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Attitude Control for a Rigid-Body With Dynamic Disturbance Based on Angular Velocity Observer

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ABSTRACT In this paper, we propose a geometric control law for attitude robust control problem of a rigid body system without angular velocity measurement. We suppose there is a disturbance related to angular velocity and attitude in the system. Under appropriate controllers, the estimated values of attitude and angular velocity obtained by an angular velocity observer can converge to the true values exponentially by the Lyapunov stability theorem. Furthermore, by modifying an adaptive law for the inertia matrix, the exponential stabilization of the system can still be guaranteed without the knowledge of the true inertia matrix with the same way. The angular velocity observer we designed ensures the global exponential stability in the system. Finally, the proposed theoretical results are verified by our simulation examples.

INDEX TERMS Attitude control, rigid body, disturbance, angular velocity observer.


I. INTRODUCTION

Rigid body doesn't change shape, size and the distances between the mass points under pressure, which is popular in defense and civilian fields such as underwater vehicles, the unmanned aerial vehicle and gossamer spacecraft [1], [2]. In recent years, the research on the rigid body system has been a hot topic. Among them, the attitude control is always the focus and difficulty in rigid body research.

Attitude control problem is to control the attitude of a rigid body so that it can keep in within the allowable range. The attitude control of a rigid body can be divided into two categories: attitude tracking and attitude stabilization [3]. Attitude control problem especially for the full attitude states (i.e., the attitude and the angular velocity) has been extensively studied in the past, such as robust attitude control [4] and adaptive attitude control [5], which usually requires that the angular velocity and attitude have to be measured [6], [7]. Attitude control is a very important part of rigid body control, which is the basis for tracking, positioning and other flight tasks.

In practical applications, the measurement of angular velocity may not be available because of the size, quality, reliability and technical limitations of the rigid body system [8], [9]. Moreover, due to the complexity of the angular

velocity sensor, it often fails in the task and can not get the information of the angular velocity in time, so that many rigid body systems are not equipped with angular velocity sensors [10]–[12]. When the velocity measurements are not available, we usually use estimators of angular velocity instead of the real values. Nonlinear observer is often used to solve angular velocity measurement problems, which can avoid turning rigid body nonlinear system into linear one. Using the relationship between attitude and angular velocity, angular velocity information can be obtained from attitude information, which is called two-step. The first step is to determine the attitude information through appropriate measurements. The second step is to estimate the angular velocity information by differentiating the attitude time, which produces the high frequency noise easily. Based on this problem, scholars of various countries put forward different methods. It is common to design angular velocity observer based on Euler equation. Another commonly used method is to bypass the first step of attitude estimation and reconstruct the angular velocity of the rotating rigid body directly using the results of vector measurement. An angular velocity observer with separate attributes is defined to get the estimated value of the angular velocity, e.g., [13], [14]. Introducing an auxiliary dynamical system to generate damping terms in the absence of the actual angular velocities [13], [15], [16], which is equivalent to the angular velocity observer in some cases.

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In these literatures, attitude tracking control system has good exponential stability, i.e., the estimated values of angular velocity and attitude tend to the true values exponentially.

The rigid body system is highly nonlinear [17] and subject to interference including internal and external environment and time delays, whose influence in the accuracy and stability of a rigid body system leads to the degradation of system performance [18]–[20]. Therefore, the attitude dynamic function should be built with unknown parameter and disturbance [21], [22]. To study the stability of a dynamic system with disturbances is a hot research topic [23]. Design the active disturbance rejection controller (ADRC) can resist external interference and improve the robustness of the system [24]. Introducing an adaptive control method also can deal with this problem [16], [25]. However, most literatures assume that disturbance is bounded and which does not have generality.

To avoid singularities and ambiguities, we use rotation matrix denoting attitude rather than quaternion, Euler angle and modified Rodrigues parameter. The attitude of a rigid body can be modeled by a linear transformation between inertial frame and fixed frame. The rotation matrix is unique and global, i.e., the attitude of each rigid body corresponds to the only one of rotation matrix exactly [26], [27]. Therefore, in recent years, researchers have begun to focus on the stability and tracking of rotation matrices. It can be seen from the above discussion, the attitude control for a rigid body system without angular velocity measurement is a very important research problem. However, this problem is difficult when the disturbances exist. In addition, it is more challenging to solve this problem by using a rotation matrix and there is no result, to the best of our knowledge.

Motivated by the above discussion, we study the attitude control for a rigid body system without angular velocity measurement in the presence of disturbance. Contributions of this paper are given as follows.

(1) For unknown external disturbance in the system, when the disturbance is a quantity related to angular velocity and attitude error functions rather than a unknown bounded number, we discuss the exponential stability of the error functions.

(2) We define an angular velocity observer such that the estimated values of the angular and attitude approximate exponentially to the true values (Theorem 1).

(3) We propose an adaptive attitude control and prove that the estimated values of attitude, angular velocity and inertia matrix tend to the true values (Theorem 2), when the inertia matrix is unknown.

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This paper is organized as follows. The rigid body system and an angular velocity observer are formulated in Section II. An adaptive control approach is proposed for the exponentially stabilization of attitude and angular velocity in Section III. On the basis of Section III, we consider the exponentially stability of attitude and angular velocity in Section IV, when the inertia matrix is unknown. Simulation results are offered in Section V. The conclusion of this paper is summarized in Section VI.

Notations: \mathbb{R} and \mathbb{R}^+ denote real number set and positive real number set. $\mathbb{R}^{3 \times 3}$, \mathbb{R}^3 indicate 3×3 real matrices and 3×1 vectors, respectively. $\lambda_m(\cdot)$ and $\lambda_M(\cdot)$ indicate the minimum and maximum eigenvalues of a matrix. Similarly, $\max\{\cdot\}$ and $\min\{\cdot\}$ are the maximum and the minimum of the given numbers. Suppose that a matrix A is reversible, A^{-1} expresses the inverse of A . A^T and $\text{tr}(A)$ are the transpose and the trace of the matrix A , respectively. For any $x \in \mathbb{R}^n$, $A \in \mathbb{R}^{n \times n}$, the 2-norm of x and A are defined as $\|x\| = \sqrt{x^T x}$, and $\|A\|_2 = \sqrt{\lambda_M(A^T A)}$, and the Frobenius norm of A is $\|A\|_F = \sqrt{\text{tr}(A^T A)}$. $\text{diag}(\cdot)$ denotes a diagonal matrix. Let $SO(3)$ be the rotation attitude set, $SO(3) = \{R \in \mathbb{R}^{3 \times 3}\}$. “ \cdot ” and “ \times ” are the inner and outer products of two vectors, respectively. the hat map $(\cdot)^\wedge : \mathbb{R}^3 \rightarrow \mathfrak{so}(3)$ transforms a vector to a skew-symmetric matrix, where $\mathfrak{so}(3)$ is the Lie algebra of $SO(3)$ satisfying $\mathfrak{so}(3) = \{A \in \mathbb{R}^{3 \times 3} | A^T = -A\}$. For any $x = (x_1, x_2, x_3)$, $y = (y_1, y_2, y_3) \in \mathbb{R}^3$, one has $\hat{x} = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}$, and $\hat{x}y = x \times y = -y \times x = -\hat{y}x$. The inverse of the hat map is defined as $(\cdot)^\vee : \mathfrak{so}(3) \rightarrow \mathbb{R}^3$.

II. BACKGROUND

A. ATTITUDE DYNAMICS OF A RIGID BODY

We consider the attitude dynamics of a fully-actuated rigid body. We define two coordinate frames: an inertial reference frame and a body-fixed frame. The attitude of the rigid body is denoted by $R \in SO(3)$ that represents the transformation of a vector from the body-fixed frame to the inertial reference frame.

The rotation matrix R satisfies the following equations:

$$R^T R = R R^T = I_{3 \times 3}, \tag{1}$$

$$\det(R) = 1. \tag{2}$$

Let $SO(3)$ represents the set of rotation matrix, i.e.

$$SO(3) = \{R \in \mathbb{R}^{3 \times 3} | R^T R = R R^T = I_{3 \times 3}, \det(R) = 1\} \tag{3}$$

The set $SO(3)$ can be regarded as a Lie group with rotation matrix as the identity element and matrix multiplication as the group operation rule, and called the set $SO(3)$ three-dimensional rotation group. The equations of motion are

presented as follows [28], [29],

$$J\dot{\Omega} + \Omega \times J\Omega = u + \Delta, \quad (4)$$

$$\dot{R} = R\hat{\Omega}, \quad (5)$$

where $J \in \mathbb{R}^{3 \times 3}$ is the inertia matrix, $u \in \mathbb{R}^3$ is the control input, $\Omega \in \mathbb{R}^3$ is the angular velocity of the rigid body in the body-fixed frame, and $\Delta \in \mathbb{R}^3$ expresses unknown disturbance.

B. ANGULAR VELOCITY OBSERVER

Now an observer is constructed such that the value of angular velocity is estimated when the attitude measurements and the control input are available. Let the estimators of the attitude R and the angular velocity Ω be $\hat{R} \in SO(3)$, and $\hat{\Omega} \in \mathbb{R}^3$, respectively. It's worth noting that the angular velocity observer is designed in the body-fixed frame.

We define the error between the true attitude and the estimate attitude as $Q \in SO(3)$, i.e., $Q = R\hat{R}^T$. It can be noted that if $R = \hat{R}$, then $Q = I_{3 \times 3}$.

We define an attitude error function Ψ , an attitude error vector e_R , and an angular velocity error vector e_Ω as follows:

$$\Psi = \frac{1}{2} \text{tr}[G(I - Q)], \quad (6)$$

$$e_R = \frac{1}{2}(GQ^T - QG)^\vee, \quad (7)$$

$$e_\Omega = \Omega - Q\hat{\Omega}, \quad (8)$$

where $G = \text{diag}(g_1, g_2, g_3)$, $g_1, g_2, g_3 \in \mathbb{R}$ are positive constants. From [30], we can get the following Lemmas:

Lemma 1:

$$\dot{\Psi} = e_R^T e_\Omega, \quad (9)$$

Lemma 2:

$$\dot{e}_R = E_c(Q)e_\Omega, \quad (10)$$

where $E_c(Q) = \frac{1}{2} \text{tr}([QG]I - QG)$ satisfies $\|E_c(Q)\| < \frac{1}{\sqrt{2}} \text{tr}[G]$.

Lemma 3:

$$\dot{e}_\Omega = J^{-1}(u + \Delta - \Omega \times J\Omega) - \alpha, \quad (11)$$

where $\alpha = -\hat{\Omega}Q\hat{\Omega} + Q\dot{\hat{\Omega}}$.

Lemma 4: Assuming $\Psi \leq \psi \leq n_1$, then $\frac{n_1}{n_2+n_3} \|e_R\|^2 \leq \Psi \leq \frac{n_1 n_4}{n_5(n_1-\psi)} \|e_R\|^2$, where

$$\begin{aligned} n_1 &= \min\{g_1 + g_2, g_2 + g_3, g_3 + g_1\}, \\ n_2 &= \max\{(g_1 - g_2)^2, (g_2 - g_3)^2, (g_3 - g_1)^2\}, \\ n_3 &= \max\{(g_1 + g_2)^2, (g_2 + g_3)^2, (g_3 + g_1)^2\}, \\ n_4 &= \max\{g_1 + g_2, g_2 + g_3, g_3 + g_1\}, \\ n_5 &= \min\{(g_1 + g_2)^2, (g_2 + g_3)^2, (g_3 + g_1)^2\}. \end{aligned}$$

From [31], we get the dynamics of the estimators of the angular velocity as follows:

$$\dot{\hat{\Omega}} = J^{-1}(u + \Delta - \hat{\Omega} \times J\hat{\Omega}) - \alpha + J^{-1}(u + \Delta - \Omega \times J\Omega), \quad (12)$$

$$\dot{\hat{R}} = \hat{R}\hat{\hat{\Omega}}. \quad (13)$$

The angular velocity observer is designed in the body-fixed frame, the dynamics function can be converted into each other in body-fixed frame and inertial reference.

III. ATTITUDE TRACKING CONTROL

In this section, we will design controllers such that the zero equilibrium of the estimated error variables is exponentially stable.

Lemma 5: Assume that a matrix A is a symmetric matrix, then for any vector $x \in \mathbb{R}^n$, the following inequality is satisfied [32], $\lambda_m(A) \|x\|^2 \leq x^T A x \leq \lambda_M(A) \|x\|^2$.

Assumption 1: Let $e_A = e_\Omega + ce_R$ be an augmented vector, where c is a positive constant. Assume that the disturbance is related to e_A , i.e., $\Delta = ke_A$, $k \in \mathbb{R}^+$. Therefore, the rigid body dynamics equations (4), (5) are related to the estimation error dynamics (7), (8).

Remark 1: In [25], [33], [34], the disturbance is assumed to be related to angular velocity. Motivated by the above results and considering that the attitude error is related to the angular velocity, we assume that the disturbance is linearly related to angular velocity error vector.

Theorem 1: Suppose that assumption 1 is satisfied. Consider the system (4),(5), the control input is chosen as

$$u = -k_R e_R - k_\Omega e_\Omega + \Omega \times J\Omega + J\alpha + \nu. \quad (14)$$

When c satisfies

$$c < \min\left\{ \frac{1}{\lambda_M} \sqrt{\frac{2\lambda_m n_1 k_R}{n_2 + n_3}}, \sqrt{\frac{2k_R n_1 n_4}{\lambda_M n_5 (n_1 - \psi)}}, \frac{\sqrt{2}(k_R - M)}{\lambda_M \text{tr}[G]}, \frac{4k_\Omega k_R - 4Mk_R}{2\sqrt{2}k_R \lambda_M \text{tr}[G] + k_\Omega^2 + 8k_\Omega M} \right\}, \quad (15)$$

where parameters $k_R, k_\Omega \in \mathbb{R}^+$, $\nu = \frac{\delta^2 e_A}{\delta \|e_A\| + \epsilon}$, $M = \frac{\delta^2 + k\delta \|e_A\| + k\epsilon}{\delta \|e_A\| + \epsilon}$, $\delta, k, \epsilon \in \mathbb{R}^+$, the angular velocity observers (12), (13) guarantee that the zero equilibrium of the tracking error (e_R, e_Ω) is exponentially stable.

Proof 1: Consider the following Lyapunov function:

$$V = \frac{1}{2} e_\Omega^T J e_\Omega + k_R \Psi + c e_\Omega^T J e_R. \quad (16)$$

Firstly, we can obtain

$$\xi^T W_1 \xi \leq V \leq \xi^T W_2 \xi, \quad (17)$$

where $\xi = (\|e_\Omega\|, \|e_R\|)^T$. In addition, the matrices $W_1, W_2 \in \mathbb{R}^{2 \times 2}$ are given as follows

$$\begin{aligned} W_1 &= \begin{pmatrix} \frac{1}{2}\lambda_m & -\frac{1}{2}c\lambda_M \\ -\frac{1}{2}c\lambda_M & \frac{n_1 k_R}{n_2 + n_3} \end{pmatrix}, \\ W_2 &= \begin{pmatrix} \frac{1}{2}\lambda_M & \frac{1}{2}c\lambda_M \\ \frac{1}{2}c\lambda_M & \frac{k_R n_1 n_4}{n_5 (n_1 - \psi)} \end{pmatrix}, \end{aligned}$$

which are positive definite according to (15). The proof of (17) is given in Appendix A.

Substituting (14) into (11), we can get

$$J\dot{e}_\Omega = -k_R e_R - k_\Omega e_\Omega + \Delta + v. \quad (18)$$

The first derivation of V is given by

$$\dot{V} = \frac{1}{2}(J\dot{e}_\Omega)^T e_\Omega + \frac{1}{2}e_\Omega^T J\dot{e}_\Omega + k_R \dot{\Psi} + c(J\dot{e}_\Omega)^T e_R + ce_\Omega^T J\dot{e}_R. \quad (19)$$

From (9), (10) and (18), (19) can be written as

$$\begin{aligned} \dot{V} &= \frac{1}{2}(-k_R e_R - k_\Omega e_\Omega + \Delta + v)^T e_\Omega + \frac{1}{2}e_\Omega^T (-k_R e_R - k_\Omega e_\Omega + \Delta + v) + k_R e_R^T e_\Omega + c(-k_R e_R - k_\Omega e_\Omega + \Delta + v)^T e_R + ce_\Omega^T J E_c(Q) e_\Omega \\ &= -\frac{1}{2}k_R e_R^T e_\Omega - \frac{1}{2}k_\Omega e_\Omega^T e_\Omega + \frac{1}{2}(\Delta + v)^T e_\Omega - \frac{1}{2}k_R e_\Omega^T e_R - \frac{1}{2}k_\Omega e_\Omega^T e_\Omega + \frac{1}{2}e_\Omega^T (\Delta + v) + k_R e_R^T e_\Omega - ck_R e_R^T e_R - ck_\Omega e_\Omega^T e_R + c(\Delta + v)^T e_R + ce_\Omega^T J E_c(Q) e_\Omega \\ &= -k_\Omega e_\Omega^T e_\Omega - ck_R e_R^T e_R - ck_\Omega e_\Omega^T e_R + (\Delta + v)^T (e_\Omega + ce_R) + ce_\Omega^T J E_c(Q) e_\Omega. \end{aligned} \quad (20)$$

From assumption 1, we can get

$$\begin{aligned} \Delta + v &= ke_A + \frac{\delta^2 e_A}{\delta \|e_A\| + \epsilon} \\ &= \frac{\delta^2 e_A + k\delta \|e_A\| e_A + k\epsilon e_A}{\delta \|e_A\| + \epsilon} \\ &= \frac{\delta^2 + k\delta \|e_A\| + k\epsilon}{\delta \|e_A\| + \epsilon} e_A. \end{aligned} \quad (21)$$

Let $M = \frac{\delta^2 + k\delta \|e_A\| + k\epsilon}{\delta \|e_A\| + \epsilon}$, i. e.,

$$\Delta + v = Me_A. \quad (22)$$

From (22), \dot{V} can be written as

$$\begin{aligned} \dot{V} &\leq -k_\Omega \|e_\Omega\|^2 - ck_R \|e_R\|^2 + ck_\Omega \|e_\Omega\| \|e_R\| + M \|e_\Omega\|^2 + 2Mc \|e_\Omega\| \|e_R\| + Mc^2 \|e_R\|^2 + \frac{c\lambda_M \text{tr}[G]}{\sqrt{2}} \|e_\Omega\|^2 \\ &= -\xi^T W_3 \xi, \end{aligned} \quad (23)$$

where

$$W_3 = \begin{pmatrix} k_\Omega - \frac{c\lambda_M \text{tr}[G]}{\sqrt{2}} - M & -\frac{ck_\Omega}{2} - Mc \\ -\frac{ck_\Omega}{2} - Mc & ck_R - Mc^2 \end{pmatrix}$$

is positive definite according to (15). From [35], we can know that the zero equilibrium of the tracking (e_R, e_Ω) is exponentially stable.

IV. ADAPTIVE ATTITUDE TRACKING

In the section, we suppose that the inertia matrix is unknown. \tilde{J} denotes the estimated value of the inertia matrix J , and the estimation error $\tilde{J} \in \mathbb{R}^{3 \times 3}$ is defined as $\tilde{J} = J - \tilde{J}$.

Theorem 2: Suppose that assumption 1 is satisfied. Consider the system (4), (5), the control input and the dynamics of the estimation of inertia matrix are chosen as

$$u = -k_R e_R - k_\Omega e_\Omega + \Omega \times \tilde{J}\Omega + \tilde{J}\alpha + v, \quad (24)$$

$$\dot{\tilde{J}} = \frac{k_J}{2}(-\alpha e_A^T - e_A \alpha^T + \tilde{\Omega} \tilde{\Omega}^T \hat{e}_A - \hat{e}_A \tilde{\Omega} \tilde{\Omega}^T - 2\sigma \tilde{J}), \quad (25)$$

where parameters $k_J, \sigma \in \mathbb{R}^+$. When c satisfies

$$c < \min\left\{\frac{1}{\lambda_M} \sqrt{\frac{2\lambda_m n_1 k_R}{n_2 + n_3}}, \sqrt{\frac{2k_R n_1 n_4}{\lambda_m n_5 (n_1 - \psi_E)}}, \frac{\sqrt{2}(k_R - M)}{\lambda_M \text{tr}[G]}, \frac{4k_\Omega k_R - 4Mk_R}{2\sqrt{2}k_R \lambda_M \text{tr}[G] + k_\Omega^2 + 8k_\Omega M}\right\}, \quad (26)$$

angular velocity observers (12), (13) guarantee that the zero equilibrium of tracking $(e_R, e_\Omega, \tilde{J})$ is exponentially stable.

Proof 2: Consider the following Lyapunov function:

$$V = \frac{1}{2}e_\Omega^T J e_\Omega + k_R \Psi + cJ e_\Omega^T e_R + \frac{1}{2k_J} \|\tilde{J}\|_F^2. \quad (27)$$

Similar to the proof of (17), we can obtain

$$z^T W_4 z \leq V \leq z^T W_5 z, \quad (28)$$

where

$$z = (\|e_\Omega\|, \|e_R\|, \|\tilde{J}\|_F)^T, \quad W_4 = \begin{pmatrix} \frac{1}{2}\lambda_m & -\frac{1}{2}c\lambda_M & 0 \\ -\frac{1}{2}c\lambda_M & \frac{n_1 k_R}{n_2 + n_3} & 0 \\ 0 & 0 & \frac{1}{2k_J} \end{pmatrix},$$

and

$$W_5 = \begin{pmatrix} \frac{1}{2}\lambda_M & \frac{1}{2}c\lambda_M & 0 \\ \frac{1}{2}c\lambda_M & \frac{k_R n_1 n_4}{n_5 (n_1 - \psi)} & 0 \\ 0 & 0 & \frac{1}{2k_J} \end{pmatrix}$$

are positive definite according to (26).

Substituting (24) into (11), we get

$$J\dot{e}_\Omega = -k_R e_R - k_\Omega e_\Omega - \Omega \times \tilde{J}\Omega - \tilde{J}\alpha + v + \Delta. \quad (29)$$

The first derivation of V is given by

$$\begin{aligned} \dot{V} &= \frac{1}{2}(J\dot{e}_\Omega)^T e_\Omega + \frac{1}{2}e_\Omega^T J\dot{e}_\Omega + k_R \dot{\Psi} + c(J\dot{e}_\Omega)^T e_R + cJ e_\Omega^T \dot{e}_R + \frac{1}{k_J} \text{tr}[\tilde{J}\dot{\tilde{J}}]. \end{aligned} \quad (30)$$

From (9), (10) and (29), (30) can be written as

$$\begin{aligned} \dot{V} &= -\frac{1}{2}k_R e_R^T e_R - \frac{1}{2}k_\Omega e_\Omega^T e_\Omega - \frac{1}{2}(\Omega \times \tilde{J}\Omega)^T e_\Omega - \frac{1}{2}(\tilde{J}\alpha)^T \\ &\quad \times e_\Omega + \frac{1}{2}(v + \Delta)^T e_\Omega - \frac{1}{2}k_R e_\Omega^T e_R - \frac{1}{2}k_\Omega e_\Omega^T e_\Omega - \frac{1}{2}e_\Omega^T \\ &\quad \times (\Omega \times \tilde{J}\Omega) - \frac{1}{2}e_\Omega^T \tilde{J}\alpha + \frac{1}{2}e_\Omega^T (v + \Delta) + k_R e_R^T e_\Omega - ck_R \\ &\quad \times e_R^T e_R - ck_\Omega e_\Omega^T e_R - c(\Omega \times \tilde{J}\Omega)^T e_R - c\tilde{J}\alpha^T e_R \\ &\quad + c(v + \Delta)^T e_R + cJ e_\Omega^T Ec(Q)e_\Omega + \frac{1}{k_J}(\text{tr}[\tilde{J}\dot{\tilde{J}}]) \\ &= -k_\Omega e_\Omega^T e_\Omega - ck_R e_R^T e_R - (\Omega \times \tilde{J}\Omega)^T e_A - (\tilde{J}\alpha)^T e_A \\ &\quad + (v + \Delta)^T e_A - ck_\Omega e_\Omega^T e_R + cJ e_\Omega^T Ec(Q)e_\Omega + \frac{1}{k_J} \\ &\quad \times \text{tr}[\tilde{J}\dot{\tilde{J}}] \\ &= -k_\Omega e_\Omega^T e_\Omega - ck_R e_R^T e_R - ck_\Omega e_\Omega^T e_R + cJ E_c(Q)e_\Omega \\ &\quad + \sigma \|\tilde{J}\|_F^2 + (v + \Delta)^T e_A, \end{aligned} \quad (31)$$

where the proof of the last step of (31) is presented in Appendix B.

$$\begin{aligned} \dot{V} &\leq -k_\Omega \|e_\Omega\|^2 - ck_R \|e_R\|^2 + ck_\Omega \|e_\Omega\| \|e_R\| \\ &\quad + \frac{c\lambda_M \text{tr}(G)}{\sqrt{2}} \|e_\Omega\|^2 + \sigma \|\tilde{J}\|_F^2 + M \|e_\Omega\|^2 \\ &\quad + 2Mc \|e_\Omega\| \|e_R\| + Mc^2 \|e_R\|^2 \\ &= -z^T W_6 z \end{aligned} \quad (32)$$

where

$$W_6 = \begin{pmatrix} k_\Omega - \frac{c\lambda_M \text{tr}(G)}{\sqrt{2}} - M & -\frac{ck_\Omega}{2} - Mc & 0 \\ -\frac{ck_\Omega}{2} - Mc & ck_R - Mc^2 & 0 \\ 0 & 0 & -\sigma \end{pmatrix},$$

is positive definite according to (26). So, we can know that the zero equilibrium $(e_R, e_\Omega, \tilde{J})$ is exponentially stable.

V. NUMERICAL SIMULATIONS

In this section, the performance of the proposed control laws will be investigated through numerical simulations. We consider the results obtained in Theorem 1 and Theorem 2. To illustrate the influence of the the inertia matrix in system, the simulation selects the same parameters to verify deliberately. The initial attitude, angular velocity and the inertia matrix of rigid body are given by $R(0) = I_{3 \times 3}$, $\Omega = [1, 3, 2]$, and $J(0) = \text{diag}(1, 2, 3)$. The controller parameters are given as follows: $G = \text{diag}(1, 1, 1)$; $k_R = 5$; $k_\Omega = 5$; $c = 0.02$; $\delta = 3$; $\epsilon = 3$; $k = 0.001$; and $k_J = 3$. Thus we can see that c satisfies (15) and (26).

Simulation results of Theorem 2 are shown in Figure 1, where the inertia matrix is known. Simulation results of Theorem 2 are shown in Figure 2, where the inertia matrix is unknown, and we introduce the estimated values of it. In Figures 1 and 2, the red, blue, and green lines express the first, second and the third elements of the tracking error

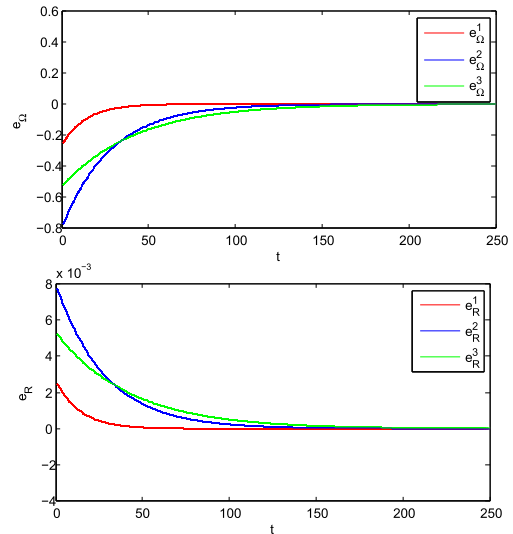


FIGURE 1. Angular error vector e_Ω and attitude error vector e_R obtained by using Theorem 1.

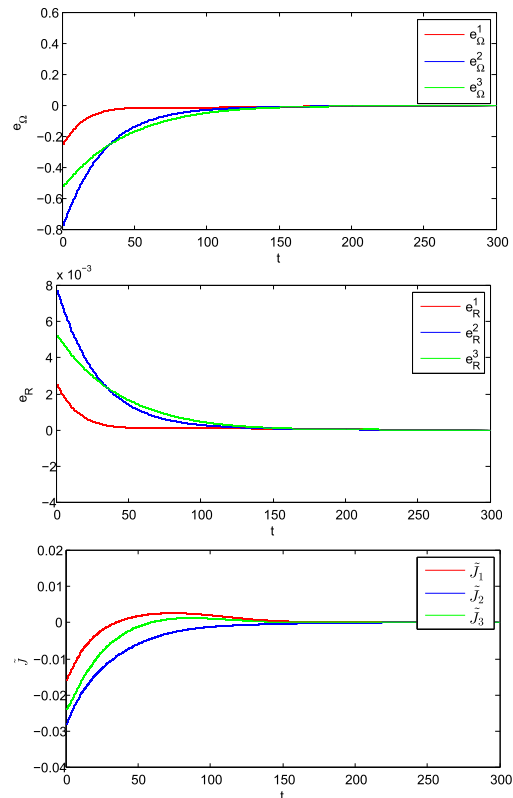


FIGURE 2. Angular velocity e_Ω , attitude error vector e_R , and inertia matrix error \tilde{J} obtained by using Theorem 2.

vectors. For example, e_Ω^i is the i th element of e_Ω . From these figures, we can find that the values of each error vector converge to zero exponentially.

In Figures 1 and 2, the angular velocity and attitude error simulation results are similar or even consistent. For further compare the converge of Theorem 1 and Theorem 2, we take

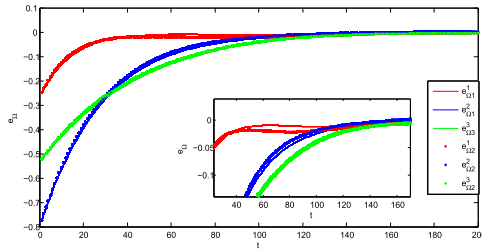


FIGURE 3. Comparison of e_Ω obtained by using the control laws of Theorem 1 and Theorem 2.

e_Ω as an example. The comparison results are shown in Figure 3, where line and dot lines represent the results of convergence affected by controller (14) and (24), respectively. In order to compare expediently, we chose the graph from 0 to 200. As can be seen from the Figure 3, the convergence speed of the dots lines is obviously faster in the latter half, i.e., the convergence result of Theorem 2 is faster than Theorem 1. Therefore, whether the inertia matrix is known or not, the adaptive controller proposed in this paper can guarantee the stability of the rigid body system (4)-(5).

VI. CONCLUSION

In this paper, an angular velocity observer is defined to obtain the estimated values of angular velocity, and attitude estimation is obtained by using the relationship between attitude and angular velocity. The errors of the estimated and true values can be proved to be zero exponentially by Lyapunov functions. On this basis, assuming we can't get the true value of inertia matrix, then it is also proved that the errors between attitude and angular velocity tend to zero exponentially by modifying the control input and other variables in the system, which further improves that the estimated value of the inertia matrix to the true value. The simulation results are provided to illustrate our proposed results. In this paper, the system disturbance is not limited to 0 or bounded, but related to angular velocity and attitude error variables. However, which is a very special model, so we need to make more general assumptions about the system disturbance in the future, and also prove which satisfies system stability.

APPENDIX A
PROOF OF EQUATION (17)

From Lemma 5, the first term of (16) satisfies

$$\frac{1}{2}\lambda_m \| e_\Omega \|^2 \leq \frac{1}{2}e_\Omega^T J e_\Omega \leq \frac{1}{2}\lambda_M \| e_\Omega \|^2. \quad (33)$$

From Lemma 4, the second term of (16) satisfies

$$\frac{k_R n_1}{n_2 + n_3} \| e_R \|^2 \leq k_R \Psi \leq \frac{k_R n_1 n_4}{n_5(n_1 - \psi)} \| e_R \|^2. \quad (34)$$

For any symmetric matrix M and vectors $x, y \in \mathbb{R}^3$, it can be obtained that

$$-\lambda_M(M) \| x \|^2 \| y \|^2 \leq Mx \cdot y = x^T M y \leq \lambda_M(M) \| x \|^2 \| y \|^2. \quad (35)$$

In fact, one has

$$-\| Mx \|^2 \| y \|^2 \leq Mx \cdot y = \| Mx \|^2 \| y \|^2 \cos\theta \leq \| Mx \|^2 \| y \|^2. \quad (36)$$

It can be noted that

$$\| Mx \|^2 = \sqrt{x^T M M x} = \sqrt{x^T M^2 x} \leq \lambda_M(M) \| x \|^2, \quad (37)$$

according to Lemma 5. From (36) and (37), (35) is satisfied. Thus, the third term of (16) satisfies

$$-c\lambda_M \| e_\Omega \|^2 \| e_R \|^2 \leq c e_\Omega^T J e_R \leq c\lambda_M \| e_\Omega \|^2 \| e_R \|^2. \quad (38)$$

From (33), (34), and (38), one can obtain that $\frac{1}{2}\lambda_m \| e_\Omega \|^2 + \frac{k_R n_1}{n_2 + n_3} \| e_R \|^2 - c\lambda_M \| e_\Omega \|^2 \| e_R \|^2 \leq V \leq \frac{1}{2}\lambda_M \| e_\Omega \|^2 + \frac{k_R n_1 n_4}{n_5(n_1 - \psi)} \| e_R \|^2 + c\lambda_M \| e_\Omega \|^2 \| e_R \|^2$.

APPENDIX B
PROOF OF EQUATION (31)

For any vectors x, y, z , the following equalities are satisfied, $\hat{x}y = x \times y, x \cdot (y \times z) = z \cdot (x \times y)$, and $x \cdot y = \text{tr}(xy^T)$. Thus, one has $(\Omega \times \tilde{J}\Omega)^T e_A = (\tilde{\Omega} \times \tilde{J}\Omega) \cdot e_A = e_A \cdot (\Omega \times \tilde{J}\Omega) = \tilde{J}\Omega \cdot (e_A \times \Omega) = \text{tr}[\tilde{J}\Omega(e_A \times \Omega)^T]$. Meanwhile, it can be noted that $(\tilde{J}\alpha)^T e_A = \text{tr}(\tilde{J}\alpha e_A^T)$.

Since, $\dot{\tilde{J}} = -\tilde{J}$, the sum of the 3rd, 4th and 8th terms of the last second equality of (31) is

$$-(\Omega \times \tilde{J}\Omega)^T e_A - (\tilde{J}\alpha)^T e_A + \frac{1}{k_J} \text{tr}(\tilde{J}\dot{\tilde{J}}) = \text{tr}[\tilde{J}(-\Omega(e_A \times \Omega)^T - \alpha e_A^T + \frac{1}{k_J} \cdot \frac{k_J}{2}(\alpha e_A^T + e_A \alpha^T - \Omega \Omega^T \hat{e}_A + \hat{e}_A \Omega \Omega^T + 2\sigma \tilde{J}))]. \quad (39)$$

It is noting that,

$$\text{tr}[\Omega \Omega^T \hat{e}_A] = \text{tr}[\Omega \Omega^T \hat{e}_A^T] = \text{tr}[\hat{e}_A^T \Omega \Omega^T] = -\text{tr}[\hat{e}_A \Omega \Omega^T] = -\text{tr}[\hat{e}_A \times \Omega \Omega^T] = -\text{tr}[\Omega \cdot (e_A \times \Omega)^T], \quad (40)$$

where, the 4th equality of (40) holds true since $\hat{e}_A^T = -\hat{e}_A$. In addition, the 5th equality of (40) holds true due to $\hat{x}y = x \times y$. Substitute (40) into (39), one can obtain the following equality

$$-(\Omega \times \tilde{J}\Omega)^T e_A - (\tilde{J}\alpha)^T e_A + \frac{1}{k_J} \text{tr}(\tilde{J}\dot{\tilde{J}}) = \sigma \text{tr}(\tilde{J}^2) = \sigma \| \tilde{J} \|_F^2,$$

which further proves the last equality of (31).

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