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An Area-Based Metrics to Evaluate Risk in Failure Mode and Effects Analysis Under Uncertainties

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ABSTRACT Failure mode and effects analysis (FMEA) is a widely used, powerful tool to identify and assess potential failure modes in products and to make products more reliable. Due to the complexity of products and lack of knowledge, FMEA involves many uncertainties in practice. In previous studies, numerous modified FMEA methods based on fuzzy logic and Dempster-Shafer (D-S) evidence theory have been employed to address these uncertainties. These studies focus on how to handle uncertainties and to identify a more reliable prioritization of risk priority numbers (RPNs). However, studies have not sufficiently examined how many uncertainties are present in resulting RPNs. To better model and process various types of uncertainties in FMEA, two new area-based metrics are constructed in this paper. One is the interval area metric (IAM), which is used in RPN representation. The other is the dimensionless uncertainty metric (DUM), which is used to measure how many uncertainties there are in RPN. IAM is used to rank the risks in failure modes, and DUM is used to rank the uncertainties in failure modes. Then, an expert system is presented to qualitatively evaluate the DUM, which can help FMEA users intuitively judge whether further investigation should be performed to alleviate the epistemic uncertainties in each failure mode. Finally, a practical risk evaluation case regarding the grinding wheel system of a numerically controlled (NC) machine is provided to demonstrate the application and effectiveness of the proposed FMEA. The case study shows that the calculation programs of IAM and DUM do not require any assumptions or need to address conflict among experts. In addition, proposed method can not only give a more accurate rating of each failure mode, but also help designers intuitively see the uncertainty grade of each RPN, which is useful to help them understand FMEA results.

INDEX TERMS Failure mode and effects analysis, D-S evidence theory, area-based metrics, epistemic uncertainty metric, risk priority number.

I. INTRODUCTION

Failure mode and effects analysis (FMEA) is a systematic procedure for analyzing a system/process to identify potential failure modes, their causes and their effects on system performance [1]. In engineering practice, F-FMEA (Function FMEA) is applied in the concept phase, D-FMEA (Design FMEA) in the design phase, and P-FMEA (Process FMEA) in the manufacturing phase. A good FMEA can help analysts identify known or potential failure modes and their causes

and effects so that effective actions with respect to the failure modes can be carried out to improve product reliability. Generally, a complex product contains hundreds or thousands of failure modes. Only a small part of the identified failure modes need to be considered for corresponding prevention measures to be proposed. Therefore, identifying the most serious failure modes is vitally important. The risk priority number (RPN) is a powerful and widely used tool to prioritize identified failure modes. It is the product of the occurrence (O), severity (S) and detection (D) of a failure, namely,

$$RPN = O \times S \times D \quad (1)$$

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The three factors O , S and D are all evaluated using scores from 1 to 10 [1]; the higher the RPN of a failure mode is, the greater priority it has.

Since its introduction, FMEA has proven to be one of the most important early preventative initiatives in the conceptualization, design and manufacturing phases. In recent decades, it has been widely used in various fields, such as the electric power system [2], logistics system [3], shipbuilding [4], software [5], and aircraft [6] industries. However, despite its wide application, the conventional RPN method for failure mode prioritization has received some criticism [7]–[10]. One criticism is that it fails to address uncertainties in risk evaluation.

In real-life engineering, due to the complexity of products and lack of knowledge in certain aspects, FMEA users often have difficulty evaluating failure modes with precise values. Instead, imperfect or uncertain judgments are more common. More flexible representations, such as fuzzy logic, intervals, linguistic terms, and evidence structures, have been introduced into the FMEA process in recent years. If there are uncertainties in O , S and D , then there also have uncertainties in the result of RPN. Although above methods can handle uncertainties in judgments of failure modes, however, how many uncertainties are included in the RPN results has not attracted enough attention. For example, if the RPN of a failure mode (denoted as FM_1) is interval [102, 136], and another is [115, 125], then expected RPN value of FM_2 is bigger than FM_1 . This means that the risk in FM_2 is greater than that in FM_1 . However, in the worst case, the RPN value of FM_1 reaches 136, exceeding 125 of FM_1 . If interval is used to measure the uncertainty in each failure mode, then uncertainty in FM_1 is 34, and FM_2 is 10. This means that more analysis, simulation and even testing of FM_1 are needed to reduce the uncertainty of FM_1 , and obtain more accurate risk assessment results. This is just a simple example. For more complex uncertainty (such as evidence bodies, hesitant evidence bodies, etc.), we should not only pay attention to the size of the RPN converted into an accurate value, but also pay attention to how much uncertainty is in the RPN evaluation results. This is very important for product design improvement and test verification in the next stage.

With the model presented this study, experts can flexibly express their opinions with crisp values, evidence bodies, hesitant evidence bodies, and interval values. To better model and process uncertainty in risk analysis, two new area metrics are constructed. One is the interval area metric (IAM), which is used in RPN representation. The other is the dimensionless uncertainty metric (DUM), which is used to measure in the number of uncertainties in an RPN. IAM is used to rank the risks in failure modes, and DUM is used to rank the uncertainties in failure modes.

The rest of this paper is organized as follows. The related works are reviewed in Section 2. In Section 3, the basic concepts of D-S evidence theory and the interval algorithm are briefly introduced. In Section 4, a new risk prioritization model is proposed to calculate novel RPNs and their

uncertainties. In Section 5, a practical risk evaluation case regarding the grinding wheel system of a numerically controlled (NC) machine is provided to demonstrate the application and effectiveness of the proposed FMEA. Finally, the paper concludes in Section 6.

II. LITERATURE REVIEW

In general, there are two popular ways to model uncertainties: fuzzy logic and Dempster Shafer (D-S) evidence theory. So in this section, literature review will be discussed from these two aspects.

A. FUZZY LOGIC APPROACH IN FMEA

Fuzzy logic is the oldest and most widely discussed method for processing uncertainties in FMEA. Bowles and Peláez [11] represented O , S and D as members of a fuzzy set and evaluated RPNs with min-max inferencing. In [12], the three parameters were represented as members of a fuzzy set fuzzified by using appropriate membership functions and were evaluated in a fuzzy inference engine that used a well-defined rule base and fuzzy logic operations to determine the criticality/risk level of failure. Tay and Lim [13] used fuzzy rule interpolation and reduction techniques to design a new fuzzy RPN model. Renjith *et al.* [14] used fuzzy FMEA to prioritize the failure modes in LNG storage facilities, and Jong *et al.* [15] applied fuzzy FMEA to edible bird nest processing. Although fuzzy FMEA is effective, there remain two issues pertaining to the practical implementation of classical FIS_RPN models: 1) the fulfillment of the monotonicity property between the FIS_RPN score (output) and the risk factors (inputs) and 2) difficulty in obtaining a complete and monotone fuzzy rule base [16]. To solve these problems, Kerk *et al.* [16] proposed an analytical interval fuzzy inference system for risk evaluation and prioritization in FMEA. The intuitionistic fuzzy approach has been another popular approach in the last few years. This approach offers some advantages over earlier models, as it accounts for degrees of uncertainty in relationships among various criteria or options, specifically when relations cannot be expressed in definite numbers [17], [18]. To better model uncertainty and impreciseness in experts' opinions, double upper approximated rough number (DUARN) is proposed to improve the utilization the risk assessment information [19]. To express FMEA users' judgments more flexibly, interval-valued intuitionistic fuzzy sets (IVIFSs) are applied in FMEA to handle uncertainty, vagueness, and incomplete information. New models have been constructed by means of Complex Proportional Assessment (COPRAS) and analytic network process (ANP) [20], TOPSIS and entropy [21] and the IVIF-MULTIMOORA method [22]. In [23], a novel fuzzy rough number extended multi-criteria group decision-making (FR-MCGDM) strategy to determine a more rational rank of failure modes by integrating the fuzzy rough number, AHP (analytic hierarchy process), and VIKOR (Serbian: ViseKriterijumska Optimizacija I Kompromisno Resenje) is

studied. Additionally, some scholars have studied RPN evaluation under hesitant fuzzy linguistic environments [24], [25].

In addition to the above literature, Bian *et al.* [26] and Liu and Deng [27] proposed a new risk priority model based on D number. D numbers can process various types of uncertainties, such as imprecision, fuzziness, and ignorance, in the failure analysis process.

B. D-S EVIDENCE THEORY IN FMEA

D-S evidence theory is another extensively studied approach to uncertainty modeling in FMEA. Evidence theory is a convenient framework for modeling imperfections in data and for combining information. Crisp values, intervals, incomplete distributions, multiple probabilities, and P-boxes can all be handled in an evidence structure [28]. In evidence theory, uncertainty is divided into two parts: aleatory uncertainty and epistemic uncertainty [29], [30]. Aleatory uncertainty, also called inherent uncertainty and irreducible uncertainty, cannot be reduced even if there are enough samples. Epistemic uncertainty is also called reducible uncertainty and subjective uncertainty. In FMEA, as the evaluation information considered by experts is always subjective, there are many epistemic uncertainties. Chin *et al.* [31] presented FMEA using evidential reasoning (ER) to handle different types of information. The proposed FMEA was examined with an illustrative application to a fishing vessel and proved to be useful and practical. However, the ER method still has drawbacks. Du *et al.* [32] asserted that in the ER approach, the number of frames of discernment is 210 in FMEA, which heavily increases the computational load. To solve this problem, Du *et al.* [32] proposed an evidential downscaling method to greatly reduce the computational complexity in FMEA. Yang *et al.* proposed a modified evidence theory to deal with different opinions of multiple experts, multiple failure modes and three risk factors in RPN analysis of FMEA. In real-world practice, when there are highly conflicting opinions among experts, evidence cannot be fused by Dempster's combination rule. Addressing this problem, Su *et al.* [33] proposed a method to transform experts' opinions into uncertain opinions by univariate normal distribution, while Suo *et al.* [34] proposed an abnormality test and a weighted average method to alleviate the conflicts among experts' opinions. From another point of view, Certa *et al.* [35] used belief and plausibility distributions to synthesize interval-valued judgments without requiring an aggregation stage; consequently, input information was not forced to have nonempty intersections. Zheng and Tang [36] proposed a triangular distribution-based basic probability assignment (TDBPA) method to model and fuse the conflict risk level coming from different experts' assessments in the framework of evidence theory. To extend the evidence theory-based FMEA, the gray relational projection method (GRPM) was used to rank the risk priorities of failure modes [37]. Huang *et al.* applied evidence theory using the evidential downscaling method and belief entropy function in fuzzy FMEA [38]. Kalathil *et al.* [39] compared evidence theory-based and fuzzy FMEA by a case study applied to an

LNG storage facility. Qin *et al.* [40] presented a way to combine interval type-2 fuzzy sets (IT2FSs) with the evidential reasoning (ER) method.

C. SUMMARY

As discussed above, there have been many valuable research efforts in FMEA under uncertain environments. Previous studies have focused on how to handle these uncertainties and obtain a more reliable prioritization of RPNs. Nevertheless, previous studies have not sufficiently considered how many uncertainties are presented in the resulting RPNs. If one failure mode's RPN is larger than another's, while the former mode's amount of uncertainty is also larger, it is difficult to determine whether the risk in the former failure mode is greater than that in the latter. That is, if FMEA users can obtain more information about a product, then they can include more accurate evaluations in their judgments. In this situation, the uncertainty in the first failure mode can be reduced and a more accurate RPN, which may be smaller than that of the last failure mode, can be obtained. Therefore, measuring how many uncertainties there are in evaluated risks is very important. It can not only reflect the reliability of prioritization but also provide direction to reduce uncertainties in judgments of failure modes.

III. PRELIMINARIES

A. D-S EVIDENCE THEORY

D-S evidence theory is a convenient framework for modeling imperfections in data and for combining information. Due to its outstanding performance in uncertainty modeling and processing, D-S evidence theory has been widely used in various fields, such as reliability analysis [42], [43], decision-making [44], [45], pattern recognition [46], and fault diagnosis [47].

In this section, the basic notations of evidence theory are introduced, and the main concepts that are essential for understanding the rest of the paper are briefly discussed.

Definition 1 (BPA and Focal Elements): A basic measure in evidence theory is a basic probability assignment (BPA) [41]. Let Θ be a finite set of mutually exclusive and exhaustive hypotheses, called the frame of discernment. Let 2^Θ be the set of all subsets of Θ . For a given evidential event A , BPA is represented by $m(A)$, which defines a mapping of 2^Θ to the interval between 0 and 1, i.e., $m:2^\Theta \rightarrow [0, 1]$. $m(A)$ expresses the proportion of all relevant and available evidence that supports the claims that certain particular elements of Θ belong to set A and makes no additional claims about any subsets of A . The value of $m(A)$ only belongs to set A and makes no additional claims about any subsets of A . For example, if $B \subset A$, then $m(B)$ is another BPA. Generally, $m(A)$ must satisfy the following constraints:

$$\begin{cases} m(A) \geq 0 & \text{for all } A \in 2^\Theta \\ m(\emptyset) = 0 \\ \sum_{A \subset 2^\Theta} m(A) = 1 \end{cases} \quad (2)$$

Definition 2 (Dempster’s Rule of Combination): Dempster’s rule of combination [41] is an operation that plays a key role in evidence theory. BPAs introduced by several sources are aggregated using this rule to yield a global BPA that synthesizes the knowledge of different sources. Let m_1, m_2, \dots, m_n be n distinct BPAs to combine, and their corresponding focal elements are $A_i (i = 1, 2, \dots, n)$. Then, Dempster’s rule is defined as follows:

$$m(A) = \begin{cases} \frac{1}{1-K} \sum_{\substack{\sim \\ A_i=A}} \prod_{1 \leq i \leq n} m_i(A_i) & A \neq \emptyset \\ 0 & A = \emptyset \end{cases} \quad (3)$$

where

$$K = \sum_{\substack{\sim \\ A_i=\emptyset}} \prod_{1 \leq i \leq n} m_i(A_i) \quad (4)$$

Coefficient K represents the mass that the combination assigns to \emptyset and reflects the conflict among sources. If there are no conflicts in the evidence, K is assigned the minimal value of 0. If K arrives at its maximum value of 1, the evidence is completely conflicting and cannot be combined by Eq.(3). On most occasions, the range of the K value is $0 < K < 1$.

Definition 3 (Belief and Plausibility Functions): Θ denotes a frame of discernment, which is a nonempty set. If m is a BBA on Θ , then function $Bel:2^\Theta \rightarrow [0, 1]$ defined by

$$Bel(A) = \sum_{B \subseteq A} m(B) \quad (5)$$

$Bel(A)$ is a belief function, and function $Pl:2^\Theta \rightarrow [0, 1]$ defined by

$$Pl(A) = \sum_{B \sim A \neq \emptyset} m(B) \quad (6)$$

$Pl(A)$ is a plausibility function, where $A \in 2^\Theta$ and $A \neq \emptyset$.

These two functions can be derived from each other. For example, the belief function can be derived from the plausibility function as follows:

$$Bel(A) = 1 - Pl(\bar{A}) \quad (7)$$

The relationship between the belief and plausibility functions is

$$Bel(A) \leq Pl(A) \quad (8)$$

Eq.(4) shows that as a measure of “event A is true,” if $P(A)$ is the true value of the measure of set $\{A \text{ is true}\}$, then $Pl(A)$ is the upper bound of $P(A)$ and $Bel(A)$ is the lower bound. Thus,

$$Bel(A) \leq P(A) \leq Pl(A) \quad (9)$$

B. INTERVAL ALGORITHM

\mathbf{R} denotes a real number field. For two specified real numbers $\underline{a}, \bar{a} \in \mathbf{R}$ and $\underline{a} \leq \bar{a}$,

$$[a] = [\underline{a}, \bar{a}] = \{A : A \in \mathbf{R}, \underline{a} \leq A \leq \bar{a}\} \quad (10)$$

$[a]$ is called a bounded closed interval, and interval number or interval for short.

Let $\odot \in \{+, -, \cdot, /\}$ be the operation of intervals; then, we define the corresponding operations for intervals $[a]$ and $[b]$ by

$$[a] \odot [b] = \{A \odot B | A \in [a], B \in [b]\} \quad (11)$$

where $A, B \in \mathbf{R}$, and we assume $0 \notin [b]$ in the case of division.

When \odot is one of the symbols “+,” “-,” “ \cdot ” or “/,” the following rules hold [48]:

$$[a] + [b] = [\underline{a} + \underline{b}, \bar{a} + \bar{b}] \quad (12)$$

$$[a] - [b] = [\underline{a} - \bar{b}, \bar{a} - \underline{b}] \quad (13)$$

$$[a] \cdot [b] = [\min\{\underline{a}\underline{b}, \underline{a}\bar{b}, \bar{a}\underline{b}, \bar{a}\bar{b}\}, \max\{\underline{a}\underline{b}, \underline{a}\bar{b}, \bar{a}\underline{b}, \bar{a}\bar{b}\}] \quad (14)$$

If we define

$$\frac{1}{b} = \left\{ \frac{1}{B} | B \in [b] \right\} \text{ if } 0 \notin [b] \quad (15)$$

where $B \in \mathbf{R}$, Then

$$[a]/[b] = [a] \cdot \frac{1}{[b]} \quad (16)$$

If a is a constant value, it can also be expressed as special interval $[a] = [a, a]$.

IV. PROPOSED RISK PRIORITIZATION MODEL

A. FMEA USERS’ ASSESSMENTS

Due to various subjective and objective conditions, such as lack of knowledge, lack of familiarity with products, and inadequate historical data, there are many epistemic uncertainties in the assessments of FMEA users. Different users may have different opinions. In this paper, their opinions can be represented in the following ways:

- Evidence bodies. For example, there are confidence degrees of 40% and 60% for values “4” and “5” in experts’ subjective judgments, which can be written as “ $m(4) = 0.4, m(5) = 0.6$ ” in the evidence structure.
- Hesitant evidence bodies. For example, the confidence degrees for ratings “{6, 7}” and “8” are 60% and 40%, respectively, and can be written as “ $m(\{6, 7\}) = 0.6, m(8) = 0.4$ ” in the evidence structure. $\{6, 7\}$ means an expert hesitates to give a specific judgment between the ratings of 6 and 7.
- A crisp value such as “3” can be written as “ $m(3) = 1.$ ”
- Interval values such as “[3, 4]” can be written as “ $m([3, 4]) = 1.$ ”

B. BPA ASSIGNMENT

Assume that there are L experts (denoted as E_1, E_2, \dots, E_L) performing FMEA, and N failure modes (denoted as F_1, F_2, \dots, F_N) of the product are considered. Generally, different experts have different weights, which are denoted as w_l for l_{th} experts. If there is no information to determine the weights, it suggests that each expert is equally credible and has an equal weight of $1/L$. In this paper, multiple sources of judgments need not be combined. Keeping the conventional definition of RPN as the product of risk factors S, O and $D,$

TABLE 1. Experts' judgments on risk factors.

Expert	weight	S	O	D
1	0.4	4	[7,8]	m(3)=0.8,m(4)=0.2
2	0.6	m(4)=0.7,m(5)=0.3	8	[2,3]

all possible RPN values among L judgments related to each failure mode are calculated.

If there are a total of $L_i (i = S, O, D)$ single-atom BPAs in each risk factor, then the number of RPN values is $L_S \times L_O \times L_D$. As the judgments are all transferred to evidence bodies as described in Section 3.1, the value of each RPN can be obtained by the following equation:

$$RPN_n^j = S_n^{k_1} \cdot O_n^{k_2} \cdot D_n^{k_3} \quad (17)$$

where $n = 1, 2, \dots, N, k_i \in [0, L_i] (i = S, O, D)$, and $j = 1, 2, \dots, L_S \times L_O \times L_D$.

Note that there may be interval values in experts' judgments, so the classical interval arithmetic rules are used in Eq.(17).

For l_{th} expert, The BPA of j_{th} RPN for F_n is

$$m_{RPN,n}^j = w_r w_p w_q \cdot m_{nS}^l(k_1) m_{nO}^l(k_2) m_{nD}^l(k_3) \quad (18)$$

where $r, p, q = 1, 2, \dots, L$, and w_r, w_p, w_q are corresponding expert weights of factors S, O and D; $k_1, k_2, k_3 \in [1, 10]$; $j = 1, 2, \dots, L_S \times L_O \times L_D$.

For clarity, an example involving two FMECA users is discussed here.

Example 1: The judgments of the experts on three risk factors are shown in Table 1.

All possible values of RPN are calculated, as shown in Table 2. For example, in the second line of Table 2, as all comments are come from expert 1, the weights of S, O, D are all 0.4. In the comments of D for 1st expert, m(3) = 0.8. So BPA of this combination is $0.4 \times 0.4 \times 0.4 \times 0.8 = 0.0512$. The calculation program of the remaining combinations is similar. As there are 3 single atoms in S, and 2 in O, and 3 in D, the number of single-atom RPNs is $3 \times 2 \times 3 = 18$.

C. AREA METRIC FOR RPN

As shown in Table 2, all possible values of RPN are intervals (precise values can also be expressed as intervals). The lower bound of for failure mode n is

$$LB_{RPN,n} = \min(\inf_{i=1,2,\dots,L^3}(RPN_n^i)) \quad (19)$$

and the upper bound is

$$UB_{RPN,n} = \max(\sup_{i=1,2,\dots,L^3}(RPN_n^i)) \quad (20)$$

From Eq.(5), the belief function can be obtained as

$$Bel(RPN_n < RPN^*) = \begin{cases} \sum_{\sup(RPN_n^j) < RPN^*} m_{RPN,n}^j & RPN^* \in [LB_{RPN,n}, UB_{RPN,n}] \\ 0 & RPN^* < LB_{RPN,n} \\ 1 & RPN^* > UB_{RPN,n} \end{cases} \quad (21)$$

TABLE 2. All possible values of RPN and their BPAs.

Item	S	O	D	RPN	BPA
1	4	[7,8]	3	[84,96]	$0.4 \times 0.4 \times 0.4 \times 0.8 = 0.0512$
2	4	[7,8]	4	[112,128]	$0.4 \times 0.4 \times 0.4 \times 0.2 = 0.0128$
3	4	[7,8]	[2,3]	[56,96]	$0.4 \times 0.4 \times 0.6 = 0.0960$
4	4	8	3	96	$0.4 \times 0.6 \times 0.4 \times 0.8 = 0.0768$
5	4	8	4	128	$0.4 \times 0.6 \times 0.4 \times 0.2 = 0.0192$
6	4	8	[2,3]	[64,96]	$0.4 \times 0.6 \times 0.6 = 0.1440$
7	4	8	[2,3]	[64,96]	$0.6 \times 0.6 \times 0.6 \times 0.7 = 0.1512$
8	5	8	[2,3]	[80,120]	$0.6 \times 0.6 \times 0.6 \times 0.3 = 0.0648$
9	4	[7,8]	[2,3]	[56,96]	$0.6 \times 0.4 \times 0.6 \times 0.7 = 0.1008$
10	5	[7,8]	[2,3]	[70,120]	$0.6 \times 0.4 \times 0.6 \times 0.3 = 0.0432$
11	4	[7,8]	3	[84,96]	$0.6 \times 0.4 \times 0.4 \times 0.7 \times 0.8 = 0.0538$
12	4	[7,8]	4	[112,128]	$0.6 \times 0.6 \times 0.4 \times 0.3 \times 0.8 = 0.0134$
13	5	[7,8]	3	[105,120]	$0.6 \times 0.6 \times 0.4 \times 0.3 \times 0.8 = 0.0230$
14	5	[7,8]	4	[140,160]	$0.6 \times 0.6 \times 0.4 \times 0.3 \times 0.8 = 0.0058$
15	4	8	3	96	$0.6 \times 0.6 \times 0.4 \times 0.7 \times 0.8 = 0.0806$
16	4	8	4	128	$0.6 \times 0.6 \times 0.4 \times 0.7 \times 0.2 = 0.0202$
17	5	8	3	120	$0.6 \times 0.6 \times 0.4 \times 0.3 \times 0.8 = 0.0346$
18	5	8	4	160	$0.6 \times 0.6 \times 0.4 \times 0.3 \times 0.2 = 0.0086$

and the plausibility function is

$$PI(RPN_n < RPN^*) = \begin{cases} \sum_{\inf(RPN_n^j) < RPN^*} m_{RPN,n}^j & RPN^* \in [LB_{RPN,n}, UB_{RPN,n}] \\ 0 & RPN^* < LB_{RPN,n} \\ 1 & RPN^* > UB_{RPN,n} \end{cases} \quad (22)$$

Then

$$Bel(RPN_n > RPN^*) = 1 - Bel(RPN_n < RPN^*) = \begin{cases} 1 - \sum_{\sup(RPN_n^j) < RPN^*} m_{RPN,n}^j & RPN^* \in [LB_{RPN,n}, UB_{RPN,n}] \\ 1 & RPN^* < LB_{RPN,n} \\ 0 & RPN^* > UB_{RPN,n} \end{cases} \quad (23)$$

$$PI(RPN_n > RPN^*) = 1 - PI(RPN_n < RPN^*) = \begin{cases} 1 - \sum_{\inf(RPN_n^j) < RPN^*} m_{RPN,n}^j & RPN^* \in [LB_{RPN,n}, UB_{RPN,n}] \\ 1 & RPN^* < LB_{RPN,n} \\ 0 & RPN^* > UB_{RPN,n} \end{cases} \quad (24)$$

Example 2: The known information about S, O and D is the same as that in Example 1. Then, from Table 5, the lower and upper bounds of RPN_n^j are $LB_{RPN,n} = 56$ and

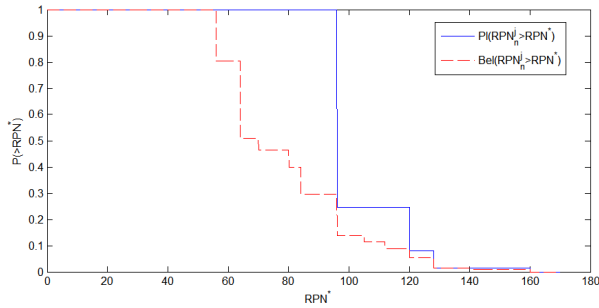


FIGURE 1. Belief and plausibility functions of Example 2.

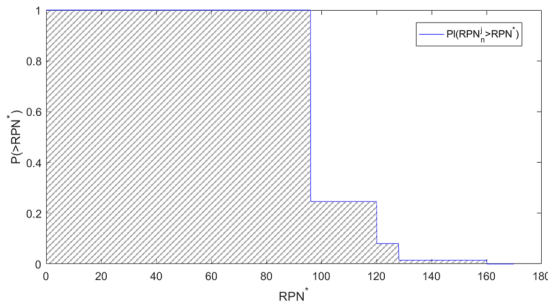


FIGURE 2. Lower bound of area metric for failure mode n .

$UB_{RPN,n} = 160$, respectively. According to the data in columns 5 and 6, the belief function and plausibility function can be calculated by Eq. (23) and (24), and the results are shown in Figure 1.

From the definitions of the belief and plausibility functions, it can be seen that the belief function is a conservative estimate of probability $P(RPN_n > RPN^*)$ and the plausibility function is an overestimate. As the RPN value of failure mode n is not a precise value but a distribution, area metrics are proposed to measure and prioritize the risks.

For the plausibility function, as shown in Figure 2, the upper bound area metrics for the n th failure mode are defined as

$$\bar{A}_n = \int_0^1 Pl^{-1}(P)dP \quad (25)$$

where P is the probability within domain $[0, 1]$, and $Pl_n^{-1}(P)$ is the inverse of the plausibility function for the n th failure mode. Obviously, \bar{A}_n is the upper bound expected value of RPN_n . For easier calculation, Eq.(25) can also be expressed as

$$\bar{A}_n = \int_0^{UB_{RPN,n}} Pl(RPN_n > x^*)dx^* \quad (26)$$

Correspondingly, the lower bound area metrics for the n th failure mode are defined as

$$\underline{A}_n = \int_0^1 Bel^{-1}(P)dP \quad (27)$$

where P is the probability within domain $[0, 1]$, and $Bel_n^{-1}(P)$ is the inverse of the belief function for the n th failure

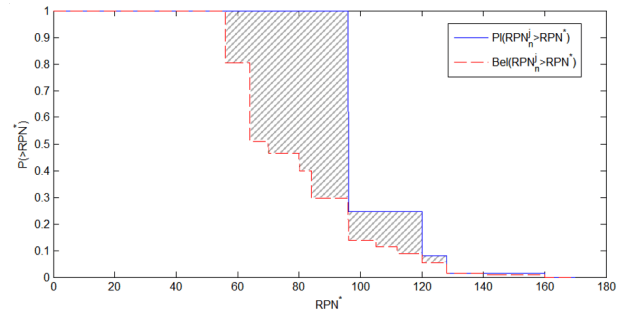


FIGURE 3. Epistemic uncertainty measure of RPN in failure mode n .

mode. \underline{A}_n is can also be expressed as

$$\underline{A}_n = \int_0^{UB_{RPN,n}} Bel(RPN_n > x^*)dx^* \quad (28)$$

Obviously, \underline{A}_n is the upper bound expected value of RPN_n . The real expected value of RPN_n is within the interval $A_n = [\underline{A}_n, \bar{A}_n]$, i.e., $RPN_n \in [\underline{A}_n, \bar{A}_n]$.

D. EPISTEMIC UNCERTAINTY METRIC OF RPN

As shown in Figure 1, there are epistemic uncertainties in the failure modes. If the uncertainties are too large, then the RPN result of this failure mode is not credible. Therefore, uncertainty ranking is necessary when there are epistemic uncertainties in experts' judgments. Figure 1 shows that if the plausibility and belief curves overlap, then there is no epistemic uncertainty in the n th failure mode. Otherwise, the greater the distance between the two curves is, the more uncertainty there is in the failure mode. Therefore, the area between the belief and plausibility curves (as shown in Figure 3) can express the amount of uncertainty in an RPN, which is defined as

$$U_n = \int_{LB_{RPN,n}}^{UB_{RPN,n}} (Pl(RPN_n^j > x^*) - Bel(RPN_n^j > x^*))dx^* \quad (29)$$

U_n is assigned its minimum value $U_n^{min} = 0$ when $Bel(RPN_n > RPN^*) = Pl(RPN_n > RPN^*)$. When $Bel(RPN_n > RPN^*) = 0$, U_n is assigned its maximum value of

$$U_n^{max} = \int_{LB_{RPN,n}}^{UB_{RPN,n}} Pl(RPN_n^j > x^*)dx^* \quad (30)$$

As U_n is a metric with dimensions, to measure the uncertainties among different failure modes, the DUM is proposed and is defined as

$$\rho_n = U_n / \text{mid}(A_n) \quad (31)$$

where $\text{mid}(A_n) = (\underline{A}_n + \bar{A}_n)/2$. Obviously, $\rho_n = 0$ when $U_n = 0$, and $\rho_n = 1$ when $U_n = A_n$. Therefore, the domain of ρ_n is $\rho_n \in [0, 1]$.

In engineering, decision makers usually want to obtain a qualitative assessment (e.g., “large,” “moderate,” or

TABLE 3. Expert system of epistemic uncertainty measurement in RPN ranking.

DUM	$\rho < 10\%$	$10\% \leq \rho < 25\%$	$25\% \leq \rho < 50\%$	$\rho \geq 50\%$
Uncertainty degree	Minor	Small	Moderate	Big

“small”) of the DUM to determine whether to accept a risk ranking. Hence, in this paper, an expert system is presented to qualitatively evaluate the DUM, in which four grades are considered, as shown in Table 3.

E. RISK PRIORITIZATION

With the area metric for an RPN, the failure modes can be ordered. As described in Section 3.3, the expected value of the RPN for F_n is interval $[\underline{A}_n, \bar{A}_n]$. To compare two overlapping intervals, many methods have been proposed [49]–[52]. In this section, a simple and effective method of interval number ranking based on the possibility degree is used to order these interval RPNs.

Definition 1 [51]: Considering two interval numbers $\tilde{a}_n = [\underline{A}_n, \bar{A}_n]$ and $\tilde{a}_m = [\underline{A}_m, \bar{A}_m]$, the possibility degree of $\tilde{a}_n > \tilde{a}_m$ is defined as

$$p_{nm} = p(\tilde{a}_n > \tilde{a}_m) = \frac{1}{2} \left(1 + \frac{(\bar{A}_n - \bar{A}_m) + (\underline{A}_n - \underline{A}_m)}{|\bar{A}_n - \bar{A}_m| + |\underline{A}_n - \underline{A}_m| + l_{nm}} \right) \quad (32)$$

where l_{nm} is the length of overlap. If $[\underline{A}_n, \bar{A}_n] \cap [\underline{A}_m, \bar{A}_m] = \emptyset$, then $l_{nm} = 0$. If \tilde{a}_n and \tilde{a}_m are crisp values, then

$$p_{nm} = p(\tilde{a}_n > \tilde{a}_m) = \begin{cases} 1 & \text{if } \tilde{a}_n > \tilde{a}_m \\ 0 & \text{if } \tilde{a}_n < \tilde{a}_m \\ \frac{1}{2} & \text{if } \tilde{a}_n = \tilde{a}_m \end{cases} \quad (33)$$

Based on Definition 1, RPN interval set $\tilde{\mathbf{a}} = \{\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_N\} = \{[\underline{A}_1, \bar{A}_1], [\underline{A}_2, \bar{A}_2], \dots, [\underline{A}_N, \bar{A}_N]\}$ can be ranked by the following steps:

(1) Calculate the possibility degree between each pair of intervals, and then, the possibility degree matrix is obtained as $\mathbf{P} = (p_{nm})_{N \times N}$.

(2) Calculate the ranking vector $\lambda = \{\lambda_1, \lambda_2, \dots, \lambda_N\}$ by the following equation [50]:

$$\lambda_n = \sum_{m=1}^N p_{nm}, \quad n = 1, 2, \dots, N \quad (34)$$

Then, we can rank the RPN intervals $\tilde{a}_n (n = 1, 2, \dots, N)$ in descending order in accordance with the value of $\lambda_n (n = 1, 2, \dots, N)$. If there are moderate or large degrees of uncertainty in the top 5 (which can also be set to 10 or another number, depending on the complexity of the product and the decision makers) failure modes, then further investigation should be performed to alleviate epistemic uncertainties.

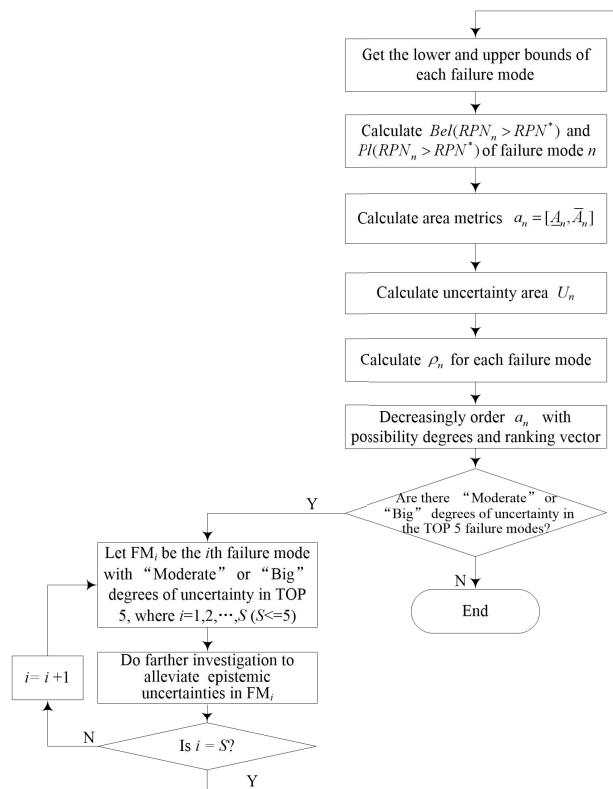


FIGURE 4. Flowchart of the prioritization procedure.

Examples of measures to reduce uncertainties include simulation, analysis, and tests.

For clarity, a flowchart related to the failure modes and their epistemic uncertainty prioritization procedure is shown in Figure 4.

V. CASE STUDY

In this section, a practical risk evaluation case regarding the grinding wheel system of an NC machine [53] is provided to demonstrate the application and effectiveness of the proposed FMEA. The grinding wheel system is an important part of NC machine MK2120, which affects the processing quality. In reality, there are many failure modes and causes related to the grinding wheel system. However, only 5 failure modes with high risk are selected for further analysis; these modes are named FM_1, FM_2, \dots, FM_5 in this study.

In the FMECA of the grinding wheel system, there are three experts named E_1, E_2 and E_3 , each giving different opinions on risk factors S, O and D . Considering the experts’ professional background and familiarity levels, they are assigned distinct weights in the risk analysis, i.e., $w = (0.35, 0.4, 0.25)$. Each expert evaluates the failure modes and identifies the rating information of the three risk factors on the 5 failure modes, as shown in Table 4. In order to reduce the length of contents, the expression of experts’ opinions in Table 4 is different from that in Table 1. For example, “8:80%

TABLE 7. FMEA results of five failure modes.

Failure mode	A_n	U_n	ρ_n	λ_n	Uncertainty rating	RPN rating	Uncertainty degree
FM_1	[221.28, 282.48]	61.20	24.30%	2.95	2	2	Small
FM_2	[188.47, 223.79]	35.32	17.13%	1.87	4	4	Minor
FM_3	[205.32, 288.06]	82.75	33.54%	3.06	1	1	Moderate
FM_4	[197.95, 231.07]	33.12	15.44%	2.04	5	3	Minor
FM_5	[165.72, 205.73]	40.01	21.54%	1.5546	3	5	Small

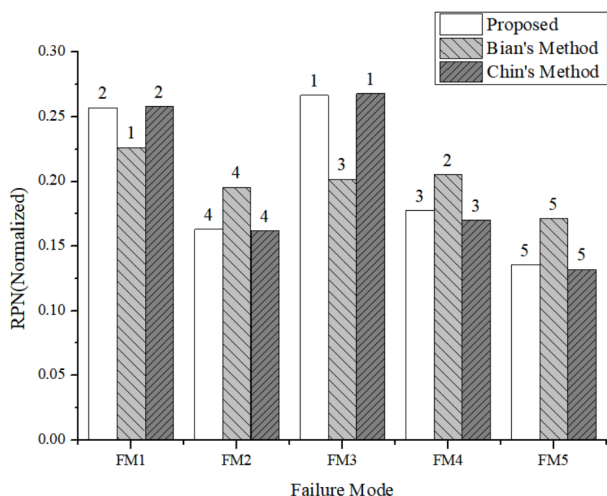


FIGURE 6. Contrast between the proposed method and Chin's method.

The RPN rating of the proposed method is also compared with that of Bian [26] and Chin [31]. As the three methods have different dimensions, the RPN values are normalized to the interval [0, 1], as shown in Figure 6. Figure 6 shows that the RPN rating of the new method is the same as that in Chin's method, but different with the result of Bian's. In proposed method, the rating is $FM_3 > FM_1 > FM_4 > FM_2 > FM_5$. However, in Bian's method, the result is $FM_1 > FM_4 > FM_3 > FM_2 > FM_5$. In other words, FM_3 and FM_4 are in the opposite order of the two methods. The risk ranking of other failure modes is the same.

According to Table 4, if the uncertainty in the evaluation information is handled by a simple average method, it can be obtained that RPN of FM_3 and FM_4 are 244.9824 and 214.3800 respectively. In other word, the rating is $FM_3 > FM_4$, which is the same as that of the proposed method and Chin's method. However, Chin's method cannot measure how many uncertainties are presented in the RPN results. So proposed method is correct and can get more effective information for decision makers.

If more investigations on FM_3 are performed and more information is obtained about this mode's effects when it fails, then uncertainties in experts' judgments can be reduced. Consider two instances in which experts give two different

TABLE 8. Second-round evaluation information on FM_3 .

Instance	S			O			D		
	E1	E2	E3	E1	E2	E3	E1	E2	E3
1	8	9	[8,9]	6	[6,7]	6	[3,4]	4	5
2	8	9	[9,10]	6	[7,8]	6	[4,5]	5	5

TABLE 9. FMEA results for FM_3 under second-round evaluation information.

Instance	A_n	U_n	ρ_n	λ_n	Uncertainty rating	RPN rating	Uncertainty degree
1	[189.70, 228.89]	39.19	18.72%	1.91	3	3	Minor
2	[254.98, 280.30]	25.31	9.46%	3.34	5	1	Minor

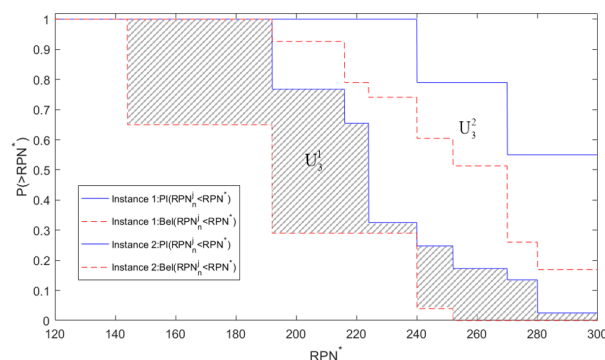


FIGURE 7. Belief and plausibility functions of RPN for FM_3 under second-round evaluation information.

opinions, as shown in Table 8. In the same way, we can obtain the new results of FM_3 , as shown in Table 9 and Figure 7.

Table 9 shows that when the uncertainties in FM_3 are decreased in instance 1, the RPN rating decreases from 1 to 3. Otherwise, in instance 2, the RPN rating is not changed. That is, if there are enormous uncertainties in FMEA, the results may mislead designers to address an unimportant failure mode or to ignore a very important failure mode, both of which are unacceptable in engineering practice.

VI. CONCLUSION

FMEA under uncertain environments has received extensive attention in recent years. In this paper, an area metric based on belief and plausibility functions is proposed to represent the RPN results, and the DUM is proposed to represent how many uncertainties are presented in RPNs. Moreover, an expert system is presented to qualitatively evaluate the DUM based on four grades ("minor," "small," "moderate," and "large"). The effectiveness of the proposed model has been illustrated by a practical risk evaluation case regarding the grinding wheel system of a numerically controlled (NC) machine. The results are consistent with the practical engineering background.

The FMEA proposed in this paper has the following advantages:

(1) The calculation program of the RPN interval area metric (IAM) does not require any assumptions (such as fuzzy distribution or uniform distribution) and does not need to address conflict among experts.

(2) The DUM can express how many uncertainties are presented in RPN results, which can help FMEA users judge whether the results are reliable. If the DUM of a failure mode is large, then more research needs to be performed to reduce the uncertainties and obtain a more reliable risk evaluation result.

(3) The expert system in Table 3 can help designers intuitively see the uncertainty grade of each failure mode, which is useful to help them understand FMEA results.

The presented method of this study has some drawbacks and limitations. First, the paper only studied three types of uncertainties, which were intervals, evidence bodies and hesitant evidence bodies. Other types of uncertainties, such as interval fuzzy, interval-valued intuitionistic fuzzy, hesitant fuzzy linguistic, etc., have not been addressed. Second, the amount of calculation of the new method is large. Therefore, the follow-up research can try to apply the proposed method to other occasions (such as interval-valued intuitionistic fuzzy, hesitant fuzzy linguistic, etc.), and propose more efficient improved algorithms.

COMPLIANCE WITH ETHICAL STANDARDS

AUTHOR CONTRIBUTIONS

Conceptualization, methodology, and writing-original draft preparation, B.S.; Methodology, investigation, software, writing-reviewing and editing, Y.Y.; Data preparation, Z.L. All authors have read and agreed to the published version of the manuscript.

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Not applicable.

INFORMED CONSENT STATEMENT

Not applicable.

DATA AVAILABILITY STATEMENT

Data are contained within the article.

CONFLICT OF INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work re-reported in this paper.

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