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Computing Antimagic Labeling of Lattically Designed Symmetric Networks

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ABSTRACT In this article, we address the super edge-antimagic total labeling of the hexagonal lattice $HTT_{m,n}$ and two non-isomorphic forms of prismatic lattice $PTT_{m,n}$. The aforementioned classes are symmetric lattices involving the finite chain of tripartite networks. Our article further closes with the summary, 3*D*- graphical illustrations and a practical example of our findings.

INDEX TERMS Super $(a, 0)$ edge-antimagic total labeling, star S_n, tripartite network, lattice.

I. INTRODUCTION

The antimagic labeling on networks is designed due to its vast applications in different branches of sciences, such as security plans, networking projects, robotics and interference free signal processing. The article in hands deals with the super $(a, 0)$ edge-antimagic labeling of the lattice networks involving finite chain of tripartite networks. Once we design the aforesaid labeling on the lattice networks, these labelings will serve as test ready for their usage in any security, industrial or networking project where the connection scheme being designed is similar. Recently in 2020, in [1], Kumar and Amit have discussed the prominent applications of antimagic network labeling in the mega industry of the crystallography.

A. DEFINITIONS AND PRELIMINARIES

Some useful definitions and preliminary results in the context of this article shall be discussed in this subsection. We will also mention some relevant study previously done in this field.

We define an ordered 2-tuple $G = (V(\Gamma), E(\Gamma))$ as a network with $V(\Gamma)$ as its vertex set and $E(\Gamma) \subseteq V(\Gamma) \times V(\Gamma)$ as its edge set. When we take the number of vertices in $\Gamma = |V(\Gamma)| = p$ and number of edges in $\Gamma = |E(\Gamma)| = q$, respectively, the network Γ is referred as a (p, q)-network. Throughout our discussion, we will consider simple and connected (p, q)-networks. We refer to [2] for further insight into the network related terminologies.

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A function or a correspondence is called labeling if it takes non-zero positive integers onto the components (vertices, edges or both) of Γ under particular constraints. If both sets of vertices and edges are included within the domain of the labeling function, it takes the terminology of total labeling. If the vertex set $V(\Gamma)$ or the edge set $E(\Gamma)$ covered within the domain merely, the labeling is termed as vertex or edge labeling respectively. The antimagic labeling is very prominent among various types of labelings. As per the definition, the distinct edge or vertex weights in a network point towards the antimagic type of labeling.

Definition 1: A bijection δ form $V(\Gamma) \cup E(\Gamma)$ onto $\{1, 2, \ldots, p + q\}$ is termed as (a, d) edge-antimagic total labeling on Γ under an attribute that the edge-weights $\delta(\alpha) + \delta(\alpha\beta) + \delta(\beta)$, for each edge $\alpha\beta$ within the network Γ, generates a sequence of positive integers which are consecutive. Where a is minimum among all the edge-weights and d is the common difference. Further, when such a labeling exists for a network Γ, it is termed as (a, d) edge-antimagic total network.

Definition 2: When minimum positive labels $1, 2, \ldots, p$ are allocated to the vertices of the network Γ, the (a, d) edge-antimagic total labeling becomes a super (a, d) edge-antimagic total labeling. Whereas, Γ in this case is termed as a super (a, d) edge-antimagic total network.

In the definitions 1 & 2, at $d = 0$, the minimum edge weight a acts as a constant, denoted by c, $\forall$ edges $\alpha\beta \in \Gamma$. This constant c is termed to be the magic constant or magic sum for the network Γ.

Onwards in the article, the following abbreviations given in Table 1 shall be used.

TABLE 1. Abbreviations to be used onwards in this article.

Terminology	Abbreviation
(a, d) edge-antimagic total	$(a, d) - EAMT$
Super (a, d) edge-antimagic total	$S - (a, d) - EAMT$

Sadlácek, in 1963 [3], identified the notion of magic labeling, Motivated from the idea of magic squares in discrete mathematics. In [4], Hartsfield postulated the idea of antimagic labeling, as distinct vertex-sums for each vertex of the network Γ. In 1966 [5], Stewart further pointed out if vertex-sums constitute a set of consecutive integers, then magic labeling is referred as super magic. In the mid nineties, Ringel and A. Llado [6] established the concept of $(a, 0)-EAMT$ labeling. The study of particularly $(a, 0)$ edge-antimagic total labeling of a network Γ was brought into the light by A. Kotzig and A. Rosa [7] who gave it the terminology of magic valuation. Hikoe Enomoto *et al.*, [8] designated the minimum positive labels to the vertices and called an $(a, 0) - EAMT$ labeling as $S - (a, 0) - EAMT$ labeling. R. Simanjantuk *et al.*, later in the year 2000, identified the notion of $(a, d) - EAMT$ labeling [9].

The realm of $S - (a, 0) - EAMT$ labeling on trees (connected and acyclic networks) covers the following useful conjectures.

Conjecture 1: There exists an $(a, 0) - EAMT$ labeling $\forall$ trees [7].

Conjecture 2: There exists an $S - (a, d) - EAMT$ labeling $\forall$ trees [8].

Many classes of trees have been discussed by researchers to support the conjecture 2. A verification for trees having at most 17 vertices was provided by Lee and Shah [10] with the help of computer programming. The results found for stars, subdivided stars [11]–[15], W- trees [16]–[18], caterpillars [19], banana trees [20], subdivided caterpillars [21] and disjoint combination of books and trees [22] are noteworthy. Further related study can be seen in [23], [24] and [25]. In [8] Enomoto *et al.* proved $K_{m,n}$ to be $S - (a, 0) - EAMT$ if and only if m or n is 1. $K_{1,m} \cup K_{1,n}$ is proven to be $S - (a, 0) - EAMT$ if either $m = \kappa_1(n + 1)$ or $n = \kappa_2(m + 1)$ [26]. H. Enomoto *et al.* [8] proved that C_n is $S-(a, 0)-EAMT \Leftrightarrow n$ is odd. In [27], it is proven that $C_3 \cup C_n$ is $S-(a, 0)-EAMT$ if and only if $n \geq 6$ and n is even (also see [28]). In [29] Figueroa-Centeno *et al.* revealed that the prism, studied as cartesian product of $C_\imath$ and $P_\jmath$, is $S-(a, 0)-EAMT$ for every odd integer $\imath$ and for all positive integers $\jmath$. The following two lemmas are quite useful in the premises of $S-(a, 0)-EAMT$ networks.

Lemma 1 [29]: A $(\wp, \Im)$-network Γ is $S - (a, 0) - EAMT$ $\Leftrightarrow \exists$ a bijection $\delta : V(\Gamma) \rightarrow \{1, 2, \cdots, \wp\}$ such that the set M consisting of edge-sums, for all edges in the network, constitutes $\Im$ consecutive integers. As a result, δ extends to an $S - (a, 0) - EAMT$ labeling of the network Γ admitting magic constant $\Re = \wp + \Im + m$. Where m is the minimum element of the set M.

Lemma 2 [30]: If a $(\wp, \Im)$-network Γ is $S-(a, d)-EAMT$, then it is $S - (a - \Im + 1, 2) - EAMT$ always.

B. RESEARCH METHODOLOGY

• While proving our results, we will proceed as follows.

1) Label the vertices of the (p, q)-network only with the help of the labeling function $\delta : V(\Gamma) \rightarrow \{1, 2, \cdots, p\}$.
2) Ensure that the edge-sums $\delta(\alpha) + \delta(\beta)$ are consecutive integers, $\forall \alpha\beta \in E(\Gamma)$.
3) For $m = min\{\delta(\alpha) + \delta(\beta)\}$, δ extends to an $S - (a, 0) - EAMT$ labeling of the network Γ admitting magic constant $a = p + q + m$, (with reference to Lemma 1).
4) The function δ also extends to an $S - (a', 2) - EAMT$ labeling of the network Γ admitting the minimum edge weight $a' = a - q + 1$ (with reference to Lemma 2).

• Our choice in this article are lattice networks containing chain of tripartite networks. These lattical chains have not been discussed in the literature before. Therefore, discussion on their antimagic labeling makes a novel contribution in the area of networking and discrete mathematics.

C. APPLICATIONS OF NETWORK LABELING IN SCIENCES

1) SOFTWARE ENGINEERING

The role of antimagic labeling on networks in the field of software engineering has continuously been revolutionary. A few examples include making the repeated labels in data mining negligible, saving precious data from hackers attacks by designing security codes with the assistance of coding of data. Further, configurations in the development of latest versions of various softwares have considerably been enhanced by test ready and reference labels. To label the connected component in binary graphics to produce the raster form of a picture, the antimagic labeling based two-scan algorithms are also getting attention. Its use make the graphics look even clearer [31]. Furthermore, Optimization and functioning of the networks are crucial in network engineering that requires hardcore planning and network management from the core. Networking is primarily done in two forms i.e., wired and wireless. Although the wireless network is progressively taking a toll in the networking strata, but wired network is still very useful. On the other hand, the apparent increase in the use of the wireless networking is hard to deny and demands the application of robust tools, for instance antimagic or magic labeling of networks, to obtain more accuracy in the network engineering [32].

2) NETWORKING

We are living in an era where satellite communication, radio transmission, and use of mobile towers are common. However, each of these networks keep facing interferences that make the channel assignment a hard task to be fulfilled [33]. The unconstrained simultaneous network

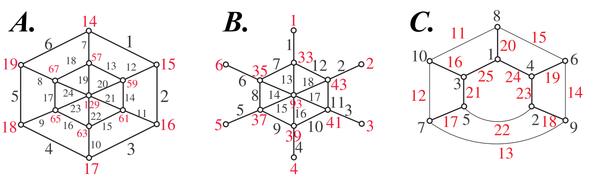

FIGURE 1. *(A)* **Edge antimagic labeling of wheel W_6;** *(B)* **Edge antimagic labeling of Helm H_6;** *(C)* **(29, 0)-Edge antimagic total labeling of the Prism D_5.**

transmission largely affects the quality of a voice call and create a background noise, even when highly sensitive communication equipment is used. The real reason of this unwanted interruption are the constrained free concurrent transmissions that admit same instance appearances [34]. This interruption is avoided by assigning different weights to distinct transmissions with the help of antimagic labeling, as this labeling produces distinct weights for all edges. In [36] and [37], the authors have discussed further applications of various labelings, including antimagic labeling, in the field of networking.

3) TELECOMMUNICATION

In the present era, the most successful commercial application of antimagic labeling can be seen in telecommunication engineering. In a cellular network, a service coverage area is divided into smaller quadrilateral or hexagonal areas, considered as a cell. Here each cell is working as a distinct station. The base cell has the ability to communicate with mobile stations such as cellular telephones, using its radio transceiver. Mobile switching center connects with another base station with the help of public switched telephone. To avoid a blocking, the challenge concerning channel assignment is to give maximum channel re-use without violating the constraints. In such type of cases, one can assign antimagic labels to each user, designated as vertex and their communication links designated by edges receiving distinct labels.

4) URBAN PLANNING

Let us consider a framework of urban planning as per the schemes shown in Figure 1. In this figure, the antimagic labelings on the networks wheel W_6, helm H_6 and prism D_5 can be observed (see [39], [40]).

In these graphical representations, rooms are represented by vertices and the weighted edges represent passages or routes to approach these rooms, where routes defined towards these rooms are the only legitimate ones. A complete disruption will occur with the violation of just a single route in the whole antimagic network. This disruption will work as an alarm for the concerned security to react instantly. Thus such antimagic networks, either vertex-antimagic or edge-antimagic, can serve as a specimen for the security design of any sensitive area of a building. In other words, these antimagic networks play their part as surveillance or security model for dissimilar types of buildings as well [41].

Further, for the assignment or earmarking of the resources and persons under certain constraints, the bipartite networks with antimagic labeling can be used. As another application, the antimagic bipartite trees provide us a straightforward interconnection of demand and supply of different quantities to build a business scheme between retailers and buyers [38]. The same kind of applications can be considered in the production factories and some modern restaurants where accurate robotic components play a very major role.

II. MAIN RESULTS

This section speaks about our main derivations. In subsection II-A, we define an $S-(a,0)-EAMT$ labeling on lattically symmetric networks namely hexagonal lattice $HTT_{m,n}$ and prismatic lattice $PTT_{m,n}$. Whereas, in Subsection II-B, we shall provide illustrative examples of the aforesaid networks. The Subsection II-C contains discussion on our findings which will lead us to the conclusion.

A. HEXAGONAL LATTICE $HTT_{m,n}$ & PRISMATIC LATTICE $PTT_{m,n}$

In this subsection, we define two simple networks termed as hexagonal lattice, denoted by $HTT_{m,n}$ and prismatic lattice, denoted by $PTT_{m,n}$.

Definition 3: We define a tripartite network Γ_n as follows for odd $n \geq 3$.

$$V(\Gamma_n) = \{x_i : 1 \leq i \leq n\} \cup \{c\}. \tag{1}$$

$$E(\Gamma_n) = \{cx_i : 1 \leq i \leq n\} \cup \{x_i x_{\frac{n+1}{2}} : 1 \leq i \leq n \text{ and } i \neq \frac{n+1}{2}\}. \tag{2}$$

Both $HTT_{m,n}$ and $PTT_{m,n}$ contain m copies of the tripartite network Γ_n.

Definition 4: The hexagonal lattice $HTT_{m,n}$ is a simple network of order $m(n+1)$ and size $2m(n+1)-3$, having vertex and edge sets for $3 \leq n \equiv 1 \pmod 2$ and for all m as follows:

- For $n \equiv 1 \pmod 4$ & $n \geq 5$: (3) and (4), as shown at the bottom of the next page.

Figure 2 shows the formation of the hexagonal lattice $HTT_{m,n}$.

- For $n \equiv 3 \pmod 4$ & $n \geq 3$: (5) and (6), as shown at the bottom of the page 5.

Theorem 1: The hexagonal lattice $HTT_{m,n}$ admits an $S-(3m(n+1),0)-EAMT$ labeling for every positive integer m and odd n.

Proof: The order and size of the network $HTT_{m,n}$ are $p = m(n+1)$ and $q = 2m(n+1)-3$, respectively.

- For $n \equiv 1 \pmod 4$:

The labeling $f : V(HTT_{m,n}) \rightarrow \{1, 2, \ldots, m(n+1)\}$ defined as (7)–(9), shown at the bottom of the page 7.

f extends to an $S-(a,0)-EAMT$ labeling of $HTT_{m,n}$ (using Lemma 1) with magic sum $a = 3m(n+1)$, where edge- sums being constituted as per the given scheme produce a consecutive sequence of integers $3, 4, 5, \ldots, 2m(n+1)-1$.

- For $n \equiv 3 \pmod 4$:

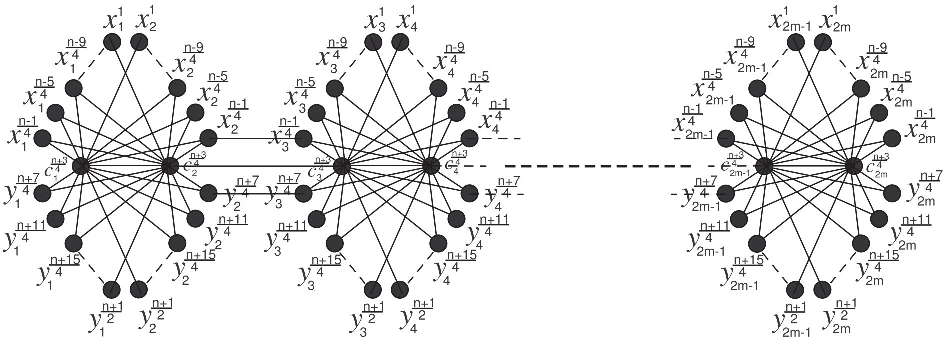

FIGURE 2. The hexagonal lattice $HTT_{m,n}$.

We define a labeling $g : V(HTT_{m,n}) \rightarrow \{1, 2, \ldots, m(n+1)\}$ as (10)–(14), shown at the bottom of the page 8. This labeling g extends to an $S-(a,0)-EAMT$ labeling of $HTT_{m,n}$ applying Lemma 1 with magic sum $a = 3m(n+1)$, where edge- sums being constituted as per the given scheme produce a consecutive sequence of integers $3, 4, 5, \ldots, 2m(n+1)-1$.

Definition 5: The prismatic lattice $PTT_{m,n}$ is a simple network of order $m(n+5)-4$ and size $2m(n+5)-11$, having vertex and edge sets for $3 \leq n \equiv 1 \pmod 2$ and for all m as follows:

- For $n \equiv 1 \pmod 4$ & $n \geq 5$: (15) and (16), as shown at the bottom of the page 8.

See the formation of the prismatic lattice $PTT_{m,n}$ in Figure 3.

- For $n \equiv 3 \pmod 4$ & $n \geq 3$: (17) and (18), as shown at the bottom of the page 9.

Theorem 2: The prismatic lattice $PTT_{m,n}$ admits an $S-(3m(n+5)-12, 0)-EAMT$ labeling for every positive integer m and odd n.

Proof: The order and size of the network $PTT_{m,n}$ are $p = m(n+5)-4$ and $q = 2m(n+5)-11$, respectively.

- For $n \equiv 1 \pmod 4$:

We define a labeling $f : V(PTT_{m,n}) \rightarrow \{1, 2, \ldots, m(n+5)-4\}$ defined as (19)–(23), shown at the bottom of the page 10. This labeling f extends to an $S-(a,0)-EAMT$ labeling of $PTT_{m,n}$ (using Lemma 1) with magic sum $a = 3m(n+5)-12$, where edge- sums being constituted as per the given scheme produce a consecutive sequence of integers $3, 4, 5, \ldots, 2m(n+5)-9$.

$$
\begin{aligned}
V(HTT_{m,n}) = {} & \{x_i^l : 1 \leq i \leq 2m,\ 1 \leq l \leq \frac{n-1}{4}\} \\
& \cup \{c_i^{\frac{n+3}{4}} : 1 \leq i \leq 2m\} \\
& \cup \{y_i^l : 1 \leq i \leq 2m,\ \frac{n+7}{4} \leq l \leq \frac{n+1}{2}\}
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
E(HTT_{m,n}) = {} & \{c_i^{\frac{n+3}{4}} y_{i+1}^l : 1 \leq i \leq 2m-1,\ i \equiv 1 \pmod 2,\ \frac{n+7}{4} \leq l \leq \frac{n+1}{2}\} \\
& \cup \{c_i^{\frac{n+3}{4}} y_i^l : 1 \leq i \leq 2m-1,\ i \equiv 1 \pmod 2,\ \frac{n+7}{4} \leq l \leq \frac{n+1}{2}\} \\
& \cup \{c_i^{\frac{n+3}{4}} y_{i-1}^l : 2 \leq i \leq 2m,\ i \equiv 0 \pmod 2,\ \frac{n+7}{4} \leq l \leq \frac{n+1}{2}\} \\
& \cup \{c_i^{\frac{n+3}{4}} y_i^l : 2 \leq i \leq 2m,\ i \equiv 0 \pmod 2,\ \frac{n+7}{4} \leq l \leq \frac{n+1}{2}\} \\
& \cup \{c_i^{\frac{n+3}{4}} x_{i+1}^l : 1 \leq i \leq 2m-1,\ i \equiv 1 \pmod 2,\ 1 \leq l \leq \frac{n-1}{4}\} \\
& \cup \{c_i^{\frac{n+3}{4}} x_i^l : 1 \leq i \leq 2m-1,\ i \equiv 1 \pmod 2,\ 1 \leq l \leq \frac{n-1}{4}\} \\
& \cup \{c_i^{\frac{n+3}{4}} x_{i-1}^l : 2 \leq i \leq 2m,\ i \equiv 0 \pmod 2,\ 1 \leq l \leq \frac{n-1}{4}\} \\
& \cup \{c_i^{\frac{n+3}{4}} x_i^l : 2 \leq i \leq 2m,\ i \equiv 0 \pmod 2,\ 1 \leq l \leq \frac{n-1}{4}\} \\
& \cup \{x_i^{\frac{n-1}{4}} x_{i+1}^{\frac{n-1}{4}},\ y_i^{\frac{n+7}{4}} y_{i+1}^{\frac{n+7}{4}} : 2 \leq i \leq 2(m-1),\ i \equiv 0 \pmod 2\} \\
& \cup \{c_i^{\frac{n+3}{4}} c_{i+1}^{\frac{n+3}{4}} : 1 \leq i \leq 2m-1\}
\end{aligned} \tag{4}
$$

• For $n \equiv 3 \pmod 4$:

We define a bijection $g : V(PTT_{m,n}) \rightarrow \{1, 2, \ldots, m(n+5)-4\}$ defined as (24)–(30), shown at the bottom of the page 10. This labeling g extends to an $S-(a,0)-EAMT$ labeling of $PTT_{m,n}$ by Lemma 1, having magic sum $a = 3m(n+5)-12$, where edge- sums being constituted as per the given scheme produce a consecutive sequence of integers $3, 4, 5, \ldots, 2m(n+5)-9$.

We now define another form of $PTT_{m,n}$, non-isomorphic to the network's from given in Definition 5. To avoid ambiguity, we denote this form of the prismatic lattice by $\Lambda_{m,n}$. In Definition 6, $V(PTT_{m,n})$ and $E(PTT_{m,n})$ will denote the vertex and edge sets respectively given in Definition 5.

Definition 6: The prismatic lattice $\Lambda_{m,n}$ is a simple network of order $m(n+5)-4$ and size $2m(n+5)-11$, having vertex and edge sets for $3 \leq n \equiv 1 \pmod 2$ and for all m as follows:

• For $n \equiv 1 \pmod 4$ & $n \geq 5$: (31) and (32), as shown at the top of the page 11.

See Figure 4 for the formation of the prismatic lattice $\Lambda_{m,n}$.

• For $n \equiv 3 \pmod 4$ & $n \geq 3$: (33) and (34), as shown at the top of the page 11.

Theorem 3: The prismatic lattice $\Lambda_{m,n}$ (non- isomorphic to $PTT_{m,n}$ given in Definition 5) admits an $S-(3m(n+5)-12, 0)-EAMT$ labeling for every positive integer m and odd n.

Proof: The labeling scheme is similar as designed in Theorem 2.

• The following results from Theorems 1, 2 and 3 are direct consequences of Lemma 2.

Theorem 4: The hexagonal lattice $HTT_{m,n}$ admits an $S-(m(n+1)+4, 2)-EAMT$ labeling for every positive integer m and odd n.

Theorem 5: The prismatic lattice networks $PTT_{m,n}$ and $\Lambda_{m,n}$ admit an $S-(m(n+5), 2)-EAMT$ labeling for every positive integer m and odd n.

B. ILLUSTRATION THROUGH EXAMPLES

An $S-(210, 0)-EAMT$ labeling of the network $HTT_{m,n}$ is being presented in Figure 5, corresponding to the parameters $m = 5$ and $n = 13$. Further, Figure 6 presents an $S-(300, 0)-EAMT$ labeling of $HTT_{m,n}$ corresponding to $m = 5$ and $n = 19$. Here, it can be observed that the value of the magic constant is perfectly as per our depiction in Theorem 1.

C. DISCUSSION

• In this article, we have discussed the $S-(a,0)-EAMT$ and $S-(a',2)-EAMT$ labelings of the hexagonal lattice $HTT_{m,n}$ and two non-isomorphic forms of the prismatic lattice network $PTT_{m,n}$. Following is Table 2 that briefs about the numerical results of our findings corresponding to the

$$
\begin{aligned}
V(HTT_{m,n}) = {} & \{x_i^l : 1 \leq i \leq 2m, 2 \leq l \leq \frac{n+1}{4}\} \cup \{c_i^{\frac{n+5}{4}} : 1 \leq i \leq 2m\} \\
& \cup \{y_i^l : 1 \leq i \leq 2m, \frac{n+9}{4} \leq l \leq \frac{n+1}{2}\} \cup \{u_i^1, v_i^{\frac{n+3}{2}} : 1 \leq i \leq m\}
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
E(HTT_{m,n}) = {} & \{c_i^{\frac{n+5}{4}} y_{i+1}^l : 1 \leq i \leq 2m-1, i \equiv 1 \pmod 2, \frac{n+9}{4} \leq l \leq \frac{n+1}{2}\} \\
& \cup \{c_i^{\frac{n+5}{4}} y_i^l : 1 \leq i \leq 2m-1, i \equiv 1 \pmod 2, \frac{n+9}{4} \leq l \leq \frac{n+1}{2}\} \\
& \cup \{c_i^{\frac{n+5}{4}} y_{i-1}^l : 2 \leq i \leq 2m, i \equiv 0 \pmod 2, \frac{n+9}{4} \leq l \leq \frac{n+1}{2}\} \\
& \cup \{c_i^{\frac{n+5}{4}} y_i^l : 2 \leq i \leq 2m, i \equiv 0 \pmod 2, \frac{n+9}{4} \leq l \leq \frac{n+1}{2}\} \\
& \cup \{c_i^{\frac{n+5}{4}} x_{i+1}^l : 1 \leq i \leq 2m-1, i \equiv 1 \pmod 2, 2 \leq l \leq \frac{n+1}{4}\} \\
& \cup \{c_i^{\frac{n+5}{4}} x_i^l : 1 \leq i \leq 2m-1, i \equiv 1 \pmod 2, 2 \leq l \leq \frac{n+1}{4}\} \\
& \cup \{c_i^{\frac{n+5}{4}} x_{i-1}^l : 2 \leq i \leq 2m, i \equiv 0 \pmod 2, 2 \leq l \leq \frac{n+1}{4}\} \\
& \cup \{c_i^{\frac{n+5}{4}} x_i^l : 2 \leq i \leq 2m, i \equiv 0 \pmod 2, 2 \leq l \leq \frac{n+1}{4}\} \\
& \cup \{x_i^{\frac{n+1}{4}} x_{i+1}^{\frac{n+1}{4}}, y_i^{\frac{n+9}{4}} y_{i+1}^{\frac{n+9}{4}} : 2 \leq i \leq 2(m-1), i \equiv 0 \pmod 2\} \\
& \cup \{c_i^{\frac{n+5}{4}} c_{i+1}^{\frac{n+5}{4}} : 1 \leq i \leq 2m-1\} \\
& \cup \{c_i^{\frac{n+5}{4}} u_{\frac{i+1}{2}}^1, c_i^{\frac{n+5}{4}} v_{\frac{i+1}{2}}^{\frac{n+3}{2}} : 1 \leq i \leq 2m-1, i \equiv 1 \pmod 2\} \\
& \cup \{c_i^{\frac{n+5}{4}} u_{\frac{i}{2}}^1, c_i^{\frac{n+5}{4}} v_{\frac{i}{2}}^{\frac{n+3}{2}} : 2 \leq i \leq 2m, i \equiv 0 \pmod 2\}
\end{aligned} \tag{6}
$$

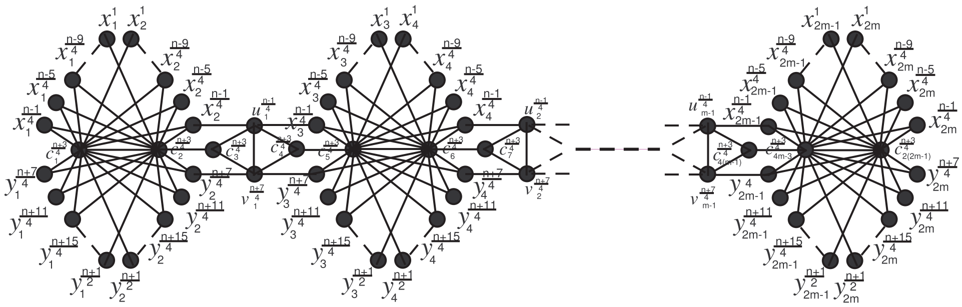

FIGURE 3. The prismatic lattice $PTT_{m,n}$.

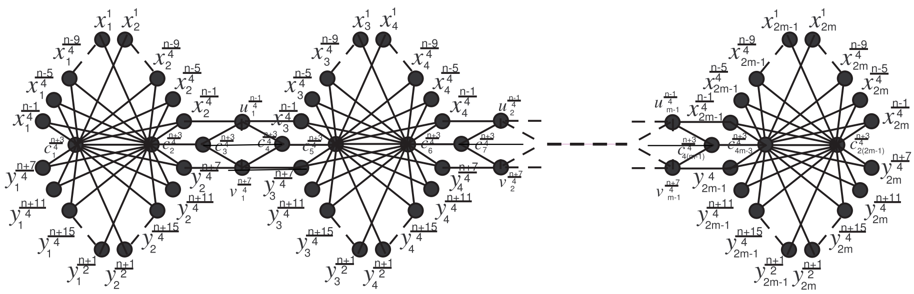

FIGURE 4. The prismatic lattice $\Lambda_{m,n}$.

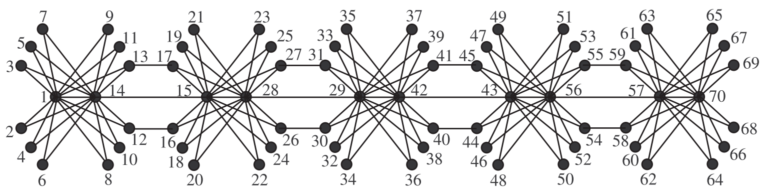

FIGURE 5. An $S-(210,0)-EAMT$ labeling of the network $HTT_{5,13}$.

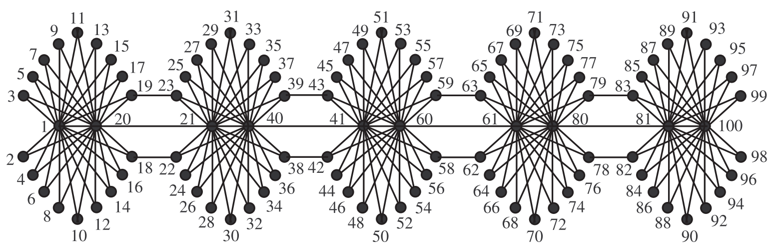

FIGURE 6. An $S-(300,0)-EAMT$ labeling of the network $HTT_{5,19}$.

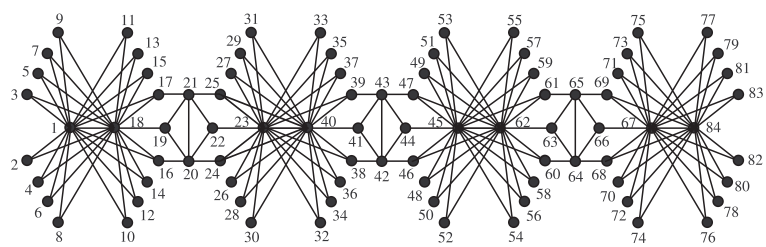

FIGURE 7. An $S-(252,0)-EAMT$ labeling of the network $PTT_{4,17}$.

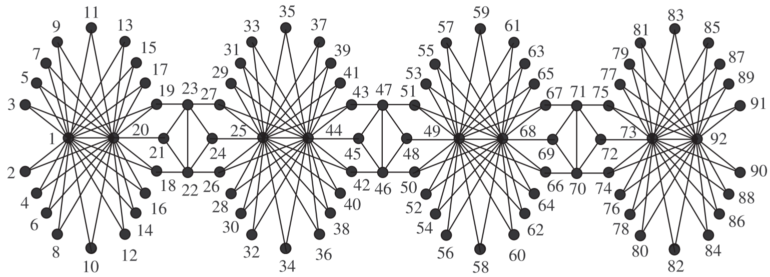

FIGURE 8. An $S-(276,0)-EAMT$ labeling of the network $PTT_{4,19}$.

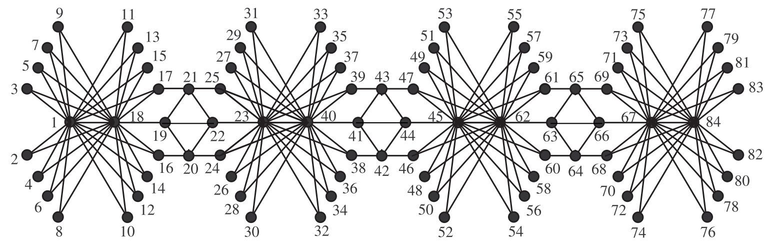

FIGURE 9. An $S-(252,0)-EAMT$ labeling of the network $\Lambda_{4,17}$.

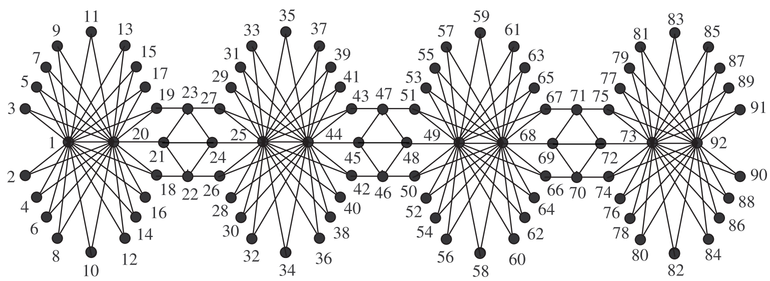

FIGURE 10. An $S-(276,0)-EAMT$ labeling of the network $\Lambda_{4,19}$.

parameters m and n. Here a represents the magic constant and a' represents the minimum edge weight of the labelings, as per standard notation.

Figures 7 and 8 illustrate Theorem 2 by providing $S-(252,0)-EAMT$ and $S-(276,0)-EAMT$ labeling of the prismatic lattice.

Similarly, Figures 9 and 10 are the illustrative examples of Theorem 3.

Graphically, the comparison of the magic constants and minimum edge weights of all three discussed lattices can be observed in Figure 11. Note that the lattices $PTT_{m,n}$ and $\Lambda_{m,n}$ have same graphical trends.

$$f(x_i^l) = \begin{cases} \frac{1}{2}(4(l-1)+i(n+1)) : & 1 \le i \le 2m-1,\ i \equiv 1 \pmod 2 \quad \text{and } 1 \le l \le \frac{n-1}{4} \\ \frac{1}{2}(4l-n-1+i(n+1)) : & 2 \le i \le 2m,\ i \equiv 0 \pmod 2 \quad \text{and } 1 \le l \le \frac{n-1}{4} \end{cases} \tag{7}$$

$$f(y_i^l) = \begin{cases} \frac{1}{2}(2(2l-n-2)+i(n+1)) : & 1 \le i \le 2m-1,\ i \equiv 1 \pmod 2 \quad \text{and } \frac{n+7}{4} \le l \le \frac{n+1}{2} \\ \frac{1}{2}(n-4l+3+i(n+1)) : & 2 \le i \le 2m,\ i \equiv 0 \pmod 2 \quad \text{and } \frac{n+7}{4} \le l \le \frac{n+1}{2} \end{cases} \tag{8}$$

$$f(c_i^{\frac{n+3}{4}}) = \begin{cases} \frac{1}{2}(i(n+1)-n+1) : & 1 \le i \le 2m-1 \quad \text{and } i \equiv 1 \pmod 2 \\ \frac{1}{2}(i(n+1)) : & 2 \le i \le 2m \quad \text{and } i \equiv 0 \pmod 2 \end{cases} \tag{9}$$

• In 2020, Xinqiang Ma *et al.*, have exhibited that stacked book graphs admit antimagic labeling [42]. These networks contain the chain of C_4, which is bipartite. Similarly in [43], Agustin *et al.*, exhibit antimagic labeling of ladder and ladder related networks, all such network classes again contain chain of bipartite network C_4. Here, we have presented three non-isomorphic forms of networks, thrice of them are antimagic and contain the chain of tripartite networks

$$g(x_i^l) = \begin{cases} \frac{1}{2}(2(3-2l)+i(n+1)): & 1 \leq i \leq 2m-1, i \equiv 1 \pmod 2 \quad \text{and } 2 \leq l \leq \frac{n+1}{4} \\ \frac{1}{2}(4l-n-3+i(n+1)): & 2 \leq i \leq 2m, i \equiv 0 \pmod 2 \quad \text{and } 2 \leq l \leq \frac{n+1}{4} \end{cases} \tag{10}$$

$$g(y_i^l) = \begin{cases} \frac{1}{2}(2(2l-n-3)+i(n+1)): & 1 \leq i \leq 2m-1, i \equiv 1 \pmod 2 \quad \text{and } \frac{n+9}{4} \leq l \leq \frac{n+1}{2} \\ \frac{1}{2}(n-4l+5+i(n+1)): & 2 \leq i \leq 2m, i \equiv 0 \pmod 2 \quad \text{and } \frac{n+9}{4} \leq l \leq \frac{n+1}{2} \end{cases} \tag{11}$$

$$g(c_i^{\frac{n+5}{4}}) = \begin{cases} \frac{1}{2}(i(n+1)-n+1): & 1 \leq i \leq 2m-1 \quad \text{and } i \equiv 1 \pmod 2 \\ \frac{1}{2}(i(n+1)): & 2 \leq i \leq 2m \quad \text{and } i \equiv 0 \pmod 2 \end{cases} \tag{12}$$

$$g(u_i^1) = \frac{1}{2}(2i(n+1)-n+1): \quad 1 \leq i \leq m \tag{13}$$

$$g(v_i^{\frac{n+3}{2}}) = \frac{1}{2}(2i(n+1)-n-1): \quad 1 \leq i \leq m \tag{14}$$

$$\begin{aligned} V(PTT_{m,n}) = &\{x_i^l : 1 \leq i \leq 2m, 1 \leq l \leq \frac{n-1}{4}\} \cup \{c_i^{\frac{n+3}{4}} : 1 \leq i \leq 2(2m-1)\} \\ &\cup \{y_i^l : 1 \leq i \leq 2m, \frac{n+7}{4} \leq l \leq \frac{n+1}{2}\} \cup \{u_i^{\frac{n-1}{4}}, v_i^{\frac{n+7}{4}} : 1 \leq i \leq m-1\} \end{aligned} \tag{15}$$

$$\begin{aligned} E(PTT_{m,n}) = &\{c_i^{\frac{n+3}{4}} y_{\frac{i+2}{2}}^l : 2 \leq i \leq 2(2m-1), i \equiv 2 \pmod 4, \frac{n+7}{4} \leq l \leq \frac{n+1}{2}\} \\ &\cup \{c_i^{\frac{n+3}{4}} y_{\frac{i}{2}}^l : 2 \leq i \leq 2(2m-1), i \equiv 2 \pmod 4, \frac{n+7}{4} \leq l \leq \frac{n+1}{2}\} \\ &\cup \{c_i^{\frac{n+3}{4}} y_{\frac{i+1}{2}}^l : 1 \leq i \leq 4m-3, i \equiv 1 \pmod 4, \frac{n+7}{4} \leq l \leq \frac{n+1}{2}\} \\ &\cup \{c_i^{\frac{n+3}{4}} y_{\frac{i+3}{2}}^l : 1 \leq i \leq 4m-3, i \equiv 1 \pmod 4, \frac{n+7}{4} \leq l \leq \frac{n+1}{2}\} \\ &\cup \{c_i^{\frac{n+3}{4}} x_{\frac{i+2}{2}}^l : 2 \leq i \leq 2(2m-1), i \equiv 2 \pmod 4, 1 \leq l \leq \frac{n-1}{4}\} \\ &\cup \{c_i^{\frac{n+3}{4}} x_{\frac{i}{2}}^l : 2 \leq i \leq 2(2m-1), i \equiv 2 \pmod 4, 1 \leq l \leq \frac{n-1}{4}\} \\ &\cup \{c_i^{\frac{n+3}{4}} x_{\frac{i+1}{2}}^l : 1 \leq i \leq 4m-3, i \equiv 1 \pmod 4, 1 \leq l \leq \frac{n-1}{4}\} \\ &\cup \{c_i^{\frac{n+3}{4}} x_{\frac{i+3}{2}}^l : 1 \leq i \leq 4m-3, i \equiv 1 \pmod 4, 1 \leq l \leq \frac{n-1}{4}\} \\ &\cup \{c_i^{\frac{n+3}{4}} c_{i+1}^{\frac{n+3}{4}} : 2 \leq i \leq 4(m-1), i \equiv 0 \pmod 4 \ \& \ i \equiv 2 \pmod 4\} \\ &\cup \{x_i^{\frac{n-1}{4}} u_{\frac{i}{2}}^{\frac{n-1}{4}}, y_i^{\frac{n+7}{4}} v_{\frac{i}{2}}^{\frac{n+7}{4}} : 2 \leq i \leq 2(m-1), i \equiv 0 \pmod 2\} \\ &\cup \{c_i^{\frac{n+3}{4}} c_{i+1}^{\frac{n+3}{4}} : 1 \leq i \leq 4m-3, i \equiv 1 \pmod 4\} \\ &\cup \{u_i^{\frac{n-1}{4}} x_{2i+1}^{\frac{n-1}{4}}, v_i^{\frac{n+7}{4}} y_{2i+1}^{\frac{n+7}{4}} : 1 \leq i \leq m-1\} \\ &\cup \{u_i^{\frac{n-1}{4}} c_{4i-1}^{\frac{n+3}{4}}, v_i^{\frac{n+7}{4}} c_{4i-1}^{\frac{n+3}{4}} : 1 \leq i \leq m-1\} \\ &\cup \{u_i^{\frac{n-1}{4}} c_{4i}^{\frac{n+3}{4}}, v_i^{\frac{n+7}{4}} c_{4i}^{\frac{n+3}{4}} : 1 \leq i \leq m-1\} \\ &\cup \{u_i^{\frac{n-1}{4}} v_i^{\frac{n+7}{4}} : 1 \leq i \leq m-1\} \end{aligned} \tag{16}$$

TABLE 2. The networks $HTT_{m,n}$, $PTT_{m,n}$, $\Lambda_{m,n}$ and the computed magic constants and minimum edge weights of their edge-antimagic total labelings.

Network	Parameters	a $(d=0)$	a' $(d=2)$
$HTT_{m,n}$	$\forall\, m$ & odd $n \geq 3$	$3m(n+1)$	$m(n+1)+4$
$PTT_{m,n}$	$\forall\, m$ & odd $n \geq 3$	$3m(n+5)-12$	$m(n+5)$
$\Lambda_{m,n}$	$\forall\, m$ & odd $n \geq 3$	$3m(n+5)-12$	$m(n+5)$

given in Definition 3. Hence, we have taken a step forward from antimagic bipartite chains to antimagic tripartite chains. Thus, we encourage the researchers to work towards the antimagic labeling of multipartite chains, as the scope of working on them is vast.

- To have a glance on the applications of our findings. Let us consider a software based automatic security design which is installed in the sensitive area of a building X. Also consider that this area of the building X is designed in the similar manner as the hexagonal lattice $HTT_{2,13}$ network. This area within the building is further divided into two parts $B1$ and $B2$. A legitimate passage has been

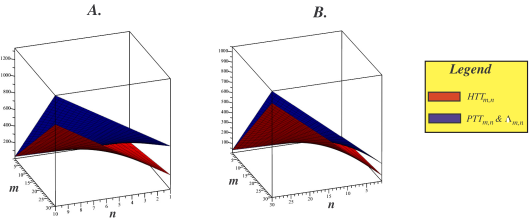

FIGURE 11. *(A)* Relative comparison of the magic constants of the networks $HTT_{m,n}$, $PTT_{m,n}$ and $\Lambda_{m,n}$ *(B)* Relative comparison of the minimum edge weights of the networks $HTT_{m,n}$, $PTT_{m,n}$ and $\Lambda_{m,n}$.

$$\begin{aligned}
V(PTT_{m,n}) = &\{x_i^l : 1 \leq i \leq 2m, 2 \leq l \leq \frac{n+1}{4}\} \cup \{c_i^{\frac{n+5}{4}} : 1 \leq i \leq 2(2m-1)\}\\
&\cup \{x_i^1, y_i^{\frac{n+3}{2}} : 1 \leq i \leq m\} \cup \{u_i^{\frac{n+1}{4}}, v_i^{\frac{n+9}{4}} : 1 \leq i \leq m-1\}\\
&\cup \{y_i^l : 1 \leq i \leq 2m, \frac{n+9}{4} \leq l \leq \frac{n+1}{2}\}
\end{aligned} \tag{17}$$

$$\begin{aligned}
E(PTT_{m,n}) = &\{c_i^{\frac{n+5}{4}} y^l_{\frac{i+2}{2}} : 2 \leq i \leq 2(2m-1), i \equiv 2 \ (\mathrm{mod}\ 4), \frac{n+9}{4} \leq l \leq \frac{n+1}{2}\}\\
&\cup \{c_i^{\frac{n+5}{4}} y^l_{\frac{i}{2}} : 2 \leq i \leq 2(2m-1), i \equiv 2 \ (\mathrm{mod}\ 4), \frac{n+9}{4} \leq l \leq \frac{n+1}{2}\}\\
&\cup \{c_i^{\frac{n+5}{4}} y^l_{\frac{i+1}{2}} : 1 \leq i \leq 4m-3, i \equiv 1 \ (\mathrm{mod}\ 4), \frac{n+9}{4} \leq l \leq \frac{n+1}{2}\}\\
&\cup \{c_i^{\frac{n+5}{4}} y^l_{\frac{i+3}{2}} : 1 \leq i \leq 4m-3, i \equiv 1 \ (\mathrm{mod}\ 4), \frac{n+9}{4} \leq l \leq \frac{n+1}{2}\}\\
&\cup \{c_i^{\frac{n+5}{4}} x^l_{\frac{i+2}{2}} : 2 \leq i \leq 2(2m-1), i \equiv 2 \ (\mathrm{mod}\ 4), 2 \leq l \leq \frac{n+1}{4}\}\\
&\cup \{c_i^{\frac{n+5}{4}} x^l_{\frac{i}{2}} : 2 \leq i \leq 2(2m-1), i \equiv 2 \ (\mathrm{mod}\ 4), 2 \leq l \leq \frac{n+1}{4}\}\\
&\cup \{c_i^{\frac{n+5}{4}} x^l_{\frac{i+1}{2}} : 1 \leq i \leq 4m-3, i \equiv 1 \ (\mathrm{mod}\ 4), 2 \leq l \leq \frac{n+1}{4}\}\\
&\cup \{c_i^{\frac{n+5}{4}} x^l_{\frac{i+3}{2}} : 1 \leq i \leq 4m-3, i \equiv 1 \ (\mathrm{mod}\ 4), 2 \leq l \leq \frac{n+1}{4}\}\\
&\cup \{c_i^{\frac{n+5}{4}} c_{i+1}^{\frac{n+5}{4}} : 2 \leq i \leq 4(m-1), i \equiv 0 \ (\mathrm{mod}\ 4) \ \& \ i \equiv 2 \ (\mathrm{mod}\ 4)\}\\
&\cup \{c_i^{\frac{n+5}{4}} c_{i+1}^{\frac{n+5}{4}} : 1 \leq i \leq 4m-3, i \equiv 1 \ (\mathrm{mod}\ 4)\}\\
&\cup \{x_i^{\frac{n+1}{4}} u_{\frac{i}{2}}^{\frac{n+1}{4}}, y_i^{\frac{n+9}{4}} v_{\frac{i}{2}}^{\frac{n+9}{4}} : 2 \leq i \leq 2(m-1), i \equiv 0 \ (\mathrm{mod}\ 2)\}\\
&\cup \{u_i^{\frac{n+1}{4}} x_{2i+1}^{\frac{n+1}{4}}, v_i^{\frac{n+9}{4}} y_{2i+1}^{\frac{n+9}{4}} : 1 \leq i \leq m-1\}\\
&\cup \{u_i^{\frac{n+1}{4}} c_{4i-1}^{\frac{n+5}{4}}, v_i^{\frac{n+9}{4}} c_{4i-1}^{\frac{n+5}{4}} : 1 \leq i \leq m-1\}\\
&\cup \{u_i^{\frac{n+1}{4}} c_{4i}^{\frac{n+5}{4}}, v_i^{\frac{n+9}{4}} c_{4i}^{\frac{n+5}{4}} : 1 \leq i \leq m-1\} \cup \{u_i^{\frac{n+1}{4}} v_i^{\frac{n+9}{4}} : 1 \leq i \leq m-1\}\\
&\cup \{c_i^{\frac{n+5}{4}} x^1_{\frac{i+3}{4}}, c_i^{\frac{n+5}{4}} y^{\frac{n+3}{2}}_{\frac{i+3}{4}} : 1 \leq i \leq 4m-3, i \equiv 1 \ (\mathrm{mod}\ 4)\}\\
&\cup \{c_i^{\frac{n+5}{4}} x^1_{\frac{i+2}{4}}, c_i^{\frac{n+5}{4}} y^{\frac{n+3}{2}}_{\frac{i+2}{4}} : 2 \leq i \leq 2(2m-1), i \equiv 2 \ (\mathrm{mod}\ 4)\}
\end{aligned} \tag{18}$$

allotted by the building security in order to reach from the entrance $E1$ to the highly secured room $R6$ and then to the exit $E2$. In order to ensure certain checks on the entering person, the legitimate passage assigned here is $\{E1 \rightarrow R1 \rightarrow R2 \rightarrow R3 \rightarrow R4 \rightarrow R5 \rightarrow R6 \rightarrow E2\}$ (indicated by green vertices and edges in Figure 12). This task can be achieved by using our test ready labels. Note that the corresponding particular network $HTT_{2,13}$ is $S-(84,0)-EAMT$. The magic constants corresponding to the legitimate passage is obtained as per the labels sequence $\{1+76+7, 7+63+14, 14+57+13, 13+54+17, 17+52+15, 15+42+27, 27+29+28\}$. As long as the entering person follow the legitimate passage protocol, no disruption will occur. And for instance, from $R2$ he moves towards $S4$, which is illegitimate, the allotted sequence will get totally disrupted. As it will become

$$f(x_i^l) = \begin{cases} \frac{1}{2}(i(n+5)-4l): & 1 \le i \le 2m-1, i \equiv 1 \pmod 2 \quad \text{and } 1 \le l \le \frac{n-1}{4} \\ \frac{1}{2}(i(n+5)+4l-n-9): & 2 \le i \le 2m, i \equiv 0 \pmod 2 \quad \text{and } 1 \le l \le \frac{n-1}{4} \end{cases} \tag{19}$$

$$f(y_i^l) = \begin{cases} \frac{1}{2}(i(n+5)+2(2l-n-4)): & 1 \le i \le 2m-1, i \equiv 1 \pmod 2 \quad \text{and } \frac{n+7}{4} \le l \le \frac{n+1}{2} \\ \frac{1}{2}(i(n+5)+n-4l-5): & 2 \le i \le 2m, i \equiv 0 \pmod 2 \quad \text{and } \frac{n+7}{4} \le l \le \frac{n+1}{2} \end{cases} \tag{20}$$

$$f(c_i^{\frac{n+3}{4}}) = \begin{cases} \frac{1}{4}(i(n+5)-n-1): & 1 \le i \le 4m-3 \quad \text{and } i \equiv 1 \pmod 4 \\ \frac{1}{4}(i(n+5)+2(n-3)): & 2 \le i \le 4m-2 \quad \text{and } i \equiv 2 \pmod 4 \\ \frac{1}{4}(i(n+5)+n-7): & 3 \le i \le 4m-5 \quad \text{and } i \equiv 3 \pmod 4 \\ \frac{1}{4}(i(n+5)): & 4 \le i \le 4(m-1) \quad \text{and } i \equiv 0 \pmod 4 \end{cases} \tag{21}$$

$$f(u_i^{\frac{n-1}{4}}) = i(n+5)-1: \quad 1 \le i \le m-1 \tag{22}$$

$$f(v_i^{\frac{n+7}{4}}) = i(n+5)-2: \quad 1 \le i \le m-1 \tag{23}$$

$$g(x_i^l) = \begin{cases} \frac{1}{2}(i(n+5)-2(2l-1)): & 1 \le i \le 2m-1, i \equiv 1 \pmod 2 \quad \text{and } 2 \le l \le \frac{n+1}{4} \\ \frac{1}{2}(i(n+5)+4l-n-11): & 2 \le i \le 2m, i \equiv 0 \pmod 2 \quad \text{and } 2 \le l \le \frac{n+1}{4} \end{cases} \tag{24}$$

$$g(y_i^l) = \begin{cases} \frac{1}{2}(i(n+5)+4l-2n-10): & 1 \le i \le 2m-1, i \equiv 1 \pmod 2 \quad \text{and } \frac{n+9}{4} \le l \le \frac{n+1}{2} \\ \frac{1}{2}(i(n+5)+n-4l-3): & 2 \le i \le 2m, i \equiv 0 \pmod 2 \quad \text{and } \frac{n+9}{4} \le l \le \frac{n+1}{2} \end{cases} \tag{25}$$

$$g(c_i^{\frac{n+5}{4}}) = \begin{cases} \frac{1}{4}(i(n+5)-n-1): & 1 \le i \le 4m-3 \quad \text{and } i \equiv 1 \pmod 4 \\ \frac{1}{4}(i(n+5)+2(n-3)): & 2 \le i \le 4m-2 \quad \text{and } i \equiv 2 \pmod 4 \\ \frac{1}{4}(i(n+5)+n-7): & 3 \le i \le 4m-5 \quad \text{and } i \equiv 3 \pmod 4 \\ \frac{1}{4}(i(n+5)): & 4 \le i \le 4(m-1) \quad \text{and } i \equiv 0 \pmod 4 \end{cases} \tag{26}$$

$$g(u_i^{\frac{n+1}{4}}) = i(n+5)-1: \quad 1 \le i \le m-1 \tag{27}$$

$$g(v_i^{\frac{n+9}{4}}) = i(n+5)-2: \quad 1 \le i \le m-1 \tag{28}$$

$$g(x_i^1) = \frac{1}{2}(2i(n+5)-n-7): \quad 1 \le i \le m \tag{29}$$

$$g(y_i^{\frac{n+3}{2}}) = \frac{1}{2}(2i(n+5)-n-9): \quad 1 \le i \le m \tag{30}$$

$$V(\Lambda_{m,n}) = V(PTT_{m,n}) \tag{31}$$

$$\begin{aligned} E(\Lambda_{m,n}) = &\ E(PTT_{m,n}) \cup \{c_i^{\frac{n+3}{4}} c_{i+1}^{\frac{n+3}{4}} : 1 \leq i \leq 4m-3\} \\ &- \{c_i^{\frac{n+3}{4}} c_{i+1}^{\frac{n+3}{4}} : 1 \leq i \leq 4m-3, i \equiv 1 \ (\mathrm{mod}\ 4)\} \\ &- \{c_i^{\frac{n+3}{4}} c_{i+1}^{\frac{n+3}{4}} : 2 \leq i \leq 4(m-1), i \equiv 0 \ (\mathrm{mod}\ 4) \ \&\ i \equiv 2 \ (\mathrm{mod}\ 4)\} \\ &- \{u_i^{\frac{n-1}{4}} v_i^{\frac{n+7}{4}} : 1 \leq i \leq m-1\} \end{aligned} \tag{32}$$

$$V(\Lambda_{m,n}) = V(PTT_{m,n}) \tag{33}$$

$$\begin{aligned} E(\Lambda_{m,n}) = &\ E(PTT_{m,n}) \cup \{c_i^{\frac{n+5}{4}} c_{i+1}^{\frac{n+5}{4}} : 1 \leq i \leq 4m-3\} \\ &- \{c_i^{\frac{n+5}{4}} c_{i+1}^{\frac{n+5}{4}} : 2 \leq i \leq 4(m-1), i \equiv 0 \ (\mathrm{mod}\ 4) \ \&\ i \equiv 2 \ (\mathrm{mod}\ 4)\} \\ &- \{c_i^{\frac{n+5}{4}} c_{i+1}^{\frac{n+5}{4}} : 1 \leq i \leq 4m-3, i \equiv 1 \ (\mathrm{mod}\ 4)\} - \{u_i^{\frac{n-1}{4}} v_i^{\frac{n+7}{4}} : 1 \leq i \leq m-1\} \end{aligned} \tag{34}$$

then $\{1 + 76 + 7, 14 + 58 + 12 \ \times \times\}$. This disturbance will promptly make the alarm to start ring, indicating the security breach to the manual and computer based security. This will promptly shut the concerned gates attached in the network as well. See Figure 12 below.

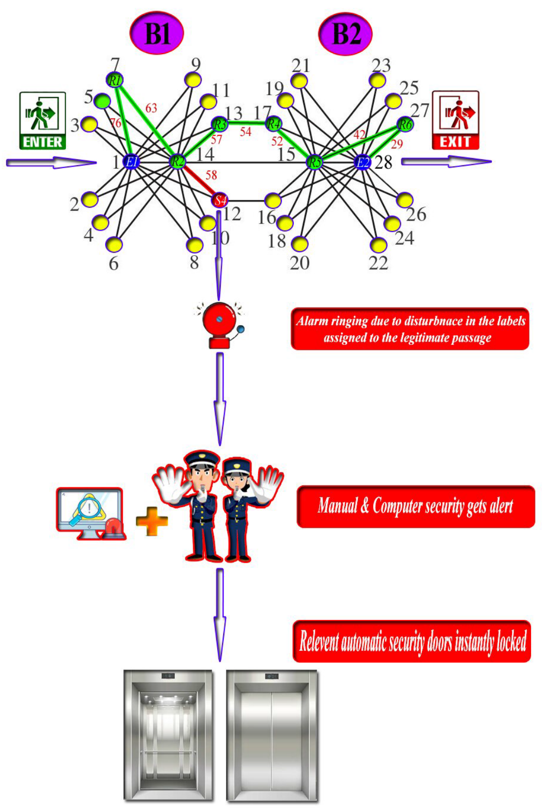

FIGURE 12. **Antimagic labeling based security plan of a sensitive building *X*.**

The same application can be used in forming a local area network or for a production machine that make different types of glass designs at different times. Our test ready labels are general and are applicable in all such practical situations.

III. CONCLUSION

In the present research article, we have successfully obtained an $S-(a,0)-EAMT$ and $S-(a',2)-EAMT$ labeling of symmetric classes of networks termed as hexagonal lattice $HTT_{m,n}$ and prismatic lattice $PTT_{m,n}$. Both of these classes contain chain of tripartite networks. Both the antimagic labeling are general and are applicable in many networking related practical scenarios (as mentioned in the Subsection II-C). Therefore, we encourage and motivate the researchers to work towards the antimagic labeling of multipartite chains for different values of the minimum edge weights, as computer related applicability and the scope of working on such network families is enormous.

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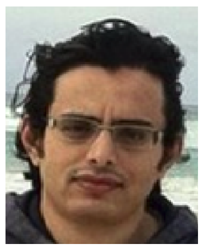

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