over the other. This method considers the outranking flows for evaluating alternatives. The concept is built on pairwise comparison between alternatives, and calculates two outranking flows for each alternative, namely positive and negative outranking flow.

ii) TOPSIS (Technique of Preference by similarity to the ideal solution) is the goal, aspiration and reference level model. This technique measure how good alternatives reach determined goals and aspirations. TOPSIS' key principle is to choose the solution that has the shortest distance from the positive ideal solution and the farthest distance from the ideal

negative solution. To measure the relative closeness levels of alternatives to the positive and negative ideal solution, Euclidean distance access is used.

B. ADVANTAGES OF THE PROPOSED MODEL

In general, real-world MCDM and MCGDM problems arise in a complicated environment under uncertain and imprecise data, which is hard to address. The proposed technique is exceptionally appropriate for the scenario when the data is complex, vague, and uncertain. Especially, when the existing data is depending on the bipolar information by decision-makers. A few benefits of proposed techniques (Promethee and TOPSIS) are listed below:

- i) The proposed approach considers positive and negative aspects of each individual alternative in the form of a bipolar soft set. This hybrid model is more generalized and appropriate to deal with aggressive decision making.
- ii) Classical Promethee and TOPSIS techniques do not provide a clear framework for assigning the weights. But, our proposed techniques are effective in solving MCGDM problems when the weights information of criteria is completely unknown.
- iii) The proposed MCGDM technique is more effective for discrete data problems.
- iv) The proposed method takes into account not only the opinions of key decision-makers, but also previous experiences with bipolar soft covering approximations in actual scenarios. As a result, it is a more comprehensive method for better interpreting available information and, as a result, making decisions using artificial intelligence.
- v) The proposed MCGDM techniques are simple to comprehend and may be applied to decision making real life situations.

C. DISADVANTAGES OF THE PROPOSED TECHNIQUE

Some minor flaws are there in the proposed techniques which are discussed below:

- i) Although there are some differences among the optimal decision-making results (the optimal alternatives) and the ranking results determined by these two decision making methods, this phenomenon is normal in decision-making theory. Decision-makers can select a method according to actual requirements and their own interest.
- ii) These techniques have complicated structure, the large data in the form of bipolar information. Such large data is hard to deal with, due to massive calculations, which are not so natural to perform. However, one could create a MATLAB programming code to make these complicated calculations simpler.

D. COMPARISON WITH SOME EXISTING METHODS

There are several approaches in the literature that can be used to solve MCGDM problems. Each of these MCGDM techniques has its own set of advantages and disadvantages. The capability of every technique relies on the problem under consideration. In this section, we compare the proposed

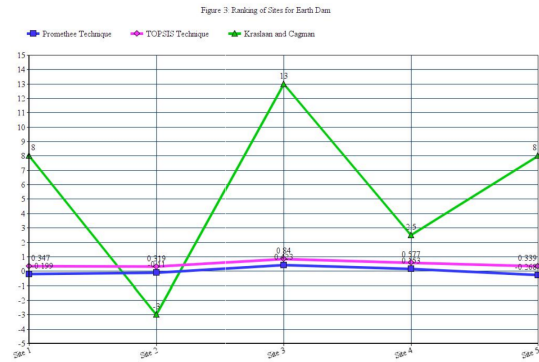


FIGURE 4. Ranking of sites for earth dam.

MCGDM technique to some current MCGDM techniques in fuzzy and bipolar fuzzy environments, and we discuss the significance of the proposed MCGDM strategies.

We talk about comparative analysis of proposed strategy with soft covering-based rough sets [83], fuzzy soft set [3], covering-based rough fuzzy set [43], picture fuzzy set [7], generalized hesitant fuzzy rough sets [63]. All these techniques have their own value in the literature. If we compare all these techniques with our proposed strategies, we investigate the following points.

- (i) The previously-mentioned techniques cannot catch bipolarity in decision making which is a fundamental aspect of human thinking and behavior.
- (ii) Besides, these techniques do not ensure harmony in the opinions of decision-makers.
- (iii) The models presented in [7], and [43] are well known for their ability to solve some decision making problems by describing the idea of decision-makers with a crisp number. They fail to handle some group decision making problems due to the uncertainty of the objective world and the complexity of the decision-making problems. For example, several experts disagree about the degree to which an element belongs to a set and cannot compromise one another. One prefers to assign 0.4, whereas the other prefers 0.6. In this situation, a rough set model based on bipolar soft covering could be an excellent solution.

(iv) When we compare our proposed result to the technique described in [4], we can see that the optimal alternative in this method is obtained simply by using the tabular form of bipolar soft sets, whereas the optimal alternative in our proposed model is obtained by using the bipolar soft covering based rough approximations.

(v) If we apply the recent approach proposed in [31] to our Example 5, we get the following ranking among the alternatives (shown in following Table) and the corresponding pictorial depiction is given in Figure 3.

VI. CONCLUSION AND FUTURE WORK

The rough set theory is arising as an incredible theory and has different applications in numerous fields. On the other hand, the bipolar soft sets are the appropriate mathematical

model to deal with the uncertainty as well as the bipolarity of the data. In this paper, we presented a general approach for the bipolar soft covering based rough bipolar soft set. Some algebraic properties of fuzzy bipolar soft covering approximations have been studied as well. We discussed a decision making problem with the information having uncertainty as well as bipolarity and applied the bipolar soft covering approximations to iron out this problem. In the real world, in a complex environment where competing systems of reasoning, ambiguous and imprecise knowledge must be taken into account, decision-making problems take place. Multi-criteria approaches for decision-making are used to face such uncertainty. PROMETHEE-II and TOPSIS are two of these processes. We have defined the process, methodology and significance of two well-known MCGDM methods in this research paper, namely, the PROMETHEE-II method and TOPSIS method by using bipolar soft covering based rough bipolar soft set. These approaches have been used to address site selection-related decision-making problems. This algorithm provides three key benefits over the present algorithms. Firstly, it manipulates the bipolarity of the data, endowed with uncertainty. Secondly, this algorithm accommodates the opinions of any (finite) number of decision-makers about any (finite) number of alternatives. Thirdly, with the best decision, it also yields the worst decision. Furthermore, a practical application demonstrates the validity of this methodology. Finally, a comparison analysis of the proposed model is performed.

There are several study topics that need to further exploration. Firstly, it is a potential topic to study some theoretical aspects on CB-BSRSs, such as attribute reductions [74], [75], granular structures [18], [74], and others. Secondly, the combination of CB-BSRSs with other important traditional MCDM methods [24], [26], [49] is also a promising research direction. We will investigate these topics in the future.

REFERENCES

- [1] J. C. R. Alcantud, "Revealed indifference and models of choice behavior," *J. Math. Psychol.*, vol. 46, no. 4, pp. 418–430, 2002.
- [2] E. Abo-Tabl, "On links between rough sets and digital topology," *Appl. Math.*, vol. 5, no. 6, pp. 941–948, 2014.
- [3] M. I. Ali, "A note on soft sets, rough soft sets and fuzzy soft sets," *Appl. Soft Comput.*, vol. 11, no. 4, pp. 3329–3332, Jun. 2011.
- [4] M. I. Ali, M. K. El-Bably, and E.-S.-A. Abo-Tabl, "Topological approach to generalized soft rough sets via near concepts," *Soft Comput.*, vol. 26, no. 2, pp. 499–509, Jan. 2022, doi: [10.1007/s00500-021-06456-z](https://doi.org/10.1007/s00500-021-06456-z).
- [5] S. Abdullah, M. Aslam, and K. Ullah, "Bipolar fuzzy soft sets and its applications in decision making problem," *J. Intell. Fuzzy Syst.*, vol. 27, no. 2, pp. 729–742, 2014.
- [6] A. Albadvi, S. K. Chaharsooghi, and A. Esfahanipour, "Decision making in stock trading: An application of promethee," *Eur. J. Oper. Res.*, vol. 177, no. 2, pp. 673–683, Mar. 2007.
- [7] S. Ashraf, T. Mahmood, S. Abdullah, and Q. Khan, "Different approaches to multi-criteria group decision making problems for picture fuzzy environment," *Bull. Brazilian Math. Soc.*, vol. 50, no. 2, pp. 373–397, Jun. 2019.
- [8] M. I. Ali, M. Shabir, and M. Naz, "Algebraic structures of soft sets associated with new operations," *Comput. Math. Appl.*, vol. 61, no. 9, pp. 2647–2654, May 2011.
- [9] J. Brans and P. Vincke, "A preference ranking organization method," *Magt. Sci.* vol. 31, no. 6, pp. 647–656, 1985.
- [10] R. Bilsel, G. Buyukozkan, and D. Ruan, "A fuzzy preference ranking model for a quality evaluation of hospital web sites," *Int. J. Intell. Syst.*, vol. 21, no. 11, pp. 1181–1197, 2006.
- [11] Z. Bonikowski, E. Bryniarski, and U. Wybraniec-Skardowska, "Extensions and intentions in the rough set theory," *Inf. Sci.*, vol. 107, nos. 1–4, pp. 149–167, Jun. 1998.
- [12] J. Brans, B. Mareschal, and P. Vincke, "PROMETHEE: A new family of outranking methods in MCDM," in *Operation Research*, J. Brans, Ed. Amsterdam, The Netherlands: North-Holland, 1984, pp. 477–490.
- [13] R. Benayoun and B. Roy, "Manual de reference du programme electre, note de synthese et formation 25," *Amer. J. Oper. Res.*, 1966.
- [14] C. Chen, "Extension of the TOPSIS for group decision-making under fuzzy environment," *Fuzzy Sets Syst.*, vol. 114, pp. 1–9, Oct. 2000.
- [15] I. Couso and D. Dubois, "Rough sets, coverings and incomplete information," *Fundam. Inf.* vol. 108, nos. 3–4, pp. 223–247, 2011.
- [16] T.-Y. Chen and C.-Y. Tsao, "The interval-valued fuzzy TOPSIS method and experimental analysis," *Fuzzy Sets Syst.*, vol. 159, no. 11, pp. 1410–1428, Jun. 2008.
- [17] N. Çağman, S. Enginoğlu, and F. Çitak, "Fuzzy soft set theory and its applications," *Iranian J. Fuzzy Syst.*, vol. 8, no. 3, pp. 137–147, 2011.
- [18] D. Chen, Y. Yang, and H. Wang, "Granular computing based on fuzzy similarity relations," *Soft Comput.*, vol. 15, pp. 1161–1172, May 2011.
- [19] L. D'eer and C. Cornelis, "A comprehensive study of fuzzy covering-based rough set models: Definitions, properties and interrelationships," *Fuzzy Sets Syst.*, vol. 336, pp. 1–26, Apr. 2018.
- [20] L. D'eer, C. Cornelis, and L. Godo, "Fuzzy neighborhood operators based on fuzzy coverings," *Fuzzy Sets Syst.*, vol. 312, pp. 17–35, Apr. 2017.
- [21] T. Deng, Y. Chen, W. Xu, and Q. Dai, "A novel approach to fuzzy rough sets based on a fuzzy covering," *Inf. Sci.*, vol. 177, no. 11, pp. 2308–2326, 2007.
- [22] D. Dubois and H. Prade, "Rough fuzzy sets and fuzzy rough sets," *Int. J. General Syst.*, vol. 17, nos. 2–3, pp. 191–209, 1990.
- [23] M. K. El-Bably, M. I. Ali, and E.-S.-A. Abo-Tabl, "New topological approaches to generalized soft rough approximations with medical applications," *J. Math.*, vol. 2021, pp. 1–16, Dec. 2021.
- [24] J. Harsanyi, "Cardinal welfare, individualistic ethics, and interpersonal comparisons of utility," *J. Political Econ.* vol. 63, pp. 309–321, Aug. 1955.
- [25] C. Huang, M. Lin, and Z. Xu, "Pythagorean fuzzy MULTIMOORA method based on distance measure and score function: Its application in multicriteria decision making process," *Knowl. Inf. Syst.*, vol. 62, no. 11, pp. 4373–4406, Nov. 2020.
- [26] C. Hwang, *Multiple Attributes Decision Making Methods and Applications*. Berlin, Germany: Springer, 1985.
- [27] F. Feng, C. Li, B. Davvaz, and M. Ali, "Soft sets combined with fuzzy sets and rough sets: A tentative approach," *Soft Comput.*, vol. 14, no. 9, pp. 899–911, 2010.
- [28] F. Feng, X. Liu, V. Leoreanu-Fotea, and Y. B. Jun, "Soft sets and soft rough sets," *Inf. Sci.*, vol. 181, no. 6, pp. 1125–1137, Mar. 2011.
- [29] T. Feng, S.-P. Zhang, and J.-S. Mi, "The reduction and fusion of fuzzy covering systems based on the evidence theory," *Int. J. Approx. Reasoning*, vol. 53, no. 1, pp. 87–103, 2012.
- [30] H. Jiang and J. Zhan, "Covering-based variable precision (I, T) -fuzzy rough sets with applications to multiattribute decision-making," *IEEE Trans. Fuzzy Syst.*, vol. 27, no. 8, pp. 1558–1572, Aug. 2019, doi: [10.1109/TFUZZ.2018.2883023](https://doi.org/10.1109/TFUZZ.2018.2883023).
- [31] F. Karaaslan and N. Çağman, "Bipolar soft rough sets and their applications in decision making," *Afrika Math.*, vol. 29, nos. 5–6, pp. 823–839, Sep. 2018.
- [32] F. Karaaslan and S. Karatas, "A new approach to bipolar soft sets and its applications," 2014, *arXiv:1406.2274*.
- [33] M. J. Khan, P. Kumam, P. Liu, W. Kumam, and S. Ashraf, "A novel approach to generalized intuitionistic fuzzy soft sets and its application in decision support system," *Mathematics*, vol. 7, no. 8, p. 742, Aug. 2019.
- [34] Q. Kong and Z. Wei, "Covering based fuzzy rough sets," *J. Intell. Fuzzy Syst.* vol. 26, pp. 2405–2411, Oct. 2015.
- [35] K. Lee, "Bipolar valued fuzzy sets and their basic operations," in *Proc. Int. Conf. Intell. Technol.*, Bangkok, Thailand, 2000, pp. 307–317.
- [36] M. Lin, C. Huang, Z. Xu, and R. Chen, "Evaluating IoT platforms using integrated probabilistic linguistic MCDM method," *IEEE Internet Things J.*, vol. 7, no. 11, pp. 11195–11208, Nov. 2020.

- [37] T. J. Li, Y. Leung, and W.-X. Zhang, "Generalized fuzzy rough approximation operators based on fuzzy coverings," *Int. J. Approx. Reason.*, vol. 48, no. 3, pp. 836–856, 2008.
- [38] J. Li, "Fuzzy approximation operators based on coverings," in *Proc. Joint Rough Set Symp.*, 2007, pp. 55–62.
- [39] J.-H. Li, C.-L. Mei, and Y.-J. Lv, "Incomplete decision contexts: Approximate concept construction, rule acquisition and knowledge reduction," *Int. J. Approx. Reasoning*, vol. 54, no. 1, pp. 149–165, 2013.
- [40] G. L. Liu and Y. Sai, "A comparison of two types of rough sets induced by coverings," *Int. J. Approx. Reasoning*, vol. 50, pp. 521–528, Mar. 2009.
- [41] Z. Li, N. Xie, and G. Wen, "Soft coverings and their parameter reductions," *Appl. Soft Comput.*, vol. 31, pp. 48–60, Jun. 2015.
- [42] M. Lin, Z. Xu, Y. Zhai, and Z. Yao, "Multi-attribute group decision-making under probabilistic uncertain linguistic environment," *J. Oper. Res. Soc.*, vol. 69, no. 2, pp. 157–170, Feb. 2018.
- [43] L. Ma, "On some types of neighborhood-related covering rough sets," *Int. J. Approx. Reason.*, vol. 53, pp. 901–911, Sep. 2012.
- [44] L. Ma, "Some twin approximation operators on covering approximation spaces," *Int. J. Approx. Reason.*, vol. 56, pp. 59–70, Jan. 2015.
- [45] L. Ma, "Two fuzzy covering rough set models and their generalizations over fuzzy lattices," *Fuzzy Sets Syst.*, vol. 294, pp. 1–17, Jul. 2016.
- [46] D. Molodtsov, "Soft set theory—first results," *Comput. Math. Appl.*, vol. 37, nos. 4–5, pp. 19–31, 1999.
- [47] P. K. Maji, R. Biswas, and A. R. Roy, "Soft set theory," *Comput. Math. Appl.*, vol. 45, pp. 555–562, Feb. 2003.
- [48] Z. M. Ma and B. Q. Hu, "Topological and lattice structures of -fuzzy rough sets determined by lower and upper sets," *Inf. Sci.*, vol. 218, pp. 194–204, Jan. 2013.
- [49] A. Mardani, A. Jusoh, and E. K. Zavadskas, "Fuzzy multiple criteria decision-making techniques and applications—two decades review from 1994 to 2014," *Expert Syst. Appl.*, vol. 42, no. 8, pp. 4126–4148, May 2015.
- [50] S. Murat, H. Kazan, and S. Coskun, "An application for measuring performance quality of schools by using the PROMETHEE multi criteria decision making method," *Proc. Soc. Behav. Sci.*, vol. 195, no. 1, pp. 729–738, 2015.
- [51] P. Majumdar and S. K. Samanta, "On soft mappings," *Comput. Math. Appl.*, vol. 60, no. 9, pp. 2666–2672, Nov. 2010.
- [52] N. Malik and M. Shabir, "A consensus model based on rough bipolar fuzzy approximations," *J. Intell. Fuzzy Syst.*, vol. 36, no. 4, pp. 3461–3470, Apr. 2019.
- [53] N. Malik and M. Shabir, "Rough fuzzy bipolar soft sets and application in decision-making problems," *Soft Comput.*, vol. 23, no. 5, pp. 1603–1614, Mar. 2019.
- [54] M. Naz and M. Shabir, "On fuzzy bipolar soft sets, their algebraic structures and applications," *J. Intell. Fuzzy Syst.*, vol. 26, no. 4, pp. 1645–1656, 2014.
- [55] X. Ma, Q. Liu, and J. Zhan, "A survey of decision making methods based on certain hybrid soft set models," *Artif. Intell. Rev.*, vol. 47, no. 4, pp. 507–530, 2017.
- [56] X. Ma, J. Zhan, M. I. N. Ali, and N. Mehmood, "A survey of decision making methods based on two classes of hybrid soft set models," *Artif. Intell. Rev.*, vol. 49, no. 4, pp. 511–529, 2018.
- [57] S. Opricovic, "A fuzzy compromise solution for multicriteria problems," *Int. J. Uncertainty, Fuzziness Knowl.-Based Syst.*, vol. 15, no. 3, pp. 363–380, Jun. 2007.
- [58] Z. Pawlak, "Rough sets," *Int. J. Comput. Inf. Sci.*, vol. 11, no. 5, pp. 341–356, Oct. 1982.
- [59] Z. Pawlak and A. Skowron, "Rudiments of rough sets," *Inf. Sci.*, vol. 177, no. 1, pp. 3–27, Jan. 2007.
- [60] J. A. Pomykala, "Approximation operations in approximation space," *Bull. Pol. Acad. Sci.*, vol. 35, nos. 9–10, pp. 653–662, 1987.
- [61] S. Roy and S. Bera, "Approximation of rough soft set and its application to lattice," *Fuzzy Inf. Eng.*, vol. 7, Sep. 2015, Art. no. 379387.
- [62] M. Shabir, M. Ali, and T. Shaheen, "Another approach to soft rough sets," *Knowl. Based Syst.*, vol. 40, pp. 72–80, Mar. 2013.
- [63] T. Shaheen, M. Ali, and M. Shabir, "Generalized hesitant fuzzy rough sets (GHFRS) and their application in risk analysis," *Soft Comput.*, vol. 24, no. 18, pp. 14005–14107, Sep. 2020.
- [64] B. Sun, X. Chen, L. Zhang, and W. Ma, "Three-way decision making approach to conflict analysis and resolution using probabilistic rough set over two universes," *Inf. Sci.*, vol. 507, pp. 809–822, Jan. 2020.
- [65] M. Shabir and M. Naz, "On bipolar soft sets," 2013, *arXiv:1303.1344*.
- [66] E. C. C. Tsang, C. Degang, and D. S. Yeung, "Approximations and reducts with covering generalized rough sets," *Int. J. Comput. Math. Appl.*, vol. 56, pp. 279–289, Jul. 2008.
- [67] G. Tuzkaya, C. Kahraman, and D. Özgen, "An integrated fuzzy multicriteria decision making methodology for material handling equipment selection problem and an application," *Expert Syst. Appl.*, vol. 37, no. 4, pp. 2853–2863, Apr. 2010.
- [68] H. Tlig and A. Rebai, "A TOPSIS method based on intuitionistic fuzzy values: A case study of north African airports," *Manage. Sci. Lett.*, vol. 7, pp. 351–358, Jan. 2017.
- [69] N. Tozlu, S. University, S. Yuksel, and T. H. Dizman, "A topological approach to soft covering approximation space," *Int. J. Math. Trends Technol.*, vol. 29, no. 1, pp. 33–38, Jan. 2016.
- [70] F. Ülengin, Y. Aker Topcu, and A. Ö. Ahin, "An integrated decision aid system for bosphorus water-crossing problem," *Eur. J. Oper. Res.*, vol. 134, no. 1, pp. 179–192, Oct. 2001.
- [71] B. Vahdani and H. Hadipour, "Extension of the ELECTRE method based on interval-valued fuzzy sets," *Soft Comput.*, vol. 15, no. 3, pp. 569–579, Mar. 2011.
- [72] M.-C. Wu and T.-Y. Chen, "The ELECTRE multicriteria analysis approach based on intuitionistic fuzzy sets," in *Proc. IEEE Int. Conf. Fuzzy Syst.*, Aug. 2009, pp. 1383–1388, doi: [10.1109/FUZZY.2009.5276880](https://doi.org/10.1109/FUZZY.2009.5276880).
- [73] C. Wang, D. Chen, B. Sun, and Q. Hu, "Communication between information systems with covering based rough sets," *Inf. Sci.*, vol. 216, pp. 17–33, Dec. 2012.
- [74] C. Y. Wang and B. Q. Hu, "Granular variable precision fuzzy rough sets with general fuzzy relations," *Fuzzy Sets Syst.*, vol. 275, pp. 39–57, Sep. 2015.
- [75] C. Wang, Y. Huang, M. Shao, and X. Fan, "Fuzzy rough set-based attribute reduction using distance measures," *Knowl.-Based Syst.*, vol. 164, pp. 205–212, Jan. 2019.
- [76] W. Xu, Q. Wang, and S. Luo, "Multigranulation fuzzy rough sets," *J. Intell. Fuzzy Syst.*, vol. 26, pp. 1323–1340, Jan. 2014.
- [77] W.-H. Xu and W.-X. Zhang, "Measuring roughness of generalized rough sets induced by a covering," *Fuzzy Sets Syst.*, vol. 158, pp. 2443–2455, Nov. 2007.
- [78] B. Yang and B. Q. Hu, "A fuzzy covering-based rough set model and its generalization over fuzzy lattice," *Inf. Sci.*, vol. 367, pp. 463–486, Nov. 2016.
- [79] B. Yang and B. Q. Hu, "On some types of fuzzy covering-based rough sets," *Fuzzy Sets Syst.*, vol. 312, pp. 36–65, Apr. 2017.
- [80] Y. Yao, "Relational interpretations of neighborhood operators and rough set approximation operators," *Inf. Sci.*, vol. 101, pp. 239–259, Oct. 1998.
- [81] Y. Yao, "Perspectives of granular computing," in *Proc. IEEE Int. Conf. Granular Comput.*, Jul. 2005, pp. 85–90.
- [82] Y. Yao and B. Yao, "Covering based rough sets approximations," *Inf. Sci.*, vol. 200, pp. 91–107, Oct. 2012.
- [83] S. Yüksel, Z. G. Ergül, and N. Tozlu, "Soft covering based rough sets and their application," *Sci. World J.*, vol. 2014, pp. 1–9, Dec. 2014.
- [84] L. A. Zadeh, "Fuzzy sets," *Inf. Control*, vol. 8, no. 3, pp. 338–353, Jun. 1965.
- [85] W. Zakowski, "Approximations in the (μ, π) space," *Demonstr. Math.*, vol. 16, pp. 761–769, May 1983.
- [86] J. Zhan and J. C. R. Alcantud, "A novel type of soft rough covering and its application to multicriteria group decision making," *Artif. Intell. Rev.*, vol. 52, no. 4, pp. 2381–2410, Dec. 2019, doi: [10.1007/s10462-018-9617-3](https://doi.org/10.1007/s10462-018-9617-3).
- [87] J. Zhan and J. C. R. Alcantud, "A survey of parameter reduction of soft sets and corresponding algorithms," *Artif. Intell. Rev.*, vol. 52, no. 3, pp. 1839–1872, Oct. 2019.
- [88] J. Zhan, M. I. Ali, and N. Mehmood, "On a novel uncertain soft set model: Z-soft fuzzy rough set model and corresponding decision making methods," *Appl. Soft Comput.*, vol. 56, pp. 446–457, Jul. 2017.
- [89] J. Zhan, Q. Liu, and T. Herawan, "A novel soft rough set: Soft rough hemirings and corresponding multicriteria group decision making," *Appl. Soft Comput.*, vol. 54, pp. 393–402, May 2017.
- [90] J. Zhan and W. Xu, "Two types of coverings based multigranulation rough fuzzy sets and applications to decision making," *Artif. Intell. Rev.*, vol. 53, no. 1, pp. 167–198, Jan. 2020, doi: [10.1007/s10462-018-9649-8](https://doi.org/10.1007/s10462-018-9649-8).
- [91] J. Zhan and Q. Wang, "Certain types of soft coverings based rough sets with applications," *Int. J. Mach. Learn. Cybern.*, vol. 10, no. 5, pp. 1065–1076, May 2019.

- [92] J. Zhan and K. Zhu, "Reviews on decision making methods based on (fuzzy) soft sets and rough soft sets," *J. Intell. Fuzzy Syst.*, vol. 29, pp. 1169–1176, Jan. 2015.
- [93] J. Zhan and K. Zhu, "A novel soft rough fuzzy set: Z-soft rough fuzzy ideals of hemirings and corresponding decision making," *Soft Comput.*, vol. 21, no. 8, pp. 1923–1936, Apr. 2017.
- [94] W.-R. Zhang, "Bipolar fuzzy sets and relations: A computational framework for cognitive modeling and multiagent decision analysis," in *Proc. 1st Int. Joint Conf. North Amer. Fuzzy Inf. Process. Soc.*, Dec. 1994, pp. 305–309.
- [95] X. Zhang, C. Mei, D. Chen, and J. Li, "Feature selection in mixed data: A method using a novel fuzzy rough set-based information entropy," *Pattern Recognit.*, vol. 56, pp. 1–15, Dec. 2016.
- [96] X. Zhang, "Fuzzy anti-grouped filters and fuzzy normal filters in pseudo-BCI algebras," *J. Intell. Fuzzy Syst.*, vol. 33, no. 3, pp. 1767–1774, Aug. 2017.
- [97] X. Zhang, D. Miao, C. Liu, and M. Le, "Constructive methods of rough approximation operators and multigranulation rough sets," *Knowl.-Based Syst.*, vol. 91, pp. 114–125, Jan. 2016.
- [98] L. Zhang and J. Zhan, "Fuzzy soft β -covering based fuzzy rough sets and corresponding decision making applications," *Int. J. Mach. Learn.* vol. 10, pp. 1487–1502, Jun. 2019.
- [99] J. Zhan, B. Sun, and J. C. R. Alcantud, "Covering based multigranulation(I,T)-fuzzy rough set models and applications in multi-attribute group decision-making," *Inf. Sci.*, vol. 476, pp. 290–318, Feb. 2019.
- [100] L. Zhang, J. Zhan, and Z. Xu, "Covering-based generalized IF rough sets with applications to multi-attribute decision-making," *Inf. Sci.*, vol. 478, pp. 275–302, Apr. 2019, doi: 10.1016/j.ins.2018.11.033.
- [101] K. Zhang, J. Zhan, W. Wu, and J. Alcantud, "Fuzzy β -covering based (I, T)-fuzzy rough set models and applications to multiattribute decision-making," *Comput. Ind. Eng.*, vol. 128, pp. 605–621, Feb. 2019.
- [102] K. Zhang, J. Zhan, and Y. Yao, "TOPSIS method based on a fuzzy covering approximation space: An application to biological nano-materials selection," *Inf. Sci.*, vol. 502, pp. 297–329, Oct. 2019.
- [103] Z. Zhao, "On some types of covering rough sets from topological points of view," *Int. J. Approx. Reson.*, vol. 68, pp. 1–14, Jan. 2016.
- [104] P. Zhu, "Covering rough set based on neighbourhood: An approach without using neighbourhoods," *Int. J. Approx. Reason.* vol. 52, pp. 461–472, Apr. 2011.
- [105] W. Zhu, "Generalized rough sets based on relations," *Inf. Sci.*, vol. 177, no. 22, pp. 4997–5011, 2007.
- [106] W. Zhu, "Topological approaches to covering rough sets," *Inf. Sci.*, vol. 177, pp. 1499–1508, Mar. 2007.
- [107] W. Zhu, "Relationships among basic concepts in coveringbased rough sets," *Inf. Sci.*, vol. 179, pp. 2478–2486, Jun. 2009.
- [108] W. Zhu and F.-Y. Wang, "Reduction and axiomization of covering generalized rough sets," *Inf. Sci.*, vol. 152, pp. 217–230, Jun. 2003.
- [109] W. Zhu, "On three types of covering-based rough sets," *IEEE Trans. Knowl. Data Eng.* vol. 19, no. 8, pp. 114–1131, Aug. 2007.



FAIZA TUFAIL received the M.Sc. degree in mathematics from the University of Gujrat, Gujarat, Pakistan, in 2016, and the M.Phil. degree in pure mathematics from Quaid-i-Azam University Islamabad, Islamabad, Pakistan, in 2018, where she is currently pursuing the Ph.D. degree with the Department of Mathematics. Her research interests include covering, fuzzy algebraic structures, soft sets, decision theory, and rough sets and their hybrid algebraic structures with their applications.



MUHAMMAD SHABIR received the M.Sc., M.Phil., and Ph.D. degrees in mathematics from Quaid-i-Azam University Islamabad, Islamabad, Pakistan, in 1981, 1984, and 1996, respectively. He is currently a Professor with the Department of Mathematics, Quaid-i-Azam University Islamabad. He has produced 16 Ph.D. and 118 M.Phil. students. He has published more than 100 research articles in international journals. He has also coauthored the book *Fuzzy Semirings With Applications to Automata Theory* (Springer, New York, NY, USA). His current research interests include fuzzy algebraic structures, and soft algebraic structures and their applications.



EL-SAYED A. ABO-TABL received the B.S. and M.S. degrees in mathematics and the Ph.D. degree in philosophy of science (mathematics) from the College of Science, Assiut University, in 1996, 2002, and 2008, respectively. He is currently working as a Professor and the Head of the Department of Mathematics, Qassim University, Al Mithnab, Saudi Arabia. He has published more than 30 research articles in international journals. His current research interests include topology, rough sets, Granular computing, and pure mathematics.

...