IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received January 25, 2022, accepted March 1, 2022, date of publication March 10, 2022, date of current version March 22, 2022.

Digital Object Identifier 10.1109/ACCESS.2022.3158345

Networked Security Observer-Based Reference Tracking Control of Stochastic Quadrotor UAV System Under Cyber-Attack:T-S Fuzzy Approach

MIN-YEN LEE[1], HAN CHIU[2], AND BOR-SEN CHEN[1,3], (Life Fellow, IEEE)

[1]Department of Electrical Engineering, National Tsing Hua University, Hsinchu 30013, Taiwan
[2]MediaTek Inc. (Headquarters), Hsinchu 30078, Taiwan
[3]Department of Electrical Engineering, Yuan Ze University, Chung-Li, Taoyuan City 32003, Taiwan

Corresponding author: Bor-Sen Chen (bschen@ee.nthu.edu.tw)

This work was supported by the Ministry of Science and Technology of Taiwan under Grant MOST 108-2221-E-007-099-MY3.

ABSTRACT In this study, a robust H_∞ networked security observer-based reference tracking control scheme is proposed for the stochastic quadrotor unmanned aerial vehicle (UAV) system under malicious attacks on the actuator and sensor of network control system (NCS). To reduce the computational burden on UAV system, the UAV system is connected with a remote computing unit and the complicated tracking control command can be calculated by remote computing unit. By using the novel discrete smoothed model, the model of attack signals on actuator and sensor can be embedded in system state of UAV and thus the attack signals as well as the quadrotor system state can be simultaneously estimated through a conventional Luenberger observer. Further, the corruption of attack signals on state estimation of UAV is also avoided. To eliminate the effect of unavailable external disturbance and intrinsic fluctuation during the reference tracking control process, a robust H_∞ networked security observer-based reference tracking control scheme is introduced to attenuate their effects on the NCS of quadrotor UAV. By using the characteristic of convex Lyapunov function, the design condition of robust H_∞ networked security observer-based tracking control is derived in terms of the nonlinear functional inequalities. Since the nonlinear functional inequalities are not easy to be solved analytically or numerically, the Takagi-Sugeno (T-S) fuzzy interpolation technique is employed to interpolate the nonlinear stochastic quadrotor NCS by a set of linear local systems via fuzzy bases. In this case, the nonlinear functional inequalities can be converted to a set of linear matrix inequalities (LMIs) which can be easily solved by the MATLAB LMI TOOLBOX. Simulation results are provided to validate the effectiveness of the proposed method in comparison with conventional robust observer-based T-S fuzzy tracking control scheme.

INDEX TERMS Network control system, security observer-based tracking control, linear matrix inequalities, UAV reference tracking control, T-S fuzzy interpolation technique.

I. INTRODUCTION

In recent years, the unmanned aerial vehicle (UAV) has attracted more attention from researchers due to its wide utilization and convenience of use. Through these advantages, UAV has extensive applications such as humanitarian relief [1], topographic survey [2], and military reconnaissance [3]. To successfully complete the tasks mentioned above, an UAV is required to track a desired trajectory. Therein, the trajectory tracking control is a popular issue for the control of UAV's flight [4], [5]. Among several types of UAV, quadrotor UAV has the capability of vertical take-off and landing (VTOL) [6]. Based on this maneuverability, a quadrotor UAV can track more kinds of trajectories than other types of UAV. Though quadrotor UAV has many appealing advantages in practical applications, the power consumption of quadrotor UAV during the flight process is a critical problem needed to be further considered. Clearly, highly power consumption limits the flight distance and using time of quadrotor UAV which may restrict its applications [7], [8].

Recently, along with the advance of communication techniques, e.g., 5G and 6G in the future smart cities [9], [10],

The associate editor coordinating the review of this manuscript and approving it for publication was Shen Yin.

the control commands of physical plants do not calculate on time by itself but by a remote computing unit through wireless network in future smart cities, e.g., network-based sampled data control. In detail, network control system separates original quadrotor UAV into two parts: local side (physical plant) and remote side (computing unit). By departing the control unit from physical systems, the weight burden and power consumption can be efficiently decreased. Even the network-based control scheme brings some advantages, it also generates several issues to be addressed. At first, due to the uncertain quality of service (QoS) in wireless communication, it may cause the occurrence of packet dropout [11]– [15]. When the packet dropout happens, the signal can not be transmitted through the wireless network channel and the receiver will not access any signal [13]. On the other hand, if the signal suffers the influence from network-induced delay, it can not be transmitted to the receiver on time [15]. In these two cases, the overall network-based system is delay-dependent and the corresponding control issues are more difficult than the delay-free network.

On the other hand, due to the vigorous development of network communication and its wide application in real world, network security has become a significant issue and has gained many interests of researchers and institutes. Even multiple methods have been developed to maintain the safety of network [16]– [18], malicious attack signals may enter the network via security breach. These attacks will disturb the transmission of signals in network channel, consequently interfering the operation of whole network system. Since these attacks are unavailable signals for the designer, the effects of malicious attacks are hard to be eliminated. Conventionally, to estimate these attack signals, the singular descriptor-based observers are widely applied in the field of fault estimation (FE) [19]– [21]. Other than the conventional descriptor system for FE, some modified attack signal models are provided and it has several fruitful results, e.g., the neural network based fault tolerant control (FTC) for Markovian jumps system in [22] and FTC design with sliding-mode observer for cyber-physicals systems in [23]. However, due to the complicated algebraic equation constraints, these descriptor-based observers are hard to be implemented for most of practical applications.

In spite of malicious attacks, quadrotor UAV will also suffer from unavailable external disturbances in physical plant, e.g., a wind gust against the quadrotor UAV. Otherwise, the continuous perturbation from the motors of quadrotor can be described as a stochastic intrinsic fluctuation of the system. These effects will degrade the performance of the system and may lead to the instability of system. As a result, the tracking control scheme should further consider the attenuation of these effects to improve the tracking control performance of the NCS of quadrotor UAV. To the best of authors' knowledge, the network control design for the quadrotor is still very few. Besides, the effect of malicious attack signals on the network is always neglected in the previous studies for the simplicity of the design. Motivated by the above discussions, the authors address on the estimation of malicious attack signals and the consequent attack–tolerant tracking control for the stochastic network-based quadrotor UAV system under cyber-attack and external disturbances.

In this study, the robust H_∞ networked security observer-based reference tracking control scheme is proposed to guarantee that the networked control quadrotor UAV can gradually approach to the desired attitude and reference path under the influence of malicious attacks, external disturbances and intrinsic random perturbations. By introducing the structure of network system, the stochastic dynamic model of networked control quadrotor UAV is constructed. With the help of discrete smoothed model, malicious attacks on actuator and sensor of quadrotor UAV through network communication channels can be effectively described and embedded in the augmented states of an augmented system consisted of quadrotor model and smoothed models of attack signals. Therefore, the attack signals and states of quadrotor can be estimated by a conventional Luenberger observer. Then, the estimated state variables and attack signals are used for the networked security tracking control of stochastic NCS of quadrotor UAV under cyber-attack. By utilizing the convex Lyapunov function, the reference tracking control design is transformed to equivalent nonlinear functional inequalities problem. Since the nonlinear functional inequalities problem for robust H_∞ networked security reference tracking control design problem is not easily to be solved, the Takagi-Sugeno (T-S) fuzzy model [27] is utilized to approximate the nonlinear stochastic system of quadrotor by interpolating a set of several linearized local systems. After applying the T-S fuzzy model, the H_∞ networked security reference tracking control problem can be transformed to a linear matrix inequalities (LMIs)-constrained problem, which can be efficiently solved by the MATLAB LMI TOOLBOX. A simulation example of a single UAV to track a upwards round-shape trajectory is provided to validate the performance of proposed robust H_∞ networked security observer-based reference tracking scheme. Further, the conventional T-S fuzzy observer-based tracking control scheme is used for performance comparison.

The main contributions of this work are described as follows:

1) A novel discrete smoothed model is utilized for modeling unavailable cyber-attack signals on the network control system. As a result, through a conventional Luenberger observer, the attack signals on NCS of quadrotor UAV can be estimated with the system states simultaneously and thus the networked security observer-based reference tracking control scheme of quadrotor UAV NCS can be guaranteed.

2) A H_∞ networked security observer-based tracking control design is proposed for quadrotor NCS through the estimated attack signals. In order to avoid solving highly complicate nonlinear observer-based H_∞ network reference tracking control design problem of stochastic quadrotor NCS, T-S fuzzy model is proposed to interpolate the nonlinear stochastic UAV networked system via several local linearized systems to simplify the design of controller and observer in

the H_∞ networked observer-based security tracking control of quadrotor UAV system under cyber-attack and external disturbance. We only need to solve a set of LMIs via MATLAB LMI TOOLBOX to obtain fuzzy control gains and observer gains of H_∞ fuzzy networked security observer-based reference tracking control strategy of quadrotor UAV NCS.

The study is organized as follows. Preliminaries and the problem formulation of the network observer-based tracking control system of UAV under cyber-attack are presented in Section II. In Section III, the stochastic H_∞ networked observer-based security tracking analysis is provided. T-S fuzzy model is introduced to cope with the nonlinear stochastic H_∞ networked observer-based security tracking control problem in Section IV. The stochastic robust H_∞ fuzzy networked observer-based security tracking control design scheme is also proposed in this section. A simulation example of quadrotor UAV flighting along a reference path under attack signals and external disturbances is given to validate the effectiveness of the proposed robust H_∞ fuzzy networked security observer-based tracking control scheme in Section V. The conclusion is made in Section VI.

Notation: A^T: the transpose of matrix A; $A \geq 0 (A > 0)$: symmetric positive semi-definite (symmetric positive definite) matrix A, respectively; I_n: the n-dimensional identity matrix; $\|x\|_2$: the Euclidean norm for the given vector x; $l^2_{\mathcal{F}}(\mathbb{R}_{\geq 0}, \mathbb{R}^n) = \{\upsilon(k) : \mathbb{R}_{\geq 0} \to \mathbb{R}^n \mid (E\{\sum_{k=0}^{k_f} \upsilon^T(k)\upsilon(k)\})^{\frac{1}{2}} < \infty\}$; $E\{\cdot\}$: the expectation operator; $diag(A, B) \triangleq \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$; $\begin{bmatrix} A & C^T \\ C & B \end{bmatrix} \triangleq \begin{bmatrix} A & C^T \\ * & B \end{bmatrix}$, $eig\{A\}$ denotes the set of eigenvalues of A; col$[D]$ denotes the column space of D.

II. SYSTEM DESCRIPTION AND PRELIMINARIES

A. DYNAMIC MODEL OF QUADROTOR UAV

In this study, for more practical applications, the position and attitude of the quadrotor UAV are simultaneously considered in the dynamic model. The position of the quadrotor UAV is described by three coordinates (x, y, z) of its mass center w.r.t an inertial frame associated with the unit vector basis (e_x, e_y, e_z) in Fig. 1. The attitude of the quadrotor is denoted by three Euler angles (ϕ, θ, ψ). These three angles are roll angle $(-\frac{\pi}{2} < \phi < \frac{\pi}{2})$, pitch angle $(-\frac{\pi}{2} < \theta < \frac{\pi}{2})$, and yaw angle $(-\pi < \psi < \pi)$ which define the orientation vector of quadrotor w.r.t a body frame associated with the unit vector basis (e_{bx}, e_{by}, e_{bz}) as shown in Fig. 1. By Newton-Euler method, the quadrotor UAV dynamic model in Fig. 1 can be represented as the following equations [28]:

$$\begin{aligned} \dot{X}(t) &= f(X(t)) + g(X(t))U(t) + \upsilon(t) \\ Y(t) &= C(X(t)) + n(t) \end{aligned} \tag{1}$$

where $X(t) = [x_1(t), x_2(t), y_1(t), y_2(t), z_1(t), z_2(t), \phi_1(t), \phi_2(t), \theta_1(t), \theta_2(t), \psi_1(t), \psi_2(t)]^T$ is the system state, $x_1(t), y_1(t), z_1(t) \in \mathbb{R}$ are the positions of the quadrotor UAV in the inertial frame, $x_2(t), y_2(t), z_2(t) \in \mathbb{R}$ are the velocities of the quadrotor UAV in the inertial frame, $\phi_1(t), \theta_1(t), \psi_1(t) \in \mathbb{R}$ are the attitudes of the quadrotor UAV in the body frame and $\phi_2(t), \theta_2(t), \psi_2(t) \in \mathbb{R}$ are the angular velocities of the quadrotor UAV in the body frame. $U(t) = [F(t), \tau_\phi(t), \tau_\theta(t), \tau_\psi(t)]^T$ is the control input of the system, $F(t) \in \mathbb{R}$ denotes total thrust and $\tau_\phi(t), \tau_\theta(t), \tau_\psi(t) \in \mathbb{R}$ denote rotational torques of Euler angles $\phi(t)$, $\theta(t)$, and $\psi(t)$, respectively, $\upsilon(t) = [0, \upsilon_x(t), 0, \upsilon_y(t), 0, \upsilon_z(t), 0, \upsilon_\phi(t), 0, \upsilon_\theta(t), 0, \upsilon_\psi(t)]^T$ is the external disturbance of the system, $C(X(t)) : \mathbb{R}^n \to \mathbb{R}^m$ is the nonlinear output matrix and $n(t)$ is the measurement noise. The system matrices in (1) are given as:

$$\begin{aligned} &f(X(t)) \\ &= [x_2(t), -\frac{d_x}{m}x_2(t), y_2(t), -\frac{d_y}{m}y_2(t), z_2(t), \\ &\quad -g - \frac{d_z}{m}z_2(t), \phi_2(t), \frac{J_\theta - J_\psi}{J_\phi}\theta_2(t)\psi_2(t) - \frac{d_\phi}{J_\phi}\phi_2(t), \\ &\quad \theta_2(t), \frac{J_\psi - J_\phi}{J_\theta}\psi_2(t)\phi_2(t) - \frac{d_\theta}{J_\theta}\theta_2(t), \\ &\quad \psi_2(t), \frac{J_\phi - J_\theta}{J_\psi}\phi_2(t)\theta_2(t) - \frac{d_\psi}{J_\psi}\psi_2(t)]^T \end{aligned}$$

$$g(X(t)) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ (\cos\phi_1(t)\sin\theta_1(t)\cos\psi_1(t) + \sin\phi_1(t)\sin\psi_1(t))\frac{1}{m} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ (\cos\phi_1(t)\sin\theta_1(t)\sin\psi_1(t) - \sin\phi_1(t)\cos\psi_1(t))\frac{1}{m} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \cos\phi_1(t)\cos\theta_1(t)\frac{1}{m} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{1}{J_\phi} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{J_\theta} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{J_\psi} \end{bmatrix}$$

where $m \in \mathbb{R}_{\geq 0}$ is the total mass of the quadrotor UAV, $g \in \mathbb{R}_{\geq 0}$ is the gravitational acceleration, $J_\phi, J_\theta, J_\psi \in \mathbb{R}_{\geq 0}$ are the moments of inertia of $\phi(t), \theta(t)$, and $\psi(t)$, respectively, and $d_x, d_y, d_z, d_\phi, d_\theta, d_\psi \in \mathbb{R}_{\geq 0}$ represent aerodynamic damping coefficients of the quadrotor UAV.

In the real world, the quadrotor UAV will be disturbed by not only external disturbances but also internal fluctuations. To make the quadrotor UAV system more practical, the Wiener process is used to formulate the internal random parametric fluctuations in the quadrotor UAV. Thus, the nonlinear stochastic quadrotor UAV can be written as follows:

$$\begin{aligned} dX(t) &= (f(X(t)) + g(X(t))U(t) + \upsilon(t))dt \\ &\quad + \sigma(X(t))dW(t) \\ Y(t) &= C(X(t)) + n(t) \end{aligned} \tag{2}$$

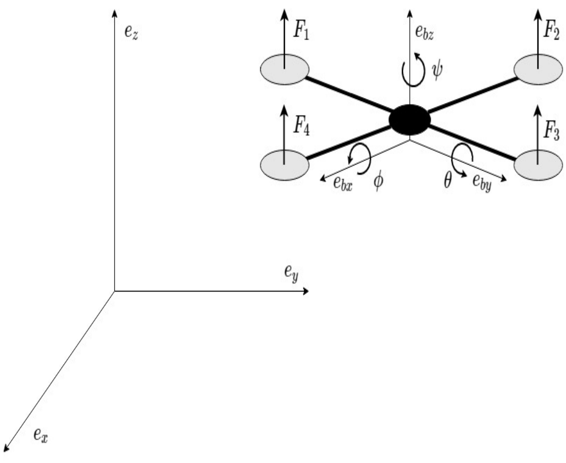

FIGURE 1. Structure of quadrotor UAV.

where $W(t)$ denotes the 1-D Wiener process and $\sigma(X(t))dW(t)$ denotes the nonlinear state-dependent parametric fluctuations. The Wiener process $W(t)$ is defined on the complete filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\in\mathbb{R}_{\geq 0}}, \mathcal{P})$ where Ω denotes the sample space, σ-field $\mathcal{F}_t$ is generated by Wiener process $W(s)$ for $s < t$, $\mathcal{F} = \cup_{t\geq 0}\mathcal{F}_t$ and $\mathcal{P}$ is the probability measure [37]. It is assumed that the nonlinear functions $f(X(t))$, $g(X(t))$ and $\sigma(X(t))$ are local-Lipschiz [37].

B. NETWORK CONTROL SYSTEM

Under the framework of network control system (NCS), the quadrotor is controlled through a remote computing unit (controller). The schematic of NCS of quadrotor UAV is shown in Fig. 2. Through the sensor, output data of quadrotor UAV is sampled and transmitted to the observer on the remote side. Then, the control commands are computed at each computing unit and transmitted via wireless network channel to zero-order holder (ZOH) at every sampling period. ZOH will transmit the control commands to actuator and this makes the control commands become a continuous control signal within a sampling period. It is worth to point out that the control signals stored in ZOH will be updated until new control commands are received.

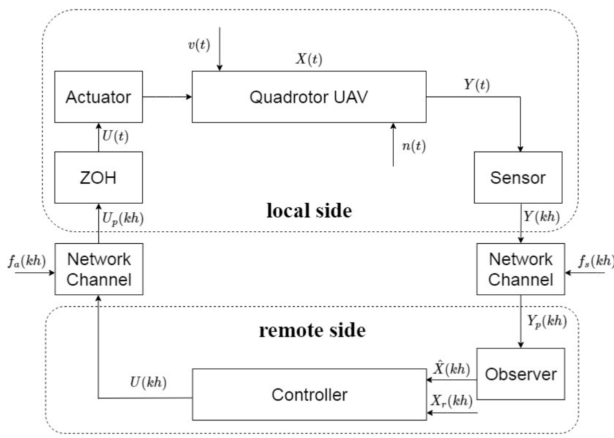

FIGURE 2. Structure of NCS of quadrotor UAV.

Based on the NCS framework in Fig. 2, the remote computing unit only receives the plant information at each sampling period. In this case, instead of using a continuous-time observer, a discrete-time observer is more appealing to be utilized to save the computation resource in the remote side. However, due to the fact that the local UAV plant in (2) is a continuous-time system and the observer is a discrete-time system, it is almost impossible to simultaneously analyze the estimation performance of observer and tracking performance of local UAV system in (2). In this situation, the digital redesign [31] should be utilized to discretilize the continuous plant for the simplicity of design. Nevertheless, due to the stochastic Wiener process and nonlinearity system characteristics in UAV system, it is impossible to obtain the exact form of system matrices of the UAV system by using conventional discretization method. As a result, the Euler-Maruyama method in [30] is applied to discretilize the UAV system and the nonlinear stochastic UAV dynamic system can be approximately represented by the following difference equation

$$\begin{aligned} X((k+1)h) &= X(kh) + h(f(X(kh)) + g(X(kh))U(kh) \\ &\quad + \upsilon(kh)) + \sigma(X(kh))\Delta W(kh) \\ Y(kh) &= C(X(kh)) + n(kh) \end{aligned} \tag{3}$$

where $\Delta W(kh) = W((k+1)h) - W(kh)$ $h > 0$ is the sampling period, $Y(kh)$ denotes the sampled system output, $n(kh)$ denotes the measurement noise of $Y(kh)$ by sensor.

Remark 1: Some properties of $\Delta W(kh)$ are given as follows [41]:

$$E\{\Delta W(kh)\} = 0, \quad E\{\Delta W^2(kh)\} = h$$

Remark 2: For the proposed NCS in (3), the control scheme can be regarded as conventional time-trigger NCS mechanism. Recently, some advanced NCS control mechanisms have been proposed such as event-trigger NCS mechanism or self-trigger NCS mechanism [32]. By applying these advanced mechanisms, it can further improve the control performance and save the limited communication resource. At the same time, it also increases the analysis difficulty during the controller/observer design.

For NCS, time-varying delay is an important issue that needs to be considered. According to Fig. 2, there are uplink and downlink networked communication channels in the network control system. As signals transmit through these two channels, signals may suffer influence from time-varying delays. These delays can be represented as follows: $\tau_1(k)$ is the downlink delay from controller to ZOH and $\tau_2(k)$ is the uplink delay from sensor to observer. By taking these two time-varying delays into consideration, the NCS-based control system of quadrotor UAV can be formulated as [33]:

$$\begin{aligned} X((k+1)h) &= X(kh) + h(f(X(kh)) + g(X(kh)) \\ &\quad \times U_p(kh) + \upsilon(kh)) + \sigma(X(kh))\Delta W(kh) \\ Y(kh) &= C(X(kh)) + n(kh) \\ Y_p(kh) &= (1-\beta(kh))Y(kh) + \beta(kh)Y(kh-\tau_2(k)) \end{aligned} \tag{4}$$

and

$$U_p(kh) = (1-\delta(kh))U(kh) + \delta(kh)U(kh-\tau_1(k)) \quad (5)$$

From above equations, $Y_p(kh)$ denotes the signal of system output which is received by the observer. Similarly, $U_p(kh)$ is the signal of control command which is received by ZOH. Both of them are transmitted through the wireless network channel and may be influenced by time-varying delays. To formulate the influence of delay effect, the occurrence of time-varying delays is assumed to satisfy with Bernoulli sequence. Thus, $\delta(kh)$ and $\beta(kh)$ are Bernoulli processes as follows:

$$\begin{cases} P\{\delta(kh)=1\} = E\{\delta(kh)\} = \bar{\delta} \\ P\{\delta(kh)=0\} = 1-E\{\delta(kh)\} = 1-\bar{\delta} \end{cases}$$
$$\begin{cases} P\{\beta(kh)=1\} = E\{\beta(kh)\} = \bar{\beta} \\ P\{\beta(kh)=0\} = 1-E\{\beta(kh)\} = 1-\bar{\beta} \end{cases}$$

where $\bar{\delta} > 0$ and $\bar{\beta} > 0$ are the known probability of time-varying delay occurring in the signal transmission from controller to ZOH and sensor to observer through wireless network channels, respectively.

Remark 3: As time-varying delay $\tau_1(k)$ occurs, i.e., $\delta(kh) = 1$, the control signal $U(kh)$ can not be transmitted on time. Since the quadrotor UAV can not be out of control, the previously received control signal is used to compensate the delayed control signal. If time-varying delay $\tau_1(k)$ does not occur, $\delta(kh) = 0$, $U(kh)$ can be transmitted through the wireless network successfully and the system plant can receive control command on time. According to the analysis above, $U_p(kh)$ can be constructed in the form of (5). Similarly, $Y_p(kh)$ has the same form in (4).

Assumption 1: In this study, the time-varying delays satisfy the following boundary conditions:

$$d_{1m}h \le \tau_1(k) \le d_{1M}h, \quad d_{2m}h \le \tau_2(k) \le d_{2M}h$$

where $d_{1m}, d_{1M}, d_{2m}, d_{2M}$ are non-negative integers.

As signals transmitting in the wireless network channels, there may exist some malicious attacks from attackers and it tries to degrade the performance of NCS and even let NCS become unstable. Thus, these attacks need to be considered in NCS. In Fig. 2, two malicious attacks $f_a(kh)$ and $f_s(kh)$ will influence on the reference tracking performance and the state estimation of NCS of quadrotor UAV via wireless network, respectively. In this situation, sensor and actuator will receive wrong information from network channel and may cause whole system not being controlled correctly. That is, these malicious attacks can be equivalently regarded as sensor fault and actuator fault in quadrotor UAV NCS. Then, the malicious attacks should be considered in the security control design of NCS and the NCS of quadrotor UAV should be modified as:

$$\begin{aligned} X((k+1)h) &= X(kh) + h(f(X(kh)) + g(X(kh)) \\ &\quad \times (U_p(kh) + f_a(kh)) + \upsilon(kh)) \\ &\quad + \sigma(X(kh))\Delta W(kh) \\ Y(kh) &= C(X(kh)) + n(kh) + D(X(kh))f_s(kh) \\ Y_p(kh) &= (1-\beta(kh))Y(kh) + \beta(kh)Y(kh-\tau_2(k)) \end{aligned} \quad (6)$$

where $f_a(kh)$ is the actuator attack signal, $f_s(kh)$ is the sensor attack signal and $D(X(kh))$ is the effect matrix of sensor attack signal $f_s(kh)$.

Remark 4: Since actuator attack signal $f_a(kh)$ disturbs the transmission of control signal $U_p(kh)$ through the wireless network channel, actuator will receive $f_a(kh)$ and $U(kh)$ simultaneously, i.e., the effect matrix of actuator attack signal $f_a(kh)$ is the same as the input matrix $g(X(kh))$.

Assumption 2: The system state $x(kh)$, malicious actuator attack signal $f_a(kh)$, malicious sensor attack signal $f_s(kh)$ and external disturbance $\upsilon(kh))$ are assumed to be zero before the tracking control process, i.e., $x(kh) = 0, f_a(kh) = 0, f_s(kh) = 0$ and $\upsilon(kh)) = 0$, $\forall k < 0$.

Since the actuator attack $f_a(kh)$ and sensor attack $f_s(kh)$ are unavailable signals, they can not be estimated by the conventional estimator from the discrete stochastic NCS in (6) directly. To simplify the attack signal estimation, the discrete-time smoothed models of attack signals $f_a(kh)$ and $f_s(kh)$ are constructed. By the discrete-time smoothed models of attack signals $f_a(kh)$ and $f_s(kh)$, the conventional Luenberger observer can be designed to estimate system states and attack signals simultaneously. Similar to [34], the discrete-time smoothed model of actuator attack signal $f_a(kh)$ is proposed as follows:

$$F_a((k+1)h) = A_{f_a}F_a(kh) + M_{f_a}\delta_a(kh) \quad (7)$$

where $F_a(kh) = [f_a^T(kh), f_a^T((k-1)h), \ldots, f_a^T((k-d)h)]^T$, $M_{f_a} = [I_{n_a}, 0, \ldots, 0]^T$, $\delta_a(kh) = f_a((k+1)h) - \sum_{i=0}^{d} a_i f_a((k-i)h)$ denotes the extrapolation error of $f_a((k+1)h)$, $\{a_i\}_{i=0}^{d}$ are the extrapolation coefficients and

$$A_{fa} = \begin{bmatrix} a_0 I_{n_a} & a_1 I_{n_a} & a_2 I_{n_a} & \cdots & 0 \\ I_{n_a} & 0 & \cdots & & 0 \\ 0 & I_{n_a} & \ddots & & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & \cdots & 0 & I_{n_a} & 0 \end{bmatrix} \quad (8)$$

Similar to (7), the discrete-time smoothed model of sensor attack signal $f_s(kh)$ can be described as follows:

$$F_s((k+1)h) = A_{f_s}F_s(kh) + M_{f_s}\delta_s(kh) \quad (9)$$

where $F_s(kh) = [f_s^T(kh), f_s^T((k-1)h), \ldots, f_s^T((k-d)h)]^T$, $M_{f_s} = [I_{n_s}, 0, \ldots, 0]^T$, $\delta_s(kh) = f_s((k+1)h) - \sum_{i=0}^{d} b_i f_s((k-i)h)$ denotes the extrapolation error of $f_s((k+1)h)$, $\{b_i\}_{i=0}^{d}$ are the extrapolation coefficients and

$$A_{fs} = \begin{bmatrix} b_0 I_{n_s} & b_1 I_{n_s} & b_2 I_{n_s} & \cdots & b_d I_{n_s} \\ I_{n_s} & 0 & \cdots & & 0 \\ 0 & I_{n_s} & \ddots & & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & \cdots & 0 & I_{n_s} & 0 \end{bmatrix} \quad (10)$$

Remark 5: Since the attack signals $\{f_a(kh), f_s(kh)\}$ are unavailable and their characteristics are unknown, it is not easy to update the extrapolation coefficients $\{a_i, b_i\}_{i=0}^{d}$ of $\{f_a(kh), f_s(kh)\}$ by the conventional adaptive algorithm based on the adaptive errors $f_a(kh) - \hat{f}_a(kh)$ and $f_s(kh) - \hat{f}_s(kh)$. Based on the extrapolation method, one practical selection of extrapolation parameters for two smoothed signal models in (7) and (9) can be given as: (i)$a_i \geq a_{i-1} \geq 0$, $b_i \geq b_{i-1} \geq 0, \forall i \in \{1, \cdots, d\}$ (ii) $\sum_{i=0}^{d} a_i = 1$ and $\sum_{i=0}^{d} b_i = 1$. For the first rule, it is expected that $f_a(kh)$ and $f_s(kh)$ are more related to $f_a((k-1)h)$ and $f_s((k-1)h)$, respectively, and thus the extrapolation parameters $\{a_i, b_i\}_{i=0}^{d}$ should be chosen as positive decreasing sequences. On the other side, to avoid the over-extrapolation, the summation of these extrapolation coefficients should be one.

Remark 6: The proposed smoothed signal models in (7) and (9) are the modification conventional Kalman fix-laged smoothed model [24], [25], in which the future signals are the combination of current signals and extrapolation errors, i.e., $a_0 = b_0 = 1$ and $\{a_i = 0, b_i = 0\}_{i=1}^{d}$ in the (7) and (9), respectively. To further use the lag information, the modified smoothed signal models in (7) and (9) include the extrapolation coefficients which can be selected by designer. In this case, two-side information of actuator/sensor attack signals can be utilized to achieve the better estimation of actuator/sensor attack signals in (12). Further, the more precise estimation $\hat{F}_a(kh)$ and $\hat{F}_s(kh)$ can be employed for the observer-based reference tracking control to efficiently eliminate the effect of attack signals by the proposed H_∞ observer-based reference control strategy of UAV

Remark 7: In general, the selection of these extrapolation parameters $\{a_i, b_i\}_{i=0}^{d}$ is not unique and there exist several adaptive parameter estimation methods (e.g., recursive least squares filter [26]) to estimate these extrapolation coefficients. However, there exists some difficulties to be overcome. At first, since the attack signals are unavailable, it is not easy to update these extrapolation coefficients via their adaptive estimation error for the conventional recursive algorithms. Secondly, in the networked-based control structure, the feedback information by the parameter adaptive algorithm for the UAV system of extrapolation coefficients will be delayed and it will make the proposed H_∞ network observer-based reference control strategy become more difficult to design. Moreover, it may destroy the performance of H_∞ observer-based reference control.

By combining (6), (7), and (9), we define the augmented NCS of quadrotor UAV as $\bar{X}(kh) = [X^T(kh)\ F_a^T(kh)\ F_s^T(kh)]^T$ and the corresponding augmented system state is formulated as follows:

$$\begin{aligned}\bar{X}((k+1)h) &= \bar{f}(\bar{X}(kh)) + \bar{g}(\bar{X}(kh))U_p(kh) \\ &\quad + \bar{G}\bar{\upsilon}(kh) + \bar{\sigma}(\bar{X}(kh))\Delta W(kh) \\ Y(kh) &= \bar{C}(\bar{X}(kh)) + \bar{D}\bar{\upsilon}(kh) \\ Y_p(kh) &= (1-\beta(kh))Y(kh) + \beta(kh)Y(kh-\tau_2(k))\end{aligned} \tag{11}$$

with the following system matrices:

$$\begin{aligned}\bar{f}(\bar{X}(kh)) &= \begin{bmatrix} X(kh) + h(f(X(kh)) + g(X(kh))C_{f_a}F_a(kh)) \\ A_{f_a}F_a(kh) \\ A_{f_s}F_s(kh) \end{bmatrix} \\ \bar{g}(\bar{X}(kh)) &= [g^T(X(kh))\ 0\ 0]^T \\ \bar{G} &= \begin{bmatrix} hI_{12} & 0 & 0 & 0 \\ 0 & 0 & M_{f_a} & 0 \\ 0 & 0 & 0 & M_{f_s} \end{bmatrix}, \\ \bar{\upsilon}(kh) &= \begin{bmatrix} \upsilon(kh) \\ n(kh) \\ \delta_a(kh) \\ \delta_s(kh) \end{bmatrix} \\ \bar{\sigma}(\bar{X}(kh)) &= [\sigma^T(X(kh))\ 0\ 0]^T, \quad \bar{D} = [0\ I_m\ 0\ 0], \\ \bar{C}(\bar{X}(kh)) &= C(X(kh)) + D(X(kh))C_{f_s}F_s(kh) \\ C_{fa} &= [I_{n_a}\ \ 0\ \ 0\ \ \ldots\ \ 0], \\ C_{f_s} &= [I_{n_s}\ \ 0\ \ 0\ \ \ldots\ \ 0]\end{aligned}$$

In general, due to the nonlinear system funtions in (11), the observability of augmented system in (11) can not be easily ensured. To faciliate the observer design in sequel, the following assumption is made.

Assumption 3: The augmented quadrotor NCS in (11) is observable.

Through the assumption above, the following nonlinear Luenberger observer is employed to estimate the states of the augmented system in (11):

$$\begin{aligned}\hat{\bar{X}}((k+1)h) &= \bar{f}(\hat{\bar{X}}(kh)) + \bar{g}(\hat{\bar{X}}(kh))U(kh) \\ &\quad + L(\hat{\bar{X}}(kh))(Y_p(kh) - \hat{Y}(kh)) \\ \hat{Y}(kh) &= \bar{C}(\hat{\bar{X}}(kh))\end{aligned} \tag{12}$$

where $\hat{\bar{X}}(kh)$ is the estimated state of augmented quadrotor NCS (11), $\hat{Y}(kh)$ is estimated measurement output and $L(\hat{\bar{X}}(kh))$ is the nonlinear observer gain. observer gain.

Remark 8: If we estimate quadrotor state $X(kh)$ from quadrotor NCS in (6), the attack signals $f_a(kh)$ and $f_s(kh)$ will deteriorate the state estimation. While attack signals $f_a(kh)$ and $f_s(kh)$ are embedded in the state of augmented system in (11), we not only estimate $X(kh)$, $f_a(kh)$ and $f_s(kh)$ simultaneously by the observer in (12) but also avoid the corruption of attack signals $f_a(kh)$ and $f_s(kh)$ on state estimation. This is the main merit of the proposed discrete-time smoothed model of attack signals in (7) and (9).

Remark 9: Since the control command can be directly calculated and transmitted to observer system in (12), the control command $U(kh)$ in observer system is delay-free. However, due to the effect of networked induced delay in the network communication link in Fig. 2, the control command $U_p(kh)$ in the augmented NCS of quadrotor UAV in (11) is delay-dependent. Similarly, the estimated measurement output $\hat{Y}(kh)$ is delay-free for the observer system in (12) and the real measurement output $Y_p(kh)$ received by the observer is delay-dependent.

In real application, the quadrotor UAV is controlled to track the desired path to complete some tasks. To generate the desired tracking control trajectory, the reference tracking model is used to generate desired reference state [35]:

$$X_r((k+1)h) = A_r X_r(kh) + (I - A_r)r(kh) \tag{13}$$

where $X_r(kh)$ is the reference state to be tracked, A_r denotes a specific stable matrix and $r(kh)$ is the reference input.

Remark 10: At the steady state, $X_r((k+1)h) = X_r(kh)$ and the reference model will become

$$(I - A_r)X_r(kh) = (I - A_r)r(kh)$$

If the eigenvalues of matrix A_r are all inside unit circle in the z-complex domain, i.e., $|z| < 1$, the desired trajectory $X_r(kh)$ is equal to the reference input $r(kh)$ at the steady state, i.e., $X_r(kh)$ will approach to $r(kh)$ at the steady state. Besides, A_r specifies the transient characteristics of $X_r(kh)$ to approach the desired reference $r(kh)$.

C. PROBLEM FORMULATION

In this study, the nonlinear Luenberger observer in (12) is employed to estimate quadrotor states $X(kh)$ as well as malicious attacks $f_a(kh)$ and $f_s(kh)$ in (11). If quadrotor states and malicious attack signals on actuator and sensor can be effectively estimated, their effects on the system can be reduced or even eliminated. On the other hand, in order to make the quadrotor UAV track the desired trajectory and finish its task efficiently, an observer-based tracking controller $U(kh) = K(\hat{\bar{X}}(kh), X_r(kh))$ based on $\hat{\bar{X}}(kh)$ and $X_r(kh)$ is needed. Since there are some disturbances and measurement noises during the flight of UAV which are unavoidable, the following robust H_∞ networked security observer-based reference tracking control design strategy of stochastic quadrotor NCS is proposed to effectively attenuate these undesired effects on the tracking control performance below a prescribed disturbance attenuation level ρ:

$$\begin{aligned} &H_\infty(L(\hat{\bar{X}}(kh)), U(kh)) \\ &= \sup_{\substack{\bar{v}_1(kh) \\ \in l^2_{\mathcal{F}}[0,k_f]}} \frac{\begin{aligned}E\{\textstyle\sum_{k=0}^{k_f} [(\bar{X}(kh) - \hat{\bar{X}}(kh))^T Q_1 (\bar{X}(kh) \\ -\hat{\bar{X}}(kh)) + (X(kh) - X_r(kh))^T Q_2 (X(kh) \\ -X_r(kh))] - V_1(\bar{X}(0), \hat{\bar{X}}(0), X_r(0))\}\end{aligned}}{E\{\sum_{k=0}^{k_f} \bar{v}_1^T(kh)\bar{v}_1(kh)\}} \\ &\le \rho \end{aligned} \tag{14}$$

where $V_1(\bar{X}(0), \hat{\bar{X}}(0), X_r(0))$ denotes the effect of initial condition of augmented system in (11) to be deduced on the state estimation of the observed augmented system and the tracking of reference system, $Q_1 \ge 0$ and $Q_2 \ge 0$ denote the weighting matrix of the estimation error and the tracking error, respectively, $k_f \in \mathbb{N}$ denotes the terminal time, and $\bar{v}_1(kh) = [\bar{v}^T(kh)\ r^T(kh)\ ]^T$. If one could specify a tracking controller $U(kh)$ and a nonlinear observer gain $L(\hat{\bar{X}}(kh))$ such that (14) holds, then the worst-case effect of external disturbance $\bar{v}_1(kh)$ on the estimation error $\bar{X}(kh) - \hat{\bar{X}}(kh)$ and tracking error $X(kh) - X_r(kh)$ can be attenuated to a prescribed disturbance attenuation level ρ from the viewpoint of energy.

III. H_∞ NETWORKED SECURITY OBSERVER-BASED REFERENCE TRACKING CONTROL DESIGN OF QUADROTOR UAV NCS

In this section, the robust networked security observer-based tracking control design for nonlinear quadrotor UAV NCS is investigated. To begin with, we define the augmented state estimation error vector as $e(kh) = \bar{X}(kh) - \hat{\bar{X}}(kh)$ and the corresponding dynamic of $e(kh)$ can be derived as:

$$\begin{aligned} &e((k+1)h) \\ &= \bar{f}(\bar{X}(kh)) - \bar{f}(\bar{X}(kh) - e(kh)) + \bar{G}\bar{v}(kh) \\ &\quad + \bar{g}(\bar{X}(kh))(1 - \delta(kh))U(kh) \\ &\quad + \bar{g}(\bar{X}(kh))\delta(kh)U(kh - \tau_1(k)) \\ &\quad - \bar{g}(\bar{X}(kh) - e(kh))U(kh) + \bar{\sigma}(\bar{X}(kh))\Delta W(kh) \\ &\quad - L(\bar{X}(kh) - e(kh))[(1 - \beta(kh))(\bar{C}(\bar{X}(kh)) \\ &\quad + \bar{D}\bar{v}(kh)) - \bar{C}(\bar{X}(kh) - e(kh))] \\ &\quad - L(\bar{X}(kh) - e(kh))\beta(kh)(\bar{C}(\bar{X}(kh - \tau_2(k))) \\ &\quad + \bar{D}\bar{v}(kh - \tau_2(k))) \end{aligned} \tag{15}$$

Next, since the augmented state $\bar{X}(kh)$ includes $X(kh)$, $F_a(kh)$ and $F_s(kh)$, to be consistent in dimension, the reference tracking model in (13) should be extended as follows:

$$\bar{X}_r((k+1)h) = \bar{A}_r \bar{X}_r(kh) + \bar{B}_r r(kh) \tag{16}$$

where $\bar{X}_r(kh) = [X_r^T(kh)\, 0\, 0]^T$, $\bar{A}_r = diag(A_r, 0, 0)$ and $\bar{B}_r = [(I - A_r)^T\ 0\ 0]^T$.

Then, to simplify the design procedure of robust H_∞ networked security observer-based tracking control design in (14), the dynamic system in (11), estimation error dynamic in (15) and the reference dynamic system in (16) are augmented as

$$\begin{aligned} \tilde{X}((k+1)h) = &\tilde{f}(\tilde{X}(kh), kh) + \tilde{g}(\tilde{X}(kh), kh)U(kh) \\ &+ \tilde{g}_1(\tilde{X}(kh), kh)U(kh - \tau_1(k)) \\ &+ \tilde{o}_1(\tilde{X}(kh), kh)\bar{v}_1(kh) \\ &+ \tilde{o}_2(\tilde{X}(kh), kh)\bar{v}_1(kh - \tau_2(k)) \\ &+ \tilde{\sigma}(\tilde{X}(kh))\Delta W(kh) \end{aligned} \tag{17}$$

where $\tilde{X}(kh) = [\bar{X}^T(kh)\ e^T(kh)\ \bar{X}_r^T(kh)]^T$ with system matrices

$$\tilde{f}(\tilde{X}(kh), kh) = \begin{bmatrix} \bar{f}(\bar{X}(kh)) \\ \tilde{f}_1(\tilde{X}(kh), kh) \\ \bar{A}_r \bar{X}_r(kh) \end{bmatrix},$$

$$\tilde{\sigma}(\tilde{X}(kh)) = [\bar{\sigma}^T(\bar{X}(kh)), \bar{\sigma}^T(\bar{X}(kh)), 0]^T$$

$$\begin{aligned} \tilde{g}(\tilde{X}(kh), kh) = &[((1 - \delta(kh)\bar{g}^T(\bar{X}(kh))), (1 - \delta(kh)) \\ &\times \bar{g}^T(\bar{X}(kh)) - \bar{g}^T(\bar{X}(kh) - e(kh)), 0]^T \end{aligned}$$

$$\tilde{g}_1(\tilde{X}(kh), kh) = \begin{bmatrix} \delta(kh)\bar{g}(\bar{X}(kh)) \\ \delta(kh)\bar{g}(\bar{X}(kh)) \\ 0 \end{bmatrix},$$

$$\tilde{o}_1(\tilde{X}(kh)) = \begin{bmatrix} \bar{G} & 0 \\ \bar{G}-(1-\beta(kh)) \\ \times L(\bar{X}(kh)-e(kh))\bar{D} & 0 \\ 0 & \bar{B}_r \end{bmatrix}$$

$$\tilde{o}_2(\tilde{X}(kh)) = \begin{bmatrix} 0 & 0 \\ -\beta(kh)L(\bar{X}(kh)-e(kh))\bar{D} & 0 \\ 0 & 0 \end{bmatrix}$$

and $\tilde{f}_1(\tilde{X}(kh), kh) = \bar{f}(\bar{X}(kh)) - \bar{f}(\bar{X}(kh)-e(kh)) - L(\bar{X}(kh)-e(kh))\,[(1-\beta(kh))\,\bar{C}(\bar{X}(kh)) - \bar{C}(\bar{X}(kh)-e(kh))] - L(\bar{X}(kh)-e(kh))\beta(kh)\;\bar{C}(\bar{X}(kh-\tau_2(k))$.

Based on the augmented system (17), the robust H_∞ networked security observer-based reference tracking control strategy can be rewritten as

$$H_\infty(L(\hat{\bar{X}}(kh)), \bar{U}(kh)) = \sup_{\bar{\upsilon}_1(kh)\in l^2_{\mathcal{F}}[0,\infty)} \frac{E\left\{\sum_{k=0}^{k_f}(\tilde{X}^T(kh)\tilde{Q}\tilde{X}(kh)) - V(\tilde{X}(0))\right\}}{E\left\{\sum_{k=0}^{k_f}\bar{\upsilon}_1^T(kh)\bar{\upsilon}_1(kh)\right\}} \le \rho \tag{18}$$

where $V(\tilde{X}(0)) = V_1(\bar{X}(0), \hat{\bar{X}}(0), X_r(0))$ and

$$\tilde{Q} = \begin{bmatrix} Q_2 & 0 & -Q_2 \\ 0 & Q_1 & 0 \\ -Q_2 & 0 & Q_2 \end{bmatrix}$$

Based on the above analysis, the robust H_∞ networked security observer-based reference tracking control design problem in (14) for the quadrotor NCS under attack signals and external disturbances becomes how to specify $U(kh)$ and $L(\hat{\bar{X}}(kh))$ to achieve the robust H_∞ stabilization problem in (18) for the augmented system in (17) to simplify the design procedure. On the other hand, if $\bar{\upsilon}_1(kh)$ is vanished in (17), the following definition of mean square stability in probability is given to address the stability criterion in (17)

Definition 1: The nonlinear stochastic augmented UAV system in (17) satisfies the mean-square stability in probability if the following condition holds

$$E\{\tilde{X}^T(kh)\tilde{X}(kh)\} \to 0, \quad as\ k \to \infty \tag{19}$$

Due to the nonlinear system functions and the delayed external disturbance $\bar{\upsilon}_1(kh-\tau_2(k))$ with corresponding state-dependent matrix $\tilde{o}_2(\tilde{X}(kh))$, the robust tracking control design is more difficult than the conventional case (e.g., the linear system matrix with non-delay external disturbance). Thus, the following assumption and definition are made to facilitate the robust tracking control design in this study.

Assumption 4: The state trajectory of augmented system $\tilde{X}(kh)$ in (17) lies in a compact domain Ξ, i.e., $\tilde{X}(kh) \in \Xi$.

Definition 2: A function $V(x) : \mathbb{R}^n \to \mathbb{R}_{\ge 0}$ is convex Lyapunov function if (i) $V(\cdot) \ge 0$ with $V(0) = 0$ (ii) $V(\alpha x + (1-\alpha)y) \le \alpha V(x) + (1-\alpha)V(y),\ \forall \alpha \in [0,1],\ x, y \in \mathbb{R}^n$

With the help of the convex Lyapunov function $V(\cdot)$ in above definition and Assumption 4, the design method of robust H_∞ networked security observer-based reference tracking controller is given as follows:

Theorem 1: For the observer-based security tracking control design of quadrotor UAV in (11)–(13) or in the augmented system in (17), if there exist observer gain $L(\hat{\bar{X}}(kh))$, tracking controller $U(kh)$, a prescribed disturbance attenuation level ρ and convex Lyapunov function $V(\cdot)$ satisfying the following nonlinear functional inequalities

$$\begin{aligned} &\alpha E\{V(\frac{1}{\alpha}(\tilde{f}(\tilde{X}(kh, kh)) + \tilde{g}(\tilde{X}(kh), kh)U(kh) \\ &\quad + \tilde{g}_1(\tilde{X}(kh), kh)U(kh-\tau_1(k)) \\ &\quad + \tilde{\sigma}(\tilde{X}(kh))\Delta W(kh)))\} - E\{V(\tilde{X}(kh))\} \\ &\quad + E\{\tilde{X}^T(kh)\tilde{Q}\tilde{X}(kh)\} < 0 \end{aligned} \tag{20}$$

$$\sup_{\Lambda_1(kh)} \frac{(1-\alpha)\beta E\{V(\frac{1}{1-\alpha}\frac{1}{\beta}\Lambda_1(kh))\}}{\|\Lambda_1(kh)\|^2} \le \frac{\rho}{2\gamma_1}$$

$$\Lambda_1(kh) = \tilde{o}_1(\tilde{X}(kh), kh)\bar{\upsilon}_1(kh) \tag{21}$$

$$\sup_{\Lambda_2(kh)} \frac{(1-\alpha)(1-\beta)E\{V(\frac{1}{1-\alpha}\frac{1}{1-\beta}\Lambda_2(kh)))\}}{\|\Lambda_2(kh)\|^2} \le \frac{\rho}{2\gamma_2}$$

$$\Lambda_2(kh) = \tilde{o}_2(\tilde{X}(kh), kh)\bar{\upsilon}_1(kh-\tau_2(k)) \tag{22}$$

$\{\alpha, \beta\} \in (0,1)$, $k = 0, 1, \cdots, k_f$, where $\gamma_1 = \sup_{\tilde{X}\in\Xi} E\{eig(\tilde{o}_1^T(\tilde{X}(kh), kh)\tilde{o}_1(\tilde{X}(kh), kh))$ and $\gamma_2 = \sup_{\tilde{X}\in\Xi} E\{eig(\tilde{o}_2^T(\tilde{X}(kh), kh)\tilde{o}_2(\tilde{X}(kh), kh))\}$, then the robust H_∞ networked security observer-based reference tracking control strategy in (18) of quadrotor NCS is guaranteed for a prescribed disturbance attenuation level ρ. Besides, if the external disturbance $\bar{\upsilon}_1(kh)$ is vanished, the mean square stability of $\tilde{X}(kh)$ is achieved, i.e., the mean square state estimation error and mean square tracking error will converge to 0 in probability.

Proof: Please refer to Appendix A. □

Due to the characteristics of discrete-time nonlinear system and nonlinear Lyapunov function, the system characteristic which includes deterministic parts and stochastic parts are embedded in the Lyapunov function in (20). In this situation, it is not easy to decouple the deterministic parts and the stochastic parts. Thus, it can be seen that the design condition of Theorem 1 in (20) involves the increment of Wiener process $\Delta W(kh)$. In general, it is not easy to directly find a convex Lyapunov function $V(\cdot)$ with the corresponding observer gain $L(\hat{\bar{X}}(kh))$ and tracking controller $\bar{U}(kh)$ to meet the design conditions in Theorem 1.

Remark 11: In general, due to the state-dependent nonlinear matrices $\tilde{o}_1(\tilde{X}(kh), kh)$ and $\tilde{o}_2(\tilde{X}(kh), kh)$ w.r.t. the noise terms in (17), it is not easy to decouple the noise terms and its' system matrices during the derivation. However, with the utilization of Assumption 4, these time-varying nonlinear matrices can be bounded by its' operator norm. In this case, the design conditions of robust H_∞ networked security observer-based reference tracking control strategy in (18) can be derived in terms of nonlinear difference inequality problem with operator constraints in (20)–(22).

IV. T-S FUZZY H_∞ OBSERVER-BASED SECURITY REFERENCE TRACKING CONTROL FOR NONLINEAR QUADROTOR UAV NCS UNDER CYBER-ATTACK

In general, the observer-based controller design of robust H_∞ networked security observer-based tracking control for nonlinear stochastic quadrotor UAV NCS needs to solve nonlinear functional inequalities in (20), which are not easy to be solved. Therefore, the T-S fuzzy interpolation method is introduced to represent the nonlinear stochastic NCS of quadrotor UAV by the convex combination of a set of specific local linearized systems [38]. By designing the observer-based controller of each local system, the H_∞ observer-based security tracking controller of whole nonlinear NCS of quadrotor UAV under cyber-attack can be constructed by the combination of these T-S fuzzy local observer-based controllers. To begin with, the ith T-S fuzzy rule of stochastic quadrotor UAV NCS in (6) is described as follows [38]–[40]:

Plant Rule i :

If $z_1(kh)$ *is* G_{i1} *and*, $\cdots$, *and* $z_g(kh)$ *is* G_{ig},

Then

$$\begin{aligned} X((k+1)h) &= X(kh) + h[A_iX(kh) + B_i(U_p(kh) \\ &\quad + f_a(kh)) + \upsilon(kh)] + E_iX(kh)\Delta W(kh) \\ Y(kh) &= C_iX(kh) + n(kh) + D_if_s(kh) \end{aligned} \tag{23}$$

where $\{z_i(kh)\}_{i=1}^{g}$ are the premise variables, G_{iq} denotes the ith fuzzy set of the qth premise variable, for $i = 1, \ldots, M$ and $q = 1, \ldots, g$, M is the number of fuzzy rules and g is the number of premise variables. The local linearized matrices $\{A_i, B_i, C_i, D_i, E_i\}_{i=1}^{M}$ are with appropriate dimensions. By the defuzzification process, the overall T-S fuzzy NCS of quadrotor UAV can be inferred as follows [38]:

$$\begin{aligned} X((k+1)h) &= \sum_{i=1}^{M} h_i(z(kh))\{X(kh) + h[A_iX(kh) \\ &\quad + B_i(U_p(kh) + f_a(kh)) + \upsilon(kh)] \\ &\quad + E_iX(kh)\,\Delta W(kh)\} + \Delta f(X(kh)) \\ &\quad + \Delta g(X(kh))U_p(kh) + f_a(kh)) \\ &\quad + \Delta j(X(kh))\Delta W(kh) \\ Y(kh) &= \sum_{i=1}^{M} h_i(z(kh))(C_iX(kh) + n(kh) + D_if_s(kh)) \\ &\quad + \Delta C(X(kh)) + \Delta D(X(kh))f_s(kh) \end{aligned} \tag{24}$$

where $z(kh) = [z_1(kh), \ldots, z_g(kh)]$, $\mu_i(z(kh)) = \Pi_{q=1}^{g} G_{iq}(z_q(kh))$, $h_i(z(kh)) = \frac{\mu_i(z(kh))}{\sum_{j=1}^{M}\mu_j(z(kh))}$, which satisfies with $0 \leq h_i(z(kh)) \leq 1$ and $\sum_{i=1}^{M} h_i(z(kh)) = 1$, with the fuzzy approximation errors

$$\begin{aligned} \Delta f(X(kh)) &= \{f(X(kh)) - \sum\nolimits_{i=1}^{M} h_i(z(kh))A_iX(kh)\}h \\ \Delta g(X(kh)) &= \{g(X(kh)) - \sum\nolimits_{i=1}^{M} h_i(z(kh))B_i\}h \\ \Delta j(X(kh)) &= \sigma(X(kh)) - \sum\nolimits_{i=1}^{M} h_i(z(kh))E_iX(kh) \\ \Delta C(X(kh)) &= C(X(kh)) - \sum\nolimits_{i=1}^{M} h_i(z(kh))C_iX(kh) \\ \Delta D(X(kh)) &= D(X(kh)) - \sum\nolimits_{i=1}^{M} h_i(z(kh))D_i \end{aligned}$$

In order to estimate $X(kh)$, $f_a(kh)$ and $f_s(kh)$ simultaneously, the dynamic models of cyber-attack signals in (7) and (9) need to be augmented with T-S fuzzy NCS of quadrotor UAV in (24) as the following augmented NCS of quadrotor UAV:

$$\begin{aligned} \bar{X}((k+1)h) &= \sum\nolimits_{i=1}^{M} h_i(z(kh))(\bar{A}_i\bar{X}(kh) + \bar{B}_iU_p(kh) \\ &\quad + \bar{G}\bar{\upsilon}(kh) + \bar{E}_i\bar{X}(kh)\Delta W(kh)) + \Delta\bar{f}(\bar{X}(kh)) \\ &\quad + \Delta\bar{g}_1(X(kh))U_p(kh) + \Delta\bar{g}_2(\bar{X}(kh))\bar{X}(kh) \\ &\quad + \Delta\bar{j}(\bar{X}(kh))\Delta W(kh) \\ Y(kh) &= \sum\nolimits_{i=1}^{M} h_i(z(kh))(\bar{C}_i\bar{X}(kh) + \bar{D}\bar{\upsilon}(kh)) \\ &\quad + \Delta\bar{D}(\bar{X}(kh))\bar{X}(kh) + \Delta\bar{C}(\bar{X}(kh)) \\ Y_p(kh) &= (1-\beta(k))\bar{Y}(kh) + \beta(k)\bar{Y}(kh-\tau_2(k)) \end{aligned} \tag{25}$$

where $\bar{X}(kh) = [X^T(kh)\ F_a^T(kh)\ F_s^T(kh)]^T$ and $\bar{\upsilon}(kh) = [\upsilon^T(kh)\ n^T(kh)\ \delta_a^T(kh)\ \delta_s^T(kh)]^T$ with the matrices

$$\begin{aligned} \bar{A}_i &= \begin{bmatrix} hA_i + I_{12} & hB_iC_{f_a} & 0 \\ 0 & A_{f_a} & 0 \\ 0 & 0 & A_{f_s} \end{bmatrix}, \\ \bar{B}_i &= [hB_i^T \quad 0 \quad 0]^T, \\ \Delta\bar{f}(X(kh)) &= [\Delta f^T(X(kh)) \quad 0 \quad 0]^T, \\ \Delta\bar{g}_1(X(kh)) &= [\Delta g^T(X(kh)) \quad 0 \quad 0]^T \\ \Delta\bar{j}(X(kh)) &= [\Delta j^T(X(kh)) \quad 0 \quad 0]^T, \\ \Delta\bar{g}_2(X(kh)) &= \begin{bmatrix} 0 & \Delta g(X(kh))C_{f_a} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

and

$$\begin{aligned} \Delta\bar{C}(\bar{X}(kh)) &= \Delta C(X(kh)) \\ \Delta\bar{D}(\bar{X}(kh)) &= [0 \quad 0 \quad \Delta D(X(kh))C_{f_s}] \\ \bar{C}_i &= \begin{bmatrix} C_i & 0 & D_iC_{f_s} \end{bmatrix}, \quad \bar{E}_i = \mathrm{diag}\{E_i, 0, 0\}, \\ \bar{D} &= \begin{bmatrix} 0 & I_m & 0 & 0 \end{bmatrix}, \\ \bar{G} &= \begin{bmatrix} hI_{12} & 0 & 0 & 0 \\ 0 & 0 & M_{f_a} & 0 \\ 0 & 0 & 0 & M_{f_s} \end{bmatrix} \end{aligned}$$

Before the further discussion, the following theorem is proposed to address the observability of local linear system in the augmented fuzzy system in (25):

Theorem 2: In the augmented NCS of quadrotor UAV in (25), if (A_i^F, C) is observable for each i, i.e.,

$$rank \begin{bmatrix} zI_{12} - A_i^F \\ C \end{bmatrix} = 12, \quad for\ z \in \mathbf{Z}_1,\ i = 1, \cdots, M \tag{26}$$

where $A_i^F \triangleq I_{12} + hA_i$ and the following conditions are satisfied

$$eig\{A_i^F\} \cap eig\{A_{f_a}\} = \emptyset,$$

$$
\begin{aligned}
eig\{A_i^F\} \cap eig\{A_{f_s}\} &= \emptyset,\\
eig\{A_{f_a}\} \cap eig\{A_{f_s}\} &= \emptyset \qquad (27)\\
\text{col}\begin{bmatrix} -hB_iC_{f_a} \\ 0 \end{bmatrix} \cap \text{col}\begin{bmatrix} zI_{12}-A_i^F \\ C \end{bmatrix} &= \emptyset,\\
&\quad for\ z \in eig(A_{f_a}) \qquad (28)\\
rank\begin{bmatrix} zI_{n_a(d+1)} - A_{fa} \\ -hB_iC_{f_a} \end{bmatrix} &= n_a(d+1),\\
&\quad for\ z \in \mathbf{Z}_1 \qquad (29)\\
rank\begin{bmatrix} zI_{n_s(d+1)} - A_{fs} \\ D_iC_{fs} \end{bmatrix} &= n_s(d+1),\\
&\quad for\ z \in \mathbf{Z}_1 \qquad (30)
\end{aligned}
$$

where $\mathbf{Z}_1$ is the set that collects the complex number with norm less than 1, then the ith local linearized system $(\bar{A}_i, \bar{C}_i)$ is observable in the augmented NCS, for $i = 1, \cdots, M$.

Proof: Please refer to Appendix B. □

Then, in order to estimate the augmented NCS of quadrotor UAV in (25), the ith fuzzy Luenberger observer rule is defined as:

Observer Rule i :

$$
\begin{aligned}
&If\ z_1(kh)\ is\ G_{i1},\ and \cdots and,\ z_p(kh)\ is\ G_{ig}\\
&\hat{\bar{X}}((k+1)h) = \bar{A}_i\hat{\bar{X}}(kh) + \bar{B}_iU(kh)\\
&\qquad\qquad + L_i(Y_p(kh) - \hat{Y}_p(kh))\\
&\hat{Y}(kh) = \bar{C}_i\hat{\bar{X}}(kh) \qquad (31)
\end{aligned}
$$

where L_i is the fuzzy observer gain for $i = 1, \cdots, M$. Then, the overall fuzzy observer can be represented as:

$$
\begin{aligned}
\hat{\bar{X}}((k+1)h) &= \sum\nolimits_{i=1}^{M} h_i(z(kh))(\bar{A}_i\hat{\bar{X}}(kh) + \bar{B}_iU(kh)\\
&\quad + L_i(Y_p(kh) - \hat{Y}_p(kh)))\\
\hat{Y}(kh) &= \sum\nolimits_{i=1}^{M} h_i(z(kh))(\bar{C}_i\hat{\bar{X}}(kh))\\
\hat{Y}_p(kh) &= \hat{Y}(kh) \qquad (32)
\end{aligned}
$$

According to above analysis, the observer-based fuzzy controller of the nonlinear NCS of quadrotor UAV in (25) can be inferred as follows:

Controller Rule i :

$$
\begin{aligned}
&If\ z_1(kh)\ is\ G_{i1},\ and \cdots and,\ z_p(kh)\ is\ G_{ig}\\
&U(kh) = K_{1j}\hat{\bar{X}}(kh) + K_{2j}(\hat{\bar{X}}(kh) - \bar{X}_r(kh)) \qquad (33)
\end{aligned}
$$

The overall fuzzy controller can be formulated as:

$$
\begin{aligned}
U(kh) &= \sum\nolimits_{j=1}^{M} h_j(z(kh))(K_{1j}\hat{\bar{X}}(kh)\\
&\quad + K_{2j}(\hat{\bar{X}}(kh) - \bar{X}_r(kh)))\\
U_p(kh) &= (1-\delta(kh))U(kh) + \delta(kh)U(kh-\tau_1(k)) \qquad (34)
\end{aligned}
$$

Then, the dynamic state $\bar{X}(kh)$ of augmented NCS of quadrotor UAV in (25) can be obtained as follows:

$$
\begin{aligned}
&\bar{X}((k+1)h)\\
&= \sum_{i,j,l=1}^{M} h_i(z(kh))h_j(z(kh))\\
&\quad \times h_l(z(kh-\tau_1(k)))\{\bar{A}_i\bar{X}(kh)\\
&\quad + (1-\delta(kh))\bar{B}_i(K_{1j}\hat{\bar{X}}(kh)\\
&\quad + K_{2j}(\hat{\bar{X}}(kh) - \bar{X}_r(kh))) + \delta(kh)\bar{B}_i\\
&\quad \times (K_{1l}\hat{\bar{X}}(kh-\tau_1(k)) + K_{2l}(\hat{\bar{X}}(kh-\tau_1(k))\\
&\quad - \bar{X}_r(kh-\tau_1(k))))) + \bar{G}\bar{\upsilon}(kh)\\
&\quad + \bar{E}_i\bar{X}(kh)\Delta W(kh)] + \Delta\bar{f}(\bar{X}(kh))\\
&\quad + \Delta\bar{g}_1(X(kh))[(1-\delta(kh))(K_{1j}\hat{\bar{X}}(kh) + K_{2j}\\
&\quad \times (\hat{\bar{X}}(kh) - \bar{X}_r(kh))) + \delta(kh)(K_{1l}\hat{\bar{X}}(kh-\tau_1(k))\\
&\quad + K_{2l}(\hat{\bar{X}}(kh-\tau_1(k)) - \bar{X}_r(kh-\tau_1(k)))]\\
&\quad + \Delta\bar{g}_2(\bar{X}(kh))\bar{X}(kh) + \Delta\bar{j}(\bar{X}(kh))\Delta W(kh)\}
\end{aligned}
$$

Also, the nonlinear estimation error system in (15) can be rewritten as follows:

$$
\begin{aligned}
&e((k+1)h)\\
&= \sum_{i,j,l,v=1}^{M} h_i(z(kh))h_j(z(kh))\\
&\quad \times h_l(z(kh-\tau_1(k)))h_v(z(kh-\tau_2(k)))\{\bar{A}_i\bar{X}(kh)\\
&\quad - \bar{A}_j\hat{\bar{X}}(kh) + (1-\delta(kh))\bar{B}_i(K_{1j}\hat{\bar{X}}(kh)\\
&\quad + K_{2j}(\hat{\bar{X}}(kh) - \bar{X}_r(kh))) + \delta(kh)\bar{B}_i(K_{1l}\\
&\quad \times \hat{\bar{X}}(kh-\tau_1(k)) + K_{2l}(\hat{\bar{X}}(kh-\tau_1(k))\\
&\quad - \bar{X}_r(kh-\tau_1(k))))) - \bar{B}_j(K_{1j}\hat{\bar{X}}(kh) + K_{2j}(\hat{\bar{X}}(kh)\\
&\quad - \bar{X}_r(kh))) + \bar{G}\bar{\upsilon}(kh) + \bar{E}_i\bar{X}(kh)\Delta W(kh)]\\
&\quad + \Delta\bar{f}(\bar{X}(kh)) - L_j[(1-\beta(k))(\bar{C}_i\bar{X}(kh) + \bar{D}\bar{\upsilon}(kh))\\
&\quad + \Delta\bar{D}(\bar{X}(kh))\bar{X}(kh) + \Delta\bar{C}(\bar{X}(kh)) + \beta(k)\\
&\quad \times (\bar{C}_v\bar{X}(kh-\tau_2(k)) + \bar{D}\bar{\upsilon}(kh-\tau_2(k))\\
&\quad + \Delta\bar{D}(\bar{X}(kh-\tau_2(k)))\bar{X}(kh-\tau_2(k))\\
&\quad + \Delta\bar{C}(\bar{X}(kh-\tau_2(k))) - \bar{C}_j\hat{\bar{X}}(kh))]\\
&\quad + \Delta\bar{g}_1(X(kh))[(1-\delta(kh))(K_{1j}\hat{\bar{X}}(kh) + K_{2j}(\hat{\bar{X}}(kh)\\
&\quad - \bar{X}_r(kh))) + \delta(kh)(K_{1l}\hat{\bar{X}}(kh-\tau_1(k))\\
&\quad + K_{2l}(\hat{\bar{X}}(kh-\tau_1(k)) - \bar{X}_r(kh-\tau_1(k)))]\\
&\quad + \Delta\bar{g}_2(\bar{X}(kh))\bar{X}(kh) + \Delta\bar{j}(\bar{X}(kh))\Delta W(kh)\}
\end{aligned}
$$

Finally, the augmented fuzzy observer-based reference tracking control system can be formulated as follows:

$$
\begin{aligned}
&\tilde{X}((k+1)h)\\
&= \sum_{i,j,l,v=1}^{M} h_i(z(kh))h_l(z(kh-\tau_1(k)))\\
&\quad \times h_j(z(kh))h_v(z(kh-\tau_2(k)))\{[\tilde{A}_{ijlv,D} + \tilde{A}_{ijlv,R}(kh)\\
&\quad + \Delta\tilde{g}_2(\tilde{X}(kh)) + \Delta\tilde{g}_1(\tilde{X}(kh))(\tilde{G}_{ijlv,1,D}\\
&\quad + \tilde{G}_{ijlv,1,R}(kh)) + (\tilde{D}_{x,ijlv,D} + \tilde{D}_{x,ijlv,R}(kh))\\
&\quad \times \Delta\tilde{D}(\tilde{X}(kh))]\tilde{X}(kh) + [\tilde{A}^{d_1}_{ijlv,D} + \tilde{A}^{d_1}_{ijlv,R}(kh)
\end{aligned}
$$

$$
\begin{aligned}
&+\Delta\tilde{g}_1(\tilde{X}(kh))(\tilde{G}_{ijlv,2,D}+\tilde{G}_{ijlv,2,R}(kh))]\\
&\times\tilde{X}((kh-\tau_1(k))+[\tilde{A}^{d_2}_{ijlv,D}+\tilde{A}^{d_2}_{ijlv,R}(kh)\\
&+(\tilde{D}^{d_2}_{x,ijlv,D}+\tilde{D}^{d_2}_{x,ijlv,R}(kh))\Delta\tilde{D}(\tilde{X}(kh-\tau_2(k)))]\\
&\times\tilde{X}(kh-\tau_2(k))+(\tilde{D}_{ijlv,D}+\tilde{D}_{ijlv,R}(kh))\bar{\upsilon}_1(kh)\\
&+(\tilde{D}^{d_2}_{ijlv,D}+\tilde{D}^{d_2}_{ijlv,R}(kh))\bar{\upsilon}_1(kh-\tau_2(k))+\Delta\tilde{f}(\tilde{X}(kh))\\
&+(\tilde{C}_{j,D}+\tilde{C}_{j,R}(kh))\Delta\tilde{C}(\tilde{X}(kh))\\
&+(\tilde{C}^d_{j,D}+\tilde{C}^d_{j,R}(kh))\Delta\tilde{C}(\tilde{X}(kh-\tau_2(k))\\
&+\tilde{E}_{ijlv}\tilde{X}(kh)\Delta W(kh)+\Delta\tilde{j}(\tilde{X}(kh))\Delta W(kh)\} \qquad (35)
\end{aligned}
$$

where $\tilde{X}(kh) = \left[\bar{X}^T(kh)\ e^T(kh)\ \bar{X}_r^T(kh)\right]^T$ and $\bar{\upsilon}_1(kh) = [\bar{\upsilon}^T(kh)\ r^T(kh)]^T$. The system matrices in (35) are given as follows:

$$
\tilde{A}_{ijlv,D}=\begin{bmatrix}\tilde{A}^1_{ijlv,D} & -(1-\bar{\delta})\bar{B}_i\bar{K}_j & -(1-\bar{\delta})\bar{B}_iK_{2j}\\ \tilde{A}^2_{ijlv,D} & \tilde{A}^3_{ijlv,D} & \tilde{A}^4_{ijlv,D}\\ 0 & 0 & A_r\end{bmatrix}
$$

$$
\begin{aligned}
\tilde{A}^1_{ijlv,D}&=\bar{A}_i+(1-\bar{\delta})\bar{B}_i\bar{K}_j,\\
\tilde{A}^2_{ijlv,D}&=((1-\bar{\delta})\bar{B}_i-\bar{B}_j)\bar{K}_j\\
&\quad -L_j((1-\bar{\beta})\bar{C}_i-\bar{C}_j)+\bar{A}_i-\bar{A}_j\\
\tilde{A}^3_{ijlv,D}&=\bar{A}_j-L_j\bar{C}_j-((1-\bar{\delta})\bar{B}_i-\bar{B}_j)\bar{K}_j\\
\tilde{A}^4_{ijlv,D}&=-((1-\bar{\delta})\bar{B}_i-\bar{B}_j)K_{2j}
\end{aligned}
$$

$$
\tilde{A}_{ijlv,R}(kh)=\begin{bmatrix}\tilde{A}^1_{ijlv,R}(kh) & \tilde{\delta}(kh)\bar{B}_i\bar{K}_j & \tilde{\delta}(kh)\bar{B}_iK_{2j}\\ \tilde{A}^2_{ijlv,R}(kh) & \tilde{A}^3_{ijlv,R}(kh) & \tilde{A}^4_{ijlv,R}(kh)\\ 0 & 0 & 0\end{bmatrix}
$$

$$
\begin{aligned}
\tilde{A}^1_{ijlv,R}(kh)&=-\tilde{\delta}(kh)\bar{B}_i\bar{K}_j\\
\tilde{A}^2_{ijlv,R}(kh)&=-\tilde{\delta}(kh)\bar{B}_i\bar{K}_j+\tilde{\beta}(kh)L_j\bar{C}_i\\
\tilde{A}^3_{ijlv,R}(kh)&=\tilde{\delta}(kh)\bar{B}_i\bar{K}_j\\
\tilde{A}^4_{ijlv,R}(kh)&=\tilde{\delta}(kh)\bar{B}_iK_{2j}
\end{aligned}
$$

$$
\tilde{E}_{ijlv}=\begin{bmatrix}\bar{E}_i & 0 & 0\\ \bar{E}_i & 0 & 0\\ 0 & 0 & 0\end{bmatrix},\quad \bar{K}_j=(K_{1j}+K_{2j})
$$

$$
\tilde{A}^{d_1}_{ijlv,D}=\bar{\delta}\begin{bmatrix}\bar{B}_i\bar{K}_l & -\bar{B}_i\bar{K}_l & -\bar{B}_iK_{2l}\\ \bar{B}_i\bar{K}_l & -\bar{B}_i\bar{K}_l & -\bar{B}_iK_{2l}\\ 0 & 0 & 0\end{bmatrix}
$$

$$
\tilde{A}^{d_1}_{ijlv,R}(kh)=\tilde{\delta}(kh)\begin{bmatrix}\bar{B}_i\bar{K}_l & -\bar{B}_i\bar{K}_l & -\bar{B}_iK_{2l}\\ \bar{B}_i\bar{K}_l & -\bar{B}_i\bar{K}_l & -\bar{B}_iK_{2l}\\ 0 & 0 & 0\end{bmatrix}
$$

$$
\tilde{A}^{d_2}_{ijlv,D}=\begin{bmatrix}0 & 0 & 0\\ -\bar{\beta}L_j\bar{C}_v & 0 & 0\\ 0 & 0 & 0\end{bmatrix}
$$

$$
\tilde{A}^{d_2}_{ijlv,R}(kh)=\begin{bmatrix}0 & 0 & 0\\ -\tilde{\beta}(kh)L_j\bar{C}_v & 0 & 0\\ 0 & 0 & 0\end{bmatrix}
$$

$$
\tilde{\delta}(kh)=\delta(kh)-\bar{\delta},\quad \tilde{\beta}(kh)=\beta(kh)-\bar{\beta}
$$

$$
\tilde{G}_{ijlv,2,D}=\bar{\delta}\begin{bmatrix}\bar{K}_l & -\bar{K}_l & -K_{2l}\\ \bar{K}_l & -\bar{K}_l & -K_{2l}\\ 0 & 0 & 0\end{bmatrix}
$$

$$
\tilde{G}_{ijlv,2,R}(kh)=\tilde{\delta}(kh)\begin{bmatrix}\bar{K}_l & -\bar{K}_l & -K_{2l}\\ \bar{K}_l & -\bar{K}_l & -K_{2l}\\ 0 & 0 & 0\end{bmatrix}
$$

$$
\Delta\tilde{g}_2(\tilde{X}(kh))=\begin{bmatrix}\Delta\bar{g}_2(\bar{X}(kh)) & 0 & 0\\ \Delta\bar{g}_2(\bar{X}(kh)) & 0 & 0\\ 0 & 0 & 0\end{bmatrix}
$$

$$
\tilde{D}^{d_2}_{ijlv,D}=\begin{bmatrix}0 & 0\\ -\bar{\beta}L_i\bar{D} & 0\\ 0 & 0\end{bmatrix}
$$

$$
\tilde{D}^{d_2}_{ijlv,R}(kh)=\begin{bmatrix}0 & 0\\ -\tilde{\beta}(kh)L_i\bar{D} & 0\\ 0 & 0\end{bmatrix}
$$

$$
\Delta\tilde{D}(\tilde{X}(kh))=\begin{bmatrix}0 & 0 & 0\\ \Delta\bar{D}(\bar{X}(kh) & 0 & 0\\ 0 & 0 & 0\end{bmatrix}
$$

$$
\tilde{D}_{ijlv,D}=\begin{bmatrix}\bar{G} & 0\\ \bar{G}-(1-\bar{\beta})L_j\bar{D} & 0\\ 0 & \bar{B}_r\end{bmatrix}
$$

$$
\tilde{D}_{ijlv,R}(kh)=\begin{bmatrix}0 & 0\\ \tilde{\beta}(kh)L_j\bar{D} & 0\\ 0 & 0\end{bmatrix}
$$

$$
\tilde{G}_{ijlv,1,D}=(1-\bar{\delta})\begin{bmatrix}\bar{K}_j & -\bar{K}_j & -K_{2j}\\ \bar{K}_j & -\bar{K}_j & -K_{2j}\\ 0 & 0 & 0\end{bmatrix}
$$

$$
\tilde{G}_{ijlv,1,R}(kh)=\tilde{\delta}(kh)\begin{bmatrix}-\bar{K}_j & \bar{K}_j & K_{2j}\\ -\bar{K}_j & \bar{K}_j & K_{2j}\\ 0 & 0 & 0\end{bmatrix}
$$

$$
\begin{aligned}
\tilde{D}_{x,ijlv,D}&=\mathrm{diag}\{0,-(1-\bar{\beta})L_j,0\}\\
\tilde{D}_{x,ijlv,R}(kh)&=\mathrm{diag}\{0,\tilde{\beta}(k))L_j,0\}\\
\tilde{D}^{d_2}_{x,ijlv,D}&=\mathrm{diag}\{0,-\bar{\beta}L_j,0\}\\
\tilde{D}^{d_2}_{x,ijlv,R}(kh)&=\mathrm{diag}\{0,-\tilde{\beta}(k)L_j,0\}\\
\Delta\tilde{g}_1(\tilde{X}(kh))&=\mathrm{diag}\{\Delta\bar{g}_1(X(kh)),\Delta\bar{g}_1(X(kh)),0\}\\
\Delta\tilde{f}(\tilde{X}(kh))&=[\Delta\bar{f}^T(\bar{X}(kh)),\Delta\bar{f}^T(\bar{X}(kh)),0]^T\\
\Delta\tilde{C}(\tilde{X}(kh))&=[0,\Delta\bar{C}^T(\bar{X}(kh)),0]^T,\\
\tilde{C}_{j,D}&=\mathrm{diag}\{0,-(1-\bar{\beta})L_j,0\}\\
\tilde{C}_{j,R}(kh)&=\mathrm{diag}\{0,\tilde{\beta}(k))L_j,0\}\\
\tilde{C}^d_{j,D}&=\mathrm{diag}\{0,-\bar{\beta}L_j,0\},\\
\tilde{C}^d_{j,R}(kh)&=\mathrm{diag}\{0,-\tilde{\beta}(k)L_j,0\},\\
\Delta\tilde{j}(\tilde{X}(kh))&=[\Delta\bar{j}^T(\bar{X}(kh)),\Delta\bar{j}^T(\bar{X}(kh)),0]^T
\end{aligned}
$$

Before the further discussion, the following assumption is made to deal with the fuzzy approximation error:

Assumption 5: There exists some scalars $\{\epsilon_i \geq 0\}_{i=1}^6$ such that the fuzzy approximation errors are bounded as follows

$$
\begin{aligned}
&\left\|\Delta\tilde{f}(\tilde{X}(kh))\right\|_2\leq\epsilon_1\left\|\tilde{X}(kh)\right\|_2,\quad \left\|\Delta\tilde{g}_1(\tilde{X}(kh))\right\|_2\leq\epsilon_2,\\
&\left\|\Delta\tilde{g}_2(\tilde{X}(kh))\right\|_2\leq\epsilon_3,\quad \left\|\Delta\tilde{C}(\tilde{X}(kh))\right\|_2\leq\epsilon_4\left\|\tilde{X}(kh)\right\|_2\\
&\left\|\Delta\tilde{D}(\tilde{X}(kh)\right\|_2\leq\epsilon_5,\quad \left\|\Delta\tilde{j}(\tilde{X}(kh))\right\|_2\leq\epsilon_6\left\|\tilde{X}(kh)\right\|_2
\end{aligned}
$$

Besides, the following Lemmas are introduced to facilitate the design procedure:

Lemma 1 [42]: For any matrix X and Y with appropriate dimensions, the following inequality holds:

$$X^T Y + Y^T X \le X^T P^{-1} X + Y^T P Y$$

where P is any positive definite symmetric matrix.

Lemma 2 [29]: Given a set of matrices $\{A_i\}_{i=1}^n$ *with appropriate dimension, a positive matrix P and a set of series* $\{\alpha_i \ge 0\}_{i=1}^n$ *with* $\sum_{i=1}^n \alpha_i = 1$*, the following inequality holds:*

$$\sum_{i,j=1}^{n} \alpha_i \alpha_j A_i^T P A_j \le \sum_{i=1}^{n} \alpha_i A_i^T P A_i$$

By using the above two Lemmas, the main theorem is proposed as follows:

Theorem 3: Given a set of scalars $\{\Theta_i \in \mathbb{R}^+\}_{i=1}^8$, Θ_P *and* $\rho > 0$*, if there exist fuzzy controller gains* $\{K_{1j}, K_{2j}\}_{j=1}^M$*, fuzzy observer gains* $\{L_i\}_{i=1}^M$*, positive definite matrices* W, $\{M_i\}_{i=1}^8$, $\{N_{ijlv,1}, N_{ijlv,2}, O_{ijlv,1}, O_{ijlv,2}, Y_{1,ijlv}, Y_{2,ijlv}, Y_{3,ijlv}, S_i > 0\}_{i,j,l,v=1}^M$ *such that the following LMIs hold:*

$$\begin{bmatrix} \tilde{\Lambda}_{ijlv}^* & \tilde{\Pi}_{ijlv}^T & \tilde{\Xi}_{ijlv}^T \\ * & -\tilde{W}_1 & 0 \\ * & * & -\tilde{W}_2 \end{bmatrix} \le 0 \tag{36}$$

$$\begin{bmatrix} M_1 & \tilde{D}_{x,ijlv,E}^T \\ * & W \end{bmatrix} \ge 0, \quad \begin{bmatrix} M_2 & (\tilde{D}_{ijlv,E}^{d_2})^T \\ * & W \end{bmatrix} \ge 0$$
$$\begin{bmatrix} M_3 & \tilde{C}_{j,E}^T \\ * & W \end{bmatrix} \ge 0, \quad \begin{bmatrix} M_4 & (\tilde{C}_{j,E}^d)^T \\ * & W \end{bmatrix} \ge 0, \quad \begin{bmatrix} \Theta_P I & I \\ * & W \end{bmatrix} \ge 0,$$
$$M_1 \le \Theta_1 I, \quad M_2 \le \Theta_2 I, \quad M_3 \le \Theta_3 I, \quad M_4 \le \Theta_4 I, \tag{37}$$

$$\begin{bmatrix} M_5 & \tilde{D}_{x,ijlv,D}^T \\ * & W \end{bmatrix} \ge 0, \quad \begin{bmatrix} M_6 & (\tilde{D}_{ijlv,D}^{d_2})^T \\ * & W \end{bmatrix} \ge 0,$$
$$\begin{bmatrix} M_7 & \tilde{C}_{i,D}^T \\ * & W \end{bmatrix} \ge 0, \quad \begin{bmatrix} M_8 & (\tilde{C}_{i,D}^d)^T \\ * & W \end{bmatrix} \ge 0,$$
$$M_5 \le \Theta_5 I, \quad M_6 \le \Theta_6 I, \quad M_7 \le \Theta_7 I, \quad M_8 \le \Theta_8 I, \tag{38}$$

$$\begin{bmatrix} Y_{1,ijlv} & \tilde{G}_{ijlv,1,D}^T & \tilde{G}_{ijlv,1,E}^T \\ * & \frac{1}{16\Theta_P \epsilon_2^2} I & 0 \\ * & * & \frac{1}{12\Theta_P \epsilon_2^2} I \end{bmatrix} > 0$$
$$\begin{bmatrix} Y_{2,ijlv} & \tilde{G}_{ijlv,2,D}^T & \tilde{G}_{ijlv,2,E}^T \\ * & \frac{1}{16\Theta_P \epsilon_2^2} I & 0 \\ * & * & \frac{1}{12\Theta_P \epsilon_2^2} I \end{bmatrix} > 0$$
$$\begin{bmatrix} Y_{3,ijlv} & \tilde{E}_{ijlv}^T \\ * & \frac{1}{2h} W \end{bmatrix} > 0 \tag{39}$$

where

$$\tilde{\Lambda}_{ijlv}^* = \begin{bmatrix} \tilde{\Lambda}_{ijlv,I}^* & \tilde{\Lambda}_{ijlv,II} \\ * & \tilde{\Lambda}_{ijlv,III}^* \end{bmatrix},$$
$$\tilde{\Lambda}_{ijlv,I}^* = \begin{bmatrix} \tilde{\Lambda}_{ijlv,1}^* & \tilde{\Lambda}_{ijlv,8} & \tilde{\Lambda}_{ijlv,9} \\ * & \tilde{\Lambda}_{ijlv,2}^* & 0 \\ * & * & \tilde{\Lambda}_{ijlv,3} \end{bmatrix}$$
$$\tilde{\Lambda}_{ijlv,II} = \begin{bmatrix} 0 & 0 & \tilde{\Lambda}_{ijlv,10} & \tilde{\Lambda}_{ijlv,11} \\ 0 & 0 & \tilde{\Lambda}_{ijlv,12} & 0 \\ 0 & 0 & 0 & \tilde{\Lambda}_{ijlv,13} \end{bmatrix}$$
$$\tilde{\Lambda}_{ijlv,III}^* = diag\{\tilde{\Lambda}_{ijlv,4}, \tilde{\Lambda}_{ijlv,5}, \tilde{\Lambda}_{ijlv,6}^*, \tilde{\Lambda}_{ijlv,7}^*\}$$
$$\tilde{\Pi}_{ijlv} = diag\{\Pi_{ijlv}, I, I\}, \quad \Pi_{ijlv} = \sqrt{10} diag \times \{\tilde{A}_{ijlv,E}, \tilde{A}_{ijlv,E}^{d_1}, \tilde{A}_{ijlv,E}^{d_2}, \tilde{D}_{ijlv,E}, \tilde{D}_{ijlv,E}^{d_2}\}$$
$$\tilde{\Xi}_{ijlv} = diag\{\Xi_{ijlv}, I, I\}$$
$$\Xi_{ijlv} = [\tilde{A}_{ijlv,D}\ \tilde{A}_{ijlv,D}^{d_1}\ \tilde{A}_{ijlv,D}^{d_2}\ \tilde{D}_{ijlv,D}\ \tilde{D}_{ijlv,D}^{d_2}]$$
$$\tilde{W}_1 = diag\{W, W, W, W, W, W, W\},$$
$$\tilde{W}_2 = diag\{\frac{1}{2} W, W, W\}$$
$$\tilde{\Lambda}_{ijlv,1}^* = 16(\Theta_P \epsilon_1^2 + \Theta_5 \epsilon_5^2 + \Theta_P \epsilon_3^2 + \Theta_7 \epsilon_4^2) I + Y_{1,ijlv} + 12((\Theta_1 \epsilon_5^2 + \Theta_3 \epsilon_4^2) I) + Y_{3,ijlv} + 2h\epsilon_6^2 \Theta_P I + \tilde{Q} + S_2 + S_1 - 2I + W - 2N_{ijlv,1} - 2N_{ijlv,2} + (d_{1M} - d_{1m} + 1) S_3 + (d_{2M} - d_{2m} + 1) S_4$$
$$\tilde{\Lambda}_{ijlv,2}^* = Y_{2,ijlv} - S_1,$$
$$\Lambda_{ijlv,3} = 16(\Theta_6 \epsilon_5^2 + \Theta_8 \epsilon_4^2) I + 12(\Theta_2 \epsilon_5^2 + \Theta_4 \epsilon_4^2) I) - S_2$$
$$\Lambda_{ijlv,4} = -\frac{\rho}{2} I, \quad \Lambda_{ijlv,5} = -\frac{\rho}{2} I,$$
$$\tilde{\Lambda}_{ijlv,6}^* = 2(-2I + W) - 2O_{ijlv,1},$$
$$\tilde{\Lambda}_{ijlv,7}^* = 2(-2I + W) - 2O_{ijlv,2}$$
$$\tilde{\Lambda}_{ijlv,8} = N_{ijlv,1}^T, \quad \tilde{\Lambda}_{ijlv,9} = N_{ijlv,2}^T$$
$$\tilde{\Lambda}_{ijlv,10} = O_{ijlv,1} + N_{ijlv,1}^T$$
$$\tilde{\Lambda}_{ijlv,11} = O_{ijlv,2} + N_{ijlv,2}^T$$
$$\tilde{\Lambda}_{ijlv,12} = -O_{ijlv,1}, \tilde{\Lambda}_{ijlv,13} = -O_{ijlv,2}$$

with the following matrices

$$\tilde{A}_{ijlv,E} = \begin{bmatrix} -\delta^*\bar{B}_i\bar{K}_j & \delta^*\bar{B}_i\bar{K}_j & \delta^*\bar{B}_iK_{2j} \\ -\delta^*\bar{B}_i\bar{K}_j + \beta^*L_j\bar{C}_i & \delta^*\bar{B}_i\bar{K}_j & \delta^*\bar{B}_iK_{2j} \\ 0 & 0 & 0 \end{bmatrix}$$

$$\tilde{A}^{d_1}_{ijlv,E} = \delta^* \begin{bmatrix} \bar{B}_i\bar{K}_j & -\bar{B}_i\bar{K}_j & -\bar{B}_iK_{2j} \\ \bar{B}_i\bar{K}_j & -\bar{B}_i\bar{K}_j & -\bar{B}_iK_{2j} \\ 0 & 0 & 0 \end{bmatrix}$$

$$\tilde{A}^{d_2}_{ijlv,E} = \begin{bmatrix} 0 & 0 & 0 \\ -\beta^*L_j\bar{C}_l & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\tilde{G}_{ijlv,2,E} = \delta^* \begin{bmatrix} \bar{K}_j & -\bar{K}_j & -K_{2j} \\ \bar{K}_j & -\bar{K}_j & -K_{2j} \\ 0 & 0 & 0 \end{bmatrix}$$

$$\tilde{D}^{d_2}_{ijlv,E} = \begin{bmatrix} 0 & 0 \\ -\beta^*L_j\bar{D} & 0 \\ 0 & 0 \end{bmatrix} \quad \tilde{D}_{ijlv,E} = \begin{bmatrix} 0 & 0 \\ \beta^*L_j\bar{D} & 0 \\ 0 & 0 \end{bmatrix}$$

$$\tilde{G}_{ijlv,1,E} = \delta^* \begin{bmatrix} -\bar{K}_j & \bar{K}_j & K_{2j} \\ -\bar{K}_j & \bar{K}_j & K_{2j} \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} \tilde{D}_{x,ijlv,E} &= diag\{0, \beta^*L_j, 0\}, \\ \tilde{D}^{d_2}_{x,ijlv,E} &= diag\{0, -\beta^*L_j, 0\} \\ \tilde{C}_{j,E} &= diag\{0, \beta^*L_j, 0\}, \quad \tilde{C}^d_{j,E} = diag\{0, -\beta^*L_j, 0\} \\ \delta^* &= \sqrt{\bar{\delta}(1-\bar{\delta})}, \quad \beta^* = \sqrt{\bar{\beta}(1-\bar{\beta})}, \end{aligned}$$

for $i, j, l, v = 1, \cdots, M$, then the H_∞ stochastic fuzzy security observer-based reference tracking control strategy in (14) is achieved with the disturbance attenuation level ρ. Besides, if the external disturbance $\bar{v}_1(kh)$ is vanished, the mean square stability of $\tilde{X}(kh)$ in (35) is achieved, i.e., the mean square state estimation error and mean square tracking error will converge to 0 in probability.

Proof: Please refer to Appendix C. □

According to the analysis above, the design procedure of H_∞ fuzzy networked security observer-based reference tracking control of quadrotor NCS is given as follows:

Step 1: Select fuzzy plant rules and membership function for nonlinear system in (24).

Step 2: Select the extrapolation coefficients $\{a_i, b_i\}_{i=0}^d$, in (8) and (10) to construct dynamic smoothed models of (7) and (9) of malicious attacks $f_a(kh)$ and $f_s(kh)$, respectively.

Step 3: Select probabilities $\bar{\delta}, \bar{\beta}$ and boundaries of time-varying delays $d_{1M}, d_{1m}, d_{2M}, d_{2m}$ for NCS.

Step 4: Select the weighting matrices Q_1, Q_2 and disturbance attenuation level ρ for robust H_∞ networked security observer-based reference tracking strategy in (14).

Step 5: Select the boundary parameters $\{\Theta_i > 0\}_{i=1}^8$ and Θ_P, and calculate the fuzzy approximation errors $\{\epsilon_i \geq 0\}_{i=1}^6$

Step 6: Solve the LMIs in (36)-(39) to obtain $W > 0$, $\{M_i\}_{i=1}^8$, $\{N_{ijlv,1}, N_{ijlv,2}, O_{ijlv,1}, O_{ijlv,1}, Y_{1,ijlv}, Y_{2,ijlv}, Y_{3,ijlv}\}_{i,j,l,v=1}^M$, controller gains $\{K_{1j}, K_{2j}\}_{j=1}^M$, observer gain $\{L_i\}$ and $\{S_i > 0\}_{i=1}^4$. Then, construct the fuzzy observer in (32) and construct the fuzzy controller in (34).

V. SIMULATION RESULTS

In this section, to validate the effectiveness of proposed robust H_∞ security observer-based reference tracking controller of stochastic quadrotor NCS under cyber-attack and external disturbances, a simulation example of a quadrotor NCS under cyber-attack and disturbances is given for the illustration of design procedure and validation of the desired tracking performance. On the other hand, the conventional discrete-time robust H_∞ observer-based tracking control scheme in [43] is also provided for the performance comparison.

The physical model of quadrotor UAV is illustrated in (1) and the specific simulation parameters of quadrotor UAV are given as [36]:

$$\begin{aligned} &m = 2 \text{ (kg)}, \quad g = 9.8 \text{ (m/s}^2), \ J_x = J_y = 1.25 \text{ (Ns}^2\text{/rad)} \\ &J_z = 2.2 \text{ (Ns}^2\text{/rad)}, \quad d_x = d_y = d_z = 0.01 \text{ (Ns/m)} \\ &d_\phi = d_\theta = d_\psi = 0.01 \text{ (Ns/m)} \end{aligned}$$

For the desired reference tracking trajectory in this simulation, the quadrotor is asked to track a spiral trajectory with constant velocity on the z-axis. As a result, the position of reference input $r(kh)$ in (13) is specified as follows:

$$x_d = 5\sin(0.3kh), \quad y_d = 5\cos(0.3kh), \ z_d = 0.5kh + 1 \tag{40}$$

where $h = 0.01$ denotes the sampling period.

In general, for a real quadrotor system, the attitude of quadrotor is closely related to the corresponding position trajectory. Hence, according to the desired position trajectory in (40), the attitude of reference signal $r(kh)$ in (13) is given as:

$$\begin{aligned} \phi_d &= \sin^{-1}\left(\frac{m}{F(kh)}(\ddot{x}_d \sin\psi_d - \ddot{y}_d \cos\psi_d)\right), \\ \theta_d &= \tan^{-1}\left(\frac{1}{\ddot{z}_d + g}(\ddot{x}_d \cos\psi_d + \ddot{y}_d \sin\psi_d)\right), \\ \psi_d &= 0.5\pi \end{aligned}$$

where $\ddot{x}_d, \ddot{y}_d, \ddot{z}_d$ are double derivative of x_d, y_d, z_d, respectively, which mean the reference of acceleration.

The matrix A_r in the reference tracking model in (13) is specified as:

$$A_r = 0.05I_{12}$$

From the network point of view, the observation on position will suffer the influence from sensor attack $f_s(kh)$ through the wireless communication channels. On the other hand, actuator attack signal $f_a(kh)$ will be transmitted into the actuator with control commands to corrupt the system plant of quadrotor. Thus, the effect matrix of sensor attack $D(X(kh))$ and measurement output matrix $C(X(kh))$ in (6) are given as:

$$\begin{aligned} C(X(kh)) = [&x_1(kh), x_3(kh), x_5(kh), \ldots\ldots \\ &x_7(kh), x_9(kh), x_{11}(kh)]^T \end{aligned}$$

$$D(X(kh)) = [5\ 5\ 5\ 0.5\ 0.5\ 0.5]^T$$

and the sensor attack signal and actuator attack signal are given as

$$f_s(kh) = \begin{cases} 0.1\sin(5kh), & kh \in [0, 10) \\ 0.1\sin(5kh) + 0.07, & kh \in [10, 20) \\ 0.1\sin(5kh) - 0.07, & kh \in [20, 25) \\ 0.1\sin(5kh), & kh \in [25, 30) \\ 0.2\sin(5kh), & kh \in [30, 60] \end{cases}$$

$$f_a(kh) = [f_{a,1}(kh), f_{a,2}(kh), f_{a,3}(kh), f_{a,4}(kh)]^T$$

$$f_{a,1}(kh) = \begin{cases} 15, & for\ kh \in (13, 18] \\ 22.5, & for\ kh \in (30, 35] \\ 0, & o.w. \end{cases}$$

$$f_{a,2}(kh) = f_{a,3}(kh) = 10^{-3}N(0, 0.01)$$
$$f_{a,4}(kh) = 10^{-4}N(0, 0.01)$$

For the first actuator attack signal, it will provide additional forces on the change of $x_2(kh)$, $y_2(kh)$ and $z_2(kh)$ within the specified time interval during the flight process. Also, three actuator attack signals $\{f_{a,2}(kh), f_{a,3}(kh), f_{a,4}(kh)\}$ are used to describe the random noise on three torques. Due to the limitation of torque forces $\{\tau_1(kh), \tau_2(kh), \tau_2(kh)\}$, the amplitudes of these attack signals are chosen as reasonable values. On the other hand, for the sensor attack signal, it causes oscillation effect on sensor and makes the estimation of UAV become much harder. Besides, the stochastic term $\sigma(X(kh)) = 0.05[0\ x_2(kh)\ 0\ x_4(kh)\ 0\ x_6(kh)\ 0\ 0\ 0\ 0\ 0\ 0]^T$ is formulated to describe the effect of time-varying air viscosity on three velocities.

To construct the fuzzy system in (24), the observer state variables $z_2, \phi_1, \phi_2, \theta_1, \theta_2$ are selected as premise variables and the operation points of premise variables are given as follows:

$$z^1_{op,2} = 0.499, \quad z^2_{op,2} = 9.8, \ \phi^1_{op,1} = -0.05, \ \phi^2_{op,1} = 0.05$$
$$\phi^1_{op,2} = -0.015, \quad \phi^2_{op,2} = 0.015, \ \theta^1_{op,1} = -0.05,$$
$$\theta^2_{op,1} = 0.05, \quad \theta^1_{op,2} = -0.015, \ \theta^2_{op,2} = 0.015$$

On the basis of the design procedure in the end of Section IV, some matrices and variables should be determined. First, the extrapolation coefficients of sensor attack signal $f_s(kh)$ and actuator attack signal $f_a(kh)$ are selected as follows:

$$a_1 = 0.9, \quad a_2 = 0.06, \ a_3 = 0.03, \ a_4 = 0.01$$
$$b_1 = 0.96, \quad b_2 = 0.02, \ b_3 = 0.01, \ b_4 = 0.01$$

Next, the occurrence probabilities of time-varying delays on two wireless network channels are assumed to be $\bar{\delta} = \bar{\beta} = 0.05$ and the boundaries of two time-varying delays are assumed to be $d_{1M} = d_{2M} = 0.02$, $d_{1m} = d_{2m} = 0$.

The initial states of quadrotor UAV NCS and its estimation in the simulation are assumed to be

$$\bar{X}(0) = [0.5\ 1.44\ 4.9\ \ 0.025\ 2\ 0.5\ 0\ 0 \\ -0.04\ 0\ 0.5\pi\ \ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$$

$$\hat{\bar{X}}(0) = [0.5\ 0\ 4.9\ 0\ 2\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$$

and the weighting matrices in (18) are selected as follows:

$$Q_1 = 10^{-4}\mathrm{diag}\{3I_6, 10I_6, 6I_8\},$$
$$Q_2 = 10^{-4}\mathrm{diag}\{I_{12}, 0_{8\times 8}\}$$

The fuzzy approximation errors and boundary parameters $\{\Theta_i > 0\}_{i=1}^8$, Θ_P are selected as follows:

$$\epsilon_1 = 0.05, \quad \epsilon_2 = 0.08, \ \epsilon_3 = 0.05$$
$$\epsilon_4 = \epsilon_5 = \epsilon_6 = 0, \quad \{\Theta_i = 1\}_{i=1}^8, \ \Theta_P = 10$$

The quadrotor UAV NCS may suffer interferences such as external disturbance $\upsilon(kh)$ on system plant and measurement noise $n(kh)$ on measurement output. Hence, in this study, the external disturbance is set as $\upsilon(kh) = 0.01[0\ \sin(kh)\ 0\ \sin(kh)\ 0\ \sin(kh)\ 0\ 0.01\cos(kh)\ 0\ 0.01\cos(kh)\ 0\ 0.01\cos(kh)]^T$ and measurement noise $n(kh)$ is assumed to be zero mean white noise with unit variance. Since the values of angular velocity of quadrotor UAV are very small, the external disturbances of angular velocity are also given with some very small values. By solving the LMIs in Theorem 3 with the prescribed disturbance attenuation level $\rho = 10$, we can obtain the corresponding fuzzy tracking controller gains $\{K_{1j}, K_{2j}\}_{j=1}^{32}$ and observer gain $\{L_i\}_{i=1}^{32}$.

The simulation results are shown in Figs. 3-10. Fig. 3 shows the malicious attack signals $f_s(kh)$ and $f_a(kh) = [f_{a,1}(kh)\, f_{a,2}(kh)\, f_{a,3}(kh)\, f_{a,4}(kh)]^T$ on the quadrotor NCS and their estimation. From Fig. 3, by the effect of the quadrotor's transient state response due to the initial condition, the estimation of first actuator attack signal $f_{a,1}(kh)$ also has a large transient response. After that, the malicious attack signals $f_s(kh)$ and $f_{a,1}(kh)$ can be estimated quite well by the proposed H_∞ fuzzy security Luenberger observer. Also, while these two attack signals have suddenly jumps, they cause some fluctuation on the estimation. After that, the changed attack signals can be estimated well, e.g., the sensor attack signal jumps at 20s and can be estimated precisely by the proposed observer. Since the malicious attack signals $f_s(kh)$

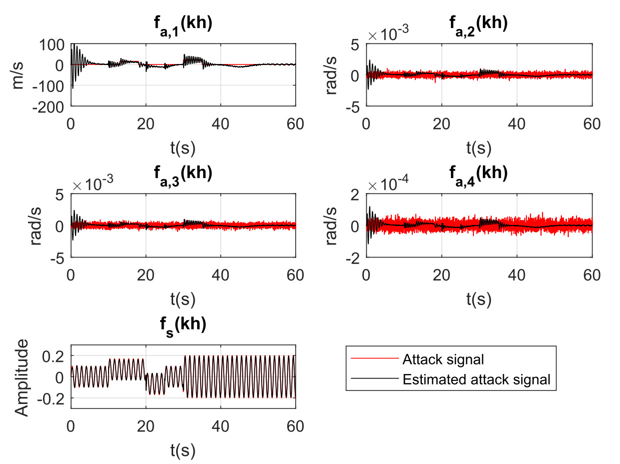

FIGURE 3. Malicious attack signals and their estimations.

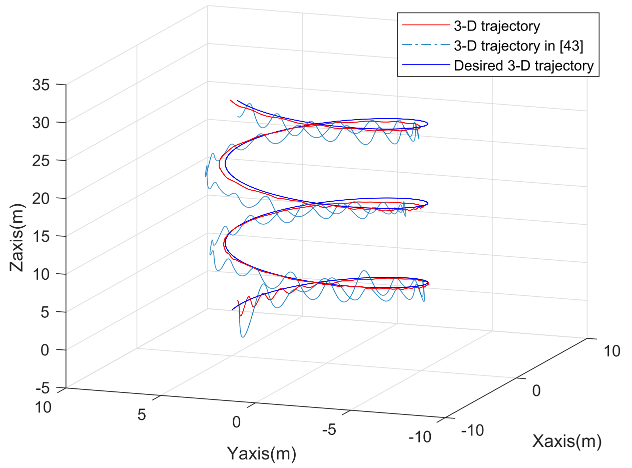

FIGURE 4. The space diagram of the desired flighting trajectory of quadrotor UAV and the 3-D trajectory by the proposed H_∞ fuzzy networked security observer-based reference tracking control method in comparison with the robust H_∞ fuzzy observer-based tracking control scheme in [43].

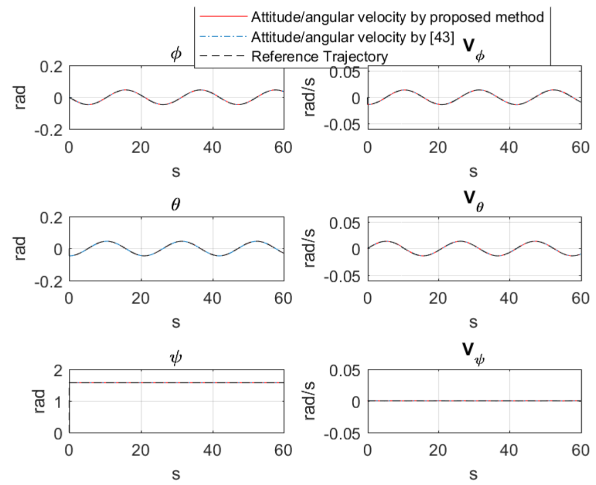

FIGURE 6. The attitude/angular velocity trajectory of quadrotor UAV by the proposed H_∞ fuzzy networked security observer-based reference tracking control method compared with the robust H_∞ fuzzy observer-based tracking control scheme in [43].

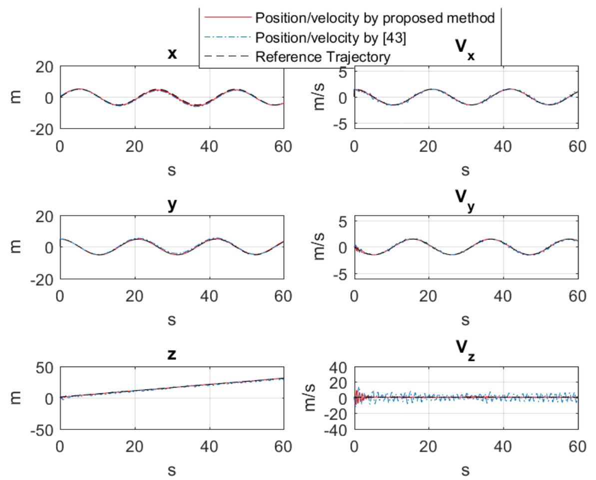

FIGURE 5. The position/velocity trajectory of quadrotor UAV by the proposed H_∞ fuzzy networked security observer-based reference tracking control method compared with the robust H_∞ fuzzy observer-based tracking control scheme in [43].

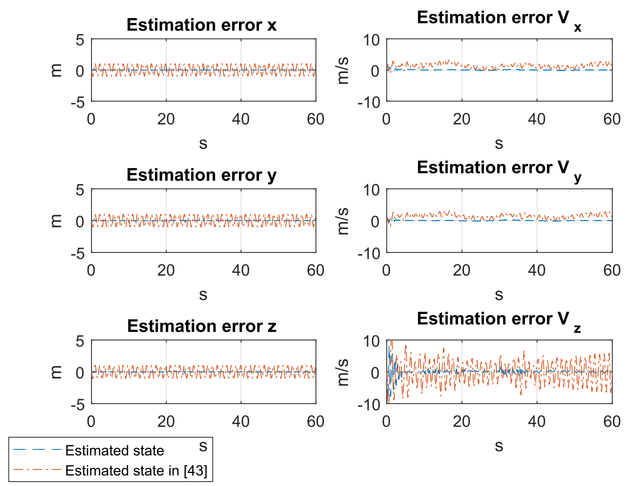

FIGURE 7. Estimation errors of the position/velocity of quadrotor UAV by the proposed H_∞ fuzzy networked security observer-based reference tracking control method compared with the robust H_∞ fuzzy observer-based tracking control scheme in [43].

and $f_{a,1}(kh)$ are estimated together in the augmented system in (11), their effects may impact with each other. For example, the estimated signal $f_{a,1}(kh)$ oscillates due to the high frequency signal $f_s(kh)$. By the fact that $f_s(kh)$ corrupts $Y_p(kh)$ in (12) directly, $Y_p(kh)$ could influence on the estimated signal $\hat{f}_{a,1}(kh)$ indirectly, which can be seen from the structure of Luenberger observer in (12) or (32). Besides, since three actuator attack signals $\{f_{a,2}(kh), f_{a,3}(kh), f_{a,4}(kh)\}$ are selected as Gaussian white noises, the corresponding estimations can not be done due to its' strong randomness. However, by the proposed H_∞ fuzzy networked security reference tracking control strategy in (14), these effects of $\delta_a(kh)$ and $\delta_s(kh)$ in $\bar{v}_1(kh)$ on the tracking can be passively attenuated during the reference tracking control process.

Fig. 4 is the 3-D plot of the flight trajectory and desired trajectory of quadrotor NCS. The trajectory of the quadrotor NCS and the corresponding desired reference trajectory is shown in Figs. 5-6. From Figs. 5-6, the trajectory of the quadrotor NCS can track the reference trajectory well with the proposed H_∞ fuzzy networked security reference tracking controller. Since the transient response of the position is bigger than the transient response of the attitude, the proposed H_∞ fuzzy network reference tracking controller has a better performance in attitude tracking than the position tracking. In Figs. 7-8, the estimation errors of system states of quadrotor UAV are shown. Since the attack signal $f_s(kh)$ is estimated precisely, its estimation can be used to compensate the effect of real sensor attack signal on the sensor. Thus, the system state estimation can be more precisely achieved by

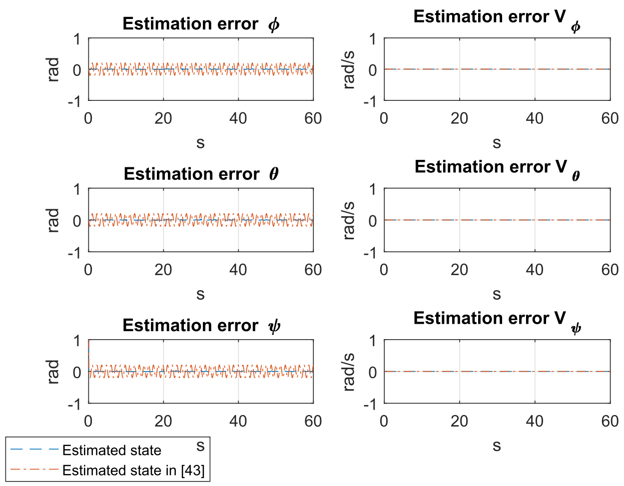

FIGURE 8. **Estimation errors of the attitude/angular velocity of quadrotor UAV by the proposed H_∞ fuzzy networked security observer-based reference tracking control method compared with the robust H_∞ fuzzy observer-based tracking control scheme in [43].**

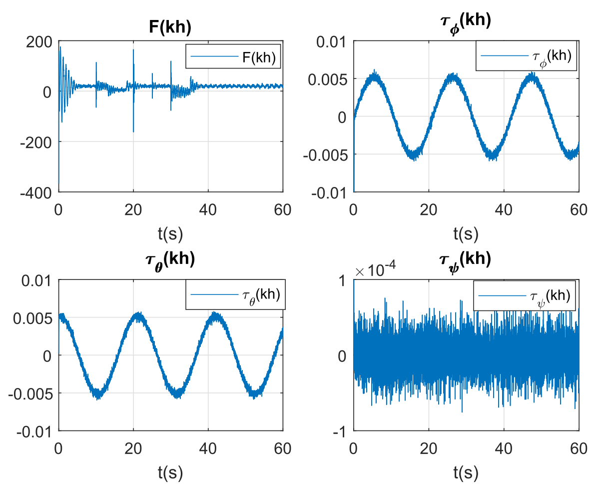

FIGURE 9. **Control inputs $U(kh) = [F(kh)\ \tau_\phi(kh)\ \tau_\theta(kh)\ \tau_\psi(kh)]^T$ are constructed by the proposed H_∞ fuzzy networked security observer-based reference tracking controller.**

the proposed H_∞ fuzzy networked security reference tracking control scheme than the conventional H_∞ fuzzy observer-based tracking control method in [43].

The control signals $U(kh) = [F(kh)\ \tau_\phi(kh)\ \tau_\theta(kh)\ \tau_\psi(kh)]^T$ of the proposed method for the NCS of quadrotor UAV are displayed in Fig. 9. To cancel out the effect of actuator attack signal $f_a(kh)$, $F(kh)$ changes its control effort when the cyber-attack signal $f_a(kh)$ attacks the actuator of NCS of quadrotor UAV. For example, as attack signal $f_a(kh)$ occurs at $13s$ in Fig. 3, $F(kh)$ gives a corresponding signal to cancel the attack signal $f_a(kh)$ on the NCS of quadrotor UAV by using the estimated fault signal $\hat{f}_a(kh)$ in Fig. 9. On the other hand, the control signals of the NCS in (34) of quadrotor UAV will also suffer from the sensor attack signal $f_s(kh)$ through $Y_p(kh)$ in (32). Since $Y_p(kh)$ is directly transmitted to the observer through wireless network channel, it suffers from the effect of sensor attack signal $f_s(kh)$. Then, the sensor attack signal $f_s(kh)$ will affect observer state $\hat{\bar{X}}(kh)$ and then affect $\bar{U}(kh)$. For example, $\tau_\psi(kh)$ shows its high frequency oscillation caused by the high frequency sensor attack signal $f_s(kh)$ in Fig. 9. However, due to the good sensor attack signal estimation for compensation, these effects on control signals are attenuated efficiently by the proposed H_∞ security observer-based reference tracking control strategy.

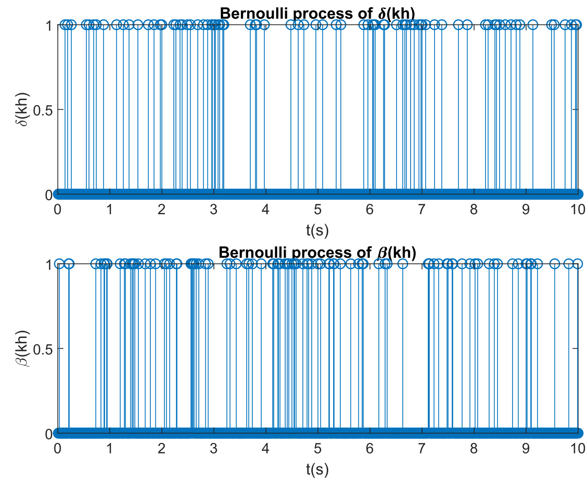

FIGURE 10. **Bernoulli sequence of $\delta(kh)$ and $\beta(kh)$.**

The time sequences, due to the occurrence of the time-varying delays, $\tau_1(k)$, $\tau_2(k)$ are displayed in Fig. 10,. When the time-varying delay $\tau_1(k)$ occurs, the value of Bernoulli process $\delta(kh)$ is equal to 1. Similarly, when the time-varying delay $\tau_2(k)$ occurs, $\beta(kh)$ is equal to 1. Even the control inputs and measurement outputs suffer from these delay effects, the proposed H_∞ fuzzy networked security reference tracking controller can achieve the desired tracking control performance with the robustness for the attenuation of these network-induced effects.

For the performance comparison, the conventional robust H_∞ fuzzy discrete-time observer-based tracking control scheme in [43] is carried out and the results are also shown in Figs. 4–8. Without the attack signal estimation, these attack signals are regarded as a kind of disturbance and their effects on the state estimation and reference tracking performance are passively attenuated by the conventional robust H_∞ fuzzy discrete-time observer-based tracking control strategy in [43]. Due to the large influence of attack signals, the estimation error of position/velocity variables in Fig. 7 will not converge and it also deteriorate the performance of observer-based controller. However, due to the selection of $D(X(kh))$, the effect of sensor attack signal on three angular positions is relatively minor and thus the estimation error of angular velocity/angular position is relatively small. Even the three attitudes are well controlled and estimated in Figs. 6 and 8, the sinusoidal sensor attack signal and actuator attack signal severely influence the control strategy on three position velocities. Moreover, since the networked-induced effects (e.g., delay effect) are not well considered, the conventional control strategy is more deteriorated and thus it cause some fluctuations on the velocity/position variables. Thus, for the 3-D reference tracking trajectory of UAV controlled by the conventional robust H_∞ fuzzy observer-based tracking controller in [43], the reference tracking trajectory is up-and-down around the desired trajectory as shown in Fig. 4. In this situation, once the UAV reference tracking trajectory is deviated from the desired trajectory, it may fail to complete the pre-designed task or even cause collision during the flight process.

Also, the following performance indices about the modified integral square error (MISE), and the modified integral time square error (MITSE) (i.e., discrete version of ISE and ITSE) are respectively given in the following for the tracking/estimation performance evaluations about our method and the method in [43]

$$\begin{aligned}
MISE_T(kh) &= \sum_{k=0}^{T/h} E_{T,error}^T(kh)E_{T,error}(kh)\\
MITSE_T(kh) &= \sum_{k=0}^{T/h} khE_{T,error}^T(kh)E_{T,error}(kh)\\
E_{T,error}(kh) &= X(kh) - X_r(kh)\\
MISE_O(kh) &= \sum_{k=0}^{T/h} E_{S,error}^T(kh)E_{S,error}(kh)\\
MITSE_O(kh) &= \sum_{k=0}^{T/h} khE_{S,error}^T(kh)E_{S,error}(kh)\\
E_{S,error}(kh) &= X(kh) - \hat{X}(kh)\\
MISE_a(kh) &= \sum_{k=0}^{T/h} E_{F,error}^T(kh)E_{F,error}(kh)\\
MITSE_a(kh) &= \sum_{k=0}^{T/h} khE_{F,error}^T(kh)E_{F,error}(kh)\\
E_{F,error}(kh) &= F_{att}(k) - \hat{F}_{att}(k)
\end{aligned}$$

where $X(kh)$ is the UAV state in (3), $X_r(kh)$ is the reference trajectory to be tracked in (13), $\hat{X}(kh)$ is the estimation of UAV state in the fuzzy Luenberger-observer in (32), $F_{att}(k) = [F_a^T(kh)\ F_s^T(kh)]^T$ is the augmented attack signal with augmented actuator attack signal $F_a(kh)$ in (7), and augmented sensor attack signal $F_s(kh)$ in (9), $\hat{F}_{att}(k)$ is the estimation of augmented attack signal $F_{att}(k)$ in fuzzy Luenberger-observer in (32), $T = 60$ is the terminal time and $h = 0.01s$ is the sampling time. It is worth to point out that the attack signal estimation is not considered in the observer design in [43]. For equality, we only compare the performances of state estimation between our method and the estimation method in [43]. On the other hand, the performance of attack signal estimation is independently calculated.

The evaluation results are shown in Figs. 11–13. For the tracking performance evaluation, due to the effect of actuator/sensor attack signals, the MISE of the conventional robust H_∞ fuzzy discrete-time observer-based tracking control scheme in [43] is increasing as time increase as shown in Fig. 11–(a). This fact implies the effect of actuator/sensor attack signals on tracking performance can not be passively attenuated by the conventional robust H_∞ fuzzy discrete-time observer-based tracking control scheme. On the contrary, since the actuator/sensor attack signals are embedded

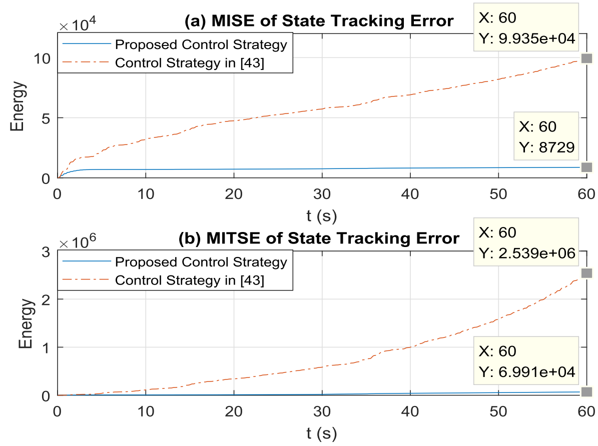

FIGURE 11. The MISE of state tracking between our method and [43] in (a) and the MITSE of state tracking between our method and [43] in (b).

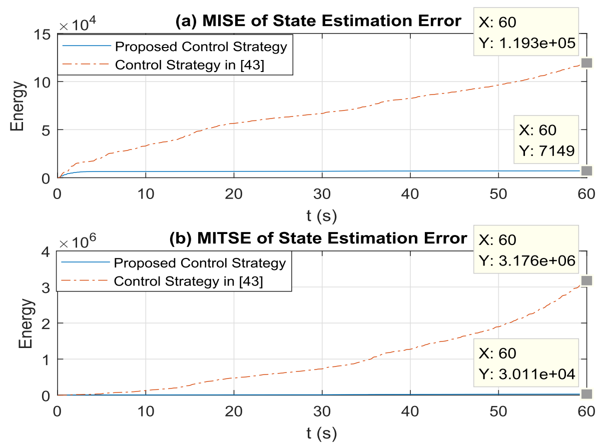

FIGURE 12. The MISE of state estimation between our method and [43] in (a) and the MITSE of state estimation between our method and [43] in (b).

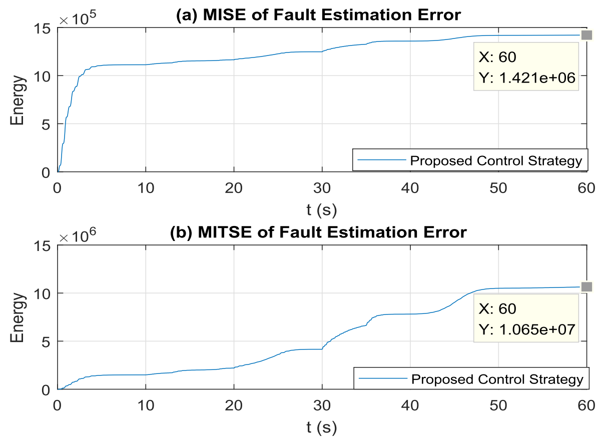

FIGURE 13. The MISE of attack signal estimation in (a) and the MITSE of attack signal estimation in (b).

in the augmented state to avoid their corruption on state estimation, the MISE of the proposed H_∞ fuzzy networked security reference tracking control scheme almost converges to a constant at 10s in Fig. 11–(a), i.e., the energy of tracking error almost approaches to zero at 10s. However, due to the external disturbance, the measurement noise and the Wiener process in UAV system in (3), there exist small fluctuations in the MISE of the proposed H_∞ fuzzy networked security reference tracking control design. According to MISE results of two methods, for the MITSE tracking performance evaluation in Fig. 11–(b), the MITSE of the conventional robust H_∞ fuzzy discrete-time observer-based tracking control scheme exponentially increases and the MITSE of the proposed H_∞ fuzzy networked security reference tracking control design increases slightly. From Fig. 11–(a), the final value of the MISE of the conventional robust H_∞ fuzzy discrete-time observer-based tracking control scheme is about the ten times of the final value of the MISE of proposed method. Similar to the discussion of the tracking performance evaluation, from Fig. 12–(a),(b), it reveals the UAV state estimation performance by the proposed method is better than the state estimation performance by the conventional robust H_∞ fuzzy discrete-time observer-based tracking control scheme.

For the attack signal estimation in Fig. 13-(a), the quickly increasing phenomenon of MISE for attack signal estimation at the initial is caused by the first actuator signal estimation $f_{a,1}(k)$ in Fig. 3. After that, due to the amplitude change of actuator/sensor attack signals, the MISE for attack signal estimation is smoothly increased. At 50s, the MISE almost converges to a constant and it implies the first actuator signal $f_{a,1}(k)$ and sensor attack signal $f_s(k)$ are efficiently estimated. However, since three actuator attack signals $\{f_{a,2}(k), f_{a,3}(k), f_{a,4}(k)\}$ are Gaussian white noises and they can not be precisely estimated, it casues a slight increment in MITSE of attack signal estimation for $t > 50$ in Fig. 13–(b).

VI. CONCLUSION

In this study, a robust H_∞ networked security observer-based reference tracking control scheme is proposed for quadrotor UAV NCS under cyber-attack. To estimate the cyber-attack on the actuator and sensor of quadrotor UAV NCS, a discrete smoothed dynamic model is introduced to describe these unavailable attack signals to simplify the state and attack signal estimation for the observer-based tracking control design of quadrotor UAV NCS. Then, by embedding the smoothed model of cyber-attack signals in quadrotor system, a conventional Lunberger observer can be applied to estimate attack signals as well as quadrotor UAV state simultaneously. By the robust H_∞ networked observer-based security reference tracking control scheme, the proposed security controller and observer could also minimize the effect of external disturbances and measurement noise to achieve the robust tracking performance and observation performance of quadrotor UAV NCS. By using the convex Lyapunov functional, the observer-based security tracking control design problem in the quadrotor UAV NCS is transformed to an equivalent nonlinear functional inequality. Further, by using T-S fuzzy interpolation method, the nonlinear quadrotor UAV NCS can be interpolated by a set of local linearized systems via fuzzy bases. Then, the robust H_∞ fuzzy observer-based security reference tracking control design of quadrotor UAV NCS can be transformed to solving a set of LMIs. A simulation example of model reference tracking control of quadrotor UAV NCS under actuator and sensor attack is given to validate the effectiveness of the proposed method in comparison with the conventional H_∞ fuzzy discrete-time robust observer-based tracking control method in [43]. Since the attack signals could be estimated effectively by the proposed discrete smoothed model, their effects on the system and sensor can be cancelled out by their estimation to improve the performance of observer-based security reference tracking controller of quadrotor UAV NCS. As a result, the security for the networked observer-based reference tracking control of the network-based quadrotor UAV NCS under actuator and sensor attack can be guaranteed. In future researches, if we want to adapt the parameters of two smoothed models by some adaptive algorithm in every time step to improve the two smoothed models, then the assumption of persistence of excitation on the input signals in (7) and (9) (i.e., $\delta_a(kh)$,$\delta_s(kh)$) should be addressed to support the proposed approach.

APPENDIX A
PROOF OF THEOREM 1

By selecting the convex Lyapunov function $V(\tilde{X}(kh))$ w.r.t. the augmented system in (17), the deviation from $V(\tilde{X}((k+1)h))$ to $V(\tilde{X}(kh))$ can be derived as follows:

$$
\begin{aligned}
&E\{\Delta V(\tilde{X}(kh))\}\\
&\quad = E\{V(\tilde{X}((k+1)h)) - V(\tilde{X}(kh))\}\\
&\quad = E\{V(\alpha\frac{1}{\alpha}[\tilde{f}(\tilde{X}(kh), kh) + \tilde{g}(\tilde{X}(kh), kh)U(kh)\\
&\qquad + \tilde{g}_1(\tilde{X}(kh), kh)U(kh-\tau_1(k)) + \tilde{\sigma}(X(kh))\Delta W(kh)]\\
&\qquad + (1-\alpha)\frac{1}{1-\alpha}[\tilde{o}_1(\tilde{X}(kh), kh)\bar{v}_1(kh)\\
&\qquad + \tilde{o}_2(\tilde{X}(kh), kh)\bar{v}_1(kh-\tau_2(k))])\} - E\{V(\tilde{X}(kh))\}\\
&\quad \le \alpha E\{V(\frac{1}{\alpha}[\tilde{f}(\tilde{X}(kh), kh) + \tilde{g}(\tilde{X}(kh), kh)U(kh)\\
&\qquad + \tilde{g}_1(\tilde{X}(kh), kh)U(kh-\tau_1(k)) + \tilde{\sigma}(X(kh))\Delta W(kh)]\}\\
&\qquad + (1-\alpha)E\{V(\frac{1}{1-\alpha}[\tilde{o}_1(\tilde{X}(kh), kh)\bar{v}_1(kh)\\
&\qquad + \tilde{o}_2(\tilde{X}(kh), kh)\bar{v}_1(kh-\tau_2(k)))]\} - E\{V(\tilde{X}(kh))\} \qquad (41)
\end{aligned}
$$

where α denotes a constant within (0,1). By applying the property of convex Lyapunov function again, (41) can be further written as

$$
\begin{aligned}
&E\{\Delta V(\tilde{X}(kh))\}\\
&\quad \le \alpha E\{V(\frac{1}{\alpha}[\tilde{f}(\tilde{X}(kh), kh) + \tilde{g}(\tilde{X}(kh), kh)U(kh)\\
&\qquad + \tilde{g}_1(\tilde{X}(kh), kh)U(kh-\tau_1(k))\\
&\qquad + \tilde{\sigma}(X(kh))\Delta W(kh)]\} - E\{V(\tilde{X}(kh))\}\\
&\qquad + (1-\alpha)E\{V(\frac{1}{1-\alpha}[\beta\frac{1}{\beta}\tilde{o}_1(\tilde{X}(kh), kh)\bar{v}_1(kh)
\end{aligned}
$$

$$
\begin{aligned}
&+(1-\beta)\frac{1}{1-\beta}\tilde{o}_2(\tilde{X}(kh),kh)\bar{v}_1(kh-\tau_2(k))]\} \\
&\leq \alpha E\{V(\frac{1}{\alpha}(\tilde{f}(\tilde{X}(kh),kh)+\tilde{g}(\tilde{X}(kh),kh)U(kh) \\
&\quad+\tilde{g}_1(\tilde{X}(kh),kh)U(kh-\tau_1(k)) \\
&\quad+\tilde{\sigma}(X(kh))\Delta W(kh))\}-V(\tilde{X}(kh)) \\
&\quad+(1-\alpha)\beta V(\frac{1}{1-\alpha}\frac{1}{\beta}\tilde{o}_1(\tilde{X}(kh),kh)\bar{v}_1(kh)) \\
&\quad+(1-\alpha)(1-\beta)V(\frac{1}{1-\alpha}\frac{1}{1-\beta} \\
&\quad\times\tilde{o}_2(\tilde{X}(kh),kh)\bar{v}_1(kh-\tau_2(k)))\}
\end{aligned} \tag{42}
$$

where β denotes a constant within (0,1).

Based on Assumption 4, if the trajectory of $\tilde{X}(kh)$ lies in a compact domain Ξ, then there exists the upper bound parameters γ_1 and γ_2 for $eig(o_1^T(\tilde{X}(kh),kh)o_1(\tilde{X}(kh),kh))$ and $eig(o_2^T(\tilde{X}(kh),kh)o_2(\tilde{X}(kh),kh))$, respectively, such that the following equations hold

$$
\begin{aligned}
&\sup_{\tilde{X}(kh)\in\Xi} E\{eig(\tilde{o}_1^T(\tilde{X}(kh),kh)\tilde{o}_1(\tilde{X}(kh),kh))\}=\gamma_1 \\
&\sup_{\tilde{X}(kh)\in\Xi} E\{eig(\tilde{o}_2^T(\tilde{X}(kh),kh)\tilde{o}_2(\tilde{X}(kh),kh))\}=\gamma_2
\end{aligned} \tag{43}
$$

for $k \in \{0,\cdots,k_f\}$

Then, to decouple the noise terms $\bar{v}_1(kh)$ and $\bar{v}_1(kh-\tau_2(k))$ in (42), the following sufficient conditions are constructed

$$
\sup_{\Lambda_1(kh)} \frac{(1-\alpha)\beta E\{V(\frac{1}{1-\alpha}\frac{1}{\beta}\Lambda_1(kh))\}}{\|\Lambda_1(kh)\|^2}\leq\frac{\rho}{2\gamma_1} \tag{44}
$$

$$
\sup_{\Lambda_2(kh)} \frac{(1-\alpha)(1-\beta)E\{V(\frac{1}{1-\alpha}\frac{1}{1-\beta}\Lambda_2(kh))\}}{\|\Lambda_2(kh)\|^2}\leq\frac{\rho}{2\gamma_2} \tag{45}
$$

where $\Lambda_1(kh) = \tilde{o}_1(\tilde{X}(kh),kh)\bar{v}_1(kh)$, $\Lambda_2(kh) = \tilde{o}_2(\tilde{X}(kh),kh)\bar{v}_1(kh-\tau_2(k))$ and ρ is a positive number.

Based on the inequalities in (43), (44), (45), the following inequalities hold

$$
\begin{aligned}
&(1-\alpha)\beta E\{V(\frac{1}{1-\alpha}\frac{1}{\beta}\tilde{o}_1(\tilde{X}(kh),kh)\bar{v}_1(kh))\} \\
&\leq E\{\frac{\rho}{2\gamma_1}\bar{v}_1^T(kh)\tilde{o}_1^T(\tilde{X}(kh),kh)\tilde{o}_1(\tilde{X}(kh),kh)\bar{v}_1(kh)\} \\
&\leq\frac{\rho}{2}E\{\bar{v}_1^T(kh)\bar{v}_1(kh)\} \\
&\quad\times(1-\alpha)(1-\beta)E\{V(\frac{1}{1-\alpha}\frac{1}{1-\beta}\tilde{o}_2(\tilde{X}(kh),kh) \\
&\quad\times\bar{v}_1(kh-\tau_2(k)))\} \\
&\leq E\{\frac{\rho}{2\gamma_2}\bar{v}_1^T(kh-\tau_2(k))\tilde{o}_2^T(\tilde{X}(kh),kh) \\
&\quad\times\tilde{o}_2(\tilde{X}(kh),kh)\bar{v}_1(kh-\tau_2(k))\} \\
&\leq\frac{\rho}{2}E\{\bar{v}_1^T(kh-\tau_2(k))\bar{v}_1(kh-\tau_2(k))\} \\
&\qquad\qquad\forall\tilde{X}(kh)\in\Xi
\end{aligned} \tag{46}
$$

By the inequalities in (46), (42) can be relaxed as follows

$$
\begin{aligned}
&E\{\Delta V(\tilde{X}(kh))\} \\
&\leq\alpha E\{V(\frac{1}{\alpha}(\tilde{f}(\tilde{X}(kh),kh)+\tilde{g}(\tilde{X}(kh))\bar{U}(kh) \\
&\quad+\tilde{g}_1(\tilde{X}(kh),kh)U(kh-\tau_1(k)) \\
&\quad+\tilde{\sigma}(X(kh))\Delta W(kh)))\}-E\{V(\tilde{X}(kh)) \\
&\quad+\frac{\rho}{2}\bar{v}_1^T(kh-\tau_2(k))\bar{v}_1(kh-\tau_2(k)) \\
&\quad+\frac{\rho}{2}\bar{v}_1^T(kh)\bar{v}_1(kh)\}
\end{aligned} \tag{47}
$$

Further, if the following condition holds

$$
\begin{aligned}
&\alpha E\{V(\frac{1}{\alpha}(\tilde{f}(\tilde{X}(kh),kh)+\tilde{g}(\tilde{X}(kh),kh)U(kh) \\
&\quad+\tilde{g}_1(\tilde{X}(kh),kh)U(kh-\tau_1(k))+\tilde{\sigma}(X(kh))\Delta W(kh)))\} \\
&\quad-E\{V(\tilde{X}(kh))\}+E\{\tilde{X}^T(kh)\tilde{Q}\tilde{X}(kh)\}<0
\end{aligned} \tag{48}
$$

the inequality in (47) can be written as follows

$$
\begin{aligned}
&E\{\Delta V(\tilde{X}(kh))\} \\
&\quad\leq E\{-\tilde{X}^T(kh)\tilde{Q}\tilde{X}(kh)+E\{\frac{\rho}{2}\bar{v}_1^T(kh-\tau_2(k)) \\
&\qquad\times\bar{v}_1(kh-\tau_2(k))\}+E\{\frac{\rho}{2}\bar{v}_1^T(kh)\bar{v}_1(kh)\}
\end{aligned}
$$

By performing summation on both sides of the above inequality from $k=0$ to $k=k_f$, we have

$$
\begin{aligned}
&\sum_{k=0}^{k_f}E\{\Delta V(\tilde{X}(kh))\} \\
&\quad=E\{V(\tilde{X}(k_fh))-V(\tilde{X}(0))\} \\
&\quad\leq\sum_{k=0}^{k_f}E\{(-\tilde{X}^T(kh)\tilde{Q}\tilde{X}(kh)+\frac{\rho}{2}\bar{v}_1^T(kh)\bar{v}_1(kh) \\
&\qquad+\frac{\rho}{2}\bar{v}_1^T(kh-\tau_2(k))\bar{v}_1(kh-\tau_2(k))\}
\end{aligned} \tag{49}
$$

By Assumption 2 that $\bar{v}_1(kh)=0$, for $k<0$, the following inequality holds

$$
\sum_{k=0}^{k_f}E\{\bar{v}_1^T(kh-\tau_2(k))\bar{v}_1(kh-\tau_2(k))\}
\leq\sum_{k=0}^{k_f}E\{\bar{v}_1^T(kh)\bar{v}_1(kh)\} \tag{50}
$$

As a result, by (50) with the fact that $V(\tilde{X}(k_fh))\geq0$, we immediately obtain the following inequality

$$
E\left\{\sum_{k=0}^{k_f}(\tilde{X}^T(kh)\tilde{Q}\tilde{X}(kh))-V(\tilde{X}(0))\right\}
\leq\rho E\left\{\sum_{k=0}^{k_f}\|\bar{v}_1(kh)\|^2\right\} \tag{51}
$$

As a result, if the inequalities in (20)–(22) hold, then the robust H_∞ networked security observer-based tracking performance in (18) of stochastic quadrotor NCS can be guaranteed for a prescribed level ρ.

On the other hand, if $\bar{v}_1(kh)=0$, the above inequality implies $E\{\tilde{X}^T(kh)\tilde{X}(kh)\}\to0$ as $k\to\infty$ due to the fact that the initial value $E\{V(\tilde{X}(0))\}$ is bounded, i.e., the mean square state/estimation error and mean square tracking error will converge to 0 in probability. The proof is completed.

APPENDIX B
PROOF OF THEOREM 2

By the rank test, the ith augmented system in (25) is observable if

$$\begin{aligned} & rank\begin{bmatrix} zI_{12+(n_a+n_s)(d+1)}-\bar{A}_i \\ \bar{C}_i \end{bmatrix} \\ &= rank\begin{bmatrix} zI_{12}-A_i^F & -hB_iC_{f_a} & 0 \\ 0 & zI_{n_a(d+1)}-A_{fa} & 0 \\ 0 & 0 & zI_{n_s(d+1)}-A_{fs} \\ C_i & 0 & D_iC_{fs} \end{bmatrix} \\ &= 12+(n_a+n_s)(d+1), \quad \forall z\in \mathbf{Z}_1 \end{aligned} \tag{52}$$

where $\mathbf{Z}_1$ collects the elements in the unit circle of complex z domain and $A_i^F = I_{12}+A_i$. To prove the rank conditions above, the proof is separated into following two cases:

$$\begin{aligned} &(i)\ z\in \mathbf{Z}_1 \setminus (eig\{A_i^F\}\cup eig\{A_{fa}\}\cup eig\{A_{fs}\}) \\ &(ii)\ z\in eig\{A_i^F\}\cup eig\{A_{fa}\}\cup eig\{A_{fs}\} \end{aligned}$$

In the first case (i), the following rank conditions can be obtained immediately:

$$\begin{aligned} & rank[zI_{12}-A_i^F] = 12, \\ & rank[zI_{n_a(d+1)}-A_{fa}] = n_a(d+1) \\ & rank[zI_{n_s(d+1)}-A_{fs}] = n_s(d+1) \\ & \qquad for\ z\in \mathbf{Z}_1\setminus (eig\{A_i^F\}\cup eig\{A_{fa}\} \cup eig\{A_{fs}\}) \end{aligned} \tag{53}$$

From the result above, (52) is satisfied for $z \in \mathbf{Z}_1 \setminus (eig\{A_i^F\}\cup eig\{A_{fa}\}\cup eig\{A_{fs}\})$, i.e.,

$$\begin{aligned} & rank\begin{bmatrix} zI_{12}-A_i^F & -hB_iC_{f_a} & 0 \\ 0 & zI_{n_a(d+1)}-A_{fa} & 0 \\ 0 & 0 & zI_{n_s(d+1)}-A_{fs} \\ C_i & 0 & D_iC_{fs} \end{bmatrix} \\ &= 12+(n_a+n_s)(d+1), \\ & \quad for\ z\in \mathbf{Z}_1\setminus (eig\{A_i^F\}\cup eig\{A_{fa}\}\cup eig\{A_{fs}\}), \\ & \quad i=1,\cdots,M \end{aligned} \tag{54}$$

In the second case (ii), by the assumptions in (27) and (28), the rank condition in (52) can be decoupled as

$$\begin{aligned} & rank\begin{bmatrix} zI_{12}-A_i^F & -hB_iC_{f_a} & 0 \\ 0 & zI_{n_a(d+1)}-A_{fa} & 0 \\ 0 & 0 & zI_{n_s(d+1)}-A_{fs} \\ C_i & 0 & D_iC_{fs} \end{bmatrix} \\ &= rank\begin{bmatrix} zI_{12}-A_i^F \\ C_i \end{bmatrix} + rank\begin{bmatrix} zI_{n_a(d+1)}-A_{fa} \\ -hB_iC_{f_a} \end{bmatrix} \\ & \quad + rank\begin{bmatrix} zI_{n_s(d+1)}-A_{fs} \\ D_iC_{fs} \end{bmatrix} \\ & \quad for\ z\in eig\{hA_i+I_{12}\}\cup eig\{A_{fa}\}\cup eig\{A_{fs}\}, \\ & \quad i=1,\cdots,M \end{aligned} \tag{55}$$

By applying the rank conditions in (26)–(30), the rank condition in (55) can be rewritten as:

$$\begin{aligned} & rank\begin{bmatrix} zI_{n_a(d+1)}-A_{fa} \\ -hB_iC_{f_a} \end{bmatrix} + rank\begin{bmatrix} zI_{12}-A_i^F \\ C_i \end{bmatrix} \\ & \quad + rank\begin{bmatrix} zI_{n_s(d+1)}-A_{fs} \\ D_iC_{fs} \end{bmatrix} \\ &= 12+n_a(d+1)+n_s(d+1), \\ & \quad for\ z\in eig\{hA_i+I_{12}\}\cup eig\{A_{fa}\}\cup eig\{A_{fs}\}, \\ & \quad i=1,\cdots,M \end{aligned}$$

Therefore, the observability for the ith local augmented quadrotor NCS in (25) is guaranteed.

APPENDIX C
PROOF OF THEOREM 3

The proof of Theorem 3 is separated as two parts. In the first part, we derive the sufficient condition for the robust H_∞ networked security observer-based tracking control design of stochastic quadrotor NCS. Then, in the second part, the sufficient conditions for the design are transformed to solvable linear matrix inequalities (LMIs).

A. DERIVATION OF SUFFICIENT CONDITIONS

To begin with, the following Lyapunov functional is selected:

$$\begin{aligned} & V(\tilde{X}(kh)) \\ &= E\{\tilde{X}^T(kh)P\tilde{X}(kh) + \sum_{e=k-\bar{\tau}_1(k)}^{(k-1)} \tilde{X}^T(eh)S_1\tilde{X}(eh) \\ & \quad + \sum_{r=k-\bar{\tau}_2(k)}^{(k-1)} \tilde{X}^T(rh)S_1\tilde{X}(rh)\} \\ & \quad + \sum_{r=d_{1m}}^{d_{1M}} \sum_{e=k-r}^{(k-1)} \tilde{X}^T(eh)S_3\tilde{X}(eh) \\ & \quad + \sum_{r=d_{2m}}^{d_{2M}} \sum_{e=k-\bar{\tau}_2(k)}^{(k-1)} \tilde{X}^T(eh)S_4\tilde{X}(eh) \end{aligned} \tag{56}$$

where $\{P, S_1, S_2, S_3, S_4\}$ are positive definite matrices, i.e., $P > 0$ and $\{S_i > 0\}_{i=1}^4$, d_{1M} is the upper bound of $\tau_1(k)$, d_{1m} is the lower bound of $\tau_1(k)$, d_{2M} is the upper bound of $\tau_2(k)$, d_{2m} is the lower bound of $\tau_2(k)$, and $\{\bar{\tau}_1(k)\in \mathbb{N}, \bar{\tau}_2(k)\in\mathbb{N}\}$ are positive sequences which satisfy $\bar{\tau}_1(k)h = \tau_1(k)$ and $\bar{\tau}_2(k)h=\tau_2(k)$, respectively. Then, the difference of Lyapunov function in (56) from $(k+1)h$ to kh can be derived as:

$$\begin{aligned} & V(\tilde{X}((k+1)h)) - V(\tilde{X}(kh)) \\ &= E\{\tilde{X}^T((k+1)h)P\tilde{X}((k+1)h) \\ & \quad -\tilde{X}^T(kh)P\tilde{X}(kh) + \tilde{X}^T(kh)S_1\tilde{X}(kh) \\ & \quad -\tilde{X}^T(kh-\tau_1(k))S_1\tilde{X}(kh-\tau_1(k)) \\ & \quad +\tilde{X}^T(kh)S_2\tilde{X}(kh) - \tilde{X}^T(kh-\tau_2(k))S_2\tilde{X}(kh-\tau_2(k)) \end{aligned}$$

$$
\begin{aligned}
&+(d_{1M}-d_{1m}+1)\tilde{X}^T(kh)S_3\tilde{X}(kh)\\
&-\sum_{r=d_{1m}}^{d_{1M}}\tilde{X}^T((k-r)h)S_3\tilde{X}((k-r)h)\\
&+(d_{2M}-d_{2m}+1)\tilde{X}^T(kh)S_4\tilde{X}(kh)\\
&-\sum_{r=d_{2m}}^{d_{2M}}\tilde{X}^T((k-r)h)S_4\tilde{X}((k-r)h)\}
\end{aligned}\tag{57}
$$

Then, with the difference of Lypunov function in (57), the numerator of the robust H_∞ networked security observer-based tracking performance in (18) can be rewritten as

$$
\begin{aligned}
&E\{\sum_{k=0}^{k_f}[\tilde{X}^T(kh)\tilde{Q}\tilde{X}(kh)]-V(\tilde{X}(0))\}\\
&\quad=E\{\sum_{k=0}^{k_f}[\tilde{X}^T(kh)\tilde{Q}\tilde{X}(kh)+V(\tilde{X}((k+1)h))\\
&\qquad-V(\tilde{X}(kh))]+V(\tilde{X}(0))\\
&\qquad-V(\tilde{X}((k_f+1)h))-V(\tilde{X}(0))\}\\
&\quad\le E\{\sum_{k=0}^{k_f}[\tilde{X}^T(kh)\tilde{Q}\tilde{X}(kh)\\
&\qquad+\tilde{X}^T((k+1)h)P\tilde{X}((k+1)h)\\
&\qquad-\tilde{X}^T(kh)P\tilde{X}(kh)+\tilde{X}^T(kh)S_1\tilde{X}(kh)\\
&\qquad-\tilde{X}^T(kh-\tau_1(k))S_1\tilde{X}(kh-\tau_1(k))\\
&\qquad+\tilde{X}^T(kh)S_2\tilde{X}(kh)\\
&\qquad-\tilde{X}^T(kh-\tau_2(k))S_2\tilde{X}(kh-\tau_2(k))\\
&\qquad+(d_{1M}-d_{1m}+1)\tilde{X}^T(kh)S_3\tilde{X}(kh)\\
&\qquad+(d_{2M}-d_{2m}+1)\tilde{X}^T(kh)S_4\tilde{X}(kh)]
\end{aligned}\tag{58}
$$

Before the further discussion, the representation of quadratic form x^TA^TPAx is denoted as $x^TA^TP[*]$ for the simplicity. Then, by substituting the dynamic of the augmented fuzzy observer-based tracking control system in (35) with Lemma 2 and the fact that $E\{\Delta W(kh)\}=0$, (57) can be written as follows:

$$
\begin{aligned}
&E\{V(\tilde{X}((k+1)h))-V(\tilde{X}(kh))\}\\
&\ \le\sum_{i,j,l,v=1}^{M}h_i(z(kh))h_j(z(kh))h_k(z(kh-\tau_1(k)))\\
&\quad\times h_l(z(kh-\tau_2(k)))\{E\{(\tilde{A}_{ijlv,D}+\tilde{A}_{ijlv,R}(kh)\\
&\quad+\Delta\tilde{g}_2(\tilde{X}(kh))+\Delta\tilde{g}_1(\tilde{X}(kh))(\tilde{G}_{ijlv,1,D}\\
&\quad+\tilde{G}_{ijlv,1,R}(kh))+(\tilde{D}_{x,ijlv,D}+\tilde{D}_{x,ijlv,R}(kh))\\
&\quad\times\Delta\tilde{D}(\tilde{X}(kh))]\tilde{X}(kh)+[\tilde{A}^{d_1}_{ijlv,D}+\tilde{A}^{d_1}_{ijlv,R}(kh)\\
&\quad+\Delta\tilde{g}_1(\tilde{X}(kh))(\tilde{G}_{ijlv,2,D}+\tilde{G}_{ijlv,2,R}(kh))]\\
&\quad\times\tilde{X}((kh-\tau_1(k))+[\tilde{A}^{d_2}_{ijlv,D}+\tilde{A}^{d_2}_{ijlv,R}(kh)\\
&\quad+(\tilde{D}^{d_2}_{x,ijlv,D}+\tilde{D}^{d_2}_{x,ijlv,R}(kh))\Delta\tilde{D}(\tilde{X}(kh-\tau_2(k)))]\\
&\quad\times\tilde{X}(kh-\tau_2(k))+(\tilde{D}_{ijlv,D}+\tilde{D}_{ijlv,R}(kh))\bar{v}_1(kh)\\
&\quad+(\tilde{D}^{d_2}_{ijlv,D}+\tilde{D}^{d_2}_{ijlv,R}(kh))\bar{v}_1(kh-\tau_2(k))+\Delta\tilde{f}(\tilde{X}(kh))\\
&\quad+(\tilde{C}_{j,D}+\tilde{C}_{j,R}(kh))\Delta\tilde{C}(\tilde{X}(kh))+(\tilde{C}^d_{j,D}+\tilde{C}^d_{j,R}(kh))\\
&\quad\times\Delta\tilde{C}(\tilde{X}(kh-\tau_2(k)))^TP[*]+(\tilde{E}_{ijlv}\tilde{X}(kh)\Delta W(kh)\\
&\quad+\Delta\tilde{j}(\tilde{X}(kh))\Delta W(kh))^TP(\tilde{E}_{ijlv}\tilde{X}(kh)\Delta W(kh)\\
&\quad+\Delta\tilde{j}(\tilde{X}(kh))\Delta W(kh))\}-E\{\tilde{X}^T(kh)P\tilde{X}(kh)\\
&\quad+\tilde{X}^T(kh)S_1\tilde{X}(kh)-\tilde{X}^T(kh-\tau_1(k))S_1\tilde{X}(kh-\tau_1(k))\\
&\quad+\tilde{X}^T(kh)S_2\tilde{X}(kh)-\tilde{X}^T(kh-\tau_2(k))S_2\tilde{X}(kh-\tau_2(k))\\
&\quad+(d_{1M}-d_{1m}+1)\tilde{X}^T(kh)S_3\tilde{X}(kh)\\
&\quad+(d_{2M}-d_{2m}+1)\tilde{X}^T(kh)S_4\tilde{X}(kh)\}
\end{aligned}\tag{59}
$$

By the fact of Bernoulli process that $E\{\delta(kh)-\bar{\delta}\}=0$, $E\{\beta(kh)-\bar{\beta}\}=0$, (59) can be separated as follows:

$$
\begin{aligned}
&E\{V(\tilde{X}((k+1)h))-V(\tilde{X}(kh))\}\\
&\ \le\sum_{i,j,l,v=1}^{M}h_i(z(kh))h_j(z(kh))h_k(z(kh-\tau_1(k)))\\
&\quad\times h_l(z(kh-\tau_2(k)))\{E\{([\tilde{A}_{ijlv,D}+\Delta\tilde{g}_2(\tilde{X}(kh))\\
&\quad+\Delta\tilde{g}_1(\tilde{X}(kh))\tilde{G}_{ijlv,1,D}+\tilde{D}_{x,ijlv,D}\Delta\tilde{D}(\tilde{X}(kh))]\tilde{X}(kh)\\
&\quad+[\tilde{A}^{d_1}_{ijlv,D}+\Delta\tilde{g}_1(\tilde{X}(kh))\tilde{G}_{ijlv,2,D}]\tilde{X}((kh-\tau_1(k))\\
&\quad+[\tilde{A}^{d_2}_{ijlv,D}+\tilde{D}^{d_2}_{x,ijlv,D}\Delta\tilde{D}(\tilde{X}(kh-\tau_2(k)))]\\
&\quad\times\tilde{X}(kh-\tau_2(k))+\tilde{D}_{ijlv,D}\bar{v}_1(kh)+\Delta\tilde{f}(\tilde{X}(kh))\\
&\quad+\tilde{D}^{d_2}_{ijlv,D}\bar{v}_1(kh-\tau_2(k))+\tilde{C}_{j,D}\Delta\tilde{C}(\tilde{X}(kh))\\
&\quad+\tilde{C}^d_{j,D}\Delta\tilde{C}(\tilde{X}(kh-\tau_2(k)))^TP[*]\\
&\quad+([\tilde{A}_{ijlv,R}(kh)+\Delta\tilde{g}_1(\tilde{X}(kh))\tilde{G}_{ijlv,1,R}(kh)\\
&\quad+\tilde{D}_{x,ijlv,R}(kh)\Delta\tilde{D}(\tilde{X}(kh))]\tilde{X}(kh)+[\tilde{A}^{d_1}_{ijlv,R}(kh)\\
&\quad+\Delta\tilde{g}_1(\tilde{X}(kh))(\tilde{G}_{ijlv,2,R}(kh))]\tilde{X}((kh-\tau_1(k))\\
&\quad+[\tilde{A}^{d_2}_{ijlv,R}(kh)+\tilde{D}^{d_2}_{x,ijlv,R}(kh)\Delta\tilde{D}(\tilde{X}(kh-\tau_2(k)))]\\
&\quad\times\tilde{X}(kh-\tau_2(k))+\tilde{D}_{ijlv,R}(kh)\bar{v}_1(kh)+\tilde{D}^{d_2}_{ijlv,R}(kh)\\
&\quad\times\bar{v}_1(kh-\tau_2(k))+\tilde{C}_{j,R}(kh)\Delta\tilde{C}(\tilde{X}(kh))+\tilde{C}^d_{j,R}(kh)\\
&\quad\times\Delta\tilde{C}(\tilde{X}(kh-\tau_2(k)))^TP[*]+[\tilde{E}_{ijlv}\tilde{X}(kh)\Delta W(kh)\\
&\quad+\Delta\tilde{j}(\tilde{X}(kh))\Delta W(kh)]^TP[\tilde{E}_{ijlv}\tilde{X}(kh)\Delta W(kh)\\
&\quad+\Delta\tilde{j}(\tilde{X}(kh))\Delta W(kh)]\}-\tilde{X}^T(kh)P\tilde{X}(kh)\\
&\quad+\tilde{X}^T(kh)S_1\tilde{X}(kh)-\tilde{X}^T(kh-\tau_1(k))S_1\tilde{X}(kh-\tau_1(k))\\
&\quad+\tilde{X}^T(kh)S_2\tilde{X}(kh)-\tilde{X}^T(kh-\tau_2(k))S_2\tilde{X}(kh-\tau_2(k))\\
&\quad+(d_{1M}-d_{1m}+1)\tilde{X}^T(kh)S_3\tilde{X}(kh)\\
&\quad+(d_{2M}-d_{2m}+1)\tilde{X}^T(kh)S_4\tilde{X}(kh)
\end{aligned}\tag{60}
$$

By using Lemma 1, the terms associated with the time-varying matrices in (60) can be relaxed as:

$$
\begin{aligned}
&E\{([\tilde{A}_{ijlv,R}(kh)+\Delta\tilde{g}_1(\tilde{X}(kh))\tilde{G}_{ijlv,1,R}(kh)\\
&\quad+\tilde{D}_{x,ijlv,R}(kh)\Delta\tilde{D}(\tilde{X}(kh))]\tilde{X}(kh)\\
&\quad+[\tilde{A}^{d_1}_{ijlv,R}(kh)+\Delta\tilde{g}_1(\tilde{X}(kh))(\tilde{G}_{ijlv,2,R}(kh))]\\
&\quad\times\tilde{X}((kh-\tau_1(k))+[\tilde{A}^{d_2}_{ijlv,R}(kh)+\tilde{D}^{d_2}_{x,ijlv,R}(kh)\\
&\quad\times\Delta\tilde{D}(\tilde{X}(kh-\tau_2(k)))]\tilde{X}(kh-\tau_2(k))\\
&\quad+\tilde{D}_{ijlv,R}(kh)\bar{v}_1(kh)+\tilde{D}^{d_2}_{ijlv,R}(kh)\bar{v}_1(kh-\tau_2(k))
\end{aligned}
$$

$$
\begin{aligned}
&+\tilde{C}_{j,R}(kh)\Delta\tilde{C}(\tilde{X}(kh))+\tilde{C}^{d}_{j,R}(kh)\\
&\times\Delta\tilde{C}(\tilde{X}(kh-\tau_2(k)))^T P[*]\}\\
\le\ & 2E\{[\tilde{A}_{ijlv,R}(kh)\tilde{X}(kh)+\tilde{A}^{d_1}_{ijlv,R}(kh)\tilde{X}((kh-\tau_1(k))\\
&+\tilde{A}^{d_2}_{ijlv,R}(kh)\tilde{X}(kh-\tau_2(k))+\tilde{D}_{ijlv,R}(kh)\bar{\upsilon}_1(kh)\\
&+\tilde{D}^{d_2}_{ijlv,R}(kh)\bar{\upsilon}_1(kh-\tau_2(k))]^T P[*]\}\\
&+2E\{[\Delta\tilde{g}_1(\tilde{X}(kh))\tilde{G}_{ijlv,1,R}(kh)+\tilde{D}_{x,ijlv,R}(kh)\\
&\times\Delta\tilde{D}(\tilde{X}(kh))]\tilde{X}(kh)+[\Delta\tilde{g}_1(\tilde{X}(kh))(\tilde{G}_{ijlv,2,R}(kh))]\\
&\times\tilde{X}((kh-\tau_1(k))+[\tilde{D}^{d_2}_{x,ijlv,R}(kh)\\
&\times\Delta\tilde{D}(\tilde{X}(kh-\tau_2(k)))]\tilde{X}(kh-\tau_2(k))\\
&+\tilde{C}_{j,R}(kh)\Delta\tilde{C}(\tilde{X}(kh))\\
&+\tilde{C}^{d}_{j,R}(kh)\Delta\tilde{C}(\tilde{X}(kh-\tau_2(k))]^T P[*]\}\\
\le\ & 10E\{[\tilde{X}^T(kh)\tilde{A}^T_{ijlv,R}(kh)P[*]\\
&+\tilde{X}^T((kh-\tau_1(k))(\tilde{A}^{d_1}_{ijlv,R})^T P[*]\\
&+\tilde{X}^T(kh-\tau_2(k))(\tilde{A}^{d_2}_{ijlv,R}(kh))^T P[*]\\
&+\bar{\upsilon}_1^T(kh)\tilde{D}^T_{ijlv,R}(kh)P[*]\\
&+\bar{\upsilon}_1^T(kh-\tau_2(k))(\tilde{D}^{d_2}_{ijlv,R}(kh))^T P[*]\}\\
&+12E\{[\tilde{X}^T(kh)\tilde{G}^T_{ijlv,1,R}(kh)\Delta\tilde{g}_1^T(\tilde{X}(kh))P[*]\\
&+\tilde{X}^T(kh)\Delta\tilde{D}^T(\tilde{X}(kh))\tilde{D}^T_{x,ijlv,R}(kh)P[*]\\
&+\tilde{X}^T(kh-\tau_1(k))\tilde{G}^T_{ijlv,2,R}(kh)\Delta\tilde{g}_1^T(\tilde{X}(kh))P[*]\\
&+\tilde{X}^T(kh-\tau_2(k))\Delta\tilde{D}^T(\tilde{X}(kh-\tau_2(k)))(\tilde{D}^{d_2}_{x,ijlv,R}(kh))^T\\
&\times P[*]+\Delta\tilde{C}^T(\tilde{X}(kh))\tilde{C}^T_{j,R}(kh)P[*]\\
&+\Delta\tilde{C}^T(\tilde{X}(kh-\tau_2(k))(\tilde{C}^{d}_{j,R}(kh))^T P[*]\}
\end{aligned}\tag{61}
$$

To decouple the fuzzy approximation errors, the following eigenvalue constraint of P and slack variables are introduced.:

$$
\begin{aligned}
E\{\tilde{D}^T_{x,ijlv,R}(kh)P\tilde{D}^T_{x,ijlv,R}(kh)\}&\le M_1,\\
E\{(\tilde{D}^{d_2}_{x,ijlv,R}(kh))^T P\tilde{D}^{d_2}_{x,ijlv,R}(kh)\}&\le M_2\\
E\{\tilde{C}^T_{j,R}(kh)P\tilde{C}_{j,R}(kh)\}&\le M_3,\\
E\{(\tilde{C}^{d}_{j,R}(kh))^T P\tilde{C}^{d}_{j,R}(kh)\}&\le M_4\\
P\le\Theta_P I,\quad M_1&\le\Theta_1 I,\ M_2\le\Theta_2 I,\\
M_3\le\Theta_3 I,\quad M_4&\le\Theta_4 I,
\end{aligned}\tag{62}
$$

where $\{M_i\}_{i=1}^4$ are positive definite matrices to be designed and $\{\Theta_P,\Theta_1,\Theta_2,\Theta_3,\Theta_4,\Theta_5\}$ denotes the 1-D predefined positive scalar.

By applying the expectation operator with the constraints in (62), (61) can be relaxed as:

$$
\begin{aligned}
&E\{[\tilde{A}_{ijlv,R}(kh)+\Delta\tilde{g}_1(\tilde{X}(kh))\tilde{G}_{ijlv,1,R}(kh)\\
&\quad+\tilde{D}_{x,ijlv,R}(kh)\Delta\tilde{D}(\tilde{X}(kh))]\tilde{X}(kh)+[\tilde{A}^{d_1}_{ijlv,R}(kh)\\
&\quad+\Delta\tilde{g}_1(\tilde{X}(kh))(\tilde{G}_{ijlv,2,R}(kh))]\tilde{X}((kh-\tau_1(k))\\
&\quad+[\tilde{A}^{d_2}_{ijlv,R}(kh)+\tilde{D}^{d_2}_{ijlv,R}(kh)\Delta\tilde{D}(\tilde{X}(kh-\tau_2(k)))]\\
&\quad\times\tilde{X}(kh-\tau_2(k))+\tilde{D}_{ijlv,R}(kh)\bar{\upsilon}_1(kh)\\
&\quad+\tilde{D}^{d_2}_{ijlv,R}(kh)\bar{\upsilon}_1(kh-\tau_2(k))+\tilde{C}_{j,R}(kh)\Delta\tilde{C}(\tilde{X}(kh))\\
&\quad+\tilde{C}^{d}_{j,R}(kh)\Delta\tilde{C}(\tilde{X}(kh-\tau_2(k)))^T P(*)\}\\
&\le 10[\tilde{X}^T(kh)\tilde{A}^T_{ijlv,E}P[*]+\tilde{X}^T((kh-\tau_1(k))(\tilde{A}^{d_1}_{ijlv,E})^T\\
&\quad\times P[*]+\tilde{X}^T(kh-\tau_2(k))(\tilde{A}^{d_2}_{ijlv,E})^T P[*]\\
&\quad+\bar{\upsilon}_1^T(kh)\tilde{D}^T_{ijlv,E}P[*]+\bar{\upsilon}_1^T(kh-\tau_2(k))(\tilde{D}^{d_2}_{ijlv,E})^T\\
&\quad\times P[*]]+\tilde{X}^T(kh)[12(\Theta_P\epsilon_2^2\tilde{G}^T_{ijlv,1,E}\tilde{G}_{ijlv,1,E}\\
&\quad+(\Theta_1\epsilon_5^2+\Theta_3\epsilon_4^2)I)]\tilde{X}(kh)\\
&\quad+\tilde{X}^T(kh-\tau_2(k))[12(\Theta_2\epsilon_5^2+\Theta_4\epsilon_4^2)I)]\\
&\quad\times\tilde{X}(kh-\tau_2(k))+\tilde{X}^T(kh-\tau_1(k))\\
&\quad\times[12\Theta_P\epsilon_2^2\tilde{G}^T_{ijlv,2,E}\tilde{G}_{ijlv,2,E}]\tilde{X}(kh-\tau_1(k))
\end{aligned}\tag{63}
$$

with the following deterministic matrices

$$
\tilde{A}_{ijlv,E}=\begin{bmatrix}-\delta^*\bar{B}_i\bar{K}_j & \delta^*\bar{B}_i\bar{K}_j & \delta^*\bar{B}_iK_{2j}\\ -\delta^*\bar{B}_i\bar{K}_j+\beta^*L_j\bar{C}_i & \delta^*\bar{B}_i\bar{K}_j & \delta^*\bar{B}_iK_{2j}\\ 0&0&0\end{bmatrix}
$$

$$
\tilde{A}^{d_1}_{ijlv,E}=\delta^*\begin{bmatrix}\bar{B}_i\bar{K}_j & -\bar{B}_i\bar{K}_j & -\bar{B}_iK_{2j}\\ \bar{B}_i\bar{K}_j & -\bar{B}_i\bar{K}_j & -\bar{B}_iK_{2j}\\ 0&0&0\end{bmatrix}
$$

$$
\tilde{A}^{d_2}_{ijlv,E}=\begin{bmatrix}0&0&0\\-\beta^*L_j\bar{C}_l&0&0\\0&0&0\end{bmatrix},
$$

$$
\tilde{G}_{ijlv,2,E}=\delta^*\begin{bmatrix}\bar{K}_j & -\bar{K}_j & -K_{2j}\\ \bar{K}_j & -\bar{K}_j & -K_{2j}\\ 0&0&0\end{bmatrix}
$$

$$
\tilde{D}^{d_2}_{ijlv,E}=\begin{bmatrix}0&0\\-\beta^*L_i\bar{D}&0\\0&0\end{bmatrix}
$$

$$
\tilde{D}_{ijlv,E}=\begin{bmatrix}0&0\\\beta^*L_j\bar{D}&0\\0&0\end{bmatrix}
$$

$$
\tilde{G}_{ijlv,1,E}=\delta^*\begin{bmatrix}-\bar{K}_j & \bar{K}_j & K_{2j}\\ -\bar{K}_j & \bar{K}_j & K_{2j}\\ 0&0&0\end{bmatrix}
$$

$$
\begin{aligned}
\tilde{D}_{x,ijlv,E}&=diag\{0,\beta^*L_j,0\},\quad \delta^*=\sqrt{\bar{\delta}(1-\bar{\delta})},\\
\tilde{D}^{d_2}_{x,ijlv,E}&=diag\{0,-\beta^*L_j,0\}\quad \tilde{C}_{j,E}=diag\{0,\beta^*L_j,0\}\\
\tilde{C}^{d}_{j,E}&=diag\{0,-\beta^*L_j,0\},\quad \beta^*=\sqrt{\bar{\beta}(1-\bar{\beta})}
\end{aligned}
$$

On the other hand, by using Lemma 1, the terms associated with the time-invariant matrices in (60) can be relaxed as:

$$
\begin{aligned}
&[\tilde{A}_{ijlv,D}+\Delta\tilde{g}_2(\tilde{X}(kh))+\Delta\tilde{g}_1(\tilde{X}(kh))\tilde{G}_{ijlv,1,D}\\
&\quad+\tilde{D}_{x,ijlv,D}\Delta\tilde{D}(\tilde{X}(kh))]\tilde{X}(kh)+[\tilde{A}^{d_1}_{ijlv,D}\\
&\quad+\Delta\tilde{g}_1(\tilde{X}(kh))\tilde{G}_{ijlv,2,D}]\tilde{X}((kh-\tau_1(k))\\
&\quad+[\tilde{A}^{d_2}_{ijlv,D}+\tilde{D}^{d_2}_{ijlv,D}\Delta\tilde{D}(\tilde{X}(kh-\tau_2(k)))]\\
&\quad\times\tilde{X}(kh-\tau_2(k))+\tilde{D}_{ijlv,D}\bar{\upsilon}_1(kh)\\
&\quad+\Delta\tilde{f}(\tilde{X}(kh))+\tilde{D}^{d_2}_{ijlv,D}\bar{\upsilon}_1(kh-\tau_2(k))+\tilde{C}_{j,D}\\
&\quad\times\Delta\tilde{C}(\tilde{X}(kh))+\tilde{C}^{d}_{j,D}\Delta\tilde{C}(\tilde{X}(kh-\tau_2(k)))^T P[*]
\end{aligned}
$$

$$\begin{aligned}
&\leq 2[\tilde{A}_{ijlv,D}\tilde{X}(kh) + \tilde{A}^{d_1}_{ijlv,D}\tilde{X}((kh-\tau_1(k)) \\
&\quad + \tilde{A}^{d_2}_{ijlv,D}\tilde{X}(kh-\tau_2(k)) + \tilde{D}_{ijlv,D}\bar{\upsilon}_1(kh) \\
&\quad + \tilde{D}^{d_2}_{ijlv,D}\bar{\upsilon}_1(kh-\tau_2(k))]^T P[*] \\
&\quad + 2\{[\Delta\tilde{g}_2(\tilde{X}(kh)) + \Delta\tilde{g}_1(\tilde{X}(kh))\tilde{G}_{ijlv,1,D} \\
&\quad + \tilde{D}_{x,ijlv,D}\Delta\tilde{D}(\tilde{X}(kh))]\tilde{X}(kh) + \Delta\tilde{f}(\tilde{X}(kh)) \\
&\quad + \Delta\tilde{g}_1(\tilde{X}(kh))\tilde{G}_{ijlv,2,D}\tilde{X}((kh-\tau_1(k)) \\
&\quad + \tilde{D}^{d_2}_{ijlv,D}\Delta\tilde{D}(\tilde{X}(kh-\tau_2(k)))\tilde{X}(kh-\tau_2(k)) \\
&\quad + \tilde{C}_{j,D}\Delta\tilde{C}(\tilde{X}(kh)) + \tilde{C}^d_{j,D}\Delta\tilde{C}(\tilde{X}(kh-\tau_2(k))\}P[*] \qquad (64)
\end{aligned}$$

To decouple the fuzzy approximation errors, the following slack variables are introduced.:

$$\begin{aligned}
&\tilde{D}^T_{x,ijlv,D}P\tilde{D}_{x,ijlv,D} \leq M_5 \\
&(\tilde{D}^{d_2}_{x,ijlv,D})^T P\tilde{D}^{d_2}_{x,ijlv,D} \leq M_6 \\
&\tilde{C}^T_{j,D}P\tilde{C}_{j,D} \leq M_7, \quad (\tilde{C}^d_{j,D})^T P\tilde{C}^d_{j,D} \leq M_8 \\
&M_5 \leq \Theta_5 I, \quad M_6 \leq \Theta_6 I \\
&M_7 \leq \Theta_7 I, \quad M_8 \leq \Theta_8 I \qquad (65)
\end{aligned}$$

where $\{M_i\}_{i=5}^{8}$ are positive definite matrices to be designed and $\{\Theta_5, \Theta_6, \Theta_7, \Theta_8\}$ denote the 1-D predefined positive scalars.

Similar to the derivation in (63), by using the inequalities in (65) and Lemma 1, the terms associated with the fuzzy approximation errors of time-invariant matrices in (64) can be relaxed as:

$$\begin{aligned}
&2\{[\Delta\tilde{g}_2(\tilde{X}(kh)) + \Delta\tilde{g}_1(\tilde{X}(kh))\tilde{G}_{ijlv,1,D} \\
&\quad + \tilde{D}_{x,ijlv,D}\Delta\tilde{D}(\tilde{X}(kh))]\tilde{X}(kh) + \Delta\tilde{f}(\tilde{X}(kh)) \\
&\quad + \Delta\tilde{g}_1(\tilde{X}(kh))\tilde{G}_{ijlv,2,D}\tilde{X}((kh-\tau_1(k)) \\
&\quad + \tilde{D}^{d_2}_{ijlv,D}\Delta\tilde{D}(\tilde{X}(kh-\tau_2(k)))\tilde{X}(kh-\tau_2(k)) \\
&\quad + \tilde{C}_{j,D}\Delta\tilde{C}(\tilde{X}(kh)) + \tilde{C}^d_{j,D}\Delta\tilde{C}(\tilde{X}(kh-\tau_2(k))\}P[*] \\
&\leq 16\{\Delta\tilde{f}^T(\tilde{X}(kh))P\Delta\tilde{f}(\tilde{X}(kh)) \\
&\quad + \tilde{X}^T(kh)\Delta^T\tilde{D}(\tilde{X}(kh))\tilde{D}^T_{x,ijlv,D}P[*] \\
&\quad + \tilde{X}^T(kh)\Delta\tilde{g}_2^T(\tilde{X}(kh))P[*] \\
&\quad + \tilde{X}^T(kh)\tilde{G}^T_{ijlv,1,D}\Delta\tilde{g}_1^T(\tilde{X}(kh))P[*] \\
&\quad + \tilde{X}^T((kh-\tau_1(k))\tilde{G}^T_{ijlv,2,D}\Delta\tilde{g}_1^T(\tilde{X}(kh))P[*] \\
&\quad + \tilde{X}^T(kh-\tau_2(k))\Delta\tilde{D}^T(\tilde{X}(kh-\tau_2(k)))(\tilde{D}^{d_2}_{ijlv,D})^T \\
&\quad \times P[*] + \Delta\tilde{C}^T(\tilde{X}(kh))\tilde{C}_{j,D}P[*] \\
&\quad \times \Delta\tilde{C}^T(\tilde{X}(kh-\tau_2(k))(\tilde{C}^d_{j,D})^T P[*]\} \\
&\leq \tilde{X}^T(kh)[16((\Theta_P\epsilon_1^2 + \Theta_5\epsilon_5^2 + \Theta_P\epsilon_3^2 + \Theta_7\epsilon_4^2)I \\
&\quad + \Theta_P\epsilon_2^2\tilde{G}^T_{ijlv,1,D}\tilde{G}_{ijlv,1,D})]\tilde{X}(kh) + \tilde{X}^T((kh-\tau_1(k)) \\
&\quad \times [16\Theta_P\epsilon_2^2\tilde{G}^T_{ijlv,2,D}\tilde{G}_{ijlv,2,D}]\tilde{X}((kh-\tau_1(k)) \\
&\quad + \tilde{X}^T(kh-\tau_2(k))[16(\Theta_6\epsilon_5^2 + \Theta_8\epsilon_4^2)I]\tilde{X}(kh-\tau_2(k)) \qquad (66)
\end{aligned}$$

Also, by Lemma 1 with the fact $E\{\Delta W^2(kh)\} = h$, the terms associated with stochastic process can be relaxed as:

$$\begin{aligned}
&E\{(\tilde{E}_{ijlv}\tilde{X}(kh)\Delta W(kh) + \Delta\tilde{j}(\tilde{X}(kh))\Delta W(kh)^T P[*]\} \\
&\leq h(2\tilde{X}^T(kh)\tilde{E}^T_{ijlv}P\tilde{E}_{ijlv}\tilde{X}(kh) \\
&\quad + 2\Delta\tilde{j}^T(\tilde{X}(kh))P\Delta\tilde{j}(\tilde{X}(kh))) \\
&\leq \tilde{X}^T(kh)[2h\tilde{E}^T_{ijlv}P\tilde{E}_{ijlv} + 2h\epsilon_6^2\Theta_P I]\tilde{X}(kh) \qquad (67)
\end{aligned}$$

By using (63), (66), (67), the difference of Lyapunov function in (59) can be written as:

$$\begin{aligned}
&E\{V(\tilde{X}((k+1)h)) - V(\tilde{X}(kh))\} \\
&\leq \sum_{i,j,l,v=1}^{M} h_i(z(kh))h_j(z(kh))h_k(z(kh-\tau_1(k))) \\
&\quad \times h_l(z(kh-\tau_2(k)))\{E\{2[\tilde{A}_{ijlv,D}\tilde{X}(kh) \\
&\quad + \tilde{A}^{d_1}_{ijlv,D}\tilde{X}((kh-\tau_1(k)) + \tilde{A}^{d_2}_{ijlv,D}\tilde{X}(kh-\tau_2(k)) \\
&\quad + \tilde{D}_{ijlv,D}\bar{\upsilon}_1(kh) + \tilde{D}^{d_2}_{ijlv,D}\bar{\upsilon}_1(kh-\tau_2(k))]^T P[*] \\
&\quad + \tilde{X}^T(kh)[16((\Theta_P\epsilon_1^2 + \Theta_5\epsilon_5^2 + \Theta_P\epsilon_3^2 + \Theta_7\epsilon_4^2)I \\
&\quad + \Theta_P\epsilon_2^2\tilde{G}^T_{ijlv,1,D}\tilde{G}_{ijlv,1,D})]\tilde{X}(kh) \\
&\quad + \tilde{X}^T((kh-\tau_1(k))[16\Theta_P\epsilon_2^2\tilde{G}^T_{ijlv,2,D}\tilde{G}_{ijlv,2,D}] \\
&\quad \times \tilde{X}((kh-\tau_1(k)) + \tilde{X}^T(kh-\tau_2(k))[16(\Theta_6\epsilon_5^2 \\
&\quad + \Theta_8\epsilon_4^2)I]\tilde{X}(kh-\tau_2(k)) + 10[\tilde{X}^T(kh)\tilde{A}^T_{ijlv,E}P[*] \\
&\quad + + \tilde{X}^T((kh-\tau_1(k))(\tilde{A}^{d_1}_{ijlv,E})^T P[*] + \tilde{X}^T(kh-\tau_2(k)) \\
&\quad \times (\tilde{A}^{d_2}_{ijlv,E})^T P[*] + \bar{\upsilon}_1^T(kh)\tilde{D}^T_{ijlv,E}P[*] \\
&\quad + \bar{\upsilon}_1^T(kh-\tau_2(k))(\tilde{D}^{d_2}_{ijlv,E})^T P[*]] + \tilde{X}^T(kh) \\
&\quad \times [12(\Theta_P\epsilon_2^2\tilde{G}^T_{ijlv,1,E}\tilde{G}_{ijlv,1,E} + (\Theta_1\epsilon_5^2 + \Theta_3\epsilon_4^2)I)] \\
&\quad \times \tilde{X}(kh) + \tilde{X}^T(kh-\tau_2(k))[12(\Theta_2\epsilon_5^2 \\
&\quad + \Theta_4\epsilon_4^2)I)]\tilde{X}(kh-\tau_2(k)) + \tilde{X}^T(kh-\tau_1(k)) \\
&\quad \times [12\Theta_P\epsilon_2^2\tilde{G}^T_{ijlv,2,E}\tilde{G}_{ijlv,2,E}]\tilde{X}(kh-\tau_1(k)) \\
&\quad + \tilde{X}^T(kh)[2h\tilde{E}^T_{ijlv}P\tilde{E}_{ijlv} + 2h\epsilon_6^2\Theta_P I]\tilde{X}(kh) \\
&\quad - \tilde{X}^T(kh)P\tilde{X}(kh) + \tilde{X}^T(kh)S_1\tilde{X}(kh) \\
&\quad - \tilde{X}^T(kh-\tau_1(k))S_1\tilde{X}(kh-\tau_1(k)) \\
&\quad + \tilde{X}^T(kh)S_2\tilde{X}(kh) \\
&\quad - \tilde{X}^T(kh-\tau_2(k))S_2\tilde{X}(kh-\tau_2(k)) \\
&\quad + (d_{1M} - d_{1m} + 1)\tilde{X}^T(kh)S_3\tilde{X}(kh) \\
&\quad + (d_{2M} - d_{2m} + 1)\tilde{X}^T(kh)S_4\tilde{X}(kh) \qquad (68)
\end{aligned}$$

After some manipulation, (68) can be rewritten as the following compact form:

$$\begin{aligned}
&E\{V(\tilde{X}((k+1)h)) - V(\tilde{X}(kh))\} \\
&\leq \sum_{i,j,l,v=1}^{M} h_i(z(kh))h_j(z(kh))h_k(z(kh-\tau_1(k))) \\
&\quad \times h_l(z(kh-\tau_2(k)))\{\Gamma_1^T(kh) \\
&\quad \times [2\Xi^T_{ijlv}P\Xi_{ijlv} + \Pi^T_{ijlv}\bar{P}\Pi_{ijlv} + \Lambda_{ijlv}]\Gamma_1(kh)\} \\
&\quad + \frac{\rho}{2}\bar{\upsilon}_1^T(kh)\bar{\upsilon}_1(kh) \\
&\quad + \frac{\rho}{2}\bar{\upsilon}_1^T(kh-\tau_2(k))\bar{\upsilon}_1(kh-\tau_2(k)) \qquad (69)
\end{aligned}$$

where $\Gamma_1(kh) = [\tilde{X}^T(kh)\ \tilde{X}^T((kh-\tau_1(k))\ \tilde{X}^T(kh-\tau_2(k))\ \bar{\upsilon}_1^T(kh)\ \bar{\upsilon}_1^T(kh-\tau_2(k))]^T$ with the matrices

$$\begin{aligned}
\Xi_{ijlv} &= [\tilde{A}_{ijlv,D}\ \tilde{A}^{d_1}_{ijlv,D}\ \tilde{A}^{d_2}_{ijlv,D}\ \tilde{D}_{ijlv,D}\ \tilde{D}^{d_2}_{ijlv,D}],\\
\bar{P} &= diag\{P,P,P,P,P\}\\
\Pi_{ijlv} &= \sqrt{10}diag\{\tilde{A}_{ijlv,E},\tilde{A}^{d_1}_{ijlv,E},\tilde{A}^{d_2}_{ijlv,E}\cdots\cdots,\\
&\qquad \tilde{D}_{ijlv,E},\tilde{D}^{d_2}_{ijlv,E}\}\\
\Lambda_{ijlv} &= diag\{\Lambda_{ijlv,1},\Lambda_{ijlv,2},\Lambda_{ijlv,3},\Lambda_{ijlv,4},\Lambda_{ijlv,5}\}\\
\Lambda_{ijlv,1} &= 16((\Theta_P\epsilon_1^2+\Theta_5\epsilon_5^2+\Theta_P\epsilon_3^2+\Theta_7\epsilon_4^2)I\\
&\quad +\Theta_P\epsilon_2^2\tilde{G}^T_{ijlv,1,D}\tilde{G}_{ijlv,1,D})\\
&\quad +12(\Theta_P\epsilon_2^2\tilde{G}^T_{ijlv,1,E}\tilde{G}_{ijlv,1,E}+(\Theta_1\epsilon_5^2+\Theta_3\epsilon_4^2)I)\\
&\quad +2h\tilde{E}^T_{ijlv}P\tilde{E}_{ijlv}+2h\epsilon_6^2\Theta_P I+S_2+S_1-P\\
&\quad +(d_{1M}-d_{1m}+1)S_3+(d_{2M}-d_{2m}+1)S_4\\
\Lambda_{ijlv,2} &= 16\Theta_P\epsilon_2^2\tilde{G}^T_{ijlv,2,D}\tilde{G}_{ijlv,2,D}\\
&\quad +12\Theta_P\epsilon_2^2\tilde{G}^T_{ijlv,2,E}\tilde{G}_{ijlv,2,E}-S_1\\
\Lambda_{ijlv,3} &= 16(\Theta_6\epsilon_5^2+\Theta_8\epsilon_4^2)I+12(\Theta_2\epsilon_5^2+\Theta_4\epsilon_4^2)I)-S_2\\
\Lambda_{ijlv,4} &= -\frac{\rho}{2}I,\quad \Lambda_{ijlv,5}=-\frac{\rho}{2}I
\end{aligned}$$

By (69), the numerator of the H_∞ observer-based security tracking performance in (58) can be rewritten as

$$\begin{aligned}
&E\{\sum_{k=0}^{k_f}[\tilde{X}^T(kh)\tilde{Q}\tilde{X}(kh)]-V(\tilde{X}(0))\}\\
&= E\{\sum_{k=0}^{k_f}[\tilde{X}^T(kh)\tilde{Q}\tilde{X}(kh)\\
&\quad +V(\tilde{X}((k+1)h))-V(\tilde{X}(kh))]\\
&\quad +V(\tilde{X}(0)))-V(\tilde{X}(k_f+1)h))-V(\tilde{X}(0))\}\\
&\le \sum_{k=0}^{k_f}\{\sum_{i,j,l,v=1}^{M} h_i(z(kh))h_j(z(kh))h_k(z(kh-\tau_1(k)))\\
&\quad \times h_l(z(kh-\tau_2(k)))\{\Gamma_1^T(kh)[2\Xi^T_{ijlv}P\Xi_{ijlv}\\
&\quad +\Pi^T_{ijlv}\bar{P}\Pi_{ijlv}+\bar{\Lambda}_{ijlv}]\Gamma_1(kh)\}+\frac{\rho}{2}\bar{\upsilon}_1^T(kh)\bar{\upsilon}_1(kh)\\
&\quad +\frac{\rho}{2}\bar{\upsilon}_1^T(kh-\tau_2(k))\bar{\upsilon}_1(kh-\tau_2(k))\}
\end{aligned} \tag{70}$$

where $\bar{\Lambda}_{ijlv} = diag\{\bar{\Lambda}_{ijlv,1},\Lambda_{ijlv,2},\Lambda_{ijlv,3},\Lambda_{ijlv,4},\Lambda_{ijlv,5}\}$ and $\bar{\Lambda}_{ijlv,1} = 16((\Theta_P\epsilon_1^2+\Theta_5\epsilon_5^2+\Theta_P\epsilon_3^2+\Theta_7\epsilon_4^2)I+\Theta_P\epsilon_2^2\tilde{G}^T_{ijlv,1,D}\tilde{G}_{ijlv,1,D})+12(\Theta_P\epsilon_2^2\tilde{G}^T_{ijlv,1,E}\tilde{G}_{ijlv,1,E}+(\Theta_1\epsilon_5^2+\Theta_3\epsilon_4^2)I+2h\tilde{E}^T_{ijlv}P\tilde{E}_{ijlv}+2h\epsilon_6^2\Theta_P I+S_2+S_1-P+\tilde{Q}+(d_{1M}-d_{1m}+1)S_3+(d_{2M}-d_{2m}+1)S_4$.

If the following inequalities hold:

$$\Gamma_1^T(kh)[2\Xi^T_{ijlv}P\Xi_{ijlv}+\Pi^T_{ijlv}\bar{P}\Pi_{ijlv}+\bar{\Lambda}_{ijlv}]\Gamma_1(kh)\le 0 \quad for\ i,j,l,v=1,\cdots,M \tag{71}$$

then, with the fact $\bar{\upsilon}_1(-kh)=0$, for k=1,2,3..., (70) can be written as:

$$\begin{aligned}
&E\{\sum_{k=0}^{k_f}[\tilde{X}^T(kh)\tilde{Q}\tilde{X}(kh)]-V(\tilde{X}(0)))\}\\
&\le \sum_{k=0}^{k_f}\frac{\rho}{2}\bar{\upsilon}_1^T(kh)\bar{\upsilon}_1(kh)\\
&\quad +\frac{\rho}{2}\bar{\upsilon}_1^T(kh-\tau_2(k))\bar{\upsilon}_1(kh-\tau_2(k))\\
&\le \sum_{k=0}^{k_f}\rho\bar{\upsilon}_1^T(kh)\bar{\upsilon}_1(kh)
\end{aligned} \tag{72}$$

which shows the robust H_∞ fuzzy networked security observer-based reference tracking performance is satisfied under a disturbance attenuation level ρ. Similarly, if $\bar{\upsilon}_1(kh)=0$, the above inequality implies $E\{\tilde{X}^T(kh)\tilde{X}(kh)\}\to 0$ as $k\to\infty$ due to the fact that the initial value $E\{V(\tilde{X}(0))\}$ is bounded, i.e., the mean square state/estimation error and mean square tracking error will converge to 0 in probability.

B. TRANSFORMATION OF BILINEAR MATRIX INEQUALITIES

From the above discussion, the H_∞ observer-based security tracking control design is transformed to equivalent matrix inequalities in (62), (65), (71). However, due to the coupling of design variables, these matrix inequalities are hard to be solved via current optimization methods. To deal with this problem, some matrix inequality formulas are used to transform these matrix inequalities into linear matrix inequalities (LMIs) and it can be easily solved via MATLAB LMI TOOLBOX. Furthermore, some slack variables are introduced to reduce the conservative in the derived LMIs.

To begin with, the following difference equations hold:

$$\begin{aligned}
\tilde{X}(kh)-\tilde{X}((kh-\tau_1(k)) &= \sum\nolimits_{s=k-\bar{\tau}_1(k)}^{k-1}\eta(sh)\\
\tilde{X}(kh)-\tilde{X}((kh-\tau_2(k)) &= \sum\nolimits_{s=k-\bar{\tau}_2(k)}^{k-1}\eta(sh)
\end{aligned} \tag{73}$$

where $\eta(sh)=\tilde{X}((s+1)h)-\tilde{X}(sh)$.

Then, by the fact in (73), the following equations hold:

$$\begin{aligned}
&2(\tilde{X}(kh)-\tilde{X}((kh-\tau_1(k))-\sum\nolimits_{s=k-\bar{\tau}_1(k)}^{k-1}\eta(sh))\\
&\quad\times(-N_{ijlv,1}\tilde{X}(kh)+O_{ijlv,1}\sum\nolimits_{s=k-\bar{\tau}_1(k)}^{k-1}\eta(sh))=0\\
&2(\tilde{X}(kh)-\tilde{X}((kh-\tau_2(k))-\sum\nolimits_{s=k-\bar{\tau}_2(k)}^{k-1}\eta(sh))\\
&\quad\times(-N_{ijlv,2}\tilde{X}(kh)+O_{ijlv,2}\sum\nolimits_{s=k-\bar{\tau}_1(k)}^{k-1}\eta(sh))=0
\end{aligned} \tag{74}$$

where $\{N_{ijlv,1},N_{ijlv,2},O_{ijlv,1},O_{ijlv,2}\}_{i,j,l,v=1}^M$ are slack variables to be designed.

By (74), the matrix inequalities in (71) are equivalent to following inequalities:

$$\Gamma_2^T(kh)[\tilde{\Xi}^T_{ijlv}\tilde{P}_2\tilde{\Xi}_{ijlv}+\tilde{\Pi}^T_{ijlv}\tilde{P}_1\tilde{\Pi}_{ijlv}+\tilde{\Lambda}_{ijlv}]\Gamma_2(kh)\le 0 \quad for\ i,j,l,v=1,\cdots,M \tag{75}$$

where $\Gamma_2(kh) = [\Gamma_1^T(kh)\ \sum_{s=k-\bar{\tau}_2(k)}^{k-1}\eta^T(sh))\ \sum_{s=k-\bar{\tau}_2(k)}^{k-1}\eta^T(sh))]^T$, $\tilde{\Xi}_{ijlv} = diag\{\Xi_{ijlv},I,I\}$, $\tilde{P}_2 = diag\{2P,P,P\}$,

$\tilde{P}_1 = diag\{P, P, P, P, P, P, P\}$, $\tilde{\Pi}_{ijlv} = diag\{\Pi_{ijlv}, I\ , I\}$ and

$$\tilde{\Lambda}_{ijlv} = \begin{bmatrix} \tilde{\Lambda}_{ijlv,I} & \tilde{\Lambda}_{ijlv,II} \\ * & \tilde{\Lambda}_{ijlv,III} \end{bmatrix}$$

$$\tilde{\Lambda}_{ijlv,I} = \begin{bmatrix} \tilde{\Lambda}_{ijlv,1} & \tilde{\Lambda}_{ijlv,8} & \tilde{\Lambda}_{ijlv,9} \\ * & \tilde{\Lambda}_{ijlv,2} & 0 \\ * & * & \tilde{\Lambda}_{ijlv,3} \end{bmatrix}$$

$$\tilde{\Lambda}_{ijlv,II} = \begin{bmatrix} 0 & 0 & \tilde{\Lambda}_{ijlv,10} & \tilde{\Lambda}_{ijlv,11} \\ 0 & 0 & \tilde{\Lambda}_{ijlv,12} & 0 \\ 0 & 0 & 0 & \tilde{\Lambda}_{ijlv,13} \end{bmatrix}$$

$$\begin{aligned}
\tilde{\Lambda}_{ijlv,III} &= diag\{\tilde{\Lambda}_{ijlv,4}, \tilde{\Lambda}_{ijlv,5}, \tilde{\Lambda}_{ijlv,6}, \tilde{\Lambda}_{ijlv,7}\} \\
\tilde{\Lambda}_{ijlv,1} &= 16(\Theta_P\epsilon_1^2 + \Theta_5\epsilon_5^2 + \Theta_P\epsilon_3^2 + \Theta_7\epsilon_4^2)I \\
&\quad + 16\Theta_P\epsilon_2^2\tilde{G}_{ijlv,1,D}^T\tilde{G}_{ijlv,1,D} + 12(\Theta_P\epsilon_2^2\tilde{G}_{ijlv,1,E}^T \\
&\quad \times \tilde{G}_{ijlv,1,E} \\
&\quad + (\Theta_1\epsilon_5^2 + \Theta_3\epsilon_4^2)I) + 2h\tilde{E}_{ijlv}^TP\tilde{E}_{ijlv} + 2h\epsilon_6^2\Theta_PI \\
&\quad + S_2 + S_1 - P - 2N_{ijlv,1} - 2N_{ijlv,2} \\
&\quad + \tilde{Q} + (d_{1M} - d_{1m} + 1)S_3 + (d_{2M} - d_{2m} + 1)S_4 \\
\tilde{\Lambda}_{ijlv,2} &= 16\Theta_P\epsilon_2^2\tilde{G}_{ijlv,2,D}^T\tilde{G}_{ijlv,2,D} \\
&\quad + 12\Theta_P\epsilon_2^2\tilde{G}_{ijlv,2,E}^T\tilde{G}_{ijlv,2,E} - S_1 \\
\tilde{\Lambda}_{ijlv,3} &= 16(\Theta_6\epsilon_5^2 + \Theta_8\epsilon_4^2)I + 12(\Theta_2\epsilon_5^2 + \Theta_4\epsilon_4^2)I) - S_2, \\
\tilde{\Lambda}_{ijlv,4} &= -\frac{\rho}{2}I, \quad \tilde{\Lambda}_{ijlv,5} = -\frac{\rho}{2}I, \\
\tilde{\Lambda}_{ijlv,6} &= -2P - 2O_{ijlv,1}, \quad \tilde{\Lambda}_{ijlv,7} = -2P - 2O_{ijlv,2}, \\
\tilde{\Lambda}_{ijlv,8} &= N_{ijlv,1}^T, \quad \tilde{\Lambda}_{ijlv,9} = N_{ijlv,2}^T, \\
\tilde{\Lambda}_{ijlv,10} &= O_{ijlv,1} + N_{ijlv,1}^T, \quad \tilde{\Lambda}_{ijlv,11} = O_{ijlv,2} + N_{ijlv,2}^T \\
\tilde{\Lambda}_{ijlv,12} &= -O_{ijlv,1}, \quad \tilde{\Lambda}_{ijlv,13} = -O_{ijlv,2}
\end{aligned}$$

Clearly, the inequalities (75) hold if the following matrix inequalities hold,

$$\tilde{\Xi}_{ijlv}^T\tilde{P}_1\tilde{\Xi}_{ijlv} + \tilde{\Pi}_{ijlv}^T\tilde{P}_2\tilde{\Pi}_{ijlv} + \tilde{\Lambda}_{ijlv} \le 0 \quad for\ i, j, l, v = 1, \cdots, M \tag{76}$$

Further, by the fact that P is positive definite matrix, the following inequality holds:

$$\begin{aligned} &(I - W)P(I - W) > 0 \\ &\Rightarrow -2I + W > -P \end{aligned} \tag{77}$$

where $W = P^{-1}$. On the other hand, to decouple the bilinear terms $\{\Theta_P\epsilon_2^2\tilde{G}_{ijlv,1,D}^T\tilde{G}_{ijlv,1,D}, \Theta_P\epsilon_2^2\tilde{G}_{ijlv,1,E}^T \times\tilde{G}_{ijlv,1,E}\}_{i,j,l,v=1}^M$ and $\{16\Theta_P\epsilon_2^2\tilde{G}_{ijlv,2,D}^T\tilde{G}_{ijlv,2,D}, 12\Theta_P\ \epsilon_2^2 \tilde{G}_{ijlv,2,E}^T\tilde{G}_{ijlv,2,E}\}_{i,j,l,v=1}^M$ in $\Lambda_{ijlv,1}$ and $\Lambda_{ijlv,2}$, respectively, and $2h\tilde{E}_{ijlv}^TP\tilde{E}_{ijlv}$ in $\Lambda_{ijlv,1}$, the following slack variables and constraints are introduced:

$$\begin{aligned}
&16\Theta_P\epsilon_2^2\tilde{G}_{ijlv,1,D}^T\tilde{G}_{ijlv,1,D} \\
&\quad + 12\Theta_P\epsilon_2^2\tilde{G}_{ijlv,1,E}^T\tilde{G}_{ijlv,1,E} < Y_{1,ijlv} \\
&16\Theta_P\epsilon_2^2\tilde{G}_{ijlv,2,D}^T\tilde{G}_{ijlv,2,D} \\
&\quad + 12\Theta_P\epsilon_2^2\tilde{G}_{ijlv,2,E}^T\tilde{G}_{ijlv,2,E} < Y_{2,ijlv} \\
&2h\tilde{E}_{ijlv}^TP\tilde{E}_{ijlv} < Y_{3,ijlv}
\end{aligned} \tag{78}$$

where $\{Y_{1,ijlv}, Y_{2,ijlv}, Y_{3,ijlv}\}_{i,j,l,v=1}^M$ are positive matrices to be designed.

By applying Schur complement to (76) with (77) and (78), the matrix inequalities in (76) are relaxed to following LMIs:

$$\begin{bmatrix} \tilde{\Lambda}_{ijlv}^* & \tilde{\Pi}_{ijlv}^T & \tilde{\Xi}_{ijlv}^T \\ * & -\tilde{W}_1 & 0 \\ * & * & -\tilde{W}_2 \end{bmatrix} \le 0 \quad for\ i, j, l, v = 1, \cdots, M \tag{79}$$

where $\tilde{W}_1 = diag\{W, W, W, W, W, W, W\}$, $\tilde{W}_2 = diag\{\frac{1}{2}W, W, W\}$ and

$$\tilde{\Lambda}_{ijlv}^* = \begin{bmatrix} \tilde{\Lambda}_{ijlv,I}^* & \tilde{\Lambda}_{ijlv,II} \\ * & \tilde{\Lambda}_{ijlv,III}^* \end{bmatrix}$$

$$\tilde{\Lambda}_{ijlv,I} = \begin{bmatrix} \tilde{\Lambda}_{ijlv,1}^* & \tilde{\Lambda}_{ijlv,8} & \tilde{\Lambda}_{ijlv,9} \\ * & \tilde{\Lambda}_{ijlv,2}^* & 0 \\ * & * & \tilde{\Lambda}_{ijlv,3} \end{bmatrix}$$

$$\begin{aligned}
\tilde{\Lambda}_{ijlv,III}^* &= diag\{\tilde{\Lambda}_{ijlv,4}, \tilde{\Lambda}_{ijlv,5}, \tilde{\Lambda}_{ijlv,6}^*, \tilde{\Lambda}_{ijlv,7}^*\} \\
\tilde{\Lambda}_{ijlv,1}^* &= 16(\Theta_P\epsilon_1^2 + \Theta_5\epsilon_5^2 + \Theta_P\epsilon_3^2 + \Theta_7\epsilon_4^2)I \\
&\quad + Y_{1,ijlv} + 12((\Theta_1\epsilon_5^2 + \Theta_3\epsilon_4^2)I) \\
&\quad + Y_{3,ijlv} + 2h\epsilon_6^2\Theta_PI + S_2 + S_1 - 2I \\
&\quad + W - 2N_{ijlv,1} - 2N_{ijlv,2} + \tilde{Q} \\
&\quad + (d_{1M} - d_{1m} + 1)S_3 + (d_{2M} - d_{2m} + 1)S_4 \\
\tilde{\Lambda}_{ijlv,2}^* &= Y_{2,ijlv} - S_1, \quad \tilde{\Lambda}_{ijlv,6}^* = 2(-2I + W) - 2O_{ijlv,1}, \\
\tilde{\Lambda}_{ijlv,7}^* &= 2(-2I + W) - 2O_{ijlv,2}
\end{aligned}$$

On the other hand, by taking expectation operator to (62), (62) can be represented as follows

$$\begin{aligned}
\tilde{D}_{x,ijlv,E}^TP\tilde{D}_{x,ijlv,E} &\le M_1, \\
(\tilde{D}_{ijlv,E}^{d_2})^TP\tilde{D}_{ijlv,E}^{d_2} &\le M_2 \\
\tilde{C}_{i,E}^TP\tilde{C}_{i,E}\} &\le M_3, \\
(\tilde{C}_{i,E}^d)^TP\tilde{C}_{i,E}^d &\le M_4 \\
P \le \Theta_PI, \quad M_1 &\le \Theta_1I, \ M_2 \le \Theta_2I, \\
M_3 \le \Theta_3I, \quad M_4 &\le \Theta_4I,
\end{aligned} \tag{80}$$

where $\tilde{D}_{x,ijlv,E} = diag\{0, \beta^*L_i, 0\}$, $\tilde{D}_{ijlv,E}^{d_2} = diag\{0, -\beta^*L_i, 0\}\tilde{C}_{i,E} = diag\{0, \beta^*L_i, 0\}\tilde{C}_{i,E}^d = diag\{0, -\beta^*L_i, 0\}$.

Then, by using Schur complement to (65), (78) and (80), these constraints can be transformed to the following LMIs

$$\begin{bmatrix} M_1 & \tilde{D}_{x,ijlv,E}^T \\ * & W \end{bmatrix} \ge 0,$$

$$\begin{bmatrix} M_2 & (\tilde{D}_{ijlv,E}^{d_2})^T \\ * & W \end{bmatrix} \ge 0$$

$$\begin{bmatrix} M_3 & \tilde{C}_{j,E}^T \\ * & W \end{bmatrix} \ge 0,$$

$$\begin{bmatrix} M_4 & (\tilde{C}_{j,E}^d)^T \\ * & W \end{bmatrix} \ge 0$$

$$\begin{bmatrix} \Theta_P I & I \\ * & W \end{bmatrix} \geq 0,$$
$$M_1 \leq \Theta_1 I, \quad M_2 \leq \Theta_2 I, \; M_3 \leq \Theta_3 I, \; M_4 \leq \Theta_4 I, \tag{81}$$

$$\begin{bmatrix} M_5 & \tilde{D}^T_{x,ijlv,D} \\ * & W \end{bmatrix} \geq 0,$$
$$\begin{bmatrix} M_6 & (\tilde{D}^{d_2}_{ijlv,D})^T \\ * & W \end{bmatrix} \geq 0,$$
$$\begin{bmatrix} M_7 & \tilde{C}^T_{j,D} \\ * & W \end{bmatrix} \geq 0,$$
$$\begin{bmatrix} M_8 & (\tilde{C}^d_{j,D})^T \\ * & W \end{bmatrix} \geq 0,$$
$$M_5 \leq \Theta_5 I, \quad M_6 \leq \Theta_6 I, \; M_7 \leq \Theta_7 I, \; M_8 \leq \Theta_8 I, \tag{82}$$

$$\begin{bmatrix} Y_{1,ijlv} & \tilde{G}^T_{ijlv,1,D} & \tilde{G}^T_{ijlv,1,E} \\ * & \frac{1}{16\Theta_P \epsilon_2^2} I & 0 \\ * & * & \frac{1}{12\Theta_P \epsilon_2^2} I \end{bmatrix} > 0$$
$$\begin{bmatrix} Y_{2,ijlv} & \tilde{G}^T_{ijlv,2,D} & \tilde{G}^T_{ijlv,2,E} \\ * & \frac{1}{16\Theta_P \epsilon_2^2} I & 0 \\ * & * & \frac{1}{12\Theta_P \epsilon_2^2} I \end{bmatrix} > 0$$
$$\begin{bmatrix} Y_{3,ijlv} & \tilde{E}^T_{ijlv} \\ * & \frac{1}{2h} W \end{bmatrix} > 0 \tag{83}$$

Based on above discussion, the H_∞ observer-based security reference tracking control design is transformed to LMIs in (79), (81), (82) and (83). The proof is done.

REFERENCES

[1] M. Zhu, X. Du, X. Zhang, H. Luo, and G. Wang, "Multi-UAV rapid-assessment task-assignment problem in a post-earthquake scenario," *IEEE Access*, vol. 7, pp. 74542–74557, 2019.

[2] N. Chen, N. Ni, P. Kapp, J. Chen, A. Xiao, and H. Li, "Structural analysis of the Hero Range in the Qaidam Basin, northwestern China, using integrated UAV, terrestrial LiDAR, Landsat 8, and 3-D seismic data," *IEEE J. Sel. Topics Appl. Earth Observ. Remote Sens.*, vol. 8, no. 9, pp. 4581–4591, Sep. 2015.

[3] D. Orfanus, E. P. de Freitas, and F. Eliassen, "Self-organization as a supporting paradigm for military UAV relay networks," *IEEE Commun. Lett.*, vol. 20, no. 4, pp. 804–807, Apr. 2016.

[4] M. E. Campbell and W. W. Whitacre, "Cooperative tracking using vision measurements on SeaScan UAVs," *IEEE Trans. Control Syst. Technol.*, vol. 15, no. 4, pp. 613–626, Jul. 2007.

[5] H. Zhou, H. Kong, L. Wei, D. Creighton, and S. Nahavandi, "Efficient road detection and tracking for unmanned aerial vehicle," *IEEE Trans. Intell. Transp. Syst.*, vol. 16, no. 1, pp. 297–309, Feb. 2015.

[6] F. Liao, R. Teo, J. L. Wang, X. Dong, F. Lin, and K. Peng, "Distributed formation and reconfiguration control of VTOL UAVs," *IEEE Trans. Control Syst. Technol.*, vol. 25, no. 1, pp. 270–277, Jan. 2017.

[7] Y. Zeng and R. Zhang, "Energy-efficient UAV communication with trajectory optimization," *IEEE Trans. Wireless Commun.*, vol. 16, no. 6, pp. 3747–3760, Jun. 2017.

[8] F. Wu, D. Yang, L. Xiao, and L. Cuthbert, "Energy consumption and completion time tradeoff in rotary-wing UAV enabled WPCN," *IEEE Access*, vol. 7, pp. 79617–79635, 2019.

[9] L. Chettri and R. Bera, "A comprehensive survey on Internet of Things (IoT) toward 5G wireless systems," *IEEE Internet Things J.*, vol. 7, no. 1, pp. 16–32, Jan. 2020.

[10] W. Cheng, X. Zhang, and H. Zhang, "Statistical-QoS driven energy-efficiency optimization over green 5G mobile wireless networks," *IEEE J. Sel. Areas Commun.*, vol. 34, no. 12, pp. 3092–3107, Dec. 2016.

[11] Y. Shi and B. Yu, "Output feedback stabilization of networked control systems with random delays modeled by Markov chains," *IEEE Trans. Autom. Control*, vol. 54, no. 7, pp. 1668–1674, Jul. 2009.

[12] G.-P. Liu, Y. Xia, D. Rees, and W. Hu, "Design and stability criteria of networked predictive control systems with random network delay in the feedback channel," *IEEE Trans. Syst., Man, Cybern. C, Appl. Rev.*, vol. 37, no. 2, pp. 173–184, Mar. 2007.

[13] H. Gao and T. Chen, "Network-based H_∞ output tracking control," *IEEE Trans. Autom. Control*, vol. 53, no. 3, pp. 655–667, Apr. 2008.

[14] B. Xue, S. Li, and Q. Zhu, "Moving horizon state estimation for networked control systems with multiple packet dropouts," *IEEE Trans. Autom. Control*, vol. 57, no. 9, pp. 2360–2366, Sep. 2012.

[15] R. Lu, Y. Xu, and R. Zhang, "A new design of model predictive tracking control for networked control system under random packet loss and uncertainties," *IEEE Trans. Ind. Electron.*, vol. 63, no. 11, pp. 6999–7007, Nov. 2016.

[16] B. Genge, P. Haller, and I. Kiss, "Cyber-security-aware network design of industrial control systems," *IEEE Syst. J.*, vol. 11, no. 3, pp. 1373–1384, Sep. 2017.

[17] H. Zhang, P. Cheng, L. Shi, and J. Chen, "Optimal DoS attack scheduling in wireless networked control system," *IEEE Trans. Control Syst. Technol.*, vol. 24, no. 3, pp. 843–852, May 2016.

[18] N. Liu, J. Zhang, H. Zhang, and W. Liu, "Security assessment for communication networks of power control systems using attack graph and MCDM," *IEEE Trans. Power Del.*, vol. 25, no. 3, pp. 1492–1500, Jul. 2010.

[19] H. Yang and S. Yin, "Descriptor observers design for Markov jump systems with simultaneous sensor and actuator faults," *IEEE Trans. Autom. Control*, vol. 64, no. 8, pp. 3370–3377, Aug. 2019.

[20] Q. Jia, W. Chen, Y. Zhang, and H. Li, "Fault reconstruction and fault-tolerant control via learning observers in Takagi–Sugeno fuzzy descriptor systems with time delays," *IEEE Trans. Ind. Electron.*, vol. 62, no. 6, pp. 3885–3895, Jun. 2015.

[21] Z. Gao and S. X. Ding, "Fault estimation and fault-tolerant control for descriptor systems via proportional, multiple-integral and derivative observer design," *IET Control Theory Appl.*, vol. 1, no. 5, pp. 1208–1218, Sep. 2007.

[22] H. Yang, S. Yin, and O. Kaynak, "Neural network-based adaptive fault-tolerant control for Markovian jump systems with nonlinearity and actuator faults," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 51, no. 6, pp. 3687–3698, Jun. 2021.

[23] H. Yang, S. Yin, H. Han, and H. Sun, "Sparse actuator and sensor attacks reconstruction for linear cyber-physical systems with sliding mode observer," *IEEE Trans. Ind. Informat.*, vol. 18, no. 6, pp. 3873–3884, Jun. 2022, doi: 10.1109/TII.2021.3111221.

[24] X. Lu, H. Wang, and M. Li, "Kalman fixed-interval and fixed-lag smoothing forwireless sensor systems with multiplicative noises," in *Proc. 24th Control Decis. Conf.*, Chinese, May 2012, pp. 3023–3026.

[25] A. S. Leong, S. Dey, and J. S. Evans, "On Kalman smoothing with random packet loss," *IEEE Trans. Signal Process.*, vol. 56, no. 7, pp. 3346–3351, Jul. 2008.

[26] G. C. Goodwin and K. S. Sin, *Adaptive Filtering Prediction and Control*. Englewood Cliffs, NJ, USA: Prentice-Hall, 1984.

[27] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Trans. Syst., Man, Cybern.*, vol. SMC-15, no. 1, pp. 116–132, Jan. 1985.

[28] X. Zhang, Y. Wang, G. Zhu, X. Chen, Z. Li, C. Wang, and C.-Y. Su, "Compound adaptive fuzzy quantized control for quadrotor and its experimental verification," *IEEE Trans. Cybern.*, vol. 51, no. 3, pp. 1121–1133, Mar. 2021, doi: 10.1109/TCYB.2020.2987811.

[29] B.-S. Chen and C.-F. Wu, "Robust scheduling filter design for a class of nonlinear stochastic Poisson signal systems," *IEEE Trans. Signal Process.*, vol. 63, no. 23, pp. 6245–6257, Dec. 2015.

[30] P. E. Kloeden and R. A. Pearson, "The numerical solution of stochastic differential equations," *ANZIAM J.*, vol. 20, no. 1, pp. 8–12, Jun. 1977.

[31] H. C. Sung, J. B. Park, and Y. H. Joo, "Robust control of observer-based sampled-data systems: Digital redesign approach," *IET Control Theory Appl.*, vol. 6, no. 12, pp. 1842–1850, Aug. 2012.

[32] W. P. M. H. Heemels, K. H. Johansson, and P. Tabuada, "An introduction to event-triggered and self-triggered control," in *Proc. IEEE 51st IEEE Conf. Decis. Control (CDC)*, Dec. 2012, pp. 3270–3285.

[33] X. Luan, P. Shi, and F. Liu, "Stabilization of networked control systems with random delays," *IEEE Trans. Ind. Electron.*, vol. 58, no. 9, pp. 4323–4330, Sep. 2011.

[34] B. S. Chen, M. Y. Lee, and X. H. Chen, "Security-enhanced filter design for stochastic systems under malicious attack via smoothed signal model and multiobjective H_2/H_∞ estimation method," *IEEE Trans. Signal Process.*, vol. 68, pp. 4971–4986, 2020.

[35] C.-S. Tseng, "Model reference output feedback fuzzy tracking control design for nonlinear discrete-time systems with time-delay," *IEEE Trans. Fuzzy Syst.*, vol. 14, no. 1, pp. 58–70, Feb. 2006.

[36] B.-S. Chen, C.-P. Wang, and M.-Y. Lee, "Stochastic robust team tracking control of multi-UAV networked system under Wiener and Poisson random fluctuations," *IEEE Trans. Cybern.*, vol. 51, no. 12, pp. 5786–5799, Dec. 2021, doi: 10.1109/TCYB.2019.2960104.

[37] W. Zhang, L. Xie, and B. S. Chen, *Stochastic H_2/H_∞ Control: A Nash Game Approach*. Boca Raton, FL, USA: CRC Press, 2017.

[38] C.-S. Tseng and B.-S. Chen, "Robust fuzzy observer-based fuzzy control design for nonlinear discrete-time systems with persistent bounded disturbances," *IEEE Trans. Fuzzy Syst.*, vol. 17, no. 3, pp. 711–723, Jun. 2009.

[39] K. Tanaka, T. Ikeda, and H. O. Wang, "Robust stabilization of a class of uncertain nonlinear systems via fuzzy control: Quadratic stabilizability, H_∞ control theory, and linear matrix inequalities," *IEEE Trans. Fuzzy Syst.*, vol. 4, no. 1, pp. 1–13, Feb. 1996.

[40] H. O. Wang, K. Tanaka, and M. F. Griffin, "An approach to fuzzy control of nonlinear systems: Stability and design issues," *IEEE Trans. Fuzzy Syst.*, vol. 4, no. 1, pp. 14–23, Feb. 1996.

[41] J. Liang, Z. Wang, Y. Liu, and X. Liu, "Robust synchronization of an array of coupled stochastic discrete-time delayed neural networks," *IEEE Trans. Neural Netw.*, vol. 19, no. 11, pp. 1910–1921, Nov. 2008.

[42] S. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*. Philadelphia, PA, USA: SIAM, 1994.

[43] C.-S. Tseng, "Model reference output feedback fuzzy tracking control design for nonlinear discrete-time systems with time-delay," *IEEE Trans. Fuzzy Syst.*, vol. 14, no. 1, pp. 58–70, Feb. 2006.

MIN-YEN LEE received the B.S. degree in bio-industrial mechatronics engineering from the National Chung Hsing University, Taichung, Taiwan, in 2014. He is currently pursuing the Ph.D. degree with the Department of Electrical Engineering, National Tsing Hua University, Hsinchu, Taiwan. His current research interests include robust control, fuzzy control, and non-linear stochastic systems.

HAN CHIU received the B.S. degree in interdisciplinary program of electrical engineering and computer science and the M.S. degree from the Department of Electrical Engineering, National Tsing Hua University, Hsinchu, Taiwan, in 2018 and 2020, respectively. He is currently working at Mediatek.

BOR-SEN CHEN (Life Fellow, IEEE) received the B.S. degree in electrical engineering from the Tatung Institute of Technology, Taipei, Taiwan, in 1970, the M.S. degree in geophysics from the National Central University, Taiwan, in 1973, and the Ph.D. degree in electrical engineering from the University of Southern California at Los Angeles, Los Angeles, CA, USA, in 1982. He has been a Lecturer, an Associate Professor, and a Professor at the Tatung Institute of Technology, from 1973 to 1987. He is currently the Tsing Hua Distinguished Chair Professor of electrical engineering and computer science at the National Tsing Hua University, Hsinchu, Taiwan. His current research interests include control engineering, signal processing, and systems biology. He has received the Distinguished Research Award from the National Science Council of Taiwan four times. He is the National Chair Professor of the Ministry of Education of Taiwan.